Catalog

CHAPTER 15

Introduction to Graphs

0852CH15

15.1 Introduction

Have you seen graphs in the newspapers, television, magazines, books etc.? The purpose of the graph is to show numerical facts in visual form so that they can be understood quickly, easily and clearly. Thus graphs are visual representations of data collected. Data can also be presented in the form of a table; however a graphical presentation is easier to understand. This is true in particular when there is **a trend** or **comparison** to be shown. We have already seen some types of graphs. Let us quickly recall them here.

15.1.1 A Bar graph

A bar graph is used to show comparison among categories. It may consist of two or more parallel vertical (or horizontal) bars (rectangles).

The bar graph in Fig 15.1 shows Anu's mathematics marks in the three terminal examinations. It helps you to compare her performance easily. She has shown good progress.

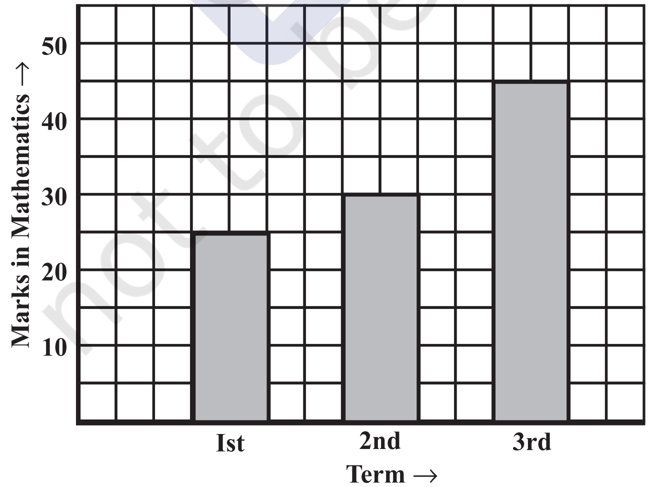


Fig 15.1

Bar graphs can also have double bars as in Fig 15.2. This graph gives a comparative account of sales (in ₹) of various fruits over a two-day period. How is Fig 15.2 different from Fig 15.1? Discuss with your friends.

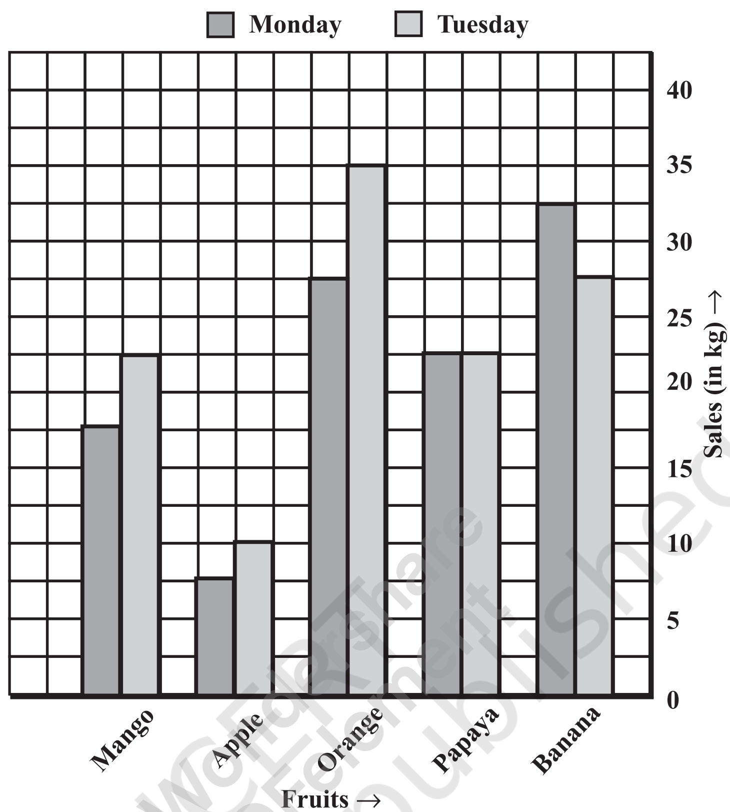


Fig 15.2

15.1.2 A Pie graph (or a circle-graph)

A pie-graph is used to compare parts of a whole. The circle represents the whole. Fig 15.3 is a pie-graph. It shows the percentage of viewers watching different types of TV channels.

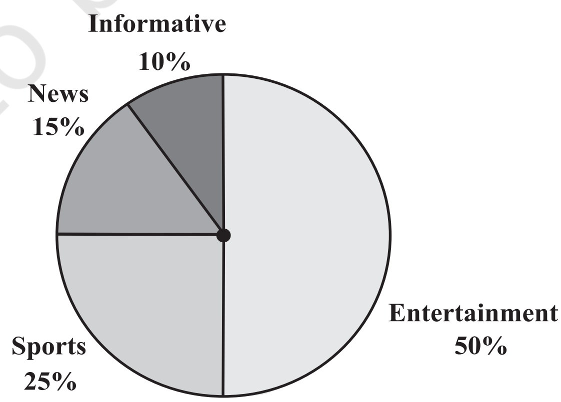


Fig 15.3

15.1.3 A histogram

A Histogram is a bar graph that shows data in intervals. It has adjacent bars over the intervals.

The histogram in Fig 15.4 illustrates the distribution of weights (in kg) of 40 persons of a locality.

Weights (kg)	40-45	45-50	50-55	55-60	60-65
No. of persons	4	12	13	6	5

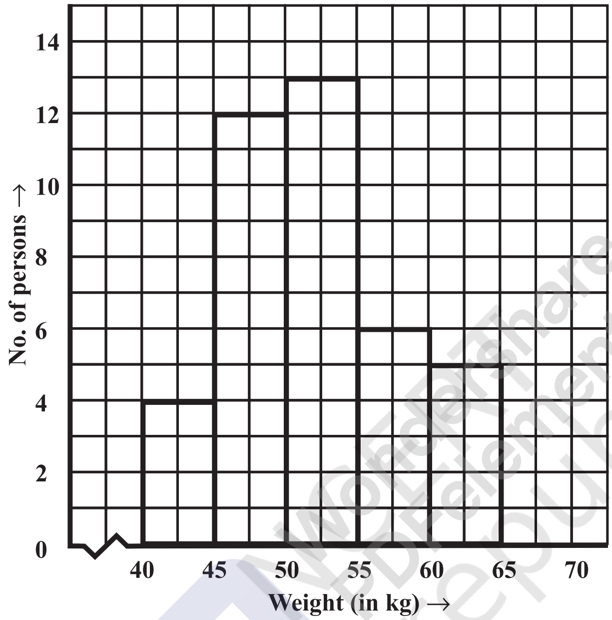


Fig 15.4

In Fig 15.4 a jagged line (⌇) has been used along horizontal line to indicate that we are not showing numbers between 0 and 40.

There are no gaps between bars, because there are no gaps between the intervals. What is the information that you gather from this histogram? Try to list them out.

15.1.4 A line graph

A **line graph** displays data that changes continuously over periods of time.

When Renu fell sick, her doctor maintained a record of her body temperature, taken every four hours. It was in the form of a graph (shown in Fig 15.5 and Fig 15.6).
We may call this a "time-temperature graph".
It is a pictorial representation of the following data, given in tabular form.

Time	6 a.m.	10 a.m.	2 p.m.	6 p.m.
Temperature(°C)	37	40	38	35

The horizontal line (usually called the *x*-axis) shows the timings at which the temperatures were recorded. What are labelled on the vertical line (usually called the *y*-axis)?

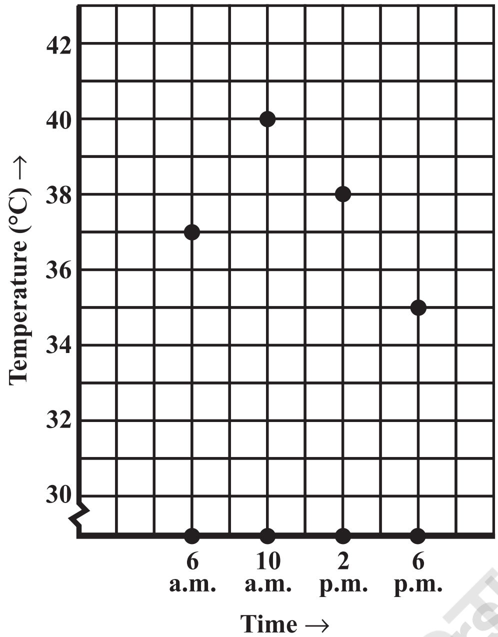


Fig 15.5

Each piece of data is shown by a point on the square grid.

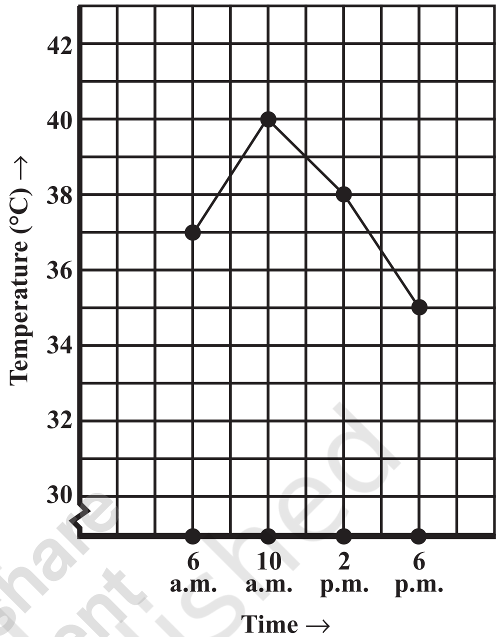


Fig 15.6

The points are then connected by line segments. The result is the **line graph**.

What all does this graph tell you? For example you can see the pattern of temperature; more at 10 a.m. (see Fig 15.5) and then decreasing till 6 p.m. Notice that the temperature increased by 3° C(= 40° C – 37° C) during the period 6 a.m. to 10 a.m.

There was no recording of temperature at 8 a.m., however the graph *suggests* that it was more than 37 °C (How?).

Example 1: **(A graph on "performance")**

The given graph (Fig 15.7) represents the total runs scored by two batsmen A and B, during each of the ten different matches in the year 2007. Study the graph and answer the following questions.

(i) What information is given on the two axes?
(ii) Which line shows the runs scored by batsman A?
(iii) Were the run scored by them same in any match in 2007? If so, in which match?
(iii) Among the two batsmen, who is steadier? How do you judge it?

Solution:

(i) The horizontal axis (or the *x*-axis) indicates the matches played during the year 2007. The vertical axis (or the *y*-axis) shows the total runs scored in each match.

(ii) The dotted line shows the runs scored by Batsman A. (This is already indicated at the top of the graph).

(iii) During the 4th match, both have scored the same number of 60 runs. (This is indicated by the point at which both graphs meet).

(iv) Batsman A has one great "peak" but many deep "valleys". He does not appear to be consistent. B, on the other hand has never scored below a total of 40 runs, even though his highest score is only 100 in comparison to 115 of A. Also A has scored a zero in two matches and in a total of 5 matches he has scored less than 40 runs. Since A has a lot of ups and downs, B is a more consistent and reliable batsman.

