Ex - 31.1

Probability Ex 31.1 Q1

The sample space $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Let,

A = Number on the card drawn is even number

$A = \{2, 4, 6, 8, 10\}$

B = Number on the card greater than 4

$B = \{4, 5, 6, 7, 8, 9, 10\}$

$A \cap B = \{4, 6, 8, 10\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{4}{7}$$

Required probability $= \frac{4}{7}$

Probability Ex 31.1 Q2

Let b and g represent the boy and the girl child respectively. If a family has two children, the sample space will be

S = {(b, b), (b, g), (g, b), (g, g)}

Let A be the event that both children are girls.

$$\therefore A = \{(g, g)\}$$

(i) Let B be the event that the youngest child is a girl.

$$\therefore B = [(b, g), (g, g)]$$

$$\Rightarrow A \cap B = \{(g, g)\}$$

$$\therefore P(B) = \frac{2}{4} = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{4}$$

The conditional probability that both are girls, given that the youngest child is a girl, is given by P (A|B).

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Therefore, the required probability is $\frac{1}{2}$.

Probability Ex 31.1 Q3

A = Two numbers on two dice are different

$$= \{(1,2),(1,3),(1,4),(1,5),(1,6)\\ (2,1),(2,3),(2,4),(2,5),(2,6)\\ (3,1),(3,2),(3,4),(3,5),(3,6)\\ (4,1),(4,2),(4,3),(4,5),(4,6)\\ (5,1),(5,2),(5,3),(5,4),(5,6)\\ (6,1),(6,2),(6,3),(6,4),(6,5)\}$$

B = Sum of numbers on the dice is 4

$B = \{(1,3),(2,2),(3,1)\}$

$A \cap B = \{(1,3),(3,1)\}$

Required probability $= P\left(\frac{B}{A}\right)$

$$= \frac{n(A \cap B)}{n(A)}$$

$$= \frac{2}{30}$$

Required probability $= \frac{1}{15}$

(ii) Let C be the event that at least one child is a girl.

$\therefore C = \{(b, g),(g, b),(g, g)\}$

$\Rightarrow A \cap C = \{g, g\}$

$\Rightarrow P(C) = \frac{3}{4}$

$P(A \cap C) = \frac{1}{4}$

The conditional probability that both are girls, given that at least one child is a girl, is given by P(A|C).

Therefore, $P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

Probability Ex 31.1 Q4

A = Head on the first two toss on three tosses of coin

$A = \{HHT, HHH\}$

B = Getting ahead on third toss

$B = \{HHH, HTH, THH, TTH\}$

$A \cap B = \{HHH\}$

$$\text{Required probability } = P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)}$$

$$\text{Required probability } = \frac{1}{2}$$

Probability Ex 31.1 Q5

A = 4 appears on third toss, if a die is thrown three times

$$= \{(1,1,4),(1,2,4),(1,3,4),(1,4,4),(1,5,4),(1,6,4)$$
$$(2,1,4),(2,2,4),(2,3,4),(2,4,4),(2,5,4),(2,6,4)$$
$$(3,1,4),(3,2,4),(3,3,4),(3,4,4),(3,5,4),(3,6,4)$$
$$(4,1,4),(4,2,4),(4,3,4),(4,4,4),(4,5,4),(4,6,4)$$
$$(5,1,4),(5,2,4),(5,3,4),(5,4,4),(5,5,4),(5,6,4)$$
$$(6,1,4),(6,2,4),(6,3,4),(6,4,4),(6,5,4),(6,6,4)\}$$

B = 6 and 5 appears respectively on first two tosses, if die is tosses three times

$B = \{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$

$A \cap B = \{(6,5,4)\}$

$$\text{Required probability } = P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{1}{6}$$

$$\text{Required probability } = \frac{1}{6}$$

Probability Ex 31.1 Q6

Given, $P(B) = 0.5,\ P(A \cap B) = 0.32$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{0.32}{0.5}$$
$$= \frac{32}{50}$$
$$= \frac{16}{25}$$

$$P\left(\frac{A}{B}\right) = \frac{16}{25}$$

Probability Ex 31.1 Q7

Given, $P(A) = 0.4,\ P(B) = 0.3$ and $P\left(\frac{B}{A}\right) = 0.5$

We know that,

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$0.5 = \frac{P(A \cap B)}{0.4}$$
$$P(A \cap B) = 0.5 \times 0.4$$

$$P(A \cap B) = 0.2$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{0.2}{0.3}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3}$$

$(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$, $P(A \cup B) = \frac{11}{30}$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{11}{30} = \frac{1}{3} + \frac{1}{5} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{11}{30}$$

$$= \frac{10 + 6 - 11}{30}$$

$$= \frac{5}{30}$$

$$P(A \cap B) = \frac{1}{6}$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{6}}{\frac{1}{5}}$$

$$P\left(\frac{A}{B}\right) = \frac{5}{6}$$

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$

$$P\left(\frac{B}{A}\right) = \frac{\frac{1}{6}}{\frac{1}{3}}$$

$$= \frac{1}{6} \times \frac{3}{1}$$

$$= \frac{1}{2}$$

$$P\left(\frac{A}{B}\right) = \frac{5}{6},\ P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Given, Couple has two children.

(i)

A = Both are male

$A = \{M_1M_2\}$

B = Atleast one is male

$B = \{M_1M_2, M_1F_2, F_1M_2\}$

$A \cap B = \{M_1M_2\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{3}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{3}$$

(ii)

A = Both are Females

$A = \{F_1F_2\}$

B = Elder child is female

$B = \{F_1M_2, F_1F_2\}$

$A \cap B = \{F_1F_2\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{1}{2}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{2}$$

Ex 31.2

Probability Ex 31.2 Q1

A = first card is king
B = second card is also king

Probability of getting two kings (without replacement)

$$= P(A)P\left(\frac{B}{A}\right)$$

$$= \frac{4}{52} \times \frac{3}{51} \qquad \text{[Since, 4 kings out of 52 cards.]}$$

$$= \frac{1}{13} \times \frac{1}{17}$$

$$= \frac{1}{221}$$

Required probability $= \frac{1}{221}$

Probability Ex 31.2 Q2

A = first card Ace
B = second card Ace
C = third card Ace
D = fourth card Ace

P (All four drawn are Ace, without replacement)

$$= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)P\left(\frac{D}{A \cap B \cap C}\right)$$

$$= \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \qquad \text{[Since, there are four Ace in 52 cards]}$$

$$= \frac{1}{270725}$$

Required probability $= \frac{1}{270725}$

Probability Ex 31.2 Q3

Bag contains 5 red and 7 white balls

A = first ball white

B = second ball white

P (2 white balls drawn without replacement)

$$= P(A)P\left(\frac{B}{A}\right)$$

$$= \frac{7}{12} \times \frac{6}{11}$$

$$= \frac{7}{22}$$

Required probability $= \frac{7}{22}$

Probability Ex 31.2 Q4

Tickets are numbered from 1 to 25

$\Rightarrow$ Total number of tickets = 25

Number of tickets with even numbers on it

$= 12 \qquad \{2,4,6,8,10,12,14,16,18,20,22,24\}$

A = first ticket with even number

B = second ticket with even number

P (Both tickets will show even number, without replacement)

$$= P(A)P\left(\frac{B}{A}\right)$$

$$= \frac{12}{25} \times \frac{11}{24}$$

$$= \frac{11}{50}$$

Required probability $= \frac{11}{50}$

Probability Ex 31.2 Q5

We know that, Deck of 52 cards contains 13 spades.

A = first card is spade

B = second card spade

C = third card spade

P (3 cards drawn without replacement are spade)

$$= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)$$

$$= \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50}$$

$$= \frac{11}{850}$$

Required probability $= \frac{11}{850}$

Probability Ex 31.2 Q6(i)

In a deck of 52 cards, there are 4 kings. Two cards are drawn without replacement

A = first card is king

B = second card is king

P (Both drawn cards are king)

$$= P(A)P\left(\frac{B}{A}\right)$$

$$= \frac{4}{52} \times \frac{3}{51}$$

$$= \frac{1}{221}$$

Required probability $= \frac{1}{221}$

Probability Ex 31.2 Q6(ii)

We know that, there are 4 kings and 4 ace in a pack of 52 cards.

Two cards are drawn without replacement

A = first card is king

B = second card an ace

P (The first card is a king and second is an ace)

$$= P(A)P\left(\frac{B}{A}\right)$$

$$= \frac{4}{52} \times \frac{4}{51}$$

$$= \frac{4}{663}$$

Required probability $= \frac{4}{663}$

Probability Ex 31.2 Q6(iii)

There are 13 heart and 26 red cards

Hearts are also red .

A = first card is heart

B = second card is red

P (first card is heart and second is red)

$$= P(A).P\left(\frac{B}{A}\right)$$

$$= \frac{13}{52} \times \frac{25}{51}$$

$$= \frac{25}{204}$$

Required probability $= \frac{25}{204}$

Probability Ex 31.2 Q7

Total number of tickets are 20 numbered from 1,2,3,...20.

Number of tickets with even numbers
$= 10$ $\quad [\text{Since, even numbers are } 2,4,6,8,10,12,14,16,18,20]$

Number of tickets with odd numbers
$= 10$ $\quad [\text{Since, odd numbers are } 1,3,5,7,9,11,13,15,17,19]$

Two cards are drawn without replacement.
A = tickets with even numbers
B = tickets with odd numbers

P (first ticket has even number and second has odd number)

$$= P(A)P\left(\frac{B}{A}\right)$$

$$= \frac{10}{20}\cdot\frac{10}{19}$$

$$= \frac{5}{19}$$

Required probability $= \frac{5}{19}$

Probability Ex 31.2 Q8

Urn contains 3 white, 4 red and 5 black balls. Total balls $= 12$

Two balls are drawn without replacement
A = first ball is black
B = second ball is black

P (Atleast one ball is black)

$$= P(A \cup B)$$

$$= 1 - P\left(\overline{A \cup B}\right)$$

$$= 1 - P\left(\overline{A} \cap \overline{B}\right)$$

$$= 1 - P\left(\overline{A}\right)P\left(\overline{B}/\overline{A}\right)$$

$$= 1 - \left(\frac{7}{12}\times\frac{6}{12}\right)$$

$$= 1 - \frac{7}{22}$$

$$= \frac{15}{22}$$

Required probability $= \frac{15}{22}$

Probability Ex 31.2 Q9

Bag contains 5 white, 7 red and 3 black balls.

Total number of balls $= 15$

Three balls are drawn without replacement

A = first ball is red

B = second ball is red

C = Third balls is red

P (Three balls are drawn, non is red)

$= P(\bar{A})P\left(\frac{\bar{B}}{\bar{A}}\right)P\left(\frac{\bar{C}}{\bar{A}\cap\bar{B}}\right)$

$= \frac{8}{15}\times\frac{7}{14}\times\frac{6}{13}$ [Since, number of non red balls $= 5+3=8$]

$= \frac{8}{65}$

Required probability $= \frac{8}{65}$

Probability Ex 31.2 Q10

Two cards are drawn from a pack of 52 cards without replacement.

There are 13 heart and 13 diamond in pack

A = first card is heart

B = second card is diamond

P (first card heart and second diamond)

$= P(A)P\left(\frac{B}{A}\right)$

$= \frac{13}{52}\times\frac{13}{51}$

$= \frac{13}{204}$

Required probability $= \frac{13}{204}$

Probability Ex 31.2 Q11

Let E and F denote respectively the events that first and second ball drawn are black. We have to find $P(E \cap F)$ or $P(EF)$.

Now $P(E) = P$ (black ball in first draw) $= \frac{10}{15}$

Also given that the first ball drawn is black, i.e, event E has occurred, now there are 9 black balls and five white balls left in the urn. Therefore, the probability that the second ball drawn is black, given that the ball in the first draw is black, is nothing but the conditional probability of F given that E has occurred.

i.e., $P(F \mid E) = \frac{9}{14}$

By multiplication rule of probability, we have

$P(E \cap F) = P(E)\, P(F \mid E)$

$= \frac{10}{15}\times\frac{9}{14} = \frac{3}{7}$

Multiplication rule of probability for more than two events if E.F and G are three events of sample space, we have

$P(E \cap F \cap G) = P(E)\, P(F|E)\, P(E)\, P(G|)\, (G \cap F) = P(E)\, P(F|E)\, P(G|EF)$

Similarly, the multiplication rule of probability can be extended for four or more events.

The following example illustrates the extension of multiplication rule of probability for three events.

Probability Ex 31.2 Q12

Let K denote the event that the card drawn is king
and A be the event that the card drawn is an ace.
We are to find $P(KKA)$.

Now, $P(K) = \frac{4}{52}$

Also, $P(K/K)$ is the probability of second king with the condition that one king has already been drawn.
Now, there are 3 kings in (52−1) = 51 cards.

$\therefore \quad P(K/K) = \frac{3}{51}$

Lastly, $P(A/KK)$ is the probability of third drawn card to be an ace, with the condition that two kings have already been drawn.
Now, there are four aces in left 50 cards.

$\therefore \quad P(A/KK) = \frac{4}{50}$

By multiplication law of probability, we have

$$P(KKA) = P(K)P(K/K)P(A/KK)$$
$$= \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} = \frac{2}{5525}$$

Probability Ex 31.2 Q13

There are 15 oranges out of which 12 are good and 3 are bad.

Three oranges selected without replacement are drawn and if they found good the box is approved for sale.

A = first orange good
B = second orange good
B = third orange good

P(All three oranges are good)

$$= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)$$
$$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13}$$
$$= \frac{44}{91}$$

Required probability $= \frac{44}{91}$

Probability Ex 31.2 Q14

Given bag contains 4 white, 7 black and 5 red balls.

Total number of balls = 16

Three balls are drawn without replacement

A = first ball is white
B = second ball is black
C = Third balls is red

P(Three balls drawn are white, Black, red respectively)

$$= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)$$
$$= \frac{4}{16} \times \frac{7}{15} \times \frac{5}{14}$$
$$= \frac{1}{24}$$

Required probability $= \frac{1}{24}$

Ex 31.3

Probability Ex 31.3 Q1

Given,

$$P(A) = \frac{7}{13}, P(B) = \frac{9}{13} \text{ and } P(A \cap B) = \frac{4}{13}$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{4}{13}}{\frac{9}{13}}$$

$$= \frac{4}{9}$$

$$P\left(\frac{A}{B}\right) = \frac{4}{9}$$

Probability Ex 31.3 Q2

Given,

$$P(A) = 0.6, P(B) = 0.3 \text{ and } P(A \cap B) = 0.2$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.2}{0.3}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3}$$

and, $$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.2}{0.6}$$

$$P\left(\frac{B}{A}\right) = \frac{1}{3}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3}, P\left(\frac{B}{A}\right) = \frac{1}{3}$$

Probability Ex 31.3 Q3

Given,

$P(A \cap B) = 0.32$ and $P(B) = 0.5$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{0.32}{0.5}$$
$$= \frac{16}{25}$$
$$= 0.64$$

$$P\left(\frac{A}{B}\right) = 0.64$$

Probability Ex 31.3 Q4

Given,

$P(A) = 0.4,\ P(B) = 0.8,\ P\left(\frac{B}{A}\right) = 0.6$

We know that,

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$0.6 = \frac{P(A \cap B)}{0.4}$$
$$P(A \cap B) = 0.6 \times 0.4$$
$$P(A \cap B) = 0.24$$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{0.24}{0.8}$$
$$P\left(\frac{A}{B}\right) = 0.3$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.4 + 0.8 - 0.24$$
$$P(A \cap B) = 0.96$$

$$P\left(\frac{A}{B}\right) = 0.3,\quad P(A \cap B) = 0.96$$

Probability Ex 31.3 Q5(ii)

Given,

$$P(A)=\frac{6}{11},\ P(B)=\frac{5}{11} \text{ and } P(A\cup B)=\frac{7}{11}$$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$P(A\cap B)=P(A)+P(B)-P(A\cup B)$$
$$=\frac{6}{11}+\frac{5}{11}+\frac{7}{11}$$
$$P(A\cap B)=\frac{4}{11}$$

We know that,

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$$
$$=\frac{\frac{4}{11}}{\frac{5}{11}}$$
$$=\frac{4}{5}$$

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$$
$$=\frac{\frac{4}{11}}{\frac{6}{11}}$$
$$=\frac{2}{3}$$

Hence,

$$P\left(\frac{A}{B}\right)=\frac{4}{5},\ P\left(\frac{B}{A}\right)=\frac{2}{3}$$

Probability Ex 31.3 Q5(iii)

Given,

$$P(A)=\frac{7}{13},\ P(B)=\frac{9}{13},\ P(A\cap B)=\frac{4}{13}$$

Since, $P(A'\cap B)=P(B)-P(A\cap B)$

$$=\frac{9}{13}-\frac{4}{13}$$
$$P(A'\cap B)=\frac{5}{13}$$

$$P\left(\frac{A'}{B}\right)=\frac{P(A'\cap B)}{P(B)}$$
$$=\frac{\frac{5}{13}}{\frac{9}{13}}$$
$$=\frac{5}{9}$$

$$P\left(\frac{A'}{B}\right)=\frac{5}{9}$$

Probability Ex 31.3 Q7

Given,

$$P(A) = \frac{6}{11},\ P(B) = \frac{5}{11},\ P(A \cup B) = \frac{7}{11}$$

(i)

Since, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{6}{11} + \frac{5}{11} - \frac{7}{11}$$

$$P(A \cap B) = \frac{4}{11}$$

(ii)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{4}{11}}{\frac{5}{11}}$$

$$P\left(\frac{A}{B}\right) = \frac{4}{5}$$

(iii)

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{4}{11}}{\frac{6}{11}}$$

$$P\left(\frac{B}{A}\right) = \frac{2}{3}$$

Probability Ex 31.3 Q8

Sample space for three coins is given by

$\{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

(i)

A = Head on third toss

$A = \{HHH, HTH, THH, TTH\}$

B = Head on first two toss

$B = \{HHH, HHT\}$

$(A \cap B) = \{HHH\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{2}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{1}{2}$

(ii)

A = At least two heads

$A = \{HHH, HHT, HTH, THH\}$

B = At most two heads

$B = \{HHT, HTT, THT, TTT, HTH, THH, TTH\}$

$(A \cap B) = \{HHT, HTH, THH\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$P\left(\frac{A}{B}\right) = \frac{3}{7}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{3}{7}$

(iii)

A = At most two tails

$A = \{HHH, HTH, THT, TTH, HHT, THT, HTT\}$

B = At least one tail

$B = \{HTH, THH, TTH, HHT, HTT, THT, TTT\}$

$(A \cap B) = \{HTH, THT, TTH, HHT, THT, HTT\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{6}{7}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{6}{7}$

Probability Ex 31.3 Q9

Sample space of two coins

$\{HH, HT, TH, TT\}$

(i)

A = Tail appears on one coin

$A = \{HT, TH\}$

B = One coin shows head

$B = \{HT, TH\}$

$(A \cap B) = \{HT, TH\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{2}{2}$$

$$P\left(\frac{A}{B}\right) = 1$$

Hence, $P\left(\frac{A}{B}\right) = 1$

(ii)

A = No tail appears

$A = \{HH\}$

B = No head appears

$B = \{TT\}$

$(A \cap B) = \{\ \}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0}{1}$$

$$= 0$$

$$P\left(\frac{A}{B}\right) = 0$$

Probability Ex 31.3 Q10

Die is thrown three times.

