

## Straight Line in Space Ex 28.1 Q1

Vector equation of a line

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

The Cartesian equation of a line is

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{z - z_1}{a_3}$$

Using the above formula,

Vector equation of the line,

$$\vec{r} = (5\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$$

The Cartesian equation of the line

$$\frac{x - 5}{3} = \frac{y - 2}{2} = \frac{z + 4}{-8}$$

## Straight Line in Space Ex 28.1 Q2

The direction ratios of the line are

$$(3 + 1, 4 - 0, 6 - 2) = (4, 4, 4)$$

Since the line passes through  $(-1, 0, 2)$

The vector equation of the line,

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\Rightarrow \vec{r} = \vec{a} + \lambda \vec{b}$$

$$\Rightarrow \vec{r} = (-\hat{i} + 0\hat{j} + 2\hat{k}) + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$$

$\therefore$  The vector equation of the line,

$$\vec{r} = (-\hat{i} + 0\hat{j} + 2\hat{k}) + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$$

## Straight Line in Space Ex 28.1 Q3

We know that, vector equation of line passing through a fixed point  $\vec{a}$  and parallel to vector  $\vec{b}$  is

$$\vec{r} = \vec{a} + \lambda \vec{b}, \text{ where } \lambda \text{ is scalar}$$

$$\text{Here, } \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{a} = 5\hat{i} - 2\hat{j} + 4\hat{k}$$

So, equation of required line is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\vec{r} = (5\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$$

Put  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ , so

$$(x\hat{i} + y\hat{j} + z\hat{k}) = (5 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (4 + 3\lambda)\hat{k}$$

Comparing the coefficients of  $\hat{i}, \hat{j}, \hat{k}$ , so

$$x = 5 + 2\lambda, y = -2 - \lambda, z = 4 + 3\lambda$$

$$\Rightarrow \frac{x - 5}{2} = \lambda, \frac{y + 2}{-1} = \lambda, \frac{z - 4}{3} = \lambda$$

Cartesian form of equation of the line is,

$$\frac{x - 5}{2} = \frac{y + 2}{-1} = \frac{z - 4}{3}$$



### Straight Line in Space Ex 28.1 Q4

We know that, equation of line passing through a vector  $\vec{a}$  and parallel to a vector  $\vec{b}$  is given by,

$$\vec{r} = \vec{a} + \lambda \vec{b}, \text{ where } \lambda \text{ is scalar,}$$

$$\text{Here, } \vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k} \text{ and } \vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$$

Required equation of line is,

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\vec{r} = (2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(3\hat{i} + 4\hat{j} - 5\hat{k})$$

$$\text{Put } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$x\hat{i} + y\hat{j} + z\hat{k} = (2 + 3\lambda)\hat{i} + (-3 + 4\lambda)\hat{j} + (4 - 5\lambda)\hat{k}$$

On equating coefficients of  $\hat{i}, \hat{j}$  and  $\hat{k}$ ,

$$\Rightarrow 2 + 3\lambda = x, -3 + 4\lambda = y, 4 - 5\lambda = z$$

$$\Rightarrow \frac{x-2}{3} = \lambda, \frac{y+3}{4} = \lambda, \frac{z-4}{-5} = \lambda$$

So, cartesian form of equation of the line is

$$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$$

### Straight Line in Space Ex 28.1 Q5

$ABCD$  is a parallelogram.

$$\Rightarrow AC \text{ and } BD \text{ bisect each other at point } O \text{ (say).}$$

$$\text{Position vector of point } O = \frac{\vec{a} + \vec{c}}{2}$$

$$= \frac{(4\hat{i} + 5\hat{j} - 10\hat{k}) + (-\hat{i} + 2\hat{j} + \hat{k})}{2}$$

$$= \frac{3\hat{i} + 7\hat{j} - 9\hat{k}}{2}$$

Let position vector of point  $O$  and  $B$  are represented by  $\vec{o}$  and  $\vec{b}$ .

Equation of the line  $BD$  is the line passing through  $O$  and  $B$  is given by

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}) \quad \left[ \begin{array}{l} \text{Since equation of the line passing through} \\ \text{two points } \vec{a} \text{ and } \vec{b} \end{array} \right]$$

$$\vec{r} = \vec{b} + \lambda(\vec{o} - \vec{b})$$

$$= (2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda \left( \frac{3\hat{i} + 7\hat{j} - 9\hat{k}}{2} - 2\hat{i} - 3\hat{j} + 4\hat{k} \right)$$

$$\vec{r} = (2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(3\hat{i} + 7\hat{j} - 9\hat{k} - 4\hat{i} + 6\hat{j} - 8\hat{k})$$

$$\vec{r} = (2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(-\hat{i} + 13\hat{j} - 17\hat{k})$$

$$\text{Put } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) = (2 - \lambda)\hat{i} + (-3 + 13\lambda)\hat{j} + (4 - 17\lambda)\hat{k}$$

Equation the coefficients of  $\hat{i}, \hat{j}, \hat{k}$ , so

$$\Rightarrow x = 2 - \lambda, y = -3 + 13\lambda, z = 4 - 17\lambda$$

$$\Rightarrow \frac{x-2}{-1} = \lambda, \frac{y+3}{13} = \lambda, \frac{z-4}{-17} = \lambda$$

So equation of the line  $BD$  in cartesian form,

$$\frac{x-2}{-1} = \frac{y+3}{13} = \frac{z-4}{-17}$$

**Straight Line in Space Ex 28.1 Q6**

We know that, equation of line passing through two points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \quad \text{--- (i)}$$

Here,  $(x_1, y_1, z_1) = A(1, 2, -1)$

$(x_2, y_2, z_2) = B(2, 1, 1)$

Using equation (i), equation of line AB,

$$\frac{x - 1}{2 - 1} = \frac{y - 2}{1 - 2} = \frac{z + 1}{1 + 1}$$

$$\frac{x - 1}{1} = \frac{y - 2}{-1} = \frac{z + 1}{2} = \lambda \text{ (say)}$$

$$x = \lambda + 1, y = -\lambda + 2, z = 2\lambda - 1$$

Vector form of equation of line AB is,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 1)\hat{i} + (-\lambda + 2)\hat{j} + (2\lambda - 1)\hat{k}$$

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k})$$

**Straight Line in Space Ex 28.1 Q7**

We know that vector equation of a line passing through  $\vec{a}$  and parallel to vector  $\vec{b}$  is given by,

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Here,  $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

So, required vector equation of line is,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

Now,

$$(x\hat{i} + y\hat{j} + z\hat{k}) = (1 + \lambda)\hat{i} + (2 - 2\lambda)\hat{j} + (3 + 3\lambda)\hat{k}$$

Equating the coefficients of  $\hat{i}, \hat{j}, \hat{k}$ ,

$$\Rightarrow x = 1 + \lambda, y = 2 - 2\lambda, z = 3 + 3\lambda$$

$$\Rightarrow x - 1 = \lambda, \frac{y - 2}{-2} = \lambda, \frac{z - 3}{3} = \lambda$$

So, required equation of line is cartesian form,

$$\frac{x - 1}{1} = \frac{y - 2}{-2} = \frac{z - 3}{3}$$



### Straight Line in Space Ex 28.1 Q8

We know that, equation of a line passing through a point  $(x_1, y_1, z_1)$  and having direction ratios proportional to  $a, b, c$  is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \quad \dots (i)$$

Here,  $(x_1, y_1, z_1) = (2, -1, 1)$  and

Given line  $\frac{x-3}{2} = \frac{y+1}{7} = \frac{z-2}{-3}$  is parallel to required line.

$$\Rightarrow a = 2\mu, b = 7\mu, c = -3\mu$$

So, equation of required line using equation (i),

$$\frac{x-2}{2\mu} = \frac{y+1}{7\mu} = \frac{z-1}{-3\mu}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y+1}{7} = \frac{z-1}{-3} = \lambda \text{ (say)}$$

$$\Rightarrow x = 2\lambda + 2, y = 7\lambda - 1, z = -3\lambda + 1$$

$$\begin{aligned} \text{So, } x\hat{i} + y\hat{j} + z\hat{k} &= (2\lambda + 2)\hat{i} + (7\lambda - 1)\hat{j} + (-3\lambda + 1)\hat{k} \\ \vec{r} &= (2\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} + 7\hat{j} - 3\hat{k}) \end{aligned}$$

### Straight Line in Space Ex 28.1 Q9

The Cartesian equation of the line is

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2} \quad \dots (1)$$

The given line passes through the point  $(5, -4, 6)$ . The position vector of this point is

$$\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$$

Also, the direction ratios of the given line are 3, 7, and 2.

This means that the line is in the direction of vector,  $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$

It is known that the line through position vector  $\vec{a}$  and in the direction of the vector  $\vec{b}$  is given by the equation,  $\vec{r} = \vec{a} + \lambda\vec{b}, \lambda \in \mathbb{R}$

$$\Rightarrow \vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

This is the required equation of the given line in vector form.

### Straight Line in Space Ex 28.1 Q10

We know that, equation of a line passing through a point  $(x_1, y_1, z_1)$  and having direction ratios proportional to  $a, b, c$  is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \quad \dots (i)$$

Here,  $(x_1, y_1, z_1) = (1, -1, 2)$  and

Given line  $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$  is parallel to required line, so

$$\Rightarrow a = \mu, b = 2\mu, c = -2\mu$$

So, equation of required line using equation (i) is,

$$\frac{x-1}{\mu} = \frac{y+1}{2\mu} = \frac{z-2}{-2\mu}$$

$$\Rightarrow \frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{-2} = \lambda \text{ (say)}$$

$$x = \lambda + 1, y = 2\lambda - 1, z = -2\lambda + 2$$

$$\text{So, } x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 1)\hat{i} + (2\lambda - 1)\hat{j} + (-2\lambda + 2)\hat{k}$$

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$$

### Straight Line in Space Ex 28.1 Q11

line is,

$$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$$

$$\Rightarrow \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda \text{ (say)}$$

$$x = -2\lambda + 4, y = 6\lambda, z = -3\lambda + 1$$

$$\text{So, } x\hat{i} + y\hat{j} + z\hat{k} = (-2\lambda + 4)\hat{i} + (6\lambda)\hat{j} + (-3\lambda + 1)\hat{k}$$

$$\vec{r} = (4\hat{i} + \hat{k}) + \lambda(-2\hat{i} + 6\hat{j} - 3\hat{k})$$

Direction ratios of the line are  $= -2, 6, -3$

Direction cosines of the line are,

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$\Rightarrow \frac{-2}{\sqrt{(-2)^2 + (6)^2 + (-3)^2}}, \frac{6}{\sqrt{(-2)^2 + (6)^2 + (-3)^2}}, \frac{-3}{\sqrt{(-2)^2 + (6)^2 + (-3)^2}}$$

$$\Rightarrow \frac{-2}{7}, \frac{6}{7}, \frac{-3}{7}$$

### Straight Line in Space Ex 28.1 Q12

$$x = ay + b,$$

$$z = cy + d$$

$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c} = \lambda \text{ (say)}$$

So DR's of line are  $(a, 1, c)$

From above equation, we can write

$$x = a\lambda + b$$

$$y = \lambda$$

$$z = c\lambda + d$$

So vector equation of line is

$$x\hat{i} + y\hat{j} + z\hat{k} = (b\hat{i} + d\hat{k}) + \lambda(a\hat{i} + \hat{j} + c\hat{k})$$

### Straight Line in Space Ex 28.1 Q13

We know that, equation of a line passing through  $\vec{a}$  and parallel to vector  $\vec{b}$  is,

$$\vec{r} = \vec{a} + \lambda\vec{b} \quad \text{--- (i)}$$

$$\text{Here, } \vec{a} = \hat{i} - 2\hat{j} - 3\hat{k}$$

$$\text{and, } \vec{b} = \text{line joining } (\hat{i} - \hat{j} + 4\hat{k}) \text{ and } (2\hat{i} + \hat{j} + 2\hat{k})$$

$$= (2\hat{i} + \hat{j} + 2\hat{k}) - (\hat{i} - \hat{j} + 4\hat{k})$$

$$= 2\hat{i} - \hat{i} + \hat{j} + \hat{j} + 2\hat{k} - 4\hat{k}$$

$$= \hat{i} + 2\hat{j} - 2\hat{k}$$

Equation of the line is

$$\vec{r} = (\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$$

For cartesian form of equation put  $x\hat{i} + y\hat{j} + z\hat{k}$ ,

$$x\hat{i} + y\hat{j} + z\hat{k} = (1 + \lambda)\hat{i} + (-2 + 2\lambda)\hat{j} + (-3 - 2\lambda)\hat{k}$$

Equating coefficients of  $\hat{i}, \hat{j}, \hat{k}$ , so

$$x = 1 + \lambda, y = -2 + 2\lambda, z = -3 - 2\lambda$$

$$\Rightarrow \frac{x-1}{1} = \lambda, \frac{y+2}{2} = \lambda, \frac{z+3}{-2} = \lambda$$

$$\text{So, } \frac{x-1}{1} = \frac{y+2}{2} = \frac{z+3}{-2}$$



### Straight Line in Space Ex 28.1 Q14

Distance of point  $P$  from  $Q = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$$PQ = \sqrt{(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2}$$

$$\begin{aligned} \Rightarrow (5)^2 &= (3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2 \\ \Rightarrow 25 &= 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2 \\ \Rightarrow 17\lambda^2 - 34\lambda &= 0 \\ \Rightarrow 17\lambda(\lambda - 2) &= 0 \\ \Rightarrow \lambda &= 0 \text{ or } 2 \end{aligned}$$

So, points on the line are  $(3(0) - 2, 2(0) - 1, 2(0) + 3)$

$$(3(2) - 2, 2(2) - 1, 2(2) + 3)$$

$$= (-2, -1, 3), (4, 3, 7)$$

### Straight Line in Space Ex 28.1 Q15

Let the given points are  $A, B, C$  with position vectors  $\vec{a}, \vec{b}, \vec{c}$  respectively, so

$$\vec{a} = -2\hat{i} + 3\hat{j}, \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{c} = 7\hat{i} - \hat{k}$$

We know that, equation of a line passing through  $\vec{a}$  and  $\vec{b}$  are,

$$\begin{aligned} \vec{r} &= \vec{a} + \lambda(\vec{b} - \vec{a}) \\ &= (-2\hat{i} + 3\hat{j}) + \lambda((\hat{i} + 2\hat{j} + 3\hat{k}) - (-2\hat{i} + 3\hat{j})) \\ &= (-2\hat{i} + 3\hat{j}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k} + 2\hat{i} - 3\hat{j}) \\ \vec{r} &= (-2\hat{i} + 3\hat{j}) + \lambda(3\hat{i} - \hat{j} + 3\hat{k}) \quad \text{--- (i)} \end{aligned}$$

If  $A, B, C$  are collinear then  $\vec{c}$  must satisfy equation (i),

$$7\hat{i} - \hat{k} = (-2 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + (3\lambda)\hat{k}$$

Equation the coefficients of  $\hat{i}, \hat{j}, \hat{k}$ ,

$$-2 + 3\lambda = 7 \Rightarrow \lambda = 3$$

$$3 - \lambda = 0 \Rightarrow \lambda = 3$$

$$3\lambda = -1 \Rightarrow \lambda = -\frac{1}{3}$$

Since, value of  $\lambda$  are not equal, so,

Given points are not collinear.

