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Additional Exercises

Question 13.23:

In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are $^{24}_{12}Mg$ (23.98504u), $^{25}_{12}Mg$ (24.98584u) and $^{26}_{12}Mg$ (25.98259u).

The natural abundance of $^{24}_{12}Mg$ is 78.99% by mass. Calculate the abundances of other two isotopes.

Answer

Average atomic mass of magnesium, m = 24.312 u

Mass of magnesium $^{24}_{12}Mg$ isotope, m_1 = 23.98504 u

Mass of magnesium $^{25}_{12}Mg$ isotope, m_2 = 24.98584 u

Mass of magnesium $^{26}_{12}Mg$ isotope, m_3 = 25.98259 u

Abundance of , $^{24}_{12}Mg$ n1 η = 78.99%

Abundance of $^{25}_{12}Mg$, $\eta_2 = x\%$

Hence, abundance of $^{26}_{12}Mg$, $\eta_3 = 100 - x - 78.99\% = (21.01 - x)\%$

We have the relation for the average atomic mass as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$24.312 = \frac{23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 - x)}{100}$$

$$2431.2 = 1894.5783096 + 24.98584x + 545.8942159 - 25.98259x$$

$$0.99675x = 9.2725255$$

$$\therefore x \approx 9.3\%$$

$$\text{And } \ 21.01 - x = 11.71\%$$

Hence, the abundance of $^{25}_{12}Mg$ is 9.3% and that of $^{26}_{12}Mg$ is 11.71%.

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Question 13.24:

The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{41}_{20}\mathrm{Ca}$ and ${}^{27}_{13}\mathrm{Al}$ from the following data:

$m\left({}^{40}_{20}\mathrm{Ca}\right)$ = 39.962591 u

$m\left({}^{41}_{20}\mathrm{Ca}\right)$ = 40.962278 u

$m\left({}^{26}_{13}\mathrm{Al}\right)$ = 25.986895 u

$m\left({}^{27}_{13}\mathrm{Al}\right)$ = 26.981541 u

Answer

For ${}^{41}_{20}\mathrm{Ca}$: Separation energy $= 8.363007$ MeV

${}^{27}_{13}\mathrm{Al}$: Separation energy $= 13.059$ MeV

For A neutron $\left({}_0n^1\right)$ is removed from a ${}^{41}_{20}\mathrm{Ca}$ nucleus. The corresponding nuclear reaction can be written as:

$${}^{41}_{20}\mathrm{Ca} \longrightarrow {}^{40}_{20}\mathrm{Ca} + {}^{1}_{0}\mathrm{n}$$

It is given that:

$m\left({}^{40}_{20}\mathrm{Ca}\right)$ Mass = 39.962591 u

$m\left({}^{41}_{20}\mathrm{Ca}\right)$ Mass) = 40.962278 u

$m\left({}_0n^1\right)$ Mass = 1.008665 u

The mass defect of this reaction is given as:

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$\Delta m = m\left({}^{40}_{20}\text{Ca}\right) + \left({}^{1}_{0}\text{n}\right) - m\left({}^{41}_{20}\text{Ca}\right)$

$= 39.962591 + 1.008665 - 40.962278 = 0.008978 \text{ u}$

But $1 \text{ u} = 931.5 \text{ MeV}/c^2$

$\therefore \Delta m = 0.008978 \times 931.5 \text{ MeV}/c^2$

Hence, the energy required for neutron removal is calculated as:

$E = \Delta mc^2$

$= 0.008978 \times 931.5 = 8.363007 \text{ MeV}$

For ${}^{27}_{13}\text{Al}$, the neutron removal reaction can be written as:

${}^{27}_{13}\text{Al} \longrightarrow {}^{26}_{13}\text{Al} + {}^{1}_{0}\text{n}$

It is given that:

$m\left({}^{27}_{13}\text{Al}\right)$ Mass = 26.981541 u

$m\left({}^{26}_{13}\text{Al}\right)$ Mass = 25.986895 u

The mass defect of this reaction is given as:

$\Delta m = m\left({}^{26}_{13}\text{Al}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{27}_{13}\text{Al}\right)$

