

Scalar or Dot Product Ex 24.1 Q1

(i)

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (\hat{i} - 2\hat{j} + \hat{k}) \cdot (4\hat{i} - 4\hat{j} + 7\hat{k}) \\ &= (1)(4) + (-2)(-4) + (1)(7) \\ &= 4 + 8 + 7 \\ &= 19\end{aligned}$$

$$\vec{a} \cdot \vec{b} = 19$$

(ii)

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (\hat{j} + 2\hat{k}) \cdot (2\hat{i} + \hat{k}) \\ &= (0 \times \hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} + 0 \times \hat{j} + \hat{k}) \\ &= (0)(2) + (1)(0) + (2)(1) \\ &= 0 + 0 + 2\end{aligned}$$

$$\vec{a} \cdot \vec{b} = 2$$

(iii)

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (\hat{j} - \hat{k}) \cdot (2\hat{i} + 3\hat{j} - 2\hat{k}) \\ &= (0 \times \hat{i} + \hat{j} - \hat{k}) \cdot (2\hat{i} + 3\hat{j} - 2\hat{k}) \\ &= (0)(2) + (1)(3) + (-1)(-2) \\ &= 0 + 3 + 2\end{aligned}$$

$$\vec{a} \cdot \vec{b} = 5$$

Scalar or Dot Product Ex 24.1 Q2

(i)

$\vec{a}$ and $\vec{b}$ are perpendicular

$$\begin{aligned}\Rightarrow \vec{a} \cdot \vec{b} &= 0 \\ \Rightarrow (\lambda\hat{i} + 2\hat{j} + \hat{k}) \cdot (4\hat{i} - 9\hat{j} + 2\hat{k}) &= 0 \\ \Rightarrow (\lambda)(4) + (2)(-9) + (1)(2) &= 0 \\ \Rightarrow 4\lambda - 18 + 2 &= 0 \\ \Rightarrow 4\lambda - 16 &= 0 \\ \Rightarrow 4\lambda &= 16 \\ \Rightarrow \lambda &= \frac{16}{4} \\ \Rightarrow \lambda &= 4\end{aligned}$$

(ii)

$\vec{a}$ and $\vec{b}$ are perpendicular

$$\begin{aligned}\Rightarrow \vec{a} \cdot \vec{b} &= 0 \\ \Rightarrow (\lambda\hat{i} + 2\hat{j} + \hat{k}) \cdot (5\hat{i} - 9\hat{j} + 2\hat{k}) &= 0 \\ \Rightarrow (\lambda)(5) + (2)(-9) + (1)(2) &= 0 \\ \Rightarrow 5\lambda - 18 + 2 &= 0 \\ \Rightarrow 5\lambda - 16 &= 0 \\ \Rightarrow 5\lambda &= 16 \\ \Rightarrow \lambda &= \frac{16}{5}\end{aligned}$$

(iii)

$\vec{a}$ and $\vec{b}$ are perpendicular

$$\begin{aligned}\Rightarrow \vec{a} \cdot \vec{b} &= 0 \\ \Rightarrow (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} + 2\hat{j} - \lambda\hat{k}) &= 0 \\ \Rightarrow (2)(3) + (3)(2) + (4)(-\lambda) &= 0 \\ \Rightarrow 6 + 6 - 4\lambda &= 0 \\ \Rightarrow 12 - 4\lambda &= 0 \\ \Rightarrow -4\lambda &= -12 \\ \Rightarrow \lambda &= \frac{-12}{-4} \\ \Rightarrow \lambda &= 3\end{aligned}$$

(iv)

$\vec{a}$ and $\vec{b}$ are perpendicular

$$\begin{aligned}\Rightarrow \vec{a} \cdot \vec{b} &= 0 \\ \Rightarrow (\lambda\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (\hat{i} - \hat{j} + 3\hat{k}) &= 0 \\ \Rightarrow (\lambda)(1) + (3)(-1) + (2)(3) &= 0 \\ \Rightarrow \lambda - 3 + 6 &= 0 \\ \Rightarrow \lambda + 3 &= 0 \\ \Rightarrow \lambda &= -3\end{aligned}$$

Scalar or Dot Product Ex 24.1 Q3

We know that, if θ is the angle between $\vec{a}$ and $\vec{b}$, then

$$\begin{aligned}\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{6}{4 \times 3} \\ &= \frac{6}{12} \\ \cos \theta &= \frac{1}{2} \\ \theta &= \cos^{-1} \left(\frac{1}{2} \right) \\ \theta &= \frac{\pi}{3}\end{aligned}$$

Angle between $\vec{a}$ and $\vec{b} = \frac{\pi}{3}$

Scalar or Dot Product Ex 24.1 Q4

$$\begin{aligned}(\vec{a} - 2\vec{b}) &= (\hat{i} - \hat{j}) - 2(-\hat{j} + 2\hat{k}) \\ &= (\hat{i} - \hat{j}) + 2\hat{j} - 4\hat{k} \\ &= (\hat{i} + \hat{j} - 4\hat{k})\end{aligned}$$

$$\begin{aligned}(\vec{a} + \vec{b}) &= (\hat{i} - \hat{j}) + (-\hat{j} + 2\hat{k}) \\ &= \hat{i} - \hat{j} - \hat{j} + 2\hat{k} \\ &= (\hat{i} - 2\hat{j} + 2\hat{k})\end{aligned}$$

Now,

$$\begin{aligned}(\vec{a} - 2\vec{b}) \cdot (\vec{a} + \vec{b}) &= (\hat{i} + \hat{j} - 4\hat{k}) \cdot (\hat{i} - 2\hat{j} + 2\hat{k}) \\ &= (1)(1) + (1)(-2) + (-4)(2) \\ &= 1 - 2 - 8 \\ &= -9\end{aligned}$$

$$(\vec{a} - 2\vec{b}) \cdot (\vec{a} + \vec{b}) = -9$$

Scalar or Dot Product Ex 24.1 Q5(i)

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad \dots (i)$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= (\hat{i} - \hat{j}) (\hat{j} + \hat{k}) \\ &= (\hat{i} - \hat{j} + 0 \times \hat{k}) (0 \times \hat{i} + \hat{j} + \hat{k}) \\ &= (1)(0) + (-1)(1) + (0)(1) \\ &= 0 - 1 + 0 \\ \vec{a} \cdot \vec{b} &= -1 \end{aligned}$$

$$\begin{aligned} |\vec{a}| &= |\hat{i} - \hat{j}| \\ &= |\hat{i} - \hat{j} + 0 \times \hat{k}| \\ &= \sqrt{(1)^2 + (-1)^2 + (0)^2} \\ &= \sqrt{1+1+0} \\ |\vec{a}| &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} |\vec{b}| &= |\hat{j} + \hat{k}| \\ &= |0 \times \hat{i} + \hat{j} + \hat{k}| \\ &= \sqrt{(0)^2 + (1)^2 + (1)^2} \\ &= \sqrt{0+1+1} \\ |\vec{b}| &= \sqrt{2} \end{aligned}$$

Put $\vec{a} \cdot \vec{b}$, $|\vec{a}|$ and $|\vec{b}|$ in equation (i)

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{-1}{\sqrt{2} \times \sqrt{2}} \end{aligned}$$

$$\cos \theta = \frac{-1}{2}$$

$$\theta = \cos^{-1} \left(-\frac{1}{2} \right)$$

$$\theta = \pi - \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3}$$

Angle between $\vec{a}$ and $\vec{b} = \frac{2\pi}{3}$

Scalar or Dot Product Ex 24.1 Q5(ii)

Let θ be the angle between two vector $\vec{a} = 3\hat{i} - 2\hat{j} - 6\hat{k}$ and $\vec{b} = 4\hat{i} - \hat{j} + 8\hat{k}$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \dots (1)$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= (3\hat{i} - 2\hat{j} - 6\hat{k}) (4\hat{i} - \hat{j} + 8\hat{k}) \\ &= 3 \times 4 + (-2)(-1) + (-6)8 \\ &= 12 + 2 - 48 \\ &= -34 \end{aligned}$$

$$\begin{aligned} |\vec{a}| &= \sqrt{3^2 + (-2)^2 + (-6)^2} \\ &= \sqrt{49} \\ &= 7 \end{aligned}$$

$$\begin{aligned} |\vec{b}| &= \sqrt{4^2 + (-1)^2 + 8^2} \\ &= \sqrt{81} \\ &= 9 \end{aligned}$$

Putting value of $|\vec{a}|$, $|\vec{b}|$ and $\vec{a} \cdot \vec{b}$ in equation (1)

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{-34}{7 \times 9} \\ &= \frac{-34}{63} \\ \theta &= \cos^{-1} \left(\frac{-34}{63} \right) \\ &= 122.66^\circ \end{aligned}$$

Scalar or Dot Product Ex 24.1 Q5(iii)

Let the angle between $\vec{a}$ and $\vec{b}$ be θ , then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad \dots (i)$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= (2\hat{i} - \hat{j} + 2\hat{k}) \cdot (4\hat{i} + 4\hat{j} - 2\hat{k}) \\ &= (2)(4) + (-1)(4) + (2)(-2) \\ &= 8 - 4 - 4 \\ \vec{a} \cdot \vec{b} &= 0 \end{aligned}$$

$$\begin{aligned} |\vec{a}| &= |2\hat{i} - \hat{j} + 2\hat{k}| \\ &= \sqrt{(2)^2 + (-1)^2 + (2)^2} \\ &= \sqrt{4 + 1 + 4} \\ &= \sqrt{9} \\ |\vec{a}| &= 3 \end{aligned}$$

$$\begin{aligned} |\vec{b}| &= |4\hat{i} + 4\hat{j} - 2\hat{k}| \\ &= \sqrt{(4)^2 + (4)^2 + (-2)^2} \\ &= \sqrt{16 + 16 + 4} \\ &= \sqrt{36} \\ |\vec{b}| &= 6 \end{aligned}$$

Put $\vec{a} \cdot \vec{b}$, $|\vec{a}|$ and $|\vec{b}|$ in equation (i)

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{0}{3 \times 6} \\ &= \frac{0}{18} \\ \cos \theta &= 0 \\ \theta &= \cos^{-1}(0) \end{aligned}$$

Angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$

Scalar or Dot Product Ex 24.1 Q5(iv)

Let θ be the angle between vector $\vec{a}$ and $\vec{b}$, then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad \dots (i)$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= (2\hat{i} - 3\hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} - 2\hat{k}) \\ &= (2)(1) + (-3)(1) + (1)(-2) \\ &= 2 - 3 - 2 \\ \vec{a} \cdot \vec{b} &= -3 \end{aligned}$$

$$\begin{aligned} |\vec{a}| &= |2\hat{i} - 3\hat{j} + \hat{k}| \\ &= \sqrt{(2)^2 + (-3)^2 + (1)^2} \\ &= \sqrt{4 + 9 + 1} \\ &= \sqrt{14} \end{aligned}$$

$$\begin{aligned} |\vec{b}| &= |\hat{i} + \hat{j} - 2\hat{k}| \\ &= \sqrt{(1)^2 + (1)^2 + (-2)^2} \\ &= \sqrt{1 + 1 + 4} \\ |\vec{b}| &= \sqrt{6} \end{aligned}$$