### Straight Line in Space Ex 28.1 Q16

We know that, equation of a line passing through a point  $(x_1, y_1, z_1)$  and having direction ratios proportional to  $a, b, c$  is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \quad \text{--- (i)}$$

Here,  $(x_1, y_1, z_1) = (1, 2, 3)$  and

$$\text{Given line } \frac{-x - 2}{1} = \frac{y + 3}{7} = \frac{2z - 6}{3}$$

$$\Rightarrow \frac{x + 2}{-1} = \frac{y + 3}{7} = \frac{z - 3}{\frac{3}{2}}$$

It parallel to the required line, so

$$a = \mu, b = 7\mu, c = \frac{3}{2}\mu$$

So, equation of required line using equation (i) is,

$$\frac{x - 1}{-\mu} = \frac{y - 2}{7\mu} = \frac{z - 3}{\frac{3}{2}\mu}$$

$$\Rightarrow \frac{x - 1}{-1} = \frac{y - 2}{7} = \frac{z - 3}{\frac{3}{2}}$$

### Straight Line in Space Ex 28.1 Q17

Given equation of line is,

$$3x + 1 = 6y - 2 = 1 - z$$

Dividing all by 6,

$$\frac{3x+1}{6} = \frac{6y-2}{6} = \frac{1-z}{6}$$

$$\Rightarrow \frac{3x}{6} + \frac{1}{6} = \frac{6y}{6} - \frac{2}{6} = \frac{1}{6} - \frac{z}{6}$$

$$\Rightarrow \frac{1}{2}x + \frac{1}{6} = y - \frac{1}{3} = -\frac{z}{6} + \frac{1}{6}$$

$$\Rightarrow \frac{1}{2}\left(x + \frac{1}{3}\right) = 1\left(y - \frac{1}{3}\right) = +\frac{1}{6}(z - 1)$$

$$\Rightarrow \frac{x + \frac{1}{3}}{2} = \frac{y - \frac{1}{3}}{1} = \frac{z - 1}{-6} = \lambda \text{ (say)} \quad \dots (i)$$

Comparing it with equation of line passing through  $(x_1, y_1, z_1)$  and direction ratios

$a, b, c$ ,

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\Rightarrow (x_1, y_1, z_1) = \left(-\frac{1}{3}, \frac{1}{3}, 1\right)$$

$$a = 2, b = 1, c = -6$$

So, direction ratios of the line are  $= 2, 1, -6$

From equation (i),

$$x = \left(2\lambda - \frac{1}{3}\right), y = \left(\lambda + \frac{1}{3}\right), z = (-6\lambda + 1)$$

So, vector equation of the given line is,

$$x\hat{i} + y\hat{j} + z\hat{k} = \left(2\lambda - \frac{1}{3}\right)\hat{i} + \left(\lambda + \frac{1}{3}\right)\hat{j} + (-6\lambda + 1)\hat{k}$$

$$\vec{r} = \left(-\frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k}\right) + \lambda(2\hat{i} + \hat{j} - 6\hat{k})$$

# Ex 28.2

## Straight Line in Space Ex 28.2 Q1

$$\text{Let } l_1 = \frac{12}{13}, m_1 = -\frac{3}{13}, n_1 = -\frac{4}{13}$$

$$l_2 = \frac{4}{13}, m_2 = \frac{12}{13}, n_2 = \frac{3}{13}$$

$$l_3 = \frac{3}{13}, m_3 = -\frac{4}{13}, n_3 = \frac{12}{13}$$

$$\begin{aligned} l_1 l_2 + m_1 m_2 + n_1 n_2 &= \frac{12}{13} \times \frac{4}{13} + \left(-\frac{3}{13}\right) \times \frac{12}{13} + \left(-\frac{4}{13}\right) \times \frac{3}{13} \\ &= \frac{48 - 36 - 12}{169} = 0 \end{aligned}$$

$$\begin{aligned} l_2 l_3 + m_2 m_3 + n_2 n_3 &= \frac{4}{13} \times \frac{3}{13} + \frac{12}{13} \times \left(-\frac{4}{13}\right) + \frac{3}{13} \times \frac{12}{13} \\ &= \frac{12 - 48 + 36}{169} = 0 \end{aligned}$$

$$\begin{aligned} l_1 l_3 + m_1 m_3 + n_1 n_3 &= \frac{12}{13} \times \frac{3}{13} + \left(-\frac{3}{13}\right) \times \left(-\frac{4}{13}\right) + \left(-\frac{4}{13}\right) \times \frac{12}{13} \\ &= \frac{36 + 12 - 48}{169} = 0 \end{aligned}$$

∴ The lines are mutually perpendicular.

## Straight Line in Space Ex 28.2 Q2

The direction ratios of a line passing through the points

(1, -1, 2) and (3, 4, -2) are

(3 - 1, 4 + 1, -2 - 2)

= (2, 5, -4)

The direction ratios of a line passing through the points

(0, 3, 2) and (3, 5, 6) are

(3 - 0, 5 - 3, 6 - 2)

= (3, 2, 4)

Angle between the lines

$$\cos \theta = \frac{(a_1 a_2 + b_1 b_2 + c_1 c_2)}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos \theta = \frac{[2 \times 3 + 5 \times 2 + (-4) \times 4]}{\sqrt{2^2 + 5^2 + (-4)^2} \sqrt{3^2 + 2^2 + 4^2}}$$

$$\cos \theta = \frac{0}{\sqrt{2^2 + 5^2 + (-4)^2} \sqrt{3^2 + 2^2 + 4^2}}$$

$$\cos \theta = 0$$

$$\theta = \frac{\pi}{2}$$

The lines are mutually perpendicular.

## Straight Line in Space Ex 28.2 Q3

The direction ratios of a line passing through the points

(4, 7, 8) and (2, 3, 4) are

(4 - 2, 7 - 3, 8 - 4)

= (2, 4, 4)

The direction ratios of a line passing through the points

(-1, -2, 1) and (1, 2, 5) are

(-1 - 1, -2 - 2, 1 - 5)

= (-2, -4, -4)

The direction ratios are proportional.

$$\frac{2}{-2} = \frac{4}{-4} = \frac{4}{-4}$$

Hence, the lines are mutually parallel.



**Straight Line in Space Ex 28.2 Q4**

The Cartesian equation of a line passing through  $(x_1, y_1, z_1)$

and with direction ratios  $(a_1, b_1, c_1)$

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

The Cartesian equation of a line passing through  $(-2, 4, -5)$

and parallel to the line  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$  is

$$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

**Straight Line in Space Ex 28.2 Q5**

Given equations of lines are  $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$  and  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ .

Clearly,

$$7 \times 1 + (-5) \times 2 + 1 \times 3$$

$$= 7 - 10 + 3$$

$$= 0$$

$\therefore$  Lines  $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$  and  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  are perpendicular to each other.

**Straight Line in Space Ex 28.2 Q6**

The direction ratios of a line joining the origin to the point  $(2, 1, 1)$

are  $(2 - 0, 1 - 0, 1 - 0) = (2, 1, 1)$

The direction ratios of a line joining  $(3, 5, -1)$  and  $(4, 3, -1)$

are  $(4 - 3, 3 - 5, -1 + 1) = (1, -2, 0)$

Angle between the lines

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos \theta = \frac{2 \times 1 + 1 \times (-2) + 1 \times 0}{\sqrt{2^2 + 1^2 + 1^2} \sqrt{1^2 + (-2)^2 + 0^2}}$$

$$\cos \theta = \frac{0}{\sqrt{6} \sqrt{5}}$$

$$\cos \theta = 0$$

$$\therefore \theta = \frac{\pi}{2}$$

The lines are mutually perpendicular.

**Straight Line in Space Ex 28.2 Q7**

Vector equation of a line is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

The direction cosines of the  $x$ -axis are  $(1, 0, 0)$ . Equation of a line parallel

to the  $x$ -axis and passing through the origin is

$$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(1\hat{i} + 0\hat{j} + 0\hat{k})$$

$$\vec{r} = \lambda\hat{i}$$



## Straight Line in Space Ex 28.2 Q8(i)

We know that, If  $\theta$  be the angle between two lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$ , then

$$\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| \cdot |\vec{b}_2|} \quad \text{--- (i)}$$

Here,  $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$

and,  $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 4\hat{k})$

$$\Rightarrow \quad \vec{a}_1 = 4\hat{i} - \hat{j}, \quad \vec{b}_1 = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \quad \vec{b}_2 = 2\hat{i} + 4\hat{j} - 4\hat{k}$$

$$|\vec{b}_1| = \sqrt{(1)^2 + (2)^2 + (-2)^2} = 3$$

$$|\vec{b}_2| = \sqrt{(2)^2 + (4)^2 + (-4)^2} = 6$$

Let  $\theta$  be the angle between given lines. So using equation (i),

$$\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| \cdot |\vec{b}_2|}$$

$$= \frac{(\hat{i} + 2\hat{j} - 2\hat{k}) \cdot (2\hat{i} + 4\hat{j} - 4\hat{k})}{3 \cdot 6}$$

$$= \frac{2 + 8 + 8}{18}$$

$$\cos \theta = 1$$

$$\theta = 0^\circ$$

## Straight Line in Space Ex 28.2 Q8(ii)

We know that, angle between two lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$ , is given by

$$\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| \cdot |\vec{b}_2|} \quad \text{--- (i)}$$

Given lines are,

$$\vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\vec{r} = (5\hat{j} - 2\hat{k}) + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

$$\Rightarrow \quad \vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}, \quad \vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$|\vec{b}_1| = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$$

$$|\vec{b}_2| = \sqrt{(3)^2 + (2)^2 + (6)^2} = 7$$

Let  $\theta$  be the angle between given lines, so using equation (i),

$$\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| \cdot |\vec{b}_2|}$$

$$= \frac{(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 6\hat{k})}{3 \cdot 7}$$

$$= \frac{3 + 4 + 12}{21}$$

$$= \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$



# Straight Line in Space Ex 28.2 Q8(iii)

We know that, angle between two lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$ , is given by

$$\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} \quad \text{--- (i)}$$

Equation of given lines are,

$$\vec{r} = \lambda (\hat{i} + \hat{j} + 2\hat{k}) \quad \text{and}$$

$$\vec{r} = 2\hat{j} + \mu [(\sqrt{3} - 1)\hat{i} - (\sqrt{3} + 1)\hat{j} + 4\hat{k}]$$

$$\Rightarrow \quad \vec{b}_1 = (\hat{i} + \hat{j} + 2\hat{k}), \quad \vec{b}_2 = (\sqrt{3} - 1)\hat{i} - (\sqrt{3} + 1)\hat{j} + 4\hat{k}$$

Let  $\theta$  be the angle between given lines, so using equation (i),

$$\begin{aligned} \cos \theta &= \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} \\ &= \frac{(\hat{i} + \hat{j} + 2\hat{k}) \cdot ((\sqrt{3} - 1)\hat{i} - (\sqrt{3} + 1)\hat{j} + 4\hat{k})}{\sqrt{(1)^2 + (1)^2 + (2)^2} \sqrt{(\sqrt{3} - 1)^2 + (-\sqrt{3} - 1)^2 + (4)^2}} \\ &= \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{\sqrt{6} \cdot \sqrt{3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} + 16}} \\ &= \frac{6}{\sqrt{6} \cdot 2\sqrt{6}} \\ \cos \theta &= \frac{1}{2} \end{aligned}$$

$$\theta = \frac{\pi}{3}$$

## Straight Line in Space Ex 28.2 Q9(i)

We know that, angle between two lines

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \quad \text{and} \quad \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

is given by,

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \quad \text{--- (i)}$$

Here, given lines are,

$$\frac{x + 4}{3} = \frac{y - 1}{5} = \frac{z + 3}{4} \quad \text{and} \quad \frac{x + 1}{1} = \frac{y - 4}{1} = \frac{z - 5}{2}$$

$$\Rightarrow \quad a_1 = 3, b_1 = 5, c_1 = 4, a_2 = 1, b_2 = 1, c_2 = 2$$

Let  $\theta$  be the angle between given lines, so using equation (i),

$$\begin{aligned} \cos \theta &= \frac{(3)(1) + (5)(1) + (4)(2)}{\sqrt{(3)^2 + (5)^2 + (4)^2} \sqrt{(1)^2 + (1)^2 + (2)^2}} \\ &= \frac{3 + 5 + 8}{\sqrt{50} \sqrt{6}} \\ &= \frac{16}{10\sqrt{3}} \\ \cos \theta &= \frac{8}{5\sqrt{3}} \end{aligned}$$

$$\theta = \cos^{-1} \left( \frac{8}{5\sqrt{3}} \right)$$

### Straight Line in Space Ex 28.2 Q9(ii)

We know that, angle between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \quad \text{and} \quad \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by,

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \quad \text{--- (i)}$$

Given, equation of lines are,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{-3} \quad \text{and} \quad \frac{x+3}{-1} = \frac{y-5}{8} = \frac{z-1}{4}$$

$$\Rightarrow a_1 = 2, b_1 = 3, c_1 = -3, a_2 = -1, b_2 = 8, c_2 = 4$$

Let  $\theta$  be the angle between two given lines, so using equation (i),

$$\begin{aligned} \cos \theta &= \frac{(2)(-1) + (3)(8) + (-3)(4)}{\sqrt{(2)^2 + (3)^2 + (-3)^2} \sqrt{(-1)^2 + (8)^2 + (4)^2}} \\ &= \frac{-2 + 24 - 12}{\sqrt{22} \sqrt{81}} \\ \cos \theta &= \frac{10}{9\sqrt{22}} \end{aligned}$$

$$\theta = \cos^{-1} \left( \frac{10}{9\sqrt{22}} \right)$$

### Straight Line in Space Ex 28.2 Q9(iii)

We know that, angle between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \quad \text{and} \quad \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by,

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \quad \text{--- (i)}$$

Given lines are,

$$\frac{5-x}{-2} = \frac{y+3}{1} = \frac{1-z}{3} \quad \text{and} \quad \frac{x}{3} = \frac{1-y}{-2} = \frac{z+5}{-1}$$

$$\Rightarrow \frac{x-5}{2} = \frac{y+3}{1} = \frac{z-1}{-3} \quad \text{and} \quad \frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-1}$$

$$\Rightarrow a_1 = 2, b_1 = 1, c_1 = -3, a_2 = 3, b_2 = 2, c_2 = -1$$

Let  $\theta$  be the angle between given lines, so using equation (i),

$$\begin{aligned} \cos \theta &= \frac{(2)(3) + (1)(2) + (-3)(-1)}{\sqrt{(2)^2 + (1)^2 + (-3)^2} \sqrt{(3)^2 + (2)^2 + (-1)^2}} \\ &= \frac{6 + 2 + 3}{\sqrt{14} \sqrt{14}} \\ \cos \theta &= \frac{11}{14} \end{aligned}$$

$$\theta = \cos^{-1} \left( \frac{11}{14} \right)$$



### Straight Line in Space Ex 28.2 Q9(iv)

We know that, angle between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \quad \text{and} \quad \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by,

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Equation of given lines are,

$$\frac{x-2}{3} = \frac{y+3}{-2}, z=5 \quad \text{and} \quad \frac{x+1}{1} = \frac{2y-3}{3} = \frac{z-5}{2}$$

$$\Rightarrow \frac{x-2}{3} = \frac{y+3}{-2}, z=5 \quad \text{and} \quad \frac{x+1}{1} = \frac{\frac{y-3}{2}}{\frac{3}{2}} = \frac{z-5}{2}$$

$$\Rightarrow a_1 = 3, b_1 = -2, c_1 = 0, a_2 = 1, b_2 = \frac{3}{2}, c_2 = 2$$

Let  $\theta$  be the angle between given lines, so from equation (i),

$$\begin{aligned} \cos \theta &= \frac{(3)(1) + (-2)\left(\frac{3}{2}\right) + (0)(2)}{\sqrt{(3)^2 + (-2)^2 + (0)^2} \sqrt{(1)^2 + \left(\frac{3}{2}\right)^2 + (2)^2}} \\ &= \frac{3-3+0}{\sqrt{38} \sqrt{\frac{29}{4}}} \\ \cos \theta &= 0 \end{aligned}$$

$$\theta = \frac{\pi}{2}$$

### Straight Line in Space Ex 28.2 Q9(v)

$$\frac{x-5}{1} = \frac{2y+6}{-2} = \frac{z-3}{1} \quad \text{and} \quad \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-6}{5}$$

$\hat{a} = \hat{i} - 2\hat{j} + \hat{k}, \hat{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$  are the vectors parallel to above lines

$$\therefore \text{angle between } \hat{a} \text{ and } \hat{b} \rightarrow \cos \theta = \frac{\hat{a} \cdot \hat{b}}{|\hat{a}| |\hat{b}|}$$

$$\cos \theta = \frac{(\hat{i} - 2\hat{j} + \hat{k}) \cdot (3\hat{i} + 4\hat{j} + 5\hat{k})}{|\hat{i} - 2\hat{j} + \hat{k}| |\hat{i} - 2\hat{j} + \hat{k}|} = \frac{3-8+5}{|\hat{i} - 2\hat{j} + \hat{k}| |\hat{i} - 2\hat{j} + \hat{k}|} = 0$$

$$\cos \theta = 0 \rightarrow \theta = 90^\circ$$

### Straight Line in Space Ex 28.2 Q9(vi)

$$\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3} \quad \text{and} \quad \frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4}$$

$\hat{a} = 2\hat{i} + 7\hat{j} - 3\hat{k}, \hat{b} = -1\hat{i} + 2\hat{j} + 4\hat{k}$  are the vectors parallel to above lines

$$\therefore \text{angle between } \hat{a} \text{ and } \hat{b} \rightarrow \cos \theta = \frac{\hat{a} \cdot \hat{b}}{|\hat{a}| |\hat{b}|}$$

$$\cos \theta = \frac{(2\hat{i} + 7\hat{j} - 3\hat{k}) \cdot (-1\hat{i} + 2\hat{j} + 4\hat{k})}{|2\hat{i} + 7\hat{j} - 3\hat{k}| |-1\hat{i} + 2\hat{j} + 4\hat{k}|} = 0$$

$$\cos \theta = 0 \rightarrow \theta = 90^\circ$$



### Straight Line in Space Ex 28.2 Q10(i)

We know that, angle  $(\theta)$  between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \quad \text{and} \quad \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by,

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \quad \text{--- (i)}$$

Here,  $a_1 = 5, b_1 = -12, c_1 = 13$

$a_2 = -3, b_2 = 4, c_2 = 5$

Let  $\theta$  be the required angle, so using equation (i),

$$\begin{aligned} \cos \theta &= \frac{(5)(-3) + (-12)(4) + (13)(5)}{\sqrt{(5)^2 + (-12)^2 + (13)^2} \sqrt{(-3)^2 + (4)^2 + (5)^2}} \\ &= \frac{-15 - 48 + 65}{\sqrt{169 \times 2} \sqrt{25 \times 2}} \\ &= \frac{2}{65 \times 2} \\ \cos \theta &= \frac{1}{65} \end{aligned}$$

$$\theta = \cos^{-1} \left( \frac{1}{65} \right)$$

### Straight Line in Space Ex 28.2 Q10(ii)

We know that, angle  $(\theta)$  between lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \quad \text{and} \quad \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by,