A = 4 appears on the third toss

$$A = \{(1,1,4),(1,2,4),(1,3,4),(1,4,4),(1,5,4),(1,6,4)$$
$$(2,1,4),(2,2,4),(2,3,4),(2,4,4),(2,5,4),(2,6,4)$$
$$(3,1,4),(3,2,4),(3,3,4),(3,4,4),(3,5,4),(3,6,4)$$
$$(4,1,4),(4,2,4),(4,3,4),(4,4,4),(4,5,4),(4,6,4)$$
$$(5,1,4),(5,2,4),(5,3,4),(5,4,4),(5,5,4),(5,6,4)$$
$$(6,1,4),(6,2,4),(6,3,4),(6,4,4),(6,5,4),(6,6,4)\}$$

B = 6 and 5 appear respectively on first two tosses

$$B = \{(6,5,1),(6,5,5),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$$

$$(A \cap B) = \{(6,5,4)\}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{1}{6}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{1}{36}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{1}{6}$, $P\left(\frac{B}{A}\right) = \frac{1}{36}$

Probability Ex 31.3 Q11

There are three person for photograph father (F), mother (M), son (S).

Sample space $= \{FMS, FSM, MFS, MSF, SFM, SMF\}$

A = Son on one end

$A = \{SFM, SMF, MFS, FMS\}$

B = Father in the middle

$B = \{MFS, SFM\}$

$$(A \cap B) = \{MFS, SFM\}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{2}{2}$$
$$P\left(\frac{A}{B}\right) = 1$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{2}{4}$$
$$P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Hence, $P\left(\frac{A}{B}\right) = 1$, $P\left(\frac{B}{A}\right) = \frac{1}{2}$

Probability Ex 31.3 Q12

The sample space of the experiment is $\{(1,1)(1,2)(1,3)\ldots(6,6)\}$ conisting of 36 outcomes.

$$P(A) = P(\text{Sum} = 6) = \frac{5}{36}$$

$$P(B) = P(4 \text{ appears at least once}) = \frac{11}{36}$$

Now, $P\left(\frac{B}{A}\right) = \frac{P(A \text{ and } B)}{P(A)}$

$$= \frac{P(\text{sum is 6 and 4 has appeared at least once})}{P(A)}$$

$$= \frac{\frac{2}{36}}{\frac{5}{36}}$$

$$= \frac{2}{5}$$

Probability Ex 31.3 Q13

Two dice are thrown.

A = Sum on the dice is 8

$A = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

B = Second die always exhibits 4

$B = \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)\}$

$(A \cap B) = \{(4,4)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{1}{6}$$

Required probability $= \frac{1}{6}$

Probability Ex 31.3 Q14

Here two dice are thrown

A = Getting 7 as sum on two dice

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

B = Second die exhibits an odd number

$$B = \{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1)$$
$$(1,3),(2,3),(3,3),(4,3),(5,3),(6,3)$$
$$(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$$

$(A \cap B) = \{(2,5),(4,3),(6,1)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{3}{18}$$

$$= \frac{1}{6}$$

Hence, Required probability $= \frac{1}{6}$

Probability Ex 31.3 Q15

A pair of dice is thrown.

A = Getting 7 as sum number on 2 dice.

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

B = Second die always exhibits prime number

$B = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)$
$(1,3),(2,3),(3,3),(4,3),(5,3),(6,3)$
$(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$

[Since, there are 2,3,5 prime number on a die]

$(A \cap B) = \{(2,5),(4,3),(5,2)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{3}{18} = \frac{1}{6}$$

Hence, Required probability $= \frac{1}{6}$

Probability Ex 31.3 Q16

A die is rolled.

A = A prime number on die

$A = \{2, 3, 5\}$

B = An odd number on die

$B = (1, 3, 5)$

$(A \cap B) = \{3, 5\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{2}{3}$$

Required probability $= \frac{2}{3}$

Probability Ex 31.3 Q17

A pair of dice is thrown

A = Getting sum 8 or more
= Getting sum 8,9,10,11 or 12 on the pair of dice

$A = \{(2,6),(3,5),(4,4),(5,3),(6,2),(3,6)$
$(4,5),(5,4),(6,3),(4,6),(5,5),(6,4)$
$(5,6),(6,5),(6,6)$

B = 4 on first die

$B = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$

$(A \cap B) = \{(4,4),(4,5),(4,6)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{3}{6} = \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Probability Ex 31.3 Q18

Two dice are thrown

A = Sum of the numbers on dice is 8

$A = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

B = At least one die does not show five

$$B = \{(1,1),(1,2),(1,3),(1,4),(1,6),(2,1),(2,2)\\ (2,3),(2,4),(2,6),(3,1),(3,2),(3,3),(3,4)\\ (3,6),(4,1),(4,2),(4,3),(4,4),(4,6),(6,1)\\ (6,2),(6,3),(6,4),(6,6)\}$$

$(A \cap B) = \{(2,6),(4,6),(6,2)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{3}{25}$$

Required probability $= \frac{3}{25}$

Probability Ex 31.3 Q19

Two numbers are selected at random from integers 1 through 9.

A = Both numbers are odd

$$A = \{(3,1),(5,1),(7,1),(9,1),(3,5),(3,7),(9,3),(5,3),(5,7),(5,9)\\ (7,3),(7,5),(7,9),(9,3),(9,5),(9,7)\}$$

B = Sum of both numbers is even

= Sum of both numbers is 2, 4, 6, 8, 10, 12, 14, 16 or 18

$$= \{(1,3),(1,5),(2,4),(1,7),(2,6),(3,5),(1,9),(2,8),\\ (3,7),(4,6),(7,5),(8,4),(9,3),(8,6),(9,5),(9,7)\}$$

$(A \cap B) = \{(1,3),(1,5),(1,7),(3,5),(1,9),(3,7),(7,5),(9,3),(9,5),(9,7)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{10}{16}$$

Required probability $= \frac{5}{8}$

Probability Ex 31.3 Q20

A die is thrown twice

A = The number 5 has appeared at least once

$A = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),(5,1),(5,2),(5,3),(5,4),(5,6)\}$

B = Sum of the numbers is 8

$= \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$(A \cap B) = \{(3,5),(5,3)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{2}{5}$$

Required probability $= \frac{2}{5}$

Probability Ex 31.3 Q21

Two dice are thrown

A = Sum of the numbers showing on the dice is 7

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

B = First die shows a 6

$= \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$(A \cap B) = \{(6,1)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{1}{6}$$

Required probability $= \frac{1}{6}$

Probability Ex 31.3 Q22

A pair of die is thrown

E = Sum is greater than or equal to 10

$= \{(4,6),(5,5),(6,4),(5,6),(6,5),(6,6)\}$

Case I:

F = 5 appears on first die

$= \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$E \cap F = \{(5,5),(5,6)\}$

$$P\left(\frac{E}{F}\right) = \frac{n(E \cap F)}{n(F)}$$

$$= \frac{2}{6}$$

$$P\left(\frac{E}{F}\right) = \frac{1}{3}$$

Case II:

F = 5 appears on at least one die

$= \{(1,5),(2,5),(3,5),(4,5),(6,5),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$E \cap F = \{(5,5),(5,6),(6,5)\}$

$$P\left(\frac{E}{F}\right) = \frac{n(E \cap F)}{n(F)}$$

$$= \frac{3}{11}$$

$$P\left(\frac{E}{F}\right) = \frac{3}{11}$$

Probability Ex 31.3 Q23

Given,

Probability to pass mathematics (M)

$$P(M) = \frac{4}{5}$$

Probability to pass in mathematics (M) and computer Science (C)

$$P(M \cap C) = \frac{1}{2}$$

To find, $P\left(\frac{C}{M}\right)$

We know that,

$$P\left(\frac{C}{M}\right) = \frac{P(M \cap C)}{P(M)}$$

$$= \frac{\frac{1}{2}}{\frac{4}{5}}$$

$$= \frac{1}{2} \times \frac{5}{4}$$

$$= \frac{5}{8}$$

Required probability $= \frac{5}{8}$

Probability Ex 31.3 Q24

Given,

Probability that a person buys a shirt (S) $= P(S) = 0.2$

Probability that he buys a trouser (T) $= P(T) = 0.3$

$$P\left(\frac{S}{T}\right) = 0.4$$

We know that,

$$P\left(\frac{S}{T}\right) = \frac{P(S \cap T)}{P(T)}$$

$$0.4 = \frac{P(S \cap T)}{0.3}$$

$$P(S \cap T) = 0.4 \times 0.3$$

$$P(S \cap T) = 0.12$$

Probability that he buys a shirt and a trouser both $= 0.12$

$$P\left(\frac{T}{S}\right) = \frac{P(S \cap T)}{P(S)}$$

$$= \frac{0.12}{0.2}$$

$$P\left(\frac{T}{S}\right) = \frac{12}{20}$$

$$= \frac{3}{5}$$

$$= 0.6$$

Probability that he buys a trouser given that he buys a shirt $= 0.6$

Probability Ex 31.3 Q25

Total students = 1000
Number of girls = 430
% of girls in class XII = 10%

Let A = Student chosen studies in class XII
B = Student chosen is a girl

Then $P(B) = \frac{430}{1000}$

$P(A \cap B) = \frac{43}{1000}$

$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{43}{430} = \frac{1}{10}$

Probability Ex 31.3 Q26

Total no. of cards = 10

Let A = drawn number is more than 3
B = drawn number is even

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

Now $P(A) = \frac{7}{10}$

$P(A \cap B) = \frac{4}{10}$

$\therefore \quad P\left(\frac{B}{A}\right) = \frac{4}{7}$

Probability Ex 31.3 Q27

(i) Let 'A' be the event that both the children born are girls.
Let 'B' be the event that the youngest is a girl.
We have to find conditional probability P(A/B).

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$A \subset B \Rightarrow A \cap B = A$

$\Rightarrow P(A \cap B) = P(A) = P(GG) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(B) = P(BG) + P(GG) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Hence, $P(A/B) = \frac{1/4}{1/2} = \frac{1}{2}$

(ii) Let 'A' be the event that both the children born are girls.
Let 'B' be the event that at least one is a girl.
We have to find the conditional probability P(A/B).

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$A \subset B \Rightarrow A \cap B = A$

$\Rightarrow P(A \cap B) = P(A) = P(GG) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(B) = 1 - P(BB) = 1 - \frac{1}{2} \times \frac{1}{2} = 1 - \frac{1}{4} = \frac{3}{4}$

Hence, $P(A/B) = \frac{1/4}{3/4} = \frac{1}{3}$

Ex 31.4

Probability Ex 31.4 Q1(i)

A coin is tossed thrice

Sample space $= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$

A = The first throw results in head
$A = \{HHT, HTH, HHH, HTT\}$

B = The last throw in tail
$B = \{HHT, HTT, THT, TTT\}$

$A \cap B = \{HHT, HTT\}$

$P(A) = \frac{4}{8} = \frac{1}{2}$
$P(B) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$
$P(A).P(B) = \frac{1}{2}.\frac{1}{2}$
$P(A).P(B) = \frac{1}{4}$

$P(A).P(B) = P(A \cap B)$

So, A and B are independent events.

Probability Ex 31.4 Q1(ii)

Sample space for a coin thrown thrice is
$= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$

A = the number of head is odd
$A = \{HTT, THT, TTH, HHH\}$

B = the number of tails is odd
$B = \{THH, HTH, HHT, TTT\}$

$A \cap B = \{\ \} = \varnothing$

$P(A) = \frac{4}{8} = \frac{1}{2}$
$P(B) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = \frac{0}{8} = 0$

$P(A).P(B) = \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{4}$

$P(A).P(B) \neq P(A \cap B)$

So, A and B are not independent events.

Probability Ex 31.4 Q1(iii)

Sample space for throwing a coin thrice

$= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$

A = the number of heads is two

$A = \{HHT, THH, HTH\}$

B = the last throw results in head

$B = \{HHH, HTH, THH, TTH\}$

$A \cap B = \{THH, HTH\}$

$P(A) = \frac{3}{8}$

$P(B) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$

$P(A).P(B) = \frac{3}{8} \times \frac{1}{2}$

$= \frac{3}{16}$

$P(A).P(B) \neq P(A \cap B)$

So, A and B are not independent events.

Probability Ex 31.4 Q2

A pair of dice are thrown. It has 36 elements in its sample space.

A = Occurence of number 4 on first die

$A = \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$

B = Occurence of 5 on second die

$B = \{(1,5), (2,5), (3,5), (4,5), (5,5), (6,5)\}$

$A \cap B = \{(4,5)\}$

$P(A) = \frac{6}{36} = \frac{1}{6}$

$P(B) = \frac{6}{36} = \frac{1}{6}$

$P(A \cap B) = \frac{1}{36}$

$P(A).P(B) = \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

$P(A).P(B) = P(A \cap B)$

So, A and B are independent events.

Probability Ex 31.4 Q3(i)

A card is drawn from 52 cards

It has 4 kings, 4 Queen, 4 Jack

A = the card drawn is a king or a queen

$$P(A) = \frac{4+4}{52} = \frac{8}{52}$$

$$P(A) = \frac{2}{13}$$

B = the card drawn is a queen or a jack

$$P(B) = \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$$

$A \cap B$ = The card drawn is a queen

$$P(A \cap B) = \frac{4}{52} = \frac{1}{13}$$

$$P(A)P(B) = \frac{2}{13} \times \frac{2}{13} = \frac{4}{169}$$

$$P(A)P(B) \neq P(A \cap B)$$

Hence, A and B are not independent.

Probability Ex 31.4 Q3(ii)

A card is drawn from pack of 52 cards

There are 26 black and four kings in which 2 kings are black.

A = the card drawn is black

$$P(A) = \frac{26}{52}$$

$$P(A) = \frac{1}{2}$$

B = the card drawn is a king

$$P(B) = \frac{4}{52} = \frac{1}{13}$$

$A \cap B$ = The card drawn is a black king

$$P(A \cap B) = \frac{2}{52} = \frac{1}{26}$$

$$P(A)P(B) = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26}$$

$$P(A).P(B) = P(A \cap B)$$

So, A and B are independent events.

Probability Ex 31.4 Q3(iii)

A card is drawn from a pack of 52 cards

There are 13 spades and 4 Ace in which one card is ace of spade

A = the card drawn is spade

$$P(A) = \frac{13}{52}$$
$$P(A) = \frac{1}{4}$$

B = the card drawn is an ace

$$P(B) = \frac{4}{52}$$
$$P(B) = \frac{1}{13}$$

$A \cap B$ = The card drawn is an ace of spade

$$P(A \cap B) = \frac{1}{52}$$

$$P(A).P(B) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

$$P(A).P(B) = P(A \cap B)$$

Hence, A and B are independent events.

Probability Ex 31.4 Q4

A coin is tossed three times,

Sample space $= \{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

A = first toss is Head

$A = \{HHH, HHT, HTH, HTT\}$

$$P(A) = \frac{4}{8}$$

$$P(A) = \frac{1}{2}$$

B = second toss is Head

$= \{HHH, HHT, THH, THT\}$

$$P(B) = \frac{4}{8}$$

$$P(B) = \frac{1}{2}$$

C = exactly two Head in a row

$C = \{HHT, THH\}$

$$P(C) = \frac{2}{8}$$

$$P(C) = \frac{1}{4}$$

$A \cap B = \{HHH, HHT\}$

$$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$$

$B \cap C = \{HHT, THH\}$

$$P(B \cap C) = \frac{2}{8}$$

$$P(B \cap C) = \frac{1}{4}$$

$A \cap C = \{HHT\}$

$$P(A \cap C) = \frac{1}{8}$$

(i)

$$P(A).P(B) = \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$
$$P(A).P(B) = P(A \cap B)$$

Hence, A and B are independent events.

(ii)

$$P(B)\,P(C) = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1}{8}$$
$$P(B)\,P(C) \neq P(B \cap C)$$

So, B and C are not independent events.

(iii)

$$P(A).P(C) = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1}{8}$$
$$P(A).P(C) = P(A \cap C)$$

Hence, A and C are independent events.

Probability Ex 31.4 Q5

Given,

$$P(A) = \frac{1}{4}, P(B) = \frac{1}{3} \text{ and } P(A \cup B) = \frac{1}{2}$$

We know that,

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= \frac{1}{4} + \frac{1}{3} + \frac{1}{2}$$
$$= \frac{3 + 4 - 6}{12}$$
$$P(A \cap B) = \frac{1}{12}$$

$$P(A)\,P(B) = \frac{1}{4} \times \frac{1}{3}$$
$$= \frac{1}{12}$$
$$P(A)\,P(B) = P(A \cap B)$$

Hence, A and B are independent events.

Probability Ex 31.4 Q6

Given that A and B are independent events and $P(A) = 0.3$, $P(B) = 0.6$

(i)

$$P(A \cap B) = P(A)P(B) \quad \text{[Since, } A \text{ and } B \text{ are indenpendent events]}$$
$$= 0.3 \times 0.6$$

$$P(A \cap B) = 0.18$$

(ii)

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$
$$= 0.3 - 0.18$$

$$P(A \cap \overline{B}) = 0.12$$

(iii)

$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$
$$= 0.6 - 0.18$$

$$P(\overline{A} \cap B) = 0.42$$

(iv)

$$P(\overline{A} \cap \overline{B}) = P(A)P(B)$$
$$= [1 - P(A)][1 - P(B)]$$
$$= (1 - 0.3)(1 - 0.6)$$
$$= 0.7 \times 0.4$$

$$P(\overline{A} \cap \overline{B}) = 0.28$$

(v)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.3 + 0.6 - 0.18$$

$$P(A \cup B) = 0.72$$

(vi)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{0.18}{0.6}$$
$$P\left(\frac{A}{B}\right) = 0.3$$

(vii)

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{0.18}{0.3}$$

$$P\left(\frac{B}{A}\right) = 0.6$$

Probability Ex 31.4 Q7

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Since A, B are independent

$\therefore \quad P(A \cap B) = P(A).P(B)$

Also $\quad P(\text{not } B) = 0.65 \Rightarrow \quad P(B) = 0.35$

Hence, we have

$$0.85 = P(A) + 0.35 - P(A)(0.35)$$

$$\Rightarrow \quad 0.5 = P(A)[1 - 0.35]$$

$$\Rightarrow \quad \frac{0.5}{.65} = P(A)$$

$$\Rightarrow \quad P(A) = 0.77$$

Probability Ex 31.4 Q8

We are given

$PP(\overline{A} \cap B) = \frac{2}{15}$

$P(A \cap \overline{B}) = \frac{1}{6}$

Since A, B are independent,

$$\therefore \quad P(\overline{A})P(B) = \frac{2}{15} \Rightarrow \quad [1 - P(A)]P(B) = \frac{2}{15} \quad ---(i)$$

$$\text{and} \quad P(A)P(\overline{B}) = \frac{1}{6} \Rightarrow \quad P(A)[1 - P(B)] = \frac{1}{6} \quad ---(ii)$$

