PROBABILITY (XII, R. S. AGGARWAL)

EXERCISE 29A (Pg.No.: 1268)

1. Let A and B be events such that $P(A)=\frac{7}{13}$, $P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$.

Find (i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(A\cup B)$ (iv) $P(\bar{B}/\bar{A})$.

Sol. We have $P(A)=\frac{7}{13}, P(B)=\frac{9}{13}, P(A\cap B)=\frac{4}{13}$

(i) $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{4}{13}}{\frac{9}{13}}=\frac{4}{9}$ (ii) $P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{\frac{4}{13}}{\frac{7}{13}}=\frac{4}{13}\times\frac{13}{7}=\frac{4}{7}$

(iii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{7}{13}+\frac{9}{13}-\frac{4}{13}=\frac{16}{13}-\frac{4}{13}=\frac{12}{13}$

(iv) $P(\bar{B}/\bar{A})=\frac{P(\bar{B}\cap\bar{A})}{P(\bar{A})}=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{A})}=\frac{P(\overline{A\cup B})}{P(\bar{A})}=\frac{1-P(A\cup B)}{1-P(A)}=\frac{1-\frac{12}{13}}{1-\frac{7}{13}}=\frac{\frac{13-12}{13}}{\frac{13-7}{13}}=\frac{}{6}$

2. Let A and B be events such that $P(A)=\frac{5}{11}, P(B)=\frac{6}{11}$ and $P(A\cup B)=\frac{7}{11}$

Find (i) $P(A\cap B)$ (ii) $P(A/B)$ (iii) $P(B/A)$ (iv) $P(\bar{A}/\bar{B})$.

Sol. We have $P(A)=\frac{5}{11}, P(B)=\frac{6}{11}$ and $P(A\cup B)=\frac{7}{11}$

(i) $P(A\cap B)=P(A)+P(B)-P(A\cup B)=\frac{5}{11}+\frac{6}{11}-\frac{7}{11}=\frac{11}{11}-\frac{7}{11}=\frac{4}{11}$

(ii) $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{4}{11}}{\frac{6}{11}}=\frac{4}{6}=\frac{2}{3}$

(iii) $P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{P(A\cap B)}{P(A)}=\frac{\frac{4}{11}}{\frac{5}{11}}=\frac{4}{5}$

(iv) $P(\bar{A}/\bar{B})=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}=\frac{P(\overline{A\cup B})}{P(\bar{B})}=\frac{1-P(A\cup B)}{1-P(B)}=\frac{1-\frac{7}{11}}{1-\frac{6}{11}}=\frac{\frac{4}{11}}{\frac{5}{11}}=\frac{4}{5}$

3. Let A and B be events such that $P(A)=0.3$, $P(B)=0.5$ and $P(B/A)=0.4$.

Find (i) $P(A\cap B)$ (ii) $P(A\cup B)$ (iii) $P(A/B)$.

Sol. We have $P(A)=0.3, P(B)=0.5$ and $P(B/A)=0.4$

(i) $P(A\cap B)=P(B/A).P(A)=0.4\times 0.3=.12$

(ii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)=0.3+0.5-0.12=0.80-0.12=0.68$

(iii) $P(A/B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{0.12}{0.5}=0.24$

4. Let A and B be events such that $2P(A)=P(B)=\frac{5}{13}$, and $P(A/B)=\frac{2}{5}$.

Find $P(A\cup B)$.

Sol. We have $2P(A)=P(B)=\frac{5}{13}$; $P(B)=\frac{5}{13}$, $P(A)=\frac{5}{26}$, $P(A/B)=\frac{2}{5}$

$P(A\cap B)=P(A/B).P(B)=\frac{2}{5}\times\frac{5}{13}=\frac{2}{13}$

$\therefore\ P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{5}{26}+\frac{5}{13}-\frac{2}{13}=\frac{5+10-4}{26}=\frac{11}{26}$

5. A die is rolled. If the outcome is an even number, what is the probability that it is a number greater than 2?

Sol. Let $A=\{2,4,6\}\Rightarrow n(A)=3$ and $B=\{3,4,5,6\}\Rightarrow n(B)=4$ Then $A\cap B=\{4,6\}, n(A\cap B)=2$

$\therefore\ P(B/A)=\dfrac{n(A\cap B)}{n(A)}=\dfrac{2}{3}$

6. A coin is tossed twice. If the outcome is at most one tail, what is the probability that both head and tail have appeared ?

