THE PLANE (XII, R. S. AGGARWAL)

EXERCISE 28 A [Pg. No.: 1166]

1. Find the equation of the plane passing through each set of points given below:

(i) $A(2,2,-1), B(3,4,2)$ and $C(7,0,6)$ (ii) $A(0,-1,-1), B(4,5,1)$ and $C(3,9,4)$

(iii) $A(-2,6,-6), B(-3,10,-9)$ and $C(-5,0,-6)$

Sol. (i) $A(2,2,-1), B(3,4,2)$ and $C(7,0,6)$

The general equation of a plane passing through the point $A(2,2,-1)$ is given by

$a(x-2)+b(y-2)+c(z+1)=0$ ... (i)

Since it passes through the point $B(3,4,2)$ and $C(7,0,6)$ we have $a(3-2)+b(4-2)+c(2+1)=0$

$\Rightarrow a+2b+3c=0$ ... (ii)

$a(7-2)+b(0-2)+c(6+1)=0$

$\Rightarrow 5a-2b+7c=0$ ... (iii)

Cross multiplying (ii) & (iii) we get $\dfrac{a}{14-(-6)}=\dfrac{b}{15-7}=\dfrac{c}{-2-10}=\lambda$

$\Rightarrow \dfrac{a}{20}=\dfrac{b}{8}=\dfrac{c}{-2-10}=\lambda \Rightarrow \dfrac{a}{20}=\dfrac{b}{8}=\dfrac{c}{-12}=\lambda$

$\Rightarrow \dfrac{a}{5}=\dfrac{b}{2}=\dfrac{c}{-3}=\lambda \Rightarrow a=5\lambda, b=2\lambda, c=-3\lambda$

Substituting $a=5\lambda, b=2\lambda, c=-3\lambda$ in (i) we get

$5\lambda(x-2)+2\lambda(y-2)-3\lambda(z+1)=0 \Rightarrow \lambda(5x-10+2y-4-3z-3)=0$

$\Rightarrow 5x+2y-3z-17=0$

Hence, $5x+2y-3z=17$ is the required equation of the plane.

(ii) $A(0,-1,-1), B(4,5,1)$ and $C(3,9,4)$

The general equation of a plane passing through the points $A(0,-1,-1)$ is given by

$a(x-0)+b(y+1)+c(z+1)=0$ ... (i)

Since if passes through the point $B(4,5,1)$ and $C(3,9,4)$ we have

$a(4-0)+b(5+1)+c(1+1)=0$

$\Rightarrow 4a+6b+2c=0$

$\Rightarrow 2a+3b+c=0$ ... (ii)

$a(3-0)+b(9+1)+c(4+1)=0$

$\Rightarrow 3a+10b+5c=0$ ... (iii)

Cross multiplying (ii) and (iii) we have $\dfrac{a}{15-10}=\dfrac{b}{3-10}=\dfrac{c}{20-9}=\lambda$

$\Rightarrow \frac{a}{5} = \frac{b}{-7} = \frac{c}{11} = \lambda \Rightarrow a = 5\lambda, b = -7\lambda, c = 11\lambda$

Substituting $a = 5\lambda, b = -7\lambda,$ and $c = 11\lambda$ in (i) we get

$5\lambda(x) - 7\lambda(y+1) + 11\lambda(z+1) = 0$

$\Rightarrow \lambda(5x - 7y - 7 + 11z + 11) = 0 \Rightarrow 5x - 7y + 11z + 4 = 0$

Hence $5x - 7y + 11z + 4 = 0$ is the required equation of the plane.

(iii) $A(-2,6,-6), B(-3,10,-9)$ and $C(-5,0,-6)$

The general equation of a plane passing through the point $A(-2,6,-6)$ is given by

$a(x+2) + b(y-6) + c(z+6) = 0$ ... (i)

Since it passes through the point $B(-3,10,-9)$ and $C(-5,0,-6)$

We have, $a(-3+2) + b(10-6) + c(-9+6) = 0$

$\Rightarrow -a + 4b - 3c = 0$ ... (i)

$a(-5+2) + b(0-6) + c(-6+6) = 0$

$\Rightarrow -3a - 6b + 0c = 0$

$\Rightarrow a + 2b - 0c = 0$ ... (ii)

Cross multiplying (ii) and (iii) we get $\frac{a}{0+6} = \frac{b}{-3-0} = \frac{c}{-2-4} = \lambda \Rightarrow \frac{a}{6} = \frac{b}{-3} = \frac{c}{-6} = \lambda$

$\Rightarrow \frac{a}{2} = \frac{b}{-1} = \frac{c}{-2} = k \Rightarrow a = 2k, b = -k, c = -2k$

Substituting $a = 2k, b = -k,$ and $c = -2k$

$2k(x+2) - k(y-6) - 2k(z+6) = 0$

$\Rightarrow k(2x + 4 - y + 6 - 2z - 12) = 0 \Rightarrow 2x - y - 2z - 2 = 0$

Hence, $2x - y - 2z = 2$ is the required equation of the plane.

2. Show that the four points $A(3,2,-5), B(-1,4,-3), C(-3,8,-5)$ and $D(-3,2,1)$ are coplanar. Find the equation of the plane containing them

Sol. The equation of the plane passing through the point $A(3,2,-5)$ is

$a(x-3) + b(y-2) + c(z+5) = 0$

It is passes through $B(-1,4,-3)$ and $C(-3,8,-5)$, we have

$a(1-3) + b(4-2) + c(-3+5) = 0 \Rightarrow -4a + 2b + 2c = 0 \Rightarrow 2a - b - c = 0$

$a(-3-3) + b(8-2) + c(5-+5) = 0 \Rightarrow -6a + 6b + 0c = 0 \Rightarrow a - b - 0c = 0$

Cross multiplying (ii) and (iii) we get $\frac{a}{(0-1)} = \frac{b}{(-1-0)} = \frac{c}{(-2+1)}$

$\Rightarrow \frac{a}{-1} = \frac{b}{-1} = \frac{c}{-1} \Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{1} = k$ (say)

$\Rightarrow a = k, b = k, c = k$

Putting $a = k, b = k$ and $c = k$ in (i) we get $k(x-3) + k(y-2) + k(z+5) = 0$

$\Rightarrow (x-3) + (y-2) + (z+5) = 0 \Rightarrow x + y + z = 0$

Thus, the equation of the plane passing through the points $A(3,2,-5), B(-1,4,-3)$ and $C(-3,8,-5)$ is $x+y+z=0$.

Clearly the fourth point $D(-3,2,1)$ also satisfies $x+y+z=0$

Hence the given four points are coplanar, and the equation of the plane containing them is $x+y+z=0$

3. Show that the four points $A(0,-1,0), B(2,1,-1), C(1,1,1)$ and $D(3,3,0)$ are coplanar. Find the equation of the plane containing them

Sol. P.V. of A, $\vec{a}=-\hat{j}$

P.V. of $B, \vec{b}=2\hat{i}+\hat{j}-\hat{k}$

P.V. of $C, \vec{c}=\hat{i}+\hat{j}+\hat{k}$

Now, $\vec{b}-\vec{a}=(2\hat{i}+\hat{j}-\hat{k})-(-\hat{j})$

$\vec{c}-\vec{a}=(\hat{i}+\hat{j}+\hat{k})-(-\hat{j}) \quad =\hat{i}+2\hat{j}+\hat{k}$

$$\therefore (\vec{b}-\vec{a})\times(\vec{c}-\vec{a})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & -1\\ 1 & 2 & 1\end{vmatrix} = \begin{vmatrix}2 & -1\\ 2 & 1\end{vmatrix}\hat{i}-\begin{vmatrix}2 & -1\\ 1 & 1\end{vmatrix}\hat{j}+\begin{vmatrix}2 & 2\\ 1 & 2\end{vmatrix}\hat{k}$$

$=(2+2)\hat{i}-(2+1)\hat{j}+(4-2)\hat{k} \quad =4\hat{i}-3\hat{j}+2\hat{k}$

Now $\vec{r}-\vec{a}=(x\hat{i}+y\hat{j}+z\hat{k})-(-\hat{j}) \quad =x\hat{i}+(y+1)\hat{j}+z\hat{k}$

Equation of plate passing through three non collinear points with position vectors $\vec{a}, \vec{b}, \vec{c}$ is

$(\vec{r}-a)\cdot\left[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right]=0$

$\therefore$ Equation of plane passes through A, B and C is $(x\hat{i}+(y+1)\hat{j}+z\hat{k})\cdot(4\hat{i}-3\hat{j}+2\hat{k})=0$

$\Rightarrow 4x-3(y+1)+2z=0 \quad \Rightarrow 4x-3y+2z-3=0$

Putting $x=0, y=3 \,\&\, z=0$ is the equation of plane we have $4\times3-3\times3+2\times0-3=0$

$\Rightarrow 12-9-3=0 \Rightarrow 0=0$ which is true

Hence A, B, C & D are coplanar proved

4. Write the equation of the plane whose intercepts on the coordinate axes are 2, -4 and 5 respectively

Sol. It make intercepts 2, –4, and 5 with the co–ordinates axes. Then the equation of the variable plane is

$$\Rightarrow \frac{x}{2}+\frac{y}{-4}+\frac{z}{5}=1 \Rightarrow \frac{10x-5y+4z}{20}=1 \Rightarrow 10x-5y+4z=20$$

Hence, the required equation at plane is $10x-5y+4z=20$

5. Reduce the equation of the plane $4x-3y+2=12$ to the intercept from and hence find the intercepts made by the plane with the coordinate axis

Sol. Given plane is $4x-3y+2z=12$

$$\Rightarrow \frac{4}{12}x-\frac{3}{12}y+\frac{2}{12}z=1 \quad \Rightarrow \frac{x}{3}+\frac{y}{-4}+\frac{}{6}=1$$

This is the required equation of plane in the intercept term

Here x-intercept = 3

y-intercept = - 4

z-intercept = 6

6. Find the equation of the plane which passes through the point $(2,-3,7)$ and makes equal intercept on the coordinate axes

Sol. Let the plane makes intercept on each at the co-ordinate axes

Then its equations is

$\frac{x}{a}+\frac{y}{a}+\frac{z}{a}=1 \Rightarrow x+y+z=9$

Putting $x=2, y=-3, z=7$ we get $2-3+7=9 \Rightarrow a=6$

So, the required equation of the plane is $x+y+z=6$

7. A plane meets the coordinate axes at A, B and C respectively such that the centroid of ΔABC is $(1,-2,3)$. Find the equation of the plane

Sol. Let the plane meet the coordinate axes at $A(a,0,0), B(0,b,0)$ and $C(0,0,c)$

Since the centrooid of ΔABC is $G(1,-2,3]$ we get $\frac{a+0+0}{3}=1, \frac{0+b+0}{3}=-2$

and $\frac{0+0+c}{3}=3$

$\Rightarrow a=3, b=-6$ and $c=9$ Hence equation of plane is $\frac{x}{3}+\frac{y}{-6}+\frac{c}{9}=1$

8. Find the Cartesian and vector equations of a plane passing through the point $(1,2,3)$ and perpendicular to a line with direction ratios $2,3,-4$

Sol. Any plane passing through the point $(1,2,3)$ is given by $a(x-1)+b(y-2)+c(z-3)=0$ ... (i)

Since the plane perpendicular the to a line with direction ratios $2,3,-4$

$\therefore a=2, b=3 \,\&\, c=-4$

Putting the value of a,b and c in equation (i) we value

$2(x-1)+3(y-2)-4(z-3)=0$

$\Rightarrow 2x-2+3y-6-4z+12=0 \Rightarrow 2x+3y-4z+4=0$

This is the Cartesian equation at plane equation of plane in vector form

$\vec{r}\cdot(2\hat{i}+3\hat{j}-4\hat{k})+4=0$

9. If O be the origin and $P(1,2,-3)$ be a given point, then find the equation of the plane passing through P and perpendicular to OP

Sol. Let the required equation the plane passing through the point $A(1,2,-3)$ be $a(x-1)+b(y-2)+c(z+3)=0$

D.r.'s of OP are $(1-0),(-3-0)$, i.e. $1,2,-3$

Since the plane is perpendicular to OP, so the normal to the plane is parallel to OP

$\therefore \frac{a}{1}=\frac{b}{2}=\frac{c}{-3}=k$ (say) $\Rightarrow a=k, b=2k$ and $c=-3k$

$\therefore$ required equation of the plane I $k(x-1)+2k(y-2)-3k(z+3)=0$

$\Rightarrow (x-1)+2(y-2)-3(z+3)=0 \Rightarrow (x+2y-3z)+(-1-4-9)=0$

$\Rightarrow x+2y-3z-14=0$

EXERCISE 28 B [Pg. No.: 1181]

1. Find the vector equation of a plane which is at a distance of 5 units from the origin and which has $\hat{k}$ as the unit vector normal to it.

Sol. Clearly, the required equation of the plane is $\vec{r}.\hat{k}=5$.

2. Find the vector and Cartesian equations of a plane which is at a distance of 7 units from the origin and whose normal vector from the origin is $(3\hat{i}+5\hat{j}-6\hat{k})$

Sol. Here normal vector from the origin $\vec{n}=3\hat{i}+5\hat{j}-6\hat{k}$

$\Rightarrow|\vec{n}|=\sqrt{3^2+5^2+(-6)^2}=\sqrt{9+25+36}=\sqrt{70}$

$\therefore$ unit vector normal to the plane $\hat{n}=\dfrac{\vec{n}}{|\vec{n}|}\Rightarrow\hat{n}=\dfrac{1}{\sqrt{70}}(3\hat{i}+5\hat{j}-6\hat{k})$

Distance between origin and plane P = 7 units

So the vector equation of the plane is $\vec{r}\cdot\hat{n}=P\Rightarrow\vec{r}\left(\dfrac{3}{\sqrt{70}}\hat{i}+\dfrac{5}{\sqrt{70}}\hat{j}-\dfrac{6}{\sqrt{70}}\hat{k}\right)=7$

$\Rightarrow\vec{r}(3\hat{i}+5\hat{j}-6\hat{k})=7\sqrt{70}$

Hence the required vector equation of the plane is

$\vec{r}\cdot(3\hat{i}+5\hat{j}-6\hat{k})=7\sqrt{70}$

In Cartesian from $3x+5y-6z=7\sqrt{70}$

3. Find the vector and Cartesian equations of a plane which is at a distance of $\dfrac{6}{\sqrt{29}}$ from the origin and whose normal vector from the origin is $(2\hat{i}-3\hat{j}+4\hat{k})$

Sol. Here normal vector from the origin $\vec{n}=2\hat{i}-3\hat{j}+4\hat{k}$

$\Rightarrow|\vec{n}|=\sqrt{2^2+(-3)^2+4^2}=\sqrt{29}$

Unit vector normal to the plane $\hat{n}=\dfrac{\vec{n}}{|\vec{n}|}\Rightarrow\hat{n}=\dfrac{2}{\sqrt{29}}\hat{i}-\dfrac{3}{\sqrt{29}}\hat{j}+\dfrac{4}{\sqrt{29}}\hat{k}$

Distance between origin and plane $P=\dfrac{6}{\sqrt{29}}$ units

So the vector equation of the plane is $\vec{r}\left\{\dfrac{2}{\sqrt{29}}\hat{i}-\dfrac{3}{\sqrt{29}}\hat{j}+\dfrac{4}{\sqrt{29}}\hat{k}\right\}=\dfrac{6}{\sqrt{29}}$

$\Rightarrow\vec{r}(2\hat{i}-3\hat{j}+4\hat{k})$

Hence the required vector equation of the plane is $\vec{r}(2\hat{i}-3\hat{j}+4\hat{k})=6$

In Cartesian form $2x-3y+4z=6$

4. Find the vector and Cartesian equations of the plane which is at a distance of 6 units from the origin and which has a normal with direction ratios $2,-1,-2$.

Sol. Here, $\vec{n}=(2\hat{i}-\hat{j}-2\hat{k})\Rightarrow|\vec{n}|=\sqrt{(2)^2+(-1)^2+(-2)^2}=\sqrt{4+1+4}=\sqrt{9}=3$ and $p=6$

$$\therefore \frac{\vec{r}.\vec{n}}{|\vec{n}|}=p \Rightarrow \vec{r}.\frac{(2\hat{i}-\hat{j}-2\hat{k})}{3}=6 \Rightarrow \vec{r}(2\hat{i}-\hat{j}-2\hat{k})=18$$

Hence, the required equation of the plane is Put, $\vec{r}=(x\hat{i}+y\hat{j}+z\hat{k})$

$$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(2\hat{i}-\hat{j}-2\hat{k})=18 \Rightarrow 2x-y-2z=18$$

5. Find the vector and Cartesian equations of a plane which passes through the point $(1,4,6)$ and normal vector to the plane is $(\hat{i}-2\hat{j}+\hat{k})$

Sol. Any plane passes through the point $(1,7,6)$ is given by

$$a(x-1)+b(y-4)+c(z-6)=0 \quad \text{.... (i)}$$

a/q normal vector to the plane is $\vec{n}=\hat{i}-2\hat{j}+\hat{k}$

$\therefore a=1, b=-2$ and $c=1$

Putting the value of a,b and c in equation (i) we have

$$x-1-2(t-4)+(z-6)=0 \Rightarrow x-2y+z+1=0$$

Hence Cartesian equation of plan is $x-2y++1=0$

In vector form $\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k})+1=0$

6. Find the length of perpendicular from the origin to the plane $\vec{r}\cdot(3\hat{i}-12\hat{j}-4\hat{k})+39=0$. Also write the unit normal vector from the origin to the plane

Sol. Given equation of plane is $\vec{r}(3\hat{i}-12\hat{j}-4\hat{k})+39=0$

$$\Rightarrow \vec{r}\cdot(3\hat{i}-12\hat{j}-4\hat{k})=-39 \Rightarrow \vec{r}(-3\hat{i}+12\hat{j}+4\hat{k})=39$$

$$\Rightarrow \vec{r}\frac{(-3\hat{i}+12\hat{j}+4\hat{k})}{|-3\hat{i}+12\hat{j}+4\hat{k}|}=\frac{39}{|-3\hat{i}+12\hat{j}+4\hat{k}|}$$

$$\Rightarrow \vec{r}\frac{(-3\hat{i}+12\hat{j}+4\hat{k})}{13}=\frac{39}{13} \Rightarrow \vec{r}\left(-\frac{3}{13}\hat{i}+\frac{12}{13}\hat{j}+\frac{4}{13}\hat{k}\right)=3$$

Hence unit vector of normal to the plane is $\left(-\frac{3}{19}\hat{i}+\frac{12}{13}\hat{j}+\frac{4}{19}\hat{k}\right)$

And distance of plane from the origin is 3

7. Find the Cartesian equation of the plane whose vector equation is $\vec{r}\cdot(3\hat{i}+5\hat{j}-9\hat{k})=8$

Sol. Given equation of plane is $\vec{r}(3\hat{i}+5\hat{j}-9\hat{k})=8$

$$\Rightarrow (x\hat{i}+y\hat{j}+\hat{k})(3\hat{i}+5\hat{j}-9\hat{k})=8 \Rightarrow 3x+5y-9z=8$$

Hence Cartesian equation of plane is $3x+5y-9z=8$

8. Find the vector equation of a plane whose Cartesian equation is $5x-7y+2z+4=0$

Sol. Given equation of plan eis $5x-7y+2z+4=0$

$$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})(5\hat{i}-7\hat{j}+2\hat{k})+4=0$$

$$\Rightarrow r(5\hat{i}-7\hat{j}+2\hat{k})+4=0$$

Hence the vector equation of plane is $\vec{r}\left(5\hat{i}-7\hat{j}+2\hat{k}\right)+4=0$

9. Find a unit vector normal to the plane $x-2y+2z=6$

Sol. Equation of plane is $x-2y+2z=6$

Here direction ratios normal to the plane are $1,-2,2$

$\therefore$ A vector normal to the plane $\vec{n}=\hat{i}-2\hat{j}+2\hat{k}$

$\Rightarrow |\vec{n}|=\sqrt{1^2+(-2)^2+2^2}=3$

Now, $\hat{n}=\frac{\vec{n}}{|\vec{n}|}=\frac{1}{3}\hat{i}-\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$

Hence unit vector normal to the plane is $\hat{n}=\frac{1}{3}\hat{i}-\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$