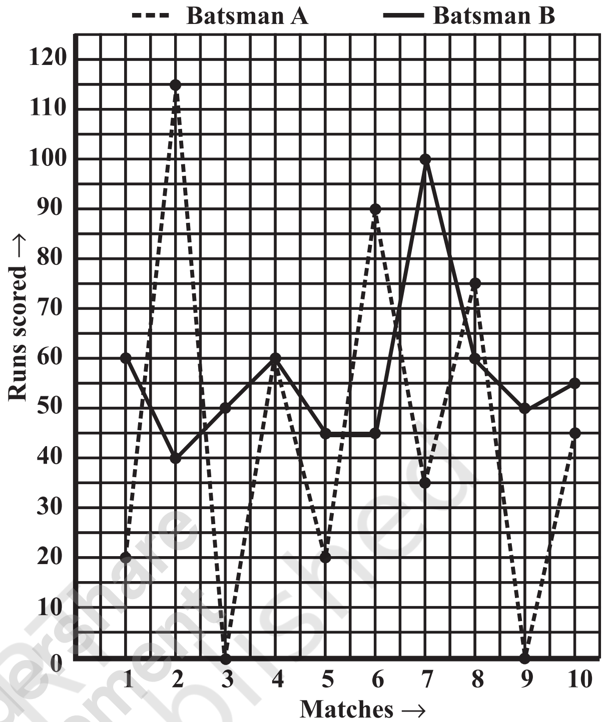


Fig 15.7

Example 2: The given graph (Fig 15.8) describes the distances of a car from a city P at different times when it is travelling from City P to City Q, which are 350 km apart. Study the graph and answer the following:

(i) What information is given on the two axes?
(ii) From where and when did the car begin its journey?
(iii) How far did the car go in the first hour?
(iv) How far did the car go during (i) the 2nd hour? (ii) the 3rd hour?
(v) Was the speed same during the first three hours? How do you know it?
(vi) Did the car stop for some duration at any place? Justify your answer.
(vii) When did the car reach City Q?

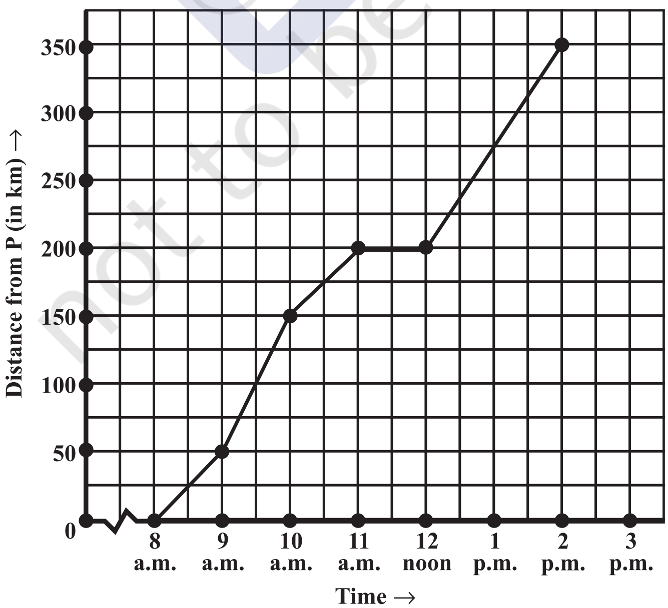


Fig 15.8

Solution:

(i) The horizontal (x) axis shows the time. The vertical (y) axis shows the distance of the car from City P.

(ii) The car started from City P at 8 a.m.

(iii) The car travelled 50 km during the first hour. [This can be seen as follows. At 8 a.m. it just started from City P. At 9 a.m. it was at the 50th km (seen from graph). Hence during the one-hour time between 8 a.m. and 9 a.m. the car travelled 50 km].

(iv) The distance covered by the car during

  (a) the 2nd hour (i.e., from 9 am to 10 am) is 100 km, (150 – 50).

  (b) the 3rd hour (i.e., from 10 am to 11 am) is 50 km (200 – 150).

(v) From the answers to questions (iii) and (iv), we find that the speed of the car was not the same all the time. (In fact the graph illustrates how the speed varied).

(vi) We find that the car was 200 km away from city P when the time was 11 a.m. and also at 12 noon. This shows that the car did not travel during the interval 11 a.m. to 12 noon. The horizontal line segment representing "travel" during this period is illustrative of this fact.

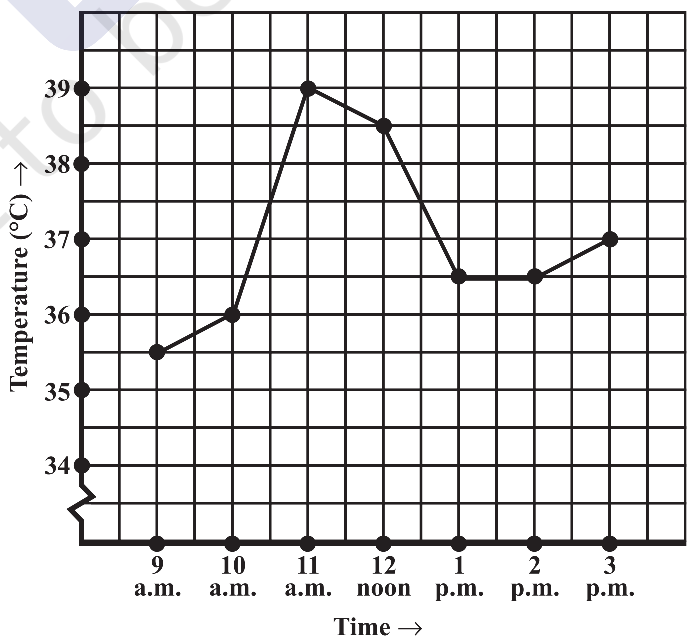
(vii) The car reached City Q at 2 p.m.

EXERCISE 15.1

1. The following graph shows the temperature of a patient in a hospital, recorded every hour.

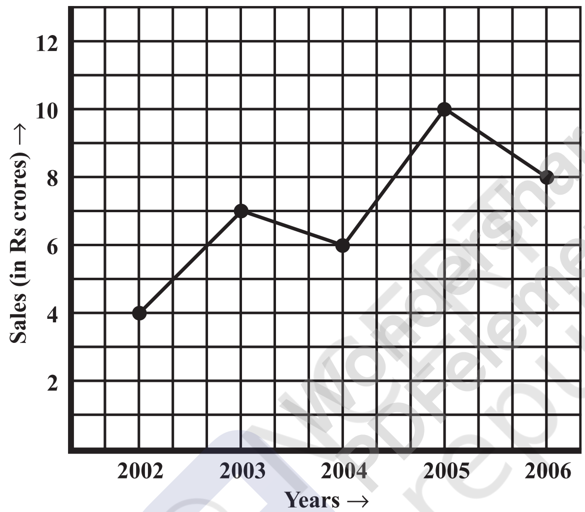
(a) What was the patient's temperature at 1 p.m. ?

(b) When was the patient's temperature 38.5° C?

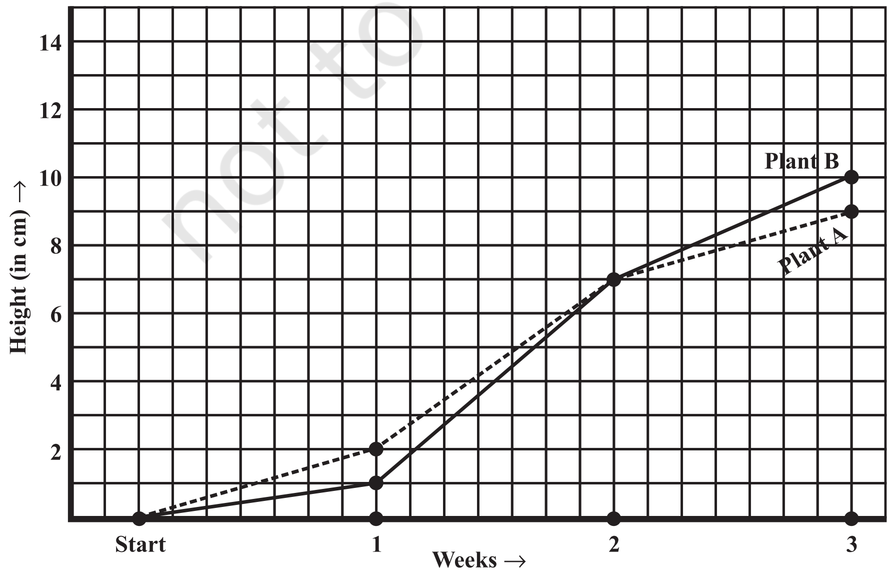


(c) The patient's temperature was the same two times during the period given. What were these two times?
(d) What was the temperature at 1.30 p.m.? How did you arrive at your answer?
(e) During which periods did the patients' temperature showed an upward trend?

2. The following line graph shows the yearly sales figures for a manufacturing company.
(a) What were the sales in (i) 2002 (ii) 2006?
(b) What were the sales in (i) 2003 (ii) 2005?
(c) Compute the difference between the sales in 2002 and 2006.
(d) In which year was there the greatest difference between the sales as compared to its previous year?

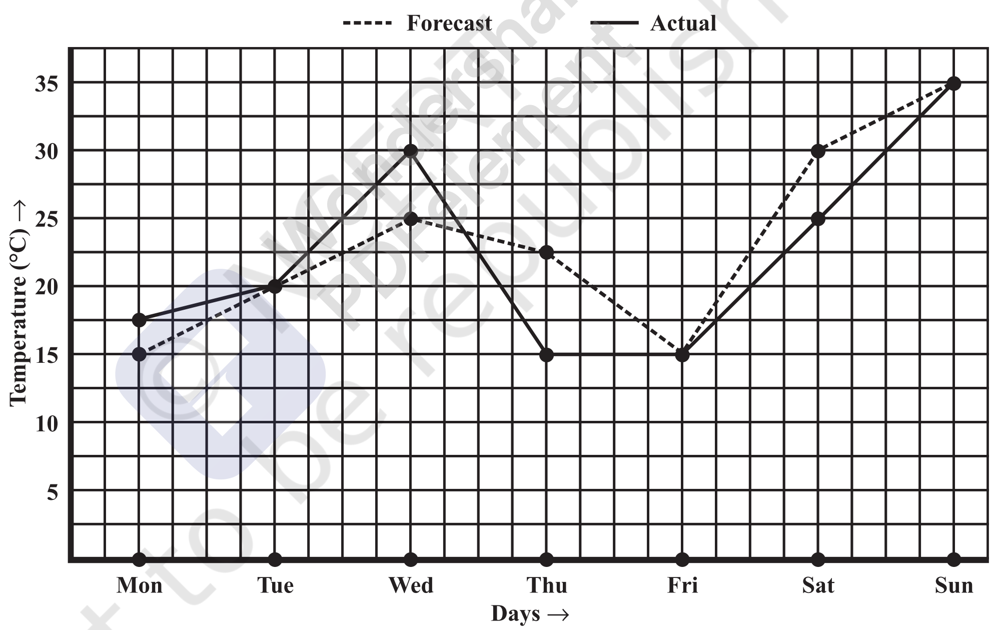


3. For an experiment in Botany, two different plants, plant A and plant B were grown under similar laboratory conditions. Their heights were measured at the end of each week for 3 weeks. The results are shown by the following graph.



(a) How high was Plant A after (i) 2 weeks (ii) 3 weeks?
(b) How high was Plant B after (i) 2 weeks (ii) 3 weeks?
(c) How much did Plant A grow during the 3rd week?
(d) How much did Plant B grow from the end of the 2nd week to the end of the 3rd week?
(e) During which week did Plant A grow most?
(f) During which week did Plant B grow least?
(g) Were the two plants of the same height during any week shown here? Specify.

4. The following graph shows the temperature forecast and the actual temperature for each day of a week.
(a) On which days was the forecast temperature the same as the actual temperature?
(b) What was the maximum forecast temperature during the week?
(c) What was the minimum actual temperature during the week?
(d) On which day did the actual temperature differ the most from the forecast temperature?



5. Use the tables below to draw linear graphs.
(a) The number of days a hill side city received snow in different years.

Year	2003	2004	2005	2006
Days	8	10	5	12

(b) Population (in thousands) of men and women in a village in different years.

Year	2003	2004	2005	2006	2007
Number of Men	12	12.5	13	13.2	13.5
Number of Women	11.3	11.9	13	13.6	12.8

6. A courier-person cycles from a town to a neighbouring suburban area to deliver a parcel to a merchant. His distance from the town at different times is shown by the following graph.