From (i) we get

$$P(B) = \frac{2}{15} \times \frac{1}{1 - P(A)}$$

Substituting this value in equation (ii) we get,

$$P(A)\left[1 - \frac{2}{15(1 - P(A))}\right] = \frac{1}{6}$$

$$\Rightarrow \quad P(A)\left[\frac{15(1 - P(A)) - 2}{15(1 - P(A))}\right] = \frac{1}{6}$$

$$\Rightarrow \quad 6P(A)(13 - 15P(A)) = 15(1 - P(A))$$

$$\Rightarrow \quad 2P(A)(13 - 15P(A)) = 5 - 5P(A)$$

$$\Rightarrow \quad 26P(A) - 30[P(A)]^2 + 5P(A) - 5 = 0$$

$$\Rightarrow \quad -30[P(A)]^2 + 31P(A) - 5 = 0$$

This is a quadratic equation in $x = P(A)$ given as

$$-30x^2 + 31x - 5 = 0$$

$$\Rightarrow \quad 30x^2 - 31x + 5 = 0$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $a = +30$, $b = -31$, $c = +5$

$$\Rightarrow \quad x = \frac{31 \pm \sqrt{(-31)^2 - 4(30)(5)}}{60}$$

$$= \frac{31 \pm \sqrt{961 - 600}}{60}$$

$$= \frac{31 \pm 19}{60}$$

$$= \frac{50}{60}, \frac{12}{60}$$

$$= \frac{5}{6}, \frac{1}{5}$$

$$\therefore \quad P(A) = \frac{5}{6} \text{ or } \frac{1}{5}$$

Now

$$P(A)[1 - P(B)] = \frac{1}{6}$$

Putting $P(A) = \frac{5}{6}$

$$\frac{5}{6}[1 - P(B)] = \frac{1}{6}$$

$$1 - P(B) = \frac{1}{5}$$

$$P(B) = 1 - \frac{1}{5}$$

$$P(B) = \frac{4}{5}$$

Putting $P(A) = \frac{1}{5}$

$$\frac{1}{5}[1 - P(B)] = \frac{1}{6}$$

$$1 - P(B) = \frac{5}{6}$$

$$P(B) = 1 - \frac{5}{6}$$

$$P(B) = \frac{1}{6}$$

Hence $P(B) = \frac{4}{5}$ or $\frac{1}{6}$

Probability Ex 31.4 Q9

Given,

$$P(A \cap B) = \frac{1}{6}$$

$$P(\bar{A} \cap \bar{B}) = \frac{1}{3}$$

We know that,

$$P(\bar{A} \cap \bar{B}) = P(A)P(B)$$

$$\frac{1}{3} = (1 - P(A))(1 - P(B))$$

$$\frac{1}{3} = 1 - P(B) - P(A) + P(A)P(B)$$

$$\frac{1}{3} = 1 - P(B) - P(A) + P(A \cap B)$$

$$\frac{1}{3} = 1 - P(B) - P(A) + \frac{1}{6}$$

$$P(A) + P(B) = \frac{1}{1} + \frac{1}{6} - \frac{1}{3}$$

$$= \frac{6 + 1 - 2}{6}$$

$$P(A) + P(B) = \frac{5}{6}$$

$$P(A) = \frac{5}{6} - P(B) \qquad \text{---(i)}$$

Given, $P(A \cap B) = \frac{1}{6}$

$$P(A)P(B) = \frac{1}{6}$$

$$\left[\frac{5}{6} - P(B)\right]P(B) = \frac{1}{6} \qquad \text{[Using equation (i)]}$$

$$\Rightarrow \quad \frac{5}{6}P(B) - \{P(B)\}^2 = \frac{1}{6}$$

$$\Rightarrow \quad \{P(B)\}^2 - \frac{5}{6}P(B) + \frac{1}{6} = 0$$

$$\Rightarrow \quad 6\{P(B)\}^2 - 5P(B) + 1 = 0$$

$$\Rightarrow \quad 6\{P(B)\}^2 - 3P(B) - 2P(B) + 1 = 0$$

$$\Rightarrow \quad 3P(B)[2P(B) - 1] - 1[2P(B) - 1] = 0$$

$$\Rightarrow \quad [2P(B) - 1][3P(B) - 1] = 0$$

$$\Rightarrow \quad 2P(B) - 1 = 0 \text{ or } 3P(B) - 1 = 0$$

Given,

$$P(A \cap B) = \frac{1}{6}$$

$$P(\bar{A} \cap \bar{B}) = \frac{1}{3}$$

We know that,

$$P(\bar{A} \cap \bar{B}) = P(A)P(B)$$

$$\frac{1}{3} = (1 - P(A))(1 - P(B))$$

$$\frac{1}{3} = 1 - P(B) - P(A) + P(A)P(B)$$

$$\frac{1}{3} = 1 - P(B) - P(A) + P(A \cap B)$$

$$\frac{1}{3} = 1 - P(B) - P(A) + \frac{1}{6}$$

$$P(A) + P(B) = \frac{1}{1} + \frac{1}{6} - \frac{1}{3}$$

$$= \frac{6 + 1 - 2}{6}$$

$$P(A) + P(B) = \frac{5}{6}$$

$$P(A) = \frac{5}{6} - P(B) \qquad ---(i)$$

Given, $P(A \cap B) = \frac{1}{6}$

$$P(A)P(B) = \frac{1}{6}$$

$$\left[\frac{5}{6} - P(B)\right]P(B) = \frac{1}{6} \qquad \text{[Using equation (i)]}$$

$$\Rightarrow \quad \frac{5}{6}P(B) - \{P(B)\}^2 = \frac{1}{6}$$

$$\Rightarrow \quad \{P(B)\}^2 - \frac{5}{6}P(B) + \frac{1}{6} = 0$$

$$\Rightarrow \quad 6\{P(B)\}^2 - 5P(B) + 1 = 0$$

$$\Rightarrow \quad 6\{P(B)\}^2 - 3P(B) - 2P(B) + 1 = 0$$

$$\Rightarrow \quad 3P(B)[2P(B) - 1] - 1[2P(B) - 1] = 0$$

$$\Rightarrow \quad [2P(B) - 1][3P(B) - 1] = 0$$

$$\Rightarrow \quad 2P(B) - 1 = 0 \text{ or } 3P(B) - 1 = 0$$

Probability Ex 31.4 Q10

Given, A and B are independent events and $P(A \cup B) = 0.60$, $P(A) = 0.2$

A and B are independent events,

So, $P(A \cap B) = P(A)P(B)$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.6 = 0.2 + P(B) - P(A)P(B)$$

$$0.6 - 0.2 = P(B) - 0.2P(B)$$

$$0.4 = 0.8P(B)$$

$$P(B) = \frac{0.4}{0.8}$$

$$P(B) = 0.5$$

Probability Ex 31.4 Q11

A die is tossed twice.

Let A = Getting a number greater than 3 on first toss
B = Getting a number greater than 3 on second toss

$P(A) = \frac{3}{6}$ [Since, number greater than 3 on die are 4, 5, 6.]

$P(A) = \frac{1}{2}$

$P(B) = \frac{3}{6}$

$P(B) = \frac{1}{2}$

P (Getting a number greater than 3 on each toss)

$= P(A \cap B)$ [Since, A and B are independent events]

$= P(A)P(B)$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

Required Probability $= \frac{1}{4}$

Probability Ex 31.4 Q12

Given,

Probability that A can solve a problem $= \frac{2}{3}$

$\Rightarrow P(A) = \frac{2}{3}$

$\Rightarrow P(\overline{A}) = 1 - \frac{2}{3}$

$P(\overline{A}) = \frac{1}{3}$

Probability that B can solve the same problem $= \frac{3}{5}$

$\Rightarrow P(B) = \frac{3}{5}$

$\Rightarrow P(\overline{B}) = 1 - \frac{3}{5}$

$P(\overline{B}) = \frac{2}{5}$

P (None of them solve the problem)

$= P(\overline{A} \cap \overline{B})$

$= P(\overline{A}).P(\overline{B})$

$= \frac{1}{3} \times \frac{2}{5}$

$= \frac{2}{15}$

Required probability $= \frac{2}{15}$

ty Ex 31.4 Q13

Given an unbiased die is tossed twice

A = Getting 4,5 or 6 on the first toss

B = 1,2,3 or 4 on second toss

$\Rightarrow \quad P(A) = \frac{3}{6}$

$P(A) = \frac{1}{2}$

and, $\quad P(B) = \frac{4}{6}$

$P(B) = \frac{2}{3}$

P (Getting 4,5 or 6 on the first toss and 1,2,3 or 4 on second toss)

$= P(A \cap B)$

$= P(A)P(B)$

$= \frac{1}{2} \times \frac{2}{3}$

$= \frac{1}{3}$

Required probability $= \frac{1}{3}$

Probabili

Given bag contains 3 red and 2 black balls.

A = Getting one red ball

$\Rightarrow \quad P(A) = \frac{3}{5}$

B = Getting one black ball

$\Rightarrow \quad P(B) = \frac{2}{5}$

(i)

P (Getting two red balls)

$= P(A)P(A)$

$= \frac{3}{5} \times \frac{3}{5}$

$= \frac{9}{25}$

P (Getting two red balls) $= \frac{9}{25}$

(ii)

P (Getting two black balls)

$= P(B).P(B)$

$= \frac{2}{5} \times \frac{2}{5}$

$= \frac{4}{25}$

P (Getting two black balls) $= \frac{4}{25}$

(iii)

P (Getting first red and second black ball)

$= P(A)P(B)$

$= \frac{3}{5} \times \frac{2}{5}$

$= \frac{6}{25}$

P (Getting first red and second black ball) $= \frac{6}{25}$

Probability Ex 31.4 Q15

Three cards are drawn with replacement consider,

A = drawing a king
B = drawing a queen
C = drawing a jack

$\Rightarrow \quad P(A) = \frac{4}{52}$ [Since there are 4 kings]

$P(A) = \frac{1}{13}$

$\Rightarrow \quad P(B) = \frac{4}{52}$ [Since there are 4 queens]

$P(B) = \frac{1}{13}$

$\Rightarrow \quad P(C) = \frac{4}{52}$ [Since there are 4 jacks]

$P(C) = \frac{1}{13}$

P (Cards drawn are king, queen and jack)

$= P(A \cap B \cap C) + P(A \cap C \cap B) + P(B \cap A \cap C)$
$P(B \cap C \cap A) + P(C \cap A \cap B) + P(C \cap B \cap A)$

[Since order of drawing them may be different]

$= P(A)P(B)P(C) + P(A)P(C)P(B) + P(B)P(A)P(C)$
$+ P(B)P(C)P(A) + P(C)P(A)P(B) + P(C)P(B)P(A)$

$= \frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13} + \frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13} + \frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13} + \frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13} + \frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13}$

$= \left(\frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13}\right) \times 6$

$= \frac{6}{2197}$

Required probability $= \frac{6}{2197}$

Probability Ex 31.4 Q16

Given,

Prat X has 9 out of 100 defective
$\Rightarrow$ Part X has 91 out of 100 non defective

Part Y has 5 out of 100 defective
$\Rightarrow$ Part Y has 95 out of 100 non defective

Consider,

X = A non defective part X
Y = A non defective part Y

$\Rightarrow \quad P(X) = \frac{91}{100}$ abd $P(Y) = \frac{95}{100}$

$= P$ (Assembled product will not be defective)
$= P$ (Niether X defective nor Y defective)
$= P(X \cap Y)$
$= P(X)P(Y)$
$= \frac{91}{100} \times \frac{95}{100}$
$= 0.8645$

Required probability $= 0.8645$

Probability Ex 31.4 Q17

Given,

Probability that A hits a target $= \frac{1}{3}$

$\Rightarrow \quad P(A) = \frac{1}{3}$

Probability that B hits the target $= \frac{2}{5}$

$\Rightarrow \quad P(B) = \frac{2}{5}$

P (Target will be hit)
$= 1 - P$ (target will not be hit)
$= 1 - P$ (Niether A non B hits the target)

$$= 1 - P\left(\bar{A} \cap \bar{B}\right)$$
$$= 1 - P\left(\bar{A}\right) P\left(\bar{B}\right)$$
$$= 1 - \left[1 - P(A)\right]\left[1 - P\left(\bar{B}\right)\right]$$
$$= 1 - \left[1 - \frac{1}{3}\right]\left[1 - \frac{2}{5}\right]$$
$$= 1 - \frac{2}{3} \cdot \frac{3}{5}$$
$$= 1 - \frac{2}{5}$$
$$= \frac{3}{5}$$

Required probability $= \frac{3}{5}$

Probability Ex 31.4 Q18

Given,

An anti aircraft gun can take a maximum 4 shots at an enemy plane

Consider,

A = Hitting the plane at first shot
B = Hetting the plane at second shot
C = Hetting the plane at third shot
D = Hetting the plane at fourth shot

$\Rightarrow \quad P(A) = 0.4,\ P(B) = 0.3,\ P(C) = 0.2,\ P(D) = 0.1$

P (Gun hits the plane)
$= 1 - P$ (Gun does not hit the plane)
$= 1 - P$ (Non of the four shots hit the plane)

$$= 1 - P\left(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}\right)$$
$$= 1 - P\left(\bar{A}\right) P\left(\bar{B}\right) P\left(\bar{C}\right) P\left(\bar{D}\right)$$
$$= 1 - \left[1 - P(A)\right]\left[1 - P\left(\bar{B}\right)\right]\left[1 - P(C)\right]\left[1 - P(D)\right]$$
$$= 1 - [1 - 0.4][1 - 0.3][1 - 0.2][1 - 0.1]$$
$$= 1 - (0.6)(0.7)(0.8)(0.9)$$
$$= 1 - 0.3024$$
$$= 0.6976$$

Required probability $= 0.6976$

Probability Ex 31.4 Q19

Given,

The odds against a certain event (say, A) are 5 to 2

$$\Rightarrow \quad P\left(\overline{A}\right) = \frac{5}{5+2}$$

$$P\left(\overline{A}\right) = \frac{5}{7}$$

The odds in favour of another event (say, B) are 6 to 5

$$\Rightarrow \quad P\left(B\right) = \frac{6}{5+6}$$

$$P\left(B\right) = \frac{6}{11}$$

$$P\left(\overline{B}\right) = 1 - \frac{6}{11}$$

$$P\left(\overline{B}\right) = \frac{5}{11}$$

(a)

P (At least one of the events will occur)

$= 1 - P$ (None of events occur)

$= 1 - P\left(\overline{A} \cap \overline{B}\right)$

$= 1 - P\left(\overline{A}\right)P\left(\overline{B}\right)$ [Since events are independent]

$= 1 - \frac{5}{7} \times \frac{5}{11}$

$= 1 - \frac{25}{77}$

$= \frac{52}{77}$

Required probability $= \frac{52}{77}$

(b)

P (None of the events will occur)

$= P\left(\overline{A} \cap \overline{B}\right)$

$= P\left(\overline{A}\right)P\left(\overline{B}\right)$

$= \frac{5}{7} \times \frac{5}{11}$

$= \frac{25}{11}$

Probability Ex 31.4 Q20

Given, A die is thrown thrice.

Consider,

A = Getting an odd number in a throw of die

$P(A) = \frac{3}{6}$ [Since there are 1,3,5 odd number on die]

$P(A) = \frac{1}{2} \quad \Rightarrow P(\bar{A}) = \frac{1}{2}$

P (Getting an odd number at least once)

$= 1 - P$ (Getting no odd number)

$= 1 - P(\bar{A} \cap \bar{A} \cap \bar{A})$

$= 1 - P(\bar{A})P(\bar{A})P(\bar{A})$

$= 1 - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$

Required probability $= \frac{7}{8}$

Probability Ex 31.4 Q21

The box contains 10 black balls and 8red balls.

Then $P(\text{black ball}) = \frac{10}{18}$

$P(\text{red ball}) = \frac{8}{18}$

(i) $P(\text{Both balls are red}) = \frac{8}{18} \times \frac{8}{18} = \frac{16}{81}$

(ii) P (First ball is black and second is red)

$= \frac{10}{18} \times \frac{8}{18} = \frac{20}{81}$

(iii) P (one of them is black and other is red)

$= \frac{10}{18} \cdot \frac{8}{18} + \frac{8}{18} \cdot \frac{10}{18}$

$= 2\left(\frac{20}{81}\right)$

$= \frac{40}{81}$

Probability Ex 31.4 Q22

Given, Urn contains 4 red and 7 black balls.
Two balls drawn at random with replacement.

Consider,

R = Getting one red ball from urn.

$P(R) = \frac{4}{11}$

B = Getting one blue ball from urn.

$P(B) = \frac{7}{11}$

(i)

P (Getting 2 red balls)

$= P(R).P(R)$

$= \frac{4}{11} \times \frac{4}{11}$

$= \frac{16}{121}$

Required probability $= \frac{16}{121}$

(ii)

P (Getting two blue balls)

$= P(B)P(B)$

$= \frac{7}{11} \times \frac{7}{11}$

$= \frac{49}{121}$

Required probability $= \frac{49}{121}$

(iii)

P (Getting one red and one blue ball)

$= P(R)P(B) + P(B)P(R)$

$= \frac{4}{11} \times \frac{7}{11} + \frac{7}{11} \times \frac{4}{11}$

$= \frac{28}{121} + \frac{28}{121}$

$= \frac{56}{121}$

Probability Ex 31.4 Q23

Given that the events 'A coming in time' and 'B coming in time' are independent.
Let 'A' denote the event of 'A coming in time'.
Then, '$\bar{A}$' denotes the complementary event of A.
Similarly we define B and $\bar{B}$.

P(only one coming in time) $= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$...(since A and B are independent events)

$= \frac{3}{7} \times \frac{2}{7} + \frac{4}{7} \times \frac{5}{7} = \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$

The advantage of coming to school in time is that you will not miss any part of the lecture and will be able to learn more.

Probability Ex 31.4 Q24

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$n(S) = 36$

E be the event of geting a total of 4.

$E = \{(1,3),(3,1),(2,2)\}$

$n(E) = 3$

$P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

F be the event of geting a total of 9 or more.

$F = \{(3,6),(6,3),(4,5),(5,4),(4,6),(6,4),(5,5),(5,6),(6,5),(6,6)\}$

$n(F) = 10$

$P(F) = \frac{n(F)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

G be the event of geting a total divisible by 5.

$G = \{(1,4),(4,1),(2,3),(3,2),(4,6),(6,4),(5,5)\}$

$n(G) = 7$

$P(G) = \frac{n(G)}{n(S)} = \frac{7}{36}$

No pair is independent.

Probability Ex 31.4 Q25

Events are said to be independent, if the occurrence or non-occurrence of one does not affect the probability of the occurrence or non-occurrence of the other.

(i) $p_1p_2 = P(A)P(B)$

$\Rightarrow$ Both A and B occour.

(ii) $(1-p_1)p_2 = (1-P(A))P(B) = P(\bar{A})P(B)$

$\Rightarrow$ Event A does not occour, but event B occours.

(iii) $1-(1-p_1)(1-p_2) = [1-(1-P(A))(1-P(B))] = (1-P(\bar{A})P(\bar{B}))$

$\Rightarrow$ At least one of the events A or B occours.

(iv) $p_1 + p_2 = 2p_1p_2$

$\Rightarrow P(A)+P(B) = 2P(A)P(B)$

$\Rightarrow P(A)+P(B)-2P(A)P(B) = 0$

$\Rightarrow P(A)-P(A)P(B)+P(B)-P(A)P(B) = 0$

$\Rightarrow P(A)(1-P(B))+P(B)(1-P(A)) = 0$

$\Rightarrow P(A)P(\bar{B})+P(B)P(\bar{A}) = 0$

$\Rightarrow P(A)P(\bar{B}) = P(B)P(\bar{A})$

$\Rightarrow$ Exactly one of A and B occours.