10. Find the direction cosines of the normal to the plane $3x-6y+2z=7$.

Sol. $3x-6y+2z=7$, The given equation may be written as

$\Rightarrow\left(\frac{3}{7}x-\frac{6}{7}y+\frac{2}{7}z\right)=\frac{7}{7}\Rightarrow\left(\frac{3}{7}x-\frac{6}{7}y+\frac{2}{7}z\right)=1$

Hence, the required direction cosine of a plane is $\left(\frac{3}{7},\frac{-6}{7},\frac{6}{7}\right)$

11. For each of the following planes find the direction cosines of the normal to the plane and the distance of the plane from the origin

(i) $2x+3y-z=5$ (ii) $z=3$ (iii) $3y+5=0$

Sol. (i) given plane is $2x+3y-z=5$

Here direction ratios normal to the plane are $2,3,-1$

Now $\sqrt{2^2+3^2(-1)^2}=\sqrt{4+9+1}=\sqrt{14}$

$\therefore \ell=\frac{2}{\sqrt{14}}, m=\frac{3}{\sqrt{14}}$ and $n=\frac{-1}{\sqrt{14}}$

Direction cosines are $\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$ and $-\frac{1}{\sqrt{14}}$

Distance from the origin $P=\frac{5}{\sqrt{14}}$

(ii) Given plane is $z=3$

Here direction ratios of normal to the plane is $0,0,1$

Now $\sqrt{0^2+0^2+1^2}=1$

$\therefore$ Direction cosines are 0,0,1

And distance from the origin P = 3

(iii) given plane is $3y+5=0$

$\Rightarrow 3y=-5 \quad \Rightarrow -y=\frac{5}{3}$

$\Rightarrow \vec{r}\left(-\hat{j}\right)=\frac{5}{3}$ this is of the form $\vec{r}\cdot\hat{n}=p$

Where $\hat{n}=-\hat{j}$ and $P=\frac{5}{3}$

Hence direction cosines of normal to the plane are $0,-1,0$

And distance from the origin $P=\frac{5}{3}$

12. Find the vector and Cartesian equation of the plane passing through the point $(2,-1,1)$ and perpendicular to the line having direction ratios $4,2,-3$

Sol. Any plane passing through the points $(2,-1,1)$ is given by

$a(x-2)+b(y+1)+c(z-1)=0$ ..... (i)

Since the plane is perpendicular to the line having direction rations $4,2,-3$

$\therefore a=4, b=2$ and $c=-3$

Putting the values at a,b and c in equation (i) we have

$4(x-2)+2(y+1)-3(z-1)=0$

$\Rightarrow 4x+2y-3z-3=0$

Hence Cartesian equation of plane is $4x+2y-3z-3=0$

To vector from $\vec{r}\cdot(4\hat{i}+2\hat{j}-3\hat{k})-3=0$

13. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane

(i) $2x+3y+4z-12=0$ (ii) $5y+8=0$

Sol. (i) equation of line passing through origin and perpendicular to the plane $2x+3y+4z-12=0$

Is given by $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}=\lambda$ (say)

The general point on the line is given by $(2\lambda,3\lambda,4\lambda)$

It the points lies on the plane we have

$2\times 2\lambda+3\times 3\lambda+4\times 4\lambda-12=0$

$\Rightarrow 29\lambda-12=0 \quad \Rightarrow \lambda=\frac{12}{29}$

Hence, the required foot is $\left(\frac{24}{29},\frac{36}{29},\frac{48}{29}\right)$

(ii) equation of line passing through origin and perpendicular to the plane $5y+8=0$ is

Given by $\frac{x}{0}=\frac{y}{5}=\frac{z}{0}=\mu$ (say)

$\Rightarrow x=0,\ y=5\mu \,\&\, z=0$

If $(0,0,5\mu)$ lies on the plane $5(5\mu)+8=0 \quad \Rightarrow \mu=-\frac{8}{25}$

Hence the required foot is $\left(0,5\left(-\frac{8}{25}\right),0\right)$

i.e. $\left(0,-\frac{8}{5},0\right)$

14. Find the coordinates of the foot of the perpendicular from the point $(2,3,7)$ to the plane $3x-y-z=7$. Also, find the length of the perpendicular.

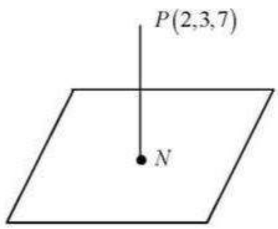
Sol. Let, the given equation of plane is $3x-y-z=7$

$\Rightarrow 3x-y-z-7=0$ ... (i)

The equation of the plane through the point $(2,3,7)$ and perpendicular to the given plane are.

$\Rightarrow \frac{x-2}{3}=\frac{y-3}{-1}=\frac{z-7}{-1}=\lambda \Rightarrow x=3\lambda+2,\ y=-\lambda+3,\ z=-\lambda+7$

$\Rightarrow$ co–ordinate of $N=(3\lambda+2,-\lambda+3,-\lambda+7)$



Satisfied the point in equation (i)

$\Rightarrow 3(3\lambda+2)-(-\lambda+3)-(-\lambda+7)-7=0$

$\Rightarrow 9\lambda+6+\lambda-3+\lambda-7-7=0 \Rightarrow 11\lambda-11=0 \therefore \lambda=1$

Putting the value of λ in co–ordinate of N, then

$\Rightarrow N=\{3(1)+2,-1+3,-1+7\} \Rightarrow N=(3+2,2,6) \therefore N=(5,2,6)$

Length of the perpendicular to the plane

$\Rightarrow PN=\sqrt{(5-2)^2+(2-3)^2+(6-7)^2}=\sqrt{(3)^2+(-1)^2+(-1)^2}=\sqrt{9+1+1}=\sqrt{11}$ units.

15. Find the length and the foot of the perpendicular from the point $(1,1,2)$ to the plane $\vec{r}\cdot(2\hat{i}-2\hat{j}+4\hat{k})+5=0$

Sol. The given point is $P(1,1,2)$

The given plane is $\vec{r}\cdot(2\hat{i}-2\hat{j}+4\hat{k})+5=0$

$\Leftarrow (x\hat{i}+y\hat{j}+z\hat{k})\cdot(2\hat{i}-2\hat{j}+4\hat{k})+5=0$

$\Leftarrow 2x-2y+4z+5=0$ ….. (i)

Any line through $P(1,1,2)$ and perpendicular to the plane (i) is given by

$\frac{x-1}{2}=\frac{y-1}{-2}=\frac{z-2}{4}=\lambda$ (say)

The coordinates of any point N on this line are $(2\lambda+1,-2\lambda+1,4\lambda+2)$. If N is the foot of the perpendicular from P to the given plane then it must lie on the plane (i)

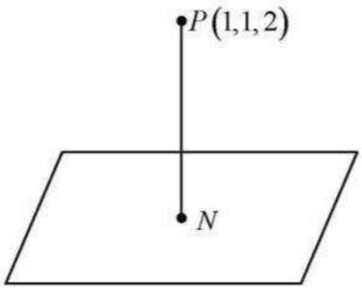
$\therefore 2(2\lambda+1)-2(-2\lambda+1)+4(4\lambda+2)+5=0 \Rightarrow \lambda=\frac{-13}{24}$

Thus, we get the point $N\left(\frac{-1}{12},\frac{25}{12},\frac{-1}{6}\right)$

Hence, the foot of the perpendicular from $P(1,1,2)$ to the given plane is $N\left(\frac{-1}{12},\frac{25}{12},\frac{-1}{6}\right)$

Length of the perpendicular from P to the given plane

$=PN=\sqrt{\left(1+\frac{1}{12}\right)^2+\left(1-\frac{25}{12}\right)^2+\left(2+\frac{1}{6}\right)^2}=\frac{13\sqrt{6}}{12}$ units



16. From the point $P(1,2,4)$ a perpendicular is drawn on the plane $2x+y-2+3=0$. Find the equation the length and the coordinates of the foot of the perpendicular

Sol. Let PN be the perpendicular drawn from the point $P(1,2,4)$ to the plane $2x+y-2z+3=0$

Then, the equation of the line PN is given by

$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-4}{-2}=\lambda \text{ (say)}$$

So, the coordinates of N are $N(2\lambda+1,\lambda+2,-2\lambda+4)$

Since N lies on the plane $2x+y-2z+3=0$, we have $2(2\lambda+1)+(\lambda+2)-2(-2\lambda+4)+3=0$

$$\Rightarrow 9\lambda=1 \Rightarrow \lambda=\frac{1}{9}$$

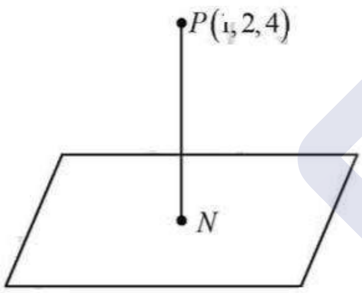
$\therefore$ Coordinates of N are $\left(\frac{2}{9}+1,\frac{1}{9}+2,\frac{-2}{9}+4\right)$, i.e. $\left(\frac{11}{9},\frac{19}{9},\frac{34}{9}\right)$

$$PN=\sqrt{\left(\frac{11}{9}-1\right)^2+\left(\frac{19}{9}-2\right)^2+\left(\frac{34}{9}-4\right)^2}$$

$$=\sqrt{\left(\frac{2}{9}\right)^2+\left(\frac{1}{9}\right)^2+\left(\frac{-2}{9}\right)^2}=\sqrt{\frac{4}{81}+\frac{1}{81}+\frac{4}{81}}=\sqrt{\frac{9}{81}}=\sqrt{\frac{1}{9}}=\frac{1}{3}$$

Thus, the required equation of PN is $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-4}{-2}$

Coordinates of the foot of the perpendicular are $N\left(\frac{11}{9},\frac{19}{9},\frac{34}{9}\right)$ and length $PN=\frac{1}{3}$ unit



17. Find the coordinates of the foot of the perpendicular and the perpendicular distance from the point $P(3,2,1)$ to the plane $2x-y+z+1=0$

Find also the image of the point P in the plane

Sol. Let M be the foot of the perpendicular form the point $P(3,2,1)$ to the plane $2x-y+z+1=0$

Now, equation of PM is $\frac{x-3}{2}=\frac{y-2}{-1}=\frac{z-1}{1}$

Let co-ordinates of M be $\{(2k+3),-k+2,k+1\}$

$\because M$ lies on the plane

$\therefore 2(2k+3)-(2-k)+k+1+1=0$

$\Rightarrow 4k+6-2+k+k+2=0 \Rightarrow 6k+6=0 \Rightarrow k=-1$

Hence the foot is $M(1,3,0)$

Distance between P and M

$PM = \sqrt{(3-1)^2 + (2-3)^2 + (1-0)^2} = \sqrt{4+1+1} = \sqrt{6}$ units

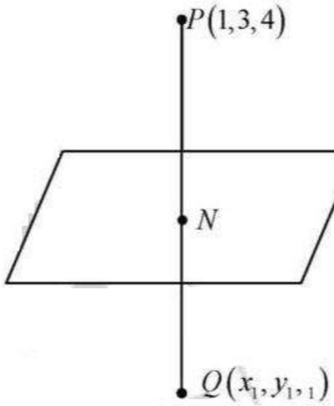
18. Find the coordinates of the image of the point $P(1,3,4)$ in the plane $2x - y + z + 3 = 0$

Sol. Let $Q(x, y_{1}, _{1})$ be the image of the point $P(1,3,4)$ in the given plane

The equations of the line through $P(1,3,4)$ and perpendicular to the given plane are

$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = k$ (say)

The coordinates of a general point on this line are $(2k+1, -k+3, k+4)$



If N is the foot of the perpendicular from P to the given plane then N lies on the plane

$\therefore 2(2k+1) - (-k+3) + (k+4) + 3 = 0$

$\Rightarrow k = -1$

Thus we get the point $N(-1,4,3)$

Now N is the midpoint of PQ

$\therefore \frac{1+x}{2} = -1, \frac{3+y_1}{2} = 4, \frac{4+z_1}{2} = 3$

$\Rightarrow x_1 = -3, y_1 = 5, z_1 = 2$

19. Find the point where the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$ meets the pane $2x + 4y - z = 1$

Sol. Given line is $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4} = k$ (let)

$\Rightarrow x = 2k+1, y = -3k+2$ and $z = 4k-3$ ..... (i)

Given plane is $2x + 4y - z = 1$ ..... (ii)

From (i) and (ii) we have $2(2k+1) + 4(-3k+2) - (4k-3) = 1$

$\Rightarrow 4k + 2 + 8 - 12k - 4k + 3 = 1 \Rightarrow -12k + 13 = 1 \Rightarrow k = 1$

$\therefore x = 3, y = -1$ and $z = 1$

Hence required point I $(3, -1, 1)$

20. Find the coordinates of the point where the line through $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane $2x + y + z = 7$

Sol. Equation of line passes through $(3,-4,-5)$ and $(2,3,-1)$

$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5} = k$ (let)

$\Rightarrow x = 3-k, y = k-4$ and $z = 6k-5$ ..... (i)

Equation of plane is $2x + y + z = 7$ ..... (ii)

From (i) and (ii) we get $2(3-k) + (k-4) + 6k - 5 = 7$

$\Rightarrow 6 - 2k + k - 4 + 6k - 5 = 7$

$\Rightarrow 5k - 3$

$\Rightarrow k = 2$

$\therefore x=1, y=-2$ and $z=7$

Hence the required point is $(1,-2,7)$

21. Find the distance of the point $(2,3,4)$ from the plane $3x+2y+2z+5=0$, measured parallel to the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$

Sol. Let l be the given line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$ and let $P(2,3,4)$ be the given point

Let $PQ \| l$

Then PQ is the line passing through $P(2,3,4)$ and having direction ratios 3,6,2

So, the equations of PQ are

$\frac{x-2}{3}=\frac{y-3}{6}=\frac{z-4}{2}=\lambda$ (say)

The coordinates of any point Q on this line are $(3\lambda+2, 6\lambda+3, 2\lambda+4)$

If this point Q lies on the given plane then $3(3\lambda+2)+2(6\lambda+3)+2(2\lambda+4)+5=0$

$\Leftrightarrow 25\lambda+25=0 \Leftrightarrow 25\lambda=-25 \Leftrightarrow \lambda=-1$

So, the coordinates of Q are $(-1,-3,2)$

$\therefore$ The required distance $= PQ = \sqrt{(2+1)^2+(3+3)^2+(4-2)^2} = \sqrt{49} = 7$ units

22. Find the distance of the point $(0,-3,2)$ from the plane $x+2y-z=1$, measured parallel to the line $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z}{3}$

Sol. Equation of line passing through $(0,-3,2)$ and parallel to the line

$\frac{x+1}{3}=\frac{y+1}{2}=\frac{z}{3}$ is $\frac{x}{3}=\frac{y+3}{2}=\frac{z-2}{3}=k$ (let

$\Rightarrow x=3k, y=2k-3$ and $z=3k+2$

Putting $x=3k, y=2k-3$

And $z=3k+2$ in $x+2y-z=1$ we have $3k+2(2k-3)-(3k+2)=1$

$\Rightarrow 3k+4k-3k-6-2=1$

$\Rightarrow 4k-8=1$

$\Rightarrow 4k=9$

$\Rightarrow k=\frac{9}{4}$

$\therefore x=3\times\frac{9}{4}=\frac{27}{4}$

$y=2\times\frac{9}{4}-3=\frac{3}{2}$

And $z=3\times\frac{9}{4}+2=\frac{35}{4}$

Hence $\left(\frac{27}{4}, \frac{3}{2}, \frac{35}{4}\right)$ is the point of intersation at line through $(0,-3,2)$ which is parallel to the line $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z}{3}$ and the plane $x+2y-z=1$

Now, Required distance

$$=\sqrt{\left(\frac{27}{4}-0\right)^2+\left(\frac{3}{2}+3\right)^2+\left(\frac{35}{4}-2\right)^2} \text{ units}$$

$$=\sqrt{\frac{729}{16}+\frac{81}{4}+\frac{729}{16}} \text{ units}$$

$$=\sqrt{\frac{729+324+729}{16}}=\sqrt{\frac{1782}{16}} \text{ units} = \frac{42.21}{4} \text{ units} = 10.55 \text{ units}$$

23. Find the equation of the line passing through the point $P(4,6,2)$ and the point of intersection of the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z+1}{7}$ and the plane $x+y-z=8$

Sol. Any points on the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z+1}{7}=k$ is given by $R(3k+1,2k,7k-1)$

If R lies on $x+y-z=8$

Then $3k+1+2k-(7k-1)=8$

$\Rightarrow 3k+1+2k-7k+1=8$

$\Rightarrow -2k+2=8 \Rightarrow -2k=8 \Rightarrow k=8$

$\therefore 3k+1=-8$

$2k=8$

$7k-1=-22$

Hence $R(-8,-6,-22)$ is the point of intersection

Now equation of line passing through $(4,6,2)$ and $(-8,-6,-22)$ is

$$\frac{x-4}{-8-4}=\frac{y-6}{-6-6}=\frac{z-2}{-22-2}$$

$$\Rightarrow \frac{x-4}{-12}=\frac{y-6}{-12}=\frac{z-2}{-24} \Rightarrow \frac{x-4}{1}=\frac{y-6}{1}=\frac{z-2}{2}$$

Hence the required equation of line is $\frac{x-4}{1}=\frac{y-6}{2}=\frac{z-2}{2}$

24. Show that the distance of the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$ from the point $(-1,-5,-10)$ is 13 units

Sol. Given line is $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=\lambda$ (let)

A general point on this line is $P(3\lambda+2,4\lambda-1,12\lambda+2)$

If this point lies on this plane $x-y+z=5$, then

$(3\lambda+2)-(4\lambda-1)+(12\lambda+2)=0$

$\Rightarrow 11\lambda+5=5 \Rightarrow \lambda=0$

$\therefore$ point P is $(2,-1,2)$

Now Distance between $Q(-1,-5,-10)$ and $P(2,-1,2)$ is

$PQ=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$ units

$=\sqrt{9+16+144}$ units $=\sqrt{169}$ units $=13$ units

25. Find the distance of the point $A(-1,-5,-10)$ from the point of intersection of the line $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

Sol. Given line is $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$

The cartesion equation of the line is $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=\lambda$

A general point on this line is $P(3\lambda+2,4\lambda-1,2\lambda+2)$

If this point lies on the plane $x-y+z=5$

Then $3\lambda+2-4\lambda+1+2\lambda+2=5$

$\Rightarrow \lambda+5=0 \Rightarrow \lambda=0$

$\therefore$ Point P is $(2,-1,2)$

Now distance between $A(-1,-5,-10)$ and $P(2,-1,2)$ is

$AP=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$ units

$=\sqrt{9+16+144}$ units $=\sqrt{169}$ units $=13$ units

26. Prove that the normals to the plants $4x+11y+2z+3=0$ and $3x-2y+5z=8$ are perpendicular to each other

Sol A vector normal to the plane $4x+11y+2z+3=0$ is

$\vec{n}_1=4\hat{i}+11\hat{j}+2\hat{k}$

A vector normal to the plane $3x-2y+5z=8$ is

$\vec{n}_2=3\hat{i}-2\hat{j}+5\hat{k}$

Now $\vec{n}_1\cdot\vec{n}_2=4\times3+11\times(-2)+2\times5 \;=12-22+10=0$

$\Rightarrow \vec{n}_1-\vec{n}_2$

Hence both the planes are perpendicular to each other

27. Show that the line $\vec{r}=\left(2\hat{i}-2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+4\hat{k}\right)$ is parallel to the plane $\vec{r}\cdot\left(\hat{i}+5\hat{j}+\hat{k}\right)=7$

Sol. A vector parallel to the line $\vec{r}=\left(2\hat{i}-2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+4\hat{k}\right)$ is given by

$\vec{b}=\hat{i}-\hat{j}+4\hat{k}$

A vector normal to the plane $\vec{r}\cdot\left(\hat{i}+5\hat{j}+\hat{k}\right)=7$ is

$\vec{n}=\hat{i}+5\hat{j}+\hat{k}$

Now, $\vec{b}\cdot\vec{n}=1\times1+(-1)\times5+4\times1=1-5+7=0$

$\Rightarrow \vec{b}\perp\vec{n} \Rightarrow$ Given line and given plane is parallel to each other

28. Find the equation of a plane which is at a distance of $3\sqrt{3}$ units from the origin and the normal to which is equally inclined to the coordinate axes

Sol. Let the required equation of the plane be $\vec{r}\cdot\hat{n}=p$, where $p=3\sqrt{3}$