Sol. When a coin is tossed twice sample space is given by

$S=\{(H\ H),(H\ T)(T\ H),(T\ T)\}$

Let $E=$ Event of getting at most one tail

And $F=$ Even that both head and tail appeared we have to find $P(F/E)$

$\therefore E=\{(H,H),(H,T),(T,H)\}$

$F=\{(H\ T),(T\ H)\}$

$\therefore E\cap F=\{(H\ T),(T\ H)\}$

$P(F/E)=\dfrac{n(E\cap F)}{n(E)}=\dfrac{2}{3}$

7. Three coins are tossed. Find the probability that all coins show heads if at least one of the coins shows a head.

Sol. The sample space S is given by, $S=\{HHH,HHT,HTH,THH,HTT,THT,TTH,TTT\}$

$n(S)=8$, E :all coins show head, $E=\{HHH\}, P(E)=\frac{1}{8}$

F :at least one coins show a head, $F=\{HHH,HHT,HTH,THH,HTT,THT,TTH\}$

$P(F)=\frac{7}{8}$, $E\cap F=\{HHH\}$, $P(E\cap F)=\frac{1}{8}$, $P(E/F)=\dfrac{P(E\cap F)}{P(F)}=\dfrac{\frac{1}{8}}{\frac{7}{8}}=\dfrac{1}{7}$

8. Two unbiased dice are thrown. Find the probability that the sum of the numbers appearing is 8 or greater, if 4 appears on the first die.

Sol. Let A = event of getting a 4 on the first die, and

B = even of getting the sum 8 or greater

$\therefore A = [(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)]$

$B = \{(2,6),(3,5),(3,6),(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$\therefore A \cap B = \{(4,4),(4,5),(4,6)\}$

$\therefore n(A) = 6,\ n(B) = 15$ and $n(A \cap B) = 3$

The required probability $= P(B/A) = \dfrac{n(A \cap B)}{n(A)} = \dfrac{3}{6} = \dfrac{1}{2}$

9. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once ?

Sol. Let A = event of getting the sum 8. and

B = event that 5 appears at least once

Then, $A = \{(2,6),(3,5),(4,4),(6,2),(5,2)\}$ and

$B = \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(1,5),(2,5),(3,5),(4,5),(6,5)\}$

$\therefore A \cap B = \{(3,5),(5,3)\}$

Thus $n(A) = 5, n(B) = 11$ and $n(A \cap B) = 2$

The required probability $= P(B/A) = \dfrac{n(A \cap B)}{n(A)} = \dfrac{2}{5}$

10. Two dice were thrown and it is known that the numbers which come up were different. Find the probability that the sum of the two numbers was 5.

Sol. Let $A =$ event that the two dice show different number,

$B =$ event that the sum is 5. Then, $n(A) = 30$ and $n(S) = 36$

$\therefore \quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{30}{36} = \dfrac{5}{6}$

Also $B = \{(2,3),(3,2),(1,4),(4,1)\}$, $n(B) = 4$ and $n(S) = 36 \quad \therefore\ P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{36} = \dfrac{1}{9}$

$\therefore \quad B \cap A = B$ [$\because$ number are different in each order pair in B]

$\Rightarrow n(B \cap A) = 4 \Rightarrow P(B \cap A) = \dfrac{n(B \cap A)}{n(S)} = \dfrac{4}{36} = \dfrac{1}{9}$

$\therefore$ required probability, $P(B/A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{\frac{1}{9}}{\frac{5}{6}} = \dfrac{1}{9} \times \dfrac{6}{5} = \dfrac{2}{15}$

11. A coin is tossed and then a die is thrown. Find the probability of obtaining a 6, given that a head came up.

Sol. The sample space S is given by

$S = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$

$n(S) = 12$

Let $A =$ event that a head comes up, $B =$ event that the die shows a 6.

Then $A = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\} \Rightarrow P(A) = \dfrac{6}{12} = \dfrac{1}{2}$, $B = \{(H,6),(T,6)\}$

$\therefore\ B\cap A=\{(H,6)\}\Rightarrow P(B\cap A)=\frac{1}{12}$

Required probability, $P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{\frac{1}{12}}{\frac{1}{2}}=\frac{1}{12}\times\frac{2}{1}=\frac{1}{6}$

12. A couple has 2 children. Find the probability that both are boys if it is known that (i) one of the children is a boy, and (ii) the elder child is a boy

Sol. We may write the sample space as $S=\{B_1B_2, B_1G_2, G_1G_2\}$

Where B and G stand for a boy and a girl respectively and the elder child appears first

(i) Let E = event that both the children are boys and

F = event that one of the children is a boy

Then, $E=\{B_1B_2\}, F=\{B_1B_2, B_1C_2, G_1B_2\}$ and $E\cap F=\{B_1, B_2\}$

Thus $n(E)=1, n(F)=3$ and $n(E\cap F)=1$

The required probability $=P(E/F)=\frac{n(E\cap F)}{n(F)}=\frac{1}{3}$

(ii) Let E = event that both the children are boys, and

H = event that the elder child is a boy

Then, $E=\{B_1B_2\}, H=\{B_1B_2, B_1G_2\}$ and $E\cap H=\{B_1B_2\}$

The required probability $=P(E/H)=\frac{n(E\cap H)}{n(H)}=\frac{1}{2}$

13. In a class 40% students study mathematics; 25% study biology and 15% study both mathematics and biology. One student is selected at random.

