BINOMIAL DISTRIBUTION (XII, R. S. AGGARWAL)

EXERCISE 32 (Pg.No.: 1337)

1. a coin is tossed 6 times. Find the probability of getting at least 3 heads

Sol. When an unbiased coin is tossed, probability of head $= p = \frac{1}{2}$, so $q = 1 - \frac{1}{2} = \frac{1}{2}$

As the coin is thrown 6 times, so there are 6 trials i.e. $n = 6$

$$P(r) = {}^6C_r p^r q^{6-r} = {}^6C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{6-r} = {}^6C_r \left(\frac{1}{2}\right)^6$$

Required probability $= P(x \geq 3) = P(3) + P(4) + P(5) + P(6) = 1 - P(0) - P(1) - P(2)$

$$= 1 - \left({}^6C_0 + {}^6C_1 + {}^6C_2\right)\left(\frac{1}{2}\right)^6 = 1 - (1 + 6 + 15)\left(\frac{1}{2}\right)^6 = 1 - \frac{22}{64} = 1 - \frac{11}{32} = \frac{32-11}{32} = \frac{21}{32}$$

2. A coin is tossed 5 times. What is the probability that a head appears an even number of times ?

Sol. Here, $p = \frac{1}{2}$, $q = \frac{1}{2}$ and $n = 5$

$$\therefore \quad P(X = r) = {}^5C_r \left(\frac{1}{2}\right)^5.$$

P (obtaining head even number of times)

$$= P(X = 0 \text{ or } 2 \text{ or } 4) = P(X = 0) + P(X = 2) + P(X = 4)$$

$$\left({}^5C_0 + {}^5C_2 + {}^5C_4\right)\left(\frac{1}{2}\right)^5 = (1 + 10 + 5)\left(\frac{1}{32}\right) = \frac{16}{32} = \frac{1}{2}$$

3. 7 coins are tossed simultaneously. What is the probability that a till appears an odd number of times ?

Sol. Let p be the probability of success i.e. of getting a head.

$$\therefore n = 7, \quad p = \frac{1}{2} \text{ and } q = 1 - p = \frac{1}{2} = \frac{1}{2}$$

Let x be the Binomial variable of "*no. of successes*".

By Binomial Distribution, $P(X = r) = {}^nC_r p^r q^{n-r}, 0 \leq r \leq n$

$$\therefore P(X = r) = {}^7C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{7-r} = {}^7C_r p^r q^{n-r}, 0 \leq r \leq n$$

$$\therefore P(X = r) = {}^7C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{7-r} = {}^7C_r \left(\frac{1}{2}\right)^7 = {}^7C_r \left(\frac{1}{128}\right), 0 \leq r \leq 7$$

Required Probability = P(head appearing an odd number of times)

$$\Rightarrow P(X = 1 \text{ or } 3 \text{ or } 5 \text{ or } 7) \Rightarrow P(X = 1) + P(X = 3) + P(X = 5) + P(X = 7)$$

$$\Rightarrow {}^7C_1\left(\frac{1}{128}\right) + {}^7C_3\left(\frac{1}{128}\right) + {}^7C_5\left(\frac{1}{128}\right) + {}^7C_7\left(\frac{1}{128}\right) \Rightarrow (7 + 35 + 21 + 1)\left(\frac{1}{128}\right) = \frac{64}{128} = \frac{1}{2}$$

4. A coin is tossed 6 times. Find the probability of getting
(i) exactly 4 heads (ii) at least 1 head (iii) at most 4 heads

Sol. If a coin is tossed 6 times.

$p =$ probability of getting a head in a single toss $= \frac{1}{2}$, so $q = 1 - \frac{1}{2} = \frac{1}{2}$.