Ex 31.5

Probability Ex 31.5 Q1

There are two bags.
One bag (1) Contain 6 black and 3 white balls
other bag (2) Contain 5 black and 4 white balls

One ball is drawn from each bag

$$P(\text{One black from bag 1}) = \frac{6}{9}$$

$$P(B_1) = \frac{2}{3}$$

$$P(\text{One black from bag 2}) = \frac{5}{9}$$

$$P(B_2) = \frac{5}{9}$$

$$P(\text{One white from bag 1}) = \frac{3}{9}$$

$$P(W_1) = \frac{1}{3}$$

$$P(\text{One white from bag 2}) = \frac{4}{9}$$

$$P(W_2) = \frac{4}{9}$$

$$P(\text{Two balls of same colour})$$
$$= P\left[(W_1 \cap W_2) \cup (B_1 \cap B_2)\right]$$
$$= P(W_1 \cap W_2) + P(B_1 \cap B_2)$$
$$= P(W_1)P(W_2) + P(B_1)P(B_2)$$
$$= \frac{1}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{5}{9}$$
$$= \frac{4}{27} + \frac{10}{27}$$
$$= \frac{14}{27}$$

Required probability $= \frac{14}{27}$

Probability Ex 31.5 Q2

There are two bags.
Bag (1) contain 3 red and 5 black balls
Bag (2) contain 6 red and 4 black balls

$$P(\text{One red ball from bag 1}) = \frac{3}{8}$$
$$P(R_1) = \frac{3}{8}$$

$$P(\text{One black ball from bag 1}) = \frac{5}{8}$$
$$P(B_1) = \frac{5}{8}$$

$$P(\text{One red ball from bag 1}) = \frac{6}{10}$$
$$P(R_2) = \frac{3}{5}$$

$$P(\text{One black ball from bag 2}) = \frac{4}{10}$$
$$P(B_2) = \frac{2}{5}$$

One ball is drawn from each bag.

$$P(\text{One ball is red and the other is black})$$
$$= P[(R_1 \cap B_2) \cup (B_1 \cap R_2)]$$
$$= P(R_1 \cap B_2) + P(B_1 \cap R_2)$$
$$= P(R_1)P(B_2) + P(B_1)P(R_2)$$
$$= \frac{3}{8} \times \frac{2}{5} + \frac{5}{8} \times \frac{3}{5}$$
$$= \frac{6}{40} + \frac{15}{40}$$
$$= \frac{21}{40}$$

$$\text{Required probability} = \frac{21}{40}$$

Probability Ex 31.5 Q3

Given, box contains 10 black and 8 red balls.
Two balls are drawn with replacement.

(i)

$$
\begin{aligned}
& P(\text{Both the balls are red}) \\
& = P(R_1 \cap R_2) \\
& = P(R_1)\, P(R_2) \\
& = \frac{8}{18} \times \frac{8}{18} \\
& = \frac{16}{81}
\end{aligned}
$$

Required probability $= \frac{16}{81}$

(ii)

$$
\begin{aligned}
& P(\text{first ball is black and second is red}) \\
& = P(B \cap R) \\
& = P(B)\, P(R) \\
& = \frac{10}{18} \times \frac{8}{18} \\
& = \frac{20}{81}
\end{aligned}
$$

Required probability $= \frac{20}{81}$

(iii)

$$
\begin{aligned}
& P(\text{one of them red and other black}) \\
& = P((B \cap R) \cup (R \cap B)) \\
& = P(B \cap R) + P(R \cap B) \\
& = P(B)P(R) + P(R)P(B) \\
& = \frac{10}{18} \times \frac{8}{18} + \frac{8}{18} \times \frac{10}{18} \\
& = \frac{20+20}{81} \\
& = \frac{40}{81}
\end{aligned}
$$

Required probability $= \frac{40}{81}$

Probability Ex 31.5 Q4

Two cards are drawn without replacement.

There are total 4 ace.

A = Getting Ace

$$
\begin{aligned}
& \mathrm{P}(\text{Exactly one ace out of 2 cards}) \\
& = P((A \cap \bar{A}) \cup (\bar{A} \cap A)) \\
& = P(A)P\left(\frac{\bar{A}}{A}\right) + P(\bar{A})P\left(\frac{A}{\bar{A}}\right) \\
& = \frac{4}{52} \cdot \frac{48}{51} + \frac{48}{52} \cdot \frac{4}{51} \\
& = \frac{96}{663} \\
& = \frac{32}{221}
\end{aligned}
$$

Required probability $= \frac{32}{221}$

Probability Ex 31.5 Q5

Given,

A speaks truth in 75% cases.
B speaks truth in 80% cases.

$$P(A)=\frac{75}{100} \quad \Rightarrow P(\overline{A})=\frac{25}{100}$$
$$P(B)=\frac{80}{100} \quad \Rightarrow P(\overline{B})=\frac{20}{100}$$

$P(A \text{ and } B \text{ contradict each other})$

$$= P\left[\left(A\cap\overline{B}\right)\cup\left(\overline{A}\cap B\right)\right]$$
$$= P\left(A\cap\overline{B}\right)+P\left(\overline{A}\cap B\right)$$
$$= P(A)P(\overline{B})+P(\overline{A})P(B)$$
$$= \frac{75}{100}\cdot\frac{20}{100}+\frac{25}{100}\cdot\frac{80}{100}$$
$$= \frac{1500}{1000}+\frac{2000}{10000}$$
$$= \frac{3500}{10000}$$
$$= 35\%$$

Required probability $= 35\%$

Probability Ex 31.5 Q6

Given,

Probability of selection of Kamal $(K) = \frac{1}{3}$

$$P(K)=\frac{1}{3}$$

Probability of selection of Monika $(M) = \frac{1}{5}$

$$P(M)=\frac{1}{5}$$

(i)

$P(\text{Both of them selected})$

$$= P(K\cap M)$$
$$= P(K)P(M)$$
$$= \frac{1}{3}\cdot\frac{1}{5}$$
$$= \frac{1}{15}$$

Required probability $= \frac{1}{15}$

(ii)

$P(\text{None of them will be selected})$

$$= P\left(\overline{K}\cap\overline{M}\right)$$
$$= P\left(\overline{K}\right)P\left(\overline{M}\right)$$
$$= \left[1-P(K)\right]\left[1-P(M)\right]$$
$$= \left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)$$
$$= \frac{2}{3}\times\frac{4}{5}$$
$$= \frac{8}{15}$$

Required probability $= \frac{8}{15}$

(iii)

$$P(\text{At least one of them selected})$$
$$= 1 - P(\text{None of them selected})$$
$$= 1 - P\left(\overline{M} \cap \overline{K}\right)$$
$$= 1 - P\left(\overline{M}\right).P\left(\overline{K}\right)$$
$$= 1 - [1 - P(M)][1 - P(K)]$$
$$= 1 - \left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{3}\right)$$
$$= 1 - \frac{4}{5}.\frac{2}{3}$$
$$= 1 - \frac{8}{15}$$
$$= \frac{7}{15}$$

Required probability $= \frac{7}{15}$

(iv)

$$P(\text{Only one of them will be selected})$$
$$= P\left[\left(K \cap \overline{M}\right) \cup \left(\overline{K} \cap M\right)\right]$$
$$= P\left(K \cap \overline{M}\right) + P\left(\overline{K} \cap M\right)$$
$$= P(K)P\left(\overline{M}\right) + P\left(\overline{K}\right)P(M)$$
$$= \frac{1}{3}[1 - P(M)] + [1 - P(K)]\frac{1}{5}$$
$$= \frac{1}{3}\left[1 - \frac{1}{5}\right] + \left[1 - \frac{1}{3}\right].\frac{1}{5}$$
$$= \frac{1}{3}.\frac{4}{5} + \frac{2}{3}.\frac{1}{5}$$
$$= \frac{4}{15} + \frac{2}{15}$$
$$= \frac{6}{15}$$
$$= \frac{2}{5}$$

Required probability $= \frac{2}{5}$

Probability Ex 31.5 Q7

Bag contain 3 white, 4 red, 5 black balls.
Two balls are drawn without replacement.

$$P(\text{One ball is white and other black})$$
$$= P[(W \cap B) \cup (B \cap W)]$$
$$= P(W \cap B) + P(B \cap W)$$
$$= P(W)P\left(\frac{B}{W}\right) + P(B)P\left(\frac{W}{B}\right)$$
$$= \frac{3}{12} \times \frac{5}{12} + \frac{5}{12} \times \frac{3}{11}$$
$$= \frac{15}{132} + \frac{15}{132}$$
$$= \frac{30}{132}$$
$$= \frac{5}{22}$$

Required probability $= \frac{5}{22}$

Probability Ex 31.5 Q8

A bag contains 8 red and 6 green balls.
Three balls are drawn without replacement

$$P(\text{at least 2 balls are green})$$
$$= P\left[(G_1 \cap G_2 \cap R_1) \cup (G_1 \cap R_1 \cap G_2) \cup (R_1 \cap G_1 \cap G_2) \cup (G_1 \cap G_2 \cap G_3)\right]$$
$$= P(G_1 \cap G_2 \cap R_1) + P(G_1 \cap R_1 \cap G_2) + P(R_1 \cap G_1 \cap G_2) + P(G_1 \cap G_2 \cap G_3)$$
$$= P(G_1)P\left(\frac{G_2}{G_1}\right)P\left(\frac{R_1}{G_1 \cap G_2}\right) + P(G_1)P\left(\frac{R_1}{G_1}\right)P\left(\frac{G_2}{R_1 \cap G_1}\right) +$$
$$P(R_1)P\left(\frac{G_1}{R_1}\right)P\left(\frac{G_2}{G_1 \cap R_1}\right) + P(G_1)P\left(\frac{G_2}{G_1}\right)P\left(\frac{G_3}{G_1 \cap G_2}\right)$$
$$= \frac{6}{14} \times \frac{5}{13} \times \frac{8}{12} + \frac{6}{14} \times \frac{8}{13} \times \frac{5}{12} + \frac{8}{14} \times \frac{6}{13} \times \frac{5}{12} + \frac{6}{14} \times \frac{5}{13} \times \frac{4}{12}$$
$$= \frac{1}{14} \times \frac{1}{13} \times \frac{1}{12} \times (240 + 240 + 240 + 120)$$
$$= \frac{840}{14 \times 13 \times 12}$$
$$= \frac{5}{13}$$

$$\text{Required probability} = \frac{5}{13}$$

Probability Ex 31.5 Q9

Given, Probability of Arun's (A) selection $= \frac{1}{4}$

$$P(A) = \frac{1}{4}$$

Probability of Tarun's (T) rejection $= \frac{2}{3}$

$$P(\bar{T}) = \frac{2}{3}$$

$$P(\bar{A}) = 1 - P(A)$$
$$\Rightarrow \quad P(\bar{A}) = 1 - \frac{1}{4}$$
$$\Rightarrow \quad P(\bar{A}) = \frac{3}{4}$$

$$P(T) = 1 - P(\bar{T})$$
$$\Rightarrow \quad P(T) = 1 - \frac{2}{3}$$
$$\Rightarrow \quad P(T) = \frac{1}{3}$$

$$P(\text{At least one of them will be selected})$$
$$= 1 - P(\text{None of them selected})$$
$$= 1 - P(\bar{A} \cap \bar{T})$$
$$= 1 - P(\bar{A})P(\bar{T})$$
$$= 1 - \frac{2}{3} \times \frac{3}{4}$$
$$= \frac{1}{2}$$

$$\text{Required probability} = \frac{1}{2}$$

Probability Ex 31.5 Q10

Let E be event of occuring head in a toss of fair coin.

$$P(E) = \frac{1}{2}$$

$$P(\bar{E}) = \frac{1}{2}$$

A wins the game in first or 3rd or 5th throw, ...

Probability that A wins in first throw

$$= P(E) = \frac{1}{2}$$

Probability that A wins in 3rd throw

$$= P(\bar{E})P(\bar{E})P(E)$$

$$= \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^3$$

Probability that A wins in 5th throw

$$= P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(E)$$

$$= \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^5$$

Hence,

Probability of winning A

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \dots$$

$$= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \dots\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\left(\frac{1}{2}\right)^2}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\frac{1}{4}}\right]$$

$$= \frac{1}{2} \times \frac{4}{3}$$

$$= \frac{2}{3}$$

Probability that B wins $= 1 - P(A \text{ wins})$

$$= 1 - \frac{2}{3}$$

$$= \frac{1}{3}$$

Required probability $= \frac{1}{3}$

Probability Ex 31.5 Q11

Two cards are drawn without replacement from a pack of 52 cards.
There are 26 black and 26 red cards

$$P\text{ (one red and other black card)}$$
$$= P\left[(R \cap B) \cup (B \cap R)\right]$$
$$= P(R \cap B) + P(B \cap R)$$
$$= P(R)P\left(\frac{B}{R}\right) + P(B)P\left(\frac{R}{B}\right)$$
$$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$$
$$= \frac{13}{51} + \frac{13}{51}$$
$$= \frac{26}{51}$$

Required probability $= \frac{26}{51}$

Probability Ex 31.5 Q12

Tickets are numbered from 1 to 10.
Two tickets are drawn.

Consider, A = Multiple of 5
B = Multiple of 4

$$P(A) = \frac{2}{10} \quad \text{[Since 5, 10 are multiple of 5]}$$
$$P(A) = \frac{1}{5}$$
$$P(B) = \frac{2}{10}$$
$$P(B) = \frac{1}{5} \quad \text{[Since 4, 8 are multiple of 4]}$$

$$P\text{ (One number multiple of 5 and other multiple of 4)}$$
$$= P\left[(A \cap B) \cup (B \cap A)\right]$$
$$= P(A \cap B) + P(B \cap A)$$
$$= P(A)P\left(\frac{B}{A}\right) + P(B)P\left(\frac{A}{B}\right)$$
$$= \frac{1}{5} \times \frac{2}{9} + \frac{1}{5} \times \frac{2}{9}$$
$$= \frac{4}{45}$$

Required probability $= \frac{4}{45}$

Probability Ex 31.5 Q13

Given, In a family Husband (H) tells a lie in 30% cases and Wife (W) tells a lie in 35%

$P(H) = 30\%,\ P(\overline{H}) = 70\%$

$P(W) = 35\%,\ P(\overline{W}) = 65\%$

P (Both contradict each other)

$= P\left[\left(H \cap \overline{W}\right) \cup \left(\overline{H} \cap W\right)\right]$

$= P\left(H \cap \overline{W}\right) + P\left(\overline{H} \cap W\right)$

$= P(H)P\left(\overline{W}\right) + P\left(\overline{H}\right)P(W)$

$= \frac{30}{100} \times \frac{65}{100} + \frac{70}{100} \times \frac{35}{100}$

$= \frac{1950 + 2450}{10000}$

$= \frac{4400}{10000}$

$= 0.44$

Required probability = 0.44

Probability Ex 31.5 Q14

Given, Probability of Husband's (H) selection $= \frac{1}{7}$

$P(H) = \frac{1}{7}$

Probability of Wife's (W) selection $= \frac{1}{5}$

$P(W) = \frac{1}{5}$

(a)

P(Both of them will be selected)

$= (H \cap W)$

$= P(H)P(W)$

$= \frac{1}{7} \times \frac{1}{5}$

$= \frac{1}{35}$

Required probability $= \frac{1}{35}$

(b)

P(Only one of them will be selected)

$= P\left[\left(H \cap \overline{W}\right) \cup \left(\overline{H} \cap W\right)\right]$

$= P\left(H \cap \overline{W}\right) + P\left(\overline{H} \cap W\right)$

$= P(H)P\left(\overline{W}\right) + P\left(\overline{H}\right)P(W)$

$= P(H)[1 - P(W)] + [1 - P(H)]P(W)$

$= \frac{1}{7}\left[1 - \frac{1}{5}\right] + \left[1 - \frac{1}{7}\right]\frac{1}{5}$

$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5}$

$= \frac{10}{35}$

$= \frac{2}{7}$

Required probability $= \frac{2}{7}$

(c)

$$P(\text{None of them selected})$$
$$= (\overline{H} \cap \overline{W})$$
$$= P(\overline{H})P(\overline{W})$$
$$= (1 - P(H))(1 - P(W))$$
$$= \left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{5}\right)$$
$$= \frac{6}{7} \times \frac{4}{5}$$
$$= \frac{24}{35}$$

Required probability $= \frac{24}{35}$

Probability Ex 31.5 Q15

A bag contains 7 white, 5 black and 4 red balls.
Four balls are drawn without replacement

$$P(\text{At least three balls are black})$$
$$= P(\text{3 black balls and one not black or 4 black balls})$$
$$= P(\text{3 black and one not black}) + P(\text{4 black balls})$$
$$= \frac{{}^5C_3 \times {}^{11}C_1}{{}^{16}C_4} + \frac{{}^5C_4}{{}^{16}C_4}$$
$$= \frac{\frac{5!}{3!2!} \times 11 + \frac{5!}{4!1!}}{\frac{16!}{4!12!}} \qquad \left[\text{Since } {}^nC_r = \frac{n!}{r!(n-r)!}\right]$$
$$= \frac{\frac{5.4}{2} \times 11 + 5}{\frac{16.15.14.13}{4.3.2}}$$
$$= \frac{(110 + 5)}{1820}$$
$$= \frac{115}{1820}$$
$$= \frac{23}{364}$$

Required probability $= \frac{23}{364}$

Probability Ex 31.5 Q16

Given,

A speaks truth 3 out of four times

B speaks truth 4 out of five times

C speads truth 5 out of six times.

$$\Rightarrow \quad P(A) = \frac{3}{4},\ P(B) = \frac{4}{5},\ P(C) = \frac{5}{6}$$

P (Reported truth fully by majority of witnesses)

$$= P\left(\left(A \cap B \cap \overline{C}\right) \cup \left(A \cap \overline{B} \cap C\right) \cup \left(\overline{A} \cap B \cap C\right) \cup \left(A \cap B \cap C\right)\right)$$

$$= P\left(A \cap B \cap \overline{C}\right) + P\left(A \cap \overline{B} \cap C\right) + P\left(\overline{A} \cap B \cap C\right) + P\left(A \cap B \cap C\right)$$

$$= P(A)P(B)P\left(\overline{C}\right) + P(A)P\left(\overline{B}\right)P(C) + P\left(\overline{A}\right)P(B)P(C) + P(A)P(B)P(C)$$

$$= P(A)P(B)(1 - P(C)) + P(A)(1 - P(B))P(C) + (1 - P(A))P(B)P(C) + P(A)P(B)P(C)$$

$$= \frac{3}{4} \times \frac{4}{5}\left(1 - \frac{5}{6}\right) + \frac{3}{4}\left(1 - \frac{4}{5}\right)\frac{5}{6} + \left(1 - \frac{3}{4}\right)\frac{4}{5}.\frac{5}{6} + \frac{3}{4}.\frac{4}{5}.\frac{5}{6}$$

$$= \frac{3}{4}.\frac{4}{5}.\frac{1}{6} + \frac{3}{4}.\frac{1}{5}.\frac{5}{6} + \frac{1}{4}.\frac{4}{5}.\frac{5}{6} + \frac{3}{4}.\frac{4}{5}.\frac{5}{6}$$

$$= \frac{1}{10} + \frac{1}{8} + \frac{1}{6} + \frac{1}{2}$$

$$= \frac{12 + 15 + 20 + 60}{120}$$

$$= \frac{107}{120}$$

Required probability $= \dfrac{107}{120}$

Probability Ex 31.5 Q17

Bag A has 4 white balls and 2 black balls;

Bag B has 3 white balls and 5 black balls.

