LINEAR PROGRAMMING (XII, R. S. AGGARWAL)

EXERCISE 31 A (Pg.No.: 1357)

Graph the solution sets of the following In equations :

1. $x+y\geq 4$

Sol. The given inequation is $x+y\geq 4$ ...(1)

The corresponding linear equation is $x+y=4$ ...(2)

Now, we shall draw the line $x+y=4$

When $x=0$, line implies $0+y=4 \Rightarrow y=4$

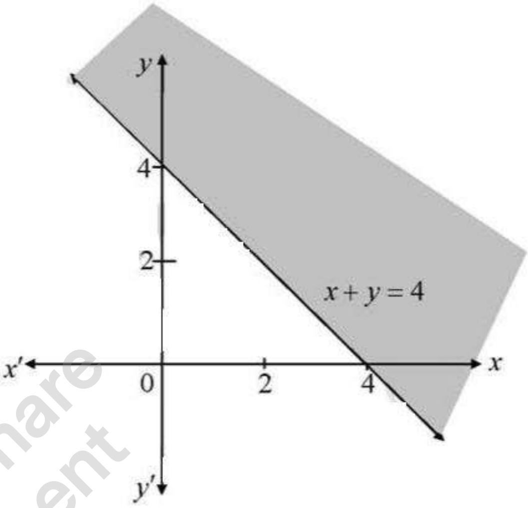
When $y=0$, line implies $x+0=4 \Rightarrow x=10$

$\therefore$ The points $(0, 4)$ and $(4, 0)$ are on the line

The line joining $(0, 4)$ and $(4, 0)$ divides the x-y plane in two half-planes.

Putting (0,0) in $x+y\geq 4$ which is false.

$\therefore$ The closed half-plane does not contain the origin is the graph of the given inequation.



2. $x-y\leq 3$

Sol. Given, $x-y\leq 3$

Taking line $x-y=3$

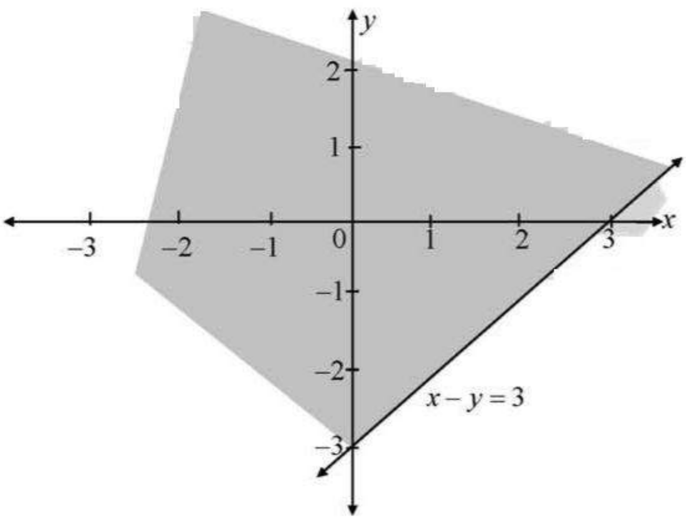
Taking line $3x-2y=12$ and putting $x=0, y=-3$

Putting $y=0,\ x=3$

Now we draw the graph of line $x-y=3$

Putting (0,0) in $x-y\leq 3$ we get $0-0\leq 3$, which is true.

Hence, shaded region towards origin including line $x-y=3$.



3. $x+2y>1$

Sol. Given, $x+2y>1$

Taking line $x+2y=1$

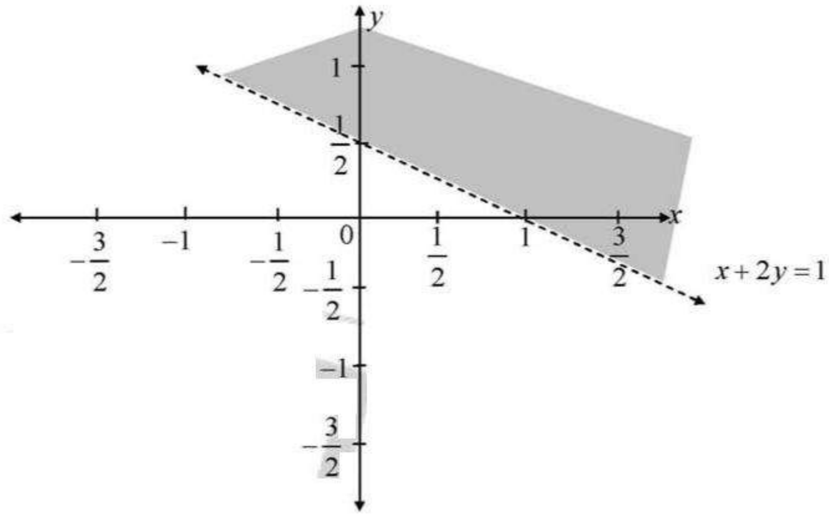
Taking line $x+2y=1$ and putting $x=0, y=\frac{1}{2}$

Putting $y=0,\ x=1$

Now we draw the graph of line $x+2y=1$

Putting (0,0) in $x+2y>1$ we get $0>1$ which is false

Hence, shaded region above the line $x+2y=1$.



4. $2x-3y<4$

Sol. Given, $2x-3y<4$

Taking line $2x-3y=4$

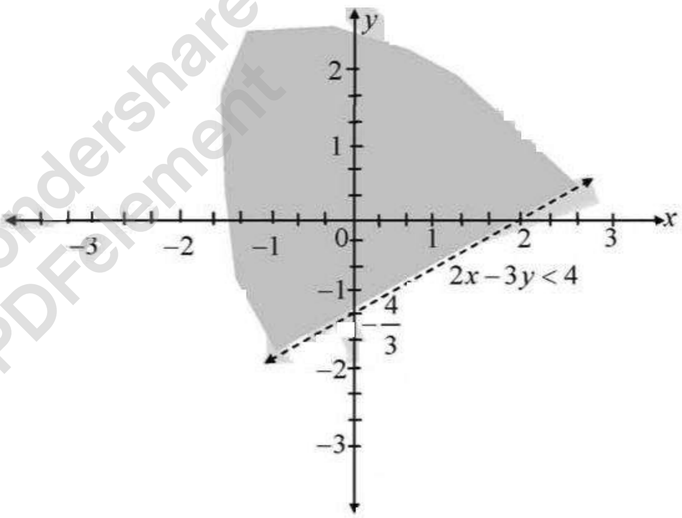
Taking line $2x-3y=4$ and putting $x=0, y=-\frac{4}{3}$

Putting $y=0,\ x=2$

Now we draw the graph of line $2x-3y=4$

Putting (0,0) in $2x-3y<4$ we get $0<4$, which is true.

Hence, shaded region above the line $2x-3y=4$.



5. $x \geq y-2$

Sol. Given, $x \geq y-2$

Taking line $x=y-2$

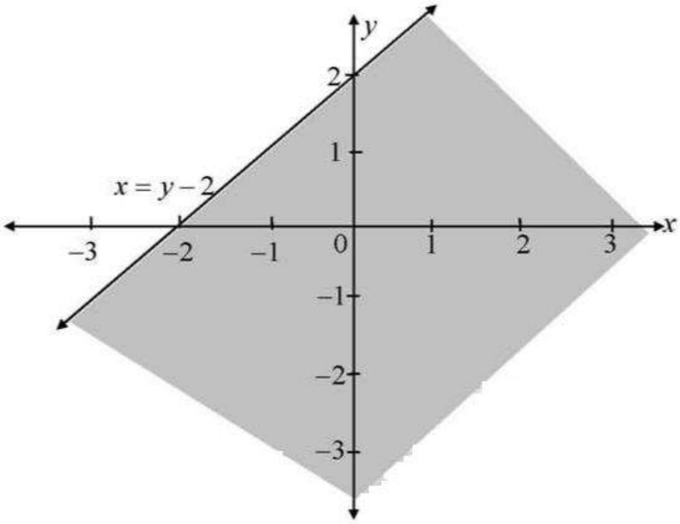
Taking line $x=y-2$ and putting $x=0, y=2$

Putting $y=0,\ x=-2$

Now we draw the graph of line $x=y-2$

Putting (0,0) in $x \geq y-2$ we get $0 \geq -2$ which is true

Hence, shaded region towards origin including line $x=y-2$.



6. $y-2 \le 3x$

Sol. Given, $y-2 \le 3x$

Taking line $y-2=3x$

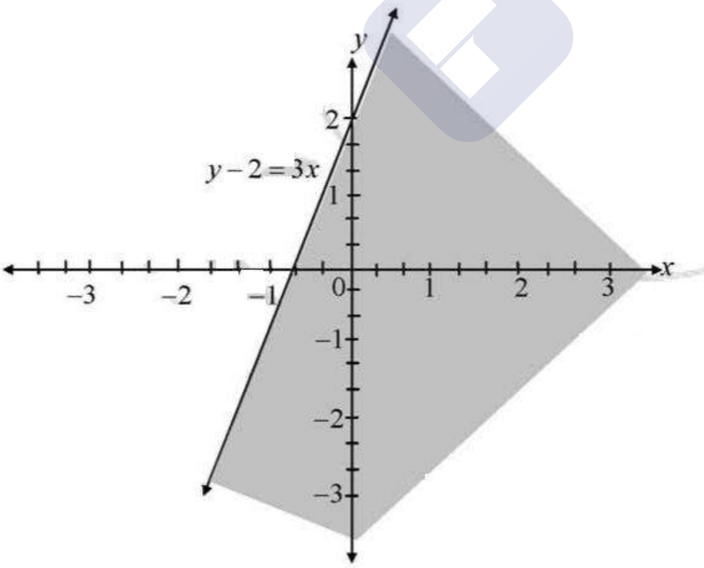
Taking line $y-2=3x$ and putting $x=0, y=2$

Putting $y=0$, $x=-\frac{2}{3}$

Now we draw the graph of line $y-2=3x$

Putting (0,0) in $y-2 \le 3x$ we get $-2 \le 0$, which is true.

Hence, shaded region towards origin including line $y-2=3x$.



Solve each of the following systems of simultaneous Inequations :

7. $2x+y>1$ and $2x-y\geq -3$

Sol. The given of inequalities is $2x+y>1$ ...(1)

and $2x-y\geq -3$ ...(2)

The equation corresponding to inequality (1) is $2x+y=1$

Putting $x=0$ in $2x+y=1, y=1$

Putting $x=\frac{1}{2}$ in $2x+y=1, y=0$

Putting (0,0) in $2x+y>1$, we have

$0>1$, which is false

Thus the shaded region is above the line.

The equation corresponding to inequality (2) is $2x-y=-3$

Putting $x=0$ in $2x-y=-3, y=3$

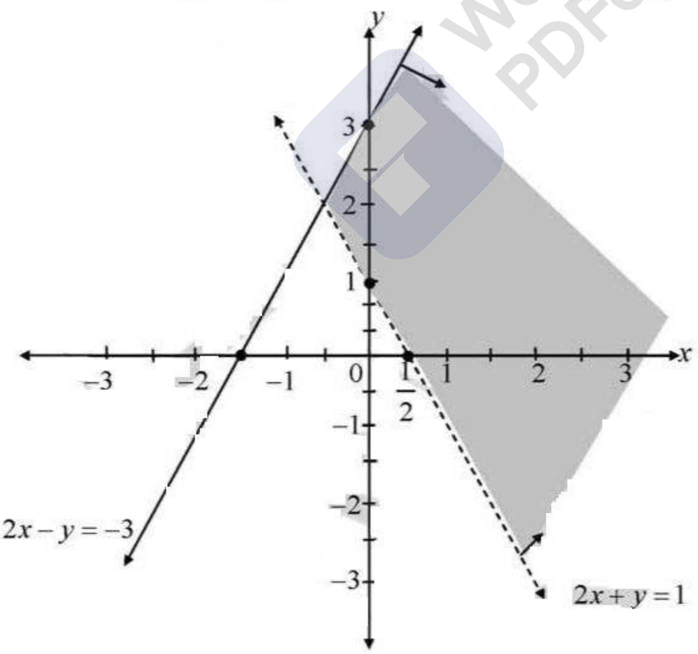
Putting $x=-\frac{3}{2}$ in $2x-y=-3, y=0$

Putting (0,0) in $2x-y\geq -3$, we have

$0>-3$, which is true

Hence, shaded region towards origin including line $2x-y=-3$

Hence, the common solution set (i.e. intersection of two solution set) is the shaded region. Any point in this shaded region represents a solution of the given system of inequalities.



8. $x-2y\geq 0, 2x-y\leq -2$

Sol. The given linear inequations are $x-2y\geq 0$ ...(1)

$2x-y\leq -2$ ...(2)

The line corresponding to (1) is $x-2y=0$ ...(3)

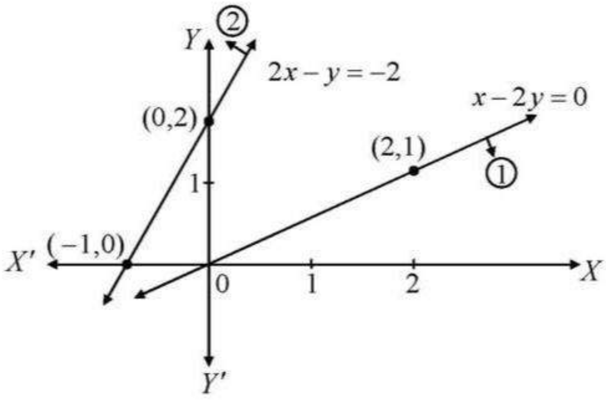
On the line (3), $x=0 \Rightarrow 0-2y=0$
$\Rightarrow y=0$

and $x=2 \Rightarrow 2-2y=0 \Rightarrow y=1$

$\therefore$ $(0, 0)$ and $(2, 1)$ are on the line (3).

$(0, 1)$ is not on this line and it lies in the half-plane of (1) if $0-2(1)\geq 0$, which is not true.

$\therefore$ The closed half-plane not containing $(0, 1)$ is the graph of (1).



The line corresponding to (2) is

$2x-y=-2$ ...(4)

On the line (4), $x=0 \Rightarrow 0-y=-2 \Rightarrow y=2$

and $y=0 \Rightarrow 2x-0=-2 \Rightarrow x=-1$

$\therefore$ $(0, 2)$ and $(-1, 0)$ are on the line (4). $(0, 0)$ is not on this line and it lies in the half-plane of (2) if $2(0)-0\leq -2$, which is not true. $\therefore$ The closed half-plane not containing $(0, 0)$ is the graph of (2).

The graph of the given system is the intersection of half-planes of the inequations in the system.
The intersection of half-planes is empty.

$\therefore$ The solution set of the given inequations is empty.

9. $3x+4y\geq 12$, $x\geq 0$, $y\geq 1$ and $4x+7y\leq 28$

Sol. $3x+4y\leq 12, 4x+7y\leq 28, x\geq 0, y\geq 1$

We take lines $3x+4y=12$ and $4x+3y=12$ taking first line as

$y=\dfrac{12-3x}{4}$ and putting

$x=0, y=3$

$x=4, y=0$

x	0	4
y	3	0

and second line as $y=\dfrac{28-4x}{7}$

Here $x=0, y=4$

$x=3, y=0$

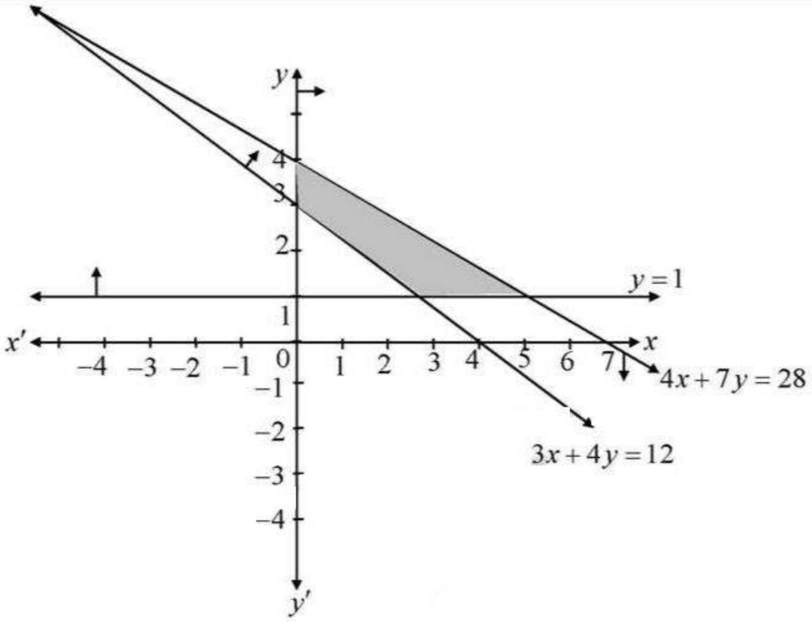
x	0	7
y	4	0

Now putting (0, 0) in $3x+4y\geq 12$ we get $0+0\geq 12$. which is false

$\therefore$ Region show by shading gives $3x+4y\geq 12$

And putting (0, 0) in $4x+7y\leq 28$ we get $0+0\leq 28$ which is True.

Region shown by shading gives $4x+7y\leq 28$. $\therefore$ Closed shaded region represents required solution.



10. Show that the solution set of the following linear constraints is empty :

$x-2y\geq 0,\ 2x-y\leq -2,\ x\geq 0$ and $y\geq 0$

Sol. The given linear inequations are $x-2y\geq 0$ ...(1)

$2x-y\leq -2$ ...(2), $x\geq 0$ ...(3)

$y\geq 0$ ...(4)

The line corresponding to (1) is $x-2y=0$ ...(5)

On the line (5), $x=0 \Rightarrow 0-2y=0 \Rightarrow y=0$

and $x=2 \Rightarrow 2-2y=0 \Rightarrow y=1$

$\therefore$ $(0, 0)$ and $(2, 1)$ are on the line (5).

$(0, 1)$ is not on this line and it lies in the half-plane of (1) if $0-2(1)\geq 0$, which is not true.