As the coin is thrown 6 times, so there are 6 trials i.e., $n = 6$

$\therefore \; P(r) = {}^nC_r p^r q^r = {}^6C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{6-r} = {}^6C_r \left(\frac{1}{2}\right)^6$

(i) Probability 4 heads $= P(X=4) = {}^6C_4 \left(\frac{1}{2}\right)^6 = \frac{6.5.4.3}{1.2.3.4} \cdot \frac{1}{64} = \frac{15}{64}$

(ii) Probability atleast one head $= P(X \geq 1) = 1 - P(X=0) = 1 - \frac{1}{64} = \frac{63}{64}$

(iii) Probability at most 4 heads

$= P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 1 - P(X=5) - P(X=6)$

$= 1 - {}^6C_5 \left(\frac{1}{2}\right)^6 - {}^6C_6 \left(\frac{1}{2}\right)^6 = 1 - \frac{6}{64} - \frac{1}{64} = \frac{57}{64}$

5. 10 coins are tossed simultaneously. Find the probability of getting
(i) exactly 3 heads (ii) not more than 4 heads (iii) at least 4 heads

Sol. 10 coins being tossed simultaneously is the same as one coin being tossed 10 times

$P(X=r) = {}^nC_r \cdot p^r \cdot q^{(n-r)} = {}^{10}C_r \cdot \left(\frac{1}{2}\right)^{10}$

(i) P(exactly 3 heads) $= {}^{10}C_3 \cdot \left(\frac{1}{2}\right)^{10}$

(ii) P(not more than 4 heads)

$= P(X \leq 4)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= \left({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_3 + {}^{10}C_4\right)\left(\frac{1}{2}\right)^{10}$

(iii) P(at least 4 heads)

= P(4 heads or 5 heads or …. Or 10 heads)

$= 1 - P$ (0 head or 1 head or 2 heads or 3 heads)

$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$

$= 1 - \left({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3\right)\left(\frac{1}{2}\right)^{10}$

6. A die is thrown 6 times. If getting an even number' is a success, find the probability of getting
(i) exactly 5 successes (ii) at least 5 successes
(iii) at most 5 successes

Sol. The repeated tosses of a die are Bernoulli trials. Let X denote the number of successes of getting odd numbers in an experiment of 6 trials.

Probability of getting an odd number in a single throw of a die is $p=\frac{3}{6}=\frac{1}{2}$

$\therefore q=1-p=\frac{1}{2}$

X has a binomial distribution

Therefore $P(X=x)={}^{n}C_{n-x}q^{n-x}p^{x}$, where $n=0,1,2.....n$

$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6-x}\cdot\left(\frac{1}{2}\right)^{x}={}^{6}C_{x}\left(\frac{1}{2}\right)^{6}$

(i) P (5 successes) $=P(X=5)$

$={}^{6}C_{5}\left(\frac{1}{2}\right)^{6}=6\cdot\frac{1}{64}=\frac{3}{32}$

(ii) P (at least 5 successes) $=P(X\geq 5)$

$P(X=5)+P(X=6)$

$={}^{6}C_{5}\left(\frac{1}{2}\right)^{6}+{}^{6}C_{6}\left(\frac{1}{2}\right)^{6}=6\cdot\frac{1}{64}+1\cdot\frac{1}{64}=\frac{7}{64}$

(iii) P (at most 5 successes) $=P(X\leq 5)$

$=1-P(X>5)=1-P(X=6)=1-{}^{6}C_{6}\left(\frac{1}{2}\right)^{6}=1-\frac{1}{64}=\frac{63}{64}$

7. A die is thrown 4 times 'Getting a 1 or a 6' is considered a success. Find the probability of getting
(i) exactly 3 successes (ii) at least 2 successes
(iii) at most 2 success

Sol. When a die thrown, sample space $=\{1,2,3,4,5,6\}$ It has six equally likely outcomes]

Let E be ht events of 'getting 1 or 6', then $p=P(E)=\frac{2}{3}=\frac{1}{3}$, so $q=1-\frac{1}{3}=\frac{2}{3}$

As the die is thrown 4 times, so there are 4 trials i.e., $n=4$

(i) Required probability $=P(X=3)={}^{4}C_{3}p^{3}q=4.\left(\frac{1}{3}\right)^{3}.\frac{2}{3}=\frac{8}{81}$