Let $\hat{n}=(\cos\alpha)\hat{i}+(\cos\alpha)\hat{j}+(\cos\alpha)\hat{k}$, where α is actue

Then $\cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1 \Rightarrow 3\cos^2\alpha=1 \Rightarrow \cos^2\alpha=\frac{1}{3} \Rightarrow \cos\alpha=\frac{1}{\sqrt{3}}$

The required equation is $\vec{r}\cdot\left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right)=3\sqrt{3}$

Hence $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=9 \Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})(\hat{i}+\hat{j}+\hat{k})=9 \Rightarrow x+y+z=9$

29. A vector $\vec{n}$ of magnitude 8 units is inclined to the x-axis at 45°, y -axis at 60° and an axute angle with the z-axis. If a plane passes through a point $(\sqrt{2},-1,1)$ and is normal to $\vec{n}$, find its equation in vector form

Sol. We know that $\hat{n}=\frac{\vec{n}}{|\vec{n}|}=(l\hat{i}+m\hat{j}+n\hat{k})$

Here $l=\cos 45°=\frac{1}{\sqrt{2}}, m=\cos 60°=\frac{1}{2}$ and $n=\cos\gamma$

Then, $l^2+m^2+n^2=1 \Rightarrow \frac{1}{2}+\frac{1}{4}+\cos^2\gamma=1$

$\Rightarrow \cos^2\gamma=\frac{1}{4} \Rightarrow \cos\gamma=\frac{1}{2}$

$\therefore \vec{n}=|\vec{n}|(l\hat{i}+m\hat{j}+n\hat{k}) \Rightarrow \vec{r}\cdot(4\sqrt{2}\hat{i}+4\hat{j}+4\hat{k})=(\sqrt{2}\hat{i}-\hat{j}+\hat{k})(4\sqrt{2}\hat{i}+4\hat{j}+4\hat{k})$

$\Rightarrow \vec{r}(4\sqrt{2}\hat{i}+4\hat{j}+4\hat{k})=(8-4+4)=8 \Rightarrow \vec{r}\cdot(\sqrt{2}\hat{i}+\hat{j}+\hat{k})=2$

30. Find the vector equation of a line passing through the point $(2\hat{i}-3\hat{j}-5\hat{k})$ and perpendicular to the plane $\vec{r}\cdot(6\hat{i}-3\hat{j}+5\hat{k})+2=0$ Also find point of intersection of the line and the plane

Sol. Clearly the required line passing through the point $(2\hat{i}-3\hat{j}-5\hat{k})$ and is parallel to the normal of the given plane which is $(6\hat{i}-3\hat{j}+5\hat{k})$

The required vector equation is $\vec{r}=(2\hat{i}-3\hat{j}-5\hat{k})+\lambda(6\hat{i}-3\hat{j}+5\hat{k})$

The general equation of the line is $\frac{x-2}{6}=\frac{y+3}{-3}=\frac{z+5}{5}=k$

A general point on this line is $P(6k+2,-3k-3,5k-3)$

For some particular value of k, let the line cut the plane $6x-3y+5z+2=0$

$\Rightarrow (36k+9k+25k)=2 \Rightarrow 70k=2 \Rightarrow k=\frac{1}{35}$

$\therefore$ required point of intersection of the line and plane is $P\left(\frac{6}{35}+2,\frac{-3}{35},-3,\frac{1}{7},-5\right)$

i.e. $P\left(\frac{76}{35},\frac{-108}{35},\frac{-34}{7}\right)$

EXERCISE 28 C [Pg. No.: 1196]

1. Find the distance of the point $(2\hat{i}-\hat{j}-4\hat{k})$ from the plane $\vec{r}\cdot(3\hat{i}-4\hat{j}+12\hat{k})=9$

Sol. We know that the perpendicular distance of a point with position vector $\vec{a}$ from the plane $\vec{r}\cdot\vec{n}=d$ is given by

$$p=\frac{|\vec{a}\cdot\vec{n}-d|}{|\vec{n}|}$$

Here $\vec{a}=2\hat{i}-\hat{j}-4\hat{k}$, $\vec{n}=3\hat{i}-4\hat{j}+12\hat{k}$, and $d=9$

$\therefore$ the required distance is given by $p=\dfrac{|(2\hat{i}-\hat{j}-4\hat{k})\cdot(3\hat{i}-4\hat{j}+12\hat{k})-9|}{\left|\sqrt{3^2+(-4)^2+(12)^2}\right|}$

$$=\frac{|(6+4-48)-8|}{|\sqrt{169}|}=\frac{47}{13} \text{ units}$$

2. Find the distance of he point $(\hat{i}+2\hat{j}+5\hat{k})$ from the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})+17=0$.

Sol. $\vec{r}.(\hat{i}+\hat{j}+\hat{k})+17=0$

$\Rightarrow(x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+\hat{j}+\hat{k})+17=0$

$\Rightarrow x+y+z+17=0 \because P=\left|\dfrac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

$\Rightarrow P=\left|\dfrac{1(1)+1(2)+1(5)+17}{\sqrt{(1)^2+(1)^2+(1)^2}}\right| \Rightarrow P=\left|\dfrac{1+2+5+17}{\sqrt{3}}\right| \Rightarrow P=\dfrac{25}{\sqrt{3}}$ units

3. Find the distance of the point $(3,4,5)$ from the plane $\vec{r}.(2\hat{i}-5\hat{j}+3\hat{k})=13$.

Sol. $\vec{r}.(2\hat{i}-5\hat{j}+3\hat{k})-13=0$

$\Rightarrow(x\hat{i}+y\hat{j}+z\hat{k}).(2\hat{i}-5\hat{j}+3\hat{k})-13=0 \Rightarrow 2x-5y+3z-13=0$

$\because P=\left|\dfrac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right| \Rightarrow P=\left|\dfrac{2(3)-5(4)+3(5)-13}{\sqrt{(2)^2+(-5)^2+(3)^2}}\right| \Rightarrow P=\left|\dfrac{6-20+15-13}{\sqrt{4+25+9}}\right|=P=\left|\dfrac{21-33}{\sqrt{38}}\right|$

$\Rightarrow P=\dfrac{12}{\sqrt{38}}$ units.

4. Find the distance of the point $(1,1,2)$ from the plane $\vec{r}\cdot(2\hat{i}-2\hat{j}+4\hat{k})+5=0$

Sol. We know that the perpendicular distance of a point with position vector $\vec{r}_1$ from the plane $\vec{r}.\vec{n}=q$ is given by $P=\dfrac{|\vec{r}_1.\vec{n}+q|}{|\vec{n}|}$, here, $\vec{r}_1=\hat{i}+\hat{j}+2\hat{k}$, $\vec{n}=2\hat{i}-2\hat{j}+4\hat{k}$ and $q=5$

$\therefore \quad P=\frac{\left|\left(\hat{i}+\hat{j}+2\hat{k}\right)\left(2\hat{i}-2\hat{j}+4\hat{k}\right)+5\right|}{\left|2\hat{i}-2\hat{j}+4\hat{k}\right|}=\left|\frac{2-2+8+5}{\sqrt{(2)^2+(-2)^2+(4)^2}}\right|=\frac{13}{2\sqrt{6}}$ units.

$=\frac{13\sqrt{6}}{12}$ units

5. Find the distance of the point $(2,1,0)$ from the plane $2x+y+2z+5=0$

Sol. $\because P=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right| \Rightarrow P=\left|\frac{2(2)+1(1)+2(0)+5}{\sqrt{(2)^2+(1)^2+(2)^2}}\right| \Rightarrow P=\left|\frac{4+1+0+5}{\sqrt{4+1+4}}\right| \Rightarrow P=\frac{10}{3}$ units

6. Find the distance of the point $(2,1,-1)$ from the plane $x-2y+4z=9$

Sol. The required distance = the length of the perpendicular from $P(2,1,-1)$ to the plane

$x-2y+4z-9=0$

$=\frac{\left|2-2\times1+4\times(-4)-9\right|}{\left|\sqrt{1^2+(-2)^2+4^2}\right|}=\frac{13}{\sqrt{21}}$ units

7. Show that the point (1,2,1) is equidistant from the planes $\vec{r}\cdot\left(\hat{i}+2\hat{j}-2\hat{k}\right)=5$ and $\vec{r}\cdot\left(2\hat{i}-2\hat{j}+\hat{k}\right)+3=0$

Sol. We know that the perpendicular distance of a point with position vector $\vec{a}$ from the plane $\vec{r}\cdot\vec{n}=q$ is given by

$P=\frac{\left|\vec{a}\cdot\vec{n}-q\right|}{\left|\vec{n}\right|}$

Position vector of (1,2,1) is $\vec{a}=\left(\hat{i}+2\hat{j}+\hat{k}\right)$

Distance between (1,2,1) and the plane $\vec{r}\cdot\left(\hat{i}+2\hat{j}-2\hat{k}\right)=5$ is

$d_1=\frac{\left|\vec{a}\cdot\left(\hat{i}+2\hat{j}-2\hat{k}\right)-5\right|}{\left|\hat{i}+2\hat{j}-2\hat{k}\right|}$

$\Rightarrow d_1=\frac{\left|\left(\hat{i}+2\hat{j}+\hat{k}\right)\cdot\left(\hat{i}+2\hat{j}-2\hat{k}\right)-5\right|}{\left|\hat{i}+2\hat{j}-2\hat{k}\right|}$

$\Rightarrow d_1=\frac{\left|1+4-2-5\right|}{\sqrt{1^2+2^2+(-2)^2}}$ units

$\Rightarrow d_1=\frac{2}{3}$ units

Now distance between (1,2,1) and the plane $\vec{r}\cdot\left(2\hat{i}-2\hat{j}+\hat{k}\right)+3=0$ is

$d_2=\frac{\left|\vec{a}\cdot\left(2\hat{i}-2\hat{j}+\hat{k}\right)+3\right|}{\left|2\hat{i}-2\hat{j}+\hat{k}\right|}$

$$\Rightarrow d_2 = \frac{\left|\left(\hat{i}+2\hat{j}+\hat{k}\right)\cdot\left(2\hat{i}-2\hat{j}+\hat{k}\right)+3\right|}{\left|2\hat{i}-2\hat{j}+\hat{k}\right|}$$

$$\Rightarrow d_2 = \frac{|2-4+1+3|}{\sqrt{4+4+1}}$$

$$\Rightarrow d_2 = \frac{2}{3} \text{ units}$$

Here $d_1 = d_2 = \frac{2}{3}$ units

Hence the point (1,2,1) is equidistant from the planes

$\vec{r}\cdot\left(\hat{i}+2\hat{j}-2\hat{k}\right)=5$ and $\vec{r}\cdot\left(2\hat{i}-2\hat{j}+\hat{k}\right)+3=0$

8. Show that the points $(-3,0,1)$ and $(1,1,1)$ are equidistant from the plane $3x+4y-12z+13=0$.

Sol. $\because P_1 = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right| \Rightarrow P_1 = \left|\frac{3(-3)+4(0)-12(1)+13}{\sqrt{(3)^2+(4)^2+(-12)^2}}\right|$

$\Rightarrow P_1 = \left|\frac{-9+0-12+13}{\sqrt{9+16+144}}\right| \Rightarrow P_1 = \left|\frac{-8}{\sqrt{169}}\right| \therefore P_1 = \frac{8}{13}$ units and

$\because P_2 = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right| \Rightarrow P_2 = \left|\frac{3(1)+4(1)-12(1)+13}{\sqrt{(3)^2+(4)^2+(-12)^2}}\right| \Rightarrow P_2 = \left|\frac{3+4-12+13}{\sqrt{9+16+144}}\right|$

$\Rightarrow P_2 = \left|\frac{8}{\sqrt{169}}\right| \therefore P = \frac{8}{13}$ units $\because P_1 = P_2 = \frac{8}{\sqrt{13}}$ units

Hence, the given two points and one line is equidistance proved.

9. Find the distance between the parallel planes $2x+3y+4=1$ and $4x+6y+8z=12$

Sol. Equations of planes are $2x+3y+4z-4=0$ ...... (i)

And $4x+6y+8z-12=0$

$\Rightarrow 2(2x+3y+4z-6)=0$

$\Rightarrow 2x+3y+4z-6=0$ ..... (ii)

We know that distance between $ax+by+cx+d_1=0$

And $ax+by+cz+d_2=0$ is $d=\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$

Required distance $=\frac{|-4-(-6)|}{\sqrt{2^2+3^2+4^2}}$ units

$=\frac{|2|}{\sqrt{4+9+16}}$ units $=\frac{2}{\sqrt{29}}$ units $=\frac{2\sqrt{29}}{29}$ units

10. Find the distance between the parallel planes $x+2y-2z+4=0$ and $x+2y-2z-8=0$

Sol. Distance between two parallel planes $x+2y-2z+4=0$ and $x+2y-2z-8=0$ is

$$d=\frac{|4-(-8)|}{\sqrt{1^2+2^2+(-2)^2}} \text{ units}$$

$\Rightarrow d=\frac{|12|}{9}$ units $\Rightarrow d=\frac{12}{3}$ units $\Rightarrow d=4$ units

11. Find the equations of the planes parallel to the plane $x-2y+2z-3=0$, each one of which is at a unit distance from the point (1,1,1)

Sol. Any plane parallel to the plane $x-2y+2z-3=0$ is given by $x-2y+2z+d=0$

According to question distance between point (1, 1, 1) and $x-2y+2z+d=0$ is

$\Rightarrow \frac{|1-2\times1+2\times1+d|}{\sqrt{1^2+(-2)^2+2^2}}=1$

$\Rightarrow \frac{|1+d|}{3}=1 \Rightarrow |1+d|=3 \Rightarrow 1+d=\pm3 \Rightarrow d=\pm3-1 \Rightarrow d=2$ or -4

Hence equations of planes are $x-2y+2z+2=0$ and $x-2y+2z-4=0$

12. Find the equation of the plane parallel to the plane $2x-3y+5z+7=0$ and passing through the point $(3,4,-1)$. Also find the distance between the two planes

Sol. Any plane parallel to the plane $2x-3y+5z+7=0$ is given by $2x-3y+5z+d=0$ ….. (i)

Since it passes through $(3,4,-1)$

$\therefore 2\times3-3\times4+5\times(-1)+d=0$

$\Rightarrow 6-12-5+d=0 \Rightarrow d=11$

Putting d = 11 in equation (ii) we have $2x-3y+5z+11=0$

$\therefore$ equation of plane is $2x-3y+5z+11=0$

Distance between the planes is

S.D. $=\frac{|11-7|}{\sqrt{2^2+(-3)^2+5^2}}$ units

$=\frac{4}{\sqrt{38}}$ units $=\frac{4}{\sqrt{38}}\times\frac{\sqrt{38}}{\sqrt{38}}$ units $=\frac{4\sqrt{38}}{38}$ units $=\frac{2}{19}\sqrt{38}$ units

13. Find the equation of the plane mid-prallel to the planes $2x-3y+6z+21=0$ and $2x-3y+6z-14=0$

Sol. Let the required equation of the plane be $2x-3y+6z+5=0$ this plane equidistance from each at the given planes

Let $P(\alpha,\beta,\gamma)$ be any on the plane $2x-3y+6z+k=0$ …. (i)

Then $2\alpha-3\beta+6\gamma+k=0$

$\because P(\alpha,\beta,\gamma)$ is equidistant from the planes $2x-3y+6z+21=0$ and $2x-3y+6z-14=0$

$\therefore \frac{|2\alpha-3\beta+6\gamma+21|}{\sqrt{2^2+(-3)^2+6^2}}=\frac{|2\alpha-3\beta+6\gamma-14|}{\sqrt{2^2+(-3)^2+6^2}}$

$\Rightarrow |-k+21|=|-k-14| \Rightarrow (-k+21)=\pm(-k-14) \Rightarrow -k+21=-k-14$

Or $-k+21=-(-k-14)$

Here $-k+21\neq-k-14$

Now $-k+21=k+14 \Rightarrow 2k=21-14 \Rightarrow k=\frac{7}{2}$

Putting $k=\frac{7}{2}$ in (i) we have $2x-3y+6z+\frac{7}{2}=0$

$\Rightarrow 4x-6y+12z+7=0$ this is required equation of plane

EXERCISE 28 D [Pg. No.: 1198]

1. Show that the planes $2x-y+6z=5$ and $5x-2.5y+15z=12$ are parallel

Sol. A vector normal to the plane $2x-y+6z=5$ is $\vec{n}_1=2\hat{i}-\hat{j}+6\hat{k}$

And A vector normal to the plane $5x-2.5j+15z=12$ is $\vec{n}_2=5\hat{i}-2.5\hat{j}+15\hat{k}$

Now $\vec{n}_2=5\hat{i}-2.5\hat{j}+15\hat{k} \;=2.5\{2\hat{i}-\hat{j}+6\hat{k}\} \;=2.5\vec{n}_1$

$\Rightarrow \vec{n}_2 \parallel \vec{n}_1$

Hence both the planes are parallel to each other

2. Find the vector equation of the plane through the point $(3\hat{i}+4\hat{j}-\hat{k})$ and parallel to the plane $\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})+5=0$

Sol. Any plane parallel to the plane $\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})+5=0$ is given by

$\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})+d=0$ ..... (i)

Since the plane passes through the point having position vector $3\hat{i}+4\hat{j}-\hat{k}$

$\therefore (3\hat{i}+4\hat{j}-\hat{k})\cdot(2\hat{i}-3\hat{j}+5\hat{k})+d=0$

$\Rightarrow 6-12-5+d=0 \;\Rightarrow d=11$

Hence required equation of plane is $\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})+11=0$

3. Find the vector equation of the plane passing through the point (a,b,c) and parallel to the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$

Sol. Position vector of the point (a,b,c) is $\vec{a}=a\hat{i}+b\hat{j}+c\hat{k}$

Any plane parallel to the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-2=0$ is given by $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})+d=0$

Since it passes through the point having position vector $a\hat{i}+b\hat{j}+c\hat{k}$

$\therefore (a\hat{i}+b\hat{j}+c\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})+d=0$

$\Rightarrow a+b+c+d=0 \;\Rightarrow d=-(a+b+c)$

Hence equation of plane is $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-(a+b+c)=0$

$\Rightarrow \vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=a+b+c$

4. Find the vector equation f the plane passing through the point $(1,1,1)$ and parallel to the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+2\hat{k})=5$

Sol. Position vector of the point $(1,1,1)$ is

$\vec{a}=\hat{i}+\hat{j}+\hat{k}$

Any plane parallel to the plane $\vec{r}\cdot\left(2\hat{i}-\hat{j}+2\hat{k}\right)-5=0$ is given by $\vec{r}\cdot\left(2\hat{i}-\hat{j}+2\hat{k}\right)+d=0$

Since it passes through the point having position vector $\hat{i}+\hat{j}+\hat{k}$

$\therefore\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\left(2\hat{i}-\hat{j}+2\hat{k}\right)+d=0$

$\Rightarrow 2-1+2+d=0 \quad \Rightarrow d=3$

Hence the required equation of plane $\vec{r}\cdot\left(2\hat{i}-\hat{j}+2\hat{k}\right)-3=0$

$\Rightarrow \vec{r}\cdot\left(2\hat{i}-\hat{j}+2\hat{k}\right)=3$

5. Find the equation of the plane passing through the point $(1,4,-2)$ and parallel to the plane $2x-y+3z+7=0$

Sol. Any plane parallel to the plane $2x-y+3z+7=0$ is given by $2x-y+3z+d=0$

Since it passes though $(1,4,-2)$

$\therefore 2\times1-4+3(-2)+d=0$

$\Rightarrow 2-4-6+d=0 \quad \Rightarrow d=8$

Hence the required equation of plane is $2x-y+3z+8=0$

6. Find the equation of the plane passing through the origin and parallel to the plane $5x-3y+7z+13=0$

Sol. Any plane parallel to the plane $5x-3y+7z+13=0$ is given by $5x-3y+7z+d=0$

Since it passes through origin $\therefore d=0$

Hence equation of plane is $5x-3y+7z=0$

7. Find the equation of the plane passing through the point $(-1,0,7)$ and parallel to the plane $3x-5y+4z=11$

Sol. Equation of plane parallel to the plane $3x-5y+4z=11$ is given by $3x-5y+4z=d$ .... (i)

Since it passes through the point $(-1,0,7)$

$\therefore 3(-1)-5\times0+4\times7=d$

$\Rightarrow -3+28=d \quad \Rightarrow d=25$

Hence equation of plane is $3x-5y+4z=25$

8. Find the equations of planes parallel to the plane $x-2y+2z=3$ which are at a unit distance from the point $(1,2,3)$

Sol. Let the required plane by $x-2y+2z+k=0$ for some constants k

Then, its distance from the point $P(1,2,3)$ is

$$\frac{|1-2\times2+2\times3+k|}{\sqrt{1^2+(-2)^2+2^2}}=\frac{|3+k|}{3}=1 \quad \Rightarrow |3+k|=3$$

$\Rightarrow 3+k=3$ or $3+k=-3 \quad \Rightarrow k=0$ or $k=-6$

Hence the required equations are $x-2y+2z=0$ or $x-2y+2z-6=0$

9. Find the distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$

Sol. Let $P(x_1,y_1,z_1)$ be any point on the plane $x+2y+3z+7=0$

Then $x_1+2y_1+3z_1=7$

$$\therefore\ p=\frac{|2x_1+4y_1+6z_1+7|}{\sqrt{2^2+4^2+6^2}}=\frac{|2(x_1+2y_1+3z_1)+7|}{\sqrt{56}}$$

$$=\frac{|2\times(-7)+7|}{\sqrt{56}}=\frac{7}{\sqrt{56}}\text{ units}$$

EXERCISE 28 E [Pg. No.: 1205]

1. Find the equation of the plane through the line of intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$, and passing through the point $(1,1,1)$.