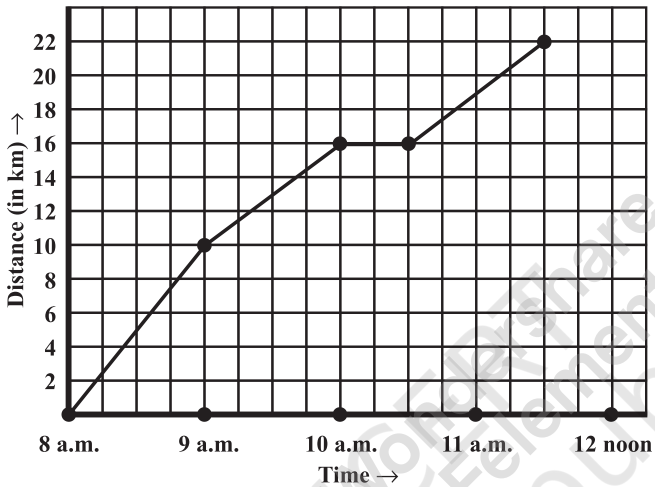
(a) What is the scale taken for the time axis?

(b) How much time did the person take for the travel?

(c) How far is the place of the merchant from the town?

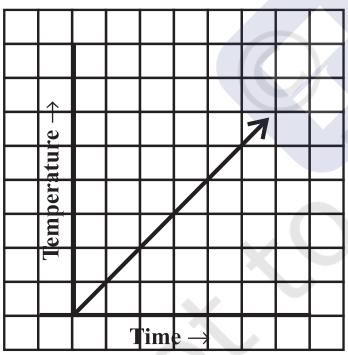
(d) Did the person stop on his way? Explain.

(e) During which period did he ride fastest?

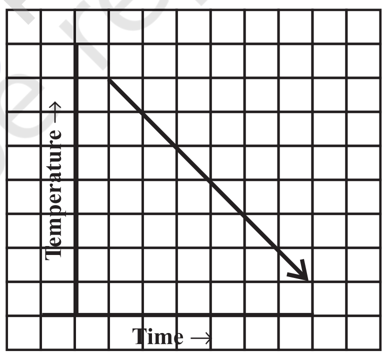


7. Can there be a time-temperature graph as follows? Justify your answer.

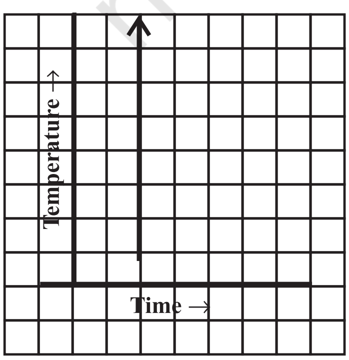
(i)



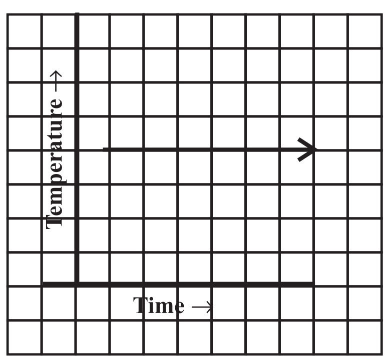
(ii)



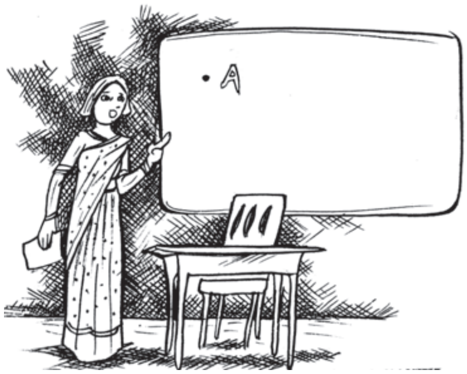
(iii)



(iv)



15.2 Linear Graphs



A line graph consists of bits of line segments joined consecutively. Sometimes the graph may be a whole unbroken line. Such a graph is called a **linear graph**. To draw such a line we need to locate some points on the graph sheet. We will now learn how to locate points conveniently on a graph sheet.

15.2.1 Location of a point

The teacher put a dot on the black-board. She asked the students how they would describe its location. There were several responses (Fig 15.9).

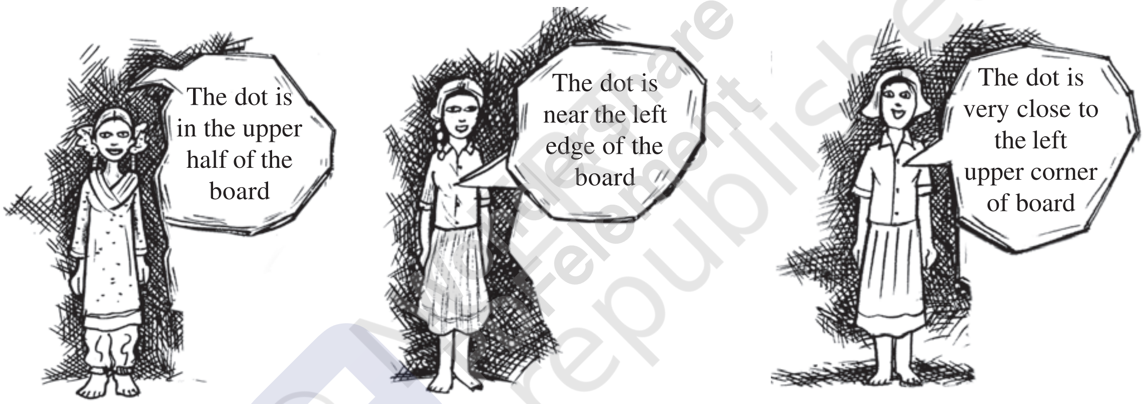


Fig 15.9

Can any one of these statements help fix the position of the dot? No! Why not? Think about it.

John then gave a suggestion. He measured the distance of the dot from the left edge of the board and said, "The dot is 90 cm from the left edge of the board". Do you think John's suggestion is really helpful? (Fig 15.10)

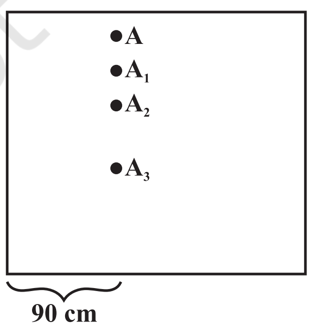


Fig 15.10

A, A_1, A_2, A_3 are all 90 cm away from the left edge.

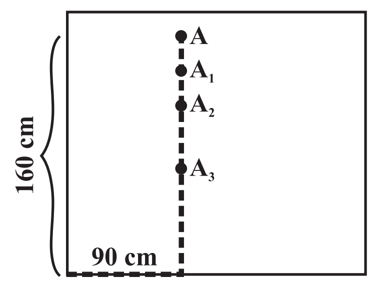
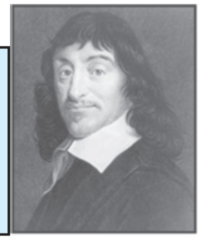


Fig 15.11

A is 90 cm from left edge and 160 cm from the bottom edge.

Rekha then came up with a modified statement : "The dot is 90 cm from the left edge and 160 cm from the bottom edge". That solved the problem completely! (Fig 15.11) The teacher said, "We describe the position of this dot by writing it as (90, 160)". Will the point (160, 90) be different from (90, 160)? Think about it.

The 17th century mathematician ***Rene Descartes****, it is said, noticed the movement of an insect near a corner of the ceiling and began to think of determining the position of a given point in a plane. His system of fixing a point with the help of two measurements, vertical and horizontal, came to be known as Cartesian system, in his honour.*



Rene Descartes (1596-1650)

15.2.2 Coordinates

Suppose you go to an auditorium and search for your reserved seat. You need to know two numbers, the row number and the seat number. This is the basic method for fixing a point in a plane.

Observe in Fig 15.12 how the point (3, 4) which is 3 units from left edge and 4 units from bottom edge is plotted on a graph sheet. The graph sheet itself is a square grid. We draw the *x* and *y* axes conveniently and then fix the required point. 3 is called the ***x*-coordinate** of the point; 4 is the ***y*-coordinate** of the point. We say that the **coordinates** of the point are (3, 4).

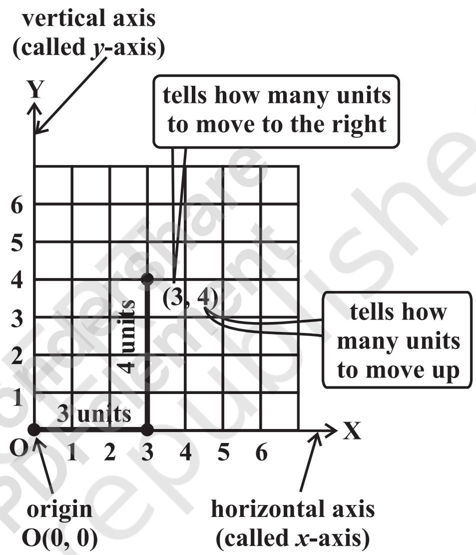


Fig 15.12

Example 3: Plot the point (4, 3) on a graph sheet. Is it the same as the point (3, 4)?

Solution: Locate the *x*, *y* axes, (they are actually number lines!). Start at O (0, 0). Move 4 units to the right; then move 3 units up, you reach the point (4, 3). From Fig 15.13, you can see that the points (3, 4) and (4, 3) are two different points.

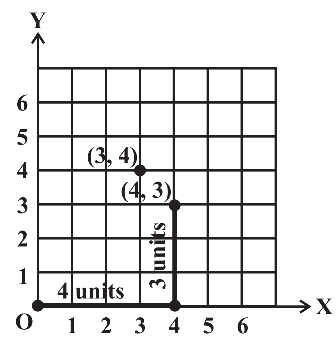


Fig 15.13

Example 4: From Fig 15.14, choose the letter(s) that indicate the location of the points given below:

(i) (2, 1)

(ii) (0, 5)

(iii) (2, 0)

Also write

(iv) The coordinates of A.

(v) The coordinates of F.

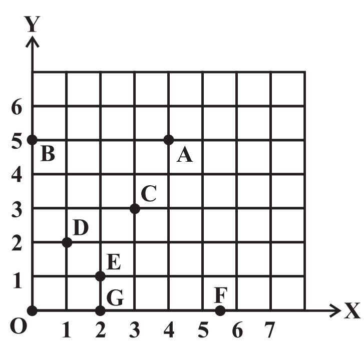


Fig 15.14

Solution:

(i) (2, 1) is the point E (It is not D!).

(ii) (0, 5) is the point B (why? Discuss with your friends!). (iii) (2, 0) is the point G.

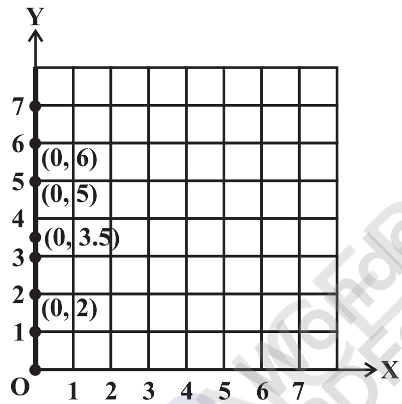
(iv) Point A is (4, 5) (v) F is (5.5, 0)

Example 5: Plot the following points and verify if they lie on a line. If they lie on a line, name it.