(ii) Required probability $=P(X\geq 2)=P(X=2)+P(X=3)+P(X=4)$

$=1-P(X=0)-P(X=1)$

$=1-{}^{4}C_{0}p^{0}q^{4}-{}^{4}C_{1}p^{1}q^{3}$

$=1-\left(\frac{2}{3}\right)^{4}-4\times\frac{2}{3}\times\left(\frac{1}{3}\right)^{3}$

$=\frac{81-16-8}{81}=\frac{57}{81}=\frac{19}{27}$

(iii) Required probability $=P(\text{atmost 2 sucesses})=P(X\leq 2)$

$=P(X=0)+P(X=1)+P(X=2)=1-(P(X=3)+P(X=4))=1-\left(\frac{8}{81}+\frac{1}{81}\right)=1-\frac{1}{9}=\frac{8}{9}$

8. Find the probability of a 4 turning up at least once in two tosses of a fair die.

Sol. If a die is tossed once

Probability of getting a four $=\frac{1}{6}$

$\therefore \quad p=\frac{1}{6}, q=\left(1-\frac{1}{6}\right)=\frac{5}{6}$ and $n(\text{No. of throws})=2$

$P(X=r)={}^{n}C_{r}\cdot p^{r}\cdot q^{(n-r)}$

P (at least one 4) $=P(X=1 \text{ or } X=2)=P(X=1)+P(X=2)$

$=1-P(X=0)$

$=1-{}^{2}C_{0}p^{0}q^{2}$

$1-\left(\frac{5}{6}\right)^{2}=1-\frac{25}{36}=\frac{11}{36}$

9. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of getting 2 successes

Sol. Total number of outcomes = 36

The possible doublets are $(1,1),(2,2)(3,3)(4,4),(5,5,)$ and $(6,6)$

Let p be the probability of success, therefore,

$p=\frac{6}{36}=\frac{1}{6}$

So $q=1-p=\ 1-\frac{1}{6}=\frac{5}{6}$

Since the dice is thrown $n=4$

We have $P(X=r)={}^{n}C_{r}\,p^{r}q^{n-r}$

10. A pair of dice is thrown 7 times. If getting a total of 7′ is considered a success, find the probability of getting

(i) no success

(ii) exactly 6 successes

(iii) at least 6 successes

(iv) at most 6 successes

Sol. Let p = probability of getting a total of $7=\frac{6}{36}=\frac{1}{6}$

$q=1-p=1-\left(\frac{1}{6}\right)=\frac{5}{6}$

Here n = 7

$P(X=r)={}^{n}C_{r}q^{n-r}p^{r}$

(a) P (no success) $=P(X=0)={}^{7}C_{0}\left(\frac{5}{6}\right)^{7}\left(\frac{1}{6}\right)^{0}=\left(\frac{5}{6}\right)^{7}$

(b) P (6 successes) $=P(X=6)={}^{7}C_{6}\left(\frac{5}{6}\right)^{1}\left(\frac{1}{6}\right)^{6}=35x\left(\frac{1}{6}\right)^{7}$

(c) P (at least 6 successes)

$= P(X=6)+P(X=7) = {}^7C_6\left(\frac{5}{6}\right)^1\left(\frac{1}{6}\right)^6 + {}^7C_7\left(\frac{5}{6}\right)^0\left(\frac{1}{6}\right)^7 = 35x\left(\frac{1}{6}\right)^7+\left(\frac{1}{6}\right)^7 = 36\left(\frac{1}{6}\right)^7$

(d) P (at most 6 successes)

$= P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)+P(X=6) = 1-P(X=7)$

$= 1-{}^7C_7p^7 = 1-\left(\frac{1}{6}\right)^7$

11. There are 6% defective items in a large bulk of items. Find the probability that a sample of 8 items will include not more than one defective item

Sol. Let X denote the number of defective items in a sample of 8 items. Then, X follws a binomial distribution with $n=8$