$= 25.986895 + 1.008665 - 26.981541$

$= 0.014019 \text{ u}$

$= 0.014019 \times 931.5 \text{ MeV}/c^2$

Hence, the energy required for neutron removal is calculated as:

$E = \Delta mc^2$

$= 0.014019 \times 931.5 = 13.059 \text{ MeV}$

Question 13.25:

A source contains two phosphorous radio nuclides ${}^{32}_{15}\text{P}$ ($T_{1/2} =$ 14.3d) and ${}^{33}_{15}\text{P}$ ($T_{1/2} =$ 25.3d). Initially, 10% of the decays come from ${}^{33}_{15}\text{P}$. How long one must wait until 90% do so?

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Answer

Half life of ${}^{32}_{15}P$, $T_{1/2}$ = 14.3 days

Half life of ${}^{33}_{15}P$, $T'_{1/2}$ = 25.3 days ${}^{33}_{15}P$ nucleus decay is 10% of the total amount of decay.

The source has initially 10% of ${}^{33}_{15}P$ nucleus and 90% of ${}^{32}_{15}P$ nucleus.

Suppose after t days, the source has 10% of ${}^{32}_{15}P$ nucleus and 90% of ${}^{33}_{15}P$ nucleus.

Initially:

Number of ${}^{33}_{15}P$ nucleus = N

Number of ${}^{32}_{15}P$ nucleus = 9 N

Finally:

Number of ${}^{33}_{15}P$ nucleus $= 9N'$

Number of ${}^{32}_{15}P$ nucleus $= N'$

For ${}^{32}_{15}P$ nucleus, we can write the number ratio as:

$$\frac{N'}{9N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$N' = 9N(2)^{\frac{-t}{14.3}} \qquad \dots (1)$$

For ${}^{33}_{15}P$, we can write the number ratio as:

$$\frac{9N'}{N} = \left(\frac{1}{2}\right)^{\frac{t}{T'_{1/2}}}$$

$$9N' = N(2)^{\frac{-t}{25.3}} \qquad \dots (2)$$

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On dividing equation (1) by equation (2), we get:

$$\frac{1}{9} = 9 \times 2^{\left(\frac{t}{25.3} - \frac{t}{14.3}\right)}$$

$$\frac{1}{81} = 2^{-\left(\frac{11t}{25.3 \times 14.3}\right)}$$

$$\log 1 - \log 81 = \frac{-11t}{25.3 \times 14.3} \log 2$$

$$\frac{-11t}{25.3 \times 14.3} = \frac{0 - 1.908}{0.301}$$

$$t = \frac{25.3 \times 14.3 \times 1.908}{11 \times 0.301} \approx 208.5 \text{ days}$$

Hence, it will take about 208.5 days for 90% decay of ${}_{15}P^{33}$.

Question 13.26:

Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$${}^{223}_{88}\text{Ra} \longrightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C}$$

$${}^{223}_{88}\text{Ra} \longrightarrow {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Answer

Take a ${}^{14}_{6}\text{C}$ emission nuclear reaction:

$${}^{223}_{88}\text{Ra} \longrightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C}$$

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We know that:

Mass of $^{223}_{88}Ra$, $m_1 = 223.01850$ u

Mass of $^{209}_{82}Pb$, $m_2 = 208.98107$ u

Mass of , $^{14}_{6}C$ $m_3 = 14.00324$ u

Hence, the Q-value of the reaction is given as:

$Q = (m_1 - m_2 - m_3)\,c^2$

$= (223.01850 - 208.98107 - 14.00324)\,c^2$

$= (0.03419\,c^2)$ u

But 1 u = 931.5 MeV/c^2

$\therefore Q = 0.03419 \times 931.5$

= 31.848 MeV

Hence, the Q-value of the nuclear reaction is 31.848 MeV. Since the value is positive, the reaction is energetically allowed.