(i) $P\left(A_W \text{ and } B_W\right) = P\left(A_W\right)P\left(B_W\right) = \frac{4}{6}.\frac{3}{8} = \frac{1}{4}$

(ii) $P\left(A_B \text{ and } B_B\right) = P\left(A_B\right)P\left(B_B\right) = \frac{2}{6}.\frac{5}{8} = \frac{5}{24}$

(iii) $P\left(A_W \text{ and } B_B \text{ or } A_B \text{ and } B_W\right) = P\left(A_W\right)P\left(B_B\right) + P\left(A_B\right)P\left(B_W\right)$

$$= \frac{4}{6}.\frac{5}{8} + \frac{2}{6}.\frac{3}{8}$$

$$= \frac{20}{48} + \frac{6}{48}$$

$$= \frac{26}{48} = \frac{13}{24}$$

Probability Ex 31.5 Q18

Number of white balls = 4

Number of black balls = 7

Number of red balls = 5

Total balls = 16

Number of ways in which 4 balls can be drawn from 16 balls = ${}^{16}C_4$

Let A = getting at least two white ball = getting 2, 3, 4 white balls

Number of ways of choosing 2 white balls = ${}^{4}C_2 \times {}^{12}C_2$

Number of ways of choosing 3 white balls = ${}^{4}C_3 \times {}^{12}C_1$

Number of ways of choosing 4 white balls = ${}^{4}C_4 \times {}^{12}C_0$

$$\therefore \quad P(A) = \frac{{}^{4}C_2 \times {}^{12}C_2 + {}^{4}C_3 \times {}^{12}C_1 + {}^{4}C_4 \times {}^{12}C_0}{{}^{16}C_4} = \frac{67}{256}$$

Probability Ex 31.5 Q19

Three cards are drawn with replacement from a pack of cards.
There are 4 Kings, 4 Queens, 5 Jacks.

$$P\left(1 \text{ King, } 1 \text{ Queen, } 1 \text{ Jack}\right)$$

$$= P\left((K \cap Q \cap J) \cup (K \cap J \cap Q) \cup (J \cap K \cap Q) \cup (J \cap Q \cap K) \cup (Q \cap K \cap L) \cup (Q \cap J \cap K)\right)$$
$$= P(K \cap Q \cap J) + P(K \cap J \cap Q) + P(J \cap K \cap Q) + P(J \cap Q \cap K) + P(Q \cap K \cap J) + P(Q \cap J \cap K)$$
$$= P(K)P(Q)P(J) + P(K)P(J)P(Q) + P(J)P(K)P(Q) + P(J)P(Q)P(K) + P(Q)P(K)P(J) + P(Q)P(J)P(K)$$
$$= \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} + \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} + \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} + \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} + \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} + \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52}$$
$$= \frac{6}{13.13.13}$$
$$= \frac{6}{2197}$$

Required probability $= \frac{6}{2197}$

Probability Ex 31.5 Q20

Given, Bag(1) contains 4 red and 5 black balls.
Bag (2) contains 3 red and 7 black balls

One ball is drawn at random from each bag.

(i)

$$P(\text{Balls are of different colours})$$
$$= P\left((R_1 \cap B_2) \cup (B_1 \cap R_2)\right)$$
$$= P(R_1 \cap B_2) + P(B_1 \cap R_2)$$
$$= P(R_1)P(B_2) + P(B_1)P(R_2)$$
$$= \frac{4}{9}\cdot\frac{7}{10} + \frac{5}{9}\cdot\frac{9}{10}$$
$$= \frac{28}{90} + \frac{15}{90}$$
$$= \frac{43}{93}$$

(ii)

$$P(\text{Balls are of the same colour})$$
$$= P\left((B_1 \cap B_2) \cup (R_1 \cap R_2)\right)$$
$$= P(B_1 \cap B_2) + P(R_1 \cap R_2)$$
$$= P(B_1)P(B_2) + P(R_1)P(R_2)$$
$$= \frac{5}{9}\cdot\frac{7}{10} + \frac{4}{9}\cdot\frac{3}{10}$$
$$= \frac{35}{90} + \frac{12}{90}$$
$$= \frac{47}{90}$$

Required probability $= \frac{47}{90}$

ility Ex 31.5 Q21

e the event that "A hits the target",
B be the event that "B hits the target" and
C be the event that "C hits the target".
Then A, B and C are independent events such that
$P(A) = \frac{3}{6} = \frac{1}{2}$; $P(B) = \frac{2}{6} = \frac{1}{3}$; $P(C) = \frac{4}{4} = 1$

The target is hit by at least 2 shots in the following mutually exclusive ways :
(i) A hits, B hits and C does not hit, i.e., $A \cap B \cap \bar{C}$
(ii) A hits, B does not hit and C hits, i.e., $A \cap \bar{B} \cap C$
(iii) A does not hit, B hits and C hits, i.e., $\bar{A} \cap B \cap C$
(iv) A hits, B hits and C hits, i.e., $A \cap B \cap C$
Hence, by the addition theorem for mutually exclusive events, the probability that at least 2 shots hit.

$= P(i) + P(ii) + P(iii) + P(iv)$
$= P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C) + P(A \cap B \cap C)$
$= P(A)\,P(B)\,P(\bar{C}) + P(A)\,P(\bar{B})\,P(C) + P(\bar{A})\,P(B)\,P(C) + P(A)\,P(B)\,P(C)$
$= P(A)\,P(B)\,[1 - P(C)] + P(A)\,[1 - P(B)]\,P(C) + [1 - P(A)]\,P(B)\,P(C) + P(A)\,P(B)\,P(C)$
$= \frac{1}{2} \times \frac{1}{3} \times (1 - 1) + \frac{1}{2} \times \left(1 - \frac{1}{3}\right) \times 1 + \left(1 - \frac{1}{2}\right) \times \frac{1}{3} \times 1 + \frac{1}{2} \times \frac{1}{3} \times 1$
$= \frac{1}{2} \times \frac{1}{3} \times 0 + \frac{1}{2} \times \left(\frac{2}{3}\right) \times 1 + \left(\frac{1}{2}\right) \times \frac{1}{3} \times 1 + \frac{1}{2} \times \frac{1}{3} \times 1$
$= 0 + \frac{2}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6}$
$= \frac{2}{3}$

Probability Ex 31.5 Q22

Given,

The probability of A passing exam $= \frac{2}{9}$

The probability of B passing exam $= \frac{5}{9}$

And they are independent.

$\Rightarrow \quad P(A) = \frac{2}{9}, \; P(B) = \frac{5}{9}$

(i)

$P(\text{Only } A \text{ passing the exam})$
$= P\left(A \cap \bar{B}\right)$
$= P(A).P(\bar{B})$
$= P(A).\left(1 - P(B)\right)$
$= \frac{2}{9}\left(1 - \frac{5}{9}\right)$
$= \frac{2}{9}\left(\frac{4}{9}\right)$

$= \frac{8}{81}$

(ii)

$P(\text{Only one of them passing exam})$
$= P\left(\left(A \cap \bar{B}\right) \cup \left(\bar{A} \cap B\right)\right)$
$= P\left(A \cap \bar{B}\right) + P\left(\bar{A} \cap B\right)$
$= P(A)\,P\left(\bar{B}\right) + P\left(\bar{A}\right)P(B)$
$= P(A)\left(1 - P(B)\right) + \left(1 - P(A)\right)P(B)$
$= \frac{2}{9}\left(1 - \frac{5}{9}\right) + \left(1 - \frac{2}{9}\right)\frac{5}{9}$
$= \frac{2}{9} \cdot \frac{4}{9} + \frac{7}{9} \cdot \frac{5}{9}$
$= \frac{8}{81} + \frac{35}{81}$
$= \frac{43}{81}$

Required probability $= \frac{43}{81}$

bility Ex 31.5 Q23

Urn A contains 4 red (R_1) and 3 black (B_1) balls
Urn B contains 5 red (R_2) and 4 black (B_2) balls
Urn C contains 4 red (R_3) and 4 black (B_3) balls.

P (3 balls drawn consists of 2 red and a black ball)

$= P\left[(R_1 \cap R_2 \cap R_3) \cup (R_1 \cap B_2 \cap R_3) \cup (B_1 \cap R_2 \cap R_3)\right]$

$= P(R_1 \cap R_2 \cap R_3) + P(R_1 \cap B_2 \cap R_3) + P(B_1 \cap R_2 \cap R_3)$

$= P(R_1) + P(R_2) + P(R_3) + P(R_1) + P(B_2) + P(R_3) + P(B_1) + P(R_2) + P(R_3)$

$= \frac{4}{7} \cdot \frac{5}{9} \cdot \frac{4}{8} + \frac{4}{7} \cdot \frac{4}{9} \cdot \frac{4}{8} + \frac{3}{7} \cdot \frac{5}{9} \cdot \frac{4}{8}$

$= \frac{80 + 64 + 60}{504}$

$= \frac{204}{504}$

$= \frac{17}{42}$

Required probability $= \frac{17}{42}$

Probability Ex 31.5 Q24

Given,

Probability of getting A grade in mathematics $(m) = 0.2$

$\Rightarrow \quad P(m) = 0.2$

Probability of getting A grade in physics $(p) = 0.3$

$\Rightarrow \quad P(p) = 0.3$

Probability of getting A grade in chemistry $(p) = 0.5$

$\Rightarrow \quad P(c) = 0.5$

(i)

P (Getting A grade in all subjects)

$= P(m \cap p \cap c)$

$= P(m) + P(p) + P(c)$

$= 0.2 \times 0.3 \times 0.5$

$= 0.03$

Required probability $= 0.03$

(ii)

P (Getting A in no subject)

$= P(\overline{m} \cap \overline{p} \cap \overline{c})$

$= P(\overline{m}) + P(\overline{p}) + P(\overline{c})$

$= (1 - P(m))(1 - P(p))(1 - P(c))$

$= (1 - 0.2)(1 - 0.3)(1 - 0.5)$

$= (0.8)(0.7)(0.5)$

$= 0.28$

Required probability $= 0.28$

(iii)

P (Getting A grade in two subjects)

$= P\left((m \cap p \cap \overline{c}) \cup (m \cap \overline{p} \cap c) \cup (\overline{m} \cap p \cap c)\right)$

$= P(m) P(p) P(\overline{c}) + P(m) P(\overline{p}) P(c) + P(\overline{m}) P(p) P(c)$

$= P(m) P(p)(1 - P(c)) + P(m)(1 - P(p)) P(c) + (1 - P(m)) P(p) P(c)$

$= (0.2)(0.3)(1 - 0.5) + (0.2)(1 - 0.3)(0.5) + (1 - 0.2)(0.3)(0.5)$

Probability Ex 31.5 Q25

Sum of 9 can be obtained by

$$E = \{(3,6),(4,5),(5,4),(6,3)\}$$

Probability of throwing 9 = $\frac{4}{36}$

$$P(E) = \frac{1}{9},\ P(\bar{E}) = \frac{8}{9}$$

$$\Rightarrow \quad P(A) = P(B) = \frac{1}{9}$$

$$\Rightarrow \quad P(\bar{A}) = P(\bar{B}) = \frac{8}{9}$$

A and B take turns in throwing two dice.
Let A starts the game.

$$P(A \text{ wins the game})$$
$$= P\left(A \cup \bar{A} \cap \bar{B} \cap A \cup \bar{A} \cap \bar{B} \cap \bar{A} \cap \bar{B} \cap A \cup \ldots\right)$$
$$= P(A) + P\left(\bar{A} \cap \bar{B} \cap A\right) + P\left(\bar{A} \cap \bar{B} \cap \bar{A} \cap \bar{B} \cap A\right) + \ldots$$
$$= P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \ldots$$
$$= \frac{1}{9} + \frac{8}{9}\cdot\frac{8}{9}\cdot\frac{1}{9} + \frac{8}{9}\cdot\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{1}{9} + \ldots$$
$$= \frac{1}{9}\left[1 + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^4 + \ldots\right]$$
$$= \frac{1}{9}\left[\frac{1}{1 - \left(\frac{8}{9}\right)^2}\right] \qquad \left[\text{Since for a G.P. with first term 9 and common ratio r, } S_\infty = \frac{a}{1-r}\right]$$
$$= \frac{1}{9}\left[\frac{1}{1 - \frac{64}{81}}\right]$$
$$= \frac{1}{9}\left[\frac{81}{81 - 64}\right]$$
$$= \frac{9}{17}$$

$$P(B \text{ wins the game}) = 1 - P(A \text{ wins the game})$$
$$= 1 - \frac{9}{17}$$
$$= \frac{8}{17}$$

Chances of winning of $A : B$

$$= \frac{9}{17} : \frac{8}{17}$$
$$= 9 : 8$$

Chances of winning $A : B = 9 : 8$

F be event of getting a head.

$P(E) = \frac{1}{2} \quad \Rightarrow P(\bar{E}) = \frac{1}{2}$

If A starts the game,

$\Rightarrow$ A wins the game in 1^{st}, 4^{th}, 7^{th},... toss of coin.

$$\begin{aligned}
&P(A \text{ wins})\\
&= P(E \cup \bar{E} \cap \bar{E} \cap \bar{E} \cap E \cup \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap E \cup \ldots)\\
&= P(\bar{E}) + P(\bar{E} \cap \bar{E} \cap \bar{E} \cap E) + P(\bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap E) + \ldots\\
&= P(E) + P(\bar{E})P(\bar{E})P(\bar{E})P(E) + P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(E) + \ldots\\
&= \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \ldots\\
&= \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^7 + \ldots\\
&= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 + \ldots\right]\\
&= \frac{1}{2}\left[\frac{1}{1-\left(\frac{1}{2}\right)^3}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]\\
&= \frac{1}{2}\left[\frac{1}{1-\frac{1}{8}}\right]\\
&= \frac{1}{2}\left[\frac{8}{7}\right]\\
&= \frac{4}{7}
\end{aligned}$$

B wins in 2^{nd}, 5^{th}, 8^{th},... toss of coin

$$\begin{aligned}
&P(B \text{ wins})\\
&= P(\bar{E} \cap E \cup \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap E \cup \ldots)\\
&= P(\bar{E} \cap E) + P(\bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap E \cup) + \ldots\\
&= P(\bar{E})P(E) + P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(E) + \ldots\\
&= \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \ldots\\
&= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^5 + \ldots\\
&= \left(\frac{1}{2}\right)^2\left[1 + \left(\frac{1}{2}\right)^3 + \ldots\right]\\
&= \frac{1}{4}\left[\frac{1}{1-\left(\frac{1}{2}\right)^3}\right] \qquad \left[\begin{array}{l}\text{Since for G.P.}\\ S_\infty = \frac{a}{1-r}\end{array}\right]\\
&= \frac{1}{4}\left[\frac{1}{1-\frac{1}{8}}\right]\\
&= \frac{1}{4}\left[\frac{8}{7}\right]\\
&= \frac{2}{7}
\end{aligned}$$

$$\begin{aligned}
&P(C \text{ wins}) = 1 - P(A \text{ wins}) - P(B \text{ wins})\\
&= 1 - \frac{4}{7} - \frac{2}{7}\\
&= \frac{1}{7}
\end{aligned}$$

Probabilities of winning A, B and C are $\frac{4}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively.

Probability Ex 31.5 Q27

Let E be the even of getting a six

$$P(E) = \frac{1}{6}$$

$$P(\bar{E}) = \frac{5}{6}$$

A wins if he gets a six in 1st or 4th or 7th... throw

A wins in first throw $= P(E) = \frac{1}{6}$

A wins in 4th throw if he fails in 1^{st}, B fails in 2^{nd}, C fails in 3^{rd} throw.

Probability of winning A in 4th throw

$$= P(\bar{E})P(\bar{E})P(\bar{E})P(E)$$

$$= \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6}$$

Similarly, Probability of winning A in 7th thrwo $= P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(E)$

$$= \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6}$$

Hence, probability of winning of A

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \dots$$

$$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \dots\right]$$

$$= \frac{1}{6}\left[\frac{1}{1 - \left(\frac{5}{6}\right)^3}\right] \qquad \left[\text{Using } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

$$= \frac{1}{6}\left[\frac{1}{1 - \frac{125}{216}}\right]$$

$$= \frac{1}{6} \times \frac{216}{91}$$

$$= \frac{36}{91}$$

B wins if he gets a six in 2nd or 5th or 8th ...throw.

B wins in 2nd throw $= P\left(\bar{E}\right)P\left(E\right)$

$= \left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$

B wins in 5th throw if A fails in first, B fails in 2nd, C fails in 3rd, A fails in 4th.

Probability of winning B in 5th throw

$= P\left(\bar{E}\right)P\left(\bar{E}\right)P\left(\bar{E}\right)P\left(\bar{E}\right)P\left(E\right)$

$= \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$

Probability of winning B in 8th thrwo

$= \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$

Hence, probability of winning B

$= \left(\frac{5}{6}\right)\frac{1}{6} + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$

$= \frac{5}{6}.\frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \ldots\right]$

$= \frac{5}{36}\left[\frac{1}{1-\left(\frac{5}{6}\right)^3}\right]$ $\left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$

$= \frac{5}{36}\left[\frac{1}{1-\frac{125}{216}}\right]$

$= \frac{5}{36} \times \left[\frac{216}{91}\right]$

$= \frac{30}{91}$

Probability of winning $C = 1 - P\left(A \text{ wins}\right) - P\left(B \text{ wins}\right)$

$= 1 - \frac{36}{91} - \frac{30}{91}$

$= \frac{25}{91}$

The respective probabilities of winning of A, B and C are $\frac{36}{91}$, $\frac{30}{91}$ and $\frac{25}{91}$.

Probability Ex 31.5 Q28

Let E be events of throwing 10 on a pair of dice,

$E = \{(4,6),(5,5),(6,4)\}$

$P(E) = \frac{3}{37}$

$P(E) = \frac{1}{12}$

$P(\bar{E}) = \frac{11}{12}$

A wins the game in first or 3rd or 5th throw, ...

Probability that A wins in first throw $= P(E) = \frac{1}{12}$

Probability that A wins in 3rd throw

$= P(\bar{E})P(\bar{E})P(E)$

$= \left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right)$

Probability that A wins in 5th throw

$= P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(E)$

$= \left(\frac{11}{12}\right)^4\left(\frac{1}{12}\right)$

Hence,

Probability of winning A

$= \frac{1}{12} + \left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right) + \left(\frac{11}{12}\right)^4\left(\frac{1}{12}\right)$

$= \frac{1}{12}\left[1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \dots\right]$

$= \frac{1}{12}\left[\frac{1}{1 - \left(\frac{11}{12}\right)^2}\right]$ $\left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$

Probability Ex 31.5 Q29

Bag A has 3 red and 5 black balls

Bag B has 2 red and 3 black balls

One ball is drawn from bag A and two from bag B.

P(One red from bag A and 2 black from bag B Or one black from bag A and 1 red and one black from bag B)

$= P(R_1 \cap (2B_2)) + P(B_1 \cap R_2 \cap B_2)$

$= P(R_1)P(2B_2) + P(B_1)P(R_2)P(B_2)$

$= \frac{3}{8}\cdot\frac{{}^3C_2}{{}^5C_2} + \frac{5}{8}\cdot\frac{2}{5}\cdot\frac{3}{4}$

$= \frac{3}{8}\cdot\frac{3}{\left(\frac{5.4}{2}\right)} + \frac{5}{8}\cdot\frac{2}{5}\cdot\frac{3}{4}$

$= \frac{18}{160} + \frac{30}{160}$

$= \frac{48}{160} = \frac{3}{10}$

Required probability $= \frac{3}{10}$

Probability Ex 31.5 Q30

Given,

Probability of Fatima's (F) selection = $\frac{1}{7}$

$P(F) = \frac{1}{7} \quad \Rightarrow P(\bar{F}) = \frac{6}{7}$

Probability of John's (J) selection = $\frac{1}{5}$

$P(J) = \frac{1}{5} \quad \Rightarrow P(\bar{J}) = \frac{4}{5}$

(i)

P (Both of them selected)

$= P(F \cap J)$

$= P(F).P(J)$

$= \frac{1}{7} \times \frac{1}{5}$

$= \frac{1}{35}$

Required probability = $\frac{1}{35}$

(ii)

P (only one of them selected)

$= P((F \cap \bar{J}) \cup (\bar{F} \cap J))$

$= P(F)P(\bar{J}) + P(\bar{F})P(J)$

$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5}$

$= \frac{4+6}{35}$

$= \frac{10}{35}$

$= \frac{2}{7}$

Required probability = $\frac{2}{7}$

(iii)

P (None of them selected)

$= P(\bar{F} \cap \bar{J})$

$= P(\bar{F})P(\bar{J})$

$= \frac{6}{7} \times \frac{4}{5}$

$= \frac{24}{35}$

Required probability = $\frac{24}{35}$

Probability Ex 31.5 Q31

Bag contains 3 blue, 5 red marble. One marble is drawn, its colour noted and replaced, then again a marble drawn and its colour is noted.