P = (probability of getting a defective item) = 0.06 and $q=1-p=0.94$

$P(X=r) = {}^8C_r(0.06)^r(0.94)^{8-r}, r=0,1,2,3,.....8$

The required probability = probability of not more than one defective item

$= P(X\le 1) = P(X=0)+P(X=1)$

$= {}^8C_0(0.06)^0(0.94)^{8-0} + {}^8C_1(0.06)^1(0.94)^{8-1}$

$= (0.94)^8 + 8(0.06)(0.94)^7 \quad = (0.94)^7\{0.94+0.48\} = 1.42(0.94)^7$

12. In a box containing 60 bulbs, 6 are defective. What is the probability that out of a sample of 5 bulbs
(i) none is defective (ii) exactly 2 are defective

Sol. P (getting a defective bulb) $= \frac{6}{60} = \frac{1}{10}$

$\therefore p = \frac{1}{10}, q = 1-\frac{1}{10} = \frac{9}{10}$

If 5 bulbs are drawn, $n=5$

we have $P(X=r) = {}^nC_r q^{n-r} p^r$

(i) P (None is defective) $= p(X=0) = {}^5C_0\, p^0\, q^5 = \left(\frac{9}{10}\right)^5$

(ii) P (exactly 2 are defective) $= p(X=2) = {}^5C_2\, p^2\, q^3 = \frac{5\times 4}{2\times 1}\times\left(\frac{1}{10}\right)^2\times\left(\frac{9}{10}\right)^3 = \frac{729}{10000}$

13. the probability that a bulb produced by a factory will fuse after 6 months of use is 0.05. Find the probability that out of 5 such bulbs
(i) none will fuse after 6 months of use
(ii) at least one will fuse after 6 months of use
(iii) not more than one will fuse after 6 months of use

Sol. Let E be the event of 'bulb will fuse after 6 months of use', then

$p = P(E) = 0.05 = \frac{5}{100} = \frac{1}{20}$, so $q = 1-\frac{1}{20} = \frac{19}{20}$

As a sample space of 5 bulb is taken, so there are 5 trials i.e. $n=5$

(i) Required probability $= P(X=0) = {}^5C_0q^5 = 1\times\left(\frac{19}{20}\right)^5 = \left(\frac{19}{20}\right)^5$

(ii) Required probability $= P(\text{atleast one}) = P(X \geq 1) = 1 - P(0) = 1 - \left(\frac{19^5}{20}\right)$

(iii) Required probability $= P(X \leq 1) = P(0) + P(1) = {}^5C_0 q^5 + {}^5C_1 pq^4$

$$= \left(\frac{19}{20}\right)^5 + 5.\frac{1}{20}.\left(\frac{19}{20}\right)^4 = \left(\frac{19}{20} + \frac{5}{20}\right)\left(\frac{19}{20}\right)^4 = \frac{6}{5}\left(\frac{19}{20}\right)^4$$

14. In the items produced by a factory, there are 10% defective items. A sample of 6 items is randomly chosen. Find the probability that this sample contains (i) exactly 2 defective items, (ii) not more than 2 defective items, (iii) at least 3 defective items

Sol. If an item is chosen,

Probability of getting a defective item $= 10\% = \frac{1}{10}$

$\therefore P(X = r) = {}^nC_r \cdot p^r \cdot q^{(n-r)} = {}^6C_r \cdot \left(\frac{1}{10}\right)^r \cdot \left(\frac{9}{10}\right)^{(6-r)}$

(i) $P(X = 2) = {}^6C_2 \cdot \left(\frac{1}{10}\right)^2 \cdot \left(\frac{9}{10}\right)^4 = \frac{3}{20} \times \left(\frac{9}{10}\right)^4$

(ii) $P(X \leq 2) = {}^6C_0 \cdot \left(\frac{9}{10}\right)^6 + {}^6C_1 \cdot \frac{1}{10} \times \left(\frac{9}{10}\right)^5 + {}^5C_2 \cdot \frac{1}{100} \times \left(\frac{9}{10}\right)^4 = \frac{3}{2} \times \left(\frac{19}{20}\right)^4$