Find the probability that

(i) he studies mathematics if it is known that he studies biology

(ii) he studies biology if it is known that he studies mathematics

Sol. Let E = even that the selected student studies mathematics

F = Even that the selected student studies biology

A/c

$P(E)=40\%, =\frac{40}{100}=\frac{2}{5}$

$P(F)=25\%\ \frac{25}{100}=\frac{1}{4}$

And $P(E\cap F)=15\%=\frac{15}{100}=\frac{3}{20}$

(i) $P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{3/20}{1/4}-\frac{3}{20}\times 4=\frac{3}{5}$

(ii) $P(F)E=\frac{P(E\cap F)}{P(E)}$

$=\frac{3/5}{2/5}=\frac{3/20}{2/5}=\frac{3}{20}\times\frac{5}{2}\ =\frac{3}{8}$

14. The probability that a student selected at random from a class will pass in Hindi is $\frac{4}{5}$ and the probability that he passes in Hindi and English is $\frac{1}{2}$. What is the probability that he will pass in English if it is known that he has passed in Hindi?

Sol. Let A = event that the student passes in Hindi, B = event that the student passes in English.

$$\therefore \; P(A)=\frac{4}{5} \text{ and } P(A\cap B)=\frac{1}{2}; \quad P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{\frac{1}{2}}{\frac{4}{5}}=\frac{1}{2}\times\frac{5}{4}=\frac{5}{8}$$

15. The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a coat is 0.3 and the probability that he will buy a shirt given that he buys a coat is 0.4. Find the probability that he will buy both a shirt and a coat.

Sol. Let A = event that the person buys a shirt, B = event that the person buys a coat.

Then, $P(A)=0.2, P(B)=0.3$ and $P(A/B)=0.4$; $P(A\cap B)=P(B).P(A/B)=0.3\times 0.4=0.12$

16. In a hostel 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random

(i) Find the probability that he reads neither Hindi nor English newspaper

(ii) If he reads hindi newspaper, what is the probability that he reads English newspaper ?

(iii) if he reads English newspaper, what is the probability that he reads Hindi newspapwer ?

Sol. Let

E = Even t that selected student reads Hindi newspaper

F = Even that selected student reads English news paper

A/Q

$$P(E)=60\%=\frac{60}{100}=\frac{3}{5}$$

$$P(F)=40\%=\frac{40}{100}=\frac{2}{5}$$

$$P(E\cap F)=20\%=\frac{20}{100}=\frac{1}{5}$$

(i) P (student reads Leither Hindi Lor English newspaper

$$=P(\bar{E}\cap \bar{F})=P(\overline{E\cup F})$$

$$1-P(E\cup F)=1-P(E)-P(F)+P(E\cap F)$$

$$=1-\frac{3}{5}-\frac{2}{5}+\frac{1}{5}$$

$$=\frac{5-3-2+1}{5}$$

$$=\frac{1}{5}$$

(ii) $P(F/E)=\dfrac{P(E\cap F)}{P(F)}$

$$=\frac{1/5}{3/5}=\frac{1}{3}$$

(iii) $P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{1/5}{2/5}=\frac{1}{2}$

17. Two integers are selected at random from integers 1 through 11. If the sum is even, find the probability that both the numbers selected are odd.

Sol. Let $A=$ event of getting both odd numbers. $B=$ event that sum of chosen number is even.
In integral from 1 to 11, these are 5 even and 6 odd integers.

$$P(A)=\frac{{}^6C_2}{{}^{11}C_2}=\frac{15}{55}=\frac{3}{11}, P(B)=\frac{{}^6C_2+{}^5C_2}{{}^{11}C_2}=\frac{15+10}{55}=\frac{25}{55}=\frac{5}{11}$$

$$P(A\cap B)=\frac{{}^6C_2}{{}^{11}C_2}=\frac{15}{55}=\frac{3}{11}.\text{ Required probability, } P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{3}{11}}{\frac{5}{11}}=\frac{3}{5}$$

EXERCISE 29B (Pg.No.: 1283)

1. A bag contains 17 tickets numbered from 1 to 17. a ticket is drawn and then another ticket is drawn without replacing the first one. Find the probability that both the tickets may show even number

Sol. Let
$E_1=$ Event that the first ticket shows even number
$E_2=$ Event that the 2nd ticket shows even number
Since the bag contains 17 tickets numbered from 1 to 17
Therefore 8 ticket shows even no. and 9 ticket shows odd no