Sol. Any plane through the intersection of two given plane

$(x+y+z-6)+\lambda(2x+3y+4z+5)=0$ ... (i)

and, its passes through the point $(1,1,1)$ then

$\Rightarrow(1+1+1-6)+\lambda\{2(1)+3(1)+4(1)+5\}=0$

$\Rightarrow-3+\lambda(2+3+4+5)=0\Rightarrow-3+14\lambda=0\therefore\lambda=\frac{3}{14}$

Putting the value of λ in equation (i), then

$\Rightarrow(x+y+z-6)+\frac{3}{14}(2x+3y+4z+5)=0$

$\Rightarrow\frac{14(x+y+z-6)+3(2x+3y+4z+5)}{14}=0$

$\Rightarrow14x+14y+14z-84+6x+9y+12z+15=0\Rightarrow20x+23y+26z-69=0$

Hence, the required equation of the plane is $20x+23y+26z-69=0$

2. Find the equation of the plane through the line of intersection of the planes $x-3y+z+6=0$ and $x+2y+3z+5=0$, and passing through the origin.

Sol. Any plane through the intersection of two given plane,

$(x-3y+z+6)+\lambda(x+2y+3z+5=0)$ ... (i)

and its passes through the giving $(0,0,0)$, then

$\{0-3(0)+0+6\}+\lambda\{0+2(0)+3(0)+5\}=0$

$\Rightarrow6+5\lambda=0\therefore\lambda=\frac{-6}{5}$

Putting the value of λ in equation (i), then

$(x-3y+z+6)-\frac{6}{5}(x+2y+3z+5)=0\Rightarrow\frac{5(x-3y+z+6)-6(x+2y+3z+5)}{5}=0$

$\Rightarrow5x-15y+5z+30-6x-12y-18z-30=0$

$\Rightarrow-x-27y-13z=0\Rightarrow-(x+27y+13z)=0$

Hence, the required equation of a plane is $x+27y+13z=0$

3. Find the equation of the plane passing through the intersection of the planes $2x+3y-z+1=0$ and $x+y-2z+3=0$, and perpendicular to the plane $3x-y-2z-4=0$.

Sol. Any plane through the intersection of two given planes

$(2x+3y-z+1)+\lambda(x+y-2z+3)=0$ ... (i)

$\Rightarrow x(2+\lambda)+y(3+\lambda)+z(-1-2\lambda)+(1+3\lambda)=0$

and its perpendicular to the plane $(3x-y-2z-4)=0$

$\Rightarrow 3(2+\lambda)-1(3+\lambda)-2(-1-2\lambda)=0$

$\Rightarrow 6+3\lambda-3-\lambda+2+4\lambda=0$

$\Rightarrow 6\lambda+5=0 \therefore \lambda=\frac{-5}{6}$

Putting the value of λ in equation (i), then

$(2x+3y-z+1)-\frac{5}{6}(x+y-2z+3)=0 \Rightarrow \frac{6(2x+3y-z+1)-5(x+y-2z+3)=0}{6}$

$\Rightarrow 12x+18y-6z+6-5x-5y+10z-15=0 \Rightarrow 7x+13y+4z-9=0$

Hence, the required equation of a plane is $7x+13y+4z=9$

4. Find the equation of the plane passing through the line of intersection of the planes $2x-y=0$ and $3z-y=0$, and perpendicular to the plane $4x+5y-3z=9$.

Sol. Any plane parallel to the given plane is

$(2x-y)+\lambda(3z-y)=0$ and, its perpendicular to the given plane $4x-5y-3z=9$, then,

$\Rightarrow \{2(4)-5\}+\lambda\{3(-3)-5\}=0$

$\Rightarrow (8-5)+\lambda(-9-5)=0 \Rightarrow 3-14\lambda=0 \Rightarrow \lambda=\frac{3}{14}$

Putting the value of λ in equation (i), then

$(2x-y)+\frac{3}{14}(3z-y)=0 \Rightarrow \frac{14(2x-y)+3(3z-y)}{14}=0$

$\Rightarrow 28x-14y+9z-3y=0 \Rightarrow 28x-17y+9z=0$

Hence the required equation of the plane is $28x-17y+9z=0$

5. Find the equation of the plane passing through the intersection of the planes $x-2y+z=1$ and $2x+y+z=8$, and parallel to the line with direction ratios $1,2,1$. Also, find the perpendicular distance of $(1,1,1)$ from the plane.

Sol. Let the required plane be

$(x-2y+z-1)+\lambda(2x+y+z-8)=0 \quad \ldots$ (i)

$\Rightarrow (1+2\lambda)x+(\lambda-2)y+(1+\lambda)z-(1+8\lambda)=0 \quad \ldots$ (ii)

The direction ratio of the Normal to this plane are $(1+2\lambda),(\lambda-2),(1+\lambda)$

The Normal to the plane (ii) is perpendicular to the line with direction ratio 1, 2, 1.

$\therefore \quad (1+2\lambda)+2(\lambda-2)+(1+\lambda)=0 \Rightarrow 1+2\lambda+2\lambda-4+1+\lambda=0 \Rightarrow 5\lambda-2=0 \Rightarrow \lambda=\frac{2}{5}$

putting the value of λ in equation (i)

$(x-2y+z-10+\frac{2}{5}(2x+y+z-8)=0$

$\Rightarrow 5x-10y+5z-5+4x+2y+2z-16=0 \quad \Rightarrow 9x-8y+7z-21=0$

length of perpendicular from the point (1, 1, 1)

$P=\frac{|9\times1-8\times1+7\times1-21|}{\sqrt{(9)^2+(-8)^2+(7)^2}}=\frac{|9-8+7-21|}{\sqrt{81+64+49}}=\frac{13}{\sqrt{194}}$ units

6. Find the equation of the plane passing through the line intersection of the planes $x+2y+3z-5=0$ and $3x-2y-z+1=0$ and cutting off equal intercepts on the x-axis and z-axis

Sol. Any plane passing through the intersection of two planes $x+2y+3z-5=0$ and $3x-2y-z+1=0$ is given by

$(x+2y+3z-5)+k(3x-2y-z+1)=0$

Then $(1+3k)x+(2-2k)y+(3-k)z-5+k=0$

$\Rightarrow (1+3k)x+(2-2k)y+(3-k)z=5-k$

$\Rightarrow \frac{x}{\frac{5-k}{1+3k}}+\frac{y}{\frac{5-k}{2-2k}}+\frac{z}{\frac{5-k}{3-k}}=1$

Since intercepts on the x-axis and z axis are eagal we have $\frac{5-k}{1+3k}=\frac{5-k}{3-k}$

$\Rightarrow 3-k=1+3k \quad \Rightarrow 4k=2 \quad \Rightarrow k=\frac{1}{2}$

Hence equation of plase is $(x+2y+3z-5)+\frac{1}{2}(3x-2y-z+1)=0$

$\Rightarrow 2x+4y+6z-10+3x-2y-z+1=0 \quad \Rightarrow 5x+2y+5z-9=0$

7. Find the equation of the plane through the intersection of the planes $3x-4y+5z=10$ and $2x+2y-3z=4$ and parallel to the line $x=2y=3z$

Sol. Any place through the intersection of the planes $3x-4y+5z=10$

And $2x+2y-3z=4$ is given by $(3x-4y+5z-10)+k(2x+2y-3z-4)=0$

$\Rightarrow (3+2k)x+(2k-4)y+(5-3k)z-10-4k=0$

D.r's of normal to the plane are $3+2k, 2k-4, 5-3k$

Given line is $x=2y=3z$

i.e. $\frac{x}{6}=\frac{y}{3}=\frac{z}{2}$

D.r's of line are $6,3,2$

$\because$ The line is perpendicular to the plane

$\therefore 6(3+2k)+3(2k-4)+2(5-3k)=0$

$\Rightarrow 18+12k+6k-12+10-6k=0$

$\Rightarrow 12k+16=0 \quad \Rightarrow k=-\frac{16}{12} \quad \Rightarrow k=-\frac{4}{3}$

Hence the required equation of plane is

$(3x-4y+5z-10)-\frac{4}{3}(2x+2y-3z-4)=0 \quad \Rightarrow x-20y+27z=14$

8. Find the vector equation of the plane through the intersection of the planes $\vec{r}.(\hat{i}+3\hat{j}-\hat{k})=0$ and $\vec{r}.(\hat{j}+2\hat{k})=0,$ and passing through the point $(2,1,-1)$.

Sol. Here, $\vec{r}(\hat{i}+3\hat{j}-\hat{k})=0$, The Cartesian equation of the plane is, put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+3\hat{j}-\hat{k})=0 \Rightarrow x+3y-z=0$ and, $\vec{r}.(\hat{j}+2\hat{k})=0$

$\Rightarrow\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(\hat{j}+2\hat{k}\right)=0\Rightarrow y+2z=0$

Any plane through the intersection of the planes

$(x+3y-z)+\lambda(y+2z)=0$ ... (i)

and, its passes through the point $(2,1,-1)$

$\Rightarrow\{2+3(1)-(-1)\}+\lambda\{1+2(-1)\}=0$

$\Rightarrow(2+3+1)+\lambda(1-2)=0\Rightarrow 6-\lambda=0\therefore\lambda=6$

Putting the value of λ in equation (i), then

$(x+3y-z)+6(y+2z)=0\Rightarrow x+3y-z+6y+12z=0\Rightarrow x+9y+11z=0$

On vector equation, $\Rightarrow\left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0\Rightarrow\vec{r}.\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0$

Hence the required vector equation of the line is $\vec{r}.\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0$

9. Find the vector equation of the plane through the point $(1,1,1)$, and passing through the intersection of the planes $\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)+1=0$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)-5=0$.

Sol. Here $\vec{n}_1=(\hat{i}-\hat{j}+3\hat{k})$ and $\vec{n}_2=(2\hat{i}+\hat{j}-\hat{k})$;

$d_1=-1$ and $d_2=5$

Required vector equation is $\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_2+\lambda d_2$

i.e. $\vec{r}.\left\{(\hat{i}-\hat{j}+3\hat{k})+\lambda(2\hat{i}+\hat{j}-\hat{k})\right\}=-1+\lambda.5$

$\vec{r}.\left\{(1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(3-\lambda)\hat{k}\right\}=5\lambda-1$

where λ is some real number

Taking $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, we get

$(x\hat{i}+y\hat{j}+z\hat{k})\left\{(1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(3-\lambda)\hat{k}\right\}=5\lambda-1$

$\Rightarrow(1+2\lambda)x+(-1+\lambda)y+(3-\lambda)z=5\lambda-1\Rightarrow(x-y+3z+1)+\lambda(2x+y-z-5)=0$

Since the plane passing through the point (1, 1, 1)

$\Rightarrow(1-1+3+1)+\lambda(2+1-1-5)=0$

$\Rightarrow 4-3\lambda=0\quad\Rightarrow\lambda=\frac{4}{3}$

Putting the value of λ in equation (i)

$(x-y+3z+1)+\frac{4}{3}(2x+y-z-5)=0$

$\Rightarrow 3x-3y+9z+3+8x+4y-4z-20=0\Rightarrow 11x+y+5z-17=0$

its vector equation be $\vec{r}.(11\hat{i}+\hat{j}+5\hat{k})-17=0$

10. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(2\hat{i}-7\hat{j}+4\hat{k}\right)=3$ and $\vec{r}.\left(3\hat{i}-5\hat{j}+4\hat{k}\right)+11=0$, and passing through the point $(-2,1,3)$.

Sol. We have $\vec{r}.\left(2\hat{i}-7\hat{j}+4\hat{k}\right)-3=0$ Now, the Cartesian equation of the plane is, put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\Rightarrow\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}-7\hat{j}+4\hat{k}\right)-3=0\Rightarrow 2x-7y+4z-3=0$

and $\vec{r}.\left(3\hat{i}-5\hat{j}+4\hat{k}\right)+11=0$ Now, the Cartesian equation of the plane is, put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})(3\hat{i}-5\hat{j}+4\hat{k})+11=0 \Rightarrow 3x-5y+4z+11=0$

Any plane passing through the intersection of the planes.

$(2x-7y+4z-3)+\lambda(3x-5y+4z+11)=0$ ... (i)

and, its passes through the point $(-2,1,3)$, then

$\Rightarrow \{2(-2)+7(1)+4(3)-3\}+\lambda\{3(-2)-5(1)+4(3)+11\}=0$

$\Rightarrow (-4-7+12-3)+\lambda(-6-5+12+11)=0 \Rightarrow -2+12\lambda=0 \Rightarrow \lambda=\frac{2}{12}=\frac{1}{6}$

Putting the value of λ in equation (i), then

$\Rightarrow (2x-7y+4z-3)+\frac{1}{6}(3x-5y+4z+11)=0 \quad \Rightarrow \frac{6(2x-7y+4z-3)+(3x-5y+4z+11)}{6}=0$

$\Rightarrow 12x-42y+24z-18+3x-5y+4z+11=0 \Rightarrow 15x-47y+28z-7=0$

Now, vector equation is $\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})(15\hat{i}-47\hat{j}+28\hat{k})=7 \Rightarrow \vec{r}.(15\hat{i}-47\hat{j}+28\hat{k})=7$

Hence the required vector equation of the plane is $\vec{r}.(15\hat{i}-47\hat{j}+28\hat{k})=7$

11. Find the equation of the plane through the line of intersection of the planes $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-\hat{j})+4=0$ and perpendicular to the planes $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+8=0$

Sol. Any plane through the line of intersection of the two given planes is

$[\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})-1]+\lambda[\vec{r}\cdot(\hat{i}-\hat{j})+4]=0$

$\Rightarrow \vec{r}\cdot[(2+\lambda)\hat{i}-(3+\lambda)\hat{j}+4\hat{k}]=1-4\lambda$ .... (i)

If this plane is perpendicular to the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+8=0$

We have $2(2+\lambda)+(3+\lambda)+4=0 \Leftrightarrow 3\lambda+11=0 \Leftrightarrow \lambda=\frac{-11}{3}$

Putting $\lambda=\frac{-11}{3}$ in (i) we get the required equation of the plane as $\vec{r}\cdot(-5\hat{i}+2\hat{j}+12\hat{k})=47$

12. Find the cortication and vector equations of the planes through the line of intersection of the planes $\vec{r}\cdot(\hat{i}-\hat{j})+6=0$ and $\vec{r}\cdot(3\hat{i}+3\hat{j}-4\hat{k})=0$ which are at a unit distance from the origin

Sol. The equation of the given plane are $(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}-\hat{j})+6=0$ and $(x\hat{i}+y\hat{j}+z\hat{k})\cdot(3\hat{i}+3\hat{j}-4\hat{k})=0$

$\Rightarrow x-y+6=0$ and $3x+3y-4z=0$

Any plane through their intersection is $(x-y+6)+\lambda(3x+3y-4z)=0$

$\Rightarrow (1+3\lambda)x+(3\lambda-1)y-4\lambda x+6=0$ ..... (i)

$\therefore \frac{6}{\sqrt{(1+3\lambda)^2+(3\lambda-1)^2+(-4\lambda)^2}}=1 \quad \Rightarrow 34\lambda^2+2=36 \quad \Rightarrow \lambda^2=1 \Rightarrow \lambda=\pm 1$

So, the required planes are $2x+y-2z+3=0$ and $x+2y-2z-3=0$

In vector form they are $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})+3=0$ and $\vec{r}\cdot(\hat{i}+2\hat{j}-2\hat{k})-3=0$

EXERCISE 28 F [Pg. No.: 1217]

1. Find the acute angle between the planes:

(i) $\vec{r}\left(\hat{i}+\hat{j}-2\hat{k}\right)=5$ and $\vec{r}.\left(2\hat{i}+2\hat{j}-\hat{k}\right)=9$ $\because \cos\theta=\dfrac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

(ii) $\vec{r}.\left(\hat{i}+2\hat{j}-\hat{k}\right)=6$ and $\vec{r}\left(2\hat{i}-\hat{j}-\hat{k}\right)+3=0$

(iii) $\vec{r}.\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=1$ and, $\vec{r}.\left(-\hat{i}+\hat{j}\right)=4$

(iv) $\vec{r}.\left(2\hat{i}-3\hat{j}+6\hat{k}\right)=8$ and $\vec{r}.\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+7=0$

Sol. (i) $\vec{r}\left(\hat{i}+\hat{j}-2\hat{k}\right)=5$ and $\vec{r}.\left(2\hat{i}+2\hat{j}-\hat{k}\right)=9$ $\Rightarrow$ We know that the angle between the plane

$\vec{r}.\vec{n}_1=a_1$ *and* $\vec{r}.\vec{n}_2=a_2$ is given by $\cos\theta=\dfrac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

Here, $\vec{n}_1=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{n}_2=2\hat{i}+2\hat{j}-\hat{k}$

$\Rightarrow|\vec{n}_1|=\sqrt{(1)^2+(1)^2+(-2)^2}=\sqrt{1+1+4}=\sqrt{6}$

and $|\vec{n}_2|=\sqrt{(2)^2+(2)^2+(-1)^2}=\sqrt{4+4+1}=\sqrt{9}=3$

$$\cos\theta=\frac{\left(\hat{i}+\hat{j}-2\hat{k}\right).\left(2\hat{i}+2\hat{j}-\hat{k}\right)}{3\sqrt{6}}\Rightarrow\cos\theta=\frac{2+2+2}{3\sqrt{6}}=\frac{6}{3\sqrt{6}}=\frac{\sqrt{6}}{3}\quad\therefore\theta=\cos^{-1}\left(\frac{\sqrt{6}}{3}\right)$$

Hence, the angle between the given planes is $\cos^{-1}\left(\dfrac{\sqrt{6}}{3}\right)$

(ii) $\vec{r}.\left(\hat{i}+2\hat{j}-\hat{k}\right)=6$ and $\vec{r}\left(2\hat{i}-\hat{j}-\hat{k}\right)+3=0$ $\Rightarrow$ We know that the angle between the plane

$\vec{r}.\vec{n}_1=a_1$ *and* $\vec{r}.\vec{n}_2=a_2$ is given by $\cos\theta=\dfrac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

Here $\vec{n}_1=\hat{i}+2\hat{j}-\hat{k}$ *and* $\vec{n}_2=2\hat{i}-\hat{j}-\hat{k}$

$\Rightarrow|\vec{n}_1|=\sqrt{(1)^2+(2)^2+(-1)^2}=\sqrt{1+4+1}=\sqrt{6}$

and $|\vec{n}_2|=\sqrt{(2)^2+(-1)^2+(-1)^2}=\sqrt{4+1+1}=\sqrt{6}$

$$\Rightarrow\cos\theta=\frac{\left(\hat{i}+2\hat{j}-\hat{k}\right).\left(2\hat{i}-\hat{j}-\hat{k}\right)}{\sqrt{6}.\sqrt{6}}\Rightarrow\cos\theta=\frac{2-2+1}{6}=\frac{1}{6}\Rightarrow\theta=\cos^{-1}\left(\frac{1}{6}\right)$$

Hence, the required angle between the plane is $\cos^{-1}\left(\dfrac{1}{6}\right)$