Put $\vec{a} \cdot \vec{b}$, $|\vec{a}|$ and $|\vec{b}|$ in equation (i),

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{-3}{\sqrt{14} \times \sqrt{6}} \\ \cos \theta &= \frac{-3}{\sqrt{84}} \\ \theta &= \cos^{-1}\left(\frac{-3}{\sqrt{84}}\right) \end{aligned}$$

Angle between vector $\vec{a}$ and $\vec{b}$ = $\cos^{-1}\left(\frac{-3}{\sqrt{84}}\right)$

Scalar or Dot Product Ex 24.1 Q5(v)

Let θ be the angle between vector $\vec{a}$ and $\vec{b}$, then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad \dots (i)$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (\hat{i} + 2\hat{j} - \hat{k}) (\hat{i} - \hat{j} + \hat{k}) \\ &= (1)(1) + (2)(-1) + (-1)(1) \\ &= 1 - 2 - 1 \\ \vec{a} \cdot \vec{b} &= -2\end{aligned}$$

$$\begin{aligned}|\vec{a}| &= |\hat{i} + 2\hat{j} - \hat{k}| \\ &= \sqrt{(1)^2 + (2)^2 + (-1)^2} \\ &= \sqrt{1 + 4 + 1} \\ &= \sqrt{6}\end{aligned}$$

$$\begin{aligned}|\vec{b}| &= |\hat{i} - \hat{j} + \hat{k}| \\ &= \sqrt{(1)^2 + (-1)^2 + (1)^2} \\ &= \sqrt{1 + 1 + 1} \\ |\vec{b}| &= \sqrt{3}\end{aligned}$$

Put $\vec{a} \cdot \vec{b}$, $|\vec{a}|$, $|\vec{b}|$ in equation (i),

$$\begin{aligned}\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{-2}{\sqrt{6}\sqrt{3}} \\ &= \frac{-2}{\sqrt{18}} \\ &= \frac{-2 \times \sqrt{2}}{3\sqrt{2} \times \sqrt{2}} \\ &= \frac{-2\sqrt{2}}{3 \times 2} \\ \cos \theta &= \frac{-\sqrt{2}}{3} \\ \theta &= \cos^{-1}\left(\frac{-\sqrt{2}}{3}\right)\end{aligned}$$

$$\text{Angle between vector } \vec{a} \text{ and } \vec{b} = \cos^{-1}\left(\frac{-\sqrt{2}}{3}\right)$$

Scalar or Dot Product

Remove Watermark



Wondershare
PDFelement

Component along x-, y- and z-axis are i, j and k respectively.

Let θ_1 be the angle between $\vec{a}$ and $\hat{i}$.

$$\begin{aligned}\cos \theta_1 &= \frac{\vec{a} \cdot \hat{i}}{|\vec{a}| |\hat{i}|} \\ &= \frac{(\hat{i} - \hat{j} + \sqrt{2}\hat{k}) (\hat{i} + 0\hat{j} + 0\hat{k})}{|\hat{i} - \hat{j} + \sqrt{2}\hat{k}| |\hat{i} + 0\hat{j} + 0\hat{k}|} \\ &= \frac{(1)(1) + (-1)(0) + (\sqrt{2})(0)}{\sqrt{(1)^2 + (-1)^2 + (\sqrt{2})^2} \cdot \sqrt{(1)^2 + (0)^2 + (0)^2}} \\ &= \frac{1 + 0 + 0}{\sqrt{4}\sqrt{1}} \\ \cos \theta_1 &= \frac{1}{2}\end{aligned}$$

$$\theta_1 = \frac{\pi}{3}$$

Let θ_2 be the angle between $\vec{a}$ and $\hat{j}$.

$$\begin{aligned}\cos \theta_2 &= \frac{\vec{a} \cdot \hat{j}}{|\vec{a}| |\hat{j}|} \\ &= \frac{(\hat{i} - \hat{j} + \sqrt{2}\hat{k}) (0\hat{i} + \hat{j} + 0\hat{k})}{\sqrt{(1)^2 + (-1)^2 + (\sqrt{2})^2} \cdot \sqrt{(0)^2 + (1)^2 + (0)^2}} \\ &= \frac{(1)(0) + (-1)(1) + (\sqrt{2})(0)}{\sqrt{1+1+2} \sqrt{1}} \\ &= \frac{-1}{\sqrt{4}\sqrt{1}} \\ &= \frac{-1}{2} \\ \cos \theta_2 &= -\frac{1}{2}\end{aligned}$$

$$\theta_2 = \pi - \frac{\pi}{3}$$

$$\theta_2 = \frac{2\pi}{3}$$

Let θ_3 be the angle between $\vec{a}$ and $\hat{k}$, then

$$\begin{aligned}\cos \theta_3 &= \frac{\vec{a} \cdot \hat{k}}{|\vec{a}| |\hat{k}|} \\ &= \frac{(\hat{i} - \hat{j} + \sqrt{2}\hat{k}) (0\hat{i} + 0\hat{j} + \hat{k})}{\sqrt{(1)^2 + (-1)^2 + (\sqrt{2})^2} \cdot \sqrt{(0)^2 + (0)^2 + (1)^2}} \\ &= \frac{(1)(0) + (-1)(0) + (\sqrt{2})(1)}{\sqrt{1+1+2} \sqrt{1}} \\ &= \frac{\sqrt{2}}{\sqrt{4}\sqrt{1}} \\ \cos \theta_3 &= \frac{1}{\sqrt{2}} \\ \theta_3 &= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) \\ \theta_3 &= \frac{\pi}{4}\end{aligned}$$

So, the angle between vector $\vec{a}$ and x-axis is $\frac{\pi}{3}$, vector $\vec{a}$ and y-axis is $\frac{2\pi}{3}$,
vector $\vec{a}$ and z-axis is $\frac{\pi}{4}$.



MillionStars edu
Think, Learn & Practice

Scalar or Dot Product Ex 24.1 Q7(i)

Let the required vector be $x\hat{i} + y\hat{j} + z\hat{k}$

According to question,

$$\begin{aligned}(x\hat{i} + y\hat{j} + z\hat{k})(\hat{i} + \hat{j} - 3\hat{k}) &= 0 \\ (x)(1) + (y)(1) + (z)(-3) &= 0 \\ x + y - 3z &= 0 \quad \text{--- (i)}\end{aligned}$$

And,

$$\begin{aligned}(x\hat{i} + y\hat{j} + z\hat{k})(\hat{i} + 3\hat{j} - 2\hat{k}) &= 5 \\ (x)(1) + (y)(3) + (z)(-2) &= 5 \\ x + 3y - 2z &= 5 \quad \text{--- (ii)}\end{aligned}$$

And,

$$\begin{aligned}(x\hat{i} + y\hat{j} + z\hat{k})(2\hat{i} + \hat{j} + 4\hat{k}) &= 8 \\ (x)(2) + (y)(1) + (z)(4) &= 8 \\ 2x + y + 4z &= 8 \quad \text{--- (iii)}\end{aligned}$$

Subtracting (i) from (ii),

$$\begin{array}{r} x + 3y - 2z = 5 \\ x + y - 3z = 0 \\ (-)(-) \quad (+) \quad (-) \\ \hline 2y + z = 5 \quad \text{--- (iv)}\end{array}$$

Subtracting $2 \times$ (ii) from (iii),

$$\begin{array}{r} 2x + y + 4z = 8 \\ 2x + 6y - 4z = 10 \\ (-) \quad (-) \quad (+) \quad (-) \\ \hline -5y + 8z = -2 \quad \text{--- (v)}\end{array}$$

Subtracting $8 \times$ (iv) from (v),

$$\begin{array}{r} -5y + 8z = -2 \\ 18y + 8z = 40 \\ (-) \quad (-) \quad (-) \\ \hline -23y = -42 \quad \text{--- (vi)} \\ y = \frac{-42}{-23} \\ y = 2\end{array}$$

Put $y = 2$ in equation (iv),

$$\begin{aligned}2y + z &= 5 \\ 2(2) + z &= 5 \\ 4 + z &= 5 \\ z &= 5 - 4 \\ z &= 1\end{aligned}$$

Put $y = 2$ and $z = 1$ in equation (i),

$$\begin{aligned}x + y - 3z &= 0 \\ x + (2) - 3(1) &= 0 \\ x + 2 - 3 &= 0 \\ x - 1 &= 0 \\ x &= 1\end{aligned}$$

The required vector = $x\hat{i} + y\hat{j} + z\hat{k}$

The required vector = $\hat{i} + 2\hat{j} + \hat{k}$

Scalar or Dot Product Ex 24.1 Q8(i)

Here, $\hat{a}$ and $\hat{b}$ are unit vectors, then

$$|\hat{a}| = |\hat{b}| = 1$$

$$\begin{aligned} |\hat{a} + \hat{b}|^2 &= (\hat{a} + \hat{b})^2 \\ &= (\hat{a})^2 + (\hat{b})^2 + 2\hat{a} \cdot \hat{b} \\ &= |\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a} \cdot \hat{b} \\ &= (1)^2 + (1)^2 + 2\hat{a} \cdot \hat{b} \\ |\hat{a} + \hat{b}|^2 &= 2 + 2\hat{a} \cdot \hat{b} \end{aligned}$$

$$|\hat{a} + \hat{b}|^2 = 2 + 2 \times |\hat{a}| |\hat{b}| \cos \theta \quad \left[\text{Since } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \right]$$

$$\begin{aligned} |\hat{a} + \hat{b}|^2 &= 2 + 2 \times 1 \times 1 \times \cos \theta \\ &= 2 + 2 \cos \theta \end{aligned}$$

$$|\hat{a} + \hat{b}|^2 = 2(1 + \cos \theta)$$

$$|\hat{a} + \hat{b}|^2 = 2 \left(2 \cos^2 \frac{\theta}{2} \right) \quad \left[\text{Since } 1 + \cos \theta = 2 \cos^2 \frac{\theta}{2} \right]$$

$$|\hat{a} + \hat{b}|^2 = 4 \cos^2 \frac{\theta}{2}$$

$$|\hat{a} + \hat{b}| = \sqrt{4 \cos^2 \frac{\theta}{2}}$$

$$|\hat{a} + \hat{b}| = 2 \cos \frac{\theta}{2}$$

$$\cos \frac{\theta}{2} = \frac{1}{2} |\hat{a} + \hat{b}|$$

Scalar or Dot Product Ex 24.1 Q8(ii)