Now take a $^{4}_{2}He$ emission nuclear reaction:

$$^{223}_{88}Ra \longrightarrow {}^{229}_{86}Rn + {}^{4}_{2}He$$

We know that:

Mass of $^{223}_{88}Ra$, $m_1 = 223.01850$

Mass of $^{219}_{82}Rn$, $m_2 = 219.00948$

Mass of $^{4}_{2}He$, $m_3 = 4.00260$

Q-value of this nuclear reaction is given as:

$Q = (m_1 - m_2 - m_3)\,c^2$

$= (223.01850 - 219.00948 - 4.00260)\,C^2$

$= (0.00642\,c^2)$ u

$= 0.00642 \times 931.5 = 5.98$ MeV

Hence, the Q value of the second nuclear reaction is 5.98 MeV. Since the value is positive, the reaction is energetically allowed.

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Question 13.27:

Consider the fission of $^{238}_{92}\mathrm{U}$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are $^{140}_{58}\mathrm{Ce}$ and $^{99}_{44}\mathrm{Ru}$.

Calculate Q for this fission process. The relevant atomic and particle masses are

$\left(^{238}_{92}\mathrm{U}\right)$ m =238.05079 u

$\left(^{140}_{58}\mathrm{Ce}\right)$ m =139.90543 u

$\left(^{99}_{44}\mathrm{Ru}\right)$ m = 98.90594 u

Answer

In the fission of $^{238}_{92}\mathrm{U}$, 10 β^- particles decay from the parent nucleus. The nuclear reaction can be written as:

$$^{238}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{140}_{58}\mathrm{Ce} + {}^{99}_{44}\mathrm{Ru} + 10\,{}^{0}_{-1}e$$

It is given that:

Mass of a nucleus m_1 $^{238}_{92}\mathrm{U}$, = 238.05079 u

Mass of a nucleus m_2 $^{140}_{58}\mathrm{Ce}$, = 139.90543 u

Mass of a nucleus , m_3 $^{99}_{44}\mathrm{Ru}$ = 98.90594 u

Mass of a neutron m_4 $^{1}_{0}\mathrm{n}$, = 1.008665 u

Q-value of the above equation,

$$Q = \left[m'\left(^{238}_{92}\mathrm{U}\right) + m\left(^{1}_{0}\mathrm{n}\right) - m'\left(^{140}_{58}\mathrm{Ce}\right) - m'\left(^{99}_{44}\mathrm{Ru}\right) - 10m_e\right]c^2$$

Where,

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m′ = Represents the corresponding atomic masses of the nuclei

$m'\left({}^{238}_{92}\mathrm{U}\right)$ = $m_1 - 92m_e$

$m'\left({}^{140}_{58}\mathrm{Ce}\right)$ = $m_2 - 58m_e$

$m'\left({}^{99}_{44}\mathrm{Ru}\right)$ = $m_3 - 44m_e$

$m\left({}^{1}_{0}\mathrm{n}\right)$ = m_4

$$Q=\left[m_1-92m_e+m_4-m_2+58m_e-m_3+44m_e-10m_e\right]c^2$$
$$=\left[m_1+m_4-m_2-m_3\right]c^2$$
$$=\left[238.0507+1.008665-139.90543-98.90594\right]c^2$$
$$=\left[0.247995\ c^2\right]\mathrm{u}$$

$$\text{But } 1\ \mathrm{u}=931.5\ \mathrm{MeV}/c^2$$

$$\therefore Q=0.247995\times 931.5=231.007\ \mathrm{MeV}$$

Hence, the Q-value of the fission process is 231.007 MeV.

Question 13.28:

Consider the D−T reaction (deuterium−tritium fusion)

$${}^{2}_{1}\mathrm{H}+{}^{3}_{1}\mathrm{H}\longrightarrow{}^{4}_{2}\mathrm{He}+\mathrm{n}$$

(a) Calculate the energy released in MeV in this reaction from the data:

$m\left({}^{2}_{1}\mathrm{H}\right)$ = 2.014102 u

$m\left({}^{3}_{1}\mathrm{H}\right)$ = 3.016049 u

(b)Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles = 2(3kT/2); k = Boltzman's constant, T = absolute temperature.)