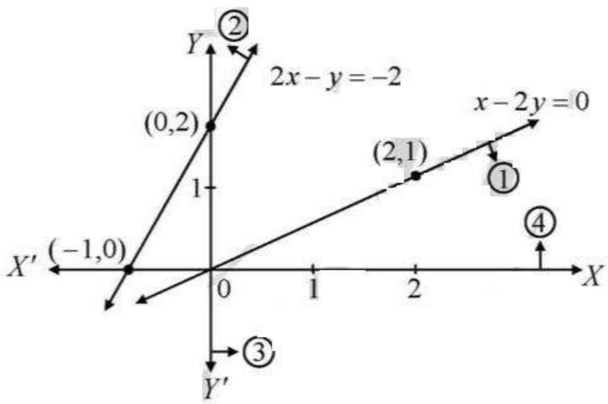
$\therefore$ The closed half-plane not containing $(0, 1)$ is the graph of (1).

The line corresponding to (2) is

$2x-y=-2$ ...(6)

On the line (6), $x=0 \Rightarrow 0-y=-2 \Rightarrow y=2$

and $y=0 \Rightarrow 2x-0=-2 \Rightarrow x=-1$



$\therefore$ $(0, 2)$ and $(-1, 0)$ are on the line (6). $(0, 0)$ is not on this line and it lies in the half-plane of (2) if $2(0)-0\leq -2$, which is not true. $\therefore$ The closed half-plane not containing $(0, 0)$ is the graph of (2).

The inequation $x\geq 0$ represent the closed half-plane on the right of y-axis.

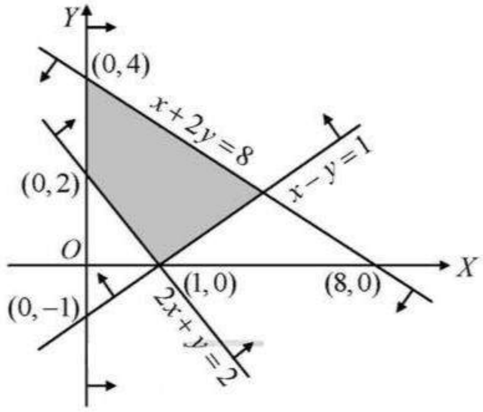
The inequation $y\geq 0$ represent the closed half-plane above x-axis.

The graph of the given system is the intersection of half-planes of the inequations in the system.

The intersection of half-planes is empty.

$\therefore$ The solution set of the given inequations is empty.

11. Find the linear constraints for which the shaded area in the figure given is the solution set.



Sol. Given $x+2y=8$ ...(1)

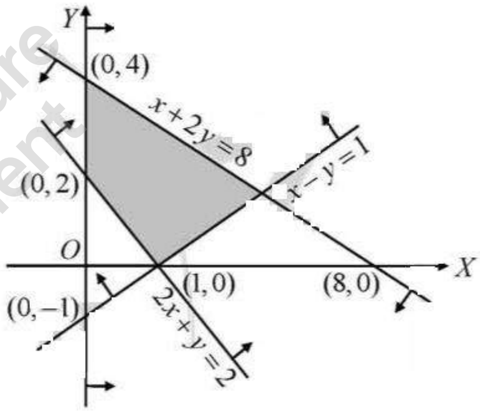
$2x+y=2$ ...(2)

$x-y=1$ ...(3)

Line (1), shaded area and origin lie on the same sides of $x+2y=8$

$\therefore$ Corresponding inequation is,

$x+2y\le 8$.



Line (2), shaded area and origin lie on the opposite side of $2x+y=2$

$\therefore$ Corresponding inequation is,

$2x+y\ge 2$

Line (3), shaded area and origin lie on the same side of $x-y=1$.

$\therefore$ Corresponding inequation is, $x-y\le 1$

$\because$ Shaded area on right side of y-axis.

$\therefore$ Corresponding inequation is $x\ge 0$.

$\because$ Shaded area on above of x-axis.

$\therefore$ Corresponding inequation is $y\ge 0$.

Hence, the linear constraints are $x\ge 0$, $y\ge 0$, $2x+y\ge 2$, $x-y\le 1$ and $x+2y\le 8$

EXERCISE 33 B (Pg.No.: 1376)

1. Find the maximum value of $Z=7x+7y$, subject to the constraints $x\geq 0$, $y\geq 0$, $x+y\geq 2$ and $2x+3y\leq 6$.

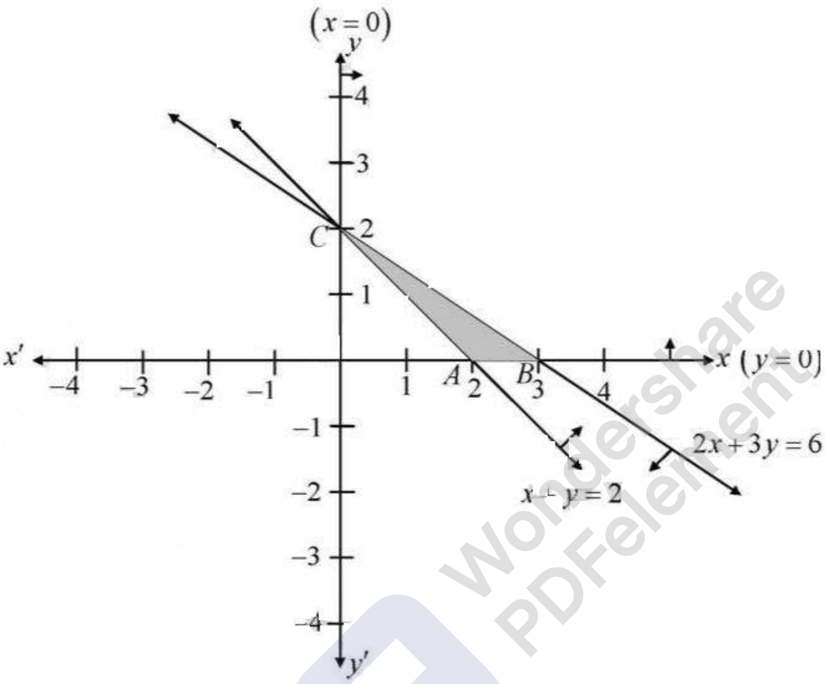
Sol. Given, $Z=7x+7y$, Subject to the constraints

$x+y\geq 2$

$2x+3y\leq 6$

and $x\geq 0$, $y\geq 0$

Now draw the line $x+y=2$ and $2x+3y=6$



and shaded region satisfied by above inequalities.

Here, the feasible region is bounded.

The corner points are given as $A(2, 0)$, $B(3,0)$ and $C(0, 2)$.

The value of Z at $A(3, 0), Z=7\times 2+7\times 0=14$, at $B(3,0), Z=7\times 3+7\times 0=21$ and at $C(0, 2), Z=7\times 0+7\times 2=14$

Thus, the maximum value of Z is 21, which occurs at $B(3,0)$.

2. Maximize $Z=4x+9y$, subject to the constraints $x\geq 0$, $y\geq 0$. $x+5y\leq 200$, $2x+3y\leq 134$.

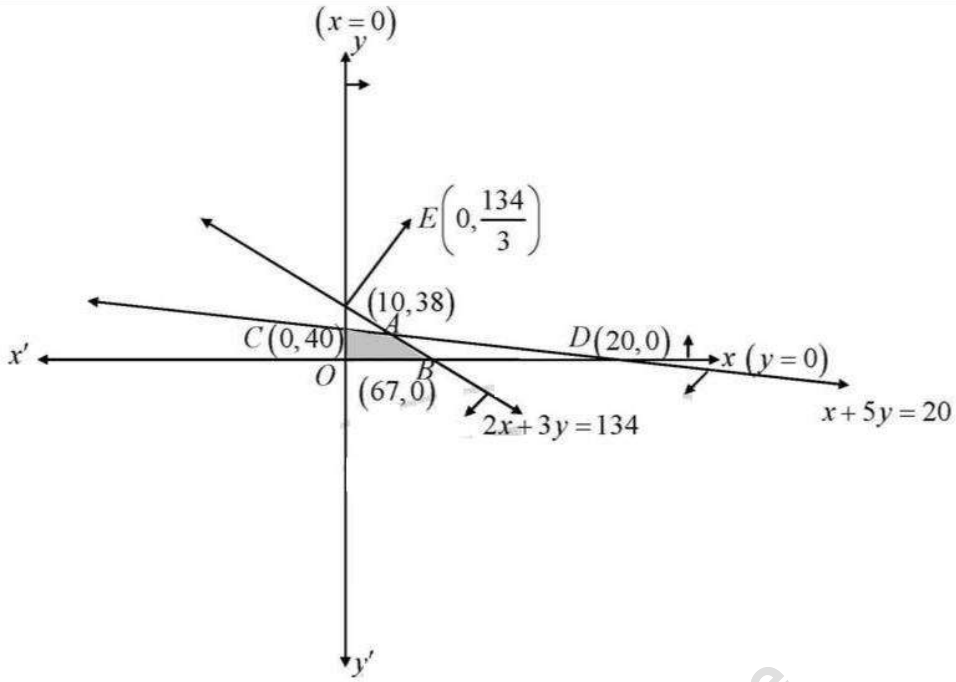
Sol. Given, $Z=4x+9y$, Subject to the constraints

$x+5y\leq 200$

$2x+3y\leq 134$.

and $x\geq 0$, $y\geq 0$

Now draw the line $x+5y=200$ and $2x+3y=134$



and shaded region satisfied by above inequalities.

Here, the feasible region is bounded.

The corner points are given as $O(0,\ 0)$, $A(10,38)$, $B(67,0)$ and $C(0,\ 40)$.

The value of Z at $O(0,\ 0), Z=0$, at $A(10,38), Z=382$ at $B(67,0), Z=268$ and at $C(0,\ 40)$, $Z=360$

Thus, the maximum value of Z is 382, which occurs at $A(10,38)$.

3. Find the maximum value of $Z=3x+5y$, subject to the constraints $-2x+y\le 4$, $x+y\ge 3$, $x-2y\le 2$, $x\ge 0$ and $y\ge 0$.

Sol. Given, $Z=3x+5y$, Subject to the constraints

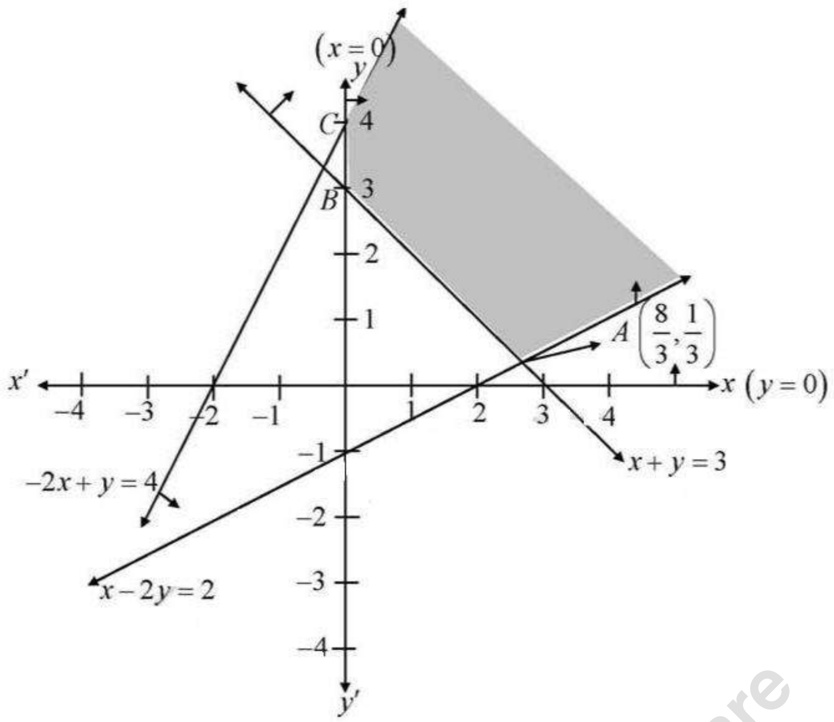
$-2x+y\le 4,$

$x+y\ge 3,$

$x-2y\le 2,$

and $x\ge 0,\ \ y\ge 0$

Now draw the line $-2x+y=4$, $x+y=3$, and $x-2y=2$,



and shaded region satisfied by above inequalities.
Here, the feasible region is unbounded.

The corner points are given as $A\left(\frac{8}{3},\frac{1}{3}\right)$, $B(0,3)$ and $C(0, 4)$.

The value of Z at $A\left(\frac{8}{3},\frac{1}{3}\right)=3\times\frac{8}{3}+5\times\frac{1}{3}=\frac{29}{3}$, at $B(0,3)=3\times0+5\times3=15$ and at $C(0, 4)=3\times0+5\times4=20$

At corner points, the maximum value of Z is 20, which occurs at $C(0, 4)$.

Since the feasible region is unbounded. Therefore the maximum value of z is undefined

4. Minimize $Z=2x+3y$, subject to the constraints $x\geq0$, $y\geq0$, $x+2y\geq1$ and $x+2y\leq10$.

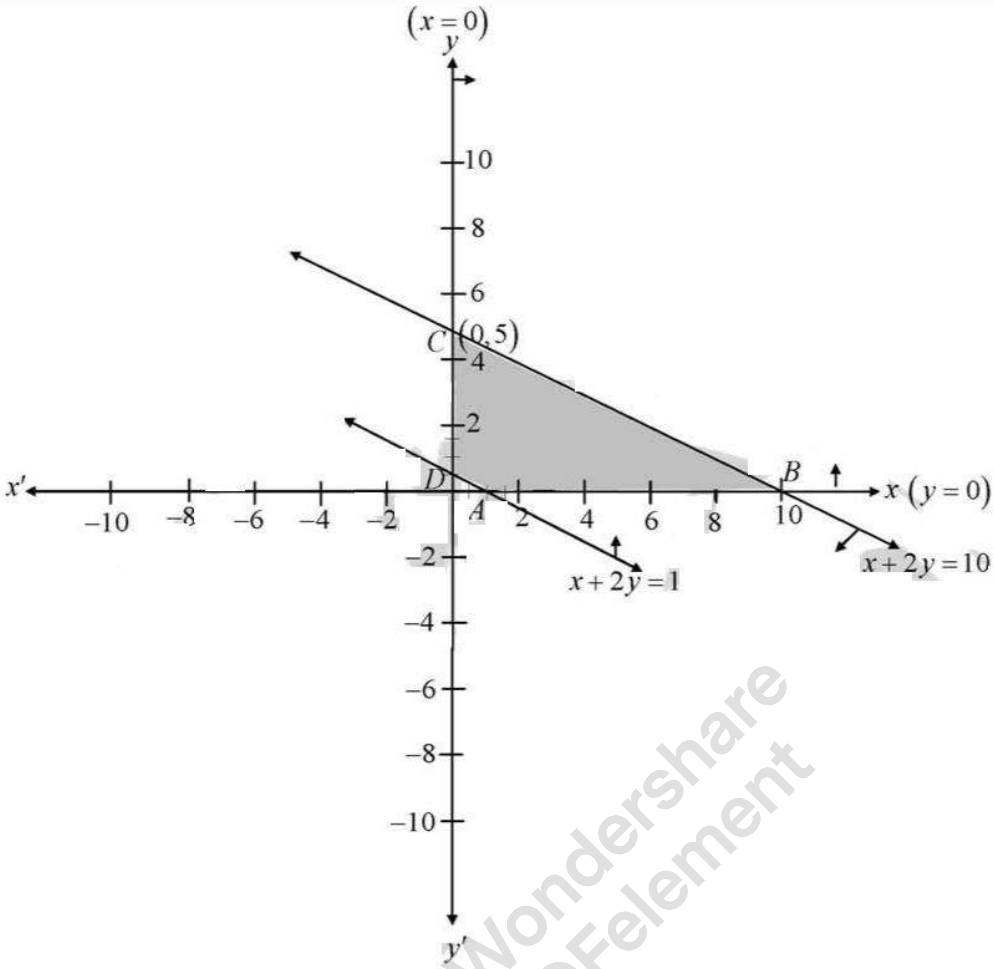
Sol. Given, $Z=2x+3y$, Subject to the constraints

$x+2y\geq1$

$x+2y\leq10$

and $x\geq0$, $y\geq0$

Now draw the line $x+2y=1$ and $x+2y=10$



and shaded region satisfied by above inequalities

Here, the feasible region is bounded.

The corner points are given as $A(1,0)$, $B(10,0)$, $C(0,\ 5)$, and $D\left(0,\frac{1}{2}\right)$

The value of Z at $A(1,0)=2$, at $B(10,0)=20$, at $C(0,\ 5)=15$ and at $D\left(0,\frac{1}{2}\right)=\frac{3}{2}$

At corner points, the minimum value of Z is $\frac{3}{2}$, which occurs at $D\left(0,\ \frac{1}{2}\right)$.