Here, $\hat{a}$ and $\hat{b}$ are unit vectors

$$|\hat{a}| = |\hat{b}| = 1$$

$$\begin{aligned} \frac{|\hat{a} - \hat{b}|^2}{|\hat{a} + \hat{b}|^2} &= \frac{(\hat{a} - \hat{b})^2}{(\hat{a} + \hat{b})^2} \\ &= \frac{(\hat{a})^2 + (\hat{b})^2 - 2\hat{a} \cdot \hat{b}}{(\hat{a})^2 + (\hat{b})^2 + 2\hat{a} \cdot \hat{b}} \\ &= \frac{|\hat{a}|^2 + |\hat{b}|^2 - 2\hat{a} \cdot \hat{b}}{|\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a} \cdot \hat{b}} \end{aligned}$$

$$\frac{|\hat{a} - \hat{b}|^2}{|\hat{a} + \hat{b}|^2} = \frac{(1)^2 + (1)^2 - 2|\hat{a}| |\hat{b}| \cos \theta}{(1)^2 + (1)^2 + 2|\hat{a}| |\hat{b}| \cos \theta} \quad \left[\text{Since } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \right]$$

$$\frac{|\hat{a} - \hat{b}|^2}{|\hat{a} + \hat{b}|^2} = \frac{1 + 1 - 2(1)(1) \cos \theta}{1 + 1 + 2(1)(1) \cos \theta}$$

$$\begin{aligned} &= \frac{2 - 2 \cos \theta}{2 + 2 \cos \theta} \\ &= \frac{2(1 - \cos \theta)}{2(1 + \cos \theta)} \end{aligned}$$

$$= \frac{2 \times \sin^2 \frac{\theta}{2}}{2 \times \cos^2 \frac{\theta}{2}} \quad \left[\text{Since } 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2}, 1 + \cos \theta = 2 \cos^2 \frac{\theta}{2} \right]$$

$$\frac{|\hat{a} - \hat{b}|^2}{|\hat{a} + \hat{b}|^2} = \tan^2 \frac{\theta}{2}$$

$$\tan \frac{\theta}{2} = \frac{|\hat{a} - \hat{b}|}{|\hat{a} + \hat{b}|}$$

Scalar or Dot Product Ex 24.1 Q9

Let $\hat{a}$ and $\hat{b}$ are two unit vectors

$$\text{Then, } |\hat{a}| = |\hat{b}| = 1$$

And sum of $\hat{a}$ and $\hat{b}$ is a unit vector, then

$$|\hat{a} + \hat{b}| = 1$$

Taking square of both the sides,

$$|\hat{a} + \hat{b}|^2 = (1)^2$$

$$(\hat{a} + \hat{b})^2 = 1$$

$$(\hat{a})^2 + (\hat{b})^2 + 2\hat{a} \cdot \hat{b} = 1$$

$$|\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a} \cdot \hat{b} = 1$$

$$(1)^2 + (1)^2 + 2\hat{a} \cdot \hat{b} = 1$$

$$2 + 2\hat{a} \cdot \hat{b} = 1$$

$$2\hat{a} \cdot \hat{b} = 1 - 2$$

$$2\hat{a} \cdot \hat{b} = -1$$

$$\hat{a} \cdot \hat{b} = \frac{-1}{2} \quad \dots (i)$$

$$|\hat{a} - \hat{b}|^2 = (\hat{a} - \hat{b})^2$$

$$= (\hat{a})^2 + (\hat{b})^2 - 2\hat{a} \cdot \hat{b}$$

$$= |\hat{a}|^2 + |\hat{b}|^2 - 2 \times \hat{a} \cdot \hat{b}$$

$$= (1)^2 + (1)^2 - 2 \times \left(\frac{-1}{2}\right)$$

Using equation (i)

$$= 1 + 1 + \frac{2}{2}$$

$$= 1 + 1 + 1$$

$$|\hat{a} - \hat{b}|^2 = 3$$

$$|\hat{a} - \hat{b}| = \sqrt{3}$$

Scalar or Dot Product Ex 24.1 Q10

Given that $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular, so,

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

and $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, so

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

Now,

$$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c})^2$$

$$= (\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + 2\vec{a}\vec{b} + 2\vec{b}\vec{c} + 2\vec{c}\vec{a}$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(0) + 2(0) + 2(0)$$

$$= (1)^2 + (1)^2 + (1)^2 + 0$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 1 + 1 + 1$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 3$$

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$$



MillionStars edu
Think, Learn & Practice



Scalar or Dot Product Ex 24.1 Q11

Here, $|\vec{a} + \vec{b}| = 60$

Squaring both the sides,

$$|\vec{a} + \vec{b}|^2 = (60)^2$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (60)^2$$

$$(\vec{a})^2 + (\vec{b})^2 + 2\vec{a} \cdot \vec{b} = 3600$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 3600 \quad \text{--- (i)}$$

Now, $|\vec{a} - \vec{b}| = 40$

Squaring both the sides,

$$|\vec{a} - \vec{b}|^2 = (40)^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 1600 \quad \text{--- (ii)}$$

Adding (i) and (ii),

$$2|\vec{a}|^2 + 2|\vec{b}|^2 + 2\vec{a} \cdot \vec{b} - 2\vec{a} \cdot \vec{b} = 3600 - 1600$$

$$2|\vec{a}|^2 + 2(46)^2 = 5200$$

$$2|\vec{a}|^2 = 5200 - 4232$$

$$2|\vec{a}|^2 = 968$$

$$|\vec{a}|^2 = \frac{968}{2}$$

$$|\vec{a}|^2 = 484$$

$$|\vec{a}| = \sqrt{484}$$

$$|\vec{a}| = 22$$

Scalar or Dot Product Ex 24.1 Q12

Let θ be the angle between $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i}$

Then,

$$\begin{aligned} \cos \theta &= \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i})}{|\hat{i} + \hat{j} + \hat{k}| |\hat{i}|} \\ &= \frac{1}{\sqrt{3}} \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

Similarly, if α and γ are angles that $\hat{i} + \hat{j} + \hat{k}$ make with $\hat{j}$ and $\hat{k}$

Then,

$$\cos \alpha = \frac{1}{\sqrt{3}}$$

$$\text{and } \cos \gamma = \frac{1}{\sqrt{3}}$$

Therefore, $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the three axes.

Scalar or Dot Product Ex 24.1 Q13

We have,

$$\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$$

$$\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$$

Then,

$$\begin{aligned} \vec{a} \cdot \vec{b} &= \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}) \\ &= \frac{1}{49}(6 - 18 + 12) = 0 \end{aligned}$$

Similarly,

$$\vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{c} = 0$$

$\therefore \vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular

Scalar or Dot Product Ex 24.1 Q14

$$\text{Let } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}| = |\vec{b}|$$

$$\text{Let } |\vec{a}| = |\vec{b}|$$

Squaring both the sides.

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$(\vec{a})^2 - (\vec{b})^2 = 0$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

Thus,

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0 \Leftrightarrow |\vec{a}| = |\vec{b}|$$

Scalar or Dot Product Ex 24.1 Q15

If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$,
find λ

Given that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$

$$\therefore \vec{a} \cdot (\lambda\vec{b} + \vec{c}) = 0$$

$$\lambda\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$$

$$\lambda(2\hat{i} - \hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} - 2\hat{k}) + (2\hat{i} - \hat{j} + \hat{k}) \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 0$$

$$\lambda(2 - 1 - 2) + (2 - 3 - 1) = 0$$

$$-\lambda - 2 = 0$$

$$\lambda = -2$$

Scalar or Dot Product Ex 24.1 Q16

$$\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k} \text{ and } \vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{p} + \vec{q}$$

$$= 5\hat{i} + \lambda\hat{j} - 3\hat{k} + \hat{i} + 3\hat{j} - 5\hat{k}$$

$$= 6\hat{i} + (\lambda + 3)\hat{j} - 8\hat{k}$$

$$\vec{p} - \vec{q}$$

$$= 5\hat{i} + \lambda\hat{j} - 3\hat{k} - \hat{i} - 3\hat{j} + 5\hat{k}$$

$$= 4\hat{i} + (\lambda - 3)\hat{j} + 2\hat{k}$$

$$(\vec{p} + \vec{q}) \cdot (\vec{p} - \vec{q}) = 0$$

$$\Rightarrow [6\hat{i} + (\lambda + 3)\hat{j} - 8\hat{k}] \cdot [4\hat{i} + (\lambda - 3)\hat{j} + 2\hat{k}] = 0$$

$$\Rightarrow 24 + (\lambda^2 - 9) - 16 = 0$$

$$\Rightarrow \lambda^2 - 9 + 8 = 0$$

$$\Rightarrow \lambda^2 - 1 = 0$$

$$\therefore \lambda = \pm 1$$

Scalar or Dot Product Ex 24.1 Q17

According to question $\vec{\beta}_1$ is parallel to $\vec{\alpha}$. So

$$\begin{aligned}\vec{\beta}_1 &= \gamma \vec{\alpha} \\ &= \gamma (3\hat{i} + 4\hat{j} + 5\hat{k})\end{aligned}$$

$$\begin{aligned}\vec{\beta} &= \vec{\beta}_1 + \vec{\beta}_2 \\ 2\hat{i} + \hat{j} - 4\hat{k} &= \gamma (3\hat{i} + 4\hat{j} + 5\hat{k}) + \vec{\beta}_2 \quad (\text{putting } \vec{\beta} \text{ and } \vec{\beta}_1) \\ \vec{\beta}_2 &= (2 - 3\gamma)\hat{i} + (1 - 4\gamma)\hat{j} - (4 + 5\gamma)\hat{k}\end{aligned}$$

Again $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. So

$$\begin{aligned}\vec{\beta}_2 \cdot \vec{\alpha} &= 0 \\ [(2 - 3\gamma)\hat{i} + (1 - 4\gamma)\hat{j} - (4 + 5\gamma)\hat{k}] \cdot (3\hat{i} + 4\hat{j} + 5\hat{k}) &= 0 \\ 6 - 9\gamma + 4 - 16\gamma - 20 - 25\gamma &= 0 \\ -50\gamma &= 10 \\ \gamma &= -\frac{1}{5}\end{aligned}$$

$$\begin{aligned}\vec{\beta}_1 &= -\frac{1}{5}(3\hat{i} + 4\hat{j} + 5\hat{k}) \\ \vec{\beta} &= \vec{\beta}_1 + \vec{\beta}_2 \\ 2\hat{i} + \hat{j} - 4\hat{k} &= -\frac{1}{5}(3\hat{i} + 4\hat{j} + 5\hat{k}) + \vec{\beta}_2 \quad (\text{putting } \vec{\beta} \text{ and } \vec{\beta}_1) \\ \vec{\beta}_2 &= \frac{1}{5}(13\hat{i} + 9\hat{j} - 15\hat{k}) \\ \vec{\beta} &= -\frac{1}{5}(3\hat{i} + 4\hat{j} + 5\hat{k}) + \frac{1}{5}(13\hat{i} + 9\hat{j} - 15\hat{k})\end{aligned}$$

Scalar or Dot Product Ex 24.1 Q18

Consider $\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$.

Then,

$$\vec{a} \cdot \vec{b} = 2 \cdot 3 + 4 \cdot 3 + 3 \cdot (-6) = 6 + 12 - 18 = 0$$

We now observe that:

$$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$$

$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Scalar or Dot Product Ex 24.1 Q19

Here,

$$\begin{aligned}\vec{b} + \vec{c} &= (\hat{i} - 3\hat{j} + 5\hat{k}) + (2\hat{i} + \hat{j} - 4\hat{k}) \\ &= 3\hat{i} + 2\hat{j} + \hat{k} \\ \vec{b} + \vec{c} &= \vec{a}\end{aligned}$$

$\therefore \vec{a}, \vec{b}, \vec{c}$ are represents the sides of a triangle.

$$\begin{aligned}|\vec{a}| &= \sqrt{(3)^2 + (-2)^2 + (1)^2} \\ &= \sqrt{9 + 4 + 1} \\ &= \sqrt{14}\end{aligned}$$

$$\begin{aligned}|\vec{b}| &= \sqrt{(1)^2 + (-3)^2 + (5)^2} \\ &= \sqrt{1 + 9 + 25} \\ |\vec{b}| &= \sqrt{35}\end{aligned}$$

$$\begin{aligned}|\vec{c}| &= \sqrt{(2)^2 + (1)^2 + (-4)^2} \\ &= \sqrt{4 + 1 + 16} \\ &= \sqrt{21}\end{aligned}$$

$$\begin{aligned}(\sqrt{21})^2 + (\sqrt{14})^2 &= (\sqrt{35})^2 \\ 21 + 14 &= 35 \\ 35 &= 35\end{aligned}$$

$$|\vec{c}|^2 + |\vec{a}|^2 = |\vec{b}|^2$$

$\therefore$ By the pythagorou theorem,

Triangle formed by $\vec{a}, \vec{b}, \vec{c}$ is a right angled triangled.