$\therefore P(E_1)=\frac{8}{17}$ and $P(E_2/E_1)=\frac{8-1}{17-1}=\frac{7}{16}$

P (both the tickets show even no)

$=P(E_1\cap E_2)$

$=P(E_1)\times P(E_2/E_1)=\frac{8}{17}\times\frac{7}{16}=\frac{7}{34}$

2. Two marbles are drawn successively from a box containing 3 black and 4 white marbles. Find the probability that both the marbles are black, if the first marbles are black, if the first marble is not replaced before the second draw

Sol. There are 3 black and 4 white marbles in a box
$\therefore$ number of marbles = 7
Let $E_1=$ Event that the first marble is black
$E_2=$ Event that the 2nd marble is black

$\therefore P(E_1)=\frac{3}{7}$

And $P(E_2/E_1)=\frac{2}{6}=\frac{1}{3}$

P(Both marbles are black)

$=P(E_1\cap E_2)=P(E_1)\times P(E_2/E_1)$

$=\frac{3}{7}\times\frac{1}{3}=\frac{1}{7}$

3. A card is drawn from a well-shuffled deck of 52 cards and without replacing this card, a second card is drawn. Find the probability that the first card is a club and the second card is a spade

Sol. $E_1 =$ Event that the first card is club

$E_2 =$ Event that the 2nd card is spade

$\therefore P(E_1) = \frac{13}{52} = \frac{1}{4}$

$P(E_2/E_1) = \frac{13}{51}$

P (the first is club and 2nd is spade)

$= P(E_1 \cap E_2)$

$= P(E_1) \times P(E_2/E_1)$

$= \frac{1}{4} \times \frac{13}{51} = \frac{13}{204}$

4. There is a box containing 30 bulbs of which 5 are defective. If two bulbs are chosen at random from the box in succession without replacing the first, what is the probability that both the bulbs chosen are defective ?

Sol. Let $E =$ Event that the first bulb is defective

F = Event that the 2nd bulb is defective

$\therefore P(E) = \frac{5}{30} = \frac{1}{6}$

$P(F/E) = \frac{4}{29}$

P (Both bulbs are defective)

$= P(E \cap F)$

$= P(E) \times P(F/E)$

$= \frac{1}{6} \times \frac{4}{29} = \frac{2}{87}$

5. A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that the first ball is white and the second is black ?

Sol. E = Event that the first ball is white

F = Event that the 2nd ball is black

$P(E) = \frac{10}{25} = \frac{2}{5}$

$P(F/E) = \frac{15}{24} = \frac{5}{8}$

P (first ball is white and 2nd ball is black)

$= P(E) \times P(F/E)$

$= \frac{2}{5} \times \frac{5}{8} = \frac{1}{4}$

6. An urn contains 5 white and 8 black balls. Two successive drawings of 3 balls at a time are made such that the balls drawn in the first draw are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and the second draw gives 3 black balls.

Sol. Let

E = Event that the first three balls are white balls

F = Event that the 2nd three balls are black balls

$P(E) = \frac{^5C_3}{^{13}C_3}$

And $P(F/E) = \frac{^8C_3}{^{10}C_3}$

P(The first draw gives 3 white balls and the 2nd draw gives 3 black balls)

$= P(E \cap F)$

$= P(E) \times P(F/E)$

$= \frac{^5C_3}{^{13}C_3} \times \frac{^8C_3}{^{10}C_3}$

$= \frac{\frac{5!}{3!\,2!}}{\frac{13!}{3!\,10!}} \times \frac{\frac{8!}{3! \times 5!}}{\frac{10!}{3!\,7!}}$

$= \frac{5!}{2!} \times \frac{10!}{13!} \times \frac{8!}{5!} \times \frac{7!}{10!} = \frac{7}{429}$

7. Let E_1 and E_2 be the events such that $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{3}{5}$

Find

(i) $P(E_1 \cup E_2)$, when E_1 and E_2 are mutually exclusive

(ii) $P(E_1 \cap E_2)$, when E_1 and E_2 are independent

Sol. (i) $\because$ E_1 and E_2 are mutually exclusive

$\therefore P(E_1 \cap E_2) = 0$

Now $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$\Rightarrow P(E_1 \cup E_2) = \frac{1}{3} + \frac{3}{5}$

$= \frac{5+9}{15} = \frac{14}{15}$

(ii) $\because$ E_1 and E_2 are independent events

$\therefore P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \frac{1}{3} \times \frac{3}{5} = \frac{1}{5}$

8. If E_1 and E_2 are the two events such that $P(E_1) = \frac{1}{4}$, $P(E_2) = \frac{1}{3}$ and $P(E_1 \cup E_2) = \frac{1}{2}$, show that E_1 and E_2 are independent events

