Ex 26.1

Scalar Triple Product Ex 26.1 Q1(i)

We have

$$\left[\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{j}\ \hat{k}\ \hat{i}\right]+\left[\hat{k}\ \hat{i}\ \hat{j}\right]=\left(\hat{i}\times\hat{j}\right)\cdot\hat{k}+\left(\hat{j}\times\hat{k}\right)\cdot\hat{i}+\left(\hat{k}\times\hat{i}\right)\cdot\hat{j}$$
$$=\hat{k}\cdot\hat{k}+\hat{i}\cdot\hat{i}+\hat{j}\cdot\hat{j}$$
$$=1+1+1$$
$$=3$$

Therefore, $\left[\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{j}\ \hat{k}\ \hat{i}\right]+\left[\hat{k}\ \hat{i}\ \hat{j}\right]=3$

Scalar Triple Product Ex 26.1 Q1(ii)

We have

$$\left[2\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{i}\ \hat{k}\ \hat{j}\right]+\left[\hat{k}\ \hat{j}\ 2\hat{i}\right]=\left(2\hat{i}\times\hat{j}\right)\cdot\hat{k}+\left(\hat{i}\times\hat{k}\right)\cdot\hat{j}+\left(\hat{k}\times\hat{j}\right)\cdot 2\hat{i}$$
$$=2\hat{k}\cdot\hat{k}+\left(-\hat{j}\right)\cdot\hat{j}+\left(-\hat{i}\right)\cdot 2\hat{i}$$
$$=2-1-2$$
$$=-1$$

Therefore, $\left[2\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{i}\ \hat{k}\ \hat{j}\right]+\left[\hat{k}\ \hat{j}\ 2\hat{i}\right]=-1$

Scalar Triple Product Ex 26.1 Q2(i)

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}2 & -3 & 0\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}$$
$$=2(-1-0)+3(-1+3)$$
$$=-2+6$$
$$=4$$

Therefore, $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=4$

Scalar Triple Product Ex 26.1 Q2(ii)

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & -1\\ 0 & 1 & 1\end{vmatrix}$$
$$=1(1+1)+2(2+0)+3(2-0)$$
$$=2+4+6$$
$$=12$$

Therefore, $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=12$

Scalar Triple Product Ex 26.1 Q3(i)

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}2 & 3 & 4\\ 1 & 2 & -1\\ 3 & -1 & 2\end{vmatrix}$$
$$=2(4-1)-3(2+3)+4(-1-6)$$
$$=6-15-28$$
$$=-9-28$$
$$=-37$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=|-37|=37$ cubic unit.

Scalar Triple Product Ex 26.1 Q3(ii)

Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k},\vec{b}=\hat{i}+2\hat{j}-\hat{k},\vec{c}=3\hat{i}-\hat{j}-2\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}2 & -3 & 4\\ 1 & 2 & -1\\ 3 & -1 & -2\end{vmatrix}$$
$$=2(-4-1)+3(-2+3)+4(-1-6)$$
$$=-10+3-28$$
$$=-10-25$$
$$=-35$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=|-35|=35$ cubic unit.

Scalar Triple Product Ex 26.1 Q3(iii)

Let $\vec{a}=11\hat{i},\vec{b}=2\hat{j},\vec{c}=13\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}11&0&0\\0&2&0\\0&0&13\end{vmatrix}$$
$$=11(26-0)+0+0$$
$$=286$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=|286|=286$ cubic unit.

Scalar Triple Product Ex 26.1 Q1(iv)

Let $\vec{a}=\hat{i}+\hat{j}+\hat{k},\vec{b}=\hat{i}-\hat{j}+\hat{k},\vec{c}=\hat{i}+2\hat{j}-\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1&1&1\\1&-1&1\\1&2&-1\end{vmatrix}$$
$$=1(1-2)-1(-1-1)+1(2+1)$$
$$=-1+2+3$$
$$=4$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=|4|=4$ cubic unit.

Scalar Triple Product Ex 26.1 Q4(i)

We know that three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar iff their scalar triple product is zero i.e. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

Here,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1&2&-1\\3&2&7\\5&6&5\end{vmatrix}$$
$$=1(10-42)-2(15-35)-1(18-10)$$
$$=-32+40-8$$
$$=0$$

Hence, the given vectors are coplanar.