Scalar or Dot Product Ex 24.1 Q20

The given vectors are $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 3\hat{i} + \hat{j}$.

Now,

$$\vec{a} + \lambda\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k}) = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$$

If $(\vec{a} + \lambda\vec{b})$ is perpendicular to $\vec{c}$, then

$$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0.$$

$$\Rightarrow [(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}] \cdot (3\hat{i} + \hat{j}) = 0$$

$$\Rightarrow (2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$$

$$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow -\lambda + 8 = 0$$

$$\Rightarrow \lambda = 8$$

Hence, the required value of λ is 8.

Scalar or Dot Product Ex 24.1 Q21

$$\vec{A} = 0\hat{i} - \hat{j} - 2\hat{k}$$

$$\vec{B} = 3\hat{i} + \hat{j} + 4\hat{k}$$

$$\vec{C} = 5\hat{i} + 7\hat{j} + \hat{k}$$

$$\vec{AB} = \vec{B} - \vec{A}$$

$$= (3\hat{i} + \hat{j} + 4\hat{k}) - (0\hat{i} - \hat{j} - 2\hat{k})$$

$$= 3\hat{i} + \hat{j} + 4\hat{k} - 0\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{AB} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{BC} = \vec{C} - \vec{B}$$

$$= (5\hat{i} + 7\hat{j} + \hat{k}) - (3\hat{i} + \hat{j} + 4\hat{k})$$

$$= 5\hat{i} + 7\hat{j} + \hat{k} - 3\hat{i} - \hat{j} - 4\hat{k}$$

$$\vec{BC} = 2\hat{i} + 6\hat{j} - 3\hat{k}$$

$$\vec{AC} = \vec{C} - \vec{A}$$

$$= (5\hat{i} + 7\hat{j} + \hat{k}) - (-\hat{j} - 2\hat{k})$$

$$= 5\hat{i} + 7\hat{j} + \hat{k} + \hat{j} + 2\hat{k}$$

$$\vec{AC} = 5\hat{i} + 8\hat{j} + 3\hat{k}$$

Angle between $\vec{AB}$ and $\vec{AC}$,

$$\cos A = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|}$$

$$= \frac{(3\hat{i} + 2\hat{j} + 6\hat{k}) \cdot (5\hat{i} + 8\hat{j} + 3\hat{k})}{\sqrt{(3)^2 + (2)^2 + (6)^2} \sqrt{(5)^2 + (8)^2 + (3)^2}}$$

$$= \frac{(3)(5) + (2)(8) + (6)(3)}{\sqrt{9 + 4 + 36} \sqrt{25 + 64 + 9}}$$

$$= \frac{15 + 16 + 18}{\sqrt{49} \sqrt{98}}$$

$$= \frac{49}{\sqrt{49} \sqrt{49 \times 2}}$$

$$\cos A = \frac{49}{49\sqrt{2}}$$

$$\cos A = \frac{1}{\sqrt{2}}$$

$$A = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\pi}{4}$$

$$\angle A = \frac{\pi}{4}$$

Angle between $\vec{BC}$ and $\vec{BA}$

$$\cos B = \frac{\vec{BC} \cdot \vec{BA}}{|\vec{BC}| |\vec{BA}|}$$

$$= \frac{(2\hat{i} + 6\hat{j} - 3\hat{k}) \cdot (-3\hat{i} - 2\hat{j} - 6\hat{k})}{\sqrt{(2)^2 + (6)^2 + (-3)^2} \sqrt{(-3)^2 + (-2)^2 + (-6)^2}}$$

$$= \frac{(2)(-3) + (6)(-2) + (-3)(-6)}{\sqrt{4 + 36 + 9} \sqrt{9 + 4 + 36}}$$

$$= \frac{-6 - 12 + 18}{\sqrt{49} \sqrt{98}}$$

$$\cos B = \frac{-18 + 18}{49}$$

$$= \frac{0}{49}$$

$$\cos B = 0$$

$$B = \cos^{-1}(0)$$

$$\angle B = \frac{\pi}{2}$$

We know that,

$$\angle A + \angle B + \angle C = \pi$$

$$\frac{\pi}{4} + \frac{\pi}{2} + \angle C = \pi$$

$$\frac{3\pi}{4} + \angle C = \pi$$

$$\angle C = \frac{\pi}{4} - \frac{3\pi}{4}$$

$$\angle C = \frac{4\pi - 3\pi}{4}$$

$$\angle C = \frac{\pi}{4}$$

$$\angle A = \frac{\pi}{4}$$

$$\angle B = \frac{\pi}{2}$$

Scalar or Dot Product Ex 24.1 Q22

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

It is given that $|\vec{a}| = |\vec{b}|$, $\vec{a} \cdot \vec{b} = \frac{1}{2}$, and $\theta = 60^\circ$ (1)

We know that $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$.

$$\therefore \frac{1}{2} = |\vec{a}| |\vec{a}| \cos 60^\circ \quad [\text{Using (1)}]$$

$$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$$

$$\Rightarrow |\vec{a}|^2 = 1$$

$$\Rightarrow |\vec{a}| = |\vec{b}| = 1$$

Scalar or Dot Product Ex 24.1 Q23

Given

$$\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\vec{c} = \hat{i} - \hat{j}$$

$\vec{AB}$ = Position vector of B - Position vector of A

$$= (2\hat{i} - 4\hat{j} + 5\hat{k}) - (4\hat{i} - 3\hat{j} + \hat{k})$$

$$= 2\hat{i} - 4\hat{j} + 5\hat{k} - 4\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{AB} = -2\hat{i} - \hat{j} + 4\hat{k}$$

$\vec{BC}$ = Position vector of C - Position vector of B

$$= (\hat{i} - \hat{j}) - (2\hat{i} - 4\hat{j} + 5\hat{k})$$

$$= \hat{i} - \hat{j} - 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$= -\hat{i} + 3\hat{j} - 5\hat{k}$$

$\vec{CA}$ = Position vector of A - Position vector of C

$$= (4\hat{i} - 3\hat{j} + \hat{k}) - (\hat{i} - \hat{j})$$

$$= 4\hat{i} - 3\hat{j} + \hat{k} - \hat{i} + \hat{j}$$

$$= 3\hat{i} - 2\hat{j} + \hat{k}$$

Now, $\vec{AB} \cdot \vec{CA}$

$$= (-2\hat{i} - \hat{j} + 4\hat{k}) \cdot (3\hat{i} - 2\hat{j} + \hat{k})$$

$$= (-2)(3) + (-1)(-2) + (4)(1)$$

$$= -6 + 2 + 4$$

$$= -6 + 6$$

$$= 0$$

So, $\vec{AB}$ is perpendicular to $\vec{CA}$
 $\angle A$ is right angle.

Hence, ABC is a right triangle

$$A = (1, 2, 3)$$

$$B = (-1, 0, 0)$$

$$C = (0, 1, 2)$$

$$\text{Position vector of } A = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\text{Position vector of } B = -\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\text{Position vector of } C = 0\hat{i} + \hat{j} + 2\hat{k}$$

$$\overrightarrow{AB} = \text{Position vector of } B - \text{Position vector of } A$$

$$= (-\hat{i} + 0\hat{j} + 0\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= -2\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\overrightarrow{BC} = \text{Position vector of } C - \text{Position vector of } B$$

$$= (0\hat{i} + \hat{j} + 2\hat{k}) - (-\hat{i} + 0\hat{j} + 0\hat{k})$$

$$= \hat{i} + \hat{j} + 2\hat{k}$$

$$\overrightarrow{AC} = \text{Position vector of } C - \text{Position vector of } A$$

$$= (0\hat{i} + \hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= -\hat{i} - \hat{j} - \hat{k}$$

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = (-2\hat{i} - 2\hat{j} - 3\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k})$$

$$= -2 - 2 - 6$$

$$= -10$$

$$\angle ABC = \frac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{|\overrightarrow{AB}| |\overrightarrow{BC}|}$$

$$= \frac{-10}{\sqrt{(-2)^2 + (-2)^2 + (-3)^2} \sqrt{1^2 + 1^2 + 2^2}}$$

$$= \frac{-10}{\sqrt{17} \sqrt{6}}$$

$$= \frac{-10}{\sqrt{102}}$$

$$\angle ABC = \cos^{-1} \left(\frac{-10}{\sqrt{102}} \right)$$

Scalar or Dot Product Ex 24.1 Q25

Given

$$A = (0, 1, 1)$$

$$B = (3, 1, 5)$$

$$C = (0, 3, 3)$$

$$\text{Position vector of } A = 0\hat{i} + \hat{j} + \hat{k}$$

$$\text{Position vector of } B = 3\hat{i} + \hat{j} + 5\hat{k}$$

$$\text{Position vector of } C = 0\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\overrightarrow{AB} = \text{Position vector of } B - \text{Position vector of } A$$

$$= (3\hat{i} + \hat{j} + 5\hat{k}) - (0\hat{i} + \hat{j} + \hat{k})$$

$$= 3\hat{i} + \hat{j} + 5\hat{k} - \hat{j} - \hat{k}$$

$$\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$$

$$\overrightarrow{BC} = \text{Position vector of } C - \text{Position vector of } B$$

$$= (0\hat{i} + 3\hat{j} + 3\hat{k}) - (3\hat{i} + \hat{j} + 5\hat{k})$$

$$\overrightarrow{BC} = 3\hat{j} + 3\hat{k} - 3\hat{i} - \hat{j} - 5\hat{k}$$

$$= -3\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\overrightarrow{AC} = \text{Position vector of } C - \text{Position vector of } A$$

$$= (0\hat{i} + 3\hat{j} + 3\hat{k}) - (0\hat{i} + \hat{j} + \hat{k})$$

$$= 3\hat{j} + 3\hat{k} - \hat{j} - \hat{k}$$

$$= 2\hat{j} + 2\hat{k}$$

Wondershare
PDFelement

MillionStars edu
Think, Learn & Practice

$$\begin{aligned}\overrightarrow{BC} \cdot \overrightarrow{AC} &= (-3\hat{i} + 2\hat{j} - 2\hat{k}) \cdot (2\hat{j} + 2\hat{k}) \\ &= (-3)(0) + (2)(2) + (-2)(2) \\ &= 0 + 4 - 4 \\ &= 0\end{aligned}$$

So, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ is perpendicular

$\Rightarrow \angle C$ is right angle.