Sol. Given $P(E_1) = \frac{1}{4}$, $P(E_2) = \frac{1}{3}$

And $P(E_1 \cup E_2) = \frac{1}{2}$

Now, $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow \frac{1}{2} = \frac{1}{4} + \frac{1}{3} - P(E_1 \cap E_2)$

$\Rightarrow P(E_1 \cap E_2) = \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$

$\Rightarrow P(E_1 \cap E_2) = \frac{3+4-6}{12}$

$\Rightarrow P(E_1 \cap E_2) = \frac{1}{12}$

Now, $P(E_1) \times P(E_2) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

$\because P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

Hence E_1 and E_2 are independent parents

9. If E_1 and E_2 are independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$ find

(i) $P(E_1 \cap E_2)$ (ii) $P(E_1 \cup E_2)$ (iii) $P(\bar{E}_1 \cap \bar{E}_2)$ (iv) $P(\bar{E}_1 \cap \bar{E}_2)$

Sol. Given $P(E_1) = 0.3$ and $P(E_2) = 0.4$

And E_1 and E_2 are independent events

(i) $P(E_1 \cap E_2)$

$= P(E_1) \times P(E_2)$

$= 0.3 \times 0.4 = 0.12$

(ii) $P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= 0.3 + 0.4 - 0.12 \quad = 0.58$

(iii) $P(\bar{E}_1 \cap \bar{E}_2) = P(\bar{E}_1) \times P(\bar{E}_2)$

$= [1 - P(E_1)] \times [1 - P(E_2)]$

$= [1 - 0.3][1 - 0.4] = 0.74 \times 0.6 = 0.42$

(iv) $P(\bar{E}_1 \cap E_2)$

$= P(\bar{E}_1) \times P(E_2)$

$= [1 - P(E_1)] \times P(E_2)$

$= [1 - 0.3] \times 0.4 = 0.7 \times 0.4 = 0.2$

10. Let A and B be events such that $P(A) = 1/2. P(B) = 7/12$ and P (not A or not B) $= 1/4$. State whether A and B are (i) mutually exclusive (ii) independent.

Sol. $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\bar{A} \cup \bar{B}) = \frac{1}{4}$

$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) \quad \Rightarrow 1 - P(A \cap B) = \frac{1}{4} \quad \Rightarrow \frac{3}{4} = P(A \cap B)$

(i) Since $P(A \cap B) \neq 0$, so A and B are not mutually exclusive.

(ii) $P(A) \times P(B) = \left(\frac{1}{2} \times \frac{7}{12}\right) = \frac{7}{24} \neq P(A \cap B) \Rightarrow$ A and B are not independents.

11. Kamal and Vimal appeared for an interview for two vacancies. The probability of Kamal's selection is 1/3 and that of Vimal's selection is 1/5. Find the probability that only one of them will be selected.

Sol. Let E_1 = events that Kamal is selected. and E_2 = events that Vimal is selected then,

$$P(E_1)=\frac{1}{3}, P(E_2)=\frac{1}{5}, P(\bar{E}_1)=1-\frac{1}{3}=\frac{2}{3} \text{ and } P(\bar{E}_2)=1-\frac{1}{5}=\frac{4}{5}$$

$\therefore$ Required probability $= P[(E_1 \text{ and not } E_2) \text{ or } (E_2 \text{ and not } E_1)]$

$$= P[(E_1 \text{ and } \bar{E}_2) \text{ or } (E_2 \text{ and } \bar{E}_1)] = P(E_1 \cap \bar{E}_2) + P(E_2 \cap \bar{E}_1)$$

$$= [P(E_1) \times P(\bar{E}_2)] + [P(E_2) \times P(\bar{E}_1)]$$

$$= \left(\frac{1}{3} \times \frac{4}{5}\right) + \left(\frac{1}{5} \times \frac{2}{3}\right) = \frac{4}{15} + \frac{2}{15} = \frac{6}{15} = \frac{2}{5}$$

12. Arun and Ved appeared for an interview for two vacancies. The probability of Arun's selection is 1/4 and that of Ved's rejection is 2/3. Find the probability that at least one of them will be selected.

Sol. Let E_1 = events that Arun is selected. and E_2 = events that Ved is selected.

Then $P(E_1)=\frac{1}{4}$ and $P(E_2)=\left(1-\frac{2}{3}\right)=\frac{1}{3}$; clearly, E_1 and E_2 are independent events.

$\therefore$ $P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

Required probability $= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2) = \frac{1}{4} + \frac{1}{3} - \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$$

13. A and B appear for an interview for two vacancies in the same post. The probability of A's selection is 1/6 and that of B's selection is 1/4. Find the probability that

(i) both of them are selected (ii) only one of them is selected

(iii) none is selected (iv) at least one of them is selected.