(i) (0, 2), (0, 5), (0, 6), (0, 3.5) (ii) A (1, 1), B (1, 2), C (1, 3), D (1, 4)

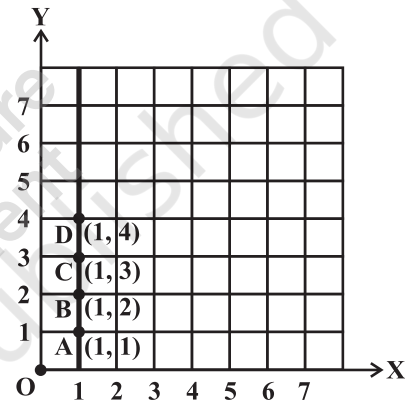
(iii) K (1, 3), L (2, 3), M (3, 3), N (4, 3) (iv) W (2, 6), X (3, 5), Y (5, 3), Z (6, 2)

Solution:



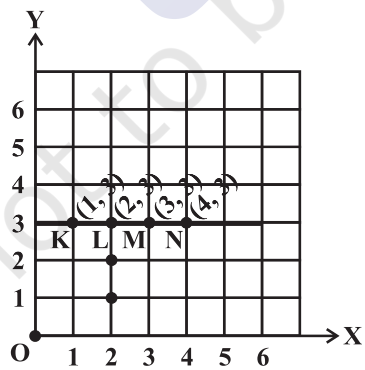
(i)

These lie on a line.
The line is *y*-axis.

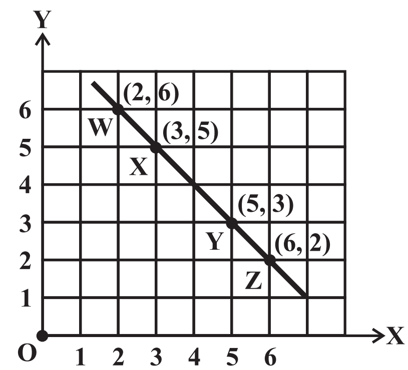


(ii)

These lie on a line. The line is AD.
(You may also use other ways of naming it). It is parallel to the *y*-axis



(iii)



(iv)

Fig 15.15

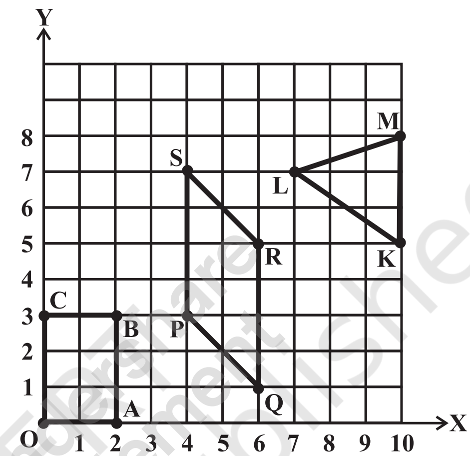
These lie on a line. We can name it as KL or KM or MN etc. It is parallel to *x*-axis

These lie on a line. We can name it as XY or WY or YZ etc.

Note that in each of the above cases, graph obtained by joining the plotted points is a line. Such graphs are called **linear graphs**.

EXERCISE 15.2

1. Plot the following points on a graph sheet. Verify if they lie on a line
   (a) A(4, 0), B(4, 2), C(4, 6), D(4, 2.5)
   (b) P(1, 1), Q(2, 2), R(3, 3), S(4, 4)
   (c) K(2, 3), L(5, 3), M(5, 5), N(2, 5)
2. Draw the line passing through (2, 3) and (3, 2). Find the coordinates of the points at which this line meets the x-axis and y-axis.
3. Write the coordinates of the vertices of each of these adjoining figures.
4. State whether True or False. Correct that are false.
   (i) A point whose x coordinate is zero and y-coordinate is non-zero will lie on the y-axis.
   (ii) A point whose y coordinate is zero and x-coordinate is 5 will lie on y-axis.
   (iii) The coordinates of the origin are (0, 0).



15.3 Some Applications

In everyday life, you might have observed that the more you use a facility, the more you pay for it. If more electricity is consumed, the bill is bound to be high. If less electricity is used, then the bill will be easily manageable. This is an instance where one quantity affects another. Amount of electric bill depends on the quantity of electricity used. We say that the quantity of electricity is an **independent variable** (or sometimes **control variable**) and the amount of electric bill is **the dependent variable**. The relation between such variables can be shown through a graph.

THINK, DISCUSS AND WRITE

The number of litres of petrol you buy to fill a car's petrol tank will decide the amount you have to pay. Which is the independent variable here? Think about it.

Example 6: **(Quantity and Cost)**

The following table gives the quantity of petrol and its cost.

No. of Litres of petrol	10	15	20	25
Cost of petrol in ₹	500	750	1000	1250

Plot a graph to show the data.

Solution: (i) Let us take a suitable scale on both the axes (Fig 15.16).

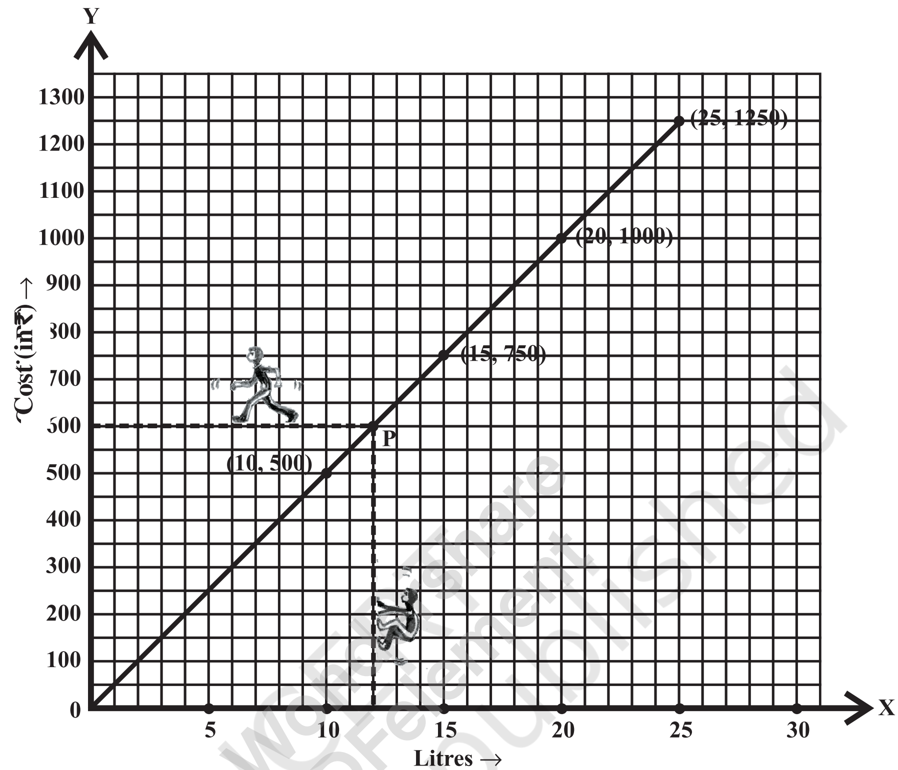


Fig 15.16

(ii) Mark number of litres along the horizontal axis.

(iii) Mark cost of petrol along the vertical axis.

(iv) Plot the points: (10,500), (15,750), (20,1000), (25,1250).

(v) Join the points.

We find that the graph is a line. (It is a linear graph). Why does this graph pass through the origin? Think about it.

This graph can help us to estimate a few things. Suppose we want to find the amount needed to buy 12 litres of petrol. Locate 12 on the horizontal axis.

Follow the vertical line through 12 till you meet the graph at P (say).

From P you take a horizontal line to meet the vertical axis. This meeting point provides the answer.

This is the graph of a situation in which two quantities, are in direct variation. (How ?). In such situations, the graphs will always be linear.

TRY THESE

In the above example, use the graph to find how much petrol can be purchased for ₹ 800.

Example 7: **(Principal and Simple Interest)**

A bank gives 10% Simple Interest (S.I.) on deposits by senior citizens. Draw a graph to illustrate the relation between the sum deposited and simple interest earned. Find from your graph

(a) the annual interest obtainable for an investment of ₹ 250.

(b) the investment one has to make to get an annual simple interest of ₹ 70.

Solution:

Sum deposited	Simple interest for a year
₹ 100	₹ $\frac{100\times1\times10}{100}$ = ₹ 10
₹ 200	₹ $\frac{200\times1\times10}{100}$ = ₹ 20
₹ 300	₹ $\frac{300\times1\times10}{100}$ = ₹ 30
₹ 500	₹ $\frac{500\times1\times10}{100}$ = ₹ 50
₹ 1000	₹ 100

Steps to follow:
1. Find the quantities to be plotted as Deposit and SI.
2. Decide the quantities to be taken on x-axis and on y-axis.
3. Choose a scale.
4. Plot points.
5. Join the points.

We get a table of values.

Deposit (in ₹)	100	200	300	500	1000
Annual S.I. (in ₹)	10	20	30	50	100

(i) Scale : 1 unit = ₹ 100 on horizontal axis; 1 unit = ₹ 10 on vertical axis.

(ii) Mark Deposits along horizontal axis.

(iii) Mark Simple Interest along vertical axis.

(iv) Plot the points : (100,10), (200, 20), (300, 30), (500,50) etc.

(v) Join the points. We get a graph that is a line (Fig 15.17).

(a) Corresponding to ₹ 250 on horizontal axis, we get the interest to be ₹ 25 on vertical axis.

(b) Corresponding to ₹ 70 on the vertical axis, we get the sum to be ₹ 700 on the horizontal axis.

TRY THESE

Is Example 7, a case of direct variation?

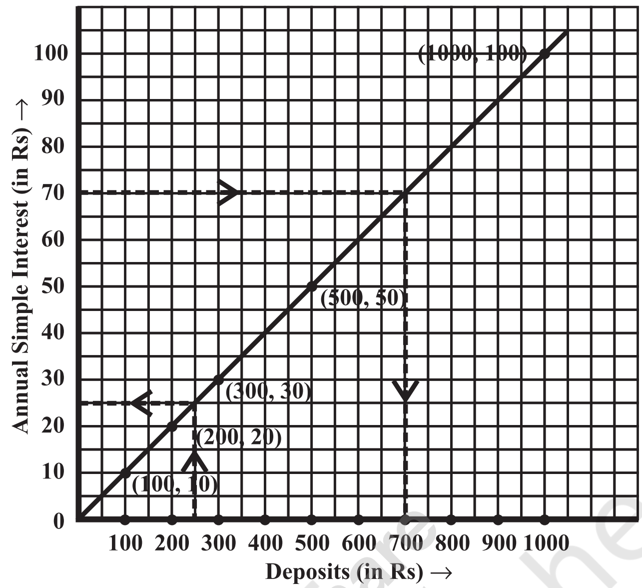


Fig 15.17

Example 8: **(Time and Distance)**

Ajit can ride a scooter constantly at a speed of 30 kms/hour. Draw a time-distance graph for this situation. Use it to find

(i) the time taken by Ajit to ride 75 km. (ii) the distance covered by Ajit in $3\frac{1}{2}$ hours.

Solution:

Hours of ride	Distance covered
1 hour	30 km
2 hours	2×30 km = 60 km
3 hours	3×30 km = 90 km
4 hours	4×30 km = 120 km and so on.

We get a table of values.

Time (in hours)	1	2	3	4
Distance covered (in km)	30	60	90	120

(i) Scale: (Fig 15.18)

Horizontal: 2 units = 1 hour

Vertical: 1 unit = 10 km

(ii) Mark time on horizontal axis.

(iii) Mark distance on vertical axis.

(iv) Plot the points: (1, 30), (2, 60), (3, 90), (4, 120).

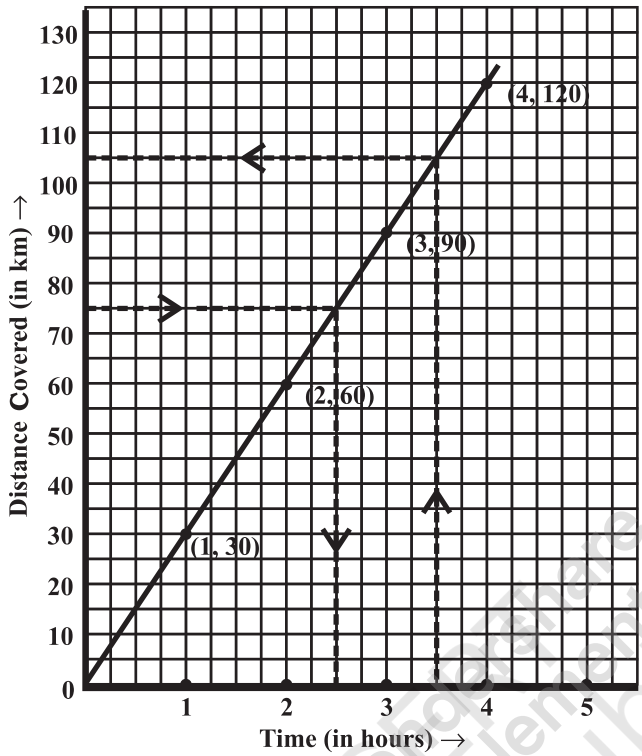


Fig 15.18

(v) Join the points. We get a linear graph.