(iii) $P(X \geq 3) = 1 - P(X < 3) = 1 - P(X \leq 2) = 1 - P(X = 0) - P(X = 1) - P(X = 2)$

$$= 1 - {}^6C_0\left(\frac{1}{10}\right)^0 \times \left(\frac{9}{10}\right)^6 - {}^6C_1\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^5 - {}^6C_2\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^4 = 1 - \left[\frac{3}{2} \times \left(\frac{19}{20}\right)^4\right]$$

15. Assume that on an average one telephone number out of 15, called between 3 p.m. on weekdays, will be busy. What is the probability that if six randomly selected telephone number are called, at least 3 of them will be busy ?

Sol. Here, $p = P$ (number is busy between 3 p.m. and 4 p.m.) $= \frac{1}{15}$

$\therefore \quad q = 1 - p = 1 - \frac{1}{15} = \frac{14}{15}$, Also, $n = 6$.

Let X denote the number of telephone numbers that will be busy

Then, we are required to determine $P(X \geq 3)$.

$P(X \geq 3) = P(X = 3 \text{ or } 4 \text{ or } 5 \text{ or } 6)$

$$= {}^6C_3\left(\frac{1}{15}\right)^3\left(\frac{14}{15}\right)^3 + {}^6C_4\left(\frac{1}{15}\right)^4\left(\frac{14}{15}\right)^2 + {}^6C_5\left(\frac{1}{15}\right)^5\left(\frac{14}{15}\right)^1 + {}^6C_6\left(\frac{1}{15}\right)^6\left(\frac{14}{15}\right)^0 = \frac{57905}{15^6}$$

16. Three cars participate in a race. The probability in a rice. The probability that any one of them has an accident is 0.1. Find the probability that all the cars reach the finishing line without any accident

Sol. $p = P(\text{ a car has an accident}) = 0.1 = \frac{1}{10}$

$\therefore q = \left(1 - \frac{1}{10}\right) = \frac{9}{10}$

Here n = 3

$P(X=r)={}^{n}C_r\cdot p^r\cdot q^{(n-r)}={}^{3}C_r\cdot\left(\frac{1}{10}\right)^r\cdot\left(\frac{9}{10}\right)^{(3-r)}$

$P(\text{all the cars reach the finishing line without any accident})=P(X=0)={}^{3}C_0\cdot\left(\frac{9}{10}\right)^3=\left(\frac{9}{10}\right)^3$

17. Past records shown that 80% of the operations performed by a certain doctor were successful. If he performs 4 operations in day, what is the probability that at least 3 operations will be successful ?

Sol. If the doctor performs an operation in a day,

Probability that operation is successful = 80%

$\therefore\ p=\frac{80}{100}=\frac{4}{5},\ q=\left(1-\frac{4}{5}\right)=\frac{1}{5}$

Here n = 4

$P(X=r)={}^{n}C_r\cdot p^r\cdot q^{(n-r)}={}^{4}C_r\cdot\left(\frac{4}{5}\right)^r\cdot\left(\frac{1}{5}\right)^{(4-r)}$

$P(X\geq 3)=P(X=3)+P(X=4)$

$=\left[{}^{4}C_3\cdot\left(\frac{4}{3}\right)^3\cdot\left(\frac{1}{5}\right)+{}^{4}C_4\cdot\left(\frac{4}{5}\right)^4\right]=4\times\frac{64}{625}+\frac{256}{625}=\frac{512}{625}$

18. the probability of a man hitting a target is (1/4). If he fires 7 times, what is the probability of his hitting the target at least twice ?