Sol. We have $P(A)=\frac{1}{6}$, $P(B)=\frac{1}{4}$

$\therefore$ $P(\bar{A}) = 1 - P(A) = \left(1-\frac{1}{6}\right) = \frac{5}{6}$ and $P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$

(i) P (both are selected) $= P(A \cap B) = P(A) \times P(B) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

(ii) P (only one of them is selected) = P[(A and not B) or (B and not A)]

$$= P(A \cap \bar{B}) + P(B \cap \bar{A}) = [P(A) \times P(\bar{B})] + [P(B) \times P(\bar{A})]$$

$$= \left(\frac{1}{6} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{5}{6}\right) = \frac{3}{24} + \frac{5}{24} = \frac{8}{24} = \frac{1}{3}$$

(iii) P (none is selected) = P (not A and not B) $= P(\bar{A} \cap \bar{B}) = P(\bar{A}) \times P(\bar{B}) = \frac{5}{6} \times \frac{3}{4} = \frac{5}{8}$

(iv) P (at least one is selected) $= 1 - P$ (none is selected) $= 1 - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{8}$

14. Given the probability that A can solve a problem is 2/3, and the probability that B can solve the same problem is 3/5, find the probability that

(i) at least one of A and B will solve the problem

(ii) none of the two will solve the problem.

Sol. Let E_1 = events that A can solve the problem and E_2 = events that B can solve the problem.

Then E_1 and E_2 are clearly independent events.

$$\therefore \quad P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$$

(i) P (at least one of A and B can solve the problem)

$$= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = \frac{2}{3} + \frac{3}{5} - \frac{2}{5} = \frac{13}{15}$$

(ii) P (none can solve the problem) $= 1 - P$ (at least one can solve the problem) $= 1 - \frac{13}{15} = \frac{2}{15}$

15. A problem is given to three students whose chances of solving it are $\frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$ respectively. Find the probability that the problem is solved.

Sol. Let A, B, C be the events of solving the problem by the 1st, 2nd and 3rd student respectively, then

$$P(A) = \frac{1}{4}, P(B) = \frac{1}{5} \text{ and } P(C) = \frac{1}{6}$$

$$\Rightarrow \Rightarrow P(\bar{A}) = 1 - \frac{1}{4} = \frac{3}{4} \quad \Rightarrow P(\bar{B}) = 1 - \frac{1}{5} = \frac{4}{5} \quad \Rightarrow P(\bar{C}) = 1 - \frac{1}{6} = \frac{5}{6}$$

$\therefore$ P (none solves the problem) $= P[(\text{not } A) \text{ and } (\text{not } B) \text{ and } (\text{not } C)]$

$$= P[\bar{A} \text{ and } \bar{B} \text{ and } \bar{C}] = P(\bar{A}) \times P(\bar{B}) \times P(\bar{C}) = \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = \frac{1}{2}$$

16. The probabilities of A, B, C solving a problem are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{6}$ respectively. If all the three try to solve he problem simultaneously, find the probability that exactly one of them will solve it.

Sol Given, $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{6}$

$$\Rightarrow P(\bar{A}) = 1 - \frac{1}{3} = \frac{2}{3} \quad \Rightarrow P(\bar{B}) = 1 - \frac{1}{4} = \frac{3}{4} \quad \Rightarrow P(\bar{C}) = 1 - \frac{1}{6} = \frac{5}{6}$$

$\therefore$ Required probability $= P$ (exactly one of them solves the problem)

$$= P[(A \cap \bar{B} \cap \bar{C}) \text{ or } (\bar{A} \cap B \cap \bar{C}) \text{ or } (\bar{A} \cap \bar{B} \cap C)]$$

$$= P[(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)]$$

$$= \{P(A) \times P(\bar{B}) \times P(\bar{C})\} + \{P(\bar{A}) \times P(B) \times P(\bar{C})\} + \{P(\bar{A}) \times P(\bar{B}) \times P(C)\}$$

$$= \left(\frac{1}{3} \times \frac{3}{4} \times \frac{5}{6}\right) + \left(\frac{2}{3} \times \frac{1}{4} \times \frac{5}{6}\right) + \left(\frac{2}{3} \times \frac{3}{4} \times \frac{1}{6}\right) = \frac{15}{72} + \frac{10}{72} + \frac{6}{72} = \frac{31}{72}$$

17. A can hit a target 4 times in 5 shots, B can hit 3 times in 4 shots, and C can hit 2 times in 3 shots. Calculate the probability that

(i) A, B and C all hit the target

(ii) B and C hit and A does not hit the target.