5. Maximize $Z=3x+5y$, subject to the constraints $x+2y\le 2000$, $x+y\le 1500$, $y\le 600$, $x\ge 0$ and $y\ge 0$.

Sol. Given, $Z=3x+5y$, Subject to the constraints

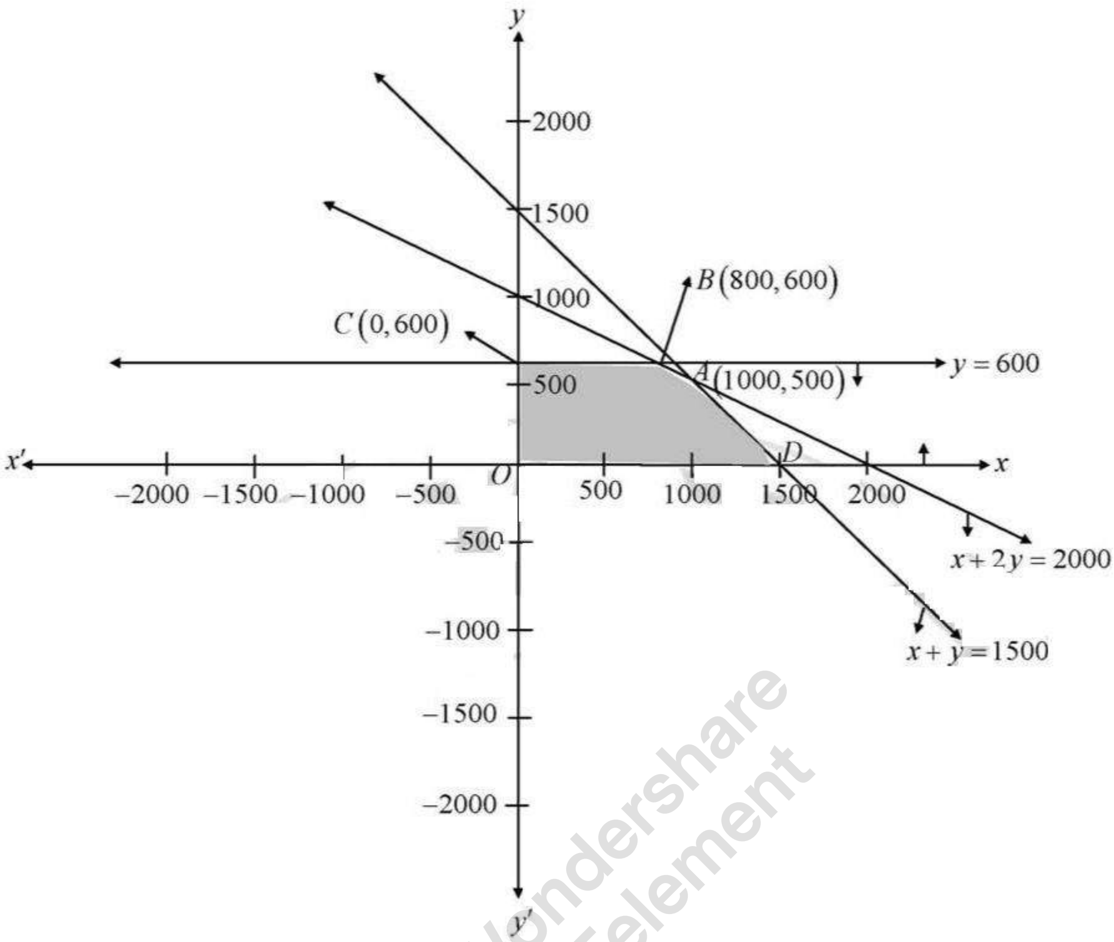
$x+2y\le 2000,$

$x+y\le 1500,$

$y\le 600,$

and $x\ge 0,\ \ y\ge 0$

Now draw the line $x+2y=2000$, $x+y=1500$, and $y=600$,



and shaded region satisfied by above inequalities.

Here, the feasible region is bounded.

The corner points are given as $O(0,0)$, $A(1000,500)$, $B(800,600)$, $C(0,600)$, and $D(1500,0)$

The value of Z at $O(0,0)=0$, at $A(1000,500)=5500$, $B(800,600)=5400$, at $C(0,600)=3000$ and at $D(1500,0)=4500$

Hence the maximum value of Z is 5500, which occurs at $A(1000,500)$.

6. Find the maximum and minimum values of $Z=2x+y$, subject to the constraints

$$x+3y\geq 6, x-3y\leq 3, 3x+4y\leq 24,$$
$$-3x+2y\leq 6, 5x+y\geq 5, x\geq 0 \text{ and } y\geq 0.$$

Sol. Given, $Z=2x+y$, Subject to the constraints

$x+3y\geq 6,$

$x-3y\leq 3,$

$3x+4y\leq 24,$

$-3x+2y\leq 6,$

$5x+y\geq 5,$

and $x\geq 0,\ y\geq 0$

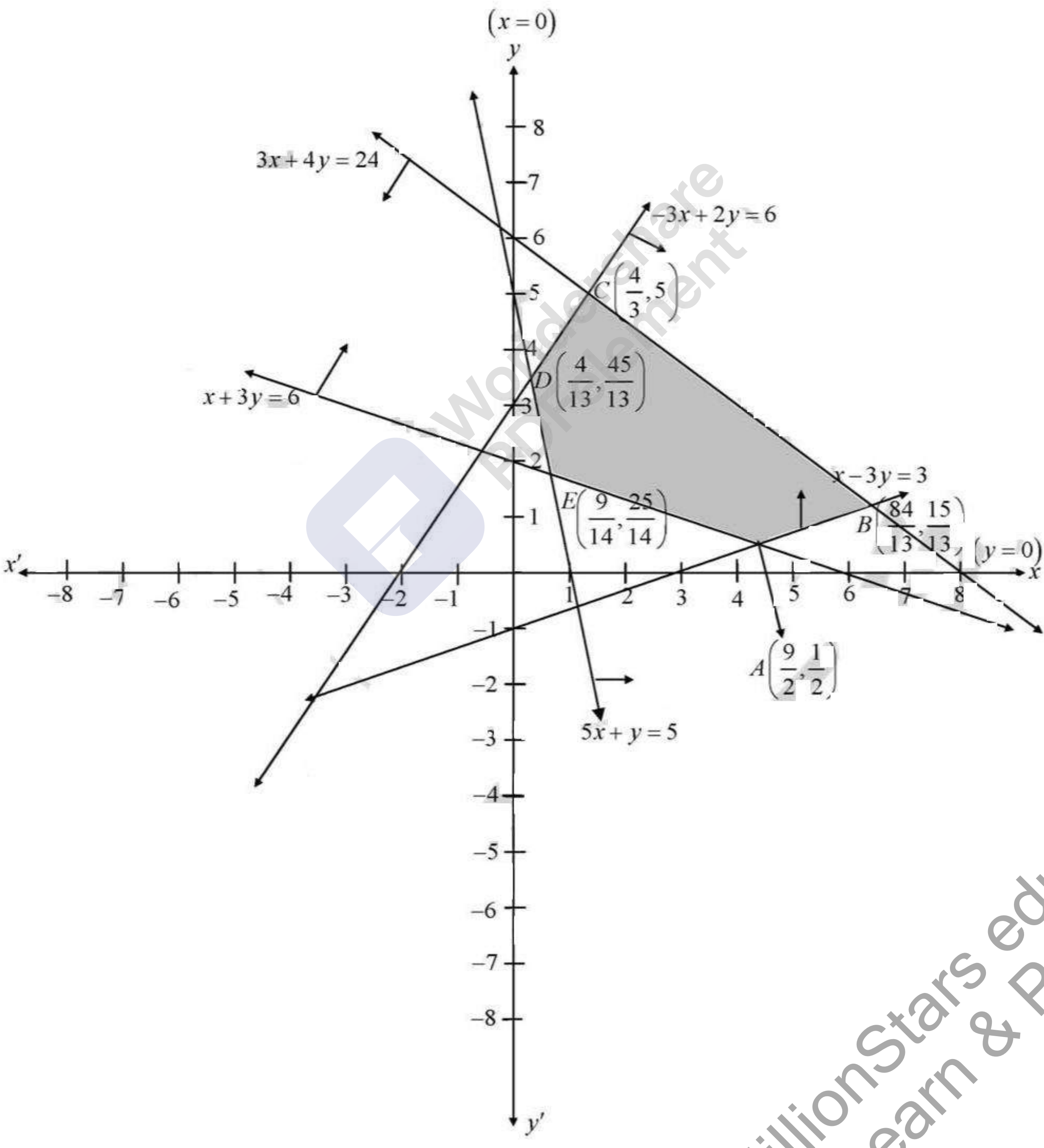
Now draw the line $x+3y=6, x-3y=3, 3x+4y=24, -3x+2y=6$ and $5x+y=5$, and shaded region satisfied by above inequalities.

Here, the feasible region is bounded.

The corner points are given as $A\left(\frac{9}{2},\frac{1}{2}\right)$, $B\left(\frac{84}{13},\frac{15}{13}\right)$, $C\left(\frac{4}{3},5\right)$, $D\left(\frac{4}{13},\frac{45}{13}\right)$ and $E\left(\frac{9}{14},\frac{25}{14}\right)$

The value of Z at $A\left(\frac{9}{2},\frac{1}{2}\right)=\frac{19}{2}$, at $B\left(\frac{84}{13},\frac{15}{13}\right)=\frac{183}{13}$, at $C\left(\frac{4}{3},5\right)=\frac{23}{3}$, at $D\left(\frac{4}{13},\frac{45}{13}\right)=\frac{53}{13}$ and at $E\left(\frac{9}{14},\frac{25}{14}\right)=\frac{43}{14}$

Hence maximum at $Z=\frac{183}{13}$ at $\left(\frac{84}{13},\frac{15}{13}\right)$ and minimum $Z=\frac{43}{14}$ at $\left(\frac{9}{14},\frac{25}{14}\right)$



7. Mr Dass wants to invest Rs 12000 in Public Provident Fund (PPF) and in National bonds. He has to invest at least Rs 1000 in PPF and at least Rs 2000 in bonds. If the rate of interest on PPF is 12% per annum and that on bonds is 15% per annum, how should he invest the money to earn maximum annual income? Also find the maximum annual income.

Sol. Let mr. Dass invest x Rs. in PPF and y Rs. in Bonds

According to question $x \geq 1000, y \geq 2000, x + y \leq 12000$

Let Z = maximum annual income

$\Rightarrow Z = \frac{12x}{100} + \frac{15y}{100}$

Draw the lines $x = 1000, y = 2000$ and $x + y = 12000$

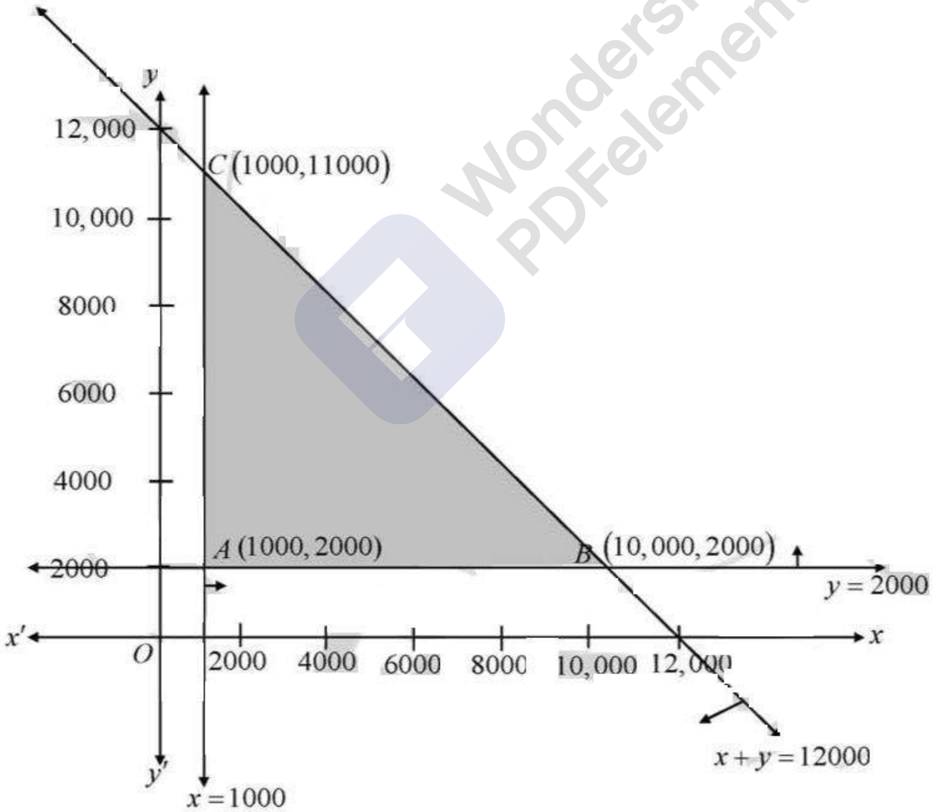
shade the region satisfied by the above inequalities.

The feasible region is $ABCA$, which is bounded

The corner points are $A(1000, 2000)$, $B(10000, 2000)$ and $C(1000, 11000)$.

The values of $Z = \frac{12x}{100} + \frac{15y}{100}$ at the points A, B and C are 420, 1500 and 1770 respectively.

Hence, the maximum annual income is Rs 1770 , he will has to invest Rs. 1000 in PPF and 11000 in bonds



8. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. it takes 1 hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. if the profit on a necklace is Rs 100 and that on a bracelet is Rs 300, how many of each should be produced daily to maximize the profit? It is being given that at least one of each must be produced.

Sol. Let no. of production of necklaces = x and no. of production of bracelets = y

According question $x+y\le 24, \frac{1}{2}x+y\le 16, x>0, y>0$

Let the profit be Z,

$\Rightarrow Z=100x+300y$

Draw the lines $x+y=24$ and $\frac{1}{2}x+y=16$

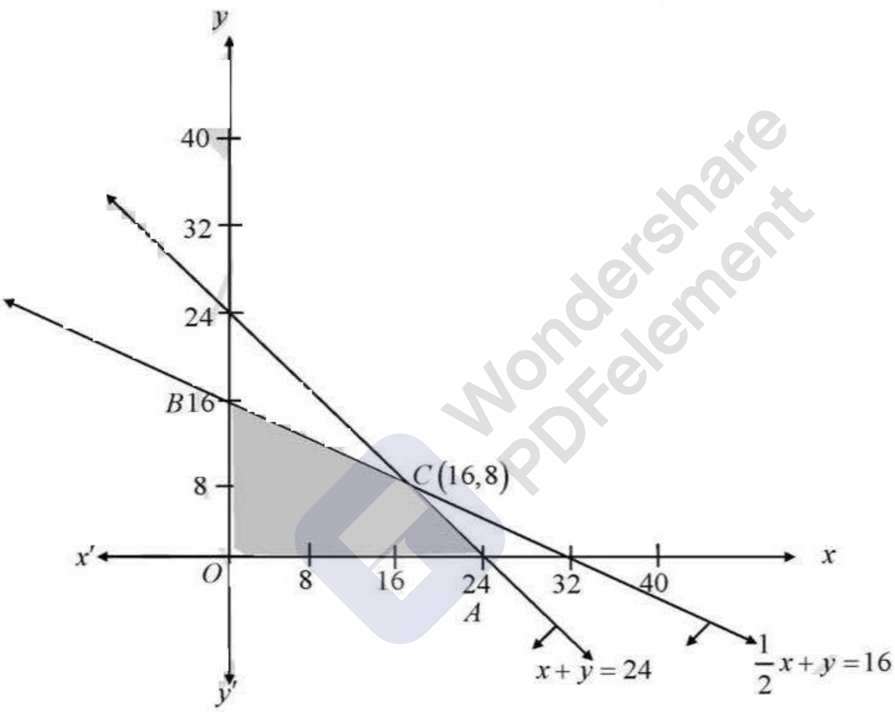
shade the region satisfied by the above inequalities.

The feasible region is $OABC$, which is bounded

The corner points are $O(0,0)$, $A(24,0)$, $B(0,16)$ and $C(16,8)$.

The values of $Z=100x+300y$ at the points O, A, B and C are 0, 2400,4800 and 4000 respectively.

Hence, the maximum annual income is Rs 4800, at B(0,16)



9. A man has Rs 1500 to purchase rise and wheat. A bag of rice and a bag of wheat cost Rs 180 and Rs 120 respectively. He has a storage capacity of 10 bags only. He earns a profit of Rs 11 and Rs 8 per bag of rice and wheat respectively. How may bags of each must he buy to make maximum profit?

Sol. Let x and y be the number of bags of rice and wheat respectively that a shopkeeper buys and sells, then the problem can be formulated as an L.P.P. as follows:

Maximize profit (in Rs) $Z=11x+8y$ subject to the constraints

$180x+120y\le 1500$ (investment constraint)

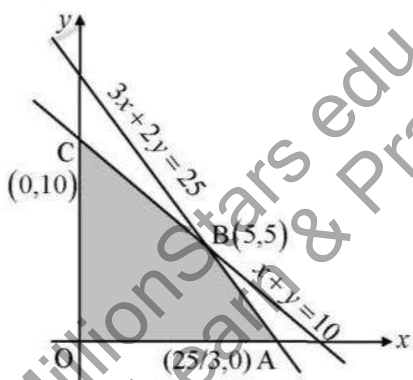
i.e. $3x+2y\le 25$

$x+y\le 10$ (capacity constraint)

$x\ge 0,\ y\ge 0$ (non-negativity constraint)

Draw the lines $3x+2y=25$ and $x+y=10$;

shade the region satisfied by the above inequalities.



The feasible region is $OABC$, which is bounded and convex.

The corner points are $O(0, 0)$, $A\left(\frac{25}{3}, 0\right)$, $B(5, 5)$ and $C(0, 10)$.

The values of $Z = 11x + 8y$ at the points O, A, B and C are 0, 275/3, 95 and 80 respectively.

Hence, the maximum profit is Rs 95 at point B, i.e., when 5 bags of rice and 5 bags of wheat are purchased and sold.

10. A manufacturer produces nuts and bolts for industrial machinery. It takes 1 hour of work on machine A and 3 hours on machine B to produce a packet of nuts while it takes 3 hours on machine A and 1 hour on machine B to produce a packet of bolts. How earns a profit of Rs 17.50 per packet on nuts and Rs 7 per packet on bolts. How many packets of each should be produced each day so as to maximize his profit if he operates his machines for at the most 12 hours a day? Also find the maximum profit.

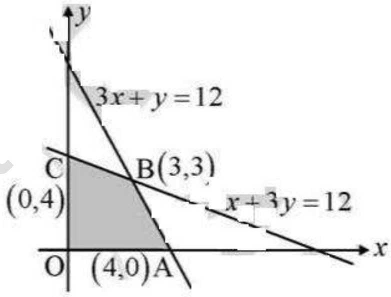
Sol. Let x and y be the number of packages of nuts and bolts respectively produced by the manufacturer, then the problem can be formulated as an L.P.P. as follows:

Maximize the profit (in Rs) $Z = 17.5x + 7y$ subject to the constraints

$x + 3y \le 12$ (machine A constraint)

$3x + y \le 12$ (machine B constraint)

$x \ge 0,\ y \ge 0$ (non-negative constraints)



Draw the lines $x + 3y = 12$ and $3x + y = 12$,

and shade the region satisfied by the above inequalities.