(a) Corresponding to 75 km on the vertical axis, we get the time to be 2.5 hours on the horizontal axis. Thus 2.5 hours are needed to cover 75 km.

(b) Corresponding to $3\frac{1}{2}$ hours on the horizontal axis, the distance covered is 105 km on the vertical axis.

EXERCISE 15.3

1. Draw the graphs for the following tables of values, with suitable scales on the axes.

(a) Cost of apples

Number of apples	1	2	3	4	5
Cost (in ₹)	5	10	15	20	25

(b) Distance travelled by a car

Time (in hours)	6 a.m.	7 a.m.	8 a.m.	9 a.m.
Distances (in km)	40	80	120	160

(i) How much distance did the car cover during the period 7.30 a.m. to 8 a.m?

(ii) What was the time when the car had covered a distance of 100 km since it's start?

(c) Interest on deposits for a year.

Deposit (in ₹)	1000	2000	3000	4000	5000
Simple Interest (in ₹)	80	160	240	320	400

(i) Does the graph pass through the origin?

(ii) Use the graph to find the interest on ₹ 2500 for a year.

(iii) To get an interest of ₹ 280 per year, how much money should be deposited?

2. Draw a graph for the following.

(i)

Side of square (in cm)	2	3	3.5	5	6
Perimeter (in cm)	8	12	14	20	24

Is it a linear graph?

(ii)

Side of square (in cm)	2	3	4	5	6
Area (in cm^2)	4	9	16	25	36

Is it a linear graph?

WHAT HAVE WE DISCUSSED?

1. Graphical presentation of data is easier to understand.

2. (i) A **bar graph** is used to show comparison among categories.

(ii) A **pie graph** is used to compare parts of a whole.

(iii) A **Histogram** is a bar graph that shows data in intervals.

3. A **line graph** displays data that changes continuously over periods of time.

4. A line graph which is a whole unbroken line is called a **linear graph**.

5. For fixing a point on the graph sheet we need, ***x*-coordinate** and ***y*-coordinate**.

6. The relation between **dependent variable** and **independent variable** is shown through a graph.

CHAPTER

16

Playing with Numbers

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16.1 Introduction

You have studied various types of numbers such as natural numbers, whole numbers, integers and rational numbers. You have also studied a number of interesting properties about them. In Class VI, we explored finding factors and multiples and the relationships among them.

In this chapter, we will explore numbers in more detail. These ideas help in justifying tests of divisibility.

16.2 Numbers in General Form

Let us take the number 52 and write it as

$$52 = 50 + 2 = 10 \times 5 + 2$$

Similarly, the number 37 can be written as

$$37 = 10 \times 3 + 7$$

In general, any two digit number *ab* made of digits *a* and *b* can be written as

$$ab = 10 \times a + b = 10a + b$$

Here *ab* does not mean $a \times b$!

What about *ba*? $$ba = 10 \times b + a = 10b + a$$

Let us now take number 351. This is a three digit number. It can also be written as

$$351 = 300 + 50 + 1 = 100 \times 3 + 10 \times 5 + 1 \times 1$$

Similarly $$497 = 100 \times 4 + 10 \times 9 + 1 \times 7$$

In general, a 3-digit number *abc* made up of digits *a*, *b* and *c* is written as

$$abc = 100 \times a + 10 \times b + 1 \times c$$
$$= 100a + 10b + c$$

In the same way,

$$cab = 100c + 10a + b$$
$$bca = 100b + 10c + a$$ and so on.

TRY THESE

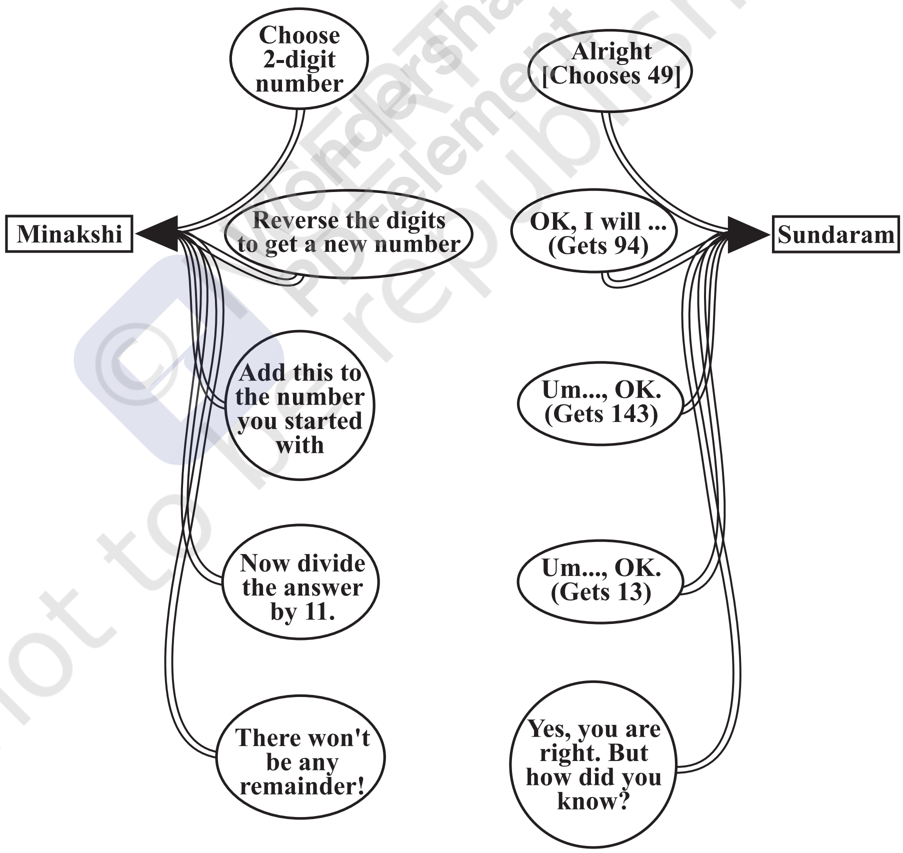
1. Write the following numbers in generalised form.
   (i) 25 (ii) 73 (iii) 129 (iv) 302
2. Write the following in the usual form.
   (i) $10 \times 5 + 6$ (ii) $100 \times 7 + 10 \times 1 + 8$ (iii) $100 \times a + 10 \times c + b$

16.3 Games with Numbers

(i) **Reversing the digits – two digit number**

Minakshi asks Sundaram to think of a 2-digit number, and then to do whatever she asks him to do, to that number. Their conversation is shown in the following figure. **Study the figure carefully before reading on**.

Conversations between Minakshi and Sundaram: First Round ...



It so happens that Sundaram chose the number 49. So, he got the reversed number 94; then he added these two numbers and got $49 + 94 = 143$. Finally he divided this number by 11 and got $143 \div 11 = 13$, with no remainder. This is just what Minakshi had predicted.

TRY THESE

Check what the result would have been if Sundaram had chosen the numbers shown below.

1. 27   2. 39   3. 64   4. 17

Now, let us see if we can **explain** Minakshi's "trick".

Suppose Sundaram chooses the number *ab*, which is a short form for the 2-digit number $10a + b$. On reversing the digits, he gets the number $ba = 10b + a$. When he adds the two numbers he gets:

$$(10a + b) + (10b + a) = 11a + 11b$$
$$= 11\,(a + b).$$

So, the sum is always a multiple of 11, just as Minakshi had claimed.

Observe here that if we divide the sum by 11, the quotient is $a + b$, which is exactly the sum of the digits of chosen number *ab*.

You may check the same by taking any other two digit number.

The game between Minakshi and Sundaram continues!

Minakshi: Think of another 2-digit number, but don't tell me what it is.

Sundaram: Alright.

Minakshi: Now reverse the digits of the number, and *subtract* the smaller number from the larger one.

Sundaram: I have done the subtraction. What next?

Minakshi: Now divide your answer by 9. I claim that there will be no remainder!

Sundaram: Yes, you are right. There is indeed no remainder! But this time I think I know how you are so sure of this!

In fact, Sundaram had thought of 29. So his calculations were: first he got the number 92; then he got $92 - 29 = 63$; and finally he did $(63 \div 9)$ and got 7 as quotient, with no remainder.

TRY THESE

Check what the result would have been if Sundaram had chosen the numbers shown below.

1. 17   2. 21   3. 96   4. 37

Let us see how Sundaram explains Minakshi's second "trick". (Now he feels confident of doing so!)

Suppose he chooses the 2-digit number $ab = 10a + b$. After reversing the digits, he gets the number $ba = 10b + a$. Now Minakshi tells him to do a subtraction, the smaller number from the larger one.

- If the tens digit is larger than the ones digit (that is, $a > b$), he does:

$$(10a + b) - (10b + a) = 10a + b - 10b - a$$
$$= 9a - 9b = 9(a - b).$$

- If the ones digit is larger than the tens digit (that is, $b > a$), he does:

$$(10b + a) - (10a + b) = 9(b - a).$$

- And, of course, if $a = b$, he gets 0.

In each case, the resulting number is divisible by 9. So, the remainder is 0. Observe here that if we divide the resulting number (obtained by subtraction), the quotient is $a - b$ or $b - a$ according as $a > b$ or $a < b$. You may check the same by taking any other two digit numbers.

(ii) **Reversing the digits – three digit number**.

Now it is Sundaram's turn to play some tricks!

Sundaram: Think of a 3-digit number, but don't tell me what it is.

Minakshi: Alright.

Sundaram: Now make a new number by putting the digits in reverse order, and subtract the smaller number from the larger one.

Minakshi: Alright, I have done the subtraction. What next?

Sundaram: Divide your answer by 99. I am sure that there will be no remainder!

In fact, Minakshi chose the 3-digit number 349. So she got:

- Reversed number: 943;
- Difference: $943 - 349 = 594$;
- Division: $594 \div 99 = 6$, with no remainder.

TRY THESE

Check what the result would have been if Minakshi had chosen the numbers shown below. In each case keep a record of the quotient obtained at the end.

1. 132 2. 469 3. 737 4. 901

Let us see how this trick works.

Let the 3-digit number chosen by Minakshi be $abc = 100a + 10b + c$.

After reversing the order of the digits, she gets the number $cba = 100c + 10b + a$. On subtraction:

- If $a > c$, then the difference between the numbers is

$$(100a + 10b + c) - (100c + 10b + a) = 100a + 10b + c - 100c - 10b - a$$
$$= 99a - 99c = 99(a - c).$$

- If $c > a$, then the difference between the numbers is

$$(100c + 10b + a) - (100a + 10b + c) = 99c - 99a = 99(c - a).$$

- And, of course, if $a = c$, the difference is 0.

In each case, the resulting number is divisible by 99. So the remainder is 0. Observe that quotient is $a - c$ or $c - a$. You may check the same by taking other 3-digit numbers.

(iii) **Forming three-digit numbers with given three-digits**.

Now it is Minakshi's turn once more.

Minakshi: Think of any 3-digit number.

Sundaram: Alright, I have done so.

Minakshi: Now use this number to form two more 3-digit numbers, like this: if the number you chose is *abc*, then

- 'the first number is *cab* (i.e., with the ones digit shifted to the "left end" of the number);
- the other number is *bca* (i.e., with the hundreds digit shifted to the "right end" of the number).