Sol. $P(A \text{ hits}) = \frac{4}{5}, P(B \text{ hits}) = \frac{3}{4}$ and $P(C \text{ hits}) = \frac{2}{3}$

(i) P (A hits and B hits and C hits) $= P(A)\times P(B)\times P(C)=\frac{4}{5}\times\frac{3}{4}\times\frac{2}{3}=\frac{2}{5}$

(ii) P (B hits and C hits and A does not hit) $= P(B)\times P(C)\times P(\bar{A})=\frac{3}{4}\times\frac{2}{3}\times\left(1-\frac{4}{5}\right)=\frac{3}{4}\times\frac{2}{3}\times\frac{1}{5}=\frac{1}{10}$

18. Neelam has offered physics, chemistry and mathematics in Class XII. She estimates that her probabilities of receiving a grade A in these courses are 0.2, 0.3 and 0.9 respectively. Find the probabilities that Neelam receives (i) all A grades (ii) no A grade (iii) exactly 2 A grades.

Sol. We have $P(A_1)=0.2 \quad \Rightarrow P(\bar{A}_1)=1-0.2=0.8$

$P(A_2)=0.3 \quad \Rightarrow P(\bar{A}_2)=1-0.3=0.7 \quad \Rightarrow P(A_3)=0.9 \quad \Rightarrow P(\bar{A}_3)=1-0.9=0.1$

(i) P (getting all A grades) $= P(A_1A_2A_3)=P(A_1)\times P(A_2)\times P(A_3)=0.2\times 0.3\times 0.9=0.054$

(ii) P (getting no A grades) $= P(\bar{A}_1\bar{A}_2\bar{A}_3) = P(\bar{A}_1)\times P(\bar{A}_2)\times P(\bar{A}_3)=0.8\times 0.7\times 0.1=0.056$

(iii) P (getting exactly two A grades) $= P\left[(\bar{A}_1A_2A_3) \text{ or } (A_1\bar{A}_2A_3) \text{ or } (A_1A_2\bar{A}_3)\right]$

$= P(\bar{A}_1A_2A_3)+P(A_1\bar{A}_2A_3)+P(A_1A_2\bar{A}_3)$

$=\{P(\bar{A}_1)\times P(A_2)\times P(A_3)\}+\{P(A_1)\times P(\bar{A}_2)\times P(A_3)\}+\{P(A_1)\times P(A_2)\times P(\bar{A}_3)\}$

$=(0.8\times 0.3\times 0.9)+(0.2\times 0.7\times 0.9)+(0.2\times 0.3\times 0.1)=0.216+0.126+0.006=0.348$

19. An article manufactured by a company consists of two parts X and Y. In the process of manufacture of part X, 8 out of 100 parts may be defective. Similarly, 5 out of 100 parts of Y may be defective. Calculate the probability that the assembled product will not be defective.

Sol. P (X is defective) $=\frac{8}{100}=\frac{2}{25}$; P (Y is defective) $=\frac{5}{100}=\frac{1}{20}$

P (X is not defective) $=\left(1-\frac{2}{25}\right)=\frac{23}{25}$; P (Y is not defective) $=\left(1-\frac{1}{20}\right)=\frac{19}{20}$

Required probability $= P$ (assembled part is not defective)

$= P$ (X is not defective and Y is not defective) $=\frac{23}{25}\times\frac{19}{20}=\frac{437}{500}$

20. A town has two fire-extinguishing engines, functioning independently. The probability of availability of each engine when needed is 0.95. what is the probability that

(i) neither of them is available when needed? (ii)exactly an engine is available when needed?

Sol. Let E_1 = events of availability of the first engine. and E_2 = events of availability of the second engine.

Then $P(E_1)=P(E_2)=0.95$ and $P(\bar{E}_1)=P(\bar{E}_2)=(1-0.95)=0.05$

(i) P (neither of them is available when needed)

$= P(\bar{E}_1 \text{ and } \bar{E}_2)=P(\bar{E}_1)\times P(\bar{E}_2)=0.05\times 0.05=0.0025$

(ii) P (exactlyan engine is available when needed)

$= P\left[(E_1 \text{ and not } E_2)\text{or}(E_2 \text{ and not } E_1)\right]$

$= P\left[(E_1 \text{ and } \bar{E}_2) \text{ or } (E_2 \text{ and } \bar{E}_1)\right]$

$= P(E_1\cap\bar{E}_2)+P(E_2\cap\bar{E}_1)=P(E_1)\times P(\bar{E}_2)+P(E_2)\times P(\bar{E}_1)$

$=0.95\times 0.05+0.95\times 0.05=2\times(0.95\times 0.05)=0.95\times 0.10=0.095$

21. A machine operates only when all of its three components function. The probabilities of the failures of the first, second and third components are 0.14, 0.10 and 0.05 respectively. What is the probability that the machine will fail?