Scalar Triple Product Ex 26.1 Q4(ii)

We know that three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar iff their scalar triple product is zero i.e. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

Here,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}-4&-6&-2\\-1&4&3\\-8&-1&3\end{vmatrix}$$
$$=-4(12+3)+6(-3+24)-2(1+32)$$
$$=-60+126-66$$
$$=0$$

Hence, the given vectors are coplanar.

Scalar Triple Product Ex 26.1 Q5(i)

We know that vectors $\vec{a},\vec{b},\vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

$\therefore\ \vec{a},\vec{b},\vec{c}$ are coplanar

$$\Rightarrow\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$$

$$\begin{vmatrix}1&-1&1\\2&1&-1\\\lambda&-1&\lambda\end{vmatrix}=0$$
$$=1(\lambda-1)+1(2\lambda+\lambda)+1(-2-\lambda)$$
$$=\lambda-1+3\lambda-2-\lambda$$
$$3=3\lambda$$
$$1=\lambda$$

Scalar Triple Product Ex 26.1 Q5(ii)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ \lambda & \lambda & 5 \end{vmatrix} = 0$$

$$= 2(10+3\lambda) + 1(5+3\lambda) + 1(\lambda - 2\lambda)$$
$$= 20 + 6\lambda + 5 + 3\lambda - \lambda$$
$$-25 = 8\lambda$$
$$\lambda = -\frac{25}{8}$$

Scalar Triple Product Ex 26.1 Q5(iii)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\begin{vmatrix} 1 & 2 & -3 \\ 3 & \lambda & 1 \\ 1 & 2 & 2 \end{vmatrix} = 0$$

$$= 1(2\lambda - 2) - 2(6-1) - 3(6-\lambda)$$
$$= 2\lambda - 2 - 12 + 2 - 18 + 3\lambda$$
$$= 5\lambda - 30$$
$$30 = 5\lambda$$
$$\lambda = 6$$

Scalar Triple Product Ex 26.1 Q5(iv)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\begin{vmatrix} 1 & 3 & 0 \\ 0 & 0 & 5 \\ \lambda & -1 & 0 \end{vmatrix} = 0$$

$$= 1(0+5) - 3(0 - 5\lambda) + 0$$
$$= 5 + 15\lambda$$
$$-5 = 15\lambda$$
$$\lambda = -\frac{1}{3}$$

Scalar Triple Product Ex 26.1 Q6

Let

$OA = 6\hat{i} - 7\hat{j},\ OB = 16\hat{i} - 19\hat{j} - 4\hat{k},\ OC = 3\hat{j} - 6\hat{k},\ OD = 2\hat{i} + 5\hat{j} + 10\hat{k}$

$AB = OB - OA = 16\hat{i} - 25\hat{j} - 4\hat{k}$

$AC = OC - OA = -16\hat{i} - 16\hat{j} + 2\hat{k}$

$CD = OD - OC = 2\hat{i} + 2\hat{j} + 16\hat{k}$

$AD = OD - OA = -4\hat{i} + 12\hat{j} + 10\hat{k}$

The four points are co-planer if vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are co-planer.

$$\begin{vmatrix} 16 & -25 & -4 \\ -16 & -16 & 2 \\ -4 & 12 & 10 \end{vmatrix} = 16(-160-24) + 25(-160+8) - 4(-144+64)$$
$$\neq 0$$

Hence the points are not coplanar.

Scalar Triple Product Ex 26.1 Q7

$AB = \text{position vector of } B - \text{position vector of } A$

$= 4\hat{i} - 2\hat{j} - 2\hat{k}$

$AC = \text{position vector of } C - \text{position vector of } A$

$= -2\hat{i} + 4\hat{j} - 2\hat{k}$

$AD = \text{position vector of } D - \text{position vector of } A$

$= -2\hat{i} - 2\hat{j} + 4\hat{k}$

The four points are co − planar if the vectors are co − planar.

$$\text{Thus, } \begin{vmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{vmatrix} = 4[16-4] + 2[-8-4] - 2[4+8] = 48 - 24 - 24 = 0$$

Hence proved.

Scalar Triple Product Ex 26.1 Q8

Let $OA = 6\hat{i} - 7\hat{j}$, $OB = 16\hat{i} - 19\hat{j} - 4\hat{k}$, $OC = 3\hat{i} - 6\hat{k}$, $OD = 2\hat{i} - 5\hat{j} + 10\hat{k}$

Thus,

$AB = OB - OA = 10\hat{i} - 12\hat{j} - 4\hat{k}$

$AC = OC - OA = -3\hat{i} + 7\hat{j} - 6\hat{k}$

$AD = OD - OA = -4\hat{i} + 2\hat{j} + 10\hat{k}$

The four points are co-planar if vectors AB, AC and AD are co-planar.