(iii) $\vec{r}.\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=1$ and, $\vec{r}.\left(-\hat{i}+\hat{j}\right)=4$

$\Rightarrow \cos\theta = \dfrac{\vec{n}_1.\vec{n}_2}{|\vec{n}||\vec{n}_2|} \Rightarrow |\vec{n}_1| = \sqrt{(2)^2+(-3)^2+(4)^2} = \sqrt{4+9+16} = \sqrt{29}$

and $|\vec{n}_2| = \sqrt{(-1)^2+(1)^2} = \sqrt{1+1} = \sqrt{2}$

$\Rightarrow \cos\theta = \dfrac{(2\hat{i}-3\hat{j}+4\hat{k}).(-\hat{i}+\hat{j})}{\sqrt{29}.\sqrt{2}} \Rightarrow \cos\theta = \dfrac{-2-3}{\sqrt{58}} \Rightarrow \cos\theta = \dfrac{-5}{\sqrt{58}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{-5}{\sqrt{58}}\right)$

Hence, the required angle between the plane is $\cos^{-1}\left(\dfrac{-5}{\sqrt{58}}\right)$

(iv) $\vec{r}.(2\hat{i}-3\hat{j}+6\hat{k}) = 8$ and $\vec{r}.(3\hat{i}+4\hat{j}-12\hat{k}) +7 = 0$ $\Rightarrow$ We know that the angle between the plane

$\vec{r}.\vec{n}_1 = a_1$ *and* $\vec{r}.\vec{n}_2 = a_2$ is given by $\cos\theta = \dfrac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

$\Rightarrow |\vec{n}_1| = \sqrt{(2)^2+(-3)^2+(6)^2} = \sqrt{4+9+36} = \sqrt{49} = 7$

and $|\vec{n}_2| = \sqrt{(3)^2+(4)^2+(-12)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$

$\Rightarrow \cos\theta = \dfrac{(2\hat{i}-3\hat{j}+6\hat{k}).(3\hat{i}+4\hat{j}-12\hat{k})}{7.13}$

$\Rightarrow \cos\theta = \dfrac{6-84}{91} \Rightarrow \cos\theta = \dfrac{-6}{7} \Rightarrow \theta = \cos^{-1}\left(\dfrac{-6}{7}\right) \Rightarrow \theta = \cos^{-1}\left(\dfrac{6}{7}\right)$

2. Show that the following planes are at right angles

(i) $\vec{r}\cdot(4\hat{i}-7\hat{j}-8\hat{k})$ and $\vec{r}\cdot(3\hat{i}-4\hat{j}+5\hat{k})+10=0$

(ii) $\vec{r}\cdot(2\hat{i}+6\hat{j}+6\hat{k})=13$ and $\vec{r}\cdot(3\hat{i}+4\hat{j}-5\hat{k})+7=0$

Sol. (i) Given plane are $\vec{r}\cdot(4\hat{i}-7\hat{j}-8\hat{k})=5$ and $\vec{r}\cdot(3\hat{i}-4\hat{j}+5\hat{k})+10=0$

Here $\vec{n}_1 = 4\hat{i}-7\hat{j}-8\hat{k}$

$\vec{n}_2 = 3\hat{i}-4\hat{j}+5\hat{k}$

Now, $\vec{n}_1\cdot\vec{n}_2 = (4\hat{i}-7\hat{j}-8\hat{k})\cdot(3\hat{i}-4\hat{j}+5\hat{k}) = 12+28-40=0$

$\Rightarrow \vec{n}_1 \perp \vec{n}_2$

Hence both the planes are perpendicular

(ii) Equation of planes are

$\vec{r}\cdot(2\hat{i}+6\hat{j}+6\hat{k})=13$

$\vec{r}\cdot(3\hat{i}+4\hat{j}-5\hat{k})+7=0$

Here $\vec{n}_1 = 2\hat{i}+6\hat{j}+6\hat{k}$

$\vec{n}_2 = 3\hat{i}+4\hat{j}-5\hat{k}$

Now, $\vec{n}_1\cdot\vec{n}_2 = (2\hat{i}+6\hat{j}+6\hat{k})\cdot(3\hat{i}+4\hat{j}-5\hat{k}) = 6+34-30=0$

$\Rightarrow \vec{n}_1 \perp \vec{n}_2$

Hence both the planes are perpendicular to each other

3. Find the value of λ for which the given planes are perpendicular to each other

(i) $\vec{r}\cdot\left(2\hat{i}-\hat{j}-\lambda\hat{k}\right)$ r= 7 and $\vec{r}\cdot\left(3\hat{i}+2\hat{j}+2\hat{k}\right)=9$

(ii) $\vec{r}\cdot\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)=5$ and $\vec{r}\cdot\left(\hat{i}+2\hat{j}-7\hat{k}\right)+11=0$

Sol. (i) we know that the plane $\vec{r}.\vec{n}_1=a_1$ and $\vec{r}.\vec{n}_2=a_2$ are perpendicular to each other only when $\vec{r}_1.\vec{n}_2=0$

Here $\vec{n}_1=2\hat{i}-\hat{j}+\lambda\hat{k}$ and $\vec{n}_2=3\hat{i}+2\hat{j}+2\hat{k}$

$\therefore$ the given plane are perpendicular to each other.

$\Rightarrow \vec{n}_1.\vec{n}_2=0 \Rightarrow (2\hat{i}-\hat{j}+\lambda\hat{k})(3\hat{i}+2\hat{j}+2\hat{k})=0 \Rightarrow 6-2+2\lambda=0$

$\Rightarrow 4+2\lambda=0 \Rightarrow \lambda=-2$

(ii) Given planes are $\vec{r}\cdot\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)=5$ and $\vec{r}\cdot\left(\hat{i}+2\hat{j}-7\hat{k}\right)+11=0$

$\because$ Both the planes are perpendicular to each other

$\therefore \left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)\cdot\left(\hat{i}+2\hat{j}-7\hat{k}\right)=0 \Rightarrow \lambda+4-21=0 \Rightarrow \lambda-17=0 \Rightarrow \lambda=17$

4. Find the acute angle between the planes

(i) $2x-y+z=5$ and $x+y+2z=7$

(ii) $x+2y+2z=3$ and $2x-3y+6z=8$

(iii) $x+y-z=4$ and $x+2y+z=9$

(iv) $x+y-2z=6$ and $2x-2y+z=11$

Sol. (i) We know that the angle between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is

given by $\cos\theta=\dfrac{a_1a_2+b_1b_2+c_1c_2}{\left(\sqrt{a_1^2+b_1^2+c_1^2}\right)\left(\sqrt{a_2^2+b_2^2+c_2^2}\right)}$

Here, $a_1=2,\ b_1=-1,\ c_1=1$ & $a_2=1,\ b_2=1,\ c_2=2$

$$\therefore \cos\theta=\frac{2\times1+(-1)\times1+1\times2}{\left(\sqrt{2^2+(-1)^2+1^2}\right)\left(\sqrt{1^2+1^2+2^2}\right)} \Rightarrow \cos\theta=\frac{2-1+2}{6} \Rightarrow \cos\theta=\frac{3}{6} \Rightarrow \cos\theta=\frac{1}{2}$$

$$\Rightarrow \theta=\cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta=\cos^{-1}\left(\cos\frac{\pi}{3}\right) \quad \therefore \theta=\frac{\pi}{3}$$

Hence, the required angle between the plane is $\frac{\pi}{3}$.

(ii) We know that the angle between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is

given by $\cos\theta=\dfrac{a_1a_2+b_1b_2+c_1c_2}{\left(\sqrt{a_1^2+b_1^2+c_1^2}\right)\left(\sqrt{a_2^2+b_2^2+c_2^2}\right)}$

Here, $a_1=1,\ b_1=2,\ c_1=2$ & $a_2=2,\ b_2=-3,\ c_2=6$

$$\cos\theta=\frac{1\times2+2\times(-3)+2\times6}{\left(\sqrt{(1)^2+(2)^2+(2)^2}\right)\left(\sqrt{(2)^2+(-3)^2+(6)^2}\right)} \Rightarrow \cos\theta=\frac{2-6+12}{\sqrt{1+4+4}\sqrt{4+9+36}}$$

$$\Rightarrow \cos\theta=\frac{8}{\sqrt{9}\sqrt{49}} \Rightarrow \cos\theta=\frac{8}{3.7} \Rightarrow \cos\theta=\frac{8}{21} \Rightarrow \theta=\cos^{-1}\left(\frac{8}{21}\right)$$

Hence, the required angle between the plane is $\cos^{-1}\left(\frac{8}{21}\right)$

(iii) We know that the angle between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is

given by $\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\left(\sqrt{a_1^2+b_1^2+c_1^2}\right)\left(\sqrt{a_2^2+b_2^2+c_2^2}\right)}$

Here, $a_1=1,\ b_1=1,\ c_1=-1$ & $a_2=1,\ b_2=2,\ c_2=1$

$$\cos\theta=\frac{1\times1+1\times2+(-1)\times1}{\left(\sqrt{(1)^2+(1)^2+(-1)^2}\right)\left(\sqrt{(1)^2+(2)^2+(1)^2}\right)}$$

$$\Rightarrow\cos\theta=\frac{1+2-1}{\sqrt{18}}\Rightarrow\cos\theta=\frac{2}{3\sqrt{2}}\Rightarrow\cos\theta=\frac{\sqrt{2}}{3}\ \therefore\theta=\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

Hence, the required angle between the plane is $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$

(iv) We know that the angle between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is

given by $\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\left(\sqrt{a_1^2+b_1^2+c_1^2}\right)\left(\sqrt{a_2^2+b_2^2+c_2^2}\right)}$

Here, $a_1=1,\ b_1=1,\ c_1=-2$ & $a_2=2,\ b_2=-2,\ c_2=1$

$$\cos\theta=\frac{1\times2+1\times(-2)+(-2)\times1}{\left(\sqrt{(1)^2+(1)^2+(-2)^2}\right)\left(\sqrt{(2)^2+(-2)^2+(1)^2}\right)}$$

$$\Rightarrow\cos\theta=\frac{2-2-2}{3\sqrt{6}}\Rightarrow\cos\theta=\left(\frac{-2}{3\sqrt{6}}\right)\Rightarrow\theta=\cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$$

Hence, the required angle between the plane is $\cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$.

5. Show that each of the following pairs of planes are at right angles:

(i) $3x+4y-5z=7$ and $2x+6y+6z+7=0$

(ii) $x-2y+4z=10$ and $18x+17y+4z=49$

Sol. (i) We know that the plane $a_1x+b_1y+c_1z+d_1=0$ & $a_2x+b_2y+c_2z+d_2=0$

are perpendicular to each other only when $a_1a_2+b_1b_2+c_1c_2=0$

Here, $a_1=3,\ b_1=4,\ c_1=-5$ & $a_2=2,\ b_2=6,\ c_2=6$

$\therefore$ the given plane are perpendicular to each other

$\Rightarrow 3\times2+4\times6+(-5)\times6=6+24-30=0$

Hence, the pairs of plane are at right angles.

(ii) We know that the plane $a_1x+b_1y+c_1z+d_1=0$ & $a_2x+b_2y+c_2z+d_2=0$

are perpendicular to each other only when $a_1a_2+b_1b_2+c_1c_2=0$

Here, $a_1=1,\ b_1=-2,\ c_1=4$ & $a_2=18,\ b_2=17,\ c_2=4$

$\therefore$ the given plane are perpendicular to each other

$\Rightarrow 1\times18+(-2)\times17+4\times4=18-34+16=0$

Hence, the pairs of plane are at right angles.

6. Prove that the plane $2x+3y-4z=9$ is perpendicular to each of the planes $x+2y+2z-7=0$ and $5x+6y+7z=23$

Sol. Given equations of planes are $2x+3y-4z=9,\ x+2y+2z-7=0$

And $5x+6y+7z=23$

Here $\vec{n}=2\hat{i}+3\hat{j}-4\hat{k}$ A vector normal to the plane $2x+3y-4z=9$

$\vec{n}_1=\hat{i}+2\hat{j}+2\hat{k}$ {a vector normal to the plane $x+2y+2z-7=0$

$\vec{n}_2=5\hat{i}+6\hat{j}+7\hat{k}$ { A vector normal to the plane $5x+6y+7z=23$

Now $\vec{n},\vec{n}_1=(2\hat{i}+3\hat{j}-4\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})=2+6-8=0$

$\Rightarrow \vec{n}\cdot\vec{n}_1$

And $\vec{n}_1\cdot\vec{n}_2=(2\hat{i}+3\hat{j}-4\hat{k})\cdot(5\hat{i}+6\hat{j}+7\hat{k})=10+18-28=0$

$\because \vec{n}\perp\vec{n}_1$ and $\vec{n}_2$

Hence the plane $2x+3y-4z=9,$ is perpendicular to each of the planes $x+2y+2z-7=0$ and $5x+6y+7z=23$

7. Show that the planes $2x-2y+4+5=0$ and $3x-3y+6z-1=0$ are parallel

Sol. A vector normal to the plane $2x-2y+4+5=0$ is $\vec{n}_1=2\hat{i}-2\hat{j}+4\hat{k}$

And a vector normal to the plane $3x-3y+6z-1=0$ is $\vec{n}_2=3\hat{i}-3\hat{j}+6\hat{k}$

$\because \vec{n}_1=\frac{2}{3}\vec{n}_2$

$\Rightarrow \vec{n}_1\,||\,\vec{n}_2$

Hence both the planes are parallel.

8. Find the value of λ for which the planes $x-4y+\lambda z+3=0$ and $2x+2y+3z=5$ are perpendicular to each other

Sol. We know that the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$

Are perpendicular to each other only when $a_1a_2+b_1b_2+c_1c_2=0$

Here $a_1=1, b_1=-4, c_1=\lambda$

And $a_2=2,\ b_2=2, c_2=3$

Since both the planes are perpendicular

$\therefore 1\times2+(-4)\times2+\lambda\times3=0 \Rightarrow 2-8+3\lambda=0$

$\Rightarrow 3\lambda=6 \Rightarrow \lambda=2$

9. Write the equation of the plane passing through the origin and parallel to the plane $5x-3y+7z+11=0$

Sol. Any plane parallel to the plane $5x-3y+7z+11=0$ is given by $5x-3y+7z+d=0$ ...(i)

Since, it passes through origin

$\therefore d=0$

Hence equation of plane is $5x-3y+7z=0$

10. Find the equation of the plane passing through the point (a,b,c) and parallel to the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$

Sol. Any plane parallel to the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$ is given by

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=d$ .... (i)

Since it passes through (a,b,c)

$\therefore (a\hat{i}+b\hat{j}+c\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})=d$

$\Rightarrow a+b+c=d$

Hence the equation of plane is $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=a+b+c$

11. Find the equation of the plane passing through the point $(1,-2,7)$ and parallel to the plane $5x+4y-11z=6$

Sol. Any plane parallel to the plane $5x+4y-11z=d$

Since it passes through $(1,-2,7)$

$\therefore 5\times1+4(-2)-11\times7=d$

$\Rightarrow 5-8-77=d$

$\Rightarrow d=-80$

Hence equation at plane is $5x+4y-11z=-80$

$\Rightarrow 5x+4y-11z+80=0$

12. Find the equation of the plane passing through the point $(-1,-1,2)$, and perpendicular to each of the planes $3x+2y-3z=1$ and $5x-4y+z=5$.

Sol. Any plane through $(-1, -1, 2)$ is

$a(x+1)+b(y+1)+c(z-2)=2$ ... (i)

Now (i), being perpendicular to each of the planes

$3x+2y-3z=1$ and $5x-4y+z=5$: we have

$3\times a+2\times b-3\times c=0$

$\Rightarrow 3a+2b-3c=0$ ... (ii)

$a\times5+b\times(-4)+c\times1=0$

$\Rightarrow 5a-4b+c=0$ ... (iii)

cross multiplying (ii) and (iii) we get

$$\frac{a}{2-12}=\frac{b}{-3-15}=\frac{c}{-12-10}=\lambda \Rightarrow \frac{a}{-10}=\frac{b}{-18}=\frac{c}{-22}=\lambda$$

$\Rightarrow a=-5k, b=-9k, c=-11k$

putting these value of (i), we have

$-5k(x+1)-9k(y+1)-11k(z-2)=0 \Rightarrow -5x-5-9y-9-11z+22=0$

$\Rightarrow -5x-9y-11z+8=0 \Rightarrow 5x+9y+11z-8=0$

13. Find the equation of the plane passing through the origin, and perpendicular to each of the planes $x+2y-z=1$ and $3x-4y+z=5$.

Sol. Any plane through 0(0, 0, 0) is

$a(x-0)+b(y-0)+c(z-0)=0$ ... (i)

Now (i), being perpendicular to each of the plane

$x+2y-z=1$ and $3x-4y+z=5$, we have

$a\times 1+b\times 2+c\times(-1)=0$

$\Rightarrow a+2b-c=0$ ... (ii)

$a\times 3+b\times(-4)+c\times 1=0$

$\Rightarrow 3a-4b+c=0$ ... (iii)

cross multiplying (ii) and (iii) we have

$$\frac{a}{2-4}=\frac{b}{-3-1}=\frac{c}{-4-6}=k \Rightarrow \frac{a}{-2}=\frac{b}{-4}=\frac{c}{-10}=k \Rightarrow \frac{a}{1}=\frac{b}{2}=\frac{c}{5}=k$$

$a=k, b=2k, c=5k$

putting the value of a, b, c in equation (i)

$k(x-0)+2k(y-0)+5k(z-0)=0 \Rightarrow x+2y+5z=0$

required equation of the plane.

14. Find the equation of the plane that contains the point $A(1,-1,2)$ and is perpendicular to both the planes $2x+3y-2z=5$ and $x+2y-3z=8$ Hence find the distance of the point $P(-2,5,5)$ from the plane obtained above

Sol. Any plane through $A(1,-1,2)$ is given by $a(x-1)+b(y+1)+c(z-2)=0$ ..... (i)

Since it is perpendicular to each of the planes $2x+3y-2z=5$ and $x+2y+3z=8$ we have

$2a+3b-2c=0$ ...... (ii)

$a+2b-3c=0$ ...... (iii)

On solving (ii) and (iii) by cross multiplication we have

$$\frac{a}{(-9+4)}=\frac{b}{(-2+6)}=\frac{c}{(4-3)}=\lambda \Rightarrow a=-5\lambda, b=4\lambda, c=\lambda$$

Putting these value in (i), we get the required equation as $-5\lambda(x-1)+4\lambda(y+1)+\lambda(z-2)=0$

$\Rightarrow 5(x-1)-4(y+1)-(z-2)=0 \Rightarrow 5x-4y-z-7=0$

Distance of the point $P(-2,5,5)$ from this plane is given by

$$d=\frac{|5\times(-2)-4\times 5-5-7|}{\sqrt{5^2+(-4)^2+(-1)^2}}=\frac{|-42|}{\sqrt{42}}=\frac{42}{\sqrt{42}}=\sqrt{42} \text{ units}$$

15. Find the equation of the plane passing through the points $A(1,-1,2)$ and $B(2,-2,2)$, and perpendicular to the plane $6x-2y+2z=9$.

Sol. Any plane through the $A(1,-1,2)$

$a(x-1)+b(y+1)+c(z-2)=0$ ... (i)

and its passes through the point $B(2,-2,2)$

$a(2-1)+b(-2+1)+c(2-2)=0 \Rightarrow a-b+0c=0$ ... (ii)

Now (i), being perpendicular to each of the plane $6x-2y+2z=9$ then we have

$a\times 6+b\times(-2)+c\times 2=0 \Rightarrow 6a-2b+2c=0$

$\Rightarrow 3a-b+c=0$ ... (iii)

cross multiplying (ii) and (iii) we get

$$\frac{a}{-1-0}=\frac{b}{0-1}=\frac{c}{-1+3} \Rightarrow \frac{a}{-1}=\frac{b}{-1}=\frac{c}{2}=k \Rightarrow a=-k, b=-k, c=2k$$

putting the value of a, b, c in equation (i)

$-k(x-1)-k(y+1)+2k(z-2)=0 \Rightarrow -x+1-y-1+2z-4=0$

$\Rightarrow -x-y+2z-4=0 \Rightarrow x+y-2z+4=0$

16. Find the equation of the plane passing through the points $A(-1,1,1)$ and $B(1,-1,1)$, and perpendicular to the plane $x+2y+2z=5$.