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \quad \text{--- (i)}$$

Here,  $a_1 = 2, b_1 = 2, c_1 = 1$

$a_2 = 4, b_2 = 1, c_2 = 8$

Let  $\theta$  be required angle, so using equation (i),

$$\begin{aligned} \cos \theta &= \frac{(2)(4) + (2)(1) + (1)(8)}{\sqrt{(2)^2 + (2)^2 + (1)^2} \sqrt{(4)^2 + (1)^2 + (8)^2}} \\ &= \frac{8+2+8}{3.9} \\ &= \frac{18}{27} \\ \cos \theta &= \frac{2}{3} \end{aligned}$$

$$\theta = \cos^{-1} \left( \frac{2}{3} \right)$$

### Straight Line in Space Ex 28.2 Q10(iii)

We know that, angle  $(\theta)$  between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \quad \text{and} \quad \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by,

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \quad \text{--- (i)}$$

Here,  $a_1 = 1, b_1 = 2, c_1 = -2$

$a_2 = -2, b_2 = 2, c_2 = 1$

Let  $\theta$  be the required angle, so using equation (i),

$$\begin{aligned} \cos \theta &= \frac{(1)(-2) + (2)(2) + (-2)(1)}{\sqrt{(1)^2 + (2)^2 + (-2)^2} \sqrt{(-2)^2 + (2)^2 + (1)^2}} \\ &= \frac{-2+4-2}{3.3} \\ &= \frac{0}{9} \\ \cos \theta &= 0 \end{aligned}$$

$$\theta = \frac{\pi}{2}$$

### Straight Line in Space Ex 28.2 Q10(vi)

$a, b, c$  and  $b-c, c-a, a-b$  are direction ratios

these are the vectors with above direction ratios

$$\hat{x} = a\hat{i} + b\hat{j} + c\hat{k}, \hat{y} = (b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}$$

are the vectors parallel to two given lines

$\therefore$  angle between the lines with above

$$\text{direction ratios are } \hat{x} \text{ and } \hat{y} \rightarrow \cos \theta = \frac{\hat{x} \cdot \hat{y}}{|\hat{x}| |\hat{y}|}$$

$$\cos \theta = \frac{(a\hat{i} + b\hat{j} + c\hat{k}) \cdot ((b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k})}{|(a\hat{i} + b\hat{j} + c\hat{k})| |(b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}|}$$

$$= \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2 + b^2 + c^2} \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}}$$

$$= \frac{ab - ac + bc - ba + ca - cb}{\sqrt{a^2 + b^2 + c^2} \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}} = 0$$

$$\cos \theta = 0 \rightarrow \theta = 90^\circ$$

### Straight Line in Space Ex 28.2 Q11

We know that, angle ( $\theta$ ) between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by,

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, Direction ratios of first line is 2, 2, 1

$$\Rightarrow a_1 = 2, b_1 = 2, c_1 = 1$$

Direction ratios of the line joining (3, 1, 4) and (7, 2, 12) is given by

$$= (7-3), (2-1), (12-4) \\ = 4, 1, 8$$

$$\Rightarrow a_2 = 4, b_2 = 1, c_2 = 8$$

Let  $\theta$  be the required angle, so using equation (i),

$$\cos \theta = \frac{(2)(4) + (2)(1) + (1)(8)}{\sqrt{(2)^2 + (2)^2 + (1)^2} \sqrt{(4)^2 + (1)^2 + (8)^2}} \\ = \frac{8+2+8}{3 \cdot 9}$$

$$\cos \theta = \frac{2}{3}$$

$$\theta = \cos^{-1} \left( \frac{2}{3} \right)$$

### Straight Line in Space Ex 28.2 Q12

We know that equation of a line passing through  $(x_1, y_1, z_1)$  and direction ratios  $a, b, c$  is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \quad \text{--- (i)}$$

Here,  $(x_1, y_1, z_1) = (1, 2, -4)$

and required line is parallel to the given line

$$\frac{x-3}{4} = \frac{y-5}{2} = \frac{z+1}{3}$$

$\Rightarrow$  Direction ratios of the required line are proportional to 4, 2, 3

$\Rightarrow a = 4\lambda, b = 2\lambda, c = 3\lambda$

So, required equation of the line is

$$\Rightarrow \frac{x-1}{4\lambda} = \frac{y-2}{2\lambda} = \frac{z+4}{3\lambda}$$

$$\Rightarrow \frac{x-1}{4} = \frac{y-2}{2} = \frac{z+4}{3}$$

### Straight Line in Space Ex 28.2 Q13

We know that, equation of a line passing through  $(x_1, y_1, z_1)$  and direction ratios are  $a, b, c$  is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \quad \text{--- (i)}$$

Here,  $(x_1, y_1, z_1) = (-1, 2, 1)$

and required line is parallel to the given line

$$\frac{2x-1}{4} = \frac{3y+5}{2} = \frac{z-2}{3}$$

$$\Rightarrow \frac{x-\frac{1}{2}}{2} = \frac{y+\frac{5}{3}}{\frac{2}{3}} = \frac{z-2}{-3}$$

$\Rightarrow$  Direction ratios of the required line are proportional to  $2, \frac{2}{3}, -3$

$\Rightarrow a = 2\lambda, b = \frac{2}{3}\lambda, c = -3\lambda$

So, required equation of the line using equation (i),

$$\frac{x+1}{2\lambda} = \frac{y-2}{\frac{2}{3}\lambda} = \frac{z-1}{-3\lambda}$$

$$\Rightarrow \frac{x+1}{2} = \frac{y-2}{\frac{2}{3}} = \frac{z-1}{-3}$$

### Straight Line in Space Ex 28.2 Q14

We know that equation of a line passing through the point  $\vec{a}$  and is the direction of vector  $\vec{b}$  is

$$\vec{r} = \vec{a} + \lambda \vec{b} \quad \text{--- (i)}$$

Here,  $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$

and given that the required line is parallel to

$$\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 5\hat{k})$$

$$\Rightarrow \vec{b} = (2\hat{i} + 3\hat{j} - 5\hat{k}), \mu$$

So, required equation of the line using equation (i) is

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 5\hat{k}), \mu$$

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda'(2\hat{i} + 3\hat{j} - 5\hat{k})$$

where  $\lambda'$  is a scalar such that  $\lambda' = \lambda\mu$





### Straight Line in Space Ex 28.2 Q15

We know that, equation of a line passing through  $(x_1, y_1, z_1)$  with direction ratios  $a, b, c$  is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

So, equation of required line passing through  $(2, 1, 3)$  is

$$\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c} \quad \text{--- (1)}$$

Given that line  $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$  is perpendicular to line (1), so

$$\begin{aligned} a_1 a_2 + b_1 b_2 + c_1 c_2 &= 0 \\ (a)(1) + (b)(2) + (c)(3) &= 0 \\ a + 2b + 3c &= 0 \quad \text{--- (2)} \end{aligned}$$

And line  $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$  is perpendicular to line (1), so

$$\begin{aligned} a_1 a_2 + b_1 b_2 + c_1 c_2 &= 0 \\ (a)(-3) + (b)(2) + (c)(5) &= 0 \\ -3a + 2b + 5c &= 0 \quad \text{--- (3)} \end{aligned}$$

Solving equation (2) and (3) by cross multiplication,

$$\begin{aligned} \frac{a}{(2)(5) - (2)(3)} &= \frac{b}{(-3)(3) - (1)(5)} = \frac{c}{(1)(2) - (-3)(2)} \\ \Rightarrow \frac{a}{10-6} &= \frac{b}{-9-5} = \frac{c}{2+6} \\ \Rightarrow \frac{a}{4} &= \frac{b}{-14} = \frac{c}{8} \\ \Rightarrow \frac{a}{2} &= \frac{b}{-7} = \frac{c}{4} = \lambda \text{ (Say)} \\ \Rightarrow a &= 2\lambda, b = -7\lambda, c = 4\lambda \end{aligned}$$

Using  $a, b, c$  in equation (1),

$$\begin{aligned} \frac{x-2}{2\lambda} &= \frac{y-1}{-7\lambda} = \frac{z-3}{4\lambda} \\ \Rightarrow \frac{x-2}{2} &= \frac{y-1}{-7} = \frac{z-3}{4} \end{aligned}$$

### Straight Line in Space Ex 28.2 Q16

We know that equation of a line passing through a point with position vector  $\vec{a}$  and perpendicular to  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$  is given by

$$\vec{r} = \vec{a} + \lambda (\vec{b}_1 \times \vec{b}_2) \quad \text{--- (i)}$$

Here,  $\vec{a} = (\hat{i} + \hat{j} - 3\hat{k})$

and required line is perpendicular to

$$\begin{aligned} \vec{r} &= \hat{i} + \lambda (2\hat{i} + \hat{j} - 3\hat{k}) \text{ and} \\ \vec{r} &= (2\hat{i} + \hat{j} - \hat{k}) + \mu (\hat{i} + \hat{j} + \hat{k}) \end{aligned}$$

$$\Rightarrow \vec{b}_1 = (2\hat{i} + \hat{j} - 3\hat{k}), \vec{b}_2 = \hat{i} + \hat{j} + \hat{k}$$

Now,

$$\begin{aligned} \vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} \\ &= \hat{i}(1+3) - \hat{j}(2+3) + \hat{k}(2-1) \\ \vec{b}_1 \times \vec{b}_2 &= 4\hat{i} - 5\hat{j} + \hat{k} \end{aligned}$$

Using equation, required equation of line is

$$\begin{aligned} \vec{r} &= \vec{a} + \lambda (\vec{b}_1 \times \vec{b}_2) \\ \vec{r} &= (\hat{i} + \hat{j} - 3\hat{k}) + \lambda (4\hat{i} - 5\hat{j} + \hat{k}) \end{aligned}$$

### Straight Line in Space Ex 28.2 Q17

We know that equation of a line passing through  $(x_1, y_1, z_1)$  and direction ratios as  $a, b, c$  is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \quad \text{---- (1)}$$

So, equation of a line passing through  $(1, -1, 1)$  is

$$\frac{x - 1}{a} = \frac{y + 1}{b} = \frac{z - 1}{c} \quad \text{--- (2)}$$

Now, Directions ratios of the line joining  $A(4, 3, 2)$  and  $B(1, -1, 0)$   
 $= (1 - 4), (-1 - 3), (0 - 2)$

$\Rightarrow$  Direction ratios of line  $AB = -3, -4, -2$

and, Directions ratios of the line joining  $C(1, 2, -1)$  and  $D(2, 1, 1)$   
 $= (2 - 1), (1 - 2), (1 + 1)$

$\Rightarrow$  Direction ratios of line  $CD = 1, -1, 2$

Given that, line  $AB$  is perpendicular to line (2), so

$$\begin{aligned} a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ (a)(-3) + (b)(-4) + (c)(-2) &= 0 \\ -3a + 4b - 2c &= 0 \\ 3a + 4b + 2c &= 0 \end{aligned} \quad \text{--- (3)}$$

and, line  $CD$  is also perpendicular to line (2), so

$$\begin{aligned} a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ (a)(1) + (b)(-1) + (c)(2) &= 0 \\ a - b + 2c &= 0 \end{aligned} \quad \text{--- (4)}$$

Solving equation (3) and (4) using cross multiplication,

$$\frac{a}{(4)(2) - (-1)(2)} = \frac{b}{(1)(2) - (3)(2)} = \frac{c}{(3)(-1) - (4)(1)}$$

$$\Rightarrow \frac{a}{8+2} = \frac{b}{2-6} = \frac{c}{-3-4}$$

$$\Rightarrow \frac{a}{10} = \frac{b}{-4} = \frac{c}{-7} = \lambda \text{ (Say)}$$

$$\Rightarrow a = 10\lambda, b = -4\lambda, c = -7\lambda$$

### Straight Line in Space Ex 28.2 Q18

We know that equation of a line passing through a point  $(x_1, y_1, z_1)$  and direction ratios  $a, b, c$  is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

So, equation of required line passing through  $(1, 2, -4)$  is

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \quad \text{--- (1)}$$

Given that, line  $\frac{x-8}{8} = \frac{y+9}{-16} = \frac{z-10}{7}$  is perpendicular to line (1), so  
 $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\begin{aligned} \Rightarrow & (a)(8) + (b)(-16) + (c)(7) = 0 \\ \Rightarrow & 8a - 16b + 7c = 0 \quad \text{---- (2)} \end{aligned}$$

also, line  $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$  is perpendicular to line (1), so  
 $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\begin{aligned} \Rightarrow & (3)(a) + (8)(b) + (-5)(c) = 0 \\ \Rightarrow & 3a + 8b - 5c = 0 \quad \text{---- (3)} \end{aligned}$$

Solving equation (2) and (3) by cross-multiplication,

$$\begin{aligned} \frac{a}{(8)(-5) - (7)(-16)} &= \frac{b}{(3)(7) - (8)(-5)} = \frac{c}{(8)(8) - (3)(-16)} \\ \Rightarrow & \frac{a}{80 - 56} = \frac{b}{21 + 40} = \frac{c}{64 + 48} \\ \Rightarrow & \frac{a}{24} = \frac{b}{61} = \frac{c}{112} = \lambda \text{ (Say)} \\ \Rightarrow & a = 24\lambda, b = 61\lambda, c = 112\lambda \end{aligned}$$

Put  $a, b, c$  in equation (1) to get required equation of the line, so

$$\begin{aligned} \frac{x-1}{24\lambda} &= \frac{y-2}{61\lambda} = \frac{z+4}{112\lambda} \\ \Rightarrow & \frac{x-1}{24} = \frac{y-2}{61} = \frac{z+4}{112} \end{aligned}$$

### Straight Line in Space Ex 28.2 Q19

Equation of lines are,

$$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$$

$$\text{and, } \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

$$\begin{aligned} \text{Now, } a_1a_2 + b_1b_2 + c_1c_2 &= (7)(1) + (-5)(2) + (1)(3) \\ &= 7 - 10 + 3 \\ &= 0 \end{aligned}$$

So, given lines are perpendicular.



### Straight Line in Space Ex 28.2 Q20

We know that, equation of a line passing through the point  $(x_1, y_1, z_1)$  and direction ratios  $a, b, c$  is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \quad \dots (1)$$

So, equation of line passing through  $(2, -1, -1)$  is

$$\frac{x - 2}{a} = \frac{y + 1}{b} = \frac{z + 1}{c} \quad \dots (2)$$

Line (2) is parallel to given line,

$$6x - 2 = 3y + 1 = 2z - 2$$

$$\Rightarrow \frac{6x - 2}{6} = \frac{3y + 1}{6} = \frac{2z - 2}{6}$$

$$\Rightarrow \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{2}}{2} = \frac{z - \frac{1}{3}}{3}$$

So,  $a = \lambda$ ,  $b = 2\lambda$ ,  $c = 3\lambda$

Using  $a, b, c$  in equation (2) to get required equation of line,

$$\frac{x - 2}{\lambda} = \frac{y + 1}{2\lambda} = \frac{z + 1}{3\lambda}$$

$$\Rightarrow \frac{x - 2}{1} = \frac{y + 1}{2} = \frac{z + 1}{3} = \lambda \text{ (Say)}$$

$$\Rightarrow x = \lambda + 2, y = 2\lambda - 1, z = 3\lambda - 1$$

So,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 2)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 1)\hat{k}$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

### Straight Line in Space Ex 28.2 Q21

The direction of ratios of the lines,  $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$  and  $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ , are  $-3, 2k, 2$  and  $3k, 1, -5$  respectively.

It is known that two lines with direction ratios,  $a_1, b_1, c_1$  and  $a_2, b_2, c_2$ , are perpendicular, if  $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore -3(3k) + 2k \times 1 + 2(-5) = 0$$

$$\Rightarrow -9k + 2k - 10 = 0$$

$$\Rightarrow 7k = -10$$

$$\Rightarrow k = \frac{-10}{7}$$

Therefore, for  $k = -\frac{10}{7}$ , the given lines are perpendicular to each other.

### Straight Line in Space Ex 28.2 Q22

The coordinates of A, B, C, and D are  $(1, 2, 3)$ ,  $(4, 5, 7)$ ,  $(-4, 3, -6)$ , and  $(2, 9, 2)$  respectively.

The direction ratios of AB are  $(4 - 1) = 3$ ,  $(5 - 2) = 3$ , and  $(7 - 3) = 4$

The direction ratios of CD are  $(2 - (-4)) = 6$ ,  $(9 - 3) = 6$ , and  $(2 - (-6)) = 8$

$$\text{It can be seen that, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

Therefore, AB is parallel to CD.

Thus, the angle between AB and CD is either  $0^\circ$  or  $180^\circ$ .