Sol. Let E be the event 'hitting a target' then $p=P(E)=\frac{1}{4},$, so $q=1-\frac{1}{4}=\frac{3}{4}$

As the man fires 7 times, so $n=7$, Required probability $=P(X\geq 2)=1-(P(0)+P(1))$

$=1-\left({}^{7}C_0q^7+{}^{7}C_1pq^6\right)=1-(q+7p)q^6=1-\left(\frac{3}{4}+7.\frac{1}{4}\right)\left(\frac{3}{4}\right)^6=1-\frac{5}{2}.\frac{3^6}{4^6}=1-\frac{3645}{8192}=\frac{4547}{8192}$

19. In a hurdles race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is (5/6). What is the probability that he will knock down fewer than 2 hurdles ?

Sol. Let "knocking down a hurdle" be a success. Then, $p=\frac{5}{6},\ q=\frac{1}{6}$ and $n=10$.

Here, $P(X=r)={}^{10}C_r\left(\frac{5}{6}\right)^r\left(\frac{1}{6}\right)^{10-r}$

$\therefore$ The required probability, $P(X<2)=P(X=0)+P(X=1)$

$={}^{10}C_0\left(\frac{5}{6}\right)^0\left(\frac{1}{6}\right)^{10}+{}^{10}C_1\left(\frac{5}{6}\right)^1\left(\frac{1}{6}\right)^9=\left(\frac{1}{6}\right)^{10}+10\times\left(\frac{5}{6^{10}}\right)=\left(\frac{1}{6}\right)^{10}(1+50)=51\times\left(\frac{1}{6}\right)^{10}$

20. A man can hit a bird, once in 3 shots. On this assumption he fires 3 shots. What is the chance that at least one bird is hit ?

Sol. P (A man can hit a bird) $=\frac{1}{3}$

$\therefore p=\frac{1}{3},q=\left(1-\frac{1}{3}\right)=\frac{2}{3}$

Here $n=3$

$P(X=r)={}^{n}C_r \cdot p^r . q^{(n-r)} = {}^{3}C_r \cdot \left(\frac{1}{3}\right)^r \cdot \left(\frac{2}{3}\right)^{(3-r)}$

Now, P(at least one bird is hit)

$=P(X\geq 1)=P(X=1)+P(X=2)+P(X=3)$

$=\left[{}^{3}C_1\times\frac{1}{3}\times\left(\frac{2}{3}\right)^2\right]+\left[{}^{3}C_2\times\left(\frac{1}{3}\right)^2\times\frac{2}{3}\right]+\left[{}^{3}C_3\times\left(\frac{1}{3}\right)^3\right] = \left(\frac{4}{9}+\frac{2}{9}+\frac{1}{9}\right)=\frac{7}{9}$

21. If the probability that a man aged 60 will live to be 70 is 0.65, what is the probability that out of 10 men, now 60, at least 8 will live to be 70 ?

Sol. Here, $p=P$ (a man now 60, will live to 70) $=0.65$, $q=0.35$ and $n=10$.

Now, $P(X\geq 8)=P(X=7)$ $=P(X=8)+P(X=9)+P(X=10)$

$={}^{10}C_8\times(0.65)^8(0.35)^2+{}^{10}C_9\times(0.65)^9(0.35)^1+{}^{10}C_{10}\times(0.65)^{10}(0.35)^0 = 0.2615$

22. A bag contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one with replacement, what is the probability that

(i) none is white, (ii) all are white (iii) at least one is white

Sol. Total number of balls $=5+7+8=20, 5$ are white

Let E be the event 'drawing a white ball', then $p=P(E)=\frac{5}{20}=\frac{1}{4}$, so $q=1-\frac{1}{4}=\frac{3}{4}$

As 4 balls are drawn one by one with replacement, so there are 4 Bernoulli an trial i.e. $n=4$

(i) P (non is white) $=P(X=0)$ $={}^{4}C_0 q^4=1.\left(\frac{3}{4}\right)^4=\left(\frac{3}{4}\right)^4$

(ii) P (all are white) $=P(X=4)$ $={}^{4}C_4 p^4=1.\left(\frac{1}{4}\right)^4=\left(\frac{1}{4}\right)^4$

(iii) P (at least one is white) $=P(X\geq 1)=1-p(X=0)=1-\left(\frac{3}{4}\right)^4$

23. A policeman fires 6 bullets at a burglar. The probability that the burglar will be hit by a bullet is 0.6. What is the probability that the burglar is still unhurt ?