Answer

(a) Take the D-T nuclear reaction: $^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + n$

It is given that:

Mass of $^{2}_{1}H$, m_1= 2.014102 u

Mass of $^{3}_{1}H$, m_2 = 3.016049 u

Mass of $^{4}_{2}He$,m_3 = 4.002603 u

Mass of $^{1}_{0}n$ m_4 = 1.008665 u

Q-value of the given D-T reaction is:

$Q = [m_1 + m_2 - m_3 - m_4]\ c^2$

$= [2.014102 + 3.016049 - 4.002603 - 1.008665]\ c^2 = [0.018883\ c^2]\ u$

But 1 u = 931.5 MeV/c^2

$\therefore Q = 0.018883 \times 931.5 = 17.59$ MeV

(b) Radius of deuterium and tritium, $r \approx 2.0$ fm $= 2 \times 10^{-15}$ m

Distance between the two nuclei at the moment when they touch each other,

$d = r + r = 4 \times 10^{-15}$ m

Charge on the deuterium nucleus = e

Charge on the tritium nucleus = e

Hence, the repulsive potential energy between the two nuclei is given as:

$$V = \frac{e^2}{4\pi \epsilon_0 (d)}$$

Where,

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ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}} = 5.76 \times 10^{-14} \text{ J}$$

$$= \frac{5.76 \times 10^{-14}}{1.6 \times 10^{-19}} = 3.6 \times 10^5 \text{ eV} = 360 \text{ keV}$$

Hence, 5.76×10^{-14} J or 360 keV of kinetic energy (KE) is needed to overcome the Coulomb repulsion between the two nuclei.

However, it is given that:

KE $= 2 \times \frac{3}{2} kT$

Where,

k = Boltzmann constant = 1.38×10^{-23} m^2 kg s^{-2} K^{-1}

T = Temperature required for triggering the reaction

$$\therefore T = \frac{\text{KE}}{3K}$$

$$= \frac{5.76 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 1.39 \times 10^9 \text{ K}$$

Hence, the gas must be heated to a temperature of 1.39×10^9 K to initiate the reaction.

Question 13.29:

Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 13.6. You are given that m (^{198}Au) = 197.968233 u m (^{198}Hg) =197.966760 u

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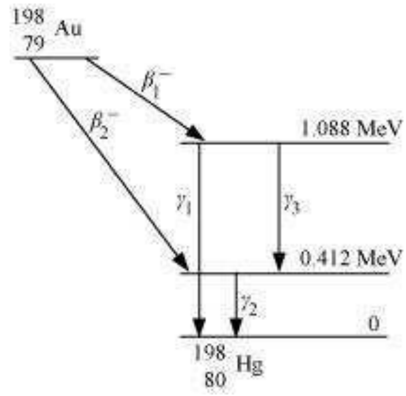

Answer

It can be observed from the given γ-decay diagram that γ_1 decays from the 1.088 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_1-decay is given as:

$E_1 = 1.088 - 0 = 1.088$ MeV $h\nu_1 = 1.088 \times 1.6 \times 10^{-19} \times 10^6$ J

Where,

h = Planck's constant = 6.6×10^{-34} Js ν_1 =

Frequency of radiation radiated by γ_1-decay

$$\therefore \nu_1 = \frac{E_1}{h}$$

$$= \frac{1.088 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 2.637 \times 10^{20} \text{ Hz}$$

It can be observed from the given γ-decay diagram that γ_2 decays from the 0.412 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_2-decay is given as:

$E_2 = 0.412 - 0 = 0.412$ MeV $h\nu_2 = 0.412 \times 1.6 \times 10^{-19} \times 10^6$ J

Where,

ν_2 = Frequency of radiation radiated by γ_2-decay

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$$\therefore \nu_2 = \frac{E_2}{h}$$
$$= \frac{0.412 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 9.988 \times 10^{19} \text{ Hz}$$

It can be observed from the given γ-decay diagram that γ_3 decays from the 1.088 MeV energy level to the 0.412 MeV energy level.