### Straight Line in Space Ex 28.2 Q23

Given equation of line are,

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1} \text{ and}$$

$$\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$$

$$\Rightarrow \frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1} \quad \text{--- (1)}$$

$$\text{and, } \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3} \quad \text{--- (2)}$$

Given that line (1) and (2) are perpendicular,

$$\text{So, } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$(5\lambda+2)(1) + (-5)(2\lambda) + (1)(3) = 0$$

$$5\lambda+2-10\lambda+3=0$$

$$-5\lambda+5=0$$

$$\lambda = \frac{5}{5}$$

$$\lambda = 1$$

### Straight Line in Space Ex 28.2 Q24

The direction ratios of the line are

$$\frac{x+2}{2} = \frac{2y-7}{6} = \frac{5-z}{6}$$

$$2, 6, 6$$

The direction cosines of the line are

$$l = \frac{2}{\sqrt{2^2+6^2+6^2}} = \frac{2}{\sqrt{76}}$$

$$m = \frac{6}{\sqrt{2^2+6^2+6^2}} = \frac{6}{\sqrt{76}}$$

$$n = \frac{6}{\sqrt{2^2+6^2+6^2}} = \frac{6}{\sqrt{76}}$$

$$\left(\frac{2}{\sqrt{76}}, \frac{6}{\sqrt{76}}, \frac{6}{\sqrt{76}}\right)$$

∴ Vector equation of the line is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\vec{r} = (-\vec{i} + 2\vec{j} + 3\vec{k}) + \lambda(2\vec{i} + 6\vec{j} + 6\vec{k})$$

# Ex 28.3

## Straight Line in Space Ex 28.3 Q1

We have equation of first line,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+3}{4} = \lambda \quad (\text{Say}) \quad \text{--- (1)}$$

General point on line (1) is

$$(\lambda, 2\lambda + 2, 3\lambda - 3)$$

Another line is,

$$\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-3}{5} = \mu \quad (\text{Say}) \quad \text{--- (2)}$$

General point on line (2) is

$$(2\mu + 2, 3\mu + 6, 4\mu + 3)$$

If lines (1) and (2) intersect then they have a common point, so for same value of  $\lambda$

and  $\mu$ , we must have,

$$\lambda = 2\mu + 2 \Rightarrow \lambda - 2\mu = 2 \quad \text{--- (3)}$$

$$2\lambda + 2 = 3\mu + 6 \Rightarrow 2\lambda - 4\mu = 4 \quad \text{--- (4)}$$

$$3\lambda - 3 = 4\mu + 3 \Rightarrow 3\lambda - 4\mu = 6 \quad \text{--- (5)}$$

Now, solving equation (3) and (4) to get  $\lambda$  and  $\mu$ ,

$$2\lambda - 4\mu = 4$$

$$\begin{array}{r} 2\lambda - 4\mu = 4 \\ (-) \quad 2\lambda - 3\mu = 4 \\ \hline -\mu = 0 \end{array}$$

$$\Rightarrow \mu = 0$$

Put  $\mu = 0$  in equation (3),

$$\lambda - 2\mu = 2$$

$$\lambda - 2(0) = 2$$

$$\lambda = 2$$

Put  $\lambda$  and  $\mu$  in equation (5),

$$3\lambda - 4\mu = 6$$

$$3(2) - 4(0) = 6$$

$$6 = 6$$

$$\text{LHS} = \text{RHS}$$

### Straight Line in Space Ex 28.3 Q2

We have equation of first line,

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda \quad (\text{Say}) \quad \text{--- (1)}$$

General point on line (1) is

$$(3\lambda + 1, 2\lambda - 1, 5\lambda + 1)$$

Another line is,

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu \quad (\text{Say}) \quad \text{--- (2)}$$

General point on line (2) is,

$$(4\mu - 2, 3\mu + 1, -2\mu - 1)$$

If lines (1) and (2) intersect, then they have a common point, so for same value of  $\lambda$  and  $\mu$ , we must have,

$$3\lambda + 1 = 4\mu - 2 \Rightarrow 3\lambda - 4\mu = -3 \quad \text{--- (3)}$$

$$2\lambda - 1 = 3\mu + 1 \Rightarrow 2\lambda - 3\mu = 2 \quad \text{--- (4)}$$

$$5\lambda + 1 = -2\mu - 1 \Rightarrow 5\lambda + 2\mu = -2 \quad \text{--- (5)}$$

Solving equation (3) and (4) to get  $\lambda$  and  $\mu$ ,

$$6\lambda - 8\mu = -6$$

$$\begin{array}{r} 6\lambda - 9\mu = 6 \\ (-) \quad (+) \quad (-) \\ \hline \end{array}$$

$$\mu = -12$$

Put the value of  $\mu$  in equation (3),

$$3\lambda - 4(-12) = -3$$

$$3\lambda + 48 = -3$$

$$3\lambda = -3 - 48$$

$$3\lambda = -51$$

$$\lambda = \frac{-51}{3}$$

$$\lambda = -17$$

Put the value of  $\lambda$  and  $\mu$  in equation (5),

$$5\lambda + 2\mu = -2$$

$$5(-17) + 2(-12) = -2$$

$$-85 - 24 = -2$$

$$-109 \neq -2$$

$$\text{LHS} \neq \text{RHS}$$

### Straight Line in Space Ex 28.3 Q3

Given equation of first line is

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \quad (\text{Say}) \quad \text{--- (1)}$$

General point on line (1) is

$$(3\lambda - 1, 5\lambda - 3, 7\lambda - 5)$$

Another equation of line is

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \quad (\text{Say}) \quad \text{--- (2)}$$

General point on line (2) is,

$$(\mu + 2, 3\mu + 4, 5\mu + 6)$$

If lines (1) and (2) are intersecting then, they have a common point. So for same value of  $\lambda$  and  $\mu$ , we must have,

$$3\lambda - 1 = \mu + 2 \Rightarrow 3\lambda - \mu = 3 \quad \text{--- (3)}$$

$$5\lambda - 3 = 3\mu + 4 \Rightarrow 5\lambda - 3\mu = 7 \quad \text{--- (4)}$$

$$7\lambda - 5 = 5\mu + 6 \Rightarrow 7\lambda - 5\mu = 11 \quad \text{--- (5)}$$

Solving equation (3) and (4) to get  $\lambda$  and  $\mu$ ,

$$15\lambda - 5\mu = 15$$

$$\begin{array}{r} 15\lambda - 9\mu = 21 \\ (-) \quad (+) \quad (-) \end{array}$$

$$4\mu = -6$$

$$\mu = \frac{-3}{2}$$

Put the value of  $\mu$  in equation (3),

$$3\lambda - \mu = 3$$

$$3\lambda - \left(-\frac{3}{2}\right) = 3$$

$$3\lambda = 3 - \frac{3}{2}$$

$$\lambda = \frac{1}{2}$$

Put the value of  $\lambda$  and  $\mu$  in equation (5),

$$7\lambda - 5\mu = 11$$

$$7\left(\frac{1}{2}\right) - 5\left(-\frac{3}{2}\right) = 11$$

$$\frac{7}{2} + \frac{15}{2} = 11$$

$$\frac{22}{2} = 11$$

$$11 = 11$$

$$\text{LHS} \neq \text{RHS}$$

Since, the values of  $\lambda$  and  $\mu$  obtained by solving (3) and (4) satisfy equation (5), Hence

Given lines intersect each other.

Point of intersection =  $(3\lambda - 1, 5\lambda - 3, 7\lambda - 5)$

$$= \left\{ \frac{3}{2} - 1, \left( \frac{5}{2} - 3 \right), \left( \frac{7}{2} - 5 \right) \right\}$$

$$= \left( \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2} \right)$$

Point of intersection is  $\left( \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2} \right)$ .



Equation of the line passing through  $A(0, -1, -1)$  and  $B(4, 5, 1)$  is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x - 0}{4 - 0} = \frac{y + 1}{5 + 1} = \frac{z + 1}{1 + 1}$$

$$\frac{x}{4} = \frac{y + 1}{6} = \frac{z + 1}{2} = \lambda \text{ (say)}$$

So, general point on line  $AB$  is

$$(4\lambda, 4\lambda, 2\lambda - 1)$$

Now, equation of the line passing through  $C(3, 9, 4)$  and  $D(-4, 4, 4)$  is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x - 3}{-4 - 3} = \frac{y - 9}{4 - 9} = \frac{z - 4}{4 - 4}$$

$$\frac{x - 3}{-7} = \frac{y - 9}{-5} = \frac{z - 4}{0} = \mu \text{ (say)}$$

So, general point on line  $CD$  is

$$(-7\mu + 3, -5\mu + 9, 0\mu + 4)$$

$$(-7\mu + 3, -5\mu + 9, 4)$$

If lines  $AB$  and  $CD$  intersect, there must be a common point to them. So we have to find

$\lambda$  and  $\mu$  such that

$$4\lambda = -7\mu + 3 \quad \Rightarrow 4\lambda + 7\mu = 3 \quad \text{--- (1)}$$

$$6\lambda - 1 = -5\mu + 9 \quad \Rightarrow 6\lambda + 5\mu = 10 \quad \text{--- (2)}$$

$$2\lambda - 1 = 4 \quad \Rightarrow 2\lambda - 1 = 4 \quad \text{--- (3)}$$

From equation (3),

$$2\lambda = 4 + 1$$

$$\lambda = \frac{5}{2}$$

Put  $\lambda = \frac{5}{2}$  in equation (2),

$$6\left(\frac{5}{2}\right) + 5\mu = 10$$

$$5\mu = 10 - 15$$

$$5\mu = -5$$

$$\mu = -1$$

Now, put values of  $\lambda$  and  $\mu$  in equation (1),

$$4\lambda + 7(\mu) = 3$$

$$4\left(\frac{5}{2}\right) + 7(-1) = 3$$

$$10 - 7 = 3$$

$$3 = 3$$

$$\text{LHS} \neq \text{RHS}$$

Since, the values of  $\lambda$  and  $\mu$  by solving (2) and (3) satisfy equation (1), so

Line  $AB$  and  $CD$  are intersecting lines

Point of intersection of  $AB$  and  $CD$

$$= (-7\mu + 3, -5\mu + 9, 4)$$

$$= (-7(-1) + 3, -5(-1) + 9, 4)$$

$$= (7 + 3, 5 + 9, 4)$$

$$= (10, 14, 4)$$

So, point of intersection of  $AB$  and  $CD$  =  $(10, 14, 4)$ .

Given equations of lines are

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$$

$$\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

If these lines intersect, they must have a common point, so, for some value of  $\lambda$  and  $\mu$  we must have,

$$(\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j}) = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

$$(1 + 3\lambda)\hat{i} + (1 - \lambda)\hat{j} - \hat{k} = (4 + 2\mu)\hat{i} + (-1 + 3\mu)\hat{k}$$

Equation the coefficients of  $\hat{i}, \hat{j}, \hat{k}$ , we get

$$1 + 3\lambda = 4 + 2\mu \Rightarrow 3\lambda - 2\mu = 3 \quad \text{---(1)}$$

$$1 - \lambda = 0 \Rightarrow \lambda = 1 \quad \text{---(2)}$$

$$-1 = -1 + 3\mu \Rightarrow \mu = 0 \quad \text{---(3)}$$

Put the value of  $\lambda$  and  $\mu$  in equation (1),

$$3\lambda - 2\mu = 3$$

$$3(1) - 2(0) = 3$$

$$3 = 3$$

$$\text{LHS} = \text{RHS}$$

The value of  $\lambda$  and  $\mu$  satisfy equation (1), so

Lines are intersecting.

Put value of  $\lambda$  in equation (1) to get point of intersection

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + (1)(3\hat{i} - \hat{j})$$

$$= \hat{i} + \hat{j} - \hat{k} + 3\hat{i} - \hat{j}$$

$$= 4\hat{i} - \hat{k}$$

So, point of intersection is  $(4, 0, -1)$ .

#### Straight Line in Space Ex 28.3 Q6(i)

Given equations of lines are

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k}) \quad \text{and}$$

$$\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{i} - \hat{k})$$

If these lines intersect each other, there must be some common point, so, we must have  $\lambda$  and  $\mu$  such that

$$(\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k}) = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{i} - \hat{k})$$

$$(1 + 2\lambda)\hat{i} - \hat{j} + \lambda\hat{k} = (2 + \mu)\hat{i} + (-1 + \mu)\hat{j} - \mu\hat{k}$$

Equation the coefficients of  $\hat{i}, \hat{j}$  and  $\hat{k}$ ,

$$1 + 2\lambda = 2 + \mu \Rightarrow 2\lambda - \mu = 1 \quad \text{---(1)}$$

$$-1 = -1 + \mu \Rightarrow \mu = 0 \quad \text{---(2)}$$

$$\lambda = -\mu \Rightarrow \lambda = 0 \quad \text{---(3)}$$

Put value of  $\lambda$  and  $\mu$  in equation (1),

$$2\lambda - \mu = 1$$

$$2(0) - (0) = 1$$

$$0 = 1$$

$$\text{LHS} \neq \text{RHS}$$

Since, the values of  $\lambda$  and  $\mu$  form equation (2) and (3) does not satisfy equation (1),

Hence, given lines do not intersect each other.

Given, equations of first line is

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \text{ (say)} \quad \text{--- (1)}$$

General point on line (1) is

$$(2\lambda + 1, 3\lambda - 1, \lambda)$$

Another equation of line is

$$\frac{x-1}{5} = \frac{y-2}{1}, z = 3 \quad \text{--- (2)}$$

$$\frac{x-1}{5} = \frac{y-2}{1} = \mu, \text{ (say)}, z = 3$$

General point on line (2) is

$$(5\mu + 1, \mu + 2, 3)$$

If line (1) and (2) intersect each other then, there is a common point to them, so, we must have value of  $\lambda$  and  $\mu$  such that

$$2\lambda + 1 = 5\mu + 1 \Rightarrow 2\lambda - 5\mu = 0 \quad \text{--- (3)}$$

$$3\lambda - 1 = \mu + 2 \Rightarrow 3\lambda - \mu = 3 \quad \text{--- (4)}$$

$$\lambda = 3 \Rightarrow \lambda = 3 \quad \text{--- (5)}$$

Put value of  $\lambda$  in equation (4),

$$3\lambda - \mu = 3$$

$$3(3) - \mu = 3$$

$$- \mu = 3 - 9$$

$$\mu = 6$$

Put the value of  $\lambda$  and  $\mu$  in equation (3), so

$$2\lambda - 5\mu = 0$$

$$2(3) - 5(6) = 0$$

$$6 - 30 = 0$$

$$-24 \neq 0$$

$$\text{LHS} \neq \text{RHS}$$

Since the values of  $\lambda$  and  $\mu$  obtained from equation (4) and (5) does not satisfy equation (3), so,

Given lines are not intersecting.

### Line in Space Ex 28.3 Q6(iii)

Equations of first line is,

$$\frac{-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda \quad (\text{say}) \quad \text{--- (1)}$$

General point on line (1) is,

$$(3\lambda + 1, -\lambda + 1, -1)$$

Another equation of line is

$$\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3} = \mu \quad (\text{say}) \quad \text{--- (2)}$$

General point on line (2) is,

$$(2\mu + 4, 0, 3\mu - 1)$$

If line (1) and (2) intersecting then there must be a common point, so, we must have the value of  $\lambda$  and  $\mu$  as

$$3\lambda + 1 = 2\mu + 4 \quad \Rightarrow 3\lambda - 2\mu = 3 \quad \text{--- (1)}$$

$$-\lambda + 1 = 0 \quad \Rightarrow \lambda = 1 \quad \text{--- (2)}$$

$$3\mu - 1 = -1 \quad \Rightarrow \mu = 0 \quad \text{--- (3)}$$

Put the value of  $\lambda$  and  $\mu$  in equation (1), so

$$3\lambda - 2\mu = 3$$

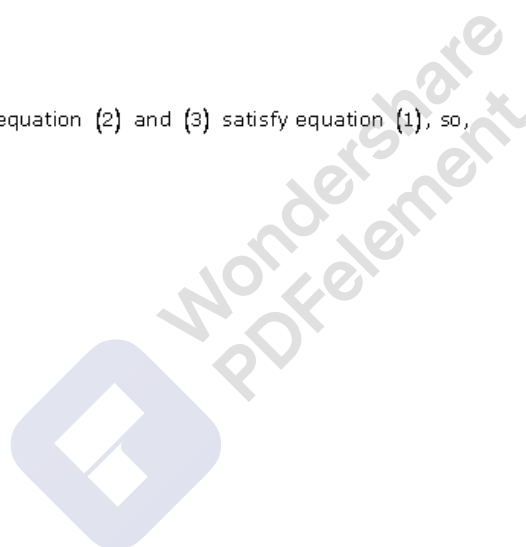
$$3(1) - 2(0) = 3$$

$$3 = 3$$

$$\text{LHS} \neq \text{RHS}$$

Since the values of  $\lambda$  and  $\mu$  obtained by equation (2) and (3) satisfy equation (1), so,

Given lines are intersecting.