(i)

P (Blue followed by red)

$= P(B \cap R)$

$= P(B)\,P(R)$

$= \frac{3}{8} \times \frac{5}{8}$

$= \frac{15}{64}$

Required probability $= \frac{15}{64}$

(ii)

P (Blue and red in any order)

$= P((B \cap R) \cup (R \cap B))$

$= P(B \cap R) + P(R \cap B)$

$= P(B)P(R) + P(R)P(B)$

$= \frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{3}{8}$

$= \frac{30}{64}$

$= \frac{15}{32}$

Required probability $= \frac{15}{32}$

(iii)

P (of the same colour)

$= P((R_1 \cap R_2) \cup (B_1 \cap B_2))$

$= P(R_1)P(R_2) + P(B_1)P(B_2)$

$= \frac{5}{8} \times \frac{5}{8} + \frac{3}{8} \times \frac{3}{8}$

$= \frac{25+9}{64}$

$= \frac{34}{64}$

$= \frac{17}{32}$

Probability Ex 31.5 Q32

An urn contains 7 red and 4 blue balls.
Two balls are drawn with replacement.

(i)

$$P\,(\text{Getting 2 red balls})$$
$$= P\,(R_1 \cap R_2)$$
$$= P\,(R_1).P\,(R_2)$$
$$= \frac{7}{11} \times \frac{7}{11}$$
$$= \frac{49}{121}$$

Required probability = $\frac{49}{121}$

(ii)

$$P\,(\text{Getting 2 blue balls})$$
$$= P\,(B_1 \cap B_2)$$
$$= P\,(B_1).P\,(B_2)$$
$$= \frac{4}{11} \times \frac{4}{11}$$
$$= \frac{16}{121}$$

Required probability = $\frac{16}{121}$

(iii)

$$P\,(\text{Getting one red and one blue ball})$$
$$= P\,((R \cap B) \cup (B \cap R))$$
$$= P\,(R)\,P\,(B) + P\,(B)\,P\,(R)$$
$$= \frac{7}{11} \times \frac{4}{11} + \frac{4}{11} \times \frac{7}{11}$$
$$= \frac{28+28}{121}$$
$$= \frac{56}{121}$$

Required probability = $\frac{56}{121}$

ty Ex 31.5 Q33

l is drawn, out come noted, the card is replaced, pack reshuffled, another card wn.

(i)

We know that, there are four suits club (C), spade (S), heart (H) diamand (D), each contains 13 cards.

$$P(\text{Both the cards are of same suit})$$
$$= P\left((C_1 \cap C_2) \cup (S_1 \cap S_2) \cup (H_1 \cap H_2) \cup (D_1 \cap D_2)\right)$$
$$= P(C_1 \cap C_2) + P(S_1 \cap S_2) + P(H_1 \cap H_2) + P(D_1 \cap D_2)$$
$$= P(C_1)P(C_2) + P(S_1)P(S_2) + P(H_1)P(H_2) + P(D_1)P(D_2)$$
$$= \frac{13}{52}\cdot\frac{13}{52} + \frac{13}{52}\cdot\frac{13}{52} + \frac{13}{52}\cdot\frac{13}{52} + \frac{13}{52}\cdot\frac{13}{52}$$
$$= \left(\frac{1}{4}\cdot\frac{1}{4}\right)^4$$
$$= \frac{1}{4}$$

Required probability $= \frac{1}{4}$

(ii)

We know that, there are four ace and 2 red queens.

$$P(\text{first card an ace and second card a red queen})$$
$$= P(\text{Getting an ace})\, P(\text{Getting a red queen})$$
$$= \frac{4}{52} \times \frac{2}{52}$$
$$= \frac{1}{338}$$

Required probability $= \frac{1}{338}$

Probability Ex 31.5 Q34

(i)

Out of 100 students two friends can enter the sections in ${}^{100}C_2$ ways.

Let $A =$ event both enter in section A(40 students)

$B =$ event both enter in section B(60 students)

$$P(A) = \frac{{}^{40}C_2}{{}^{100}C_2},\quad P(B) = \frac{{}^{60}C_2}{{}^{100}C_2}$$

$$P(A \cup B) = P(A) + P(B)$$
$$= \frac{{}^{40}C_2 + {}^{60}C_2}{{}^{100}C_2}$$
$$= \frac{\frac{40\times 39}{2} + \frac{60\times 59}{2}}{\frac{100\times 99}{2}}$$
$$= \frac{780 + 1770}{4950}$$
$$= \frac{2550}{4950}$$
$$= \frac{17}{33}$$

$$P(\text{Both enter same section}) = \frac{17}{33}$$

(ii)

$$P(\text{Both enter different section})$$
$$= 1 - P(\text{Both enter same section})$$
$$= 1 - \frac{17}{33}$$
$$= \frac{16}{33}$$

$$P(\text{Both enter different section}) = \frac{16}{32}$$

...bility Ex 31.5 Q35

...ility of getting six in any toss of a dice $= \frac{1}{6}$

Probability of not getting six in any toss of a dice $= \frac{5}{6}$

A and B toss the die alternatively.
Hence probability of A's win

$$= P(A) + P(\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}\bar{A}\bar{B}A) + \ldots\ldots.$$

$$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots..$$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + \ldots..$$

$$= \frac{1/6}{1 - (5/6)^2} = \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

Similarly, probability of B's win

$$= P(\bar{A}B) + P(\bar{A}\bar{B}\bar{A}B) + P(\bar{A}\bar{B}\bar{A}\bar{B}\bar{A}B) + \ldots\ldots$$

$$= \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots..$$

$$= \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{5}{6} \times \frac{1}{6} + \ldots..$$

$$= \frac{\frac{5}{6} \times \frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11}$$

Since the probabilities are not equal,
the decision of the refree was not a fair one.

Ex 31.6

Probability Ex 31.6 Q1

Given,

Bag A contains 5 white and 6 black balls

Bag B contains 4 white and 3 black balls.

There are two ways of transferring a ball from bag A to bag B

I- By transferring one white ball from bag A to bag B then drawing one black ball from bag B.

II- By transferring one black ball from bag A to bag B, then drawing one black from bag B.

Let, E_1, E_2 and A be events as below:-

E_1 = One white ball drawn from bag A

E_2 = One black ball drawn from bag B

A = One black ball drawn from bag B

$P(E_1) = \frac{5}{11}$

$P(E_2) = \frac{6}{11}$

$P(A \mid E_1) = \frac{3}{8}$ [Since, E_1 has increased one white ball in bag B]

$P\left(\frac{A}{E_2}\right) = \frac{4}{8}$ [Since, E_2 has increased one black ball in bag B]

By the law of total probability.

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{5}{11} \times \frac{3}{8} + \frac{6}{11} \times \frac{4}{8}$$

$$= \frac{15}{88} + \frac{24}{88}$$

$$= \frac{39}{88}$$

Required probability $= \frac{39}{88}$.

lity Ex 31.6 Q2

Contains 2 silver and 4 copper coins
) Contains 4 silver and 3 copper coins
One coin is drawn from one of the two purse and it is silver
Let, E_1, E_2 and A are defined as

E_1 = Selecting purse I
E_2 = Selecting purse II
A = Drawing a silver coin

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$ [Since, there are only 2 purses]

$P(A \mid E_1) = P(\text{A} \mid \text{silver coin from purse I})$

$= \frac{2}{6}$

$= \frac{1}{3}$

$P\left(\frac{A}{E_2}\right) = P(\text{A} \mid \text{silver coin from purse II})$

$= \frac{4}{7}$

By the law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{4}{7}$$

$$= \frac{1}{6} + \frac{4}{14}$$

$$= \frac{7+12}{42}$$

$$= \frac{19}{42}$$

Required probability $= \frac{19}{42}$.

Probability Ex 31.6 Q3

Bag I contains 4 yellow and 5 red balls
Bag II contains 6 yellow and 3 red balls
Transfer can be done in two ways:-
I- A yellow ball is transferred from bag I to bag II and then one yellow ball is drawn from bag II.
II-A red ball is transferred from bag I to bag II and then one yellow ball is drawn from bag II.
Let E_1, E_2 and A be events as:

E_1 = One yellow ball drawn from bag I
E_2 = One red ball drawn from bag I
A = One yellow ball drawn from bag II.

$P(E_1) = \frac{4}{9}$

$P(E_2) = \frac{5}{9}$

$P(A \mid E_1) = \frac{7}{10}$ [Since E_1 has increased one yellow ball in bag II]

$P\left(\frac{A}{E_2}\right) = \frac{6}{10}$ [Since E_2 has increased one red ball in bag II]

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{4}{9} \times \frac{7}{10} + \frac{5}{9} \times \frac{6}{10}$$

$$= \frac{28+30}{90}$$

$$= \frac{58}{90}$$

$$= \frac{29}{45}$$

Required probability $= \frac{29}{45}$.

Probability Ex 31.6 Q4

Bag I contains 3 white and 2 black balls
Bag II contains 2 white and 4 black balls
One bag is chosen at random, then one ball is drawn and it is white.
Let E_1, E_2 and A be events as:

E_1 = Selecting bag I
E_2 = Selecting bag II
A = Drawing one white ball

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2} \qquad \text{[Since there are only 2 bags]}$$

$$P(A \mid E_1) = P[\text{Drawing a white ball from bag I}] = \frac{3}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing a white ball from bag II}] = \frac{2}{6}$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{2}{6}$$

$$= \frac{3}{10} + \frac{2}{12}$$

$$= \frac{18+10}{60}$$

$$= \frac{28}{60}$$

$$= \frac{7}{15}$$

Required probability = $\frac{7}{15}$.

Probability Ex 31.6 Q5

Given,
Bag I contains 1 white, 2 black and 3 red balls
Bag II contains 2 white, 1 black and 1 red balls
Bag III contains 4 white, 5 black and 3 red balls.
A bag is chosen at random, then one red and one white ball is drawn.
Let E_1, E_2, E_3 and A be events as:

E_1 = Selecting bag I
E_2 = Selecting bag II
E_3 = Selecting bag III
A = Drawing one red and one white ball

$P(E_1) = \frac{1}{3}$

$P(E_2) = \frac{1}{3}$

$P(E_3) = \frac{1}{3}$ [Since there are only three bags]

$P(A \mid E_1) = P$[Drawing one red and one white ball from bag I]

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1 \times 3}{\frac{6 \times 5}{2}} = \frac{1}{5}$$

$P\left(\frac{A}{E_2}\right) = P$[Drawing one red and one white ball from bag II]

$$= \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2 \times 1}{\frac{4 \times 3}{2}} = \frac{1}{3}$$

$p\left(\frac{A}{E_3}\right) = P$[Drawing one red and one white ball from bag III]

$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{4 \times 3}{\frac{12 \times 11}{2}} = \frac{2}{11}$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$$

$$= \frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{11}$$

$$= \frac{1}{15} + \frac{1}{9} + \frac{2}{33}$$

$$= \frac{33 + 55 + 30}{495}$$

$$= \frac{118}{495}$$

Required probability $= \frac{118}{495}$.

Probability Ex 31.6 Q6

An unbiased coin is tossed, then
I:- If head occurs, pair of dice is rolled and sum on them is either 7 or 8.
II:- If tail occurs, a card is drawn from cards numbered 2,3,...,12 and is 7 or 8.

Let E_1, E_2, A be events as

E_1 = Head occurs on the coin
E_2 = Tail occurs on the coin
A = Noted number is 7 or 8

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2}$$

$P(A \mid E_1) = P$[Pair of dice shows 7 or 8 as sum]

[Sum on dice is 7 or 8 when $(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(2,6),(3,5),(4,4),(5,3),(6,2)$]

$$P(A \mid E_1) = \frac{11}{36}$$

$P\left(\frac{A}{E_2}\right) = P$[7 or 8 on card drawn from 11 cards numbered $2, 3, 4, \dots, 12$]

$$= \frac{2}{11}$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{11}$$

$$= \frac{11}{72} + \frac{2}{22}$$

$$= \frac{121 + 72}{792}$$

$$= \frac{193}{792}$$

Required probability $= \frac{193}{792}$.

Probability Ex 31.6 Q7

Let E_1, E_2, A be defined as,

E_1 = Item produced by machine A
E_2 = Item produced by machine B
A = The item drawn is defective

$$P(E_1) = 60\% = \frac{60}{100}$$

$$P(E_2) = 40\% = \frac{40}{100}$$

$P(A \mid E_1) = P$[Defective item from machine A]

$$= 2\% = \frac{2}{100}$$

$P\left(\frac{A}{E_2}\right) = P$[Defective item from machine B]

$$= 1\% = \frac{1}{100}$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{60}{100} \times \frac{2}{100} + \frac{40}{100} \times \frac{1}{100}$$

$$= \frac{120 + 40}{10000}$$

$$= \frac{160}{10000}$$

$$= 0.016$$

Required probability = 0.016.

lity Ex 31.6 Q8

ntains 8 white and 7 black balls

ntains 5 white and 4 black balls

Transfer can be done in two ways:-

I-A white ball is transferred from bag A to bag B and then one white ball is drawn from bag B.

II-A black ball is transferred from bag A to bag B, then one white ball is drawn from bag B.

Let E_1, E_2 and A be events as:-

E_1 = One white ball from bag A

E_2 = One black ball from bag A

A = One white ball from bag B

$$P(E_1) = \frac{8}{15}$$

$$P(E_2) = \frac{7}{15}$$

$$P(A \mid E_1) = \frac{6}{10} \qquad \left[\text{Since } E_1 \text{ has increased white balls in bag } B\right]$$

$$P\left(\frac{A}{E_2}\right) = \frac{5}{10} \qquad \left[\text{Since } E_2 \text{ has increased black ball in bag } B\right]$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{8}{15} \times \frac{6}{10} + \frac{7}{15} \times \frac{5}{10}$$

$$= \frac{48}{150} + \frac{35}{150}$$

$$= \frac{83}{150}$$

Required probability = $\frac{83}{150}$.

Probability Ex 31.6 Q9

There are two bags.

Bag (1) contain 4 white and 5 black balls

Bag (2) contain 3 white and 4 black balls.

A ball is taken from bag (i) and without seeing its colour is put in second bag. Then a ball is drawn from bag 2 and is white in colour.

$$P(\text{White ball from bag 1}) = \frac{4}{9}$$

$$P(W_1) = \frac{4}{9}$$

$$P(\text{Black ball from bag 1}) = \frac{5}{9}$$

$$P(B_1) = \frac{5}{9}$$

P (White ball from bag 2 given B_1 transfer)

$$P\left(\frac{W_2}{B_1}\right) = \frac{3}{8}$$

P (White from bag 2 given W_1 transfer)

$$P\left(\frac{W_2}{W_1}\right) = \frac{4}{8}$$

$$= \frac{1}{2}$$

P (White from bag 2)

$$= P(B_1)P\left(\frac{W_2}{B_1}\right) + P(W_1)P\left(\frac{W_2}{W_1}\right)$$

$$= \frac{5}{9} \times \frac{3}{8} + \frac{4}{9} \times \frac{1}{2}$$

$$= \frac{15}{72} + \frac{4}{18}$$

$$= \frac{31}{72}$$

Required probability $= \frac{31}{72}$

Probability Ex 31.6 Q10

There are two bags.

Bag (1) contain 4 white and 5 black balls

Bag (2) contain 6 white and 7 black balls.

A ball is taken from bag (1) and without seeing its colour is put in bag (2). Then a ball is drawn from bag (2) and is found white in colour.

$$P\text{(1 white ball from bag 1)} = \frac{4}{9}$$

$$P(W_1) = \frac{4}{9}$$

$$P\text{(1 black ball from bag 1)} = \frac{5}{9}$$

$$P(B_1) = \frac{5}{9}$$

P (1 white ball from bag 2 given W_1 is put in bag 2)

$$P\left(\frac{W_2}{W_1}\right) = \frac{7}{14}$$

$$P\left(\frac{W_2}{W_1}\right) = \frac{1}{2}$$

P (1 white ball from bag 2 given B_1 is put in bag 2)

$$P\left(\frac{W_2}{B_1}\right) = \frac{6}{14}$$

P (1 white from bag 2)

$$= P(W_1)P\left(\frac{W_2}{W_1}\right) + P(B_1)P\left(\frac{W_2}{B_1}\right)$$

$$= \frac{4}{9}\times\frac{1}{2} + \frac{5}{9}\times\frac{6}{14}$$

$$= \frac{4}{18} + \frac{30}{126}$$

$$= \frac{58}{126}$$

$$= \frac{29}{63}$$

$$\text{Required probability } = \frac{29}{63}$$

Probability Ex 31.6 Q11

Urn '1'	Urn '2'
10W 3B	3W 5B

Let $U1_{2W}$, $U1_{1W1B}$, $U1_{2B}$ be the events of transferring 2 white balls, 1 white & 1black ball, 2 black balls from first Urn1 to second Urn2.