Sol. Let E_1, E_2, E_3 be the respective events that the 1^{st}, 2^{nd} and 3^{rd} components function then,

$$P(\bar{E}_1)=0.14, P(\bar{E}_2)=0.10 \text{ and } P(\bar{E}_3)=0.05$$

$$\Rightarrow P(E_1)=1-0.14=0.86$$

$$\Rightarrow P(E_2)=1-0.10=0.90 \text{ and } P(E_3)=1-0.05=0.95$$

$$\Rightarrow P(\text{machine fails})=1-P(\text{machine function})$$

$$=1-P(E_1 \text{ and } E_2 \text{ and } E_3)=1-[P(E_1)\times P(E_2)\times P(E_3)]$$

$$=1-[0.86\times 0.90\times 0.95]=1-.735300=0.2647$$

22. An anti-aircraft gun can take a maximum of 4 shots at enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

Sol. Let E_1, E_2, E_3, E_4 be the respective events that true plane is hit in the 1^{st}, 2^{nd}, 3^{rd} and 4^{th} shot.

Then, $P(E_1)=0.4, P(E_2)=0.3, P(E_3)=0.2$ and $P(E_4)=0.1$

$$\therefore\ P(\bar{E}_1)=(1-0.4)=0.6, P(\bar{E}_2)=(1-0.3)=0.7,$$

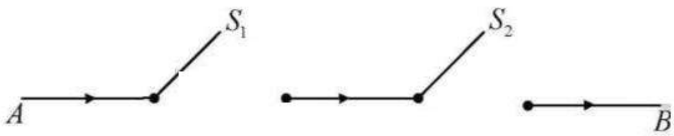
$$P(\bar{E}_3)=(1-0.2)=0.8,\ P(\bar{E}_4)=(1-0.1)=0.9$$

$$\therefore\ P(\text{at least one shot hits the plane}) = 1-P(\bar{E}_1 \text{ and } \bar{E}_2 \text{ and } \bar{E}_3 \text{ and } \bar{E}_4)$$

$$=1-\{P(\bar{E}_1)\times P(\bar{E}_2)\times P(\bar{E}_3)\times P(\bar{E}_4)\}=1-\{0.6\times 0.7\times 0.8\times 0.9\}$$

$$=1-(0.3024)=0.6976$$

23. Let S_1 and S_2 be two switches and let their probabilities of working be given by $P(S_1)=\frac{4}{5}$ and $P(S_2)=\frac{9}{10}$. Find the probability that the current flows from the terminal A to terminal B when S_1 and S_2 are installed in series, shown as follows:

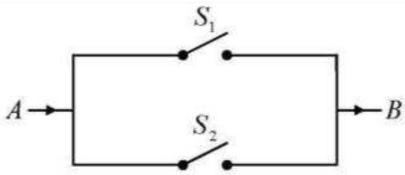


Sol. We have $P(S_1)=\frac{4}{5}$ and $P(S_2)=\frac{9}{10}$

P(current flows from A to B) = P(S_1 is closed and S_2 is closed)

$$=P(S_1 \text{ and } S_2)=P(S_1)\times P(S_2)=\frac{4}{5}\times\frac{9}{10}=\frac{18}{25}$$

24. Let S_1 and S_2 be two switches and let their probabilities of working be given by $P(S_1)=\frac{2}{3}$ and $P(S_2)=\frac{3}{4}$. Find the probability that the current flows from terminal A to terminal B, when S_1 and S_2 are installed in parallel, as shown below :



Sol. We have $P(S_1)=\frac{2}{3}$ and $P(S_2)=\frac{3}{4}$

P (the current flows) $= P(S_1 \text{ or } S_2)=P(S_1\cup S_2)=P(S_1)+P(S_2)-P(S_1\cap S_2)$

$=P(S_1)+P(S_2)-P(S_1)\times P(S_2)=\frac{2}{3}+\frac{3}{4}-\left(\frac{2}{3}\times\frac{3}{4}\right)=\frac{8+9}{12}-\frac{1}{2}=\frac{17}{12}-\frac{1}{2}=\frac{17-6}{12}=\frac{11}{12}$

25. A coin is tossed. If a head comes up, a die is thrown but if a tail comes up, the coin is tossed again. Find the probability of obtaining.

(i) two tails (ii) a head and the number 6 (iii) a head and an even number.

Sol. Let S be the sample space

$S=\{H1, H2, H3, H4, H5, H6, TT, TH\} \Rightarrow n(S)=8$

(i) P (two tails) $=\frac{1}{8}$ (ii) P (head and the number 6) $=\frac{1}{8}$

(iii) P (head and an even number) $=\frac{3}{8}$