### Straight Line in Space Ex 28.3 Q6(iv)

Given, equation of line is

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} = \lambda \quad \text{--- (1)}$$

General point on line (1) is,

$$(4\lambda + 5, 4\lambda + 7, -5\lambda - 3)$$

Another equation of line is,

$$\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3} = \mu \quad \text{--- (2)}$$

General point on line (2) is

$$(7\mu + 8, \mu + 4, 3\mu + 5)$$

If line (1) and (2) intersecting, then there must have some common point to them, so, we must have value of  $\lambda$  and  $\mu$  such that

$$4\lambda + 5 = 7\mu + 8 \quad \Rightarrow \quad 4\lambda - 7\mu = 3 \quad \text{--- (3)}$$

$$4\lambda + 5 = \mu + 4 \quad \Rightarrow \quad 4\lambda - \mu = -3 \quad \text{--- (4)}$$

$$-5\lambda - 3 = 3\mu + 5 \quad \Rightarrow \quad -5\lambda - 3\mu = 8 \quad \text{--- (5)}$$

Solving equation (3) and (4) to find  $\lambda$  and  $\mu$ ,

$$\begin{array}{r} 4\lambda - 7\mu = 3 \\ 4\lambda - \mu = -3 \\ \hline (-) \quad (+) \quad (-) \\ \hline -6\mu = 6 \\ \mu = -1 \end{array}$$

Put value of  $\lambda$  in equation (3),

$$\begin{array}{l} 4\lambda - 7\mu = 3 \\ 4\lambda - 7(-1) = 3 \\ 4\lambda = 3 - 7 \\ \lambda = -1 \end{array}$$

Put the value of  $\lambda$  and  $\mu$  in equation (5),

$$\begin{array}{l} -5\lambda - 3\mu = 8 \\ -5(-1) - 3(-1) = 8 \\ 5 + 3 = 8 \\ \text{LHS} = \text{RHS} \end{array}$$

### Straight Line in Space Ex 28.3 Q7

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

If the lines intersect each other, then the shortest distance between the lines should be zero.

Here,

$$\vec{a}_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}$$

$$\vec{a}_2 = 5\hat{i} - 2\hat{j}$$

$$\vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$(\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix}$$

$$= \hat{i}(12 - 4) - \hat{j}(6 - 6) + \hat{k}(2 - 6)$$

$$= 8\hat{i} - 0\hat{j} - 4\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) = (5\hat{i} - 2\hat{j} - 3\hat{i} - 2\hat{j} + 4\hat{k}) = (2\hat{i} - 4\hat{j} + 4\hat{k})$$

$$\text{Shortest Distance, } d = \frac{|(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \frac{|(8\hat{i} - 0\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} + 4\hat{k})|}{|8\hat{i} - 0\hat{j} - 4\hat{k}|}$$

$$= \frac{|8 \times 2 - 0 \times 4 + (-4) \times 4|}{|8\hat{i} - 0\hat{j} - 4\hat{k}|}$$

$$= \frac{0}{|8\hat{i} - 0\hat{j} - 4\hat{k}|} = 0$$

Since the shortest distance is zero, the lines are intersect each other.

Point of intersection of the lines,

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

Lines in the Cartesian form,

$$\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} = \lambda$$

$$x = \lambda + 3, y = 2\lambda + 2, z = 2\lambda - 4$$

$$\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6} = \mu$$

$$x = 3\mu + 5, y = 2\mu - 2, z = 6\mu$$

From coordinates of x,

$$\lambda + 3 = 3\mu + 5$$

$$\lambda = 3\mu + 2 \dots (i)$$

From coordinates of y,

$$2\lambda + 2 = 2\mu - 2$$

$$\lambda = \mu - 2 \dots (ii)$$

Solving (i) and (ii),

$$\lambda = -4, \mu = -2$$

Coordinates of the point of intersection,

$$x = 3(-2) + 5, y = 2(-2) - 2, z = 6(-2)$$

$$x = -1, y = -6, z = -12$$

$$(-1, -6, -12)$$



# Ex 28.4

## Straight Line in Space Ex 28.4 Q1

Let the foot of the perpendicular drawn from  $P(3, -1, 11)$  to the line

$\frac{x}{2} = \frac{y-2}{-3} = \frac{z-3}{4}$  is  $Q$ , so we have to find length of  $PQ$ .  $Q$  is general point on the line

$$\frac{x}{2} = \frac{y-2}{-3} = \frac{z-3}{4} = \lambda \quad (\text{say})$$

Co-ordinate of  $Q = (2\lambda, -3\lambda + 2, 4\lambda + 3)$

Direction ratios of the given line =  $2, -3, 4$

Since  $PQ$  is perpendicular to the given line therefore

$$\begin{aligned} a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ \Rightarrow 2(2\lambda - 3) + (-3)(-3\lambda + 2) + 4(4\lambda - 8) &= 0 \\ \Rightarrow 4\lambda - 6 + 9\lambda - 9 + 16\lambda - 32 &= 0 \\ \Rightarrow 29\lambda - 47 &= 0 \\ \Rightarrow \lambda &= \frac{47}{29} \end{aligned}$$

Therefore co-ordinates of  $Q$

$$\begin{aligned} &= 2\left(\frac{47}{29}\right), -3\left(\frac{47}{29}\right) + 2, 4\left(\frac{47}{29}\right) + 3 \\ &= \frac{94}{29}, -\frac{83}{29}, \frac{275}{29} \end{aligned}$$

Distance between  $P$  and  $Q$  is

## Straight Line in Space Ex 28.4 Q2

Let foot of the perpendicular drawn from the point  $P(1, 0, 0)$  to the line  $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$  is

$Q$ . We have to find length of  $PQ$ .

$Q$  is a general point on the line,

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda \quad (\text{say})$$

Coordinate of  $Q = (2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

Direction ratios line  $PQ$  are

$$= (2\lambda + 1 - 1), (-3\lambda - 1 - 0), (8\lambda - 10 - 0)$$

$$\Rightarrow = (2\lambda), (-3\lambda - 1), (8\lambda - 10)$$

Since, line  $PQ$  is perpendicular to the given line, so

$$\begin{aligned} a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ (2)(2\lambda) + (-3)(-3\lambda - 1) + 8(8\lambda - 10) &= 0 \\ 4\lambda + 9\lambda + 3 + 64\lambda - 80 &= 0 \\ 77\lambda - 77 &= 0 \\ \lambda &= 1 \end{aligned}$$

Therefore, coordinate of  $Q$  is  $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

$$\begin{aligned} &= (2(1) + 1, -3(1) - 1, 8(1) - 10) \\ &= (3, -4, -2) \end{aligned}$$

Therefore, coordinate of  $Q$  is  $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

$$\begin{aligned} &= (2(1) + 1, -3(1) - 1, 8(1) - 10) \\ &= (3, -4, -2) \end{aligned}$$

$$\begin{aligned} PQ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\ &= \sqrt{(1 - 3)^2 + (0 + 4)^2 + (0 + 2)^2} \\ &= \sqrt{4 + 16 + 4} \\ &= \sqrt{24} \\ &= 2\sqrt{6} \end{aligned}$$

So, foot of perpendicular =  $(3, -4, -2)$

length of perpendicular =  $2\sqrt{6}$  units



### Straight Line in Space Ex 28.4 Q3

the foot of the perpendicular drawn from  $A(1,0,3)$  to the line joining the points  $B(4,7,1)$

And  $C(3,5,3)$  be  $D$

Equation of line passing through  $B(4,7,1)$  and  $C(3,5,3)$  is

$$\begin{aligned}\frac{x-x_1}{x_2-x_1} &= \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \\ \Rightarrow \frac{x-4}{3-4} &= \frac{y-7}{5-7} = \frac{z-1}{3-1} \\ \Rightarrow \frac{x-4}{-1} &= \frac{y-7}{-2} = \frac{z-1}{2} = \lambda \text{ (say)}\end{aligned}$$

Direction ratio of  $AD$  are

$$\begin{aligned}(-\lambda+4-1), (-2\lambda+7-0), (2\lambda+1-3) \\ = (-\lambda+3), (-2\lambda+7), (2\lambda-2)\end{aligned}$$

Line  $AD$  is perpendicular to  $BC$  so

$$\begin{aligned}a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ \Rightarrow (-1)(-\lambda+3) + (-2)(-2\lambda+7) + 2(2\lambda-2) &= 0 \\ \Rightarrow \lambda - 3 + 4\lambda - 14 + 4\lambda - 4 &= 0 \\ \Rightarrow 9\lambda - 21 &= 0 \\ \Rightarrow \lambda &= \frac{21}{9}\end{aligned}$$

Co-ordinates of  $D$  are

$$\begin{aligned}&= \left( -\frac{21}{9} + 4, (-2)\left(\frac{21}{9} + 7\right), 2\left(\frac{21}{9} + 1\right) \right) \\ &= \left( \frac{15}{9}, \frac{21}{9}, \frac{51}{9} \right) \\ &= \left( \frac{5}{3}, \frac{7}{3}, \frac{17}{3} \right)\end{aligned}$$

### Straight Line in Space Ex 28.4 Q4

Given that  $D$  is the foot of perpendicular from  $A(1,0,4)$  on  $BC$ , so

Equation of line passing through  $B, C$  is

$$\begin{aligned}\frac{x-x_1}{x_2-x_1} &= \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \\ \Rightarrow \frac{x-0}{2-0} &= \frac{y+11}{-3+11} = \frac{z-3}{1-3} \\ \Rightarrow \frac{x}{2} &= \frac{y+11}{8} = \frac{z-3}{-2} = \lambda \text{ (say)}\end{aligned}$$

Coordinate of  $D = (2\lambda, 8\lambda - 11, -2\lambda + 3)$

Direction ratios of  $AD = 2\lambda - 1, 8\lambda - 11 - 0, -2\lambda + 3 - 4$   
 $= (2\lambda - 1), (8\lambda - 11), (-2\lambda - 1)$

Since, line  $AD$  is perpendicular on  $BC$ , so

$$\begin{aligned}a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ \Rightarrow (2)(2\lambda - 1) + (8)(8\lambda - 11) + (-2)(-2\lambda - 1) &= 0 \\ \Rightarrow 4\lambda - 2 + 64\lambda - 88 + 4\lambda + 2 &= 0 \\ \Rightarrow 72\lambda - 88 &= 0 \\ \Rightarrow \lambda &= \frac{88}{72} \\ \lambda &= \frac{11}{9}\end{aligned}$$

Coordinate of  $D = (2\lambda, 8\lambda - 11, -2\lambda + 3)$   
 $= \left( 2\left(\frac{11}{9}\right), 8\left(\frac{11}{9}\right) - 11, -2\left(\frac{11}{9}\right) + 3 \right)$

Coordinate of  $D = \left( \frac{22}{9}, \frac{-11}{9}, \frac{5}{9} \right)$



### Straight Line in Space Ex 28.4 Q5

Let foot of the perpendicular from  $P(2, 3, 4)$  is  $Q$  on the line  $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ , so

Equation of given line is,

$$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$$

$$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda \text{ (say)}$$

Coordinate of  $Q = (-2\lambda + 4, 6\lambda, -3\lambda + 1)$

Direction ratios of  $PQ = (-2\lambda + 4 - 2), (6\lambda - 3), (-3\lambda + 1 - 4)$   
 $= (-2\lambda + 2), (6\lambda - 3), (-3\lambda - 3)$

Line  $PQ$  is perpendicular to given line, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$(-2)(-2\lambda + 2) + (6)(6\lambda - 3) + (-3)(-3\lambda - 3) = 0$$

$$4\lambda - 4 + 36\lambda - 18 + 9\lambda + 9 = 0$$

$$49\lambda - 13 = 0$$

$$\lambda = \frac{13}{49}$$

Coordinate of  $Q = (-2\lambda + 4, 6\lambda, -3\lambda + 1)$

$$= \left( -2\left(\frac{13}{49}\right) + 4, 6\left(\frac{13}{49}\right), -3\left(\frac{13}{49}\right) + 1 \right)$$

$$= \left( \frac{-26 + 196}{49}, \frac{78}{49}, \frac{-39 + 49}{49} \right)$$

Coordinate of  $Q = \left( \frac{170}{49}, \frac{78}{49}, \frac{10}{49} \right)$

$$\begin{aligned} PQ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\ &= \sqrt{\left(\frac{170}{49} - 2\right)^2 + \left(\frac{78}{49} - 3\right)^2 + \left(\frac{10}{49} - 4\right)^2} \\ &= \sqrt{\left(\frac{72}{49}\right)^2 + \left(\frac{69}{49}\right)^2 + \left(-\frac{168}{49}\right)^2} \\ &= \sqrt{\frac{5184 + 4761 + 34596}{2401}} \\ &= \sqrt{\frac{44541}{2401}} \\ &= \sqrt{\frac{909}{49}} \\ &= \frac{3\sqrt{101}}{49} \end{aligned}$$

Perpendicular distance from  $(2, 3, 4)$  to given line is  $\frac{3\sqrt{101}}{49}$  units.

### Straight Line in Space Ex 28.4 Q6

Let  $\theta$  be the foot of the perpendicular drawn from  $P(2, 4, -1)$  to the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$$

$$\text{Given line is } \frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda \text{ (say)}$$

$$\begin{aligned} \text{Coordinate of } Q \text{ (General point on the line)} \\ = (\lambda - 5, 4\lambda - 3, -9\lambda + 6) \end{aligned}$$

$$\begin{aligned} \text{Direction ratios of } PQ \text{ are} \\ = (\lambda - 5 - 2), (4\lambda - 3 - 4), (-9\lambda + 6 + 1) \\ = \lambda - 7, 4\lambda - 7, -9\lambda + 7 \end{aligned}$$

Line  $PQ$  is perpendicular to the given line, so

$$\begin{aligned} a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ (1)(\lambda - 7) + (4)(4\lambda - 7) + (-9)(-9\lambda + 7) &= 0 \\ \lambda - 7 + 16\lambda - 28 + 81\lambda - 63 &= 0 \\ 98\lambda - 98 &= 0 \\ \lambda &= 1 \end{aligned}$$

$$\begin{aligned} \therefore \text{Coordinate of } Q &= (\lambda - 5, 4\lambda - 3, -9\lambda + 6) \\ &= (1 - 5, 4(1) - 3, -9(1) + 6) \end{aligned}$$

$$\text{Coordinate of foot of perpendicular} = (-4, 1, -3)$$

So, equation of the perpendicular  $PQ$  is

$$\begin{aligned} \frac{x-x_1}{x_2-x_1} &= \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \\ \Rightarrow \frac{x-2}{-4-2} &= \frac{y-4}{1-4} = \frac{z+1}{-3+1} \\ \Rightarrow \frac{x-2}{-6} &= \frac{y-4}{-3} = \frac{z+1}{-2} \end{aligned}$$

### Straight Line in Space Ex 28.4 Q7

Let foot of the perpendicular drawn from  $P(5, 4, -1)$  to the given line is  $Q$ , so

Given equation of line is,

$$\begin{aligned} \vec{r} &= \hat{i} + \lambda(2\hat{i} + 9\hat{j} + 5\hat{k}) \\ (x\hat{i} + y\hat{j} + z\hat{k}) &= (1 + 2\lambda)\hat{i} + (9\lambda)\hat{j} + (5\lambda)\hat{k} \end{aligned}$$

Equation the coefficients of  $\hat{i}, \hat{j}$  and  $\hat{k}$

$$\begin{aligned} \Rightarrow x &= 1 + 2\lambda, y = 9\lambda, z = 5\lambda \\ \Rightarrow \frac{x-1}{2} &= \lambda, \frac{y}{9} = \lambda, \frac{z}{5} = \lambda \\ \Rightarrow \frac{x-1}{2} &= \frac{y}{9} = \frac{z}{5} = \lambda \text{ (say)} \end{aligned}$$

$$\text{Coordinate of } Q = (2\lambda + 1, 9\lambda, 5\lambda)$$

$$\begin{aligned} \text{Direction ratios of line } PQ \text{ are} \\ (2\lambda + 1 - 5), 9\lambda - 4, 5\lambda + 1 \end{aligned}$$

$$\Rightarrow 2\lambda - 4, 9\lambda - 4, 5\lambda + 1$$

**Straight Line in Space Ex 28.4 Q8**

Let position vector of foot of perpendicular drawn from  $P(\hat{i} + 6\hat{j} + 3\hat{k})$  on

$\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$  be  $Q(\vec{q})$ . So

$Q$  is on the line  $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

So, Position vector of  $Q = (\lambda)\hat{i} + (1 + 2\lambda)\hat{j} + (2 + 3\lambda)\hat{k}$

$\vec{PQ}$  = Position vector of  $Q$  - Position vector of  $P$

$$= \{\lambda\hat{i} + (1 + 2\lambda)\hat{j} + (2 + 3\lambda)\hat{k}\} - \{\hat{i} + 6\hat{j} + 3\hat{k}\}$$

$$= (\lambda - 1)\hat{i} + (1 + 2\lambda - 6)\hat{j} + (2 + 3\lambda - 3)\hat{k}$$

$$\vec{PQ} = (\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}$$

Here,  $\vec{PQ}$  is perpendicular to given line

So,

$$\{(\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}\} \cdot \{\hat{i} + 2\hat{j} + 3\hat{k}\} = 0$$

$$\Rightarrow (\lambda - 1)(1) + (2\lambda - 5)(2) + (3\lambda - 1)(3) = 0$$

$$\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$$

$$\Rightarrow 14\lambda - 14 = 0$$

$$\Rightarrow \lambda = 1$$

$$\begin{aligned}\text{Position vector of } Q &= (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}) \\ &= (\hat{j} + 2\hat{k}) + (1)(\hat{i} + 2\hat{j} + 3\hat{k})\end{aligned}$$

$$\text{Foot of perpendicular} = \hat{i} + 3\hat{j} + 5\hat{k}$$

$\vec{PQ}$  = Position vector of  $Q$  - Position vector of  $P$

$$= (\hat{i} + 3\hat{j} + 5\hat{k}) - (\hat{i} + 6\hat{j} + 3\hat{k})$$

$$= \hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 6\hat{j} - 3\hat{k}$$

$$= -3\hat{j} + 2\hat{k}$$

$$|\vec{PQ}| = \sqrt{(-3)^2 + (2)^2}$$

$$= \sqrt{13} \text{ units}$$

$$\text{Length of perpendicular} = \sqrt{13} \text{ units}$$

Let  $Q$  be the perpendicular drawn from  $P(-\hat{i} + 3\hat{j} + 2\hat{k})$  on the line

$$\vec{r} = (2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$$

Let the position vector of  $Q$  be

$$(2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$$

$$(2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (3 + 3\lambda)\hat{k}$$

$$\begin{aligned}\overrightarrow{PQ} &= \text{Position vector of } Q - \text{Position vector of } P \\ &= \{(2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (3 + 3\lambda)\hat{k}\} - \{-\hat{i} + 3\hat{j} + 2\hat{k}\} \\ &= (2\lambda + 1)\hat{i} + (2\lambda - 3)\hat{j} + (3 + 3\lambda - 2)\hat{k} \\ \overrightarrow{PQ} &= (2\lambda + 1)\hat{i} + (\lambda - 1)\hat{j} + (3\lambda + 1)\hat{k}\end{aligned}$$