Sol. Here $n=6$, $p=0.6=\frac{6}{10}=\frac{3}{5}, q=1-p=1-\frac{3}{5}=\frac{2}{5}$

P(burglar is still unhurt) $=P(X=0)={}^{6}C_0\left(\frac{2}{5}\right)^6=1\times(0.4)^6=0.0041$

24. A die is tossed thrice. A success is 1 or 6 on a toss. Find the mean and variance of successes

Sol. If a die is tossed thrice ,we have $n=3$

P = (Probability of success in single toss) $=\frac{2}{6}=\frac{1}{3}$

$\therefore\ q=1-\frac{1}{3}=\frac{2}{3}$

Mean $\mu=np=3\times\frac{1}{3}=1$

Variance $\left(\sigma^2\right) = npq = 3 \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{3}$

25. A die is thrown 100 times. Getting an even number is considered a success. Find the mean and variance of successes

Sol. If a die is tossed 100 times , we have $n = 100$

P = (Probability of success in single toss) = $\frac{3}{6} = \frac{1}{2}$

$\therefore \quad q = 1 - \frac{1}{2} = \frac{1}{2}$

Mean $\mu = np = 100 \times \frac{1}{2} = 50$

Variance $\left(\sigma^2\right) = npq = 100 \times \frac{1}{2} \times \frac{1}{2} = 25$

26. Determine the binomial distribution whose mean is 9 and variance is 6

Sol. Given that mean $= np = 9$ and variance $= npq = 6 \Rightarrow 9q = 6 \Rightarrow q = \frac{2}{3}$

$P = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$

$\because npq = 6 \quad \Rightarrow n \times \frac{1}{3} \times \frac{2}{3} = 6 \quad \Rightarrow n = 27$

Hence, Required probability distribution is ${}^{27}C_r \cdot \left(\frac{1}{3}\right)^r \cdot \left(\frac{2}{3}\right)^{(27-r)}$, where $r = 0, 1, 2, 3, \ldots, 27$

27. Find the binomial distribution whose mean is 5 and variance is 2.5

Sol. Given that mean $= np = 5$ and variance $= npq = 2.5 \Rightarrow 5q = 2.5 \Rightarrow q = \frac{1}{2}$

$P = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$

$\because npq = 2.5 \quad \Rightarrow n \times \frac{1}{2} \times \frac{1}{2} = 2.5 \quad \Rightarrow n = 10$

Hence, Required probability distribution is ${}^{10}C_r \cdot \left(\frac{1}{2}\right)^r \cdot \left(\frac{1}{2}\right)^{(10-r)}$, $0 \le r \le 10$

i.e., ${}^{10}C_r \left(\frac{1}{2}\right)^{10}, 0 \le r \le 10$

28. The mean and variance of a binomial distribution are 4 and (4/3) respectively. Find $P(X \ge 1)$

Sol. $\because$ mean = 4 & variance $= \frac{4}{3}$

$\left(np = 4 \text{ and } npd = \frac{4}{3}\right) \Rightarrow q = \frac{1}{3}$

$\therefore p = (1 - q) = \left(1 - \frac{1}{3}\right) = \frac{2}{3}$ and $n = \frac{4}{p} = \left(4 \times \frac{3}{2}\right) = 6$

$$\therefore P(X=r)={}^{n}C_r\cdot p^r\cdot q^{(n-r)}={}^{6}C_r\cdot\left(\frac{2}{3}\right)^r\cdot\left(\frac{1}{3}\right)^{(6-r)}$$

$$P(X\geq 1)=1-P(X=0)\ =1-{}^{6}C_0\cdot\left(\frac{2}{3}\right)^0\cdot\left(\frac{1}{3}\right)^6=\left(1-\frac{1}{3^6}\right)=\frac{728}{729}$$