The feasible region is the polygon $OABC$,

which is convex and bounded.

The corner points are $O(0, 0)$, $A(4, 0)$, $B(3, 3)$ and $C(0, 4)$.

The values of $Z = 17.5x + 7y$ (in Rs) at the points O, A, B and C are 0, 70, 73.5 and 28 respectively.

$\therefore$ Maximum profit = Rs 73.50, when 3 packages of nuts and 3 packages of bolts are produced.

11. Two tailors, A and B, earn Rs 300 and Rs 400 per day respectively A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. How many days should each of them work if it is desired to produce at least 60 shits and 32 pairs of trousers at a minimum labour cost?

Sol. Let the tailor A work for x days and the tailor B work for y days, then the problem can be formulated as an L.P.P. as follows:

Minimum the labour charges (in Rs) $Z = 150x + 200y$ subject to the constraints

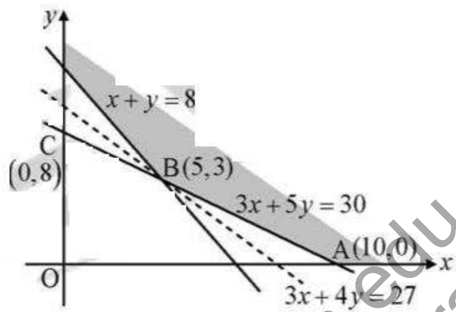
$6x + 10y \ge 60$ (number of shirts constraint)

i.e. $3x + 5y \ge 30$

$4x + 4y \ge 32$ (number of pants constraint)

i.e. $x + y \ge 8$

$x \ge 0,\ y \ge 0$ (non-negativity constraint)



Draw the line $3x + 5y = 30$ and $x + y = 8$,

and shade the region satisfied by the above inequalities.

The feasible region (convex and unbounded) is shown in the figure.

The corner points are $A(10, 0)$, $B(5, 3)$ and $C(0, 8)$.

The values (in Rs) of $Z = 300x + 400y$ at the point A, B and C are 3000, 2700 and 3200 respectively.

Among the values of Z, the least value is 2700.

As the feasible region is unbounded, we draw the half plane $300x+400y<2700$ i.e. $3x+4y\le 27$ and note that there is no common point with the feasible region, therefore, Z has minimum value = Rs 2700 at the point $B(5, 3)$. Hence, the minimum charges = Rs 2700, when the tailor A works for 5 days and the tailor B works for 3 days.

12. A dealer wishes to purchase a number of fans sewing machines. He has only Rs 5760 to invest and has space for at most 20 ites. A fan costs him Rs 360 and a sewing machine, Rs 240. He expects to gain Rs 22 on a fan and Rs 18 on a sewing machine. Assuming that he can sell all the items he can buy, how should he invest the money in order to maximize the profit?

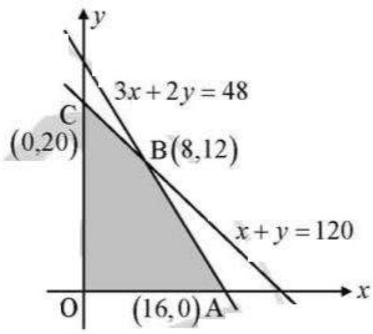
Sol. Let x and y be the number of fans and sewing machines that a dealer purchases and sells, then the problem can be formulated as an L.P.P. as follows:

Maximize the profit (in Rs) $Z=22x+18y$ subject to the constraints

$3600x+2400y\le 57600$ (investment constraint)

i.e. $3x+2y\le 48$

$x+y\le 20$ (storage constraint)



Draw the lines $3x+2y=48$ and $x+y=20$,

and shade the region satisfied by the above inequalities.

The feasible region is the polygon $OABC$,

which is convex and bounded.

The corner of points are $O(0, 0)$, $A(16, 0)$, $B(8, 12)$ and $C(0, 20)$

The value of Z (in Rs) $=22x+18y$ at the points O, A, B and C are 0, 352, 392 and 360 respectively.

$\therefore$ Maximum profit = Rs 392, when 8 fans and 12 sewing machines are purchased and sold.

13. A firm manufactures tow types of products, A and B, and sells them at a profit of Rs 2 on type A and Rs 2 on type B. Each product is processed on two machines, M_1 and M_2. Type A requires one minute of processing time on M_1 and two minutes on M_2. Type B requires one minute on M_1 and one minute on M_2. The machine M_1 is available for not more than 6 hours 40 minutes while M_2 is available for at most 10 hours a day. Find how man products of each type the firm should produce each day in order to get maximum profit.

Sol. The tabular form of the problem is

Products	Machine	
	M_1	M_2
A	1	2
B	1	1

Let firm produces x units of product A and y units of product B.

Then total profit $= Rs.(2x+2y)$

Total time available for M_1 to produce both $A\,\&\,B$ $=x+y\le 400$ (6 hrs + 40 min = 400 min)

Total time available for M_2 to produce both $A\,\&\,B$ $=2x+y\le 600$

Hence, mathematically, L.P.P. can be given as :

Maximize $z=2x+2y$ subject to constraints

$x+y\le 400$ (machine M_1 constraints)

$2x+y\le 600$ (machine M_2 constraints)

$x\ge 0,\ y\ge 0$ (non-negativity constraints)

14. A manufacturer produces two types of soap bars using two machines, A and B. A is operated for 2 minutes and B for 3 minutes to manufacture the first type, while it takes 3 minutes on machine A and 5 minutes on machine B to manufacture the second type. Each machine can be operated at the most for 8 hours per day. The two types of soap bars are sold at a profit of Rs 0.25 and Rs 0.50 each. Assuming that the manufacturer can sell all the soap bars he can manufacture, how many bars of soap of each type should be manufactured per day so as to maximize his profit?

Sol. Let the number of shop A be x and shop B be y

Hence the number of minutes machine A is working $2x+3y$

And it can be operated for 8 hours in max

So $2x+3y \le 480$ ..... (i)

And the number of minutes machine B is working is $3x+5y$

And it can be operated for 8 hours in maximum

So $3x+5y \le 480$ ..... (ii)

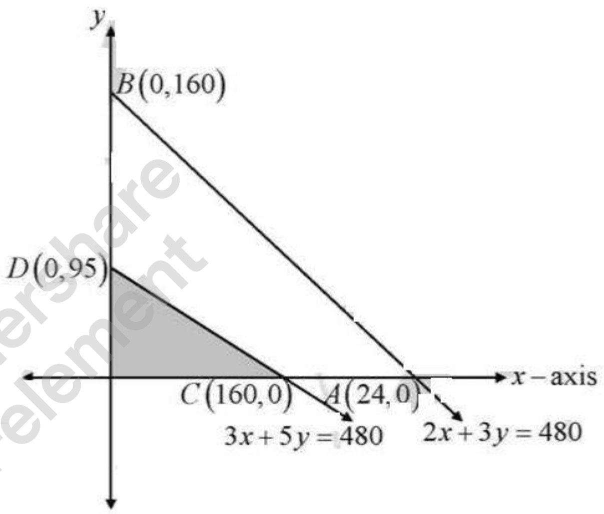
$y \ge 0$ ..... (iii)

$y \ge 0$ ..... (iv)

And the profit on type A is 0.25 and on B is 0.50

So the maximizing equation is $0.25x+0.5y=z$

Sop plotting the curve using all these equation constraint we get



Hence the corner point will be $C(0,96)$ and $D(0,96)$

So $z=0.25\times160+.5\times0=40$ for $(160,0)$

And $Z=0.25\times0+0.5\times96=48$ for (0.96)

Hence the maximum profit can be obtained at $(0,96)$

15. A manufacturer of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20000 bottles of A and 40000 bottles of B but there are only 45000 bottles into which either of the medicines can be put. Furthermore, it takes 1 hour to prepare enough material to fill 1000 bottles of B, and there are 66 hours available for this operation. The profit is Rs 8 per bottle for A and Rs 7 per bottle for B.

How should the manufacturer schedule the production in order to maximize his profit? Also, find the maximum profit.

Sol. Let manufacturer produced bottle of medicine A and y bottle of B and let Z be the total profit LPP is given by

Maximize ..... (profit)

$z=8x+7y$

Subject to contrains

$x+y \le 45,000$ .... (combined production constraint)

$x \le 20,000$ .... (bottle A constraint)

$y \le 40,000$ .... (bottle B constraint)

$\frac{13x}{1000}+\frac{1y}{1000} \le 66$ .... (time available constraint)

Or $3x+y \le 66,000$

And $x \ge 0\,; y \le 0$

Draw the lines $x+y=45,000$, $x=20,000$, $y=40,000$ and $3x+y=66,000$,

and shade the region satisfied by the above inequalities.

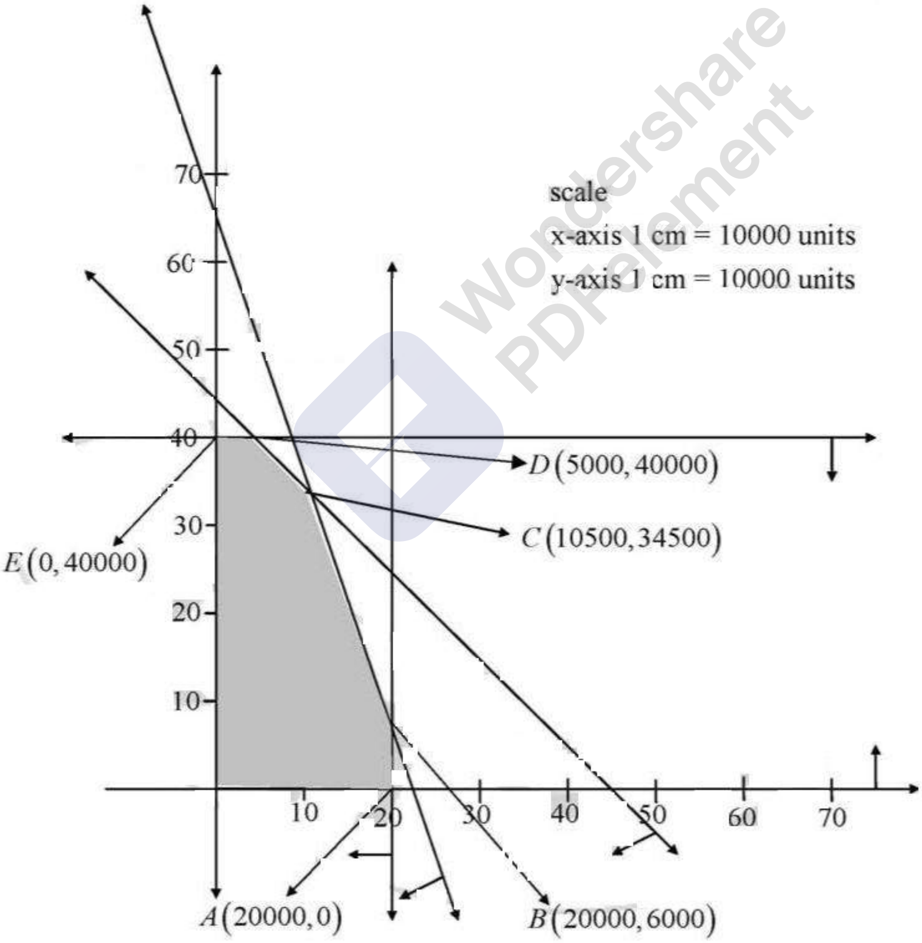
The feasible region is the polygon $OABC$,

which is convex and bounded.

Corner points	Value of the objective function $z=8x+7y$
$A(20000,0)$	$z=1,60,000$
$B(20000,6000)$	$z=1,60,000+42,000=2,02000$
$C(10500,34500)$	$z=84,000+2,41,500=32550$
$D(5000,40000)$	$z=40,000+2,80,000=3,20,000$
$E(0,40000)$	$z=2,80,000$

Z is maximizing at $x=10,500$ & $y=34,500$

$\therefore$ manufacture should fill 10,500 bottles of medicine A and 34,500 bottles of medicine B and the maximum profit is Rs. 32,5500



16. A toy company manufactures two types of dolls, A and B. Each doll of type B takes twice as long to produce as one of type A, and the company would have time to make a maximum of 2000 per day, if it produces only type A. the supply of plastic is sufficient to produces only type A. The supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). Type B requires a fancy dress of which there are only 600 per day available. If the company makes profits of Rs 3 and Rs 5 per doll respectively on dolls A and B how many of each should be produced per day in order to maximize the profit? Also, find the maximum profit.

Sol. Let the company manufacture x dolls of type A and y dolls of type B, then the problem can be formulated as an L.P.P. as follows :

Maximize profit (in Rs) $Z = 3x + 5y$ subject to the constraints

$x + 2y \leq 2000$ (time constraint)

$x + y \leq 1500$,

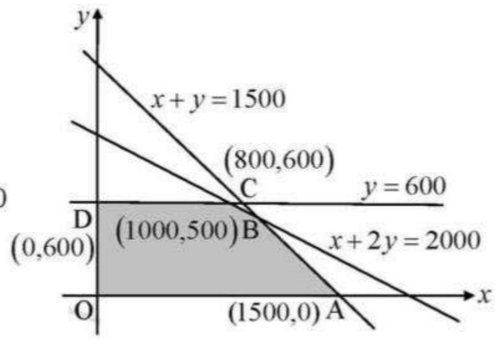
$y \leq 600$,

$x \geq 0,\ y \geq 0$

Now draw the line $x + y = 1500$, $x + 2y = 2000$ and $y = 600$ and shade the region satisfied by the given inequalities.

The feasible region in shown shaded in the figure.

It is convex and bounded.



The corner points are $O(0,\ 0)$, $A(1500,\ 0)$, $B(1000,\ 500)$, $C(800,\ 600)$ and $D(0,\ 600)$.

The values (in Rs) of $Z = 3x + 5y$ at the points $O,\ A,\ B,\ C$ and D are 0, 45000, 5500, 5400 and 3000 respectively.

So, maximum profit = Rs 5500 at $B(1000,\ 500)$ i.e when 1000 dolls of type A and 500 dolls of type B are manufactured.

17. A small manufacturer has employed 5 skilled men and 10 semiskilled men and makes an article in two qualities, a de luxe model and an ordinary model. The making of a de luxe model requires 2 hours' work by a skilled man and 2 hours' work by a semiskilled man. The ordinary model requires 1 hour by a skilled man and 3 hours by a semiskilled man. By union rules, no man can work more than 8 hours per day. The manufacturer gains Rs 15 on the de luxe model and Rs 10 on the ordinary model. How many of each type should be made in order to maximize his total daily profit? Also, find the maximum daily profit.

Sol. Let x articles of deluxe model and y articles of ordinary model be produced.

As 5 skilled men work for atmost 8 hours a day, total skilled men's working hours available ≤ 40;

and as 10 semi-skilled men work atmost 8 hours a day, total semi-skilled men's working hours available ≤ 80.

The problem can be formulated as an L.P.P. as follows :

Maximize the profit (in Rs) $Z = 15x + 10y$ subject to the constraints

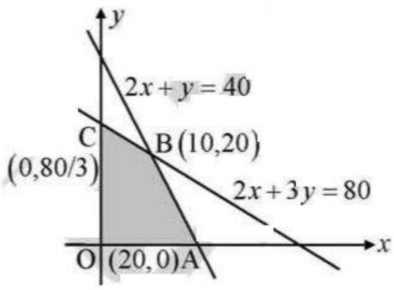
$2x + y \leq 40$, $2x + 3y \leq 80$, $x \geq 0,\ y \geq 0$

Now draw the line $2x + y = 40$ and $2x + 3y = 80$

and shade the region satisfied by the given inequalities.

The feasible region is shown shaded in the figure.

It is convex and bounded.



The corner points are $O(0,\ 0)$, $A(20,\ 0)$, $B(10,\ 20)$ and $C\left(0,\ \frac{80}{3}\right)$.

The values (in Rs) $Z = 15x + 10y$ at the points $O,\ A,\ B$ and C are 0, 300, 350 and $\frac{800}{3}$.

Hence, maximum profit = Rs 350 at $B(10,\ 20)$ i.e. when 10 articles of deluxe model and 20 articles of ordinary model are manufactured.

18. A company producing soft drinks has a contract which requires a minimum of 80 units of chemical A and 60 units of chemical B to go in each bottle of the drink. The chemicals are available in a prepared mix from two different suppliers. Supplier X has a mix of 4 units of A and 2 units of B that costs Rs 10, and the supplier Y has a mix of 1 unit of A and 1 units of B that costs Rs 4. How many mixes from X and Y should the company purchase to honour the contract requirement and yet minimize the cost?