Since,  $\overrightarrow{PQ}$  is perpendicular to the given line, so

$$\begin{aligned}\{(2\lambda + 1)\hat{i} + (\lambda - 1)\hat{j} + (3\lambda + 1)\hat{k}\} \cdot \{2\hat{i} + \hat{j} + 3\hat{k}\} &= 0 \\ (2\lambda + 1)(2) + (\lambda - 1)(1) + (3\lambda + 1)(3) &= 0 \\ 4\lambda + 2 + \lambda - 1 + 9\lambda + 3 &= 0 \\ 14\lambda + 4 &= 0 \\ \lambda &= -\frac{4}{14} \\ \lambda &= -\frac{2}{7}\end{aligned}$$

$$\begin{aligned}\text{Position vector of } Q &= (2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (3 + 3\lambda)\hat{k} \\ &= 2\left(-\frac{2}{7}\right)\hat{i} + \left(2 - \frac{2}{7}\right)\hat{j} + \left(3 + 3\left(-\frac{2}{7}\right)\right)\hat{k} \\ &= -\frac{4}{7}\hat{i} + \frac{12}{7}\hat{j} + \frac{15}{7}\hat{k}\end{aligned}$$

$$\text{Coordinates of foot of the perpendicular} = \left(-\frac{4}{7}, \frac{12}{7}, \frac{15}{7}\right)$$

Equation of  $PQ$  is

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

$$\Rightarrow \vec{r} = (-\hat{i} + 3\hat{j} + 2\hat{k}) + \lambda\left(\left(-\frac{4}{7}\hat{i} + \frac{12}{7}\hat{j} + \frac{15}{7}\hat{k}\right) - (-\hat{i} + 3\hat{j} + 2\hat{k})\right)$$

#### Straight Line in Space Ex 28.4 Q10

Let foot of the perpendicular drawn from  $(0, 2, 7)$  to the line  $\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$  be  $Q$ .

Given equation of the line is

$$\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2} = \lambda \text{ (say)}$$

$$\text{Coordinate of } Q \text{ is } \{-\lambda - 2, 3\lambda + 1, -2\lambda + 3\}$$

$$\begin{aligned}\text{Direction ratios of } PQ &\text{ are } \{-\lambda - 2 - 0\}, \{3\lambda + 1 - 2\}, \{-2\lambda + 3 - 7\} \\ &= \{-\lambda - 2\}, \{3\lambda - 1\}, \{-2\lambda - 4\}\end{aligned}$$

Since,  $PQ$  is perpendicular to given line, so

$$\begin{aligned}a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ (-1)(-\lambda - 2) + (3)(3\lambda - 1) + (-2)(-2\lambda - 4) &= 0\end{aligned}$$

$$\Rightarrow \lambda + 2 + 9\lambda - 3 + 4\lambda + 8 = 0$$

$$\Rightarrow 14\lambda + 7 = 0$$

$$\lambda = -\frac{1}{2}$$

$$\begin{aligned}\text{Foot of the perpendicular} &= \{-\lambda - 2, 3\lambda + 1, -2\lambda + 3\} \\ &= \left(-\left(-\frac{1}{2}\right) - 2, 3\left(-\frac{1}{2}\right) + 1, -2\left(-\frac{1}{2}\right) + 3\right)\end{aligned}$$

$$\text{Foot of the perpendicular} = \left(-\frac{3}{2}, -\frac{1}{2}, 4\right)$$



## Straight Line in Space Ex 28.4 Q11

Let foot of the perpendicular from  $P(1, 2, -3)$  to the line  $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$  be  $Q$

Given equation of the line is

$$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$$

$$\Rightarrow x = 2\lambda - 1, y = -2\lambda + 3, z = -\lambda$$

Coordinate of  $Q(2\lambda - 1, -2\lambda + 3, -\lambda)$

Direction ratios of  $PQ$  are

$$(2\lambda - 1 - 1), (-2\lambda + 3 - 2), (-\lambda + 3)$$

$$\Rightarrow (2\lambda - 2), (-2\lambda + 1), (-\lambda + 3)$$

Let  $PQ$  is perpendicular to given line, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow (2)(2\lambda - 2) + (-2)(-2\lambda + 1) + (-1)(-\lambda + 3) = 0$$

$$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$$

$$\Rightarrow 9\lambda - 9 = 0$$

$$\Rightarrow \lambda = 1$$

Coordinate of foot of perpendicular

$$= (2\lambda - 1, -2\lambda + 3, -\lambda)$$

$$= (2(1) - 1, -2(1) + 3, -1)$$

$$= (1, 1, -1)$$

## Straight Line in Space Ex 28.4 Q12

Equation of line  $AB$  is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \frac{x - 0}{-3 - 0} = \frac{y - 6}{-6 - 6} = \frac{z + 9}{3 + 9}$$

$$\Rightarrow \frac{x}{-3} = \frac{y - 6}{-12} = \frac{z + 9}{12} = \lambda \text{ (say)}$$

Coordinate of point  $D = (-3\lambda, -12\lambda + 6, 12\lambda - 9)$

Direction ratios of  $CD = (-3\lambda - 7), (-12\lambda + 6 - 4), (12\lambda - 9 + 1)$   
 $= (-3\lambda - 7), (-12\lambda + 2), (12\lambda - 8)$

Line  $CD$  is perpendicular to line  $AB$ , so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow (-3)(-3\lambda - 7) + (-12)(-12\lambda + 2) + (12)(12\lambda - 8) = 0$$

$$\Rightarrow 9\lambda + 21 + 144\lambda - 24 + 144\lambda - 96 = 0$$

$$\Rightarrow 297\lambda - 99 = 0$$

$$\Rightarrow \lambda = \frac{1}{3}$$

Coordinate of  $D = (-3\lambda, -12\lambda + 6, 12\lambda - 9)$

$$= \left(-3\left(\frac{1}{3}\right), -12\left(\frac{1}{3}\right) + 6, 12\left(\frac{1}{3}\right) - 9\right)$$

Coordinate of  $D = (-1, 2, -5)$

Equation of  $CD$  is,

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \frac{x - 7}{-1 - 7} = \frac{y - 4}{2 - 4} = \frac{z + 1}{-5 + 1}$$

$$\Rightarrow \frac{x - 7}{-8} = \frac{y - 4}{-2} = \frac{z + 1}{-4}$$

$$\text{or } \frac{x - 7}{4} = \frac{y - 4}{1} = \frac{z + 1}{2}$$

### Straight Line in Space Ex 28.4 Q13

Let  $P = (2, 4, -1)$ .

In order to find the distance we need to find a point  $Q$  on the line.

We see that line is passing through the point  $Q(-5, -3, 6)$ .

So, let take this point as required point.

Also line is parallel to the vector  $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$ .

$$\text{Now, } \vec{PQ} = (-5\hat{i} - 3\hat{j} + 6\hat{k}) - (2\hat{i} + 4\hat{j} - \hat{k}) = -7\hat{i} - 7\hat{j} + 7\hat{k}$$

$$\vec{b} \times \vec{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -9 \\ -7 & -7 & 7 \end{vmatrix} = -35\hat{i} + 56\hat{j} + 21\hat{k}$$

$$|\vec{b} \times \vec{PQ}| = \sqrt{1225 + 3136 + 441} = \sqrt{4802}$$

$$|\vec{b}| = \sqrt{1 + 16 + 81} = \sqrt{98}$$

$$d = \frac{|\vec{b} \times \vec{PQ}|}{|\vec{b}|} = \frac{\sqrt{4802}}{\sqrt{98}} = 7$$

### Straight Line in Space Ex 28.4 Q14

Let  $L$  be the foot of the perpendicular drawn from  $A(1, 8, 4)$  on the line joining the points  $B(0, -1, 3)$  and  $C(2, -3, -1)$ .

Equation of the line passing through the points  $B(0, -1, 3)$  and  $C(2, -3, -1)$  is given by,

$$\vec{r} = \vec{b} + \lambda(\vec{c} - \vec{b})$$

$$\vec{r} = (0 + 2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k}$$

Let position vector of  $L$  be,

$$\vec{r} = (2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k} \dots\dots\dots(i)$$

Then,  $\vec{AL}$  = Position vector of  $L$  - position vector of  $A$

$$\Rightarrow \vec{AL} = (2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k} - (1\hat{i} + 8\hat{j} + 4\hat{k})$$

$$\Rightarrow \vec{AL} = (-1 + 2\lambda)\hat{i} + (-9 - 2\lambda)\hat{j} + (-1 - 4\lambda)\hat{k}$$

Since  $\vec{AL}$  is perpendicular to the given line which is parallel to  $\vec{b} = 2\hat{i} - 2\hat{j} - 4\hat{k}$

$$\therefore \vec{AL} \cdot \vec{b} = 0$$

$$\Rightarrow 2(-1 + 2\lambda) - 2(-9 - 2\lambda) - 4(-1 - 4\lambda) = 0$$

$$\Rightarrow -2 + 4\lambda + 18 + 4\lambda + 4 + 16\lambda = 0$$

$$\Rightarrow 24\lambda = -20$$

$$\Rightarrow \lambda = \frac{-5}{6}$$

Putting value of  $\lambda = \frac{-5}{6}$  in (i) we get

$$\vec{r} = -\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{19}{3}\hat{k}$$

Coordinates of the foot of the perpendicular are  $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ .



## Ex 28.5

## Shortest Line in Space Ex 28.5 Q1(i)

We know that, shortest distance between lines

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \text{ and } \vec{r} = \vec{a}_2 + \mu \vec{b}_2 \text{ is given by}$$

$$\text{S.D.} = \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \quad \dots (i)$$

Given equations of lines are.

$$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda(3\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (-3\hat{i} - 7\hat{j} + 6\hat{k}) + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$$

$$\Rightarrow \vec{a}_1 = (3\hat{i} + 8\hat{j} + 3\hat{k}), \vec{b}_1 = 3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = (-3\hat{i} - 7\hat{j} + 6\hat{k}), \vec{b}_2 = (-3\hat{i} + 2\hat{j} + 4\hat{k})$$

Now,

$$\vec{a}_2 - \vec{a}_1 = (-3\hat{i} - 7\hat{j} + 6\hat{k}) - (3\hat{i} + 8\hat{j} + 3\hat{k})$$

$$= -3\hat{i} - 7\hat{j} + 6\hat{k} - 3\hat{i} - 8\hat{j} - 3\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$(\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$$= \hat{i}(-4 - 2) - \hat{j}(12 + 3) + \hat{k}(6 - 3)$$

$$= (-6\hat{i} - 15\hat{j} + 3\hat{k})$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (-6\hat{i} - 15\hat{j} + 3\hat{k}) \cdot (-6\hat{i} - 15\hat{j} + 3\hat{k})$$

$$= (-6)(-6) + (-15)(-15) + (3)(3)$$

$$= 36 + 225 + 9$$

$$= 270$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-6)^2 + (-15)^2 + (3)^2}$$

$$= \sqrt{36 + 225 + 9}$$

$$= \sqrt{270}$$

Substituting values of  $|\vec{b}_1 \times \vec{b}_2|$  and  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$  in equation (i) to get shortest distance between given lines, so

$$\text{S.D.} = \frac{270}{\sqrt{270}}$$

$$= \sqrt{270}$$

$$\text{S.D.} = 3\sqrt{30} \text{ units}$$



## Short Line in Space Ex 28.5 Q1(ii)

Now that, shortest distance between lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$  is given by

$$\text{S.D.} = \frac{|\vec{a}_2 - \vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \text{--- (i)}$$

Given equations of lines are,

$$\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda (\hat{i} - 2\hat{j} + 7\hat{k}) \quad \text{and}$$

$$\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu (7\hat{i} - 6\hat{j} + \hat{k})$$

$$\Rightarrow \vec{a}_1 = (3\hat{i} + 5\hat{j} + 7\hat{k}), \vec{b}_1 = (\hat{i} - 2\hat{j} + 7\hat{k})$$

$$\vec{a}_2 = (-\hat{i} - \hat{j} - \hat{k}), \vec{b}_2 = (7\hat{i} - 6\hat{j} + \hat{k})$$

$$\begin{aligned} \text{So, } \vec{a}_2 - \vec{a}_1 &= (-\hat{i} - \hat{j} - \hat{k}) - (3\hat{i} + 5\hat{j} + 7\hat{k}) \\ &= -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k} \\ &= -4\hat{i} - 6\hat{j} - 8\hat{k} = -2(2\hat{i} + 3\hat{j} + 4\hat{k}) \end{aligned}$$

$$\begin{aligned} \vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 7 \\ 7 & -6 & 1 \end{vmatrix} \\ &= \hat{i}(-2 + 42) - \hat{j}(1 - 49) + \hat{k}(-6 + 14) \\ &= 40\hat{i} + 48\hat{j} + 8\hat{k} \\ &= 8(5\hat{i} + 6\hat{j} + \hat{k}) \end{aligned}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= \{-2(2\hat{i} + 3\hat{j} + 4\hat{k})\} \cdot \{8(5\hat{i} + 6\hat{j} + \hat{k})\} \\ &= -16[(2)(5) + (3)(6) + (4)(1)] \\ &= -16[10 + 18 + 4] \\ &= -16 \times 32 \end{aligned}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = -512$$

$$\begin{aligned} |\vec{b}_1 \times \vec{b}_2| &= 8\sqrt{(5)^2 + (6)^2 + (1)^2} \\ &= 8\sqrt{25 + 36 + 1} \\ &= 8\sqrt{62} \end{aligned}$$

Substituting values of  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$  and  $|\vec{b}_1 \times \vec{b}_2|$  in equation (i) to get required shortest distance between given lines, so

$$\text{S.D.} = \frac{|-512|}{8\sqrt{62}}$$

$$\text{S.D.} = \frac{512}{\sqrt{3968}}$$



### Straight Line in Space Ex 28.5 Q1(iii)

We know that, shortest distance between lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$  is given by

$$\text{S.D.} = \frac{|\vec{a}_2 - \vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \dots (i)$$

Given equations of lines are,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda (2\hat{i} + 3\hat{j} + 4\hat{k}) \quad \text{and}$$

$$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu (3\hat{i} + 4\hat{j} + 5\hat{k})$$

$$\begin{aligned} \text{Now, } (\vec{a}_2 - \vec{a}_1) &= (2\hat{i} + 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) \\ &= 2\hat{i} + 4\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k} \\ (\vec{a}_2 - \vec{a}_1) &= \hat{i} + 2\hat{j} + 2\hat{k} \end{aligned}$$

$$\begin{aligned} (\vec{b}_1 \times \vec{b}_2) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} \\ &= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9) \\ (\vec{b}_1 \times \vec{b}_2) &= -\hat{i} + 2\hat{j} - \hat{k} \end{aligned}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) \\ &= (1)(-1) + (2)(2) + (2)(-1) \\ &= -1 + 4 - 2 \end{aligned}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 1$$

$$\begin{aligned} |\vec{b}_1 \times \vec{b}_2| &= \sqrt{(-1)^2 + (2)^2 + (-1)^2} \\ &= \sqrt{1 + 4 + 1} \end{aligned}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{6}$$

Substituting values of  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$  and  $|\vec{b}_1 \times \vec{b}_2|$  in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \frac{1}{\sqrt{6}}$$

$$\text{S.D.} = \frac{1}{\sqrt{6}} \text{ units}$$

### Straight Line in Space Ex 28.5 Q1(iv)

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-t)\hat{k}$$

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

Above equations can be rewritten as

$$\vec{r} = (i - 2j + 3k) + t(-i + j - k)$$

$$\vec{r} = (i - j - k) + s(i + 2j - 2k)$$

$$\text{Shortest distance is given by } \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$(\vec{b}_1 \times \vec{b}_2) = -3j - 3k$$

$$(\vec{a}_2 - \vec{a}_1) = j - 4k$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 9$$

$$|\vec{b}_1 \times \vec{b}_2| = 3\sqrt{2}$$

$$\text{Shortest distance is } \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$$