Sol. Any plane through the point $A(-1, 1, 1)$

$a(x+1)+b(y-1)+c(z-1)=0$ ... (i)

and its passes through the point $(1, -1, 1)$

$a(1+1)+b(-1-1)+c(1-1)=0$

$\Rightarrow 2a-2b-0c=0$ ... (ii)

Now(i), being perpendicular to each of the plane

$x+2y+2z=5$ then we have

$a\times 1+b\times 2+c\times 2=0$

$\Rightarrow a+2b+2c=0$ ... (iii)

cross multiplying (ii) and (iii) we get

$\Rightarrow \frac{a}{-4-0}=\frac{b}{0-4}=\frac{c}{4+2}=\lambda \Rightarrow \frac{a}{-4}=\frac{b}{-4}=\frac{c}{6}=\lambda \Rightarrow \frac{a}{2}=\frac{b}{2}=\frac{2}{-3}=\lambda$

$\Rightarrow a=2\lambda,\ b=2\lambda,\ c=-3\lambda$

putting the value of a, b, c in equation (i)

$\Rightarrow 2\lambda(x+1)+2\lambda(y-1)-3\lambda(z-1)=0$

$\Rightarrow 2x+2+2y-2-3z+3=0 \Rightarrow 2x+2y-3z+3=0$

17. Find the equation of the plane through the points $A(3,4,2)$ and $B(7,0,6)$ and perpendicular to the plane $2x-5y=15$

Sol. The general equation of a plane passing through the point $A(3,4,2)$

$a(x-3)+b(y-4)+c(z-2)=0$ ..... (i)

Since the point $B(7,0,6)$ Lies on the plane

$\therefore a(7-3)+b(0-4)+c(6-2)=0$

$\Rightarrow 4a-4b+4c=0 \quad \Rightarrow a-b+c=0$ .... (ii)

Since the plane is perpendicular to the plane $2x-5y=15$

$\therefore 2a-5b+0\times c=0$ .... (iii)

On cross multiplying (ii) and (iii) we have $\frac{a}{\begin{vmatrix}-1 & 1\\ -5 & 0\end{vmatrix}}=\frac{-b}{\begin{vmatrix}1 & 1\\ 2 & 0\end{vmatrix}}=\frac{c}{\begin{vmatrix}1 & -1\\ 2 & -5\end{vmatrix}}=k$ (let)

$\Rightarrow \frac{a}{5}=\frac{b}{2}=\frac{c}{-3}=k$

$\Rightarrow a=5k,\ b=2k$ and $c=-3k$

Putting $a=5k, b=2k$ and $c=-3k$ in equation (i) we have

$5k(x-3)+2k(y-4)-3k(z-2)=0$

$\Rightarrow k\{5x-15+2y-8-3z+6\}=0$

$\Rightarrow 5x+2y-3z=17$

This is the required equation of plane

18. Find the equation of the plane through the points $A(2,1,-1)$ and $B(-1,3,4)$ and perpendicular to the plane $x-2y+4z=10$. Also show that the plane thus obtained contains the line $\vec{r}=\left(-\hat{i}+3\hat{j}+4\hat{k}\right)+\lambda\left(3\hat{i}-2\hat{j}-5\hat{k}\right)$

Sol. Any plane passing through the point $A(2,1,-1)$ is given by

$a(x-2)+b(y-1)+c(z+1)=0$ .. (i)

Since it passes through $B(-1,3,4)$

$\therefore a(-1-2)+b(3-1)+c(4+1)=0$

$\Rightarrow -3a+2b+5c=0 \quad \Rightarrow 3a-2b-5c=0$ .... (ii)

Since the plane is perpendicular to the plane $x-2y+4z=10$

$\therefore a-2b+4c=0$ .... (iii)

On solving (ii) and (iii) by cross multiplying we have

$$\frac{a}{\begin{vmatrix}-2 & -5\\ -2 & 4\end{vmatrix}}=\frac{b}{\begin{vmatrix}3 & -5\\ 1 & 4\end{vmatrix}}=\frac{c}{\begin{vmatrix}3 & -2\\ 1 & -2\end{vmatrix}}=k \quad \text{(let)}$$

$$\Rightarrow \frac{a}{-8-10}=\frac{-b}{12+5}=\frac{c}{-6+2}=k$$

$\Rightarrow a=-18k, b=-17k$ and $c=-4k$

Putting $a=-18k, b=-17k$ and $c=-4k$ in equation (i) we have

$-18k(x-2)-17k(y-1)-4k(z+1)=0$

$\Rightarrow -k\{18(x-2)+17(y-1)+4(z+1)\}=0$

$\Rightarrow 18x-36+17y-17+4z+4=0$

$\Rightarrow 18x+17y+4z-49=0$

$\Rightarrow 18x+17y+4=49$ .... (iv)

This is the required equation of plane

The given line is $\vec{r}=(3\lambda-1)\hat{i}+(3-2\lambda)\hat{j}+(4-5\lambda)\hat{k}$

Co-ordinates of any point on this line are $(3\lambda-1, 3-2\lambda, 4-5\lambda)$

Now, $18(3\lambda-1)+17(3-2\lambda)+4(4-5\lambda) = 54\lambda-18+51-34\lambda+16-20\lambda=49$

This the point $(3\lambda-1, 3-2\lambda, 4-5\lambda)$

Satisfy the equation (iv)

Hence the plane contains the line

EXERCISE 28 G [Pg. No.: 1231]

1. Find the angle between the line $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and the plane $\vec{r}.(2\hat{i}-\hat{j}+\hat{k})=4$.

Sol. The angle θ between the given line and the plane is given by

$$\sin\theta=\frac{|\vec{b}.\vec{n}|}{|b||n|}=\frac{|(\hat{i}-\hat{j}+\hat{k}).(2\hat{i}-\hat{j}+\hat{k})|}{\sqrt{1^2+(-1)+1^2}\sqrt{2^2+(-1)^2+1^2}}=\frac{|2\times1+(-1)\times(-1)+1\times1|}{\sqrt{3}\sqrt{6}}=\frac{4}{\sqrt{18}}=\frac{2\sqrt{2}}{3}$$

$$\Rightarrow \theta=\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right).$$

2. Find the angle between the line $\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+\lambda(3\hat{i}-\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=3$.

Sol. We know that the angle θ between the line $\vec{r}=\vec{r}_1+\lambda\vec{m}$ and the plane $\vec{r}.\vec{n}=q$ is given by

$\sin\theta=\left|\frac{\vec{m}.\vec{n}}{|\vec{m}|.|\vec{n}|}\right|$. Here, $\vec{m}=3\hat{i}-\hat{j}+2\hat{k}$ and $\vec{n}=\hat{i}+\hat{j}+\hat{k}$.

$$\therefore\ \sin\theta=\left|\frac{(3\hat{i}-\hat{j}+2\hat{k}).(\hat{i}+\hat{j}+\hat{k})}{|3\hat{i}-\hat{j}+2\hat{k}||\hat{i}+\hat{j}+\hat{k}|}\right|=\left|\frac{3\times1+(-1)\times1+2\times1}{\sqrt{3^2+(-1)^2+2^2}\sqrt{1^2+1^2+1^2}}\right|=\left|\frac{3-1+2}{\sqrt{14}\sqrt{3}}\right|=\frac{4}{\sqrt{42}}$$

$\therefore\ \theta=\sin^{-1}\frac{4}{\sqrt{42}}$. Hence, the angle between the line and the plane is $\sin^{-1}\frac{4}{\sqrt{42}}$.

3. Find the angle between the line $\vec{r}=(3\hat{i}+\hat{k})+\lambda(\hat{j}+\hat{k})$ and the plane $\vec{r}.(2\hat{i}-\hat{j}+2\hat{k})=1$.

Sol. $\vec{r}=(3\hat{i}+\hat{k})+\lambda(\hat{j}+\hat{k})$ and $\vec{r}.(2\hat{i}-\hat{j}+2\hat{k})=1$ we know that the angle θ between the line

$\vec{r}=\vec{n}_1+\lambda\vec{m}$ and the plane $\vec{r}.\vec{n}=9$ is given by $\Rightarrow \sin\theta=\frac{\vec{m}.\vec{n}}{|\vec{m}||\vec{n}|}$

$$\Rightarrow \sin\theta=\frac{(\hat{j}+\hat{k}).(2\hat{i}-\hat{j}+2\hat{k})}{\sqrt{(1)^2+(1)^2}\sqrt{(2)^2+(-1)^2+(2)^2}}$$

$$\Rightarrow \sin\theta=\frac{-1+2}{\sqrt{1+1}\sqrt{4+1+4}}\Rightarrow \sin\theta=\frac{1}{\sqrt{2}\sqrt{9}}\Rightarrow \sin\theta=\frac{1}{3\sqrt{2}}\ \therefore \theta=\sin^{-1}\left(\frac{1}{3\sqrt{2}}\right)$$

4. Find the angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and the plane $3x+4y+z+5=0$.

Sol. The given line is $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}=\lambda$

$\Rightarrow x=3\lambda+2,\ y=-\lambda-1,\ z=2\lambda+3$

$\Rightarrow (x\hat{i}+y\hat{k}+z\hat{k})=(2\hat{i}-\hat{j}+3\hat{k})+\lambda(3\hat{i}-\hat{j}+2\hat{k})$

and, given plane is $3x+4y+z+5=0$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(3\hat{i}+4\hat{j}+\hat{k})+5=0$, The angle between the line and plane is

$\because\ \sin\theta=\frac{\vec{m}.\vec{n}}{|\vec{m}||\vec{n}|}$ Here $\vec{m}=3\hat{i}-\hat{j}+2\hat{k}$ and $\vec{n}=3\hat{i}+4\hat{j}+\hat{k}$

$$\Rightarrow \sin\theta = \frac{(3\hat{i}-\hat{j}+2\hat{k}).(3\hat{i}+4\hat{j}+\hat{k})}{\sqrt{(3)^2+(-1)^2+(2)^2}\sqrt{(3)^2+(4)^2+(1)^2}}$$

$$\Rightarrow \sin\theta = \frac{9-4+2}{\sqrt{9+1+4}\sqrt{9+16+1}} \Rightarrow \sin\theta = \frac{7}{\sqrt{14}\sqrt{24}} \Rightarrow \sin\theta = \frac{7}{2\sqrt{91}}$$

$\therefore \theta = \sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)$, Hence the required angle is $\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)$.

5. Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z=3$

Sol. A vector parallel to the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ is $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$

A vector normal to the plate $10x+2y-11z=3$ is $\vec{n}=10\hat{i}+2\hat{j}-11\hat{k}$

Let θ be the angle between given line and the plane

$$\therefore \theta = \sin^{-1}\frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$$

$$\Rightarrow \theta = \sin^{-1}\frac{|2\times10+3\times2+6\times(-11)|}{\sqrt{2^2+3^2+6^2}\sqrt{10^2+2^2+(-11)^2}}$$

$$\Rightarrow \theta = \sin^{-1}\frac{|20+6-66|}{7\times15} \Rightarrow \theta = \sin^{-1}\frac{40}{105} \Rightarrow \theta = \sin^{-1}\left(\frac{8}{21}\right)$$

6. Find the angle between the line joining the points $A(3,-4,-2)$ and $B(12,2,0)$ and the plane $3x-y+z=1$

Sol. Equation of line joining the points $A(3,-4,-2)$ and $B(12,2,0)$ is

$$\frac{x-3}{12-3}=\frac{y+4}{2+4}=\frac{z+2}{0+2}$$

$$\Rightarrow \frac{x-3}{9}=\frac{y+4}{6}=\frac{z+2}{2}$$

A vector parallel to the line is $\vec{b}=9\hat{i}+6\hat{j}+2\hat{k}$

A vector normal to the plane is $\vec{n}=3\hat{i}-\hat{j}+\hat{k}$

Let θ be the angle between the line and plane

$$\theta = \sin^{-1}\frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$$

$$\Rightarrow \theta = \sin^{-1}\frac{|9\times3+6\times(-1)+2\times1|}{\sqrt{9^2+6^2+2^2}\sqrt{3^2+(-1)^2+1^2}}$$

$$\Rightarrow \theta = \sin^{-1}\frac{|27-6+2|}{\sqrt{121}\sqrt{11}} \Rightarrow \theta = \sin^{-1}\frac{23}{11\sqrt{11}}$$

7. If the plane $2x-3y-6z=13$ makes an angle $\sin^{-1}(\lambda)$ with the x-axis then find the value of λ

Sol. D.r.'s of the x-axis are 1, 0, 0 and d.r.'s of normal to the plane are 2, –3, –6

Let ϕ be the angle between the x-axis and the given plane Then

$$\sin\phi = \frac{|1\times 2+0\times(-3)+0\times(-6)|}{\left\{\sqrt{1^2+0^2+0^2}\right\}\left\{\sqrt{2^2+(-3)^2+(-6)^2}\right\}} = \frac{2}{7} \Rightarrow \phi = \sin^{-1}\left(\frac{2}{7}\right)$$

Hence $\lambda = \frac{2}{7}$

8. Show that the line $\vec{r} = (2\hat{i}+5\hat{j}+7\hat{k})+\lambda(\hat{i}+3\hat{j}+4\hat{k})$ is parallel to the plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k}) = 7$. Also, find the distance between them.

Sol. $\vec{r} = (2\hat{i}+5\hat{j}+7\hat{k})+\lambda(\hat{i}+3\hat{j}+4\hat{k})$ and $\vec{r}.(\hat{i}+\hat{j}-\hat{k}) = 7$

We know that the line $\vec{r} = \vec{r}_1+\lambda\vec{m}$ is parallel to a plane $\vec{r}.\vec{r}_1 = P$ then , $\vec{m}.\vec{n} = 0$

$\Rightarrow (\hat{i}+3\hat{j}+4\hat{k}).(\hat{i}+\hat{j}-\hat{k}) = 0 \Rightarrow 1+3-4=0 \Rightarrow 4-4=0 \Rightarrow 0=0$

Hence, the given line is parallel to the given plane and, distance between them

$$= \left|\frac{\vec{r}_1.\vec{n}-P}{|\vec{n}|}\right| = \frac{(2\hat{i}+5\hat{j}-7\hat{k}).(\hat{i}+\hat{j}-\hat{k})-7}{\sqrt{(1)^2+(1)^2+(-1)^2}} = \left|\frac{2+5-7-7}{\sqrt{3}}\right| = \frac{7}{\sqrt{3}} \text{ units.}$$

9. Find the value of m for which the line $\vec{r} = (\hat{i}+2\hat{k})+\lambda(2\hat{i}-m\hat{j}-3\hat{k})$ is parallel to the plane $\vec{r}.(m\hat{i}+3\hat{j}+\hat{k}) = 4$.

Sol. $\vec{r} = (\hat{i}+2\hat{k})+\lambda(2i-m\hat{j}-3\hat{k})$ and $\vec{r}.(m\hat{i}+3\hat{j}+\hat{k}) = 4$

We know that a line is parallel to the plane $\vec{r}.\vec{n} = p$. then $\vec{m}.\vec{n} = 0$

$\Rightarrow (2\hat{i}-m\hat{j}-3\hat{k}).(m\hat{i}+3\hat{j}+\hat{k}) = 0$

$\Rightarrow 2m-3m-3=0 \Rightarrow -m-3=0 \therefore m=-3$

10. Find the vector equation of a line passing through the origin and perpendicular to the plane $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k}) = 3$

Sol. The required line is perpendicular to the plane

$\vec{r}.(\hat{i}+2\hat{j}+3\hat{k}) = 3$ ... (i)

So the required line is parallel to $\vec{n} = \hat{i}+2\hat{j}+3\hat{k}$

Thus, the required line passes through the point with position vector $\vec{a} = 0\hat{i}+0\hat{j}+0\hat{k}$ and parallel to $\vec{n} = \hat{i}+2\hat{j}+3\hat{k}$

Hence, the vector equation of the required line is

$\vec{r} = \vec{a}+\lambda\vec{n}$ i.e. $\vec{r} = \lambda(\hat{i}+2\hat{j}+3\hat{k})$ ... (ii)

If the line (ii) meets the plane (i) then

$\lambda(\hat{i}+2\hat{j}+3\hat{k}).(\hat{i}+2\hat{j}+3\hat{k})-3=0$

$\Rightarrow \lambda(1+4+9)-3=0 \Rightarrow \lambda = \frac{3}{14}$

putting the value of λ in equation (ii), $\vec{r} = \frac{3}{14}(\hat{i}+2\hat{j}+3\hat{k})$

11. Find the vector equation of the line passing through the point with position vector $(\hat{i}-2\hat{j}+5\hat{k})$ and perpendicular to the plane $\vec{r}.(2\hat{i}-3\hat{j}-\hat{k}) = 0$.

Sol. The required line is perpendicular to the plane

$\vec{r}.(2\hat{i}-3\hat{j}-\hat{k})=0$ ... (i)

So, the required line is parallel to $\vec{n}=2\hat{i}-3\hat{j}-\hat{k}$

Thus, the required line passing through the point with position vector $\vec{a}=\hat{i}-2\hat{j}+5\hat{k}$ and is parallel to $\vec{n}=2\hat{i}-3\hat{j}-\hat{k}$

Hence, the vector equation of the required line is

$$\vec{r}=\vec{a}+\lambda\vec{n} \Rightarrow \vec{r}=(\hat{i}-2\hat{j}+5\hat{k})+\lambda(2\hat{i}-3\hat{j}-\hat{k})$$

for some scalar value λ.

12. Show that the equation $ax+by+d=0$ represents a plane parallel to the z-axis Hence find the equation of a plane which is parallel to the z-axis and passes through the points $A(2,-3,1)$ and $B(-4,7,6)$

Sol. The given equation is $ax+by+0.z+d=0$ which is of the form $ax+by+cz+d=0$

Therefore it represents a plane

D.r.'s of normal to the plane are $a,b,0$

D.r.'s of the z-axis are $0,0,1$

Now $a\times0+b\times0+0\times1=0$

This shows that the given plane is parallel to the z-axis

Let the required plane be $ax+by+d=0$ ..... (i)

Since it passes through the points $A(2,-3,1)$ and $B(-4,7,6)$ we have

$2a-3b+d=0$ ..... (ii)

$-4a+7b+d=0$ ..... (iii)

On solving (ii) and (iii) by cross multiplication we get

$$\frac{a}{(-3-7)}=\frac{b}{(-4-2)}=\frac{c}{(14-12)}$$

$$\Rightarrow \frac{a}{-10}=\frac{b}{-6}=\frac{c}{2} \quad \Rightarrow \frac{a}{5}=\frac{b}{3}=\frac{c}{-1}=k \text{ (say)}$$

$\therefore a=5k,\ b=3k$ and $c=-k$

Putting these value in (i), we get $5kx+3ky-k=0 \Rightarrow 5x+3y-1=0$

Which is the required equation of the plane

13. Find the equation of the plane passing through the points $(1,2,3)$ and $(0,-1,0)$ and parallel to the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}$

Sol. Any plane through $(1,2,3)$ is $a(x-1)+b(y-2)+c(z-3)=0$

Since it passes through $(0,-1,0)$ we have $a(0-1)+b(-1-2)+c(0-3)=0 \Rightarrow a+3b+3c=0$

It is being given that the plane (i) is parallel to the line

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}$$

$\therefore 2a+3b-3c=0$

On solving (i) and (ii), we get $\frac{a}{(9+9)}=\frac{b}{(-3-6)}=\frac{c}{(6-3)} \Rightarrow \frac{a}{6}=\frac{b}{-3}=\frac{c}{1}$

Hence the required plane is $6(x-1)-3(y-2)+1\cdot(z-3)=0 \Rightarrow 6x-3y+z=3$

14. Find the equation of a plane passing through the point $(2,-1,5)$ perpendicular to the plane $x+2y-3z=7$ and parallel to the line $\frac{x+5}{3}=\frac{y+1}{-1}=\frac{z-2}{1}$

Sol. Any plane through the point $(2,-1,5)$ is given by $a(x-2)+b(y+1)+c(z-5)=0$ ... (i)

Since it is perpendicular to the plane $x+2y-3z=7$

$\therefore 1\times a+2\times b-3\times c=0$

$\Rightarrow a+2b-3c=0$ .... (ii)

Since the plane is parallel to the line $\frac{x+5}{3}=\frac{y+1}{-1}=\frac{z-2}{1}$

$\therefore 3a-b+c=0$ ..... (iii)

On solving (ii) and (iii) by cross multiplying we have

$$\frac{a}{\begin{vmatrix}2 & -3\\ -1 & 1\end{vmatrix}}=\frac{b}{\begin{vmatrix}1 & -3\\ 3 & 1\end{vmatrix}}=\frac{c}{\begin{vmatrix}1 & 2\\ 3 & -1\end{vmatrix}}=k \text{ (let)}$$

$$\Rightarrow \frac{a}{2-3}=\frac{-b}{1+9}=\frac{c}{-1-6}=k$$

$\Rightarrow a=-k, b=-10k$ and $c=-7k$

Putting $a=-k, b=-10k \,\&\, c=-7k$

In equation (i) we have

$-k(x-2)-10k(y+1)-7k(z-5)=0$

$\Rightarrow -5\{x-2+10(y+1)+7(z-5)\}=0$

$\Rightarrow x-2+10y+10+7z-35=0$

$\Rightarrow x+10y+7z-27=0$ this is the required equation of plane

15. Find the equation of the plane passing through the intersection of the planes $4x-y+z=10$ and $x+y-z=4$, and parallel to the line having direction ratios $2,1,1$.