Sol. Let x and y units of packet of mixes ae purchased from S and T respectively. If Z is total cost then

$Z = 10x + 4y$ ..... (i)

Is objective function which we have to minimize

Here constraints are

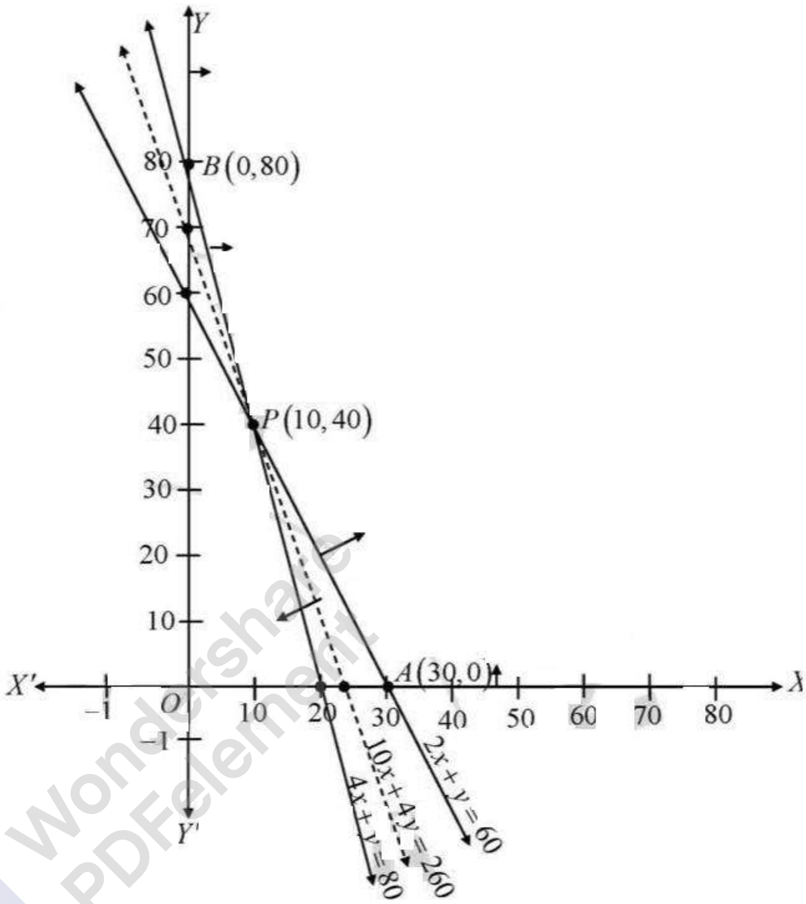
$4x + y \geq 80$ ..... (ii)

$2x + y \geq 60$ ... (iii)

And $x \geq 0$ .... (iv)

$y \geq 0$ .... (v)

On plotting graph of above constraints or inequalities (ii). (iii), (iv) and (v) we get shaded region having corner point A, P, B as fesible region



For coordinate of P

Point of intersection of

$2x + y = 60$ ... (vi)

And $4x + y = 80$ ....

(vii)

(vi) – (vii)

$\Rightarrow 2x + y - 4x - y = 60 - 80$

$\Rightarrow -2x = -20 px = 10$

$\Rightarrow y = 40$

Co-ordinate of $P \equiv (10, 40)$

Now the value of z is evaluated at corner point in the following table

← minimum

Corner point	$x = 10x + 4y$
$A(30,0)$	300
P (10,40)	260
B (0, 80)	320

Since feasible region is unbounded. Therefore we have to draw the graph of the inequality $10x + 4y < 260$ ... (viii)

Since the graph of inequality (viii) does not have any point common

So the minimum value of z is 260 at (10,40)

i.e., minimum cost of each bottle is Rs. 260 if the company purchases 10 packets of mixes from S and 40 packets of mixes from supplier T

19. A small firm manufactures gold rings and chins. The combined number of rings and chains manufactured per day is at most 24. it takes 1 hour to make a ring and half an hour for a chain. The maximum number of hours available per day is 16. if the profit on a ring is Rs 300 and that on a chin is Rs 190, how many of each should be manufactured daily so as to maximize the profit?

Sol. Let x and y be the number of gold rings and chains respectively manufactured per day, then the problem can be formulated as an L.P.P. as follows :

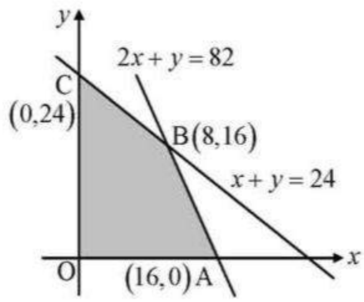
Maximize the profit (in Rs) $Z = 300x + 190y$ subject to the constraints

$x+y\le 24$ (manufacturing constraint)

$1\cdot x+\frac{1}{2}y\le 16$ (time constraint)

i.e. $2x+y\le 32$

$x\ge 0,\ y\ge 0$ (non-negativity constraints)



Draw the lines $x+y=24$ and $2x+y=32$,

and shade the region satisfied by the above inequalities.

The feasible region is the polygon $OABC$, which is convex and bounded.

Corner points are $O(0,0)$, $A(16,0)$, $B(8,16)$ and $C(0,24)$.

The values of Z (in Rs) $=300x+190y$ at $O,\ A,\ B$ and C are 0, 4800, 5440 and 4560 respectively.

$\therefore$ Maximum profit = Rs 5440, when 8 rings and 16 chains are manufactured per day.

20. A manufacturer makes two types, A and B, of teapots. Three machines are needed for the manufacture and the time required for each teapot on the machines is given below.

Machine	Time (In minutes)		
Type	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each teapot of type A is 75 paise and that on each teapot of type B is 50 paise, show that 15 teapots of type A and 30 of type B should be manufactured in a day to get the maximum profit.

Sol. Let x and y be the number of teapots to types A and B respectively manufactured by the manufacturer, then the problem can be formulated as an L.P.P. as follows:

Maximize the profit (in Rs) $Z=0.75x+0.5y$ subject to the constraints

$12x+6y\le 360$ (machine I constraint)

i.e. $2x+y\le 60$,

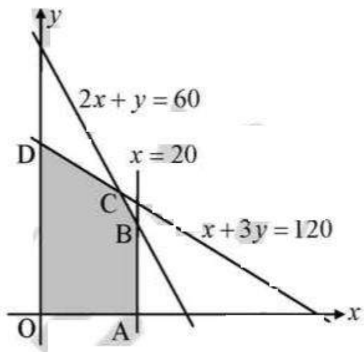
$18x+0y\le 360$ (machine II constraint)

i.e. $x\le 20$,

$6x+3y\le 360$ (machine III constraint)

i.e. $2x+3y\le 120$

$x\ge 0,\ y\ge 0$ (non-negative constraints)



Draw the lines $2x+y=60$, $x=20$ and $2x+3y=20$,

and shade the region satisfied by the above inequalities.

The feasible region is the polygon $OABCD$, which is convex and bounded.

The corner points are $O(0,0)$, $A(20,0)$, $B(20,20)$, $C(15,30)$ and $D(0,40)$.

The values (in Rs) of $Z=0.75x+0.5y$ at the points $O,\ A,\ B,\ C$ and D are 0, 15, 25, 26.25 and 20 respectively.

$\therefore$ Maximum profit = Rs 26.25, when 15 teapots of type A and 30 teapots of type B are manufactured.

21. A manufacturer makes tow products, A and B. Product A sells at Rs 200 each and takes 1/2 hour to make. Product B sells at Rs 300 each and takes 1 hour to make. There is a permanent order for 14 of product A and 16 of product B. A working week consists of 40 hours of production and the weekly turnover must not be less than Rs 10000. If the profit on each of the product A is Rs. 20 and on product B, it is Rs. 30 then how many of each should be produced so that the profit is maximum? Also, find the maximum profit.

Sol. Let the number of articles produced per week be x of A and y of B

Then, $\frac{1}{2}x + y \le 40, 200x + 300y \ge 10000, x \ge 14, y \ge 16$

Profit function is $Z = 20x + 30y$

Maximize $Z = 20x + 30y$, subject to the constraints

$x + 2y \le 80, 2x + 3y \ge 1000, x \ge 14, y \ge 16$

Draw the lines $x + 2y = 80, 2x + 3y = 1000, x = 14, y = 16$

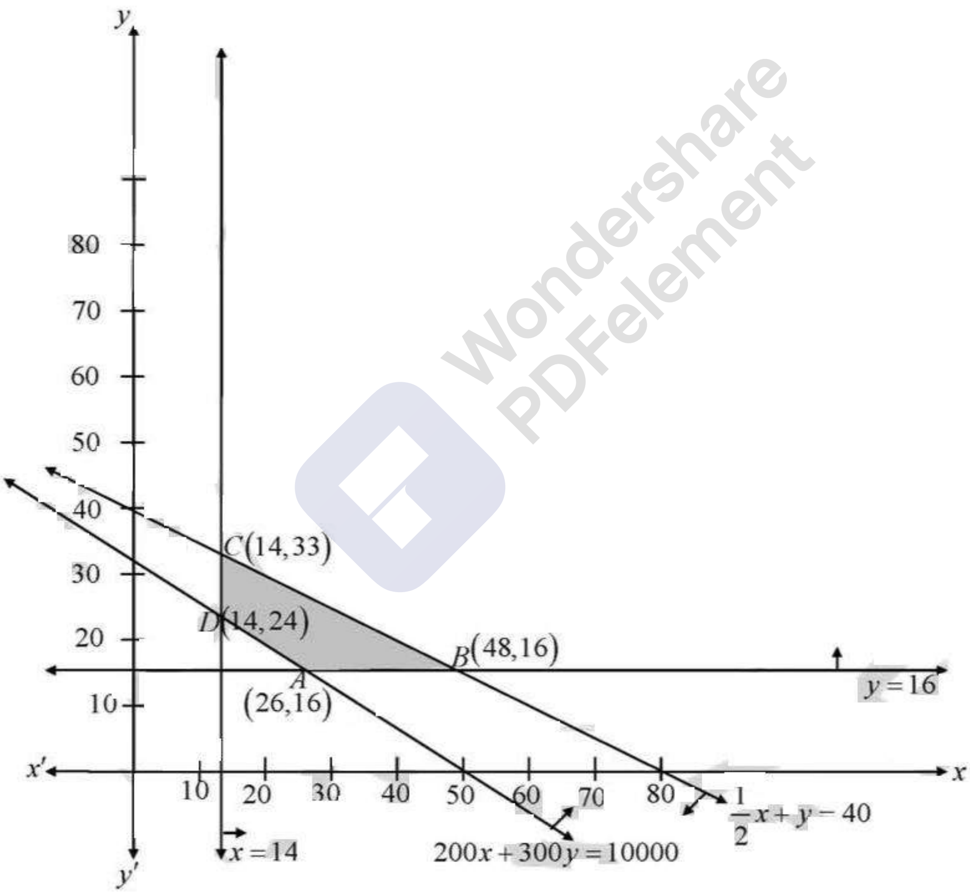
shade the region satisfied by the above inequalities.

The feasible region is ABC, which is bounded

The corner points are $A(26,16)$, $B(48,16)$, $C(14,33)$ and $D(14,24)$.

The values of $Z = 20x + 30y$, at the points A, B, C and D are 1200, 1440,1270 and 1000 respectively.

Hence, the maximum profit is Rs 1440, at C(48,16)



22. A man owns a field f area 1000 m^2. He wants to plant fruit trees in it. He has a sum of Rs 1400 to purchase young trees. He has the choice of two types of trees. Type a requires 10 m^2 of ground per tree and costs Rs 20 per tree, and type B requires 20 m^2 of ground per tree and costs Rs 25 per tree. When full grown, a type A tree produces an average of 20 kg of fruit which can be sold at a profit of Rs 2 per kg and a type B tree produces an average of 40 kg of fruit which can be sold at a profit of Rs 1.50 per kg. How many of each type should be planted to achieve maximum profit when trees are fully grown? What is the maximum profit?

Sol. Let x and y be the number of trees of types A and B respectively.

The profit on a tree of type $A = \text{Rs}\ (20 \times 2) = \text{Rs}\ 40$ and

Profit on a tree of type $B = \text{Rs}\ (40 \times 1.5) = \text{Rs}\ 60$.

The problem can be formulated as an L.P.P. as follows :

Maximize the profit (in Rs) $Z = 40x + 60y$ subject to the constraints

$20x + 25y \le 1400$ i.e. $4x + 5y \le 280$

$10x + 20y \le 1000$ i.e. $x + 2y \le 100$, $x \ge 0$, $y \ge 0$

Now draw the line $4x + 5y = 280$ and $x + 2y = 100$ and shade the region satisfied by the given inequalities. The feasible region is shown shaded in the figure.

It is convex and bounded. The corner points are $O(0, 0)$, $A(70, 0)$, $B(20, 30)$ and $C(0, 50)$.

The values (in Rs) of $Z = 40x + 60y$ at the points O, A, B and C are 0, 2800, 3200 and 3000.

Hence, maximum profit = Rs 3200 at $B(20, 40)$ i.e. when 20 trees of type A and 40 trees of type B are planted.

23. A publisher sells a hardcover edition of a book for Rs 72 and a paperback edition of the same for Rs 40. costs t the publisher are RS 56 and RS 28 respectively in addition to weekly costs of Rs 9600. Both types require 5 minutes of printing time although the hardcover edtion requires 10 minutes of binding time and the paperback edition requires only 2 minutes. Both the printing and binding operations have 4800 minutes available each week. How many of each type of books should be produced in order to maximize the profit? Also, find the maximum profit per week.

Sol. Let the sale of hand cover edition be h and that of paperback editions be t

SP of a hard cover edition of the textbook = Rs. 72

SP of a paperback edition of the textbook = Rs. 40

Cost to the publisher for hard cover edition = Rs. 56

Cost to the publisher for a paperback editon = Rs. 28

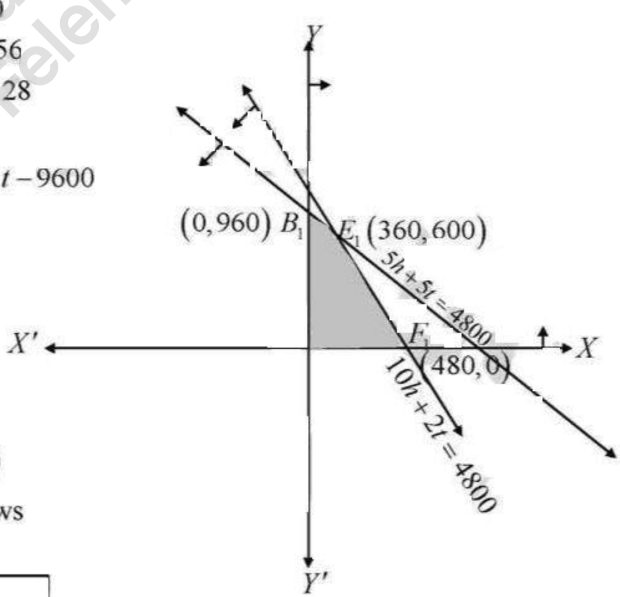
Weekly cost to the publisher = Rs. 9600

Profit to be maximized $Z = (72 - 56)h + (40 - 28)t - 9600$

$\Rightarrow Z = 16h + 12t - 9600$

$5(h + t) \le 4800$

$10h + 2t \le 4800$



The corner points are

$O(0,0), B_1(0,960), E_1(360,600)$ and $F_1(480,0)$

The values of Z at these corner points are as follows

Corner point	$z = 16h + 12t - 9600$
O (0,0)	–9600
B_1 (0,960)	1920
E_1 (360,600)	3360
F_1 (480, 0)	–1920

The maximum value of Z is 3360 which is attained at E_1 (360,600)

The maximum profit is 3360 which is obtained by selling 360 copies of hardcofer edition and 600 copies of paperback edition

24. A gardener has a supply of fertilizers of the type I which consists of 10% nitrogen and 6% phosphoric acid, and of the type II which consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, he finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type-I fertilizer costs 60 paise per kg and the type-II fertilizer costs 40 paise per kg, determine how many kilograms of each type of fertilizer should be used so that the nutrient requirements are met at a minimum cost. What is the minimum cost?

Sol. Let x kg of fartiliser type I and y kg of fertilizer type II be used, then the problem can be formulated as an L.P.P. as follows :

Minimize the cost (in Rs) $Z = 0.6x + 0.4y$ subject to the constraints

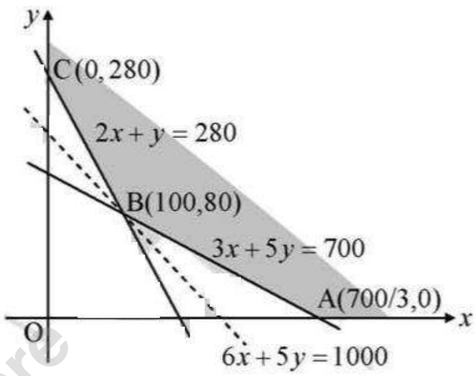
10% of x + 5% of $y \geq 14$ (nitrogen constraint)

i.e. $\frac{10}{100}x + \frac{5}{100}y \geq 14$ i.e. $2x + y \geq 280$

6% of x + 10% of $y \geq 14$ (phosphoric acid constraint)

i.e. $\frac{6}{100}x + \frac{10}{100}y \geq 14$

i.e. $3x + 5y \geq 700$



Draw the lines $2x + y = 280$ and $3x + 5y = 700$,

and shade the region satisfied by the above inequalities.

The feasible region (convex and unbounded) is shown in the figure.

The corner points are $A\left(\frac{700}{3}, 0\right)$, $B(100, 80)$ and $C(0, 280)$.

The values (in Rs) $Z = 0.6x + 0.4y$ at the points A, B and C are 140, 92 and 112 respectively.

Among the values of Z, the least value is 92. As the feasible region is unbounded, we draw the half plane $0.6x + 0.4y < 92$ and note that there is no common points with the feasible region, therefore, Z has minimum value = Rs 92 at the point $B(100, 80)$. Hence, the minimum cost = Rs 92, when 100 kg of fertilizer type I and 80 kg of fertilizer type II are mixed.

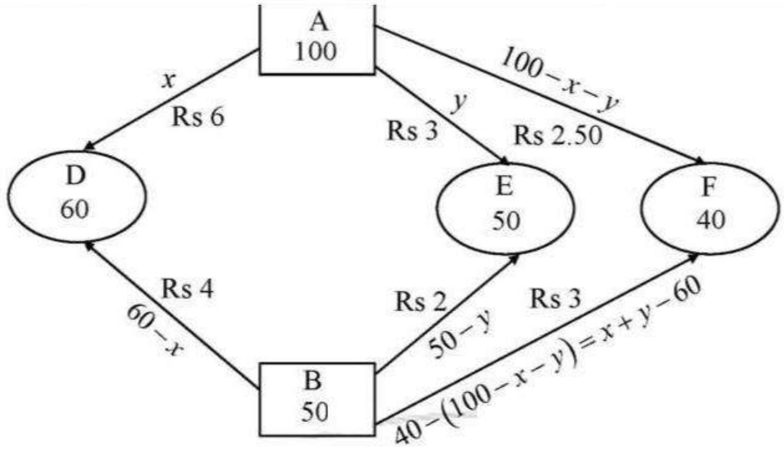
25. Two godowns, A and B, have a grain storage capacity of 100 quintals and 50, quintals respectively. Their supply goes to three ration shops, D,E and F, whose requirements are 60, 50 and 40 quintals respectively. The costs of transportation per quintal from the godowns to the shops are given in the following table.