Hence, the energy corresponding to γ_3-decay is given as:

$E_3 = 1.088 - 0.412 = 0.676$ MeV $h\nu_3 = 0.676 \times 10^{-19} \times 10^6$

J Where,

ν_3 = Frequency of radiation radiated by γ_3-decay

$$\therefore \nu_3 = \frac{E_3}{h}$$
$$= \frac{0.676 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 1.639 \times 10^{20} \text{ Hz}$$

Mass of $m\left({}^{198}_{78}\text{Au}\right)$ = 197.968233 u

Mass of $m\left({}^{198}_{80}\text{Hg}\right)$ = 197.966760 u

1 u = 931.5 MeV/c^2

Energy of the highest level is given as:

$$E = \left[m\left({}^{198}_{78}\text{Au}\right) - m\left({}^{190}_{80}\text{Hg}\right)\right]$$

$$= 197.968233 - 197.966760 = 0.001473 \text{ u}$$
$$= 0.001473 \times 931.5 = 1.3720995 \text{ MeV}$$

β_1 decays from the 1.3720995 MeV level to the 1.088 MeV level

∴Maximum kinetic energy of the β_1 particle = 1.3720995 − 1.088

= 0.2840995 MeV

β_2 decays from the 1.3720995 MeV level to the 0.412 MeV level

∴ Maximum kinetic energy of the β_2 particle = 1.3720995 − 0.412

= 0.9600995 MeV

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Question 13.30:

Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of ^{235}U in a fission reactor.

Answer

(a) Amount of hydrogen, m = 1 kg = 1000 g

1 mole, i.e., 1 g of hydrogen ($^{1}_{1}H$) contains 6.023 × 10^{23} atoms.

∴1000 g of $^{1}_{1}H$ contains 6.023 × 10^{23} × 1000 atoms.

Within the sun, four $^{1}_{1}H$ nuclei combine and form one $^{4}_{2}He$ nucleus. In this process 26 MeV of energy is released.

Hence, the energy released from the fusion of 1 kg $^{1}_{1}H$ is:

$$E_1 = \frac{6.023 \times 10^{23} \times 26 \times 10^3}{4}$$

$$= 39.1495 \times 10^{26} \text{ MeV}$$

(b) Amount of $^{235}_{92}U$ = 1 kg = 1000 g

1 mole, i.e., 235 g of $^{235}_{92}U$ contains 6.023 × 10^{23} atoms.

∴1000 g of $^{235}_{92}U$ contains $\frac{6.023 \times 10^{23} \times 1000}{235}$ atoms

It is known that the amount of energy released in the fission of one atom of $^{235}_{92}U$ is 200 MeV.

Hence, energy released from the fission of 1 kg of $^{235}_{92}U$ is:

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$$E_2 = \frac{6 \times 10^{23} \times 1000 \times 200}{235}$$

$$= 5.106 \times 10^{26} \text{ MeV}$$

$$\frac{E_1}{E_1} = \frac{39.1495 \times 10^{26}}{5.106 \times 10^{26}} = 7.67 \approx 8$$

∴

Therefore, the energy released in the fusion of 1 kg of hydrogen is nearly 8 times the energy released in the fission of 1 kg of uranium.

Question 13.31:

Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200MeV.

Answer

Amount of electric power to be generated, P = 2×10^5 MW

10% of this amount has to be obtained from nuclear power plants.

∴Amount of nuclear power, $P_1 = \frac{10}{100} \times 2 \times 10^5$

= 2×10^4 MW

= $2 \times 10^4 \times 10^6$ J/s

= $2 \times 10^{10} \times 60 \times 60 \times 24 \times 365$ J/y

Heat energy released per fission of a ^{235}U nucleus, E = 200 MeV

Efficiency of a reactor = 25%

Hence, the amount of energy converted into the electrical energy per fission is calculated as:

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$$\frac{25}{100} \times 200 = 50 \text{ MeV}$$

$$= 50 \times 1.6 \times 10^{-19} \times 10^{6} = 8 \times 10^{-12} \text{ J}$$

Number of atoms required for fission per year:

$$\frac{2 \times 10^{10} \times 60 \times 60 \times 24 \times 365}{8 \times 10^{-12}} = 78840 \times 10^{24} \text{ atoms}$$

1 mole, i.e., 235 g of U^{235} contains 6.023×10^{23} atoms.

∴Mass of 6.023×10^{23} atoms of U^{235} = 235 g = 235×10^{-3} kg

∴Mass of 78840×10^{24} atoms of U^{235}

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 78840 \times 10^{24}$$

$$= 3.076 \times 10^{4} \text{ kg}$$

Hence, the mass of uranium needed per year is 3.076×10^4 kg.