29. For a binomial distribution, the mean is 6 and the standard devitation is $\sqrt{2}$. Find the probability of getting 5 successes

Sol. $\because$ mean = 6 & variance = 2

$$np=6 \text{ and } npq=2\ \Rightarrow q=\frac{1}{3} \text{ and } p=\frac{2}{3}$$

$$\left(n\times\frac{2}{3}\right)=6\Rightarrow n=\left(6\times\frac{3}{2}\right)=9$$

$$\therefore \text{ P(5 successes) } {}^{9}C_5\cdot\left(\frac{2}{3}\right)^5\cdot\left(\frac{1}{3}\right)^4$$

30. In a binomial distribution, the sum and the product of the mean and the variance are $(25/3)$ and $(50/3)$ respectively. Find the distribution

Sol. We have, mean + variance = $\frac{25}{3}$ and mean × variance = $\frac{50}{3}$

np +npq = $\frac{25}{3}$ and np x npq = $\frac{50}{3}$

$\Rightarrow$ np + np(1-p) = $\frac{25}{3}$

$\Rightarrow$ np(1 + 1-p) = $\frac{25}{3}$

$\Rightarrow$ np(2-p) = $\frac{25}{3}$

$\Rightarrow$ np = $\frac{25}{3(2-p)}$ ..... (i)

similarly, npxnp(1-p) = $\frac{50}{3}$

$$\left(\frac{25}{3(2-p)}\right)^2\times(1-p)=\frac{50}{3}$$

$$\Rightarrow\frac{625(1-p)}{9(2-p)^2}=\frac{50}{3}$$

$$3\times625\times(1-p)=50\times9\times(2-p)^2$$

$$\Rightarrow125\times(1-p)=30\left(4+p^2-4p\right)$$

$$\Rightarrow125-125p=120+30p^2-120p$$

$$\Rightarrow30p^2+125p-120p-125+120=0$$

$$\Rightarrow30p^2+5p-5=0$$

$$\Rightarrow6p^2+p-1=0\Rightarrow6p^2+3p-2p-1=0\Rightarrow3p(2p+1)-(2p+1)=0$$

$\Rightarrow (3p-1)(2p+1)=0 \Rightarrow p=\frac{1}{3}$

Now from (i) np = $\frac{25}{3(2-p)}$

$\Rightarrow n\times\frac{1}{3}=\frac{25}{3\times\left(2-\frac{1}{3}\right)} \Rightarrow n=\frac{25\times3}{5} \Rightarrow n=15$

Now $q=1-p=1-\frac{1}{3}=\frac{2}{3}$

Hence the probability distribution is , ${}^{15}C_r\cdot\left(\frac{1}{3}\right)^r\cdot\left(\frac{2}{3}\right)^{(15-r)}$, $0\le r\le 15$

31. Obtain the binomial distribution whose mean is 10 and standard deviation is $2\sqrt{2}$

Sol. Here, we are given that mean = 10,

i.e. $np=10$ .....(1) and S.D. = $2\sqrt{2}$, i.e. $\sqrt{npq}=2\sqrt{2}$

$\Rightarrow npq=8$ .....(2)

Dividing (2) by (1) we get, $q=\frac{8}{10}=\frac{4}{5}$ and hence $p=1-q=1-\frac{4}{5}=\frac{1}{5}$

Substituting this value of p in (1), we get $\Rightarrow n\left(\frac{1}{5}\right)=10 \Rightarrow n=50.$

$\therefore$ The required binomial distribution is ${}^{50}C_r\cdot\left(\frac{1}{5}\right)^r\cdot\left(\frac{4}{5}\right)^{(50-r)}$, $0\le r\le 50$

32. Bring out the fallacy, if any, in the following statement

The mean of binomial distribution is 6 and its variance is 9

Sol. Given mean = 6 and variance = 9 ,

$\Rightarrow np=6$ and $npq=9 \Rightarrow q=\frac{npq}{np}=\frac{9}{6}=\frac{3}{2}$

But, q cannot be greater than 1.