Scalar or Dot Product Ex 24.1 Q26

Projection of $(\vec{b} + \vec{c})$ on $\vec{a}$

$$\begin{aligned}&= \frac{(\vec{b} + \vec{c}) \cdot \vec{a}}{|\vec{a}|} \\ &= \frac{\vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a}}{\sqrt{(2)^2 + (-2)^2 + (1)^2}} \\ &= \frac{(\hat{i} + 2\hat{j} - 2\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k}) + (2\hat{i} - \hat{j} + 4\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{4 + 4 + 1}} \\ &= \frac{(1)(2) + (2)(-2) + (-2)(1) + (2)(2) + (-1)(-2) + (4)(1)}{\sqrt{9}} \\ &= \frac{2 - 4 - 2 + 4 + 2 + 4}{3} \\ &= \frac{12 - 6}{3} = \frac{6}{3} = 2\end{aligned}$$

Projection of $(\vec{b} + \vec{c}) = 2$

Scalar or Dot Product Ex 24.1 Q27

$$\begin{aligned}\vec{a} + \vec{b} &= (5\hat{i} - \hat{j} - 3\hat{k}) + (\hat{i} + 3\hat{j} - 5\hat{k}) \\ &= 5\hat{i} - \hat{j} - 3\hat{k} + \hat{i} + 3\hat{j} - 5\hat{k} \\ \vec{a} + \vec{b} &= 6\hat{i} + 2\hat{j} - 8\hat{k} \quad \text{--- (i)}\end{aligned}$$

$$\begin{aligned}\vec{a} - \vec{b} &= (5\hat{i} - \hat{j} - 3\hat{k}) - (\hat{i} + 3\hat{j} - 5\hat{k}) \\ &= 5\hat{i} - \hat{j} - 3\hat{k} - \hat{i} - 3\hat{j} + 5\hat{k} \\ \vec{a} - \vec{b} &= 4\hat{i} - 4\hat{j} + 2\hat{k} \quad \text{--- (ii)}\end{aligned}$$

$$\begin{aligned}\text{Now, } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) &= (6\hat{i} + 2\hat{j} - 8\hat{k}) \cdot (4\hat{i} - 4\hat{j} + 2\hat{k}) \\ &= (6)(4) + (2)(-4) + (-8)(2) \\ &= 24 - 8 - 16 \\ &= 0\end{aligned}$$

So, $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular.

Scalar or Dot Product Ex 24.1 Q28

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

Since $\vec{a}$ is a unit vector, $|\vec{a}| = 1$.

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{4}$ with $\hat{i}$, $\frac{\pi}{3}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have:

$$\begin{aligned}\cos \frac{\pi}{4} &= \frac{a_1}{|\vec{a}|} \\ \Rightarrow \frac{1}{\sqrt{2}} &= a_1 \quad [|\vec{a}| = 1]\end{aligned}$$

$$\begin{aligned}\cos \frac{\pi}{3} &= \frac{a_2}{|\vec{a}|} \\ \Rightarrow \frac{1}{2} &= a_2 \quad [|\vec{a}| = 1]\end{aligned}$$

$$\text{Also, } \cos \theta = \frac{a_3}{|\vec{a}|}$$

$$\Rightarrow a_3 = \cos \theta$$

Now,

$$|a| = 1$$

$$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2 \theta = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2 \theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore a_3 = \cos \frac{\pi}{3} = \frac{1}{2}$$

Hence, $\theta = \frac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right)$.

Scalar or Dot Product Ex 24.1 Q29

$$\begin{aligned} & (3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}) \\ &= 3\vec{a} \cdot 2\vec{a} + 3\vec{a} \cdot 7\vec{b} - 5\vec{b} \cdot 2\vec{a} - 5\vec{b} \cdot 7\vec{b} \\ &= 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35\vec{b} \cdot \vec{b} \\ &= 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2 \\ &= \mathbf{6 \cdot 2^2 + 11 \cdot 1 - 35 \cdot 1^2} \\ &= \mathbf{35 - 35} \\ &= \mathbf{0} \end{aligned}$$

Scalar or Dot Product Ex 24.1 Q30(i)

We have,

$$\begin{aligned} & (\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8 \\ & \Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 8 \\ & \Rightarrow |\vec{x}|^2 - 1^2 = 8 \quad \text{since } |\vec{a}| = 1 \\ & \Rightarrow |\vec{x}|^2 = 8 + 1 \\ & \Rightarrow |\vec{x}|^2 = 9 \\ & \Rightarrow |\vec{x}| = 3 \end{aligned}$$

Scalar or Dot Product Ex 24.1 Q30(ii)

We have,

$$\begin{aligned} & (\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12 \\ & \Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12 \\ & \Rightarrow |\vec{x}|^2 - 1^2 = 12 \quad \text{since } |\vec{a}| = 1 \\ & \Rightarrow |\vec{x}|^2 = 12 + 1 \\ & \Rightarrow |\vec{x}|^2 = 13 \\ & \Rightarrow |\vec{x}| = \sqrt{13} \end{aligned}$$

Scalar or Dot Product Ex 24.1 Q31(i)

Here, $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 12$

$$|\vec{a}|^2 - |\vec{b}|^2 = 12$$

$$(2|\vec{b}|)^2 - |\vec{b}|^2 = 12$$

$$[\text{Using } |\vec{a}| = 2|\vec{b}|]$$

$$4|\vec{b}|^2 - |\vec{b}|^2 = 12$$

$$3|\vec{b}|^2 = 12$$

$$|\vec{b}|^2 = \frac{12}{3}$$

$$|\vec{b}|^2 = 4$$

$$|\vec{b}| = 2$$

$$|\vec{a}| = 2|\vec{b}| = 2(2)$$

$$|\vec{a}| = 4$$

$$|\vec{b}| = 2$$

Scalar or Dot Product Ex 24.1 Q31(ii)

$$(\vec{a} \cdot \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$$

$$\Rightarrow \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b} = 8$$

$$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow (8|\vec{b}|)^2 - |\vec{b}|^2 = 8$$

$$[|\vec{a}| = 8|\vec{b}|]$$

$$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow 63|\vec{b}|^2 = 8$$

$$\Rightarrow |\vec{b}|^2 = \frac{8}{63}$$

$$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}}$$

[Magnitude of a vector is non-negative]

$$\Rightarrow |\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$$

$$|\vec{a}| = 8|\vec{b}| = \frac{8 \times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$$

Scalar or Dot Product Ex 24.1 Q31(iii)

Here, $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 3$

$$|\vec{a}|^2 - |\vec{b}|^2 = 3$$

$$(2|\vec{b}|)^2 - |\vec{b}|^2 = 3$$

$$[\text{Using } |\vec{a}| = 2|\vec{b}|]$$

$$4|\vec{b}|^2 - |\vec{b}|^2 = 3$$

$$3|\vec{b}|^2 = 3$$

$$|\vec{b}|^2 = \frac{3}{3}$$

$$|\vec{b}|^2 = 1$$

$$|\vec{b}| = 1$$

$$|\vec{a}| = 2|\vec{b}|$$

$$= 2(1)$$

$$|\vec{a}| = 2$$

$$|\vec{b}| = 1$$

Scalar or Dot Product Ex 24.1 Q32(i)

$$\begin{aligned} |\vec{a} - \vec{b}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} \\ &= (2)^2 + (5)^2 - 2(8) \\ &= 4 + 25 - 16 \\ |\vec{a} - \vec{b}|^2 &= 13 \end{aligned}$$

$$|\vec{a} - \vec{b}| = \sqrt{13}$$

Scalar or Dot Product Ex 24.1 Q32(ii)

$$\begin{aligned} |\vec{a} - \vec{b}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} \\ &= (3)^2 + (4)^2 - 2(1) \\ &= 9 + 16 - 2 \\ |\vec{a} - \vec{b}|^2 &= 23 \end{aligned}$$

$$|\vec{a} - \vec{b}| = \sqrt{23}$$

Scalar or Dot Product Ex 24.1 Q32(iii)

We have

$$\begin{aligned} |\vec{a} - \vec{b}|^2 &= (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) \\ &= \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} \\ &= |\vec{a}|^2 - 2(\vec{a} \cdot \vec{b}) + |\vec{b}|^2 = (2)^2 - 2(4) + (3)^2 = 5 \end{aligned}$$

$$\therefore |\vec{a} - \vec{b}| = \sqrt{5}$$

Scalar or Dot Product Ex 24.1 Q33(i)

We have,

$$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2 \text{ and } \vec{a} \cdot \vec{b} = \sqrt{6}$$

Let θ be the angle between $\vec{a}$ and $\vec{b}$. Then

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{\sqrt{6}}{\sqrt{3} \times 2} \\ &= \frac{1}{\sqrt{2}} \\ \theta &= \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) \\ &= \frac{\pi}{4} \end{aligned}$$

Scalar or Dot Product Ex 24.1 Q33(ii)

Let the angle between $\vec{a}$ and $\vec{b}$ is θ , then

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{1}{3 \cdot 3} \\ \cos \theta &= \frac{1}{9} \end{aligned}$$

$$\theta = \cos^{-1} \left(\frac{1}{9} \right)$$

Scalar or Dot Product Ex 24.1 Q34

$$\text{Let } \vec{a} = \vec{u} + \vec{v}$$

$$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v} \quad \text{--- (i)}$$

Such that $\vec{u}$ is parallel to $\vec{b}$ and $\vec{v}$ is perpendicular to $\vec{b}$.

Now, $\vec{u}$ is parallel to $\vec{b}$

$$\vec{u} = \lambda \vec{b}$$

$$= \lambda (3\hat{i} + \hat{k})$$

$$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k} \quad \text{--- (ii)}$$

Put value of $\vec{u}$ in equation (i),

$$5\hat{i} - 2\hat{j} + 5\hat{k} = (3\lambda\hat{i} + \lambda\hat{k}) + \vec{v}$$

$$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 3\lambda\hat{i} - \lambda\hat{k}$$

$$\vec{v} = (5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}$$

$\vec{v}$ is perpendicular to $\vec{b}$

$$\text{Then, } \vec{v} \cdot \vec{b} = 0$$

$$[(5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}] \cdot (3\hat{i} + 0\hat{j} + \hat{k}) = 0$$

$$(5 - 3\lambda)(3) + (-2)(0) + (5 - \lambda)(1) = 0$$

$$15 - 9\lambda + 0 + 5 - \lambda = 0$$

$$20 - 10\lambda = 0$$

$$-10\lambda = -20$$

$$\lambda = \frac{-20}{-10}$$

$$\lambda = 2$$

Put λ in equation (ii)

$$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k}$$

$$= 3(2)\hat{i} + (2)\hat{k}$$

$$\vec{u} = 6\hat{i} + 2\hat{k}$$

Put the value of $\vec{u}$ in equation (i)

$$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$$

$$5\hat{i} - 2\hat{j} + 5\hat{k} = (6\hat{i} + 2\hat{k}) + \vec{v}$$

$$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 6\hat{i} - 2\hat{k}$$

$$\vec{v} = -\hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{a} = (6\hat{i} + 2\hat{k}) + (-\hat{i} - 2\hat{j} + 3\hat{k})$$

Scalar or Dot Product Ex 24.1 Q35

Vectors $\vec{a}$ and $\vec{b}$ have same magnitude, then

$$|\vec{a}| = |\vec{b}| = x \text{ (Say)}$$

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\cos 30^\circ = \frac{3}{x \cdot x}$$

$$\frac{\sqrt{3}}{2} = \frac{3}{x^2}$$

$$\sqrt{3}x^2 = 6$$

$$x^2 = \frac{6}{\sqrt{3}}$$

Rationalizing the denominator,

$$x^2 = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$x^2 = \frac{6\sqrt{3}}{3}$$

$$x^2 = 2\sqrt{3}$$

$$x = \sqrt{2\sqrt{3}}$$

$$|\vec{a}| = |\vec{b}| = \sqrt{2\sqrt{3}}$$



MillionStars edu
Think, Learn & Practice

Let $(2\hat{i} - \hat{j} + 3\hat{k}) = \vec{a} + \vec{b}$ --- (i)

Such that $\vec{a}$ is a vector parallel to vector $(2\hat{i} + 4\hat{j} - 2\hat{k})$ and $\vec{b}$ is a vector perpendicular to the vector $(2\hat{i} + 4\hat{j} - 2\hat{k})$.