## Straight Line in Space Ex 28.5 Q1(v)

We know that, the shortest distance between lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$  is given by

$$\vec{r} = \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \quad \dots (i)$$

Given equations of lines are,

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$$

$$\Rightarrow \vec{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k}) \text{ and}$$

$$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

$$\Rightarrow \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-\hat{i} + 2\hat{j} + \hat{k})$$

$$\text{So, } \vec{a}_1 = (-\hat{i} + \hat{j} - \hat{k}), \vec{b}_1 = (\hat{i} + \hat{j} - \hat{k}) \text{ and}$$

$$\vec{a}_2 = (\hat{i} - \hat{j} + 2\hat{k}), \vec{b}_2 = (-\hat{i} + 2\hat{j} + \hat{k})$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) &= (\hat{i} - \hat{j} + 2\hat{k}) - (-\hat{i} + \hat{j} - \hat{k}) \\ &= \hat{i} - \hat{j} + 2\hat{k} + \hat{i} - \hat{j} + \hat{k} \end{aligned}$$

$$(\vec{a}_2 - \vec{a}_1) = 2\hat{i} - 2\hat{j} + 3\hat{k}$$

$$(\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1+2) - \hat{j}(1-1) + \hat{k}(2+1)$$

$$(\vec{b}_1 \times \vec{b}_2) = 3\hat{i} + 3\hat{k}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (2\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 3\hat{k}) \\ &= (2)(3) + (-2)(0) + (3)(3) \\ &= 6 + 0 + 9 \end{aligned}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 15$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(3)^2 + (3)^2}$$

$$= \sqrt{18}$$

$$|\vec{b}_1 \times \vec{b}_2| = 3\sqrt{2}$$

Substituting values of  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$  and  $|\vec{b}_1 \times \vec{b}_2|$  in equation (i) to get the shortest distance between the given lines, so

$$\text{S.D.} = \frac{15}{3\sqrt{2}}$$

$$\text{S.D.} = \frac{5}{\sqrt{2}} \text{ units}$$



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Given lines are,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \text{ (say)}$$

$$x = 2\lambda + 1, y = 3\lambda + 2, z = 4\lambda + 3$$

$$\begin{aligned} \Rightarrow \vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (2\lambda + 1)\hat{i} + (3\lambda + 2)\hat{j} + (4\lambda + 3)\hat{k} \\ \vec{r} &= (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k}) \end{aligned}$$

$$\Rightarrow \vec{a}_1 = (\hat{i} + 2\hat{j} + 3\hat{k}), \vec{b}_1 = (2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\text{and, } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5} = \mu \text{ (say)}$$

$$x = 3\mu + 2, y = 4\mu + 3, z = 5\mu + 5$$

$$\begin{aligned} \Rightarrow \vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (3\mu + 2)\hat{i} + (4\mu + 3)\hat{j} + (5\mu + 5)\hat{k} \\ \vec{r} &= (2\hat{i} + 3\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k}) \end{aligned}$$

$$\Rightarrow \vec{a}_2 = (2\hat{i} + 3\hat{j} + 5\hat{k}), \vec{b}_2 = (3\hat{i} + 4\hat{j} + 5\hat{k})$$

We know that, the shortest distance between the lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by,

$$\text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \text{--- (i)}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) &= (2\hat{i} + 3\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) \\ &= 2\hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k} \\ (\vec{a}_2 - \vec{a}_1) &= \hat{i} + \hat{j} + 2\hat{k} \end{aligned}$$

$$\begin{aligned} \vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} \\ &= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9) \end{aligned}$$

$$(\vec{b}_1 \times \vec{b}_2) = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (\hat{i} + \hat{j} + 2\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) \\ &= (1)(-1) + (1)(2) + (2)(-1) \\ &= -1 + 2 - 2 \end{aligned}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = -1$$

$$\begin{aligned} |\vec{b}_1 \times \vec{b}_2| &= \sqrt{(-1)^2 + (2)^2 + (-1)^2} \\ &= \sqrt{1 + 4 + 1} \\ &= \sqrt{6} \end{aligned}$$

Using the values of  $(\vec{a}_2 - \vec{a}_1)$ ,  $(\vec{b}_1 \times \vec{b}_2)$  and  $|\vec{b}_1 \times \vec{b}_2|$  in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \frac{|-1|}{\sqrt{6}}$$

$$\text{S.D.} = \frac{1}{\sqrt{6}} \text{ units}$$



Given equations of line are,

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \quad (\text{say})$$

$$\Rightarrow x = 2\lambda + 1, y = 3\lambda - 1, z = \lambda$$

$$\begin{aligned}\vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (2\lambda + 1)\hat{i} + (3\lambda - 1)\hat{j} + \lambda\hat{k} \\ \vec{r} &= (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})\end{aligned}$$

$$\Rightarrow \vec{a}_1 = \hat{i} - \hat{j}, \vec{b}_1 = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\text{and, } \frac{x+1}{3} = \frac{y-2}{1} = \mu, z = 2$$

$$\Rightarrow x = 3\mu - 1, y = \mu + 2, z = 2$$

$$\begin{aligned}\vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (3\mu - 1)\hat{i} + (\mu + 2)\hat{j} + 2\hat{k} \\ \vec{r} &= (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(3\hat{i} + \hat{j})\end{aligned}$$

$$\Rightarrow \vec{a}_2 = (-\hat{i} + 2\hat{j} + 2\hat{k}), \vec{b}_2 = (3\hat{i} + \hat{j})$$

We know that, the shortest distance between two lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by,

$$\text{S.D.} = \frac{|\vec{a}_2 - \vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \dots (i)$$

$$\begin{aligned}\vec{a}_2 - \vec{a}_1 &= (\hat{i} - \hat{j}) - (-\hat{i} + 2\hat{j} + 2\hat{k}) \\ &= \hat{i} - \hat{j} + \hat{i} - 2\hat{j} - 2\hat{k} \\ \vec{a}_2 - \vec{a}_1 &= 2\hat{i} - 3\hat{j} - 2\hat{k}\end{aligned}$$

$$\begin{aligned}\vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & 1 & 0 \end{vmatrix} \\ &= \hat{i}(0 - 1) - \hat{j}(0 - 3) + \hat{k}(2 - 9)\end{aligned}$$

$$(\vec{b}_1 \times \vec{b}_2) = -\hat{i} + 3\hat{j} - 7\hat{k}$$

$$\begin{aligned}(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (2\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (-\hat{i} + 3\hat{j} - 7\hat{k}) \\ &= (2)(-1) + (-3)(3) + (-2)(-7) \\ &= -2 - 9 + 14\end{aligned}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 3$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + (3)^2 + (-7)^2} = \sqrt{59}$$

Substitute the value of  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$  and  $|\vec{b}_1 \times \vec{b}_2|$  in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \frac{3}{\sqrt{59}}$$

$$\text{S.D.} = \frac{3}{\sqrt{59}} \text{ units}$$



### Right Line in Space Ex 28.5 Q2(iii)

Then equation of lines are,

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2} = \lambda \text{ (say)}$$

$$\Rightarrow x = -\lambda + 1, y = \lambda - 2, z = -2\lambda + 3$$

$$\begin{aligned}\vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (-\lambda + 1)\hat{i} + (\lambda - 2)\hat{j} + (-2\lambda + 3)\hat{k} \\ \vec{r} &= (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})\end{aligned}$$

$$\Rightarrow \vec{a}_1 = (\hat{i} - 2\hat{j} + 3\hat{k}), \vec{b}_1 = (-\hat{i} + \hat{j} - 2\hat{k})$$

$$\text{and, } \frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2} = \mu \text{ (say)}$$

$$\Rightarrow x = \mu + 1, y = 2\mu - 1, z = -2\mu - 1$$

$$\begin{aligned}\vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} + (-2\mu - 1)\hat{k} \\ \vec{r} &= (\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})\end{aligned}$$

$$\Rightarrow \vec{a}_2 = (\hat{i} - \hat{j} - \hat{k}), \vec{b}_2 = (\hat{i} + 2\hat{j} - 2\hat{k})$$

$$\begin{aligned}(\vec{a}_2 - \vec{a}_1) &= (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k}) \\ &= \hat{i} - \hat{j} - \hat{k} - \hat{i} + 2\hat{j} - 3\hat{k} \\ &= \hat{j} - 4\hat{k}\end{aligned}$$

$$\begin{aligned}\vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} \\ &= \hat{i}(-2 + 4) - \hat{j}(2 + 2) + \hat{k}(-2 - 1) \\ (\vec{b}_1 \times \vec{b}_2) &= 2\hat{i} - 4\hat{j} - 3\hat{k}\end{aligned}$$

$$\begin{aligned}|\vec{b}_1 \times \vec{b}_2| &= \sqrt{(2)^2 + (-4)^2 + (-3)^2} \\ &= \sqrt{29}\end{aligned}$$

$$\begin{aligned}(\vec{a}_2 - \vec{a}_1)(\vec{b}_1 \times \vec{b}_2) &= (\hat{j} - 4\hat{k})(2\hat{i} - 4\hat{j} - 3\hat{k}) \\ &= (0)(2) + (1)(-4) + (-4)(-3) \\ &= 0 - 4 + 12\end{aligned}$$

$$(\vec{a}_2 - \vec{a}_1)(\vec{b}_1 \times \vec{b}_2) = 8$$

We know that, shortest distance between  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by,

$$\text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \text{--- (i)}$$

So, shortest distance between given lines is

$$\text{S.D.} = \left| \frac{8}{\sqrt{29}} \right|$$

$$\text{S.D.} = \frac{8}{\sqrt{29}} \text{ units}$$



### Straight Line in Space Ex 28.5 Q2(iv)

Given equation of lines are,

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = \lambda \quad (\text{say})$$

$$\Rightarrow x = \lambda + 3, y = -2\lambda + 5, z = \lambda + 7$$

$$\begin{aligned}\Rightarrow \vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (\lambda + 3)\hat{i} + (-2\lambda + 5)\hat{j} + (\lambda + 7)\hat{k} \\ \vec{r} &= (3\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} - 2\hat{j} + \hat{k})\end{aligned}$$

$$\Rightarrow \vec{a}_1 = (3\hat{i} + 5\hat{j} + 7\hat{k}), \vec{b}_1 = (\hat{i} - 2\hat{j} + \hat{k})$$

and,  $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} = \mu \quad (\text{say})$

$$\Rightarrow x = 7\mu - 1, y = -6\mu - 1, z = \mu - 1$$

$$\begin{aligned}\Rightarrow \vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (7\mu - 1)\hat{i} + (-6\mu - 1)\hat{j} + (\mu - 1)\hat{k} \\ \vec{r} &= (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k})\end{aligned}$$

$$\Rightarrow \vec{a}_2 = (-\hat{i} - \hat{j} - \hat{k}), \vec{b}_2 = 7\hat{i} - 6\hat{j} + \hat{k}$$

We know that, shortest distance between  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by,

$$\text{S.D.} = \frac{|\vec{a}_2 - \vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \text{--- (i)}$$

$$\begin{aligned}(\vec{a}_2 - \vec{a}_1) &= (-\hat{i} - \hat{j} - \hat{k}) - (3\hat{i} + 5\hat{j} + 7\hat{k}) \\ &= -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k} \\ (\vec{a}_2 - \vec{a}_1) &= -4\hat{i} - 6\hat{j} - 8\hat{k}\end{aligned}$$

$$\begin{aligned}\vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix} \\ &= \hat{i}(-2+6) - \hat{j}(1-7) + \hat{k}(-6+14) \\ \vec{b}_1 \times \vec{b}_2 &= 4\hat{i} + 6\hat{j} + 8\hat{k}\end{aligned}$$

$$\begin{aligned}|\vec{b}_1 \times \vec{b}_2| &= \sqrt{(4)^2 + (6)^2 + (8)^2} \\ &= \sqrt{16 + 36 + 64} \\ &= \sqrt{116} \\ &= 2\sqrt{29}\end{aligned}$$

$$\begin{aligned}(\vec{a}_2 - \vec{a}_1)(\vec{b}_1 \times \vec{b}_2) &= (-4\hat{i} - 6\hat{j} - 8\hat{k})(4\hat{i} + 6\hat{j} + 8\hat{k}) \\ &= (-4)(4) + (-6)(6) + (-8)(8) \\ &= -16 - 36 - 64 \\ &= -116\end{aligned}$$

Substituting the values of  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$  and  $|\vec{b}_1 \times \vec{b}_2|$  in equation (i) to get the shortest distance between the two given lines, so

$$\begin{aligned}\text{S.D.} &= \frac{|-116|}{2\sqrt{29}} \\ &= \frac{58}{\sqrt{29}}\end{aligned}$$



**Straight Line in Space Ex 28.5 Q3(i)**

Given equations of lines are,

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$$

$$\Rightarrow \vec{a}_1 = (\hat{i} - \hat{j}), \vec{b}_1 = (2\hat{i} + \hat{k})$$

and,  $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$

$$\Rightarrow \vec{a}_2 = (2\hat{i} - \hat{j}), \vec{b}_2 = (\hat{i} + \hat{j} - \hat{k})$$

We know that, shortest distance between lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by

$$\text{S.D.} = \frac{|\vec{a}_2 - \vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \dots (i)$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) &= (2\hat{i} - \hat{j}) - (\hat{i} - \hat{j}) \\ &= 2\hat{i} - \hat{j} - \hat{i} + \hat{j} \\ &= \hat{i} \end{aligned}$$

$$\begin{aligned} |\vec{b}_1 \times \vec{b}_2| &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} \\ &= \hat{i}(0 - 1) - \hat{j}(-2 - 1) + \hat{k}(2 - 0) \\ &= -\hat{i} + 3\hat{j} + 2\hat{k} \end{aligned}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (\hat{i}) \cdot (-\hat{i} + 3\hat{j} + 2\hat{k}) \\ &= (1)(-1) + (0)(3) + (0)(2) \\ &= -1 + 0 + 0 \end{aligned}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = -1$$

$$\begin{aligned} |\vec{b}_1 \times \vec{b}_2| &= \sqrt{(-1)^2 + (3)^2 + (2)^2} \\ &= \sqrt{1 + 9 + 4} \\ |\vec{b}_1 \times \vec{b}_2| &= \sqrt{14} \end{aligned}$$

So, shortest distance between the given lines using equation (1) is,

$$\begin{aligned} \text{S.D.} &= \frac{|-1|}{\sqrt{14}} \\ &= \frac{1}{\sqrt{14}} \text{ units} \\ \text{S.D.} &\neq 0 \end{aligned}$$

Since, shortest distance between lines is not zero, so lines are not intersecting.

**Straight Line in Space Ex 28.5 Q3(ii)**

Given equations of lines are,

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$$

$$\Rightarrow \vec{a}_1 = (\hat{i} + \hat{j} - \hat{k}), \vec{b}_1 = (3\hat{i} - \hat{j})$$

$$\text{and, } \vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

$$\Rightarrow \vec{a}_2 = (4\hat{i} - \hat{k}), \vec{b}_2 = (2\hat{i} + 3\hat{k})$$

We know that, shortest distance between two lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by

$$\text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \text{--- (i)}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) &= (4\hat{i} - \hat{k}) - (\hat{i} + \hat{j} - \hat{k}) \\ &= 4\hat{i} - \hat{k} - \hat{i} - \hat{j} + \hat{k} \\ &= 3\hat{i} - \hat{j} \end{aligned}$$

$$\begin{aligned} \vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} \\ &= \hat{i}(-3-0) - \hat{j}(9-0) + \hat{k}(0+2) \\ &= -3\hat{i} - 9\hat{j} + 2\hat{k} \end{aligned}$$

$$\begin{aligned} |\vec{b}_1 \times \vec{b}_2| &= \sqrt{(-3)^2 + (-9)^2 + (2)^2} \\ |\vec{b}_1 \times \vec{b}_2| &= \sqrt{9+81+4} \\ &= \sqrt{94} \end{aligned}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (3\hat{i} - \hat{j}) \cdot (-3\hat{i} - 9\hat{j} + 2\hat{k}) \\ &= (3)(-3) + (-1)(-9) + (0)(2) \\ &= -9 + 9 + 0 \\ &= 0 \end{aligned}$$

Using  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$  and  $|\vec{b}_1 \times \vec{b}_2|$  in equation (i) to get shortest distance between given lines, so

$$\text{S.D.} = \left| \frac{0}{\sqrt{94}} \right|$$

$$\text{S.D.} = 0$$

Since, shortest distance between the given lines is zero, so lines are intersecting.