Now add them up. Divide the resulting number by 37. I claim that there will be no remainder.

Sundaram: Yes. You are right!

In fact, Sundaram had thought of the 3-digit number 237. After doing what Minakshi had asked, he got the numbers 723 and 372. So he did:

$$\begin{array}{r} 2\ 3\ 7 \\ +\ \ 7\ 2\ 3 \\ +\ \ 3\ 7\ 2 \\ \hline 1\ 3\ 3\ 2 \\ \hline \end{array}$$

Form all possible 3-digit numbers using all the digits 2, 3 and 7 and find their sum. Check whether the sum is divisible by 37! Is it true for the sum of all the numbers formed by the digits *a*, *b* and *c* of the number *abc*?

Then he divided the resulting number 1332 by 37:

$$1332 \div 37 = 36, \quad \text{with no remainder.}$$

TRY THESE

Check what the result would have been if Sundaram had chosen the numbers shown below.

1. 417 2. 632 3. 117 4. 937

Will this trick always work?

Let us see.

$$abc = 100a + 10b + c$$

$$cab = 100c + 10a + b$$

$$bca = 100b + 10c + a$$

$$abc + cab + bca = 111(a + b + c)$$

$$= 37 \times 3(a + b + c), \text{ which is divisible by 37}$$

16.4 Letters for Digits

Here we have puzzles in which letters take the place of digits in an arithmetic 'sum', and the problem is to find out which letter represents which digit; so it is like cracking a code. Here we stick to problems of addition and multiplication.

Here are two rules we follow while doing such puzzles.

1. *Each letter in the puzzle must stand for just one digit. Each digit must be represented by just one letter.*
2. *The first digit of a number cannot be zero.* Thus, we write the number "sixty three" as 63, and not as 063, or 0063.

A rule that we would *like* to follow is that the puzzle must have just one answer.

Example 1: Find Q in the addition.

$$\begin{array}{r} 3\ 1\ Q \\ +\ 1\ Q\ 3 \\ \hline 5\ 0\ 1 \\ \hline \end{array}$$

Solution:

There is just one letter Q whose value we have to find.

Study the addition in the ones column: from Q + 3, we get '1', that is, a number whose ones digit is 1.

For this to happen, the digit Q should be 8. So the puzzle can be solved as shown below.

$$\begin{array}{r} 3\ 1\ 8 \\ +\ 1\ 8\ 3 \\ \hline 5\ 0\ 1 \\ \hline \end{array}$$

That is, Q = 8

Example 2: Find A and B in the addition.

$$\begin{array}{r} A \\ +\ \ A \\ +\ \ A \\ \hline B\ \ A \\ \hline \end{array}$$

Solution: This has *two* letters A and B whose values are to be found.

Study the addition in the ones column: the sum of *three* A's is a number whose ones digit is A. Therefore, the sum of *two* A's must be a number whose ones digit is 0.

This happens only for A = 0 and A = 5.

If A = 0, then the sum is 0 + 0 + 0 = 0, which makes B = 0 too. We do not want this (as it makes A = B, and then the tens digit of BA too becomes 0), so we reject this possibility. So, A = 5.

Therefore, the puzzle is solved as shown below.

$$\begin{array}{r} 5 \\ +\ \ 5 \\ +\ \ 5 \\ \hline 1\ \ 5 \\ \hline \end{array}$$

That is, A = 5 and B = 1.

Example 3: Find the digits A and B.

$$\begin{array}{r} B\ A \\ \times\ B\ 3 \\ \hline 5\ 7\ A \\ \hline \end{array}$$

Solution:

This also has two letters A and B whose values are to be found.
Since the ones digit of $3 \times A$ is A, it must be that $A = 0$ or $A = 5$.

Now look at B. If $B = 1$, then $BA \times B3$ would *at most* be equal to 19×19; that is, it would at most be equal to 361. But the product here is 57A, which is more than 500. So we cannot have $B = 1$.

If $B = 3$, then $BA \times B3$ would be more than 30×30; that is, more than 900. But 57A is less than 600. So, B can not be equal to 3.

Putting these two facts together, we see that $B = 2$ only. So the multiplication is either 20×23, or 25×23.

The first possibility fails, since $20 \times 23 = 460$. But, the second one works out correctly, since $25 \times 23 = 575$.

So the answer is $A = 5$, $B = 2$.

$$\begin{array}{r} 2\ 5 \\ \times\ 2\ 3 \\ \hline 5\ 7\ 5 \\ \hline \end{array}$$

DO THIS

Write a 2-digit number *ab* and the number obtained by reversing its digits i.e., *ba*. Find their sum. Let the sum be a 3-digit number *dad*

$$\text{i.e.,} \quad ab + ba = dad$$
$$(10a + b) + (10b + a) = dad$$
$$11(a + b) = dad$$

The sum $a + b$ can not exceed 18 (Why?).

Is *dad* a multiple of 11?

Is *dad* less than 198?

Write all the 3-digit numbers which are multiples of 11 upto 198.

Find the values of *a* and *d*.

EXERCISE 16.1

Find the values of the letters in each of the following and give reasons for the steps involved.

1. $$\begin{array}{r} 3\ A \\ +\ 2\ 5 \\ \hline B\ 2 \\ \hline \end{array}$$

2. $$\begin{array}{r} 4\ A \\ +\ 9\ 8 \\ \hline C\ B\ 3 \\ \hline \end{array}$$

3. $$\begin{array}{r} 1\ A \\ \times\ A \\ \hline 9\ A \\ \hline \end{array}$$

4.
$$\begin{array}{r} A\ B \\ +\ 3\ 7 \\ \hline 6\ A \\ \hline \end{array}$$

5.
$$\begin{array}{r} A\ B \\ \times\ \ 3 \\ \hline C\ A\ B \\ \hline \end{array}$$

6.
$$\begin{array}{r} A\ B \\ \times\ \ 5 \\ \hline C\ A\ B \\ \hline \end{array}$$

7.
$$\begin{array}{r} A\ B \\ \times\ \ 6 \\ \hline B\ B\ B \\ \hline \end{array}$$

8.
$$\begin{array}{r} A\ 1 \\ +\ 1\ B \\ \hline B\ 0 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 2\ A\ B \\ +\ A\ B\ 1 \\ \hline B\ 1\ 8 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 1\ 2\ A \\ +\ 6\ A\ B \\ \hline A\ 0\ 9 \\ \hline \end{array}$$

16.5 Tests of Divisibility

In Class VI, you learnt how to check divisibility by the following divisors.

10, 5, 2, 3, 6, 4, 8, 9, 11.

You would have found the tests easy to do, but you may have wondered at the same time *why* they work. Now, in this chapter, we shall go into the "why" aspect of the above.

16.5.1 Divisibility by 10

This is certainly the easiest test of all! We first look at some multiples of 10.

10, 20, 30, 40, 50, 60, ... ,

and then at some non-multiples of 10.

13, 27, 32, 48, 55, 69,

From these lists we see that if the ones digit of a number is 0, then the number is a multiple of 10; and if the ones digit is *not* 0, then the number is *not* a multiple of 10. So, we get a test of divisibility by 10.

Of course, we must not stop with just stating the test; we must also explain *why* it "works". That is not hard to do; we only need to remember the rules of place value. Take the number. ... *cba*; this is a short form for

$$\ldots + 100c + 10b + a$$

Here a is the one's digit, b is the ten's digit, c is the hundred's digit, and so on. The dots are there to say that there may be more digits to the left of c.

Since 10, 100, ... are divisible by 10, so are $10b$, $100c$, ... And as for the number a is concerned, it must be a divisible by 10 if the given number is divisible by 10. This is possible only when $a = 0$.

Hence, a number is divisible by 10 when its one's digit is 0.

16.5.2 Divisibility by 5

Look at the multiples of 5.

5, 10, 15, 20, 25, 30, 35, 40, 45, 50,

We see that *the one's digits are alternately* 5 *and* 0, *and no other digit ever appears in this list.*

So, we get our test of divisibility by 5.

If the ones digit of a number is 0 *or* 5, *then it* is *divisible by* 5.

Let us explain this rule. Any number ... *cba* can be written as:

$\ldots + 100c + 10b + a$

Since 10, 100 are divisible by 10 so are $10b$, $100c$, ... which in turn, are divisible by 5 because $10 = 2 \times 5$. As far as number a is concerned it must be divisible by 5 if the number is divisible by 5. So a has to be either 0 or 5.

TRY THESE

(The first one has been done for you.)

1. If the division $N \div 5$ leaves a remainder of 3, what might be the ones digit of N? (The one's digit, when divided by 5, must leave a remainder of 3. So the one's digit must be either 3 or 8.)
2. If the division $N \div 5$ leaves a remainder of 1, what might be the one's digit of N?
3. If the division $N \div 5$ leaves a remainder of 4, what might be the one's digit of N?

16.5.3 Divisibility by 2

Here are the even numbers.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, ... ,

and here are the odd numbers.

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, ... ,

We see that a natural number is even if its one's digit is

2, 4, 6, 8 or 0

A number is odd if its one's digit is

1, 3, 5, 7 or 9

Recall the test of divisibility by 2 learnt in Class VI, which is as follows.

If the one's digit of a number is 0, 2, 4, 6 or 8 then the number is divisible by 2.

The explanation for this is as follows.

Any number *cba* can be written as $100c + 10b + a$

First two terms namely $100c$, $10b$ are divisible by 2 because 100 and 10 are divisible by 2. So far as a is concerned, it must be divisible by 2 if the given number is divisible by 2. This is possible only when $a = 0, 2, 4, 6$ or 8.

TRY THESE

(The first one has been done for you.)

1. If the division $N \div 2$ leaves a remainder of 1, what might be the one's digit of N? (N is odd; so its one's digit is odd. Therefore, the one's digit must be 1, 3, 5, 7 or 9.)

2. If the division N ÷ 2 leaves no remainder (i.e., zero remainder), what might be the one's digit of N?
3. Suppose that the division N ÷ 5 leaves a remainder of 4, and the division N ÷ 2 leaves a remainder of 1. What must be the one's digit of N?

16.5.4 Divisibility by 9 and 3

Look carefully at the three tests of divisibility found till now, for checking division by 10, 5 and 2. We see something common to them: *they use only the one's digit of the given number; they do not bother about the 'rest' of the digits*. Thus, *divisibility is decided just by the one's digit.* 10, 5, 2 are divisors of 10, which is the key number in our place value.

But for checking divisibility by 9, this will not work. Let us take some number say 3573.

Its expanded form is: $3 \times 1000 + 5 \times 100 + 7 \times 10 + 3$

This is equal to $3 \times (999 + 1) + 5 \times (99 + 1) + 7 \times (9 + 1) + 3$

$$= 3 \times 999 + 5 \times 99 + 7 \times 9 + (3 + 5 + 7 + 3) \quad \text{... (1)}$$

We see that the number 3573 will be divisible by 9 or 3 if $(3 + 5 + 7 + 3)$ is divisible by 9 or 3.

We see that $3 + 5 + 7 + 3 = 18$ is divisible by 9 and also by 3. Therefore, the number 3573 is divisible by both 9 and 3.

Now, let us consider the number 3576. As above, we get

$$3576 = 3 \times 999 + 5 \times 99 + 7 \times 9 + (3 + 5 + 7 + 6) \quad \text{... (2)}$$

Since $(3 + 5 + 7 + 6)$ i.e., 21 is not divisible by 9 but is divisible by 3,

therefore 3576 is not divisible by 9. However 3576 is divisible by 3. Hence,

(i) A number N is divisible by 9 if the sum of its digits is divisible by 9. Otherwise it is not divisible by 9.

(ii) A number N is divisible by 3 if the sum of its digits is divisible by 3. Otherwise it is not divisible by 3.