Since, $\vec{a}$ is parallel to $(2\hat{i} + 4\hat{j} - 2\hat{k})$

$$\vec{a} = \lambda (2\hat{i} + 4\hat{j} - 2\hat{k})$$

$$\vec{a} = 2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k} \quad \text{--- (ii)}$$

Put value of $\vec{a}$ in equation (i),

$$(2\hat{i} - \hat{j} + 3\hat{k}) = (2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k}) + \vec{b}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} - 2\lambda\hat{i} - 4\lambda\hat{j} + 2\lambda\hat{k}$$

$$\vec{b} = (2 - 2\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (3 + 2\lambda)\hat{k}$$

$\vec{b}$ is a vector perpendicular to the vector $(2\hat{i} + 4\hat{j} - 2\hat{k})$, then

$$\vec{b} \cdot (2\hat{i} + 4\hat{j} - 2\hat{k}) = 0$$

$$[(2 - 2\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (3 + 2\lambda)\hat{k}] \cdot (2\hat{i} + 4\hat{j} - 2\hat{k}) = 0$$

$$(2 - 2\lambda)(2) + (-1 - 4\lambda)(4) + (3 + 2\lambda)(-2) = 0$$

$$4 - 4\lambda - 4 - 16\lambda - 6 - 4\lambda = 0$$

$$-6 - 24\lambda = 0$$

$$-24\lambda = 6$$

$$\lambda = -\frac{1}{4}$$

Put λ in equation (ii),

$$\vec{a} = 2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k}$$

$$= 2\left(-\frac{1}{4}\right)\hat{i} + 4\left(-\frac{1}{4}\right)\hat{j} - 2\left(-\frac{1}{4}\right)\hat{k}$$

$$\vec{a} = -\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}$$

Put the value of $\vec{a}$ in equation (i),

$$(2\hat{i} - \hat{j} + 3\hat{k}) = \left(-\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}\right) + \vec{b}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} + \frac{1}{2}\hat{i} + \hat{j} - \frac{1}{2}\hat{k}$$

$$= \frac{4\hat{i} - 2\hat{j} + 6\hat{k} + \hat{i} + 2\hat{j} - \hat{k}}{2}$$

$$= \frac{5\hat{i} + 5\hat{k}}{2}$$

$$\vec{b} = \frac{5}{2}(\hat{i} + \hat{k})$$

$$(2\hat{i} - \hat{j} + 3\hat{k}) = \left(-\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}\right) + \frac{5}{2}(\hat{i} + \hat{k})$$

Scalar or Dot Product Ex 24.1 Q37

$$\text{Let } (6\hat{i} - 3\hat{j} - 6\hat{k}) = \vec{a} + \vec{b} \quad \text{--- (i)}$$

Such that $\vec{a}$ is parallel to $(\hat{i} + \hat{j} + \hat{k})$ and $\vec{b}$ is perpendicular to $(\hat{i} + \hat{j} + \hat{k})$.

Since, $\vec{a}$ is parallel to $(\hat{i} + \hat{j} + \hat{k})$

$$\vec{a} = \lambda (\hat{i} + \hat{j} + \hat{k}) \quad \text{--- (ii)}$$

Put $\vec{a}$ in equation (i),

$$(6\hat{i} - 3\hat{j} - 6\hat{k}) = (\lambda\hat{i} + \lambda\hat{j} + \lambda\hat{k}) + \vec{b}$$

$$\vec{b} = 6\hat{i} - \lambda\hat{i} - 3\hat{j} - \lambda\hat{j} - 6\hat{k} - \lambda\hat{k}$$

$$\vec{b} = (6 - \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (-6 - \lambda)\hat{k}$$

$\vec{b}$ is a vector perpendicular to the vector $(\hat{i} + \hat{j} + \hat{k})$, then

$$\vec{b} \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$[(6 - \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (-6 - \lambda)\hat{k}] \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$(6 - \lambda)(1) + (-3 - \lambda)(1) + (-6 - \lambda)(1) = 0$$

$$6 - \lambda - 3 - \lambda - 6 - \lambda = 0$$

$$-3 - 3\lambda = 0$$

$$-3 = 3\lambda$$

$$\lambda = \frac{-3}{3}$$

$$\lambda = -1$$

Put value of λ in (ii),

$$\vec{a} = -1 \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$\vec{a} = -\hat{i} - \hat{j} - \hat{k}$$

Using $\vec{a}$ in equation (i),

$$(6\hat{i} - 3\hat{j} - 6\hat{k}) = (-\hat{i} - \hat{j} - \hat{k}) + \vec{b}$$

$$\vec{b} = 6\hat{i} + \hat{i} - 3\hat{j} + \hat{j} - 6\hat{k} + \hat{k}$$

$$\vec{b} = 7\hat{i} - 2\hat{j} - 5\hat{k}$$

Thus,

$$\text{Vector } \vec{a} = -\hat{i} - \hat{j} - \hat{k} \text{ and}$$

$$\vec{b} = 7\hat{i} - 2\hat{j} - 5\hat{k}$$

are required vectors.

Scalar or Dot Product Ex 24.1 Q38

Here, $(\vec{a} + \vec{b})$ is orthogonal to $(\vec{a} - \vec{b})$

$$\text{Then, } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$\left\{ \sqrt{(5)^2 + (-1)^2 + (7)^2} \right\}^2 - \left\{ \sqrt{(1)^2 + (-1)^2 + (\lambda)^2} \right\}^2 = 0$$

$$(25 + 1 + 49) - (1 + 1 + \lambda^2) = 0$$

$$75 - (2 + \lambda^2) = 0$$

$$75 - 2 - \lambda^2 = 0$$

$$-\lambda^2 = -73$$

$$\lambda = \sqrt{73}$$

Scalar or Dot Product Ex 24.1 Q39

It is given that $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$.

Now,

$$\vec{a} \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0$$

$\therefore \vec{a}$ is a zero vector.

Hence, vector $\vec{b}$ satisfying $\vec{a} \cdot \vec{b} = 0$ can be any vector

Scalar or Dot Product Ex 24.1 Q40

Given that $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, so,

$$\vec{a} \cdot \vec{c} = 0 \text{ and } \vec{b} \cdot \vec{c} = 0$$

$$\begin{aligned} \text{Now, } \vec{c} \cdot (\vec{a} + \vec{b}) &= \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} \\ &= 0 + 0 \\ &= 0 \end{aligned}$$

$\therefore \vec{c}$ is perpendicular to $(\vec{a} + \vec{b})$

$$\begin{aligned} \vec{c} \cdot (\vec{a} - \vec{b}) &= \vec{c} \cdot \vec{a} - \vec{c} \cdot \vec{b} \\ &= 0 - 0 \\ &= 0 \end{aligned}$$

$\therefore \vec{c}$ is perpendicular to $(\vec{a} - \vec{b})$

Scalar or Dot Product Ex 24.1 Q41

Here $|\vec{a}| = a$, $|\vec{b}| = b$

$$\begin{aligned} \text{LHS} &= \left(\frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2} \right)^2 \\ &= \left(\frac{\vec{a}}{a^2} \right)^2 + \left(\frac{\vec{b}}{b^2} \right)^2 - 2 \frac{\vec{a}}{a^2} \cdot \frac{\vec{b}}{b^2} \\ &= \frac{|\vec{a}|^2}{a^4} + \frac{|\vec{b}|^2}{b^4} - \frac{2\vec{a}\vec{b}}{a^2b^2} \\ &= \frac{a^2}{a^4} + \frac{b^2}{b^4} - \frac{2\vec{a}\vec{b}}{a^2b^2} \quad \left[\text{Since } |\vec{a}| = a, |\vec{b}| = b \right] \\ &= \frac{1}{a^2} + \frac{1}{b^2} - \frac{2\vec{a}\vec{b}}{a^2b^2} \\ &= \frac{b^2 + a^2 - 2\vec{a}\vec{b}}{a^2b^2} \\ &= \frac{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\vec{b}}{a^2b^2} \\ &= \frac{(\vec{a} - \vec{b})^2}{a^2b^2} \\ &= \left(\frac{\vec{a} - \vec{b}}{ab} \right)^2 \\ &= \text{RHS} \end{aligned}$$

Hence proved

$$\therefore \left(\frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2} \right)^2 = \left(\frac{\vec{a} - \vec{b}}{ab} \right)^2$$

Scalar or Dot Product Ex 24.1 Q42

Given that

$\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that
 $\vec{d} \cdot \vec{a} = \vec{d} \cdot \vec{b} = \vec{d} \cdot \vec{c} = 0$

Given that

$$\vec{d} \cdot \vec{a} = 0$$

$\Rightarrow \vec{d}$ perpendicular to $\vec{a}$

or $\vec{d} = 0$ --- (i)

$$\vec{d} \cdot \vec{b} = 0$$

$\Rightarrow \vec{d}$ is perpendicular to $\vec{b}$ or $\vec{d} = 0$ --- (ii)

$$\vec{d} \cdot \vec{c} = 0$$

$\Rightarrow \vec{d}$ is perpendicular to $\vec{c}$ or $\vec{d} = 0$ --- (iii)

From (i), (ii), (iii), we get

$\vec{d}$ is perpendicular to $\vec{a}, \vec{b}, \vec{c}$ or $\vec{d} = 0$, but $\vec{d}$ can not be perpendicular to $\vec{a}, \vec{b}$ and $\vec{c}$ because $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, so

$$\vec{d} = 0$$

Scalar or Dot Product Ex 24.1 Q43

Given that

$\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$

It means,

$$\vec{a} \cdot \vec{b} = 0 \text{ and } \vec{a} \cdot \vec{c} = 0 \quad \text{--- (i)}$$

Let $\vec{r}$ be some vector in the plane of $\vec{b}$ and $\vec{c}$

Then, $\vec{r}, \vec{b}, \vec{c}$ are coplanar

We know that,

Three vectors are coplanar if one of them is expressible as linear combination of other two vectors.

$$\text{Let } \vec{r} = x\vec{b} + y\vec{c}$$

where x and y are same scalar

$$\vec{r} \cdot \vec{a} = (x\vec{b} + y\vec{c}) \cdot \vec{a} \quad \left[\text{Taking dot product with } \vec{a} \text{ on both the side} \right]$$

$$\begin{aligned} \vec{r} \cdot \vec{a} &= x\vec{b} \cdot \vec{a} + y\vec{c} \cdot \vec{a} \\ &= x \cdot 0 + y \cdot 0 \end{aligned} \quad \left[\text{Using (i)} \right]$$

$$\vec{r} \cdot \vec{a} = 0 + 0$$

$$\vec{r} \cdot \vec{a} = 0$$

So, $\vec{r}$ is perpendicular to $\vec{a}$

Thus,

$\vec{a}$ is perpendicular to every vector in the plane of $\vec{b}$ and $\vec{c}$