$P(U1_{2W}) = {}^{10}C_2/{}^{13}C_2 = 45/78$

$P(U1_{1W1B}) = {}^{10}C_1{}^3C_1/{}^{13}C_2 = 10x3/78$

$P(U1_{2B}) = {}^3C_2/{}^{13}C_2 = 3/78$

Let $U2_W$ be the event that a white ball is drawn from the Urn 2. There are three scenarios for Urn 2 based on the events $U1_{2W}$ $U1_{1W1B}$ $U1_{2B}$

	5W	4W	3W
	5B	6B	7B
Total	10	10	10

$P(U1_{2W}U2_W) = \frac{{}^5C_1}{{}^{10}C_1} = 1/2$

$P(U1_{1W1B}U2_W) = \frac{{}^4C_1}{{}^{10}C_1} = 2/5$

$P(U1_{2B}U2_W) = \frac{{}^3C_1}{{}^{10}C_1} = 3/10$

$P(U2_W) = P(U1_{2W}U2_W) + P(U1_{1W1B}U2_W) + P(U1_{2B}U2_W)$

$= P(U1_{2W}) x P(U1_{2W}U2_W) + P(U1_{1W1B}) x P(U1_{1W1B}U2_W) +$

$P(U1_{2B}) x P(U1_{2B}U2_W)$

$= \frac{45}{78} \times \frac{1}{2} + \frac{30}{78} \times \frac{2}{5} + \frac{3}{78} \times \frac{3}{10} = \frac{114}{780} = \frac{59}{130}$

Probability Ex 31.6 Q12

Given,

Bag (1) contains 6 red (R_1) and 8 black (B_1) balls

Bag (2) contains 8 red (R_2) and 6 black (B_2) balls

A ball is drawn from the first bag and without noticing its colour is put in the bag (2).
Then a ball is drawn from second bag and it is red.

$$P(\text{One red ball from bag 2})$$
$$= P((B_1 \cap R_2) \cup (R_1 \cap R_2))$$
$$= P(B_1 \cap R_2) + P(R_1 \cap R_2)$$
$$= P(B_1) P\left(\frac{R_2}{B_1}\right) + P(R_1) P\left(\frac{R_2}{R_1}\right)$$
$$= \frac{8}{14} \cdot \frac{8}{15} + \frac{6}{14} \cdot \frac{9}{15}$$
$$= \frac{64 + 54}{210}$$
$$= \frac{118}{210}$$
$$= \frac{59}{105}$$

Required probability $= \frac{59}{105}$

Probability Ex 31.6 Q13

Let D be the event that the picked up tube is defective.
Let A_1, A_2 and A_3 be the events that the tube is produced on machines E_1, E_2 and E_3 respectively.

$$P(D) = P(A_1)P(D \mid A_1) + P(A_2)P(D \mid A_2) + P(A_3)P(D \mid A_3) \ldots\ldots(i)$$

$$P(A_1) = \frac{50}{100} = \frac{1}{2}, P(A_2) = \frac{25}{100} = \frac{1}{4}, P(A_3) = \frac{25}{100} = \frac{1}{4}$$

$$P(D \mid A_1) = P(D \mid A_2) = \frac{4}{100} = \frac{1}{25}$$

$$P(D \mid A_3) = \frac{5}{100} = \frac{1}{20}$$

Putting these values in (i), we get

$$P(D) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$

$$P(D) = \frac{17}{400}$$

Ex 31.7

Probability Ex 31.7 Q1

Urn I contains 1 white, 2 balck and 3 red balls
Urn II contains 2 white, 1 black and 1 red balls
Urn III contains 4 white, 5 black and 3 red balls.
Consider E_1, E_2, E_2 and A be events as:-

E_1 = Selecting unr I
E_2 = Selecting urn II
E_3 = Selecting urn III
A = Drawing 1 white and 1 red balls

$$P(E_1) = \frac{1}{3}$$
$$P(E_2) = \frac{1}{3}$$
$$P(E_3) = \frac{1}{3} \qquad \text{[Since there are 3 urns]}$$

$$P(A \mid E_1) = P[\text{Drawing 1 red and 1 white from urn I}]$$
$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$
$$= \frac{1\times 3}{\frac{6\times 5}{2}}$$
$$= \frac{1}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing 1 red and 1 white from urn II}]$$
$$= \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2}$$
$$= \frac{2\times 1}{\frac{4\times 3}{2}}$$
$$= \frac{1}{3}$$

$$P\left(\frac{A}{E_3}\right) = P[\text{Drawing 1 red and 1 white from urn III}]$$
$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2}$$
$$= \frac{4\times 3}{\frac{12\times 11}{2}}$$
$$= \frac{2}{11}$$

We have to find,

$$P\,(\text{They come from urn I}) = P\left(\frac{E_1}{A}\right)$$

$$P\,(\text{They come from urn II}) = P\left(\frac{E_2}{A}\right)$$

$$P\,(\text{They come from urn III}) = P\left(\frac{E_3}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{\frac{1}{5}}{\frac{36+55+30}{165}}$$

$$= \frac{1}{5}\times\frac{165}{118}$$

$$= \frac{33}{118}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{3}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{\frac{1}{3}}{\frac{33+55+30}{165}}$$

$$= \frac{1}{3}\times\frac{165}{118}$$

$$= \frac{55}{118}$$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{11}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{2}{11}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{2}{11}\times\frac{165}{118}$$

$$= \frac{30}{118}$$

Therefore, required probability $= \frac{33}{118}, \frac{55}{118}, \frac{30}{118}$.

...bility Ex 31.7 Q2

...ontains 2 white and 3 red balls

Bag B contains 4 white and 5 red balls.

Consider E_1, E_2 and A events as:-

E_1 = Selecting bag A

E_2 = Selecting bag B

A = Drawing one red ball

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$ [Since there are 2 bags]

$P(A | E_1) = P$[Drawing one red ball from bag A]

$= \frac{3}{5}$

$P\left(\frac{A}{E_2}\right) = P$[Drawing one red ball from bag B]

$= \frac{5}{9}$

To find,

P(Drawn, one red ball is from bag B) $= P\left(\frac{E_2}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}}$$

$$= \frac{\frac{5}{9}}{\frac{3}{5}+\frac{5}{9}}$$

$$= \frac{\frac{5}{9}}{\frac{27+25}{45}}$$

$$= \frac{5}{9}\times\frac{45}{52} = \frac{25}{52}$$

Required probability $= \frac{25}{52}$.

Probability Ex 31.7 Q3

Urn I contains 2 white and 3 black balls

Urn II contains 3 white and 2 black balls

Urn III contains 4 white and 1 black balls

Let E_1, E_2, E_3 and A be events as:-

E_1 = Selecting urn I

E_2 = Selecting urn II

E_3 = Selecting urn III

A = A white balls is drawn

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$

$P(E_3) = \frac{1}{2}$ [Since there are 3 urns]

$P(A | E_1) = P$[Drawing 1 white ball from urn I]

$= \frac{2}{5}$

$P\left(\frac{A}{E_2}\right) = P$[Drawing 1 white ball from urn II]

$= \frac{3}{5}$

$P\left(\frac{A}{E_3}\right) = P$[Drawing one whtie ball from urn III]

$= \frac{4}{5}$

To find,

P(Drawn one white ball from urn I) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{4}{5}}$$

$$= \frac{\frac{2}{10}}{\frac{2+3+4}{10}}$$

$$= \frac{2}{9}$$

Required probability $= \frac{2}{9}$.

Probability Ex 31.7 Q4

Urn I contains 7 white and 3 black balls
Urn II contains 4 white and 6 black balls
Urn III contains 2 white and 8 black balls
Let E_1, E_2, E_3 and A be events as:-

E_1 = Selecting urn I
E_2 = Selecting urn II
E_3 = Selecting urn III
A = Drawing 2 white balls without replacement.

Given,

$P(E_1) = 0.20$
$P(E_2) = 0.60$
$P(E_3) = 0.20$
$P(A \mid E_1) = P$[Drawing 2 white ball from urn I]

$$= \frac{{}^7C_2}{{}^{10}C_2}$$

$$= \frac{\frac{7\times 6}{2}}{\frac{10\times 9}{2}}$$

$$= \frac{7}{15}$$

$P\left(\frac{A}{E_2}\right) = P$[Drawing 2 white ball from urn II]

$$= \frac{{}^4C_2}{{}^{10}C_2}$$

$$= \frac{\frac{4\times 3}{2}}{\frac{10\times 9}{2}}$$

$$= \frac{12}{90}$$

$$= \frac{2}{15}$$

$$P\left(\frac{A}{E_3}\right) = P\,[\text{Drawing 2 whtie ball from urn III}]$$

$$= \frac{^2C_2}{^{10}C_2}$$

$$= \frac{1}{\frac{10\times 9}{2}}$$

$$= \frac{1}{45}$$

To find,

$$P\,(\text{2 white balls drawn are from urn III}) = P\left(\frac{E_3}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_3}{A}\right) = \frac{P\,(E_3)\,P\left(\frac{A}{E_3}\right)}{P\,(E_1)\,P\left(\frac{A}{E_1}\right) + P\,(E_2)\,P\left(\frac{A}{E_2}\right) + P\,(E_3)\,P\left(\frac{A}{E_3}\right)}$$

$$= \frac{0.2\times\frac{1}{45}}{0.2\times\frac{7}{15} + 0.6\times\frac{2}{15} + 0.2\times\frac{1}{45}}$$

$$= \frac{\frac{2}{450}}{\frac{14}{150} + \frac{12}{150} + \frac{2}{450}}$$

$$= \frac{\frac{2}{450}}{\frac{42+36+2}{450}}$$

$$= \frac{2}{80}$$

$$= \frac{1}{40}$$

Required probability $= \frac{1}{40}$.

Probability Ex 31.7 Q5

Consider the following events:

E_1 = Getting 1 or 2 in a throw of die,

E_2 = Getting 3, 4, 5 or 6 in a throw of die,

A = Getting exactly one tail

Clearly,

$$P(E_1) = \frac{2}{6} = \frac{1}{3}, P(E_2) = \frac{4}{6} = \frac{2}{3}, P(A/E_1) = \frac{3}{8}, P(A/E_2) = \frac{1}{2}$$

Required probablity $= P(E_2/A)$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8} + \frac{2}{3}\times\frac{1}{2}}$$

$$= \frac{8}{11}$$

Probability Ex 31.7 Q6

Consider the following events:
E_1 = First group wins, E_2 = Second group wins, A = New product is introduced.
It is given that
$$P(E_1)=0.6, P(E_2)=0.4, P(A/E_1)=0.7, P(A/E_2)=0.3$$

Required probability $= P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$
$$= \frac{0.4\times0.3}{0.6\times0.7+0.4\times0.3} = \frac{12}{54} = \frac{2}{9}$$

Hence required probability is $\frac{2}{9}$

Probability Ex 31.7 Q7

Given, 5 man out of 100 and 25 women out of 1000 are good orators.
Consider E_1, E_2 and A events as:-

E_1 = Selected person is male
E_2 = Selected perosn is female
E_3 = Selected person is an orator

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$ [Since number of males and females are equal]

$P(A \mid E_1) = P$ (Selecting a male orator)
$$= \frac{5}{100}$$
$$= \frac{1}{20}$$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting a frmale orator)
$$= \frac{25}{1000}$$
$$= \frac{1}{40}$$

To find, P (Orator selected is a male) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$$
$$= \frac{\frac{1}{2}\times\frac{1}{20}}{\frac{1}{2}\times\frac{1}{20}+\frac{1}{2}\times\frac{1}{40}}$$
$$= \frac{\frac{1}{40}}{\frac{1}{40}+\frac{1}{80}}$$
$$= \frac{1}{40}\times\frac{80}{3}$$
$$= \frac{2}{3}$$

Required probability $= \frac{2}{3}$.

Consider events E_1, E_2 and A events as:-

E_1 = Letters come from LONDON

E_2 = Letters come from CLIFTON

E_3 = Two consecutive letters visible on the envelope are ON

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2}$$

[Since letters came either from LONDON or CLIFTON]

$$P(A \mid E_1) = P(\text{Two consecutive letters ON from LONDON})$$
$$= \frac{2}{5}$$

[Since LONDON has 2-ON and 5 letters we consider one 'ON' as one letter]

$$P\left(\frac{A}{E_2}\right) = P(\text{Two consecutive letters ON from CLIFTON})$$
$$= \frac{1}{6}$$

[Since CLIFTON has one 'ON' nad 6 letters considering ON as one letter]

(i) To find, P (ON visible are from LONDON) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$
$$= \frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{6}}$$
$$= \frac{\frac{2}{10}}{\frac{2}{10}+\frac{1}{12}}$$
$$= \frac{2}{10}\times\frac{60}{17}$$
$$= \frac{12}{17}$$
$$P\left(\frac{E_1}{A}\right) = \frac{12}{17}$$

Required probability $= \frac{12}{17}$

(ii) $$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$
$$= \frac{\frac{1}{2}\times\frac{1}{6}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{6}}$$
$$= \frac{\frac{1}{12}}{\frac{2}{10}+\frac{1}{12}}$$
$$= \frac{1}{12}\times\frac{60}{17}$$
$$= \frac{5}{17}$$

Required probability $= \frac{5}{17}$.

Consider E_1, E_2 and A events as:-

E_1 = Selected student is boy

E_2 = Selected student is girl

E_3 = A student with IQ more that 150 is selected

$P(E_1) = \frac{60}{100}$

$P(E_2) = \frac{40}{100}$

$P(A \mid E_1) = P$ (Selected boy has IQ more than 150)

$= \frac{5}{100}$

$P\left(\frac{A}{E_2}\right) = P$ (Selected girl has IQ more than 150)

$= \frac{10}{100}$

To find, P (Selected student with IQ more than 150 is a boy) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{60}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{5}{100} + \frac{40}{100} \times \frac{10}{100}}$$

$$= \frac{300}{300 + 400}$$

$$= \frac{300}{700}$$

$$= \frac{3}{7}$$

Required probability $= \frac{3}{7}$.

robability Ex 31.7 Q10

Consider E_1, E_2, E_3 and A as:-

E_1 = Bolt produced by machine X

E_2 = Bolt produced by machine Y

E_3 = Bolt produced by machine Z

A = A bolt drawn is defective.

$$P(E_1) = \frac{1000}{6000} = \frac{1}{6}$$

$$P(E_2) = \frac{2000}{6000} = \frac{1}{3}$$

$$P(E_3) = \frac{3000}{6000} = \frac{1}{2}$$

$$P(A \mid E_1) = P(\text{Drawing defective bolt from machine } X) = \frac{1}{100}$$

$$P\left(\frac{A}{E_2}\right) = P(\text{Drawing defective bolt from machine } Y) = \frac{1.5}{100} = \frac{3}{200}$$

$$P\left(\frac{A}{E_3}\right) = P(\text{Drawing defective bolt from machine } Z) = \frac{2}{100}$$

To find, $P(\text{Defective bolt drawn is produced by machine } X) = P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{200} + \frac{1}{2} \times \frac{2}{100}}$$

$$= \frac{\frac{1}{600}}{\frac{1}{600} + \frac{3}{600} + \frac{1}{100}}$$

$$= \frac{1}{10}$$

Required probability $= \frac{1}{10}$.

ility Ex 31.7 Q11

Let E_1, E_2, E_3 and A be the events defined as follows

E_1 = scooters

E_2 = cars

E_3 = trucks

A = vehicle meet with an accident

Since there are 12000 vehicles, therefore

$$P(E_1)=\frac{3000}{12000}=\frac{1}{4}, P(E_2)=\frac{4000}{12000}=\frac{1}{3}, P(E_3)=\frac{5000}{12000}=\frac{5}{12}$$

It is given that $P(A/E_1)$=Probability that the accident involves a scooter $=0.02$

Similarly $P(A/E_2)=0.03$ and $P(A/E_3)=0.04$

(i)

We are required to find $P(E_1/A)$ i.e. given that the vehicle meet with an accident is a scooter

By Baye's rule

$$P(E_1/A)=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{4}\times 0.02}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04}$$

$$=\frac{3}{19}$$

(ii)

We are required to find $P(E_2/A)$ i.e. given that the vehicle meet with an accident is a car

By Baye's rule

$$P(E_2/A)=\frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{3}\times 0.03}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04}$$

$$=\frac{6}{19}$$

(iii)

We are required to find $P(E_3/A)$ i.e. given that the vehicle meet with an accident is a scooter

By Baye's rule

$$P(E_3/A)=\frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{5}{12}\times 0.04}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04}$$

$$=\frac{10}{19}$$

Probability Ex 31.7 Q12

We need to find

$$P\left(\frac{A}{Red}\right), P\left(\frac{B}{Red}\right), P\left(\frac{C}{Red}\right)$$

Now,

$$P\left(\frac{A}{Red}\right) = \frac{P\left(\frac{Red}{A}\right)P(A)}{P\left(\frac{Red}{A}\right)P(A)+P\left(\frac{Red}{B}\right)P(B)+P\left(\frac{Red}{C}\right)P(C)+P\left(\frac{Red}{D}\right)P(D)}$$

$$= \frac{\frac{1}{10}\times\frac{1}{4}}{\frac{1}{10}\times\frac{1}{4}+\frac{6}{10}\times\frac{1}{4}+\frac{8}{10}\times\frac{1}{4}+0}$$

$$= \frac{1}{1+6+8} = \frac{1}{15}$$

Similarly

$$P\left(\frac{B}{Red}\right) = \frac{6}{15}$$

$$P\left(\frac{C}{Red}\right) = \frac{8}{15}$$

Probability Ex 31.7 Q13

Let E_1, E_2, and E_3 be the respective events of the time consumed by machines A, B, and C for the job.

$$P(E_1) = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(E_3) = 20\% = \frac{20}{100} = \frac{1}{5}$$

Let X be the event of producing defective items.

$$P(X|E_1) = 1\% = \frac{1}{100}$$

$$P(X|E_2) = 5\% = \frac{5}{100}$$

$$P(X|E_3) = 7\% = \frac{7}{100}$$

The probability that the defective item was produced by A is given by P $(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|X)=\frac{P(E_1)\cdot P(X|E_1)}{P(E_1)\cdot P(X|E_1)+P(E_2)\cdot P(X|E_2)+P(E_3)\cdot P(X|E_3)}$$

$$=\frac{\frac{1}{2}\cdot\frac{1}{100}}{\frac{1}{2}\cdot\frac{1}{100}+\frac{3}{10}\cdot\frac{5}{100}+\frac{1}{5}\cdot\frac{7}{100}}$$

$$=\frac{\frac{1}{100}\cdot\frac{1}{2}}{\frac{1}{100}\left(\frac{1}{2}+\frac{3}{2}+\frac{7}{5}\right)}$$

$$=\frac{\frac{1}{2}}{\frac{17}{5}}$$

$$=\frac{5}{34}$$

Probability Ex 31.7 Q14

Consider the following events:

E_1 = Item is produced by machine A,

E_2 = Item is produced by machine B,

E_3 = Item is produced by machine C,

A = Item is defective

Clearly,

$$P(E_1)=\frac{50}{100}=\frac{1}{2}, P(E_2)=\frac{30}{100}=\frac{3}{10}, P(E_2)=\frac{20}{100}=\frac{1}{5}$$

$$P(A/E_1)=\frac{2}{100}, P(A/E_2)=\frac{2}{100}, P(A/E_2)=\frac{3}{100}$$

Re quired probablity $= P(E_1/A)$

$$=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{2}\times\frac{2}{100}}{\frac{1}{2}\times\frac{2}{100}+\frac{3}{10}\times\frac{2}{100}+\frac{1}{5}\times\frac{3}{100}}$$

$$=\frac{5}{11}$$

Probability Ex 31.7 Q15

Let E_1, E_2, E_3 be the events that we choose the first coin, second coin, and third coin respectively in a random toss.

$P(E_1)=\frac{1}{3}, P(E_2)=\frac{1}{3}, P(E_3)=\frac{1}{3}$

Let A denote the event when the toss shows heads.

It is given that

$P(A/E_1)=1, P(A/E_2)=0.75, P(A/E_3)=.60$

We have to find $P(E_1/A)$.

By Baye's theorem

$$P(E_1/A)=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}=$$

$$=\frac{\frac{1}{3}(1)}{\frac{1}{3}(1)+\frac{1}{3}(0.75)+\frac{1}{3}(0.60)}=\frac{1/3}{(1/3)+(1/4)+(1/5)}$$

$$=\frac{1/3}{47/60}=\frac{20}{47}$$

Probability Ex 31.7 Q16

Consider events E_1, E_2, E_3 and A as:-

E_1 = Selecting product from machine A

E_2 = Selecting product from machine B

E_3 = Selecting product from machine C

A = Selecting a standard quality product

$$P(E_1) = \frac{30}{100}$$

$$P(E_2) = \frac{25}{100}$$

$$P(E_3) = \frac{45}{100}$$

$$P(A \mid E_1) = P(\text{Selecting defective product from machine } A) = \frac{1}{100}$$

$$P\left(\frac{A}{E_2}\right) = P(\text{Selecting defective product from machine } B) = \frac{1.2}{100}$$

$$P\left(\frac{A}{E_3}\right) = P(\text{Selecting defective product from machine } C) = \frac{2}{100}$$

To find, P (Selecting defective product is produced by machine B)

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{25}{100} \times \frac{12}{1000}}{\frac{30}{100} \times \frac{1}{100} + \frac{25}{100} \times \frac{12}{1000} + \frac{45}{100} \times \frac{2}{100}}$$

$$= \frac{300}{300 + 300 + 900}$$

$$= \frac{300}{1500}$$

$$= \frac{1}{5}$$

Required probability $= \frac{1}{5}$.

ility Ex 31.7 Q17

2 and A be events as:-

E_1 = Selecting bicycle from first plant

E_2 = Selecting bicycle from second plant

A = Selecting a standard quality bicycle

$P(E_1) = \frac{60}{100}$

$P(E_2) = \frac{40}{100}$

$P(A \mid E_1) = P$ (Selecting standard quality bicycle from first plant)

$= \frac{80}{100}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting standard quality bicycle from second plant)

$= \frac{90}{100}$

To find, P (Selected standard quality bicycle is from second plant) $= P\left(\frac{E_2}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{40}{100} \times \frac{90}{100}}{\frac{60}{100} \times \frac{80}{100} + \frac{40}{100} \times \frac{90}{100}}$$

$$= \frac{3600}{4800 + 3600}$$

$$= \frac{3600}{8400}$$

$$= \frac{3}{7}$$

Required probability $= \frac{3}{7}$.