If the number is '*cba*', then, $100c + 10b + a = 99c + 9b + (a + b + c)$

$$= \underbrace{9(11c + b)}_{\text{divisible by 3 and 9}} + (a + b + c)$$

Hence, divisibility by 9 (or 3) is possible if $a + b + c$ is divisible by 9 (or 3).

Example 4: Check the divisibility of 21436587 by 9.

Solution: The sum of the digits of 21436587 is $2 + 1 + 4 + 3 + 6 + 5 + 8 + 7 = 36$. This number is divisible by 9 (for $36 \div 9 = 4$). We conclude that 21436587 is divisible by 9.

We can double-check:

$$\frac{21436587}{9} = 2381843 \qquad \text{(the division is exact).}$$

Example 5: Check the divisibility of 152875 by 9.

Solution: The sum of the digits of 152875 is $1 + 5 + 2 + 8 + 7 + 5 = 28$. This number is **not** divisible by 9. We conclude that 152875 is not divisible by 9.

TRY THESE

Check the divisibility of the following numbers by 9.

1. 108
2. 616
3. 294
4. 432
5. 927

Example 6: If the three digit number $24x$ is divisible by 9, what is the value of x?

Solution: Since $24x$ is divisible by 9, sum of it's digits, i.e., $2 + 4 + x$ should be divisible by 9, i.e., $6 + x$ should be divisible by 9.

This is possible when $6 + x = 9$ or 18, ...
But, since x is a digit, therefore, $6 + x = 9$, i.e., $x = 3$.

THINK, DISCUSS AND WRITE

1. You have seen that a number 450 is divisible by 10. It is also divisible by 2 and 5 which are factors of 10. Similarly, a number 135 is divisible 9. It is also divisible by 3 which is a factor of 9.
   Can you say that if a number is divisible by any number m, then it will also be divisible by each of the factors of m?
2. (i) Write a 3-digit number abc as $100a + 10b + c$
   $$= 99a + 11b + (a - b + c)$$
   $$= 11(9a + b) + (a - b + c)$$
   If the number abc is divisible by 11, then what can you say about $(a - b + c)$?
   Is it necessary that $(a + c - b)$ should be divisible by 11?

   (ii) Write a 4-digit number $abcd$ as $1000a + 100b + 10c + d$
   $$= (1001a + 99b + 11c) - (a - b + c - d)$$
   $$= 11(91a + 9b + c) + [(b + d) - (a + c)]$$
   If the number $abcd$ is divisible by 11, then what can you say about $[(b + d) - (a + c)]$?

   (iii) From (i) and (ii) above, can you say that a number will be divisible by 11 if the difference between the sum of digits at its odd places and that of digits at the even places is divisible by 11?

Example 7: Check the divisibility of 2146587 by 3.

Solution: The sum of the digits of 2146587 is $2 + 1 + 4 + 6 + 5 + 8 + 7 = 33$. This number is divisible by 3 (for $33 \div 3 = 11$). We conclude that 2146587 is divisible by 3.

Example 8: Check the divisibility of 15287 by 3.

Solution: The sum of the digits of 15287 is 1 + 5 + 2 + 8 + 7 = 23. This number is not divisible by 3. We conclude that 15287 too is not divisible by 3.

TRY THESE

Check the divisibility of the following numbers by 3.

1. 108 2. 616 3. 294 4. 432 5. 927

EXERCISE 16.2

1. If 21*y*5 is a multiple of 9, where *y* is a digit, what is the value of *y*?
2. If 31*z*5 is a multiple of 9, where z is a digit, what is the value of z?
   You will find that there are *two* answers for the last problem. Why is this so?
3. If 24*x* is a multiple of 3, where *x* is a digit, what is the value of *x*?
   (Since 24*x* is a multiple of 3, its sum of digits 6 + *x* is a multiple of 3; so 6 + *x* is one of these numbers: 0, 3, 6, 9, 12, 15, 18, ... But since *x* is a digit, it can only be that 6 + *x* = 6 or 9 or 12 or 15. Therefore, *x* = 0 or 3 or 6 or 9. Thus, *x* can have any of four different values.)
4. If 31*z*5 is a multiple of 3, where z is a digit, what might be the values of z?

WHAT HAVE WE DISCUSSED?

1. Numbers can be written in general form. Thus, a two digit number *ab* will be written as $ab = 10a + b$.
2. The general form of numbers are helpful in solving puzzles or number games.
3. The reasons for the divisibility of numbers by 10, 5, 2, 9 or 3 can be given when numbers are written in general form.

ANSWERS

EXERCISE 1.1

1. (i) 2 (ii) $\frac{-11}{28}$

2. (i) $\frac{-2}{8}$ (ii) $\frac{5}{9}$ (iii) $\frac{-6}{5}$ (iv) $\frac{2}{9}$ (v) $\frac{19}{6}$

4. (i) $\frac{-1}{13}$ (ii) $\frac{-19}{13}$ (iii) 5 (iv) $\frac{56}{15}$ (v) $\frac{5}{2}$ (vi) –1

5. (i) 1 is the multiplicative identity (ii) Commutativity

(iii) Multiplicative inverse

6. $\frac{-96}{91}$ **7.** Associativity **8.** No, because the product is not 1.

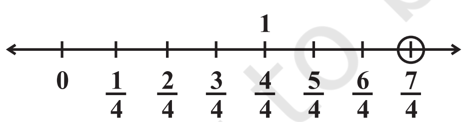
9. Yes, because $0.3 \times 3\frac{1}{3} = \frac{3}{10} \times \frac{10}{3} = 1$

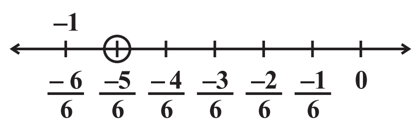
10. (i) 0 (ii) 1 and (–1) (iii) 0

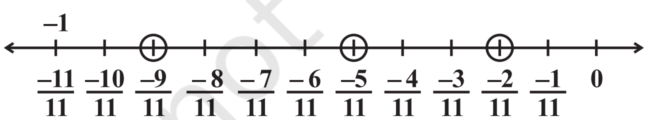
11. (i) No (ii) 1, –1 (iii) $\frac{-1}{5}$ (iv) x (v) Rational number

(vi) positive

EXERCISE 1.2

1. (i) 

(ii) 

2. 

3. Some of these are 1, $\frac{1}{2}$, 0, –1, $\frac{-1}{2}$

4. $\frac{-7}{20}, \frac{-6}{20}, \frac{-5}{20}, \frac{-4}{20}, \frac{-3}{20}, \frac{-2}{20}, \frac{-1}{20}, 0, \ldots, \frac{1}{20}, \frac{2}{20}$ (There can be many more such rational numbers)

5. (i) $\frac{41}{60}, \frac{42}{60}, \frac{43}{60}, \frac{44}{60}, \frac{45}{60}$ (ii) $\frac{-8}{6}, \frac{-7}{6}, 0, \frac{1}{6}, \frac{2}{6}$ (iii) $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$

(There can be many more such rational numbers)

6. $-\frac{3}{2}, -1, \frac{-1}{2}, 0, \frac{1}{2}$ (There can be many more such rational numbers)

7. $\frac{97}{160}, \frac{98}{160}, \frac{99}{160}, \frac{100}{160}, \frac{101}{160}, \frac{102}{160}, \frac{103}{160}, \frac{104}{160}, \frac{105}{160}, \frac{106}{160}$
(There can be many more such rational numbers)

EXERCISE 2.1

1. $x = 9$ **2.** $y = 7$ **3.** $z = 4$ **4.** $x = 2$ **5.** $x = 2$ **6.** $t = 50$

7. $x = 27$ **8.** $y = 2.4$ **9.** $x = \frac{25}{7}$ **10.** $y = \frac{3}{2}$ **11.** $p = -\frac{4}{3}$ **12.** $x = -\frac{8}{5}$

EXERCISE 2.2

1. $\frac{3}{4}$ **2.** length = 52 m, breadth = 25 m **3.** $1\frac{2}{5}$ cm **4.** 40 and 55

5. 45, 27 **6.** 16, 17, 18 **7.** 288, 296 and 304 **8.** 7, 8, 9

9. Rahul's age: 20 years; Haroon's age: 28 years **10.** 48 students

11. Baichung's age: 17 years; Baichung's father's age: 46 years;
Baichung's grandfather's age = 72 years **12.** 5 years **13** $-\frac{1}{2}$

14. ₹ 100 → 2000 notes; ₹ 50 → 3000 notes; ₹ 10 → 5000 notes

15. Number of ₹ 1 coins = 80; Number of ₹ 2 coins = 60; Number of ₹ 5 coins = 20

16. 19

EXERCISE 2.3

1. $x = 18$ **2.** $t = -1$ **3.** $x = -2$ **4.** $z = \frac{3}{2}$ **5.** $x = 5$ **6.** $x = 0$

7. $x = 40$ **8.** $x = 10$ **9.** $y = \frac{7}{3}$ **10.** $m = \frac{4}{5}$

EXERCISE 2.4

1. 4 **2.** 7, 35 **3.** 36 **4.** 26 (or 62)

5. Shobo's age: 5 years; Shobo's mother's age: 30 years

6. Length = 275 m; breadth = 100 m **7.** 200 m **8.** 72

9. Grand daughter's age: 6 years; Grandfather's age: 60 years

10. Aman's age: 60 years; Aman's son's age: 20 years

EXERCISE 2.5

1. $x = \frac{27}{10}$ 2. $n = 36$ 3. $x = -5$ 4. $x = 8$ 5. $t = 2$

6. $m = \frac{7}{5}$ 7. $t = -2$ 8. $y = \frac{2}{3}$ 9. $z = 2$ 10. $f = 0.6$

EXERCISE 2.6

1. $x = \frac{3}{2}$ 2. $x = \frac{35}{33}$ 3. $z = 12$ 4. $y = -8$ 5. $y = -\frac{4}{5}$

6. Hari's age = 20 years; Harry's age = 28 years 7. $\frac{13}{21}$

EXERCISE 3.1

1. (a) 1, 2, 5, 6, 7 (b) 1, 2, 5, 6, 7 (c) 1, 2
   (d) 2 (e) 1
2. (a) 2 (b) 9 (c) 0 3. 360°; yes.
4. (a) 900° (b) 1080° (c) 1440° (d) $(n - 2)180°$
5. A polygon with equal sides and equal angles.
   (i) Equilateral triangle (ii) Square (iii) Regular hexagon
6. (a) 60° (b) 140° (c) 140° (d) 108°
7. (a) $x + y + z = 360°$ (b) $x + y + z + w = 360°$

EXERCISE 3.2

1. (a) 360° – 250° = 110° (b) 360° – 310° = 50°
2. (i) $\frac{360°}{9} = 40°$ (ii) $\frac{360°}{15} = 24°$
3. $\frac{360}{24} = 15$ (sides) 4. Number of sides = 24
5. (i) No; (Since 22 is not a divisor of 360)
   (ii) No; (because each exterior angle is 180° – 22° = 158°, which is not a divisor of 360°).
6. (a) The equilateral triangle being a regular polygon of 3 sides has the least measure of an interior angle = 60°.
   (b) By (a), we can see that the greatest exterior angle is 120°.

EXERCISE 3.3

1. (i) BC(Opposite sides are equal) (ii) ∠DAB (Opposite angles are equal)

(iii) OA (Diagonals bisect each other)

(iv) 180° (Interior opposite angles, since $\overline{AB} \parallel \overline{DC}$)

2. (i) $x = 80°; y = 100°; z = 80°$ (ii) $x = 130°; y = 130°; z = 130°$

(iii) $x = 90°;\ y = 60°;\ z = 60°$ (iv) $x = 100°;\ y = 80°;\ z = 80°$

(v) $y = 112°; x = 28°; z = 28°$

3. (i) Can be, but need not be.

(ii) No; (in a parallelogram, opposite sides are equal; but here, AD ≠ BC).

(iii) No; (in a parallelogram, opposite angles are equal; but here, ∠A ≠ ∠C).