Scalar or Dot Product Ex 24.1 Q44

We have,

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\vec{b} + \vec{c} = -\vec{a}$$

Squaring both the sides.

$$(\vec{b} + \vec{c})^2 = (-\vec{a})^2$$

$$|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} = |\vec{a}|^2$$

$$2\vec{b} \cdot \vec{c} = |\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2$$

$$2|\vec{b}||\vec{c}|\cos\theta = |\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2 \quad \left[\text{Since } \vec{b} \cdot \vec{c} = |\vec{b}||\vec{c}|\cos\theta \right]$$

$$\cos\theta = \frac{|\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{b}||\vec{c}|}$$

Scalar or Dot Product Ex 24.1 Q45

$$\vec{u} + \vec{v} + \vec{w} = \vec{0}$$

Squaring both the sides,

$$(\vec{u} + \vec{v} + \vec{w})^2 = (\vec{0})^2$$

$$|\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2\vec{u}\vec{v} + 2\vec{v}\vec{w} + 2\vec{w}\vec{u} = 0$$

$$(3)^2 + (4)^2 + (5)^2 + 2(\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u}) = 0$$

$$9 + 16 + 25 + 2(\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u}) = 0$$

$$2(\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u}) = -50$$

$$\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u} = \frac{-50}{2}$$

$$\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u} = -25$$

Scalar or Dot Product Ex 24.1 Q46

Given

$$\vec{a} = x^2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{c} = x^2\hat{i} + 5\hat{j} - 4\hat{k}$$

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a}\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\begin{aligned} &= \frac{(x^2\hat{i} + 2\hat{j} - 2\hat{k})(\hat{i} - \hat{j} + \hat{k})}{\sqrt{(x^2)^2 + (2)^2 + (-2)^2} \sqrt{(1)^2 + (-1)^2 + (1)^2}} \\ &= \frac{(x^2)(1) + (2)(-1) + (-2)(1)}{\sqrt{x^4 + 4 + 4} \sqrt{1 + 1 + 1}} \\ &= \frac{x^2 - 2 - 2}{\sqrt{8 + x^4} \sqrt{3}} \\ \cos\theta &= \frac{x^2 - 4}{\sqrt{3}\sqrt{8 + x^4}} \end{aligned}$$

Since θ is an acute angle, so

$$\cos\theta > 0$$

$$\frac{x^2 - 4}{\sqrt{3}\sqrt{8 + x^4}} > 0$$

$$x^2 - 4 > 0$$

$$x^2 > 4$$

$$\Rightarrow x < -2 \text{ or } x > 2 \quad \text{--- (i)}$$

Again, let ϕ be the angle between $\vec{b}$ and $\vec{c}$,

$$\cos\phi = \frac{\vec{b}\vec{c}}{|\vec{b}||\vec{c}|}$$

$$\begin{aligned} &= \frac{(\hat{i} - \hat{j} + \hat{k})(x^2\hat{i} + 5\hat{j} - 4\hat{k})}{\sqrt{(1)^2 + (-1)^2 + (1)^2} \sqrt{(x^2)^2 + (5)^2 + (-4)^2}} \\ \cos\phi &= \frac{(1)(x^2) + (-1)(5) + (1)(-4)}{\sqrt{3}\sqrt{x^4 + 25 + 16}} \\ \cos\phi &= \frac{x^2 - 5 - 4}{\sqrt{3}\sqrt{x^4 + 41}} \\ \cos\phi &= \frac{x^2 - 9}{\sqrt{3}\sqrt{x^4 + 41}} \end{aligned}$$

Since ϕ is an obtuse angle, so

$$\cos\phi < 0$$

$$\frac{x^2 - 9}{\sqrt{3}\sqrt{x^4 + 41}} < 0$$

$$x^2 - 9 < 0$$

$$x^2 < 9$$

$$\Rightarrow x > -3 \text{ and } x < 3 \quad \text{--- (ii)}$$

From

$$-3 < x < -2 \text{ and } 2 < x < 3$$

$$x \in (-3, -2) \cup (2, 3)$$

re, $\vec{a}$ and $\vec{b}$ are mutually perpendicular, then

$$\vec{a} \cdot \vec{b} = 0$$

$$(3\hat{i} + x\hat{j} - \hat{k}) \cdot (2\hat{i} + \hat{j} + y\hat{k}) = 0$$

$$(3)(2) + (x)(1) + (-1)(y) = 0$$

$$6 + x - y = 0$$

$$x - y = -6 \quad \text{--- (i)}$$

Also, $\vec{a}$ and $\vec{b}$ have equal magnitude,

$$|\vec{a}| = |\vec{b}|$$

$$\sqrt{(3)^2 + (x)^2 + (-1)^2} = \sqrt{(2)^2 + (1)^2 + (y)^2}$$

$$9 + x^2 + 1 = 4 + 1 + y^2$$

$$x^2 + 10 = 5y^2$$

$$x^2 - y^2 = 5 - 10$$

$$x^2 - y^2 = -5$$

$$(x + y)(x - y) = -5$$

$$(x + y)(-6) = -5 \quad \text{[Using (i)]}$$

$$-6x - 6y = -5$$

$$-(6x + 6y) = -5$$

$$6x + 6y = 5 \quad \text{--- (ii)}$$

Solving (i) and (ii),

$$6x + 6y = 5$$

$$6x - 6y = -36 \quad \text{[(i) } \times 6]$$

$$12x = -31$$

$$x = \frac{-31}{12}$$

Put value of x in equation (i),

$$x - y = -6$$

$$\frac{-31}{12} - y = -6$$

$$-y = \frac{-6}{1} + \frac{31}{12}$$

$$-y = \frac{-72 + 31}{12}$$

$$y = \frac{41}{12}$$

Scalar or Dot Product Ex 24.1 Q48

Given

$\vec{a}$ and $\vec{b}$ are unit vectors

Then, $|\vec{a}| = |\vec{b}| = 1$

$$|\vec{a} + \vec{b}| = \sqrt{3}$$

Squaring both the sides,

$$|\vec{a} + \vec{b}|^2 = (\sqrt{3})^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 3$$

$$1 + 1 + 2\vec{a} \cdot \vec{b} = 3$$

$$2 + 2\vec{a} \cdot \vec{b} = 3$$

$$2\vec{a} \cdot \vec{b} = 3 - 2$$

$$2\vec{a} \cdot \vec{b} = 1$$

$$\vec{a} \cdot \vec{b} = \frac{1}{2}$$

Now, $(2\vec{a} - 5\vec{b}) \cdot (3\vec{a} + \vec{b})$

$$= 2\vec{a} \cdot 3\vec{a} + 2\vec{a} \cdot \vec{b} - 5\vec{b} \cdot 3\vec{a} - 5\vec{b} \cdot \vec{b}$$

$$= 6(\vec{a})^2 + 2\vec{a} \cdot \vec{b} - 15\vec{a} \cdot \vec{b} - 5(\vec{b})^2$$

$$= 6|\vec{a}|^2 - 13\vec{a} \cdot \vec{b} - 5|\vec{b}|^2$$

$$= 6(1)^2 - 13\left(\frac{1}{2}\right) - 5(1)^2$$

$$= \frac{6}{1} - \frac{13}{2} - \frac{5}{1}$$

$$= \frac{12 - 13 - 10}{2}$$

$$= \frac{12 - 23}{2}$$

$$= -\frac{11}{2}$$

$$(2\vec{a} - 5\vec{b}) \cdot (3\vec{a} + \vec{b}) = -\frac{11}{2}$$

Vector or Dot Product Ex 24.1 Q49

$$|\vec{a} + \vec{b}|^2 = |\vec{b}|^2$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = \vec{b} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} = \vec{b} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = \vec{b} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \cdot (\vec{a} + 2\vec{b}) = 0$$

$\therefore \vec{a} + 2\vec{b}$ is perpendicular to $\vec{a}$.

$$\text{Let } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}| = |\vec{b}|$$

$$\text{Let } |\vec{a}| = |\vec{b}|$$

Squaring both the sides.

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$(\vec{a})^2 - (\vec{b})^2 = 0$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

Thus,

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0 \Leftrightarrow |\vec{a}| = |\vec{b}|$$



MillionStars edu
 Think, Learn & Practice

Ex 24.2

Scalar or Dot Product Ex 24.2 Q1

Let $\vec{o}$, $\vec{a}$ and $\vec{b}$ be the position vector of the O, A and B.

P and Q are points of trisection of AB.

$$\text{Position vector of point P} = \frac{2\vec{a} + \vec{b}}{3}$$

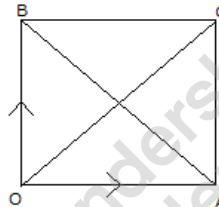
$$\text{Position vector of point Q} = \frac{\vec{a} + 2\vec{b}}{3}$$

$$\vec{OP} = \frac{2\vec{a} + \vec{b}}{3} - \vec{o} = \frac{2\vec{a} + \vec{b} - 3\vec{o}}{3} = \frac{2\vec{OA} + \vec{OB}}{3}$$

$$\vec{OQ} = \frac{\vec{a} + 2\vec{b}}{3} - \vec{o} = \frac{\vec{a} + 2\vec{b} - 3\vec{o}}{3} = \frac{\vec{OA} + 2\vec{OB}}{3}$$

$$\begin{aligned} OP^2 + OQ^2 &= \left(\frac{2\vec{OA} + \vec{OB}}{3}\right)^2 + \left(\frac{\vec{OA} + 2\vec{OB}}{3}\right)^2 \\ &= \frac{5(\vec{OA}^2 + \vec{OB}^2) + 8(\vec{OA})(\vec{OB})\cos 90^\circ}{9} \\ &= \frac{5}{9}AB^2 \dots\dots\dots [\because \vec{OA}^2 + \vec{OB}^2 = AB^2 \text{ and } \cos 90^\circ = 0] \end{aligned}$$

Scalar or Dot Product Ex 24.2 Q2



Let OACB be a quadrilateral such that its diagonal bisect each other at right angles.

We know that if the diagonals of a quadrilateral bisect each other then its a parallelogram.

$\therefore$ OACB is a parallelogram.

$\Rightarrow$ OA = BC and OB = AC.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the A and B.

AB and OC be the diagonals of quadrilateral which bisect each other at right angles.

$$\therefore \vec{OC} \cdot \vec{AB} = 0$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{b} - \vec{a}) = 0$$

$$\Rightarrow |\vec{b}|^2 = |\vec{a}|^2$$

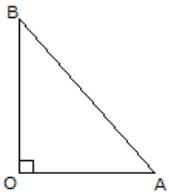
$$\Rightarrow OB = OA$$

Similarly we can show that

$$OA = OB = BC = CA$$

Hence OACB is a rhombus.

Scalar or Dot Product Ex 24.2 Q3



Let OAC be a right triangle, right angled at O.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the $\vec{OA}$ and $\vec{OB}$.