Probability Ex 31.7 Q18

Urn A contains 6 red and 4 white balls

Urn B contains 2 red and 6 white balls

Urn C contains 1 red and 5 white balls

Consider E_1, E_2, E_3 and A events as:-

E_1 = Selecting urn A

E_2 = Selecting urn B

E_3 = Selecting urn C

A = Selecting a red ball

$P(E_1) = \frac{1}{3}$

$P(E_2) = \frac{1}{3}$

$P(E_3) = \frac{1}{3}$ [Since there are three urns]

$P(A \mid E_1) = P$ (Selecting a red ball from urn A)

$= \frac{6}{10}$

$= \frac{3}{5}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting a red ball from urn B)

$= \frac{2}{8}$

$= \frac{1}{4}$

$P\left(\frac{A}{E_3}\right) = P$ (Selecting a red ball from urn C)

$= \frac{1}{6}$

To find, P (Selected red ball is from urn A) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{5}}{\frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{6}}$$

ility Ex 31.7 Q19

E_3 be the events that the people are
and non − vegetarian, smokers and vegetarian,
and non − smokers and vegetarian respectively.

$P(E_1) = \frac{2}{5}, P(E_2) = \frac{1}{4}, P(E_3) = \frac{7}{20}$

Let A denote the event that the person has the special chest disease.
It is given that

$P(A/E_1) = 0.35, P(A/E_2) = 0.20, P(A/E_3) = 0.10$

We have to find $P(E_1/A)$.

By Baye's theorem

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} =$$

$$= \frac{\frac{2}{5}(0.35)}{\frac{2}{5}(0.35) + \frac{1}{4}(0.20) + \frac{7}{20}(0.10)} = \frac{7/50}{(7/50) + (1/20) + (7/200)}$$

$$= \frac{7/50}{9/40} = \frac{28}{45}$$

Probability Ex 31.7 Q20

Let E_1, E_2, E_3 and A be events as:-

E_1 = Selecting product from machine A

E_2 = Selecting product from machine B

E_3 = Selecting product from machine C

A = Selecting a defective product

$$P(E_1) = \frac{100}{600} = \frac{1}{6}$$

$$P(E_2) = \frac{200}{600} = \frac{1}{3}$$

$$P(E_3) = \frac{300}{600} = \frac{1}{2}$$

$P(A \mid E_1) = P$(Selecting a defective item from machine A)

$$= \frac{2}{100}$$

$P\left(\frac{A}{E_2}\right) = P$(Selecting a defective item from machine B)

$$= \frac{3}{100}$$

$P\left(\frac{A}{E_3}\right) = P$(Selecting a defective item machine C)

$$= \frac{5}{100}$$

To find, P(Selected defective item is produced by machine A) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{2}{100}}{\frac{1}{6} \times \frac{2}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{5}{100}}$$

$$= \frac{\frac{2}{600}}{\frac{2}{600} + \frac{3}{300} + \frac{5}{200}}$$

$$= \frac{2}{600} \times \frac{600}{23}$$

$$= \frac{2}{23}$$

Required probability $= \frac{2}{23}$.

Probability Ex 31.7 Q21

Bag I contains 1 white and 6 red balls
Bag II contains 4 white and 3 red balls
Let E_1, E_2 and A events be:-

E_1 = Selecting bag I
E_2 = Selecting bag II
A = Selecting a white ball

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$ [Since there are two bags]

$P(A \mid E_1) = P$ (Selecting 1 white ball from bag I)

$= \frac{1}{7}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting 1 white ball from bag II)

$= \frac{4}{7}$

To find, P (Drawn white ball is from bag I) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{7}}{\frac{1}{2}\times\frac{1}{7}+\frac{1}{2}\times\frac{4}{7}}$$

$$= \frac{\frac{1}{14}}{\frac{1}{14}+\frac{4}{14}}$$

$$= \frac{1}{5}$$

Required probability $= \frac{1}{5}$.

Probability Ex 31.7 Q22

Consider the following events

$E_1 =$ The selected student is a girl
$E_2 =$ The selected student is not a girl
$A =$ The student is taller than 1.75 meters

We have,

$P(E_1) = 60\% = \frac{60}{100} = 0.6$

$P(E_2) = 1 - P(E_1) = 1 - 0.6 = 0.4$

$P(A/E_1) =$ Probability that the student is taller than 1.75 meters given that the student is a girl

$P(A/E_1) = \frac{1}{100} = 0.01$

And

$P(A/E_2) =$ Probability that the student is taller than 1.75 meters given that the student is not a girl

$P(A/E_2) = \frac{4}{100} = 0.04$

Now,
Required probability

$= P(E_1/A)$

$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$

$= \frac{0.6\times0.01}{0.6\times0.01+0.4\times0.04}$

$= \frac{\frac{6}{1000}}{\frac{22}{1000}}$

$= \frac{3}{11}$

...bility Ex 31.7 Q23

...E_2, E_3 and A be events as:-

E_1 = A is appointed

E_2 = B is appointed

E_3 = C is appointed

A = A change does take place

$P(E_1) = \frac{4}{7}$

$P(E_2) = \frac{1}{7}$

$P(E_3) = \frac{2}{7}$

$P(A \mid E_1) = P$ (Changes take place by A)

$= 0.3$

$P\left(\frac{A}{E_2}\right) = P$ (Changes take place by B)

$= 0.8$

$P\left(\frac{A}{E_3}\right) = P$ (Changes take place by C)

$= 0.5$

To find, P (Changes were taken place by B or C) $= P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{7}\times\frac{8}{10} + \frac{2}{7}\times\frac{5}{10}}{\frac{4}{7}\times\frac{3}{10} + \frac{1}{7}\times\frac{8}{10} + \frac{2}{7}\times\frac{5}{10}}$$

$$= \frac{\frac{18}{70}}{\frac{30}{70}}$$

$$= \frac{18}{30}$$

$$= \frac{3}{5}$$

Required probability $= \frac{3}{5}$.

Probability Ex 31.7 Q24

Let E_1, E_2 and A be events as:-

E_1 = Vehicle is scooter

E_2 = Vehicle is motorcycle

A = An insured met with accident

$P(E_1) = \frac{2000}{5000} = \frac{2}{5}$

$P(E_2) = \frac{3000}{5000} = \frac{3}{5}$

$P(A \mid E_1) = P$ (Accident of scooter)

$= 0.01$

$P\left(\frac{A}{E_2}\right) = P$ (Accident of motorcycle)

$= 0.02$

To find, P (Accident vehicle was motorcycle) $= P\left(\frac{E_2}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{3}{5}\times\frac{2}{100}}{\frac{2}{5}\times\frac{1}{100} + \frac{3}{5}\times\frac{2}{100}}$$

$$= \frac{\frac{6}{500}}{\frac{2}{500} + \frac{6}{500}}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

Required probability $= \frac{3}{4}$.

Probability Ex 31.7 Q25

Consider the following events

E_1 = The selected student is a hosteller

E_2 = The selected student is not a hosteller.

A = The student has an A grade.

We have,

$P(E_1) = 30\% = \frac{30}{100} = 0.3$

$P(E_2) = 20\% = \frac{20}{100} = 0.2$

$P(A/E_1)$ = Probability that the student has an A grade given that the student is a hosteller

$P(A/E_1) = \frac{60}{100} = 0.6$

And

$P(A/E_2)$ = Probability that the student has an A grade given that the student is not a hosteller

$P(A/E_2) = \frac{40}{100} = 0.4$

Now,

Required probability

$$= P(E_1/A)$$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{0.3 \times 0.6}{0.3 \times 0.6 + 0.2 \times 0.4}$$

$$= \frac{\frac{18}{100}}{\frac{26}{100}}$$

$$= \frac{9}{13}$$

Probability Ex 31.7 Q26

Let E_1, E_2, and E_3 be the respective events of choosing a two headed coin, a biased coin, and an unbiased coin.

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let A be the event that the coin shows heads.

A two-headed coin will always show heads.

$$\therefore P(A|E_1) = P(\text{coin showing heads, given that it is a two-headed coin}) = 1$$

Probability of heads coming up, given that it is a biased coin= 75%

$$\therefore P(A|E_2) = P(\text{coin showing heads, given that it is a biased coin}) = \frac{75}{100} = \frac{3}{4}$$

Since the third coin is unbiased, the probability that it shows heads is always $\frac{1}{2}$.

$$\therefore P(A|E_3) = P(\text{coin showing heads, given that it is an unbiased coin}) = \frac{1}{2}$$

The probability that the coin is two-headed, given that it shows heads, is given by

P $(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)}$$

$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{2}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3}\left(1 + \frac{3}{4} + \frac{1}{2}\right)}$$

$$= \frac{1}{\frac{9}{4}}$$

$$= \frac{4}{9}$$

robability Ex 31.7 Q27

Let A, E_1, and E_2 respectively denote the events that a person has a heart attack, the selected person followed the course of yoga and meditation, and the person adopted the drug prescription.

$$\therefore P(A) = 0.40$$
$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A|E_1) = 0.40 \times 0.70 = 0.28$$
$$P(A|E_2) = 0.40 \times 0.75 = 0.30$$

Probability that the patient suffering a heart attack followed a course of meditation and yoga is given by P (E_1|A).

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$
$$= \frac{\frac{1}{2}\times 0.28}{\frac{1}{2}\times 0.28 + \frac{1}{2}\times 0.30}$$
$$= \frac{14}{29}$$

Probability Ex 31.7 Q28

We need to find

$$P\left(\frac{\text{Box III}}{\text{Black}}\right)$$

$$= \frac{P\left(\frac{\text{Black}}{\text{Box III}}\right)P(\text{Box III})}{P\left(\frac{\text{Black}}{\text{Box III}}\right)P(\text{Box III}) + P\left(\frac{\text{Black}}{\text{Box II}}\right)P(\text{Box II}) + P\left(\frac{\text{Black}}{\text{Box I}}\right).P(\text{Box I}) + P\left(\frac{\text{Black}}{\text{Box Iv}}\right).P(\text{Box Iv})}$$

$$= \frac{\frac{1}{7}\times\frac{1}{4}}{\frac{1}{7}\times\frac{1}{4} + \frac{2}{8}\times\frac{1}{4} + \frac{3}{18}\times\frac{1}{4} + \frac{4}{13}\times\frac{1}{4}}$$
$$= \frac{\frac{1}{7}}{\frac{1}{7}+\frac{1}{4}+\frac{1}{6}+\frac{4}{13}}$$
$$= \frac{1}{7}\times\frac{7\times4\times6\times13}{4\times6\times13 + 7\times6\times13 + 7\times4\times13 + 7\times4\times6}$$
$$= \frac{4\times6\times13}{4\times6\times13 + 7\times6\times13 + 7\times4\times13 + 7\times4\times6}$$

= 0.165

Probability Ex 31.7 Q29

Let A be the event that the machine produces 2 acceptable items.

Also let B_1 be the event of correct set up and B_2 represent the event of incorrect set up.

Now, $P(B_1) = 0.8$, $P(B_2) = 0.2$

$P(A/B_1) = 0.9\times0.9$ and $P(A/B_2) = 0.4\times0.4$

Therefore, $P(B_1/A) = \frac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}$

$$= \frac{0.8\times0.9\times0.9}{0.8\times0.9\times0.9 + 0.2\times0.4\times0.4} = \frac{648}{680} = 0.95$$

ity Ex 31.7 Q30

events E_1, E_2 and A as

= The person selected is actually having T.B.

E_2 = The person selected is not having T.B.

E_3 = The person diagnosed to have T.B.

Given,

$$P(E_1) = \frac{1}{1000}$$

$$P(E_2) = \frac{999}{1000}$$

$P(A \mid E_1) = P$ (Person diagnosed to have T.B. and he is actually having T.B.)

$= 0.99$

$P\left(\frac{A}{E_2}\right) = P$ (Person diagnosed to have T.B. and he is not a actually having T.B.)

$= 0.001$

To find, P (Person diagnosed to have T.B. is actually having T.B.) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{1000} \times 0.99}{\frac{1}{1000} \times 0.99 + \frac{999}{1000} \times 0.001}$$

$$= \frac{990}{990 + 999}$$

$$= \frac{990}{1989}$$

$$= \frac{110}{221}$$

Required probability $= \frac{110}{221}$.

Probability Ex 31.7 Q31

Consider events E_1, E_2 and A as:-

E_1 = The selected person actually has disease

E_2 = The selected person has no disease

A = Selected person has disease

$$P(E_1) = \frac{0.2}{100}$$

$$= \frac{2}{1000}$$

$$P(E_2) = \frac{998}{1000}$$

$$P(A \mid E_1) = \frac{90}{100}$$

$$P\left(\frac{A}{E_2}\right) = \frac{1}{100}$$

To find, P (Person has disease is actually diseased) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{2}{1000} \times \frac{90}{100}}{\frac{2}{1000} \times \frac{90}{100} + \frac{998}{1000} \times \frac{1}{100}}$$

$$= \frac{180}{180 + 998}$$

$$= \frac{180}{1178}$$

$$= \frac{90}{589}$$

Required probability $= \frac{90}{589}$.

ability Ex 31.7 Q32

, E_2, E_3 and A be events as:-

E_1 = Patient has disease d_1

E_2 = Patient has disease d_2

E_3 = Patient has disease D_3

A = Selected patient has symptom S.

$$P(E_1) = \frac{1800}{5000} = \frac{18}{50}$$

$$P(E_2) = \frac{2100}{5000} = \frac{21}{50}$$

$$P(E_3) = \frac{1100}{5000} = \frac{11}{50}$$

$P(A \mid E_1) = P$ (Patient with disease d_1 and shows symptom S)

$$= \frac{1500}{1800} = \frac{5}{6}$$

$P\left(\frac{A}{E_2}\right) = P$ (Patient with disease d_2 and symptom S)

$$= \frac{1200}{2100} = \frac{4}{7}$$

$P\left(\frac{A}{E_3}\right) = P$ (Patient with disease d_3 and symptom S)

$$= \frac{900}{1100} = \frac{9}{11}$$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{5}{6}\times\frac{18}{50}}{\frac{5}{6}\times\frac{18}{50} + \frac{21}{50}\times\frac{4}{7} + \frac{11}{50}\times\frac{9}{11}}$$

$$= \frac{\frac{3}{10}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$

$$= \frac{3}{10}\times\frac{50}{36}$$

$$= \frac{5}{12}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{21}{50}\times\frac{4}{7}}{\frac{5}{6}\times\frac{18}{50} + \frac{21}{50}\times\frac{4}{7} + \frac{11}{50}\times\frac{9}{11}}$$

$$= \frac{\frac{6}{25}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$

$$= \frac{6}{25}\times\frac{50}{36}$$

$$= \frac{1}{3}$$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{11}{50}\times\frac{9}{11}}{\frac{5}{6}\times\frac{18}{50} + \frac{21}{50}\times\frac{4}{7} + \frac{11}{50}\times\frac{9}{11}}$$

$$= \frac{\frac{9}{50}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$

$$= \frac{9}{50}\times\frac{50}{36}$$

$$= \frac{1}{4}$$

So, probabilities of d_1, d_2, d_3 diseases are $\frac{5}{12}, \frac{1}{3}, \frac{1}{4}$ respectively.

Hence, the patient is most likely to have d_1 diseased.

bility Ex 31.7 Q33

E_2 and A be events as:-

E_1 = 1 occurs on die

E_2 = 1 does not occur on die

A = The man reports that it is one

$P(E_1) = \frac{1}{6}$

$P(E_2) = \frac{5}{6}$

$P\left(\frac{A}{E_1}\right) = P$ (He reports one when occurs on die)

$= P$ (He speaks truth)

$= \frac{3}{5}$

$P\left(\frac{A}{E_2}\right) = P$ (He reports one when 1 has not occured)

$= P$ (He does not speak truth)

$= 1 - \frac{3}{5}$

$= \frac{2}{5}$

To find, P (It is actually 1 when he reported that it is one on die) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6}\times\frac{3}{5}}{\frac{1}{6}\times\frac{3}{5}+\frac{5}{6}\times\frac{2}{5}}$$

$$= \frac{\frac{3}{30}}{\frac{3}{30}+\frac{10}{30}}$$

$$= \frac{3}{13}$$

Required probability $= \frac{3}{13}$.

Probability Ex 31.7 Q34

Let E_1, E_2 and A events be as:-

E_1 = 5 occurs on die

E_2 = 5 does not occur on die

A = He reports that it was 5

$P(E_1) = \frac{1}{6}$

$P(E_2) = \frac{5}{6}$

$P(A \mid E_1) = P$ (He reports 5 when 5 occurs on die)

$= P$ (He speaks truth)

$= \frac{8}{10}$

$= \frac{4}{5}$

$P\left(\frac{A}{E_2}\right) = P$ (He reports 5 when 5 does not occur on die)

$= P$ (He does not speak truth)

$= \frac{1}{5}$

To find, P (It was actually 5 when he reports that it is five) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{5}{6}\times\frac{1}{5}}$$

$$= \frac{\frac{4}{30}}{\frac{4}{30}+\frac{5}{30}}$$

$$= \frac{4}{9}$$

Required probability $= \frac{4}{9}$.

Probability Ex 31.7 Q35

$P(\text{Knows}) = \frac{3}{4}$

$P(\text{Guesses}) = \frac{1}{4}$

$P\left(\frac{\text{Correct}}{\text{Guesses}}\right) = \frac{1}{4}$

We need to find

$$P\left(\frac{\text{Knows}}{\text{Correctly}}\right) = \frac{P\left(\frac{\text{Correctly}}{\text{knows}}\right)P(\text{Knows})}{P\left(\frac{\text{Correctly}}{\text{knows}}\right)P(\text{Knows}) + P\left(\frac{\text{Correctly}}{\text{Guesses}}\right)P(\text{Guesses})}$$

$$= \frac{1 \times \frac{3}{4}}{1 \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}}$$

$$= \frac{\frac{3}{4}}{\frac{12+1}{16}}$$

$$= \frac{12}{13}$$

Probability Ex 31.7 Q36

Let E_1 and E_2 be the respective events that a person has a disease and a person has no disease.

Since E_1 and E_2 are events complimentary to each other,

$P(E_1) + P(E_2) = 1$

$P(E_2) = 1 - P(E_1) = 1 - 0.001 = 0.999$

Let A be the event that the blood test result is positive.

$$P(E_1) = 0.1\% = \frac{0.1}{100} = 0.001$$

$$P(A|E_1) = P(\text{result is positive given the person has disease}) = 99\% = 0.99$$

$$P(A|E_2) = P(\text{result is positive given that the person has no disease}) = 0.5\% = 0.005$$

Probability that a person has a disease, given that his test result is positive, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005}$$

$$= \frac{0.00099}{0.00099 + 0.004995}$$

$$= \frac{0.00099}{0.005985}$$

$$= \frac{990}{5985}$$

$$= \frac{110}{665}$$

$$= \frac{22}{133}$$