Thus, we have

$$\begin{vmatrix} 10 & -12 & -4 \\ -3 & 7 & -6 \\ -4 & 2 & 10 \end{vmatrix} = 10(70+12) + 12(-30-24) - 4(-6+28) = 820 - 648 - 88$$

Scalar Triple Product Ex 26.1 Q9

Let

Position vector of $A = -\hat{j} - \hat{k}$

Position vector of $B = 4\hat{i} + 5\hat{j} + \lambda\hat{k}$

Position vector of $C = 3\hat{i} + 9\hat{j} + 4\hat{k}$

Position vector of $D = -4\hat{i} + 4\hat{j} + 4\hat{k}$

The four points are coplanar if the vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar.

$\overline{AB} = 4\hat{i} + 6\hat{j} + (\lambda+1)\hat{k}$

$\overline{AC} = 3\hat{i} + 10\hat{j} + 5\hat{k}$

$\overline{AD} = -4\hat{i} + 5\hat{j} + 5\hat{k}$

$$\begin{vmatrix} 4 & 6 & (\lambda+1) \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix} = 0$$

$4(50-25) - 6(15+20) + (\lambda+1)(15+40) = 0$

$100 - 210 + 55 + 55\lambda = 0$

$55\lambda = 55$

$\lambda = 1$

Scalar Triple Product Ex 26.1 Q10

$(\bar{a}-\bar{b}).\{(\bar{b}-\bar{c})\times(\bar{c}-\bar{a})\}$

$= [(\bar{a}-\bar{b}) \quad (\bar{b}-\bar{c}) \quad (\bar{c}-\bar{a})]$

$= [a \quad (\bar{b}-\bar{c}) \quad (\bar{c}-\bar{a})] + [-b \quad (\bar{b}-\bar{c}) \quad (\bar{c}-\bar{a})]$

$= 6[a \quad b \quad c] - 6[a \quad b \quad c]$

$= 0$

Scalar Triple Product Ex 26.1 Q11

If $\bar{a}$ represents the sides AB , If $\bar{b}$ represents the sides BC, If $\bar{c}$ represents the sides AC of the triangleABC.

$\bar{a}\times\bar{b}$ is perpendicular to the plane of the triangleABC.

$\bar{b}\times\bar{c}$ is perpendicular to the plane of the triangleABC.

$\bar{c}\times\bar{a}$ is perpendicular to the plane of the triangleABC.

Hence $\bar{a}\times\bar{b} + \bar{b}\times\bar{c} + \bar{c}\times\bar{a}$ is a vector perpendicular to the plane of the triangleABC.

Scalar Triple Product Ex 26.1 Q12(i)

$\bar{a}, \bar{b}, \bar{c}$ are coplanar if

$[a \quad b \quad c] = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & c_3 \end{vmatrix} = 0$$

$0 - 1(c_3) + 1(2) = 0$

$c_3 = 2$

Scalar Triple Product Ex 26.1 Q12(ii)

$\bar{a}, \bar{b}, \bar{c}$ are coplanar if

$[a \quad b \quad c] = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ -1 & c_2 & 1 \end{vmatrix} = 0$$

$0 - 1 + 1(c_2) = 0$

$c_2 = 1$

Scalar Triple Product Ex 26.1 Q13

Let

Position vector of $OA = 3\hat{i} + 2\hat{j} + \hat{k}$

Position vector of $OB = 4\hat{i} + \lambda\hat{j} + 5\hat{k}$

Position vector of $OC = 4\hat{i} + 2\hat{j} - 2\hat{k}$

Position vector of $OD = 6\hat{i} + 5\hat{j} - \hat{k}$

The four points are coplanar if the vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar.

$$\overline{AB} = \hat{i} + (\lambda - 2)\hat{j} + 4\hat{k}$$
$$\overline{AC} = \hat{i} + 0\hat{j} - 3\hat{k}$$
$$\overline{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\begin{vmatrix} 1 & (\lambda-2) & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$$1(9) - (\lambda - 2)(-2 + 9) + 4(3 - 0) = 0$$
$$9 - 7\lambda + 14 + 12 = 0$$
$$7\lambda = 35$$
$$\lambda = 5$$