### Straight Line in Space Ex 28.5 Q3(iii)

Given equations of lines are,

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \text{ (say)}$$

$$\Rightarrow x = 2\lambda + 1, y = 3\lambda - 1, z = \lambda$$

$$\begin{aligned}\vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (2\lambda + 1)\hat{i} + (3\lambda - 1)\hat{j} + (\lambda)\hat{k} \\ \vec{r} &= (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})\end{aligned}$$

$$\Rightarrow \vec{a}_1 = (\hat{i} - \hat{j}), \vec{b}_1 = (2\hat{i} + 3\hat{j} + \hat{k})$$

$$\text{and, } \frac{x+1}{5} = \frac{y-2}{1} = \mu \text{ (say), } z = 2$$

$$\Rightarrow x = 5\mu - 1, y = \mu + 2, z = 2$$

$$\begin{aligned}\vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (5\mu - 1)\hat{i} + (\mu + 2)\hat{j} + 2\hat{k} \\ \vec{r} &= (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(5\hat{i} + \hat{j})\end{aligned}$$

$$\Rightarrow \vec{a}_2 = (-\hat{i} + 2\hat{j} + 2\hat{k}), \vec{b}_2 = (5\hat{i} + \hat{j})$$

We know that, the shortest distance between  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by

$$\text{S.D.} = \frac{|\vec{a}_2 - \vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \dots (i)$$

$$\begin{aligned}(\vec{a}_2 - \vec{a}_1) &= (-\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} - \hat{j}) \\ &= -\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} + \hat{j} \\ &= -2\hat{i} + 3\hat{j} + 2\hat{k}\end{aligned}$$

$$\begin{aligned}\vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix} \\ &= \hat{i}(0 - 1) - \hat{j}(0 - 5) + \hat{k}(2 - 15)\end{aligned}$$

$$\vec{b}_1 \times \vec{b}_2 = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$\begin{aligned}(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (-2\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 5\hat{j} - 13\hat{k}) \\ &= (-2)(-1) + (3)(5) + (2)(-13) \\ &= -9\end{aligned}$$

$$\begin{aligned}|\vec{b}_1 \times \vec{b}_2| &= \sqrt{(-1)^2 + (5)^2 + (-13)^2} \\ &= \sqrt{1 + 25 + 169} \\ &= \sqrt{195}\end{aligned}$$

Substituting the value of  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$  and  $|\vec{b}_1 \times \vec{b}_2|$  in equation (i) to get shortest distance between given lines, so

$$\begin{aligned}\text{S.D.} &= \frac{|-9|}{\sqrt{195}} \\ &= \frac{9}{\sqrt{195}} \text{ units}\end{aligned}$$

Since, shortest distance between given lines is not zero, so lines are not intersecting.



### Straight Line in Space Ex 28.5 Q3(iv)

Given lines are,

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} = \lambda \text{ (say)}$$

$$\Rightarrow x = 4\lambda + 5, y = -5\lambda + 7, z = -5\lambda - 3$$

$$\begin{aligned}\vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (4\lambda + 5)\hat{i} + (-5\lambda + 7)\hat{j} + (-5\lambda - 3)\hat{k} \\ \vec{r} &= (5\hat{i} + 7\hat{j} - 3\hat{k}) + \lambda(4\hat{i} - 5\hat{j} - 5\hat{k})\end{aligned}$$

$$\Rightarrow \vec{a}_1 = (5\hat{i} + 7\hat{j} - 3\hat{k}), \vec{b}_1 = (4\hat{i} - 5\hat{j} - 5\hat{k})$$

$$\text{and, } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3} = \mu \text{ (say)}$$

$$\Rightarrow x = 7\mu + 8, y = \mu + 7, z = 3\mu + 5$$

$$\begin{aligned}\vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (7\mu + 8)\hat{i} + (\mu + 7)\hat{j} + (3\mu + 5)\hat{k} \\ \vec{r} &= (8\hat{i} + 7\hat{j} + 5\hat{k}) + \mu(7\hat{i} + \hat{j} + 3\hat{k})\end{aligned}$$

$$\Rightarrow \vec{a}_2 = (8\hat{i} + 7\hat{j} + 5\hat{k}), \vec{b}_2 = (7\hat{i} + \hat{j} + 3\hat{k})$$

We know that, shortest distance between lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by

$$\text{S.D.} = \frac{|\vec{a}_2 - \vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \dots (i)$$

$$\begin{aligned}(\vec{a}_2 - \vec{a}_1) &= (8\hat{i} + 7\hat{j} + 5\hat{k}) - (5\hat{i} + 7\hat{j} - 3\hat{k}) \\ &= 8\hat{i} + 7\hat{j} + 5\hat{k} - 5\hat{i} - 7\hat{j} + 3\hat{k} \\ &= 3\hat{i} + 8\hat{k}\end{aligned}$$

$$\begin{aligned}\vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix} \\ &= \hat{i}(-15 + 5) - \hat{j}(12 + 35) + \hat{k}(4 + 35) \\ &= -10\hat{i} - 47\hat{j} + 39\hat{k}\end{aligned}$$

$$\begin{aligned}(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (3\hat{i} + 8\hat{k}) \cdot (-10\hat{i} - 47\hat{j} + 39\hat{k}) \\ &= (3)(-10) + (0)(-47) + (8)(39) \\ &= -30 + 312 \\ &= 282\end{aligned}$$

Using equation (i) to get the shortest distance between the given lines, so

$$\text{S.D.} = \frac{282}{|\vec{b}_1 \times \vec{b}_2|}$$

$$\text{S.D.} \neq 0$$

Since, the shortest distance between given lines is not equal to zero, so

Given lines are not intersecting.

### Straight Line in Space Ex 28.5 Q4(i)

Given, equation of lines are,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \quad \text{--- (1)}$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(-\hat{i} + \hat{j} - \hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) - \mu(\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k}) \quad \text{--- (2)}$$

These two lines pass through the points having position vectors  $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$  and  $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$  respectively and both are parallel to the vector  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

We know that, shortest distance between parallel lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}$  and  $\vec{r} = \vec{a}_2 + \mu\vec{b}$  is given by

$$\text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot \vec{b}|}{|\vec{b}|} \quad \text{--- (i)}$$

$$(\vec{a}_2 - \vec{a}_1) = (2\hat{i} - \hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= 2\hat{i} - \hat{j} - \hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) = \hat{i} - 3\hat{j} - 4\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(-3 - 4) - \hat{j}(1 + 4) + \hat{k}(-1 + 3)$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$$

$$|(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = \sqrt{(-7)^2 + (-5)^2 + (2)^2}$$

$$= \sqrt{49 + 25 + 4}$$

$$= \sqrt{78}$$

$$|\vec{b}| = \sqrt{\hat{i}^2 + \hat{j}^2 + \hat{k}^2}$$

$$= \sqrt{(1)^2 + (-1)^2 + (1)^2}$$

$$|\vec{b}| = \sqrt{3}$$

Using  $|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|$  and  $|\vec{b}|$  in equation (1) to get the shortest distance between parallel lines, so

$$\text{S.D.} = \frac{\sqrt{78}}{\sqrt{3}}$$

$$\text{S.D.} = \sqrt{\frac{78}{3}}$$

$$\text{S.D.} = \sqrt{26} \text{ units}$$

### Straight Line in Space Ex 28.5 Q4(ii)

Given, equation of lines are,

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda (2\hat{i} - \hat{j} + \hat{k}) \quad \text{--- (1)}$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu (4\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + 2\mu (2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu' (2\hat{i} - \hat{j} + \hat{k}) \quad \text{--- (2)}$$

So,  $\vec{a}_1 = (\hat{i} + \hat{j}), \vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$   
 $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

We know that, the shortest distance between the parallel lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}$  and  $\vec{r} = \vec{a}_2 + \lambda \vec{b}$  is given by

$$\text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|} \quad \text{--- (i)}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) &= (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j}) \\ &= 2\hat{i} + \hat{j} - \hat{k} - \hat{i} - \hat{j} \\ (\vec{a}_2 - \vec{a}_1) &= \hat{i} - \hat{k} \end{aligned}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix} \\ &= \hat{i}(0-1) - \hat{j}(1+2) + \hat{k}(-1-0) \\ (\vec{a}_2 - \vec{a}_1) \times \vec{b} &= -\hat{i} - 3\hat{j} - \hat{k} \end{aligned}$$

$$\begin{aligned} |(\vec{a}_2 - \vec{a}_1) \times \vec{b}| &= \sqrt{(-1)^2 + (-3)^2 + (-1)^2} \\ &= \sqrt{1+9+1} \\ |(\vec{a}_2 - \vec{a}_1) \times \vec{b}| &= \sqrt{11} \end{aligned}$$

$$\begin{aligned} |\vec{b}| &= \sqrt{(2)^2 + (-1)^2 + (1)^2} \\ &= \sqrt{4+1+1} \\ |\vec{b}| &= \sqrt{6} \end{aligned}$$

Using  $|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|$  and  $|\vec{b}|$  in equation (1) to get the shortest distance between the given lines, so

$$\text{S.D.} = \frac{\sqrt{11}}{\sqrt{6}}$$

$$\text{S.D.} = \sqrt{\frac{11}{6}} \text{ units}$$

### Straight Line in Space Ex 28.5 Q5

Equation of line passing through  $(0,0,0)$  and  $(1,0,2)$  is given by  $\vec{r} = \vec{a} + \lambda \vec{b}$

$$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda((1-0)\hat{i} + (0-0)\hat{j} + (2-0)\hat{k})$$

$$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(\hat{i} + 2\hat{k}) \quad \text{--- (1)}$$

Equation of another line passing through  $(1,3,0)$  and  $(0,3,0)$  is

$$\vec{r} = (\hat{i} + 3\hat{j} + 0\hat{k}) + \mu((0-1)\hat{i} + (3-3)\hat{j} + (0-0)\hat{k})$$

$$\vec{r} = (\hat{i} + 3\hat{j} + 0\hat{k}) + \mu(-\hat{i}) \quad \text{--- (2)}$$

From equation (1) and (2)

$$\vec{a}_1 = (0\hat{i} + 0\hat{j} + 0\hat{k}), \quad \vec{b}_1 = (\hat{i} + 2\hat{k})$$

$$\vec{a}_2 = (\hat{i} + 3\hat{j} + 0\hat{k}), \quad \vec{b}_2 = -\hat{i}$$

We know that, shortest distance between the lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  is given by

$$\text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \text{--- (3)}$$

$$(\vec{a}_2 - \vec{a}_1) = (\hat{i} + 3\hat{j} + 0\hat{k}) - (0\hat{i} + 0\hat{j} + 0\hat{k})$$

$$(\vec{a}_2 - \vec{a}_1) = (\hat{i} + 3\hat{j})$$

$$(\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -1 & 0 & 0 \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0+2) + \hat{k}(-2)$$

$$(\vec{b}_1 \times \vec{b}_2) = -2\hat{j}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (\hat{i} + 3\hat{j}) \cdot (-2\hat{j})$$

$$= (1)(0) + (3)(-2)$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = -6$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-2)^2}$$

$$|\vec{b}_1 \times \vec{b}_2| = 2$$

Using  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$  and  $|\vec{b}_1 \times \vec{b}_2|$  in equation (3) to get shortest distance between the lines, so

$$\text{S.D.} = \left| \frac{-6}{2} \right|$$

$$\text{S.D.} = 3 \text{ units}$$



# Straight Line in Space Ex 28.5 Q6

Given equations of lines are,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} = \lambda \text{ (say)}$$

$$\Rightarrow x = 2\lambda + 1, y = 3\lambda + 2, z = 6\lambda - 4$$

$$\begin{aligned} \Rightarrow \vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (2\lambda + 1)\hat{i} + (3\lambda + 2)\hat{j} + (6\lambda - 4)\hat{k} \\ \vec{r} &= (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \end{aligned}$$

$$\Rightarrow \vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Another equation of line is,

$$\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12} = \mu \text{ (say)}$$

$$\Rightarrow x = 4\mu + 3, y = 6\mu + 3, z = 12\mu - 5$$

$$\begin{aligned} \Rightarrow \vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ &= (4\mu + 3)\hat{i} + (6\mu + 3)\hat{j} + (12\mu - 5)\hat{k} \\ &= (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k}) \\ &= (3\hat{i} + 3\hat{j} - 5\hat{k}) + 2\mu(2\hat{i} + 3\hat{j} + 6\hat{k}) \\ \vec{r} &= (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k}) \end{aligned}$$

$$\Rightarrow \vec{a}_2 = (3\hat{i} + 3\hat{j} - 5\hat{k}), \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

We know that, shortest distance between parallel lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}$  and  $\vec{r} = \vec{a}_2 + \mu\vec{b}$  is given by

$$\text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|} \quad \text{--- (i)}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) &= (3\hat{i} + 3\hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} - 4\hat{k}) \\ &= 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k} \\ (\vec{a}_2 - \vec{a}_1) &= 2\hat{i} + \hat{j} - \hat{k} \end{aligned}$$

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} \\ &= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2) \\ &= 9\hat{i} - 14\hat{j} + 4\hat{k} \end{aligned}$$

$$\begin{aligned} |(\vec{a}_2 - \vec{a}_1) \times \vec{b}| &= \sqrt{(9)^2 + (-14)^2 + (4)^2} \\ &= \sqrt{81 + 196 + 16} \\ &= \sqrt{293} \end{aligned}$$

$$\begin{aligned} |\vec{b}| &= \sqrt{(2)^2 + (3)^2 + (6)^2} \\ &= \sqrt{4 + 9 + 36} \\ &= \sqrt{49} \\ |\vec{b}| &= 7 \end{aligned}$$

Using  $|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|$  and  $|\vec{b}|$  in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \frac{\sqrt{293}}{7} \text{ units}$$





### Straight Line in Space Ex 28.5 Q7(i)

Here,

$$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 2\hat{i} - \hat{j} - \hat{k} - \hat{i} - 2\hat{j} - \hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2) = -3\hat{i} + 3\hat{k}$$

The shortest distance between the two lines,

$$d = \frac{|(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$d = \frac{|(-3\hat{i} + 3\hat{k}) \cdot (\hat{i} - 3\hat{j} - 2\hat{k})|}{|-3\hat{i} + 3\hat{k}|} = \frac{|-3-6|}{\sqrt{(-3)^2 + (-3)^2}} = \frac{9}{3\sqrt{2}}$$

The shortest distance between the two lines =  $\frac{3}{\sqrt{2}}$  units

### Straight Line in Space Ex 28.5 Q7(ii)

Here,

$$\vec{a}_1 = -\hat{i} - \hat{j} - \hat{k}$$

$$\vec{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}$$

$$\vec{b}_1 = 7\hat{i} - 6\hat{j} + \hat{k}$$

$$\vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}(-6+2) - \hat{j}(7-1) + \hat{k}(-14+6)$$

$$= -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i}(3+1) + \hat{j}(5+1) + \hat{k}(7+1)$$

$$= 4\hat{i} + 6\hat{j} + 8\hat{k}$$

The shortest distance between two lines,

$$d = \frac{|(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \frac{|(-4\hat{i} - 6\hat{j} - 8\hat{k}) \cdot (4\hat{i} + 6\hat{j} + 8\hat{k})|}{\sqrt{(-4)^2 + (-6)^2 + (-8)^2}}$$

$$= \frac{|-16-36-64|}{\sqrt{116}}$$

$$= \frac{|-116|}{\sqrt{116}}$$

$$= 2\sqrt{29} \text{ units}$$



## Straight Line in Space Ex 28.5 Q7(iii)

Here,

$$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k},$$

$$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k},$$

$$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) = 4\hat{i} + 5\hat{j} + 6\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2$$

$$= (\hat{i} - 3\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} + \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3+6)$$

$$= -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\text{Shortest distance between the two lines} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$= \left| \frac{(3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot (-9\hat{i} + 3\hat{j} + 9\hat{k})}{|-9\hat{i} + 3\hat{j} + 9\hat{k}|} \right|$$

$$= \left| \frac{3 \times (-9) + 3 \times 3 + 3 \times 9}{\sqrt{(-9)^2 + 3^2 + 9^2}} \right|$$

$$= \left| \frac{-27 + 9 + 27}{\sqrt{(-9)^2 + 3^2 + 9^2}} \right|$$

$$= \left| \frac{9}{\sqrt{171}} \right| = \frac{3}{\sqrt{19}} \text{ units}$$

## Straight Line in Space Ex 28.5 Q7(iv)

Here,

$$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = -4\hat{i} - \hat{k}$$

$$\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = -4\hat{i} - \hat{k} - 6\hat{i} - 2\hat{j} - 2\hat{k}$$

$$= -10\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$$= \hat{i}(4+4) - \hat{j}(-2-6) + \hat{k}(-2+6)$$

$$= 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$$\text{Shortest Distance} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$= \left| \frac{(-10\hat{i} - 2\hat{j} - 3\hat{k}) \cdot (8\hat{i} + 8\hat{j} + 4\hat{k})}{|8\hat{i} + 8\hat{j} + 4\hat{k}|} \right|$$

$$= \left| \frac{(-10) \times 8 + (-2) \times 8 + (-3) \times 4}{\sqrt{8^2 + (-8)^2 + (-4)^2}} \right|$$

$$= \left| \frac{-80 - 16 - 12}{\sqrt{64 + 64 + 16}} \right| = \left| \frac{-108}{\sqrt{144}} \right| = 9 \text{ units}$$



### Straight Line in Space Ex 28.5 Q8

$$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$$

$$= 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2)$$

$$= 9\hat{i} - 14\hat{j} + 4\hat{k}$$

Shortest distance between 2 lines

$$= \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

$$= \frac{|9\hat{i} - 14\hat{j} + 4\hat{k}|}{\sqrt{2^2 + 3^2 + 6^2}}$$

$$= \frac{|9\hat{i} - 14\hat{j} + 4\hat{k}|}{\sqrt{49}}$$

$$= \frac{|\sqrt{9^2 + (-14)^2 + 4^2}|}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{\sqrt{49}} = \frac{\sqrt{293}}{7} \text{ units}$$