	Cost of transportation (in Rs per quintal)	
From / To	A	B
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

How should the supplies be transported in order that the transportation cost is minimum?

Sol. Total quantity of grain available $= 100 + 50$ i.e. 150 quintals and requirement $= 60 + 50 + 40$ i.e. 150 quintals. So, there is no mismatch between availability and requirement.

Let x quintals and y quintals of grain be supplied from godown A to ration shops D and E, then the following diagram represents the whole supply from godowns A and B to ration shops D, E and F : If Z (in Rs) is the total cost of transportation, then



$$Z = 6x + 3y + 2.50(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60)$$

$$\Rightarrow Z = 2.50x + 1.50y + 410$$

The problem can be formulated as an L.P.P. as follows:

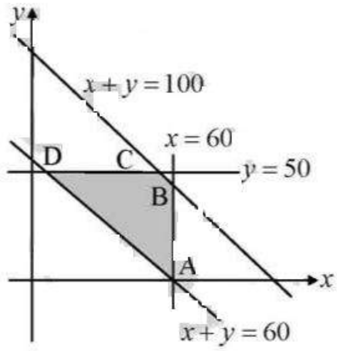
Minimize $Z = 2.5x + 1.5y + 410$ subject to the constraints

$x \le 60,\ y \le 50,$

$x + y \le 100,$

$x + y \ge 60,$

$x \ge 0,\ y \ge 0$



Draw the lines $x = 60,\ y = 50,\ x + y = 100$ and $x + y = 60$;

and shade the region satisfied by the above inequalities.

The feasible region is polygon $ABCD$, it is convex and bounded.

The corner points are $A(60,\ 0),\ B(60,\ 40),\ C(50,\ 50)$ and $D(10,\ 50)$.

The values of $Z = 2.5x + 1.5y + 410$ at the points A, B, C and D are 560, 620, 610 and 510 respectively. $\therefore$ Minimum value of $Z =$ Rs 510 at the point $D(10,\ 50)$.

Hence, supply 10, 50, 40 quintals of grain from godown A and 50, 0, 0 quintals of grain from godown B; and the minimum cost of transportation is Rs 510.

26. A brick manufacturer has tow depots, P and Q. with stocks of 30000 and 20000 bricks respectively. He receives orders from three buildings, A, B, C, for 15000, 20000 and 15000 bricks respectively. The costs of transporting 1000 bricks to the buildings from the depots are given below.

	Cost of transportation (in Rs per 1000 bricks)		
To / From	A	B	C
P	40	20	30
Q	20	60	40

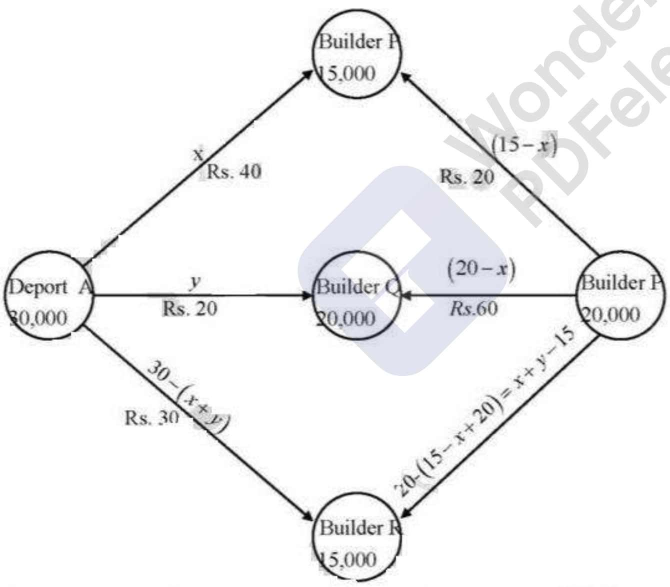
How should the manufacturer fulfil the orders so as to keep the cost of transportation minimum?

Sol. A brick manufacturer has depots. A and B, with stocks of 30,000 and 20,000 bricks respectively. He receives orders from three builders P, Q and R for 15,000, 20,000 and 15,000 bricks respectively. The cost in Rs of transporting 1000 bricks to the builders from the depots are given below

To from	P	Q	R
A	40	20	30
B	20	60	40

Let the depot A transport x thousand bricks to builder P and y thousand bricks to builder Q. Then the above LPP can be stated mathematically as follows

A manufacturer can be exhibited diagrammatically as shown in fig



Let the depot A transport x thousands bricks to builders P, y thousands to builder Q. since the depot A has stock of 30,000 bricks. Therefore, the remaining bricks i.e. $30-(x+y)$ thousands bricks will be transported to the builder R

Therefore, $x>0, y>0$ and $30-(x+y)>0$ $x \geq 0, y>0$ and $x+y<30$

Now the requirement of the builder is of 15000 bricks and x thousand bricks are transported from the depot A. therefore, the remaining $(15-x)$ thousands bricks are to be transported from the depot at B.

The requirement of the builder Q is of 20,000 bricks and y thousand bricks are transported from depot A

Therefore, the remaining $(20-y)$ thousand bricks are to be transported from depot B.

Now depot B has $20-(15-x+20-y)=x+y-15$ thousand bricks which are to be transported to the builder R

Also $15-x>0, 20-y>0$ and $x+y-15>0$

$x<15, y<20$ and $x+y>15$

The tranportion cost from the depot A to the builder P, Q and R are respectively Rs 40x, 20y and $(30-x-y)$. similarly, the transportation cost from the depot B to the builders P, Q and R are repectively Rs. $20(15-x), 60(20-y)$ and $40(x+y-15)$ respectively. Therefore, the total transportation cost Z is given by

$$Z=40x+20y+30(30-x-y)+20(15-x)+60(20-y)+40(x+y-15)$$

$$Z=30x-30y+1800$$

Minimize $Z=30x+1800$

Subject to $x+y<30$

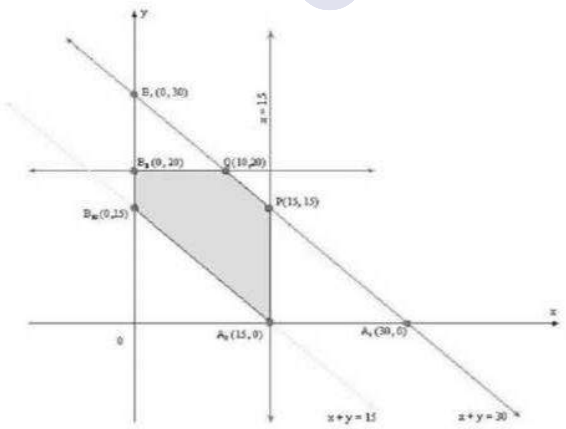
$x+y<15$

$x<20$

$y<15$

And $x<0, y>0$

To solve the LPP graphically, we first convert in equations into equations and then draw the corresponding lines. The feasible region of the LPP is shaded in fig the coordinates of the corner points of the feasible region $A_2PQB_3B_2$ are $A_2(15,0)P(15,15), Q(10,20), B_3(0,20)$ and $B_2(0,15)$. These points have been obtained by solving the cooresponding intersecting lines simultaneously



The value of the objective function at the corner points of the feasible region are given in the following table

Point (x, y)	value of the object function $X = 30x - 30y + 1800$
$A_2(15, 0)$	$Z = 30\times15 - 30\times0 + 1800 = 2250$
$P(15, 5)$	$Z = 30\times15 - 30\times15 + 1800 = 1800$
$Q(10, 20)$	$Z = 30\times10 - 30\times20 + 1800 = 1500$
$B_3(0, 20)$	$Z = 30\times0 - 30\times20 + 1800 = 1200$
$B_2(0, 15)$	$Z = 30\times0 - 30\times15 + 1800 = 1350$

Clearly Z is minimum at $x = 0, y = 20$ and the minimum value z is 1200

Thus the manufactuer should supply 0, 20000 and 10000 bricks to builders P, Q and R from depot A and 1000, 0, 5000 bricks to builders P, Q and R from depot B respectively. In this case the minimum transportation cost will be Rs. 1200

27. A medicine company has factories at tow places, X and Y, From there places, supply is made to each of its three agencies situated at P, Q and R. The monthly requirements of the agencies are respectively 40 packets, 40 packets and 50 packets of medicines, while the production capacity of the factories at X and Y are 60 packets and 70 packets respectively. The transportation cost per packet from the factories to the agencies are given as follows.

	Transportation cost per packet (in Rs)	
From / To	X	Y
P	5	4
Q	4	2
R	3	5

How many packets from each factory should be transported to each agency so that the cost of transportation is minimum? Also, find the minimum cost.

Sol. Let the number of packets sent from kitchen X to school P be x and the number of packets sent from kitchen X to school Q be y

$\therefore$ The number of packets sent from kitchen X to school R will be $(60 - x - y)$

[$\because$ Total capacity of kitchen A is 60 packets]

Now kitchen Y will sent to school $P(40 - x)$ packets [$\because$ The requirement of school P is 40 packets]

Similarly, kitchen Y will send to school $Q(40 - y)$ packets

And kitchen Y will send to school $R(50 - 60 + x + y)$ or $(x + y - 10)$ packets.

So, total transportation cost (in Rs)

i.e., $Z = 5x + 4y + 3(60 - x - y) + 4(40 - x) + 2(40 - y) + 5(x + y - 10)$

$\Rightarrow Z = 5x + 4y + 180 - 3x - 3y + 160 - 4x + 80 - 2y + 5x + 5y - 50 = 3x + 4y + 370$

Here, we have to minimize $Z = 3x + 4y + 370$, subject to the constraints

$x \ge 0,\ y \ge 0,\ x + y \le 60,\ \ x \le 40,\ y \le 40,\ \ x + y \le 10$

Draw the lines $x + y = 60,\ x = 40,\ \ y = 40$ and $x + y = 10$; and shade the region satisfied by the above inequalities. The feasible region is the polygon $ABCDEF$, which is convex and bounded.

The corner points are $A(40, 20)$; $B(20, 40)$; $C(0, 40)$; $D(0, 10)$; $E(10, 0)$ and $F(40, 0)$

The value of Z at $A(40, 20) = 3\times40 + 4\times20 + 370 = 570$

At $B(20, 40) = 3\times20 + 4\times40 + 370 = 590$

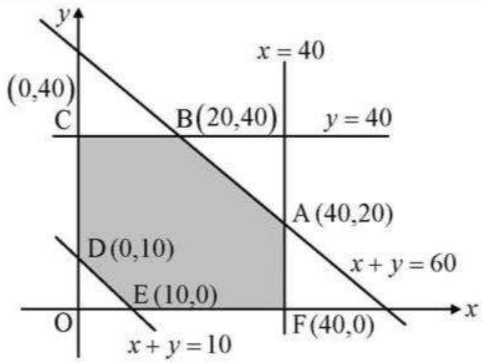
At $C(0, 40) = 3\times0 + 4\times40 + 370 = 530$

At $D(0, 10) = 3\times 0 + 4\times 40 + 370 = 410$

At $E(10, 0) = 3\times 10 + 4\times 0 + 370 = 400$

At $F(40, 0) = 3\times 40 + 4\times 0 + 370 = 490$

The minimum cost is at $(10, 0)$ i.e. Rs 400 when 10, 0, 50 packets are supplied from X and 30, 40, 0 packets are supplied from Y to the schools P, Q and R respectively.



28. An oil company has two depots, A and B, with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E, F, whose requirements are 4500 L, 3000 L and 35000 L respectively. The distances (in km) between the depots and the petrol pumps are given in the following table.

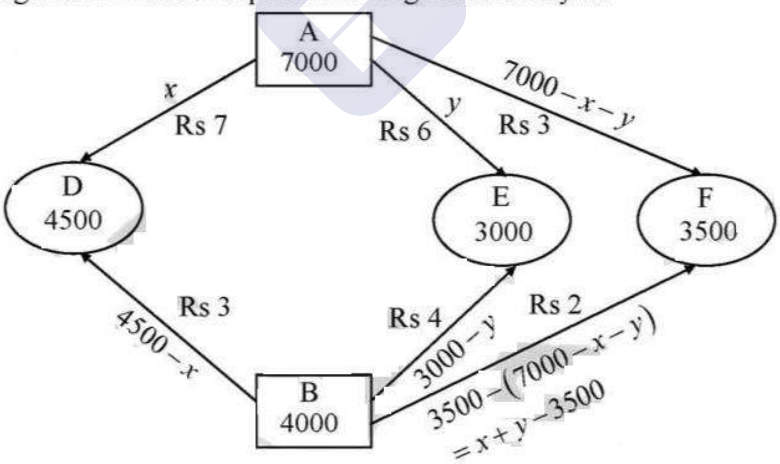
	Distance (in km)	
From / To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost per km is Re 1 per litre, how should the delivery be scheduled in order that the transportation cost is minimum?

Sol. Given that the total quantity of oil available $= (7000 + 4000)$ litre = 11000 litre and total requirement $= (4500 + 3000 + 3500)$ litre = 11000 litre, so there is no mismatch between the availability and requirement.

Let depot A supply x litre of oil to petrol pump D and y litre to E so that it supplies $(7000 - x - y)$ litre to F. Obviously $0 \le x \le 4500$, $0 \le y \le 3000$, $0 \le 7000 - x - \le 3500$.

The given date can be represented diagrammatically as:



As requirement of petrol pump D is 4500 litre and it has already received x litres from depot A, it must receive $(4500 - x)$ litre from depot B. Similarly, E receives $(3000 - y)$ litre from depot B and F receives $3500 - (7000 - x - y) = (x + y - 3500)$ litre from depot B.

The total transportation cost (in Rs)

$$= 7x + 6y + 3(7000 - x - y) + 3(4500 - x) + 4(3000 - y) + 2(x + y - 3500) = 3x + y + 39500$$

Hence, the given problem can be formulated as an L.P.P. as follows :

Minimize $Z = 3x + y + 39500$ subject to the constraints

$x \geq 0,\ y \geq 0,\ x + y \leq 7000$

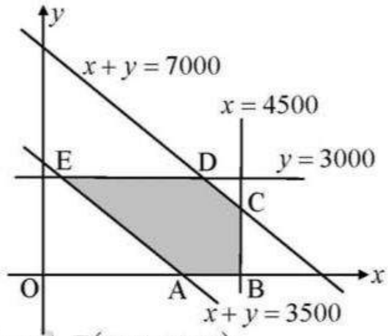
$x \leq 4500,\ y \leq 3000,$

$x + y \geq 3500.$

Draw the lines $x + y = 7000$, $x = 4500$, $y = 3000$ and $x + y = 3500$

and shade the region satisfied by the above inequalities.

The feasible region in the polygon $ABCDE$, which is convex and bounded. The corner points are



$A(3500, 0)$, $B(4500, 0)$, $C(4500, 2500)$, $D(4000, 3000)$ and $E(500, 3000)$.

The values (in Rs) of $Z = 3x + y + 39500$ at the corner points A, B, C, D and E are given in the following table:

Points	x	y	$Z = 3x + y + 39500$
A	3500	0	50000
B	4500	0	53000
C	4500	2500	55500
D	4000	3000	54500
E	500	3000	44000 ← Minimum

Here minimum value of Z = Rs 44000 at the point $E(500, 3000)$.

Hence, 500, 3000, 3500 litre are supplied from depot A and 4000, 0, 0 litre are supplied from depot B to petrol pumps D, E and F respectively; and minimum transportation cost is Rs 44000.

29. A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. They need certain nutrients, named as X, Y, Z. The pigs are fed on two products, A and B. One unit of product A contains 36 units of X, 3 units of Y and 20 units of Z, while one unit of product B contains 6 units of X, 12 units of Y and 10 units respectively. Product A costs Rs 20 per unit and product B costs Rs 40 per unit. How many units of each product must be taken to minimize the cost? Also, find the minimum cost.

Sol. Let x units of A and y units of B be taken. Then, minimize $Z = 20x + 40y$,

subject to $x \geq 0, y \geq 0, 36x + 6y \geq 108, 3x + 12y \geq 36, 20x + 10y \geq 100$

i.e. $6x + y \geq 18$

$x + 4y \geq 12$

$2x + y \geq 10$

$x \geq 0, y \geq 0$

Draw the lines $6x + y = 18$, $x + 4y = 12$ and $2x + y = 10$

and shaded the region satisfied by the above inequalities.