Find also the perpendicular distance of $(1, 1, 1)$ from this plane.

Sol. the equation of a plane passing through the intersection of the given is

$(4y-y+z-10)+\lambda(x+y-z-4)=0$

$\Rightarrow (4+\lambda)x+(-1+\lambda)y+(1-\lambda)z+(-10-4\lambda)=0$ .....(i)

Let this plane be parallel to the line with direction ratio 2, 1, 1. Then the normal to this is perpendicular to the line having the direction ratio 2, 1, 1.

$\therefore\ 2(4+\lambda)+1(-1+\lambda)+1(1-\lambda)=0$

$\Rightarrow 8+2\lambda-1+\lambda+1-\lambda=0 \Rightarrow 2\lambda=-8 \Rightarrow \lambda=-4$

Putting the value of λ in equation (i), we get the required equation of the plane as.

$(4x-y+z-10)-4(x+y-z-4)=0$

$\Rightarrow 4x-y+z-10-4x-4y+4z+16=0 \Rightarrow -5y+5z+6=0 \Rightarrow 5y-5z-6=0$

required equation of the plane.

The length of perpendicular from the point (1, 1, 1)

$$P=\frac{|5.1-5.1-6|}{\sqrt{(5)^2+(-5)^2}}=\frac{6}{\sqrt{50}}=\frac{6}{5\sqrt{2}}=\frac{3\sqrt{2}}{5}$$

EXERCISE 28 H [Pg. No.: 1237]

1. Find the vector and Cartesian equations of the plane passing through the origin and parallel to the vectors $\left(\hat{i}+\hat{j}-\hat{k}\right)$ and $\left(3\hat{i}-\hat{k}\right)$

Sol. We know that vector equation at plane passing through a point having position vector $\vec{a}$ and parallel to $\vec{b}$ and $\vec{c}$ is given by $(\vec{r}-\vec{a})\cdot\left(\vec{b}\times\vec{c}\right)=0$

Here $\vec{a}=\vec{0}$

$\vec{b}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=3\hat{i}-\hat{k}$

$$\text{Now } \vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}=\begin{vmatrix}1 & -1\\ 0 & -1\end{vmatrix}\hat{i}-\begin{vmatrix}1 & -1\\ 3 & -1\end{vmatrix}\hat{j}+\begin{vmatrix}1 & 1\\ 3 & 0\end{vmatrix}\hat{k}=-\hat{i}-2\hat{j}-3\hat{k}$$

So the required equation is $\vec{r}\left(-\hat{i}-2\hat{j}-3\hat{k}\right)=0 \Rightarrow \vec{r}\cdot\left(\hat{i}+2\hat{j}+3\hat{k}\right)=0$

2. Find the vector and Cartesian equations of the plane passing through the point $(3,-1,2)$ and parallel to the lines $\vec{r}=\left(-\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}-5\hat{j}-\hat{k}\right)$ and $\vec{r}=\left(\hat{i}-3\hat{j}+\hat{k}\right)+\mu\left(-5\hat{i}+4\hat{j}\right)$

Sol. We know that $(\vec{r}-\vec{a})\left(\vec{b}\times\vec{c}\right)=0$

Here $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}$

$\vec{b}=2\hat{i}-5\hat{j}-\hat{k}$ and $\vec{c}=-5\hat{i}+4\hat{j}$

$$\text{Now, } \vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -5 & -1\\ -5 & 4 & 0\end{vmatrix}=\begin{vmatrix}-5 & -1\\ 4 & 0\end{vmatrix}\hat{i}-\begin{vmatrix}2 & -1\\ -5 & 0\end{vmatrix}\hat{j}+\begin{vmatrix}2 & -5\\ -5 & 4\end{vmatrix}\hat{k}=4\hat{i}+5\hat{j}-17\hat{k}$$

So the required equation is $(\vec{r}-\vec{a})\cdot\left(\vec{b}\times\vec{c}\right)=0$

$\Rightarrow \left[(x-3)\hat{i}+(y+1)\hat{j}+(z-2)\hat{k}\right]\cdot\left(4\hat{i}+5\hat{j}-17\hat{k}\right)=0$

$\Rightarrow 4(x-3)+5(y+1)-17(z-2)=0 \Rightarrow 4x-12+5y+5-17z+34=0 \Rightarrow 4x+5y-17z+27=0$

This is the Cartesian equation of plane

In vector from $\vec{r}\cdot\left(4\hat{i}+5\hat{j}-17\hat{k}\right)+27=0$

3. Find the vector equation of a plane passing through the point $(1,2,3)$ and parallel to the lines whose direction ratios are $1,-1,-2$ and $-1,0,2$

Sol. The equation of the plane passing through a given point $A(x_1,y_1,z_1)$ and parallel to two given lines having direction ratios b_1,b_2,b_3 and c_1,c_2,c_3 is

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=0$$

Here $x_1=1, y_1=2, z_1=3$

$b_1=1, b_2=-1, b_3=-2$

$c_1=-1,\ c_2=0,\ c_3=2$

Hence the plane is $\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} -1 & -2 \\ 0 & 2 \end{vmatrix}(x-1) - \begin{vmatrix} 1 & -2 \\ -1 & 2 \end{vmatrix}(y-2) + \begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix}(z-3) = 0$

$\Rightarrow -2(x-1)-(2-2)(y-2)+(-1)(z-3)=0$

$\Rightarrow -2x+2-z+3=0 \Rightarrow -2x-z+5=0 \Rightarrow 2x+z-5=0 \Rightarrow 2x+z=5$

In vector form $\vec{r}\cdot\left(2\hat{i}+\hat{k}\right)=5$

4. Find the Cartesian and vector equations of a plane passing through the point $m(1,2,-4)$ and parallel to the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ and $\frac{x-1}{1}=\frac{y+3}{1}=\frac{z}{-1}$

Sol. Here, $x_1=1, y_1=2, z_1=-4$

$b_1=2, b_2=3, b_3=6$

And $c_1=1, c_2=1$ and $c_3=-1$

Hence the equation of plane is $\begin{vmatrix} x-1 & y-2 & z+4 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix} = 0$

$\Rightarrow (x-1)\begin{vmatrix} 3 & 6 \\ 1 & -1 \end{vmatrix} - (y-2)\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix} + (z+4)\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 0$

$\Rightarrow (-3-6)(x-1)-(-2-6)(y-2)+(2-3)(z+4)=0$

$\Rightarrow -9x+9+8y-16-z-4=0 \Rightarrow -9x+8y-z-11=0 \Rightarrow 9x-8y+z+11=0$

In vector from $\vec{r}\cdot\left(9\hat{i}-8\hat{j}+\hat{k}\right)+11=0$

5. Find the vector equation of the plane passing through the point $\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and parallel to the vectors $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\left(\hat{i}-\hat{j}+\hat{k}\right)$

Sol. The vector equation of a plane passing through a given point with position vector $\vec{a}$ and parallel to two given vectors $\vec{b}$ and $\vec{c}$ is

$(\vec{r}-a).\left(\vec{b}\times\vec{c}\right)=0$

Here $\vec{a}=3\hat{i}+4\hat{j}+2\hat{k}$

$\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$

Now $\vec{b}\times\vec{c}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix} \Rightarrow \vec{b}\times\vec{c}=\begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix}\hat{i}-\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}\hat{j}+\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}\hat{k}$

$\Rightarrow \vec{b}\times\vec{c}=(2+3)\hat{i}-(1-3)\hat{j}+(-1-2)\hat{k} \Rightarrow \vec{b}\times\vec{c}=5\hat{i}+2\hat{j}-3\hat{k}$

And $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)=\left(3\hat{i}+4\hat{j}+2\hat{k}\right)\cdot\left(5\hat{i}+2\hat{j}-3\hat{k}\right)=15+8-6=17$

Now $(\vec{r}-\vec{a})\cdot\left(\vec{b}\times\vec{c}\right)=0 \Rightarrow \vec{r}\cdot\left(\vec{b}\times\vec{c}\right)=\vec{a}\cdot\left(\vec{b}\times\vec{c}\right) \Rightarrow \vec{r}\cdot\left(5\hat{i}+2\hat{j}-3\hat{k}\right)=17$

This is the required equation of plane

EXERCISE 28 I [Pg. No.: 1244]

1. Show that the lines $\vec{r}=(2\hat{j}-3\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$, and $\vec{r}=(2\hat{i}+6\hat{j}+3\hat{k})+\mu(2\hat{i}+3\hat{j}+4k)$ are coplanar. Also, find the equation of the plane containing them.

Sol. $\vec{r}=(2\hat{j}-3\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$ and $\vec{r}=(2\hat{i}+6\hat{j}+3\hat{k})+\mu(2\hat{i}+3\hat{j}+4\hat{k})$

$\Rightarrow$ for coplanar, $(\vec{r}_2-\vec{r}_1).(\vec{m}_1\times\vec{m}_2)=0$

$\Rightarrow(\vec{r}_2-\vec{r}_1)=(2\hat{i}+6\hat{j}+3\hat{k})-(2\hat{j}-3\hat{k})=(2\hat{i}+4\hat{j}+6\hat{k})$

$$\Rightarrow(\vec{m}_1\times\vec{m}_2)=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&3\\2&3&4\end{vmatrix}=\hat{i}\begin{vmatrix}2&3\\3&4\end{vmatrix}-\hat{j}\begin{vmatrix}1&3\\2&4\end{vmatrix}+\hat{k}\begin{vmatrix}1&2\\2&3\end{vmatrix}=\hat{i}(8-9)-\hat{j}(4-6)+\hat{k}(3-4)$$

$=(-\hat{i}+2\hat{j}-\hat{k})$

Now, $\Rightarrow(2\hat{i}+4\hat{j}+6\hat{k})(-\hat{i}+2\hat{j}-\hat{k})=0$

$\Rightarrow\ 2+8-6=0\Rightarrow 8-8=0\Rightarrow 0=0$

Hence the given lines are coplanar and for required equation, $(\vec{r}_2-\vec{r}_1).(\vec{m}_1\times\vec{m}_2)=0$

$\Rightarrow\{\vec{r}-(2\hat{j}-3\hat{k})\}.(-\hat{i}+2\hat{j}-\hat{k})=0\Rightarrow\vec{r}.(-\hat{i}+2\hat{j}-\hat{k})-(2\hat{j}-3\hat{k})(-\hat{i}+2\hat{j}-\hat{k})=0$

$\Rightarrow\vec{r}.(\hat{i}+2\hat{j}-\hat{k})-(4+3)=0\Rightarrow\vec{r}.(-\hat{i}+2\hat{j}-\hat{k})-7=0$

$\Rightarrow\vec{r}.(-\hat{i}+2\hat{j}-\hat{k})=7\Rightarrow\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k})=-7\Rightarrow\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k})+7=0$

Hence, the required equation is $\vec{r}.(\hat{i}-2\hat{j}+k)+7=0$

2. Find the vector and Cartesian forms of the equation of the plane containing two lines $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$, and $\vec{r}=(3\hat{i}+3\hat{j}-5\hat{k})+\mu(-2\hat{i}+3\hat{j}+8\hat{k})$.

Sol. $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=(3\hat{i}+3\hat{j}-5\hat{k})+\mu(-2\hat{i}+3\hat{j}+8\hat{k})$

For required equation, $(\vec{r}_2-\vec{r}_1).(\vec{m}_1\times\vec{m}_2)=0$

$$\Rightarrow(\vec{m}_1\times\vec{m}_2)=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&6\\-2&3&8\end{vmatrix}=\hat{i}\begin{vmatrix}3&6\\3&8\end{vmatrix}-\hat{j}\begin{vmatrix}2&6\\-2&8\end{vmatrix}+\hat{k}\begin{vmatrix}2&3\\-2&3\end{vmatrix}$$

$=\hat{i}(24-18)-\hat{j}(16+12)+\hat{k}(6+6)=(6\hat{i}-28\hat{j}+12\hat{k})$

Now, $\Rightarrow\{\vec{r}-(\hat{i}+2\hat{j}-4\hat{k})\}.(6\hat{i}-28\hat{j}+12\hat{k})=0$

$\Rightarrow\vec{r}.(6\hat{i}-28\hat{j}+12\hat{k})-(\hat{i}+2\hat{j}-4\hat{k}).(6\hat{i}-28\hat{j}+12\hat{k})=0$

$\Rightarrow\vec{r}(6\hat{i}-28\hat{j}+12\hat{k})-(6-56-48)=0$

$\Rightarrow\vec{r}.(6\hat{i}-28\hat{j}+12\hat{k})+98=0$

on Cartesian form

$\Rightarrow(x\hat{i}+y\hat{j}+z\hat{k}).(6\hat{i}-28\hat{j}+12\hat{k})+98=0\Rightarrow 6x-28y+12z+98=0$

Hence the required equation is $\vec{r}.16\hat{i}-28\hat{j}+12\hat{k}+98=0$ and $6x-28y+12z+98=0$

3. Find the vector and Cartesian equations of a plane containing the two lines $\vec{r}=\left(2\hat{i}+\hat{j}-3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+5\hat{k}\right)$ and $\vec{r}=\left(3\hat{i}+3\hat{j}+2\hat{k}\right)+\mu\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$

Also show that the line $\vec{r}=\left(2\hat{i}+5\hat{j}+2\hat{k}\right)+p\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$ lies in the plane

Sol. The given lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$ where

$\vec{a}_1=\left(2\hat{i}+\hat{j}-3\hat{k}\right), \vec{a}_2=\left(3\hat{i}+3\hat{j}+2\hat{k}\right)$

$\vec{b}_1=\left(\hat{i}+2\hat{j}+5\hat{k}\right), \vec{b}_2=\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$

$$\therefore \left(\vec{b}_1\times\vec{b}_2\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 5\\ 3 & -2 & 5\end{vmatrix}=(10+10)\hat{i}-(5-15)\hat{j}+(-2-6)\hat{k}=\left(20\hat{i}+10\hat{j}-8\hat{k}\right)$$

Vector equation of the required plane is $\vec{r}\cdot\left(\vec{b}_1\times\vec{b}_2\right)=\vec{a}_1\cdot\left(\vec{b}_1\times\vec{b}_2\right)$

$\Rightarrow \vec{r}\cdot\left(20\hat{i}+10\hat{j}-8\hat{k}\right)=\left(2\hat{i}+\hat{j}-3\hat{k}\right)\cdot\left(20\hat{i}+10\hat{j}-8\hat{k}\right)=(40+10+24)=74$

$\Rightarrow \vec{r}\cdot\left(10\hat{i}+5\hat{j}-4\hat{k}\right)=37$ .... (i)

The certesian equation is $\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(10\hat{i}+5\hat{j}-4\hat{k}\right)=37 \Rightarrow 10x+5y-4z=37$ ... (ii)

The third line is $\vec{r}=\left(2\hat{i}+5\hat{j}+2\hat{k}\right)+p\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$ .... (iii)

Now the line (iii) will lie in the plane (ii) if $(2,5,2)$ lies on (ii) and $\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$ I perpendicular to the normal of (ii)

Now, $10\times2+5\times5-4\times2=37$ shows that $(2,5,2)$ lies on (ii)

Also $10\times3+5\times(-2)-4\times5=0$ shows that $\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$ is perpendicular to the normal of (ii)

Hence the line (iii) lies in plane (ii)

4. Prove that the lines $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ and $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ are coplanar .

Also find the equation of the plane containing these lines

Sol. We know that the lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$

Are coplanar $\Leftrightarrow \begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=0$

And the equation of the plane containing these lines is $\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=0$

Here $x_1=0, y_1=2, z_1=-3; x_2=2, y_2=6, z_2=3; a_1=1, b_1=2, c_1=3; a_2=2, b_2=3, c_2=4$

$$\therefore \begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=0$$

Hence the two given lines are coplanar

The equation of the plane containing both these line is

$$\begin{vmatrix} x-0 & y-2 & z+3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}=0 \Leftrightarrow \begin{vmatrix} x & y-2 & z+3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}=0$$

$$\Leftrightarrow x(8-9)-(y-2)(4-6)+(z+3)(3-4)=0$$

$$\Leftrightarrow -x+2(y-2)-(z+3)=0 \Leftrightarrow x-2y+z+7=0$$

Hence the required plane is $x-2y+z+7=0$

5. Prove that the lines $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$, and $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ are coplanar. Also, find the equation of the plane containing both these lines.

Sol. The given first line is, $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}=\lambda$

$$\Rightarrow x=\lambda+2,\ y=4\lambda+4,\ z=7\lambda+6$$

$$\Rightarrow\left(x\hat{i}+y\hat{j}+z\hat{k}\right)=\left(z\hat{i}+4\hat{j}+6\hat{k}\right)+\lambda\left(\hat{i}+4\hat{j}+7\hat{k}\right)$$

$$\Rightarrow \vec{r}=\left(2\hat{i}+4\hat{j}+6\hat{k}\right)+\lambda\left(\hat{i}+4\hat{j}+7\hat{k}\right) \quad \ldots \text{(i)}$$

and the second given line is $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}=\mu$

$$\Rightarrow x=3\mu-1,\ y=5\mu-3,\ z=7\mu-5$$

$$\Rightarrow\left(x\hat{i}+y\hat{j}+z\hat{k}\right)=\left(-\hat{i}+(-3)\hat{j}-5\hat{k}\right)+\mu\left(3\hat{i}+5\hat{j}+7\hat{k}\right)$$

$$\Rightarrow \vec{r}=\left(-\hat{i}-3\hat{j}-5\hat{k}\right)+\mu\left(3\hat{i}+5\hat{j}+7\hat{k}\right) \quad \ldots \text{(ii)}$$

For coplanar,

$$\left(\vec{r}_2-\vec{r}_1\right).\left(\vec{m}_1\times\vec{m}_2\right)=0 \Rightarrow \left(\vec{r}_2-\vec{r}_1\right)=\left(-\hat{i}-3\hat{j}-5\hat{k}\right)-\left(2\hat{i}+4\hat{j}+6\hat{k}\right)=\left(-3\hat{i}-7\hat{j}-11\hat{k}\right)$$

$$\Rightarrow\left(\vec{m}_1\times\vec{m}_2\right)=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}=\hat{i}\begin{vmatrix} 4 & 7 \\ 5 & 7 \end{vmatrix}-\hat{j}\begin{vmatrix} 1 & 7 \\ 3 & 7 \end{vmatrix}+\hat{k}\begin{vmatrix} 1 & 4 \\ 3 & 5 \end{vmatrix}$$

$$=\hat{i}(28-35)-\hat{j}(7-21)+\hat{k}(5-12)=\left(-7\hat{i}+14\hat{j}-7\hat{k}\right)$$

Now, $\Rightarrow\left(-3\hat{i}-7\hat{j}-11\hat{k}\right).\left(-7\hat{i}+14\hat{j}-7\hat{k}\right)=0 \Rightarrow 21-98+77=0 \Rightarrow 98-98=0 \Rightarrow 0=0$

Hence, the given lines are coplanar.