$\vec{OA}$ is perpendicular to $\vec{OB}$

$$\therefore \vec{OA} \cdot \vec{OB} = 0$$

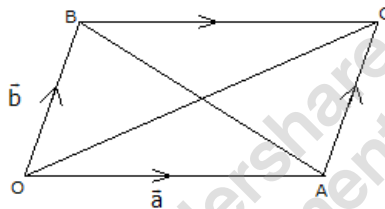
$$\vec{a} \cdot \vec{b} = 0$$

Now,

$$AB^2 = (\vec{b} - \vec{a})^2 = (\vec{a})^2 + (\vec{b})^2 - 2\vec{a} \cdot \vec{b} = (\vec{a})^2 + (\vec{b})^2 - 0 = (\vec{OA})^2 + (\vec{OB})^2$$

Hence proved.

Scalar or Dot Product Ex 24.2 Q4



Let OAC be a right triangle, right angled at O.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the $\vec{OA}$ and $\vec{OB}$.

$\vec{OA}$ is perpendicular to $\vec{OB}$

$$\therefore \vec{OA} \cdot \vec{OB} = 0$$

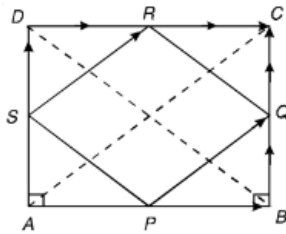
$$\vec{a} \cdot \vec{b} = 0$$

Now,

$$AB^2 = (\vec{b} - \vec{a})^2 = (\vec{a})^2 + (\vec{b})^2 - 2\vec{a} \cdot \vec{b} = (\vec{a})^2 + (\vec{b})^2 - 0 = (\vec{OA})^2 + (\vec{OB})^2$$

Hence proved.

Scalar or Dot Product Ex 24.2 Q5



ABCD be a rectangle.

Let P, Q, R and S be the midpoints of the sides AB, BC, CD and DA respectively.

Now,

$$\vec{PQ} = \vec{PB} + \vec{BQ} = \frac{1}{2}(\vec{AB} + \vec{BC}) = \frac{1}{2}\vec{AC} \dots \dots \dots (i)$$

$$\vec{SR} = \vec{SD} + \vec{DR} = \frac{1}{2}(\vec{AD} + \vec{DC}) = \frac{1}{2}\vec{AC} \dots \dots \dots (ii)$$

From (i) and (ii), we have

$\vec{PQ} = \vec{SR}$ i.e. sides PQ and SR are equal and parallel.

$\therefore$ PQRS is a parallelogram.

$$(\vec{PQ})^2 = \vec{PQ} \cdot \vec{PQ} = (\vec{PB} + \vec{BQ}) \cdot (\vec{PB} + \vec{BQ}) = |\vec{PB}|^2 + |\vec{BQ}|^2 \dots \dots \dots (iii)$$

$$(\vec{PS})^2 = \vec{PS} \cdot \vec{PS} = (\vec{PA} + \vec{AS}) \cdot (\vec{PA} + \vec{AS}) = |\vec{PA}|^2 + |\vec{AS}|^2 = |\vec{PB}|^2 + |\vec{BQ}|^2 \dots \dots \dots (iv)$$

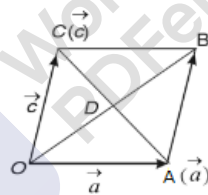
From (iii) and (iv) we get,

$$(\vec{PQ})^2 = (\vec{PS})^2 \text{ i.e. } PQ = PS$$

$\Rightarrow$ The adjacent sides of PQRS are equal.

$\therefore$ PQRS is a rhombus.

Scalar or Dot Product Ex 24.2 Q6



Let OABC be a rhombus, whose diagonals OB and AC intersect at point D.

Let O be the origin.

Let the position vector of A and C be $\vec{a}$ and $\vec{c}$ respectively then,

$$\vec{OA} = \vec{a} \text{ and } \vec{OC} = \vec{c}$$

$$\vec{OB} = \vec{OA} + \vec{AB} = \vec{OA} + \vec{OC} = \vec{a} + \vec{c} \dots \dots \dots [\because \vec{AB} = \vec{OC}]$$

$$\text{Position vector of mid-point of } \vec{OB} = \frac{1}{2}(\vec{a} + \vec{c})$$

$$\text{Position vector of mid-point of } \vec{AC} = \frac{1}{2}(\vec{a} + \vec{c})$$

$\therefore$ Midpoints of OB and AC coincide.

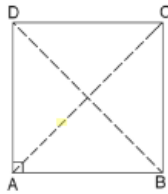
$\therefore$ Diagonal OB and AC bisect each other.

$$\vec{OB} \cdot \vec{AC} = (\vec{a} + \vec{c}) \cdot (\vec{c} - \vec{a}) = (\vec{c} + \vec{a}) \cdot (\vec{c} - \vec{a}) = |\vec{c}|^2 - |\vec{a}|^2 = \vec{OC} \cdot \vec{OA} = 0$$

[$\because$ OC and OA are sides of the rhombus]

$$\Rightarrow \vec{OB} \perp \vec{AC}$$

Scalar or Dot Product Ex 24.2 Q7



Let ABCD be a rectangle.

Take A as origin.

Let position vectors of point B, D be $\vec{a}$ and $\vec{b}$ respectively.

By parallelogram law,

$$\vec{AC} = \vec{a} + \vec{b} \text{ and } \vec{BD} = \vec{a} - \vec{b}$$

As ABCD is a rectangle, $AB \perp AD$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0 \dots \dots \dots (i)$$

Now, diagonals AC and BD are perpendicular iff $\vec{AC} \cdot \vec{BD} = 0$

$$\Rightarrow (\vec{a} + \vec{b})(\vec{a} - \vec{b}) = 0$$

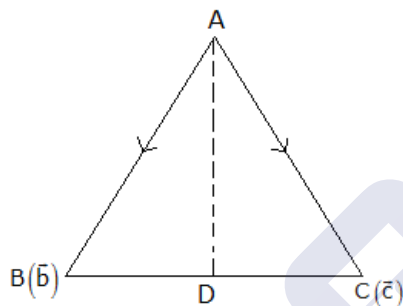
$$\Rightarrow (\vec{a})^2 - (\vec{b})^2 = 0$$

$$\Rightarrow |\vec{AB}|^2 = |\vec{AD}|^2$$

$$\Rightarrow |AB| = |AD|$$

Hence ABCD is a square.

Scalar or Dot Product Ex 24.2 Q8



Take A as origin, let the position vectors of B and C are $\vec{b}$ and $\vec{c}$ respectively.

Position vector of D = $\frac{\vec{b} + \vec{c}}{2}$, $\vec{AB} = \vec{b}$ and $\vec{AC} = \vec{c}$.

$$\vec{AD} = \frac{\vec{b} + \vec{c}}{2} - \vec{0} = \frac{\vec{b} + \vec{c}}{2}$$

Consider, $2(AD^2 + CD^2)$

$$= 2 \left[\left(\frac{\vec{b} + \vec{c}}{2} \right)^2 + \left(\frac{\vec{b} + \vec{c}}{2} - \vec{c} \right)^2 \right]$$

$$= 2 \left[\left(\frac{\vec{b} + \vec{c}}{2} \right)^2 + \left(\frac{\vec{b} - \vec{c}}{2} \right)^2 \right]$$

$$= \frac{1}{2} [(\vec{b} + \vec{c})^2 + (\vec{b} - \vec{c})^2]$$

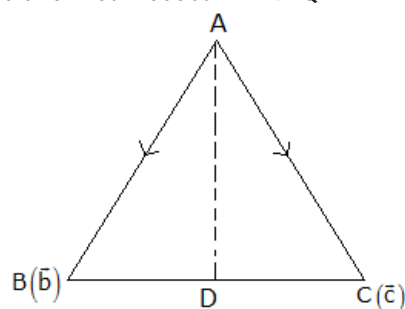
$$= (\vec{b})^2 + (\vec{c})^2$$

$$= (\vec{AB})^2 + (\vec{AC})^2$$

$$= AB^2 + AC^2$$

Hence proved.

Scalar or Dot Product Ex 24.2 Q9



Take A as origin, let the position vectors of B and C are $\vec{b}$ and $\vec{c}$ respectively.

Position vector of D = $\frac{\vec{b} + \vec{c}}{2}$, $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$.

$$\overrightarrow{AD} = \frac{\vec{b} + \vec{c}}{2} - \vec{0} = \frac{\vec{b} + \vec{c}}{2}$$

AD is perpendicular to BC

$$\Rightarrow \overrightarrow{AD} \cdot \overrightarrow{BC} = 0$$

$$\Rightarrow \left(\frac{\vec{b} + \vec{c}}{2} \right) \cdot (\vec{c} - \vec{b}) = 0$$

$$\Rightarrow (\vec{b} + \vec{c}) \cdot (\vec{c} - \vec{b}) = 0$$

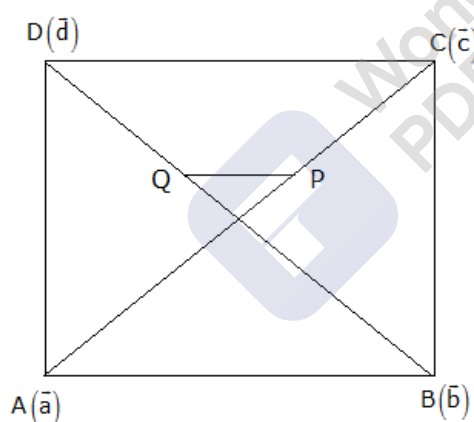
$$\Rightarrow |\vec{c}|^2 = |\vec{b}|^2$$

$$\Rightarrow |\vec{c}| = |\vec{b}|$$

$$\Rightarrow AC = AB$$

Hence $\triangle ABC$ is an isosceles triangle.

Scalar or Dot Product Ex 24.2 Q10



Take O as origin, let the position vectors of A, B, C and D are $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively.

$$\text{Position vector of P} = \frac{\vec{a} + \vec{c}}{2}$$

$$\text{Position vector of Q} = \frac{\vec{a} + \vec{d}}{2}$$

$$\begin{aligned} \text{LHS} &= AB^2 + BC^2 + CD^2 + DA^2 \\ &= (\vec{b} - \vec{a})^2 + (\vec{c} - \vec{b})^2 + (\vec{d} - \vec{c})^2 + (\vec{d} - \vec{a})^2 \\ &= 2 \left[(\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + (\vec{d})^2 - \vec{a}\vec{b}\cos\theta_1 - \vec{b}\vec{c}\cos\theta_2 - \vec{c}\vec{d}\cos\theta_3 - \vec{c}\vec{a}\cos\theta_4 \right] \end{aligned}$$

$$\begin{aligned} \text{RHS} &= AC^2 + BD^2 + 4PQ^2 \\ &= (\vec{c} - \vec{a})^2 + (\vec{d} - \vec{b})^2 + 4 \left(\frac{\vec{a} + \vec{d}}{2} - \frac{\vec{a} + \vec{c}}{2} \right)^2 \\ &= 2 \left[(\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + (\vec{d})^2 - \vec{a}\vec{b}\cos\theta_1 - \vec{b}\vec{c}\cos\theta_2 - \vec{c}\vec{d}\cos\theta_3 - \vec{c}\vec{a}\cos\theta_4 \right] \\ &= \text{LHS} \end{aligned}$$

Hence proved.