The feasible region is unbounded

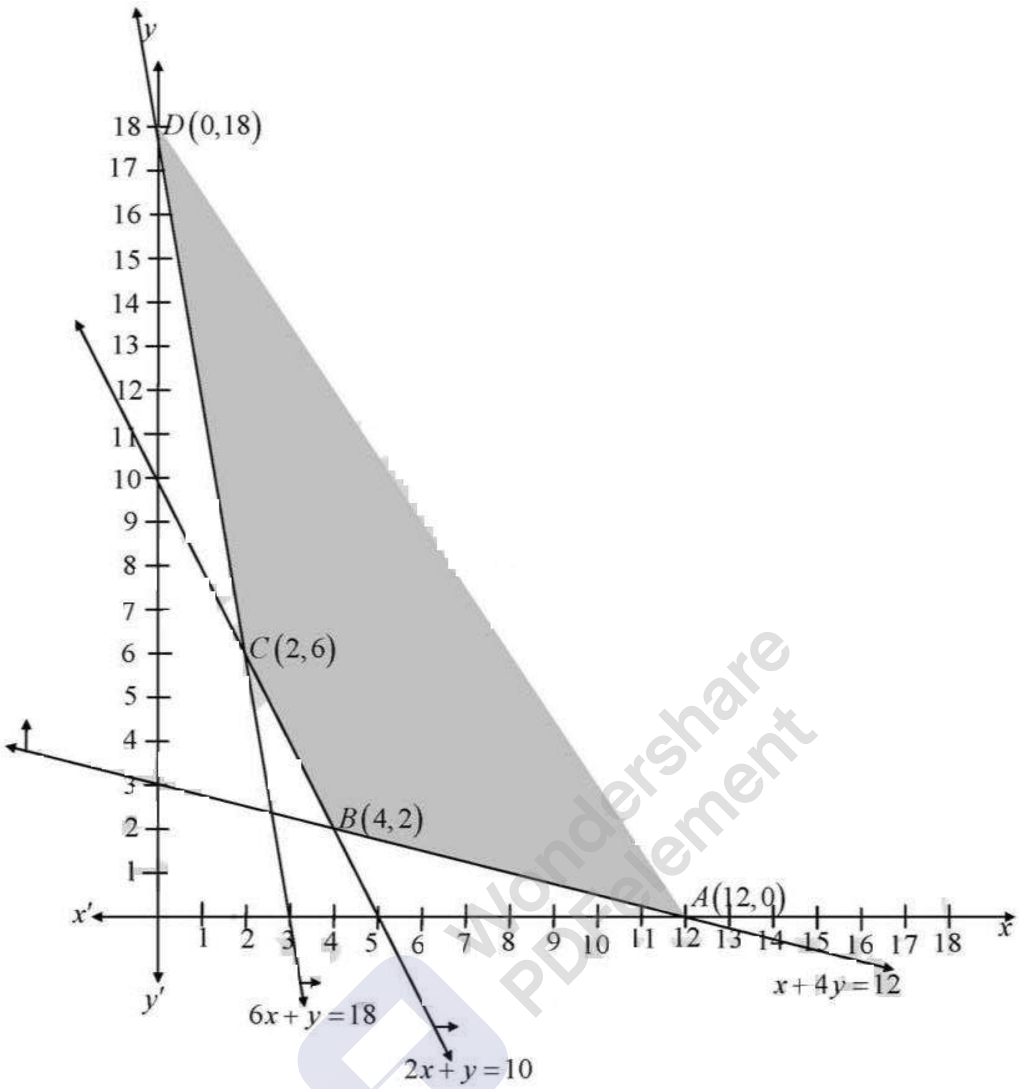
is shown shaded in the figure.

The corner points are $A(12, 0)$, $B(4, 2)$, $C(2, 6)$ and $D(0, 18)$.

The values of $Z = 20x + 40y$ at the points A, B, C and D are 240, 160, 280 and 720 respectively.

Minimum of these is 160.

Hence 2 units of a and 4 units of b are taken such that minimum cost = 160 Rs



30. A dietician wishes to mix two types of food, X and Y, in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food X contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C, while food Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 5 per kg to purchase the food X and Rs 7 per kg to purchase the food Y. Determine the minimum cost of such a mixture.

Sol. Let x kg of food X and y kg of food Y be mixed, then the problem can be formulated as an L.P.P. as follows :

Minimize the cost (in Rs) $Z = 5x + 7y$ subject to the constraints

$2x + y \geq 8$ (vitamin A constraint)

$x + 2y \geq 10$ (vitamin C constraint)

$x \geq 0,\ y \geq 0$ (non-negativity constraints)

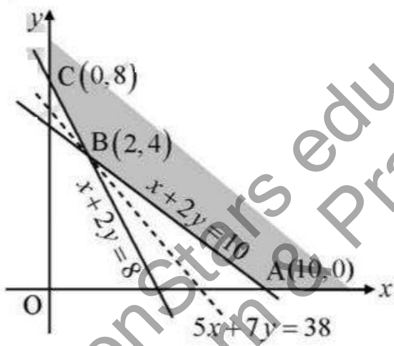
Draw the lines $2x + y = 8$ and $x + 2y = 10$,

and shaded the region satisfied by the above inequalities.

The feasible region (convex and unbounded)

is shown shaded in the figure.

The corner points are $A(10, 0)$, $B(2, 4)$ and $C(0, 8)$.



The values (in Rs) of $Z = 5x + 7y$ at the points A, B and C are 50, 38 and 56 respectively. Minimum of these is 38.

As the feasible region is unbounded, we draw the graph of the half plane $5x + 7 < 38$ i.e. $5x + 7y < 38$ and note that there is no point in common with the feasible region, therefore, Z has minimum value = Rs 38 and it occurs at $B(2, 4)$.

Hence, the minimum cost of the food is Rs 38 when 2 kg of food X and 4 kg of food Y is mixed.

31. a diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods, A and B, are available at a cost of Rs 4 and Rs 3 per unit respectively. If one unit of A contains 200 units of vitamins, 1 units of minerals and 40 calories, and 1 unit of B contains 100 units of vitamins, 2 units of minerals and 40 calories, find what combination of foods should be used to have the least cost.

Sol. We make the following table from the given data:

Resources	Food		Requirement
	A	B	
Vitamins	200	100	4000
Minerals	1	2	50
Calories	40	40	1400

If x units of food A and y units of food B are mixed, then total cost $C = 4x + 3y$ (in Rs).

Clearly $x \geq 0,\ y \geq 0$

Since there must be atleast 4000 units of vitamins in the diet, $200x + 100y \geq 4000$ i.e. $2x + y \geq 40$.

Similarly, as there must be atleast 50 units of mineral in the diet, $x + 2y \geq 50$

and as there must be atleast 1400 calories in the diet,

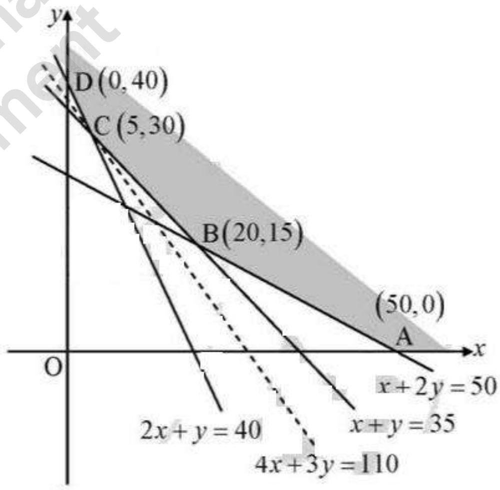
$40x + 40y \geq 1400$ i.e. $x + y \geq 35$

Hence, the given diet problem can be formulated as an L.P.P. as follows:

Find x and y which minimize

$Z = 4x + 3y$ subject to the constraints

$2x + y \geq 40,\ x + 2y \geq 50,\ x + y \geq 35,\ x \geq 0,\ y \geq 0$



To solve this L.P.P. graphically, we draw the lines $2x + y = 40$, $x + 2y = 50$, $x + y = 35$

The feasible region which is unbounded and convex, is shown shaded in the adjoining figure.

The corner points are $A(50, 0)$, $B(20, 15)$, $C(5, 30)$ and $D(0, 40)$

The value of Z (in Rs) $= 4x + 3y$ at $A(50, 0) = 4(50) + 3(0) = 200$

Similarly, the values of Z at points B, C, D are respectively Rs 125, Rs 110. Rs 120.

As the feasible region is unbounded, we draw the graph of the half plane $4x + 3y < 110$ and note that there is no point in common with the feasible region, therefore, Z has minimum value.

Minimum value of Z is Rs 110 and it occurs at the point $(5, 30)$.

So, we should mix 5 units of food A and 30 units of food B to have the least cost but still fulfill minimum nutrient and calorific requirements.

Note : To cross-check, if we mix 5 units of food A and 30 units of food B,

Vitamin content $= 5(200) + 30(100) = 4000$; Mineral content $= 5(1) + 30(2) = 65$

Calorific value $= 5(40) + 30(40) = 1400$

Note that vitamin and calorific requirement is just met, but there is surplus of mineral (65 against required minimum 50). Observe that point C lies on both $x+y=35$ (calorie constraint) and $2x+y=40$ (vitamin constraint), so there is no surplus of these two. There may be some slack in some constraint (under-utilization of resources); when there are $\geq$ constraints (as in diet problems), there may be some surplus.

32. A housewife wishes to mix together tow kinds of food, X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C.

The vitamin contents of 1 kg of each food are given below.

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

If 1 kg of food X costs Rs 6 and 1 kg of food Y costs Rs 10, find the minimum cost of the mixture which will produce the diet.

Sol. Suppose x kg of food X and y kg of food Y produce the required diet.

Then the problem can be formulated as an L.P.P. as :

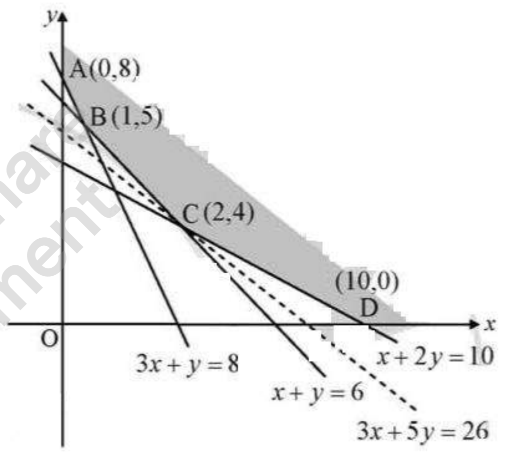
Minimize $Z=6x+10y$, Subject to the constants

$x+2y\geq 10$

$2x+2y\geq 12$ i.e. $x+y\geq 6$

$3x+y\geq 8$, $x\geq 0$, $y\geq 0$

We draw the lines $x+2y=10$, $x+y=6$, $3x+y=8$ and obtain the feasible region which is unbounded and convex, shown shaded in the figure.



The corner points are $A(0,8)$, $B(1,5)$, $C(2,4)$ and $D(10,0)$.

The values of Z (in Rs) at these points are 80, 56, 52 and 60 respectively. As the feasible region is unbounded, so on drawing the graph of the half plane $6x+10y<52$ i.e. $3x+5y<26$ and note that there is no point common with the feasible region, therefore, Z has minimum value and the minimum value is Rs 52. It occurs at the point $(2,4)$.

33. A firm manufactures two types of products, A and B, and sells them at a profit of Rs 5 per unit of type A and Rs 3 per unit of type B. Each product is processed on two machines, M_1 and M_2. One unit of type A requires one minute of processing time on M_1 and tow minutes of processing time on M_2; whereas one unit of type B requires one minute of processing time on M_1 and one minute on M_2. Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product the firm should produce a day in order to maximize the profit. Solve the problem graphically.

Sol. Let x and y be the number of units of type A and B respectively that are manufactured and sold by a firm, then the problem can be formulated as an L.P.P. as follows :

Maximize the profit (in Rs) $Z=5x+3y$ subject to the constraints

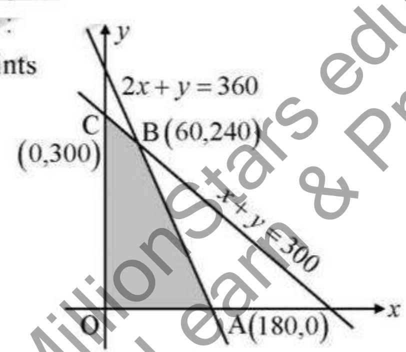
$x+y\leq 300$ (Machine M_1 constraints)

$2x+y\leq 360$ (Machine M_2 constraints)

$x\geq 0$, $y\geq 0$ (non-negativity constraints)

Draw the lines $x+y=300$ and $2x+y=360$,

and shade the region in the above inequalities.



The feasible region is the polygon $OABC$,
which is convex and bounded.

Corner points are $O(0, 0)$, $A(180, 0)$, $B(60, 240)$ and $C(0, 300)$.

The values of $Z = 5x + 3y$ (in Rs) at O, A, B and C are 0, 900, 1020 and 900 respectively.

$\therefore$ Maximum profit = Rs 1020, when 60 units of type A and 240 units of type B manufacturers.

34. A small firm manufactures items A and B. The total number of items that it can manufacture in a day is at the most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be Rs 300 and that on one unit of item B be Rs 160, how many of each type of item should be produced to maximize the profit? Solve the problem graphically.

Sol. Let the no of items manufactured by the firm of type A be x

Let the no of items manufactured by the firm of type B be y

It is said that the firm can manufacture items of both type is at most 24

$\Rightarrow x+y \le 24$

The maximum time available for both A and B is 16.

Where A takes 1 hour and B takes $\frac{1}{2}$ an hour.

$\therefore x + \frac{1}{2}y \le 16$

The profit of one unit of item A is Rs.300 and that of B is Rs160

Maximum profit to be attained is given by Z=300x+160y

Clearly $x,y \ge 0$

Now let us solve the above problem graphically :

Let us draw the lines $x + y = 24$ and $x + \frac{1}{2}y = 16$ on the graph.

Clearly the shaded portion is the feasible region.

Let us obtain the value of objective function as follows :

At the points (x,y) the value of the objective function subjected to Z=300x+160y

At O(0,0) the value of the objective function Z=0

At A(16,0) the value of the objective function $Z=300\times8+160\times0=4800$

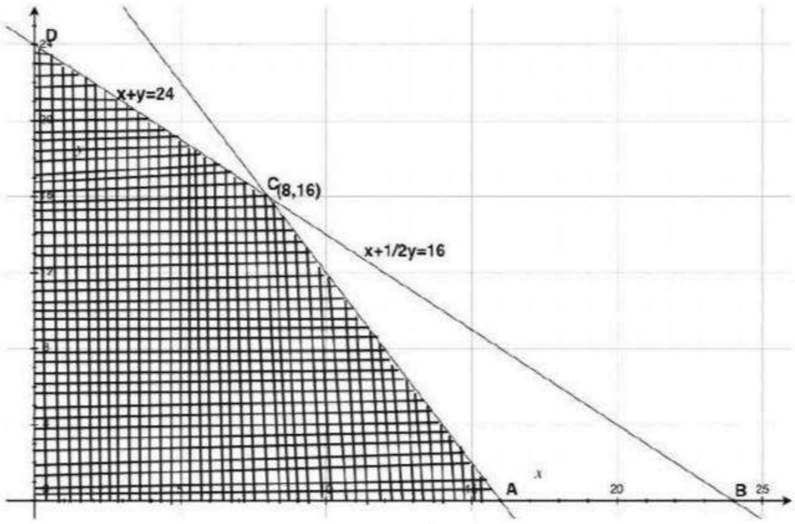
At C(8,16) the value of the objective

unction $Z=300\times8+160\times16=4960$

At D(0,24) the value of the objective function $Z=300\times0+160\times24=3840$

Clearly the maximum value is Z=4960 at C(8,16)

Hence 8 items of type A and 16 items of type B are required to acquire a maximum profit.



35. A manufacturer produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs 30 and Rs 25 per trunk of the first type and second type respectively. How many trunks of each type must he make each day to make the maximum profit?

Sol. Let x and y be the number of steel trunks of first type and second type produced by the manufacturer. As the profit on the first type of trunk is Rs 30 and on the second type of trunk is Rs 25, so the total profit $Z = 30x + 25y$ (in Rs).

Hence, the problem can be formulated as a L.P.P. as follows :

Maximize $Z = 30x + 25y$ Subject to the constraints

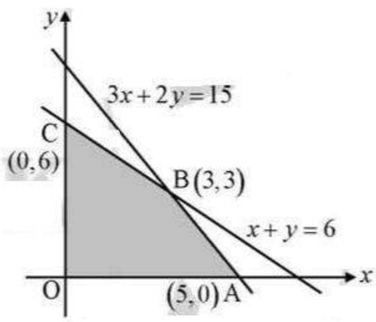
$3x + 3y \le 18$ i.e., $x + y \le 6$ (machine A constraint)

$3x + 2y \le 15$ (machine B constraint)

$x \ge 0,\ y \ge 0$ (number of trunks cannot be negative)

We draw the straight lines $x + y = 6,\ 3x + 2y = 15$

and shade the region satisfied by the above inequalities.



The shaded portion shows the feasible region which is bounded. The point of intersection of the lines $x + y = 6$ and $3x + 2y = 15$ is $B(3, 3)$.

The corner points of the feasible region $OABC$ are $O(0, 0)$, $A(5, 0)$, $B(3, 3)$ and $C(0, 6)$.

The optimal solution occurs at one of the corner points.

At $O(0, 0)$, $Z = 30.0 + 25.0 = 0$; At $A(5, 0)$, $Z = 30\,.\,5 + 25\,.\,0 = 150$

At $B(3, 3)$, $Z = 30\,.\,3 + 25\,.\,3 = 165$; At $C(0, 6)$, $Z = 30\,.\,0 + 25\,.\,6 = 150$

We find that the value of Z is maximum at $B(3, 3)$.

Hence, the manufacture should produce 3 trunks of each type to get a maximum profit of Rs 165.

36. A company manufactures two types of toys, A and B. Type A requires 5 minutes each for cutting and 10 minutes each for assembling. Type B requires 8 minutes each for cutting and 8 minutes each fro assembling there are 3 hours available for cutting and 4 hours available for assembling in a day. The profit is Rs 50 each on type A and Rs 60 each on type B. How many toys of each type should the company manufacture in a day to maximize the profit?

Sol. Let x and y be the number of toys of type A and type B respectively. Then, $5x + 8y \le 180, 10x + 8y \le 240$ and $x \ge 0, y \ge 0$

i.e. $5x + 8y \le 180$

$5x+4y \le 120$

and $x \ge 0, y \ge 0$

we have to maximize $Z = 50x + 60y$

We draw the straight lines $5x+8y=180$, $5x+4y=120$

and shade the region satisfied by the above inequalities.

The shaded portion shows the feasible region which is bounded. The point of intersection of the lines $5x+8y=180$ and $5x+4y=120$ is $A(12, 15)$.

The corner points of the feasible region $OBAC$ are $O(0, 0)$, $A(12, 15)$, $B(24, 0)$ and $C\left(0, \frac{45}{2}\right)$.