For required equation is, $\left(\vec{r}_2-\vec{r}_1\right).\left(\vec{m}_1\times\vec{m}_2\right)=0$

$$\Rightarrow\left\{\vec{r}-\left(2\hat{i}+4\hat{j}+6\hat{k}\right)\right\}.\left(-7\hat{i}+14\hat{j}-7\hat{k}\right)=0$$

$$\Rightarrow \vec{r}.\left(-7\hat{i}+14\hat{j}-7\hat{k}\right)-\left(2\hat{i}+4\hat{j}+6\hat{k}\right).\left(-7\hat{i}+14\hat{j}-7\hat{k}\right)=0$$

$$\Rightarrow \vec{r}\left(-7\hat{i}+14\hat{j}-7\hat{k}\right)-(-14+56-42)=0$$

On Cartesian equation, $\Rightarrow\left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(\hat{i}-2\hat{j}+\hat{k}\right)=0 \Rightarrow x-2y+z=0$

Hence, the required equation is, $x-2y+z=0$

6. Show that the lines $\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$ are coplanar. Find the equation of the plane containing these lines

Sol. We know that the lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_2}=\frac{z-z_2}{c_2}$ are coplanar

It $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$

Given equations of line are $\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$

i.e. $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ ..... (i)

and $\frac{x-8}{7}=\frac{27-8}{2}=\frac{z-5}{3}$

i.e. $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$

here $x_1=5,\ y_1=7,\ z_1=-3$

$x_2=8,\ y_2=4,\ z_2=5$

$a_2=7,\ b_2=1, c_2=-5$

Now $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$=\begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}=\begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}=\begin{vmatrix} 3 & 3 & 8 \\ 7 & 1 & 3 \\ 7 & 1 & 3 \end{vmatrix}\ \{R_2\to R_2+R_1\}$

$=0$ $\{\because R_2$ and R_3 are identical Hence, both the lines are coplanar$\}$

Now required equation at plane is $\begin{vmatrix} x-5 & y-7 & z+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}=0$

$\Rightarrow (x-5)\begin{vmatrix} 4 & -5 \\ 1 & 3 \end{vmatrix}-(y-7)\begin{vmatrix} 4 & -5 \\ 7 & 3 \end{vmatrix}+(z+3)\begin{vmatrix} 4 & 4 \\ 7 & 1 \end{vmatrix}=0$

$\Rightarrow (x-5)(12+5)-(y-7)(12+35)+(z+3)(y-28)=0$

$\Rightarrow 17(x-5)-47(y-7)-24(z+3)=0$

$\Rightarrow 17x-85-47y+32y-24z-72=0$

$\Rightarrow 17x-47y-24z+172=0$

This is the required equation of plane

7. Show that the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ are coplanar

Find the equation of the plane containing these lines

Sol. The given first line is, $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}=\lambda$

$\Rightarrow x=\lambda, y=-3\lambda+7,\ z=2\lambda-7 \Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right)=\left(7\hat{j}-7\hat{k}\right)+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$

$\Rightarrow \vec{r}=\left(7\hat{j}-7\hat{k}\right)+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ ... (i)

and the second given line is

$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}=\mu \Rightarrow x=-3\mu-1,\ \ y=2\mu+3,\ \ z=\mu-2$

$\Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right)=\left(-\hat{i}+3\hat{j}-2\hat{k}\right)+\mu\left(-3\hat{i}+2\hat{j}+\hat{k}\right)$

$\Rightarrow \vec{r}=\left(-\hat{i}+3\hat{j}-2\hat{k}\right)+\mu\left(-3\hat{i}+2\hat{j}+\hat{k}\right)$ ... (ii)

For coplanar, $(\vec{r}_2-\vec{r}_1).(\vec{m}_1\times\vec{m}_2)=0$

$\Rightarrow (\vec{r}_2-\vec{r}_1)=\left(-\hat{i}+3\hat{j}-2\hat{k}\right)-\left(7\hat{j}-7\hat{k}\right)\left(-\hat{i}-4\hat{j}+5\hat{k}\right)$

$\Rightarrow (\vec{m}_1\times\vec{m}_2)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 2\\ -3 & 2 & 1\end{vmatrix}=\hat{i}\begin{vmatrix}-3 & 2\\ 2 & 1\end{vmatrix}-\hat{j}\begin{vmatrix}1 & 2\\ -3 & 1\end{vmatrix}+\hat{k}\begin{vmatrix}1 & -3\\ -3 & 2\end{vmatrix}$

$\Rightarrow \hat{i}(-3-4)-\hat{j}(1+6)+\hat{k}(2-9) \Rightarrow \left(-7\hat{i}-7\hat{j}-7\hat{k}\right)$

Now, $\Rightarrow \left(-\hat{i}-4\hat{j}+5\hat{k}\right).\left(-7\hat{i}-7\hat{j}-7\hat{k}\right)=0 \Rightarrow 7+28-35=0 \Rightarrow 7-7=0 \Rightarrow 0=0$

Hence, the given lines are coplanar and for required equation,

$(\vec{r}-\vec{r}_1).(\vec{m}_1\times\vec{m}_2)=0 \Rightarrow \left\{\vec{r}-\left(7\hat{j}-7\hat{k}\right)\right\}.\left(-7\hat{i}-7\hat{j}-7\hat{k}\right)=0$

$\vec{r}.\left(-7\hat{i}-7\hat{i}-7\hat{k}\right)-\left(7\hat{j}-7\hat{k}\right).\left(-7\hat{i}-7\hat{j}-7\hat{k}\right)=0$

$\Rightarrow \vec{r}\left(-7\hat{i}-7\hat{j}-7\hat{k}\right)-(-49+49)=0$

$\Rightarrow \vec{r}.\left(7\hat{i}-7\hat{j}-7\hat{k}\right)=0 \Rightarrow \vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=0$

On Cartesian equation $\Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(\hat{i}+\hat{j}+\hat{k}\right)=0 \Rightarrow x+y+z=0$

Hence, the required equation is, $x+y+z=0$

8. Show that the line $\frac{x-1}{1}=\frac{y-3}{-1}=\frac{z}{-1}$ and $\frac{x-4}{3}=\frac{y-1}{-2}=\frac{z-1}{-1}$ are coplanar

Also find the equation of the plane containing these lines

Sol. We know that the lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are coplnar

If. $\begin{vmatrix}x_2-x_1 & y_2y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=0$

Here
$x_1=1,\quad y_1=3,\quad z_1=0$
$x_2=4,\quad y_2=1,\quad z_2=1$
$a_1=2,\quad b_1=-1\quad c_1=-1$
$a_2=3,\quad b_2=-2\quad c_2=-1$

Now $\begin{vmatrix} 4-1 & 1-3 & 1-0 \\ 2 & -1 & -1 \\ 3 & -2 & -1 \end{vmatrix} = \begin{vmatrix} 3 & -2 & 1 \\ 2 & -1 & -1 \\ 3 & -2 & -1 \end{vmatrix}$

$= \begin{vmatrix} 1 & -2 & 1 \\ 1 & -1 & -1 \\ 1 & -2 & -1 \end{vmatrix} \{C_1 \to C_2 + C_2$

$= \begin{vmatrix} -0 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & -2 & -1 \end{vmatrix} \begin{cases} R_1 + R_2 - R_2 \\ R_2 + R_2 - R_3 \end{cases}$

$= \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} = -2 \neq 0$

Hence the lines is non coplanar

9. Find the equation of the plane which contains two parallel lines given by $\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$ and $\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$

Sol. The plane which contains the two given parallel lines must pass through the point (3, −2, 0) and (4, 3, 2) and must be parallel to the line having direction ratio 1, −4, 5

Any plane passing through (3, −2, 0) is

$a(x-3) + b(y+2) + c(z-0) = 0$ - (i)

If this plane passes through the point (4, 3, 2) then

$a(4-3) + b(3+2) + c(2-0) = 0$

$\Rightarrow a + 5b + 2c = 0$ - (ii)

If the plane (i) is parallel to the line having direction ratio 1, −4, 5 then

$a - 4b + 5c = 0$ - (iii)

cross multiplying (ii) and (iii) we get

$\frac{a}{25+8} = \frac{b}{2-5} = \frac{c}{-4-5} = \lambda$

$\Rightarrow \frac{a}{33} = \frac{b}{-3} = \frac{c}{-9} = \lambda$

$\Rightarrow \frac{a}{11} = \frac{b}{-1} = \frac{c}{-3} = \lambda$

$a = 11\lambda,\ b = -\lambda,\ c = -3\lambda$

putting the value of a, b, c in equation (I)

$11\lambda(x-3) - \lambda(y+2) - 3\lambda(z-0) = 0$

$\Rightarrow 11x - 33 - y - 2 - 3z = 0$

$\Rightarrow 11x - y - 3z - 35 = 0$

$\Rightarrow 11x - y - 3z = 35$

required equation of the plane

EXERCISE 28 J [Pg. No.: 1246]

Very small answer Questions

1. Find the direction ratios of the normal to the plane $x+2y-3z=5$

Sol. The direction ratios of the normal to the plane $x+2y-3z=5$ are $1,2,-3$

2. Find the direction cosines of the normal to the plane $2x+3y-z=1$

Sol. The given plane is $2x+3y-z=4$

Direction ratios of the normal to the given plane are $2,3,-1$ and $\sqrt{2^2+3^2+(-1)^2}=\sqrt{14}$

Hence the required direction cosines are $\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}},\frac{-1}{\sqrt{14}}$

3. Find the direction cosines of the normal to the plane $y=3$

Sol. Direction ratios of the normal to the plane are $0,1,0$ and $\sqrt{0^2+1^2+0^2}=1$

Hence the required direction cosines are $0,1,0$

4. Find the direction cosines of the normal to the plane $3x+4=0$

Sol. $3x+4=0 \Rightarrow -x=\frac{4}{3}$

Direction ratios of the normal to this plane are $-1,0,0$ and $\sqrt{(-1)^2+0^2+0^2}=1$

Hence the required direction cosines are $-1,0,0$

5. Write the equation of the plane parallel to XY plane and passing through the point $(4,-2,3)$

Sol. Any plane parallel to XY plane is $z=k$

Since it passes through $(4,-2,3)$, we have $3=k$

Hence the required equation of the plane is $z=3$

6. Write the equation of the plane parallel to YZ plane and passing through the point $(-3,2,0)$

Sol. Any plane parallel to YZ plane is $x=k$

Since it passes through $(-3,2,0)$ we have $-3=k$

Hence the required equation of the plane is $x=-3$

7. Write the general equation of a plane parallel to the x-axis

Sol. Let the required equation of the plane be $ax+by+cz+d=0$

The d.r.'s of this plane are a,b,c

The d.r.'s of the x-axis are $1,0,0$

Normal of the required plane is perpendicular to the x-axis

$\therefore (a\times 1)+(b\times 0)+(c\times 0)=0 \Rightarrow a=0$

Hence the required equation is $by+ca+d=0$

8. Write the intercept cut off by the plane parallel to the x-axis

Sol. $2x+y-z=5 \Rightarrow \frac{x}{\left(\frac{5}{2}\right)}+\frac{y}{5}+\frac{z}{-5}=1$

$\therefore$ intercept cut of by the given plane on the x-axis is $\frac{5}{2}$

9. Write the intercepts made by the plane $4x-3y+2z=12$ on the coordinate axes

Sol. $4x-3y+2z=12 \Rightarrow \frac{4x}{12}+\frac{(-3y)}{12}+\frac{2z}{12}=1 \Rightarrow \frac{x}{3}+\frac{y}{-4}+\frac{z}{6}=1$

Hence the required intercepts are $3,-4,6$

10. Reduce the equation $2x-3y+5z+4=0$ to intercept form and find the intercepts made by it on the coordinate axes

Sol. The given equation may be witted as $-2x+3y-5z=4$

$$\Rightarrow \frac{(-2x)}{4}+\frac{3y}{4}+\frac{(-5z)}{4}=1 \Rightarrow \frac{x}{-2}+\frac{y}{\frac{4}{3}}+\frac{z}{\frac{-4}{5}}=1$$

$\therefore$ the required interecepts are $-2,\frac{4}{3},\frac{-4}{5}$

11. Find the equation of a plane passing through the points $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$

Sol. The equation of plane passing through the points $A(a,0,0),B(0,b,0)$ and $C(0,0,0)$

Clearly the plane cut its intercepts on the co-ordinate axes are a,b and c respectively

Hence required equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

12. Write the value of k for which the plane $2x-5y+kz=4$ and $x+2y-z=6$ are perpendicular to each other

Sol. Clearly the normals of the given planes are perpendicuarl to each other

$\therefore (2\times1)+(-5)\times2+k\times(-1)=0 \Rightarrow k=(2-10)=-8$

13. Find the angle between the planes $2x+y-2z=5$ and $3x-6y-2z=7$

Sol. D.r.'s of normals to the given planes are $2,1,-2$ and $3,-6,-2$

$$\therefore \cos\theta=\frac{|(2\times3)+1\times(-6)+(-2)\times(-2)|}{\left\{\sqrt{2^2+1^2+(-2)^2}\right\}\left\{\sqrt{3^2+(-6)^2+(-2)^2}\right\}}$$

$$=\frac{4}{(\sqrt{9})(\sqrt{49})}=\frac{4}{(3\times7)}=\frac{4}{21} \Rightarrow \theta=\cos^{-1}\left(\frac{4}{21}\right)$$

14. Find the angle between the planes $\vec{r}\cdot(\hat{i}+\hat{j})=1$ and $\vec{r}\cdot(\hat{i}+\hat{k})=3$

Sol. Given planes are $x+1=1$ and $x+z=3$

The d.r.'s of normals to these planes are $1,1,0$ and $1,0,1$

$$\therefore \cos\theta=\frac{|(1\times1)+(1\times0)+(0\times1)|}{\left\{\sqrt{1^2+1^2+0^2}\right\}\left\{\sqrt{1^2+0^2+1^2}\right\}}=\frac{1}{(\sqrt{2}\times\sqrt{2})}=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}$$

15. Find the angle between the planes $\vec{r}\cdot(3\hat{i}-4\hat{j}+5\hat{k})=0$ and $\vec{r}\cdot(2\hat{i}-\hat{j}-2\hat{k})=7$

Sol. The given planes are $3x-4y+5z=0$ and $2x-y-2z=7$

The d.r.'s of normals to these planes are $3,-4,5$ and $2,-1,-2$

$$\therefore \cos\theta=\frac{|(3\times2)+(-4)\times(-1)+5\times(-2)|}{\left\{\sqrt{3^2+(-4)^2+5^2}\right\}\left\{\sqrt{2^2+(-1)^2+(-2)^2}\right\}}=0 \Rightarrow \theta=\frac{\pi}{2}$$

16. Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z=3$

Sol. D.r.'s of the given line are $2,3,6$

D.r.'s of the normal to the given plane are $10,2,-11$

$$\therefore \sin\theta=\frac{|(2\times 10)+(3\times 2)+6\times(-11)|}{\{\sqrt{2^2+3^2+6^2}\}\{\sqrt{(10)^2+2^2+(-11)^2}\}}$$

$$=\frac{40}{\{\sqrt{49}\}\times\{\sqrt{225}\}}=\frac{40}{(7\times 15)}=\frac{8}{21}\Rightarrow\theta=\sin^{-1}\left(\frac{8}{21}\right)$$

17. Find the angle between the line $\vec{r}=(\hat{i}+\hat{j}-2\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=4$

Sol. Give line is $\vec{r}=\vec{a}+\lambda\vec{b}$ and given plane is $\vec{r}\cdot\vec{n}=p$

$$\sin\theta=\frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}=\frac{|(\hat{i}-\hat{j}+\hat{k})\cdot(2\hat{i}-\hat{j}+\hat{k})|}{\{\sqrt{1^2+(-1)^2+1^2}\}\{\sqrt{2^2+(-1)^2+1^2}\}}$$

$$=\frac{|(2\times 1)+(-1)\times 1\times 1|}{(\sqrt{3}\times\sqrt{6})}=\frac{4}{\sqrt{18}}=\left(\frac{4}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{2\sqrt{2}}{3}\Rightarrow\theta=\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

18. Find the value of λ such that the line $\frac{x-2}{6}=\frac{y-1}{\lambda}=\frac{z+5}{4}$ is perpendicular to the plane $3x-y-2z=7$

Sol. D.r.'s of the given line are $6,\lambda,-4$

D.r.'s of normal to the given plane are $3,-1,-2$

Given line is parallel to the normal of the plane $\therefore \frac{6}{3}=\frac{\lambda}{-1}=\frac{-4}{-2}\Rightarrow\lambda=-2$

19. Write the equation of the plane passing through the point (a,b,c) and parallel to the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$

Sol. Given plane is $(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})=2 \Rightarrow x+y+z=2$

Let the required plane be $x+y+z=k$, where k is a constant

Since it passes through (a,b,c) we have $k=(a+b+c)$

So, the required plane is $x+y+z=a+b+c$

In vector form it is given by $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=a+b+c$

20. Find the length of perpendicular drawn from the origin to the plane $2x-3y+6z+21=0$

Sol. We have $p=\frac{|2\times 0-3\times 0+6\times 0+21|}{\sqrt{2^2+(-3)^2+6^2}}=\frac{21}{\sqrt{49}}=\frac{21}{7}=3$ units

21. Find the direction consines of the perpendicular from the origin to the plane $\vec{r}\cdot(6\hat{i}-3\hat{j}-2\hat{k})+1=0$

Sol. The given equation is $\vec{r}\cdot(-6\hat{i}+3\hat{j}+2\hat{k})=1$

D.r.'s of normal to the plane are $-6,3,2$ and $\sqrt{(-6)^2+3^2+2^2}=\sqrt{49}=7$

$\therefore$ d.c.'s of normal to the plane are $\frac{-6}{7},\frac{3}{7},\frac{2}{7}$

22. Show that the line $\vec{r}=(4\hat{i}-7\hat{k})+\lambda(4\hat{i}-2\hat{j}+3\hat{k})$ is parallel to the plane $\vec{r}\cdot(5\hat{i}-3\hat{j}-4\hat{k})=7$

Sol. Given line is $\vec{r}=\vec{a}+\lambda\vec{b}$, where $\vec{b}=(4\hat{i}-2\hat{j}+3\hat{k})$

D.r.'s of the line are $4,-2,3$

Given plane is $\vec{r}\cdot\vec{n}=a$, where $\vec{n}=(5\hat{i}+4\hat{j}-4\hat{k})$

D.r.'s of the normal to the given plane are $5,4,-4$

So, the given line will be parallel to the given plane when this line is perpendicular to the normal to the plane

Hence we must have $(4\times5)+(-2\times4)+3\times(-4)=0$ which is true

23. Find the length of perpendicular from the origin to the plane $\vec{r}\cdot(2\hat{i}-3\hat{j}+6\hat{k})+14=0$

Sol. We have $\vec{r}\cdot(-2\hat{i}+3\hat{j}-6\hat{k})=14$

Here $\vec{n}=(2\hat{i}+3\hat{j}-6\hat{k})$ and $|\vec{n}|=\sqrt{(-2)^2+3^2+(-6)^2}=7$

$\therefore \vec{r}\cdot\frac{\vec{n}}{|\vec{n}|}=\frac{14}{|\vec{n}|}\Rightarrow \vec{r}\cdot\hat{n}=\frac{14}{7}=2$

Hence the length of perpendicular from origin to the given plane is 2 units

24. Find the value of λ for which the line $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{\lambda}$ is parallel to the plane $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=4$

Sol. Clearly, the given line must be perpendicular to the normal to the given plane

D.r.'s of the given line are $2,3,\lambda$

D.r.'s of the normal to the given plane are $2,3,4$

$\therefore (2\times2)+(3\times3)+(\lambda\times4)=0 \Rightarrow 4\lambda=-13\Rightarrow \lambda=\frac{-13}{4}$

25. Write the angle between the plane $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$ and the plane $x+y+4=0$

Sol. D.r.'s of the given line are $2,1,-2$

D.r.'s of the normal to the given plane are $1,1,0$

$\therefore \sin\theta=\frac{(2\times1)+(1\times1)+(-2)\times0}{\left\{\sqrt{2^2+1^2+(-2)^2}\right\}\left\{\sqrt{1^2+1^2+0^2}\right\}}=\frac{(2+1+0)}{(\sqrt{9})(\sqrt{2})}=\frac{3}{3\sqrt{2}}=\frac{1}{\sqrt{2}}\Rightarrow\theta=\frac{\pi}{4}$

26. Write the equation of a passing through the point $(2,-,1,1)$ and parallel to the plane $3x+2y-z=7$

Sol. Let the required equation of the plane be $a(x-2)+b(y+1)+c(z-1)=0$

Here $a=3, b=2$ and $c=-1$

So, the required equation of the plane is

$3(x-2)+2(y+1)-1\cdot(z-1)=0\Rightarrow 3x+2y-z=3$