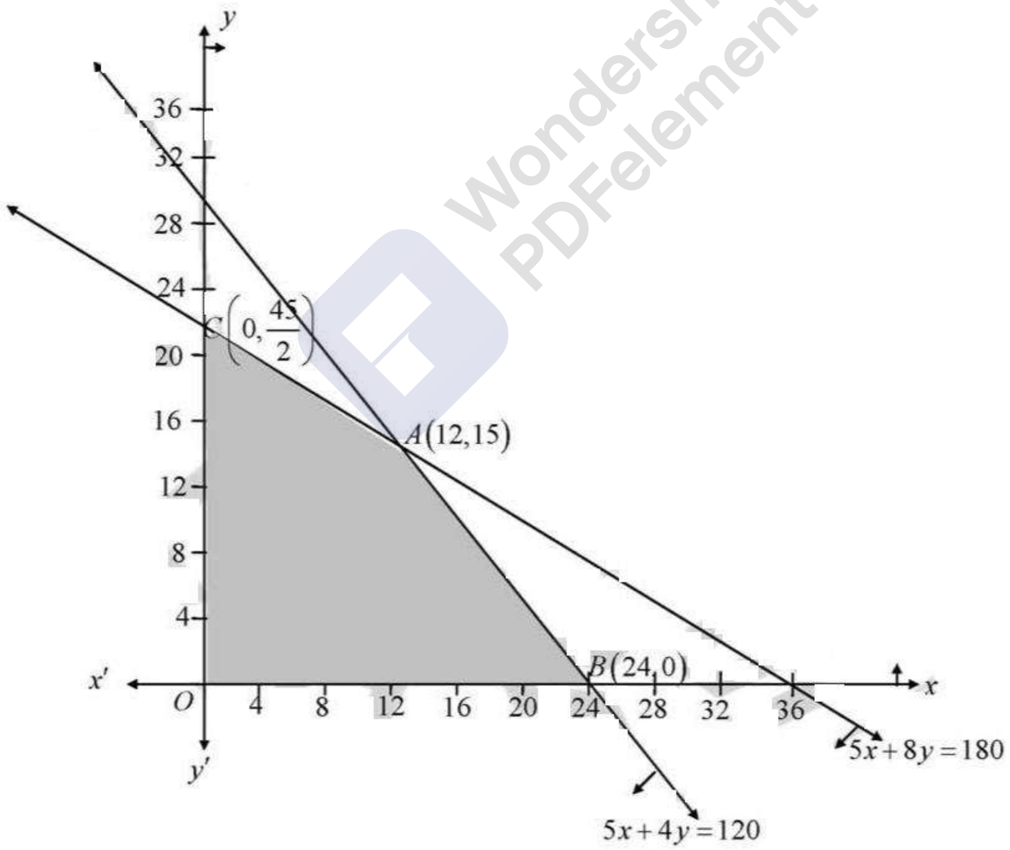
The optimal solution occurs at one of the corner points.

At $O(0, 0)$, $Z = 0$; At $A(12, 15)$, $Z = 1500$

At $B(24, 0)$, $Z = 1200$; At $C\left(0, \frac{45}{2}\right)$, $Z = 1350$

We find that the value of Z is maximum at $A(12, 15)$.

For getting a maximum profit of Rs.1500, 12 toys of type A and 15 toys of type B should be manufactured



37. Kellogg is a new cereal formed of a mixture of bran and rice, that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs Rs.5 per kilograms and rice costs Rs 4 per kilogram.

Sol. Let the cereal contains x kg bran and y kg rice

Clearly $x \geq 0$ and $y \geq 0$

According to the hypothesis the linear programming problem is

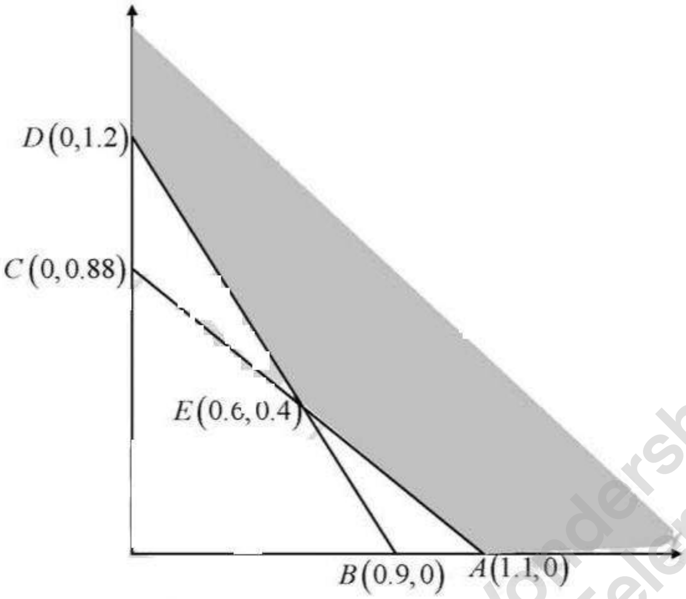
Minimize $Z = 5x + 4y$

Subject to the constrains $(80/1000)x + (100/1000)y \geq (88/1000)$ or, $20x + 25y \geq 22$ .. (i)

And $(40/1000)x + (30/1000)y \geq (36/1000)$ or $20x + 15y \geq 18$ .... (ii)

And also $x \geq 0, y \geq 0$ .... (iii)

Now the lines $20x + 2y = 22$ and $20x + 15y = 18$ are drawn



These lines meet at $E(0.6, 0.4)$

The feasible region is shaded and it is an unbounded region with vertices $A(1.1,0), E(0.6,0.4)$ and $D(0,1.2)$

Let us evaluate the objective function $Z = 10,500x + 9,000y$ at these vertices to find which one gives the maximum profit

Corner point	$Z = 5x + 4y$
$A(1.1,0)$	$5 \times 1.1 + 4 \times 0 = 5.5$
$E(0.6,0.4)$	$5 \times 0.6 + 4 \times 0.4 = 4.6 \leftarrow$ minimum
$D(0,1.2)$	$5 \times 0 + 4 \times 1.2 = 4.8$

Therefore, minimum cost of producing this cereal is Rs. 4.60 per kg

38. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5760 to invest and has space fro at most 20 items. A fan costs him Rs 360 and a sewing machine Rs 240. He expects to sell a fan at a profit of Rs 22 and a sewing machine at a profit of Rs 18. Assuming that he can sell all the items that he buys, how should he invest his money to maximize the profit? Solve graphically and find the maximum profit.

Sol. Let x and y be the number of fans and sewing machines that a dealer purchases and sells, then the problem can be formulated as an L.P.P. as follows:

Maximize the profit (in Rs) $Z = 22x + 18y$ subject to the constraints

$360x+240y \le 5760$ (investment constraint)

i.e. $3x+2y \le 48$

$x+y \le 20$ (storage constraint)

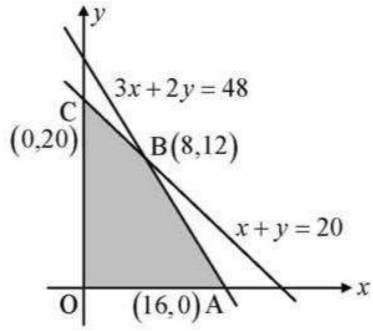
Draw the lines $3x+2y=48$ and $x+y=20$,

and shade the region satisfied by the above inequalities.

The feasible region is the polygon $OABC$,

which is convex and bounded.

The corner of points are $O(0, 0)$, $A(16, 0)$, $B(8, 12)$ and $C(0, 20)$



The value of Z (in Rs) $= 22x+18y$ at the points O, A, B and C are 0, 352, 392 and 360 respectively.

$\therefore$ Maximum profit = Rs 392, when 8 fans and 12 sewing machines are purchased and sold.

39. Anil wants to invest at the most Rs 12000 in bonds A and B. According to rules, he has to invest at least Rs 2000 in Bond A and at least Rs 4000 in Bond B. If the rate of interest of Bond A is 8% per annum and on Bond B, it is 10%per annum, how should he invest his money for maximum interest? Solve the problem graphically.

Sol. Let Anil invest Rs x in bonds A and Rs y in bonds B, then the problem can be formulated as an L.P.P. as follows:

Minimize the interest (in Rs) = 8% of $x+10\%$ of $y = \frac{8}{100}x+\frac{10}{100}y$ subject to the constraints

$x+y \le 12000$ (investment constraint)

$x \ge 2000$ (investment in bond A constraint)

$y \ge 4000$ (investment in bond B constraint)

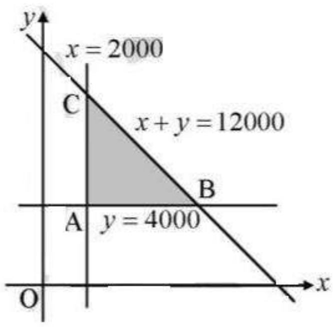
Now draw the line $x+y=12000$, $x=2000$, and $y=4000$

and shade the region satisfied by the above inequalities.

The feasible region is ABC, which is bounded and convex.

The corner points are

$A(2000, 4000)$, $B(4000, 8000)$ and $C(2000, 10000)$.



The value of $Z=\frac{8}{100}x+\frac{10}{100}y$ at the points A, B and C are 560, 1120 and 1160 respectively.

$\therefore$ Maximum interest earned = Rs 1160, when Rs 2000 are invested in bond A and Rs 10000 are invested in bond B.

40. Maximize $z=60x+15y$, subject to the constraints $x+y \le 50$, $3x+y \le 90$, $x, y \ge 0$.

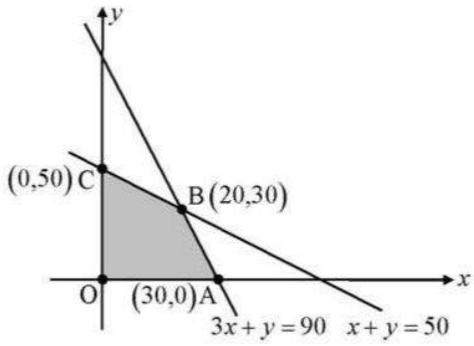
Sol. Firstly draw the lines $x+y=50$, $3x+y=90$ and shade the region satisfied by the given inequations.

The shaded region in the adjoining figure shows the feasible region determined by the given constraints.

We observe that the feasible region $OABC$ is bounded. So we use the corner point method to calculate the maximum and minimum value of Z.

The coordinates of the corner points O, A, B, C are $(0, 0)$, $(30, 0)$, $(20, 30)$ and $(0, 50)$ respectively. We evaluate $Z=60x+15y$ at each of these points.

Corner point	Value of objective function $Z = 60x + 15y$
$(0, 0)$	$0 \rightarrow$ Smallest
$(30, 0)$	$1800 \rightarrow$ Largest
$(20, 30)$	1650
$(0, 50)$	750



Hence, the minimum value of Z is at the point $(0, 0)$

and maximum value of Z is 1800 at the point $(30, 0)$.

41. A company manufactures two types of toys A and B. A requires 5 minutes each for cutting and 10 minutes each for assembling. Type B requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours available for cutting and 4 hours available for assembling in a day. He earns a profit of Rs 50 each on type A and Rs 60 each type should the company manufacture in a day to maximize the profit?

Sol. Let x and y be the number of toys of type A and type B respectively. Then, $5x + 8y \leq 180, 10x + 8y \leq 240$ and $x \geq 0, y \geq 0$

i.e. $5x + 8y \leq 180$

$5x + 4y \leq 120$

and $x \geq 0, y \geq 0$

we have to maximize $Z = 50x + 60y$

We draw the straight lines $5x + 8y = 180$, $5x + 4y = 120$

and shade the region satisfied by the above inequalities.

The shaded portion shows the feasible region which is bounded. The point of intersection of the lines $5x + 8y = 180$ and $5x + 4y = 120$ is $A(12, 15)$.

The corner points of the feasible region $OBAC$ are $O(0, 0)$, $A(12, 15)$, $B(24, 0)$ and $C\left(0, \frac{45}{2}\right)$.

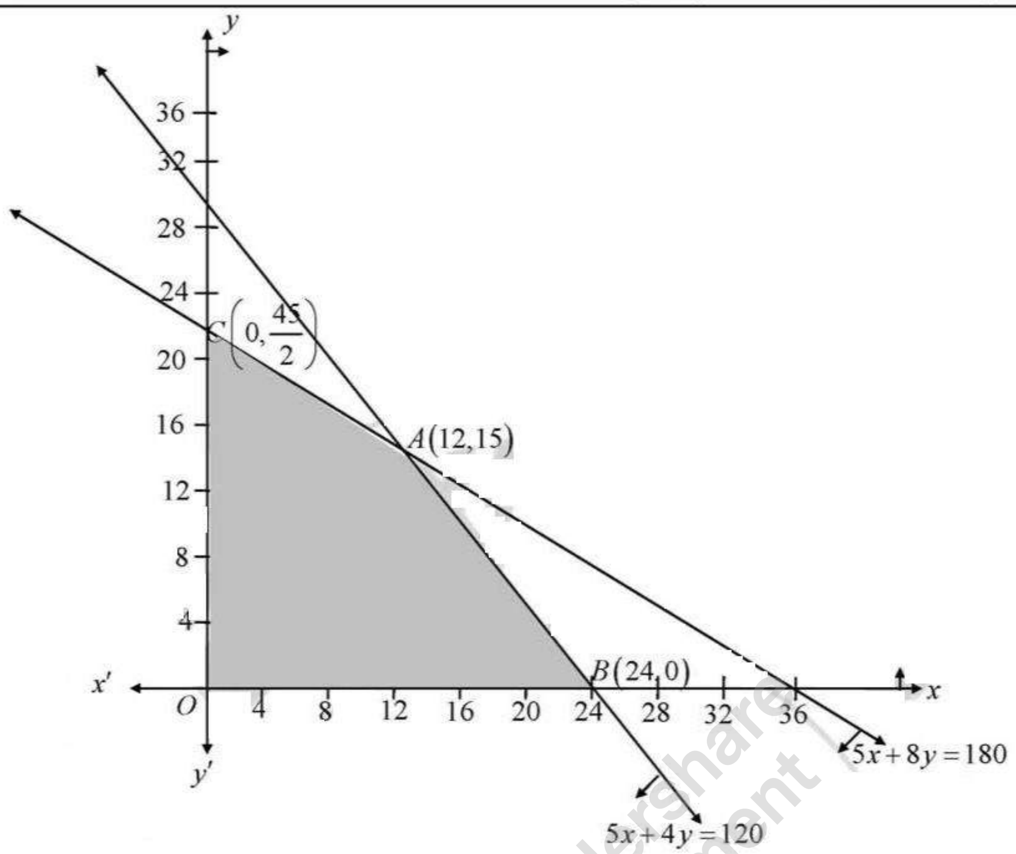
The optimal solution occurs at one of the corner points.

At $O(0, 0)$, $Z = 0$; At $A(12, 15)$, $Z = 1500$

At $B(24, 0)$, $Z = 1200$; At $C\left(0, \frac{45}{2}\right)$, $Z = 1350$

We find that the value of Z is maximum at $A(12, 15)$.

For getting a maximum profit of Rs.1500, 12 toys of type A and 15 toys of type B should be manufactured



42. One kind of cake requires 200 g of flour and 25 g of fat, another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it an LPP and solve it graphically

Sol. Let x cakes of first kind and y cakes of second kind be made respectively, then the problem can be formulated as L.P.P. as follows:

Maximize $Z = x + y$ subject to the constraints

$200x + 100y \le 5000$ (quantity of flour constraint)

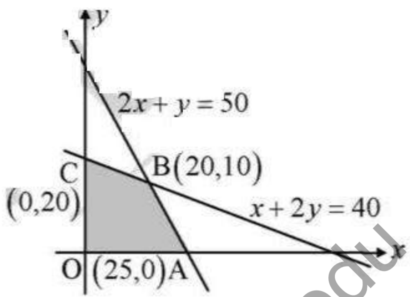
i.e. $2x + y \le 50$,

$25x + 50y \le 1000$ (quantity of fat constraint)

i.e. $x + 2y \le 40$

$x \ge 0,\ y \ge 0$ (non-negativity constraints)

Draw the lines $2x + y = 50$ and $x + 2y = 40$, and shade the region satisfied by the above inequalities. The feasible region is the polygon $OABC$, which is convex and bounded.



The corner points are $O(0, 0)$, $A(25, 0)$, $B(20, 10)$ and $C(0, 20)$.

The values of Z at the points O, A, B and C are 0, 20 30 and 20 respectively.

$\therefore$ Maximum number of cakes that can be made = 30, in which 20 of first kind and 10 of second kind.

43. A manufacturing company makes two types of teaching aids A and B of mathematics for class XII, each type of a requires 9 labour hours of fabricating and 1 labour hour for finishing each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of Rs. 80 on each piece of type A and Rs 120 on each piece of type B. How many pieces of type

A and type B should be manufactured per week to get a maximum profit? Make it as on LPP and solve graphically. What is the maximum profit per week ?

Sol. Let the pieces of type A manufactured per week be x

Let the pieces of type B manufactured per week be y

$\therefore$ maximum profit Z = 80x+120y

Fabricating hours for A is 9 and finishing hours is 1

Fabricating hours for B is 12 and finishing hours is 3

Maximum number of fabricating hours = 180

$\therefore 9x+12y \le 180$

i.e., $3x+4y \le 60$

Maximum number of finishing hours = 30

$\therefore x+3y \le 30$

Now Z is =80x+120 is subjected to

Constraints $3x+4y \le 60$

$x+3y \le 30$

$x \ge 0$

$y \ge 0$

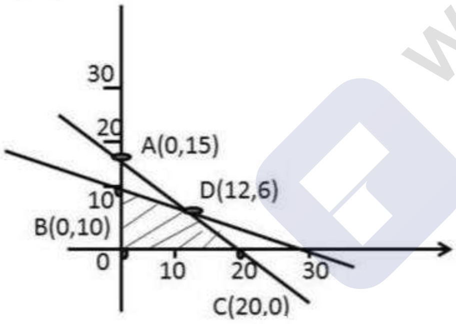
Now the area of the fesible region is as shown in the figure

$3x+4y=60 \therefore y=6$

$3x+3y=30$ and $x=12$

$3x+4y=60$

$3x+9y=30$



We can see that the points on the bounded region are

A(0, 15), B(0, 10), C(20, 0) and D(12, 6)

Now let us calculate the maximum profit

Points(x,y)	Z=80x+120y
A(0,15)	Z=80(0)+120(15)=1800
B(0,10)	Z=80(0)+120(10)=1200
C(20,0)	Z=80(20)+120(0)=1600
D(12,6)	Z=80(12)+120(6)=960+720=1680

Hence the maximum profit is 1800 at (0, 15)

$\therefore$ teaching aid A = 0 and teaching aid B = 15 has to be made.