4. A kite, for example **5.** 108°; 72°; **6.** Each is a right angle.

7. $x = 110°;\ y = 40°;\ z = 30°$

8. (i) $x = 6;\ y = 9$ (ii) $x = 3;\ y = 13;$ **9.** $x = 50°$

10. $\overline{NM} \parallel \overline{KL}$ (sum of interior opposite angles is 180°). So, KLMN is a trapezium.

11. 60° **12.** ∠P = 50°; ∠S = 90°

EXERCISE 3.4

1. (b), (c), (f), (g), (h) are true; others are false.

2. (a) Rhombus; square. (b) Square; rectangle

3. (i) A square is 4 – sided; so it is a quadrilateral.

(ii) A square has its opposite sides parallel; so it is a parallelogram.

(iii) A square is a parallelogram with all the 4 sides equal; so it is a rhombus.

(iv) A square is a parallelogram with each angle a right angle; so it is a rectangle.

4. (i) Parallelogram; rhombus; square; rectangle.

(ii) Rhombus; square (iii) Square; rectangle

5. Both of its diagonals lie in its interior.

6. $\overline{AD} \parallel \overline{BC}$; $\overline{AB} \parallel \overline{DC}$. So, in parallelogram ABCD, the mid-point of diagonal $\overline{AC}$ is O.

EXERCISE 5.1

1. (b), (d). In all these cases data can be divided into class intervals.

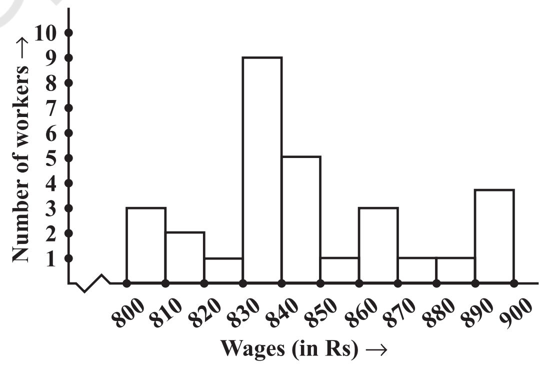
2.

Shopper	Tally marks	Number
W	卌 卌 卌 卌 卌 |||	28
M	卌 卌 卌	15
B	卌	5
G	卌 卌 ||	12

3.

Interval	Tally marks	Frequency
800 - 810	III	3
810 - 820	II	2
820 - 830	I	1
830 - 840	~~IIII~~ IIII	9
840 - 850	~~IIII~~	5
850 - 860	I	1
860 - 870	III	3
870 - 880	I	1
880 - 890	I	1
890 - 900	IIII	4
	Total	**30**

4. (i) 830 - 840 (ii) 10
(iii) 20

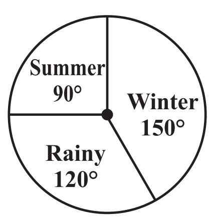


5. (i) 4 - 5 hours (ii) 34
(iii) 14

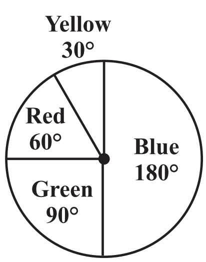
EXERCISE 5.2

1. (i) 200 (ii) Light music (iii) Classical - 100, Semi classical - 200, Light - 400, Folk - 300

2. (i) Winter (ii) Winter - 150°, Rainy - 120°, Summer - 90° (iii)

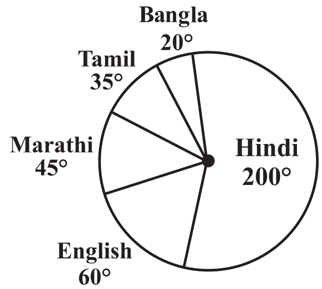


3.



4. (i) Hindi (ii) 30 marks (iii) Yes

5.



EXERCISE 5.3

1. (a) Outcomes → A, B, C, D

(b) HT, HH, TH, TT (Here HT means Head on first coin and Tail on the second coin and so on).

2. Outcomes of an event of getting

(i) (a) 2, 3, 5 (b) 1, 4, 6

(ii) (a) 6 (b) 1, 2, 3, 4, 5

3. (a) $\frac{1}{5}$ (b) $\frac{1}{13}$ (c) $\frac{4}{7}$

4. (i) $\frac{1}{10}$ (ii) $\frac{1}{2}$ (iii) $\frac{2}{5}$ (iv) $\frac{9}{10}$

5. Probability of getting a green sector = $\frac{3}{5}$; probability of getting a non-blue sector = $\frac{4}{5}$

6. Probability of getting a prime number = $\frac{1}{2}$; probability of getting a number which is not prime = $\frac{1}{2}$

Probability of getting a number greater than 5 = $\frac{1}{6}$

Probability of getting a number not greater than 5 = $\frac{5}{6}$

EXERCISE 6.1

1. (i) 1 (ii) 4 (iii) 1 (iv) 9 (v) 6 (vi) 9

(vii) 4 (viii) 0 (ix) 6 (x) 5

2. These numbers end with

(i) 7 (ii) 3 (iii) 8 (iv) 2 (v) 0 (vi) 2

(vii) 0 (viii) 0

3. (i), (iii) **4.** 10000200001, 100000020000001

5. 1020304030201, 101010101^2 **6.** 20, 6, 42, 43

7. (i) 25 (ii) 100 (iii) 144

8. (i) 1 + 3 + 5 + 7 + 9 + 11 + 13

(ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21

9. (i) 24 (ii) 50 (iii) 198

EXERCISE 6.2

1. (i) 1024 (ii) 1225 (iii) 7396 (iv) 8649 (v) 5041 (vi) 2116

2. (i) 6,8,10 (ii) 14,48,50 (iii) 16,63,65 (iv) 18,80,82

EXERCISE 6.3

1. (i) 1, 9 (ii) 4, 6 (iii) 1, 9 (iv) 5

2. (i), (ii), (iii) **3.** 10, 13

4. (i) 27 (ii) 20 (iii) 42 (iv) 64 (v) 88 (vi) 98
(vii) 77 (viii) 96 (ix) 23 (x) 90

5. (i) 7; 42 (ii) 5; 30 (iii) 7, 84 (iv) 3; 78 (v) 2; 54 (vi) 3; 48

6. (i) 7; 6 (ii) 13; 15 (iii) 11; 6 (vi) 5; 23 (v) 7; 20 (vi) 5; 18

7. 49 **8.** 45 rows; 45 plants in each row **9.** 900 **10.** 3600

EXERCISE 6.4

1. (i) 48 (ii) 67 (iii) 59 (iv) 23 (v) 57 (vi) 37
(vii) 76 (viii) 89 (ix) 24 (x) 32 (xi) 56 (xii) 30

2. (i) 1 (ii) 2 (iii) 2 (iv) 3 (v) 3

3. (i) 1.6 (ii) 2.7 (iii) 7.2 (iv) 6.5 (v) 5.6

4. (i) 2; 20 (ii) 53; 44 (iii) 1; 57 (iv) 41; 28 (v) 31; 63

5. (i) 4; 23 (ii) 14; 42 (iii) 4; 16 (iv) 24; 43 (v) 149; 81

6. 21 m **7.** (a) 10 cm (b) 12 cm

8. 24 plants **9.** 16 children

EXERCISE 7.1

1. (ii) and (iv)

2. (i) 3 (ii) 2 (iii) 3 (iv) 5 (v) 10

3. (i) 3 (ii) 2 (iii) 5 (iv) 3 (v) 11

4. 20 cuboids

EXERCISE 7.2

1. (i) 4 (ii) 8 (iii) 22 (iv) 30 (v) 25 (vi) 24
(vii) 48 (viii) 36 (ix) 56

2. (i) False (ii) True (iii) False (iv) False (v) False (vi) False
(vii) True

3. 11, 17, 23, 32

EXERCISE 8.1

1. (a) 1 : 2 (b) 1 : 2000 (c) 1 : 10
2. (a) 75% (b) $66\frac{2}{3}\%$ 3. 28% students 4. 25 matches 5. ₹ 2400
6. 10%, cricket → 30 lakh; football → 15 lakh; other games → 5 lakh

EXERCISE 8.2

1. ₹ 1,40,000 2. 80% 3. ₹ 34.80 4. ₹ 18,342.50
5. Gain of 2% 6. ₹ 2,835 7. Loss of ₹ 1,269.84
8. ₹ 14,560 9. ₹ 2,000 10. ₹ 5,000 11. ₹ 1,050

EXERCISE 8.3

1. (a) Amount = ₹ 15,377.34; Compound interest = ₹ 4,577.34
   (b) Amount = ₹ 22,869; Interest = ₹ 4869 (c) Amount = ₹ 70,304, Interest = ₹ 7,804
   (d) Amount = ₹ 8,736.20, Interest = ₹ 736.20
   (e) Amount = ₹ 10,816, Interest = ₹ 816
2. ₹ 36,659.70 3. Fabina pays ₹ 362.50 more 4. ₹ 43.20
5. (ii) ₹ 63,600 (ii) ₹ 67,416 6. (ii) ₹ 92,400 (ii) ₹ 92,610
7. (i) ₹ 8,820 (ii) ₹ 441
8. Amount = ₹ 11,576.25, Interest = ₹ 1,576.25 Yes.
9. ₹ 4,913 10. (i) About 48,980 (ii) 59,535 11. 5,31,616 (approx)
12. ₹ 38,640

EXERCISE 9.1

1.

	Term	Coefficient
(i)	$5xyz^2$ $-3zy$	5 -3
(ii)	1 x x^2	1 1 1
(iii)	$4x^2y^2$ $-4x^2y^2z^2$ z^2	4 -4 1
(iv)	3 $-pq$ qr $-rp$	3 -1 1 -1
(v)	$\frac{x}{2}$ $\frac{y}{2}$ $-xy$	$\frac{1}{2}$ $\frac{1}{2}$ -1
(vi)	$0.3a$ $-0.6ab$ $0.5b$	0.3 -0.6 0.5

2. Monomials: 1000, pqr

Binomials: $x + y$, $2y - 3y^2$, $4z - 15z^2$, $p^2q + pq^2$, $2p + 2q$

Trinomials : $7 + y + 5x$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$

Polynomials that do not fit in these categories: $x + x^2 + x^3 + x^4$, $ab + bc + cd + da$

3. (i) 0 (ii) $ab + bc + ac$ (iii) $-p^2q^2 + 4pq + 9$

(iv) $2(l^2 + m^2 + n^2 + lm + mn + nl)$

4. (a) $8a - 2ab + 2b - 15$ (b) $2xy - 7yz + 5zx + 10xyz$

(c) $p^2q - 7pq^2 + 8pq - 18q + 5p + 28$

EXERCISE 9.2

1. (i) $28p$ (ii) $-28p^2$ (iii) $-28p^2q$ (iv) $-12p^4$ (v) 0

2. pq; $50\ mn$; $100\ x^2y^2$; $12x^3$; $12mn^2p$

3.

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

4. (i) $105a^7$ (ii) $64pqr$ (iii) $4x^4y^4$ (iv) $6abc$

5. (i) $x^2y^2z^2$ (ii) $-a^6$ (iii) $1024y^6$ (iv) $36a^2b^2c^2$ (v) $-m^3n^2p$

EXERCISE 9.3

1. (i) $4pq + 4pr$ (ii) $a^2b - ab^2$ (iii) $7a^3b^2 + 7a^2b^3$

(iv) $4a^3 - 36a$ (v) 0

2. (i) $ab + ac + ad$ (ii) $5x^2y + 5xy^2 - 25xy$

(iii) $6p^3 - 7p^2 + 5p$ (iv) $4p^4q^2 - 4p^2q^4$

(v) $a^2bc + ab^2c + abc^2$

3. (i) $8a^{50}$ (ii) $-\frac{3}{5}x^3y^3$ (iii) $-4p^4q^4$ (iv) x^{10}

4. (a) $12x^2 - 15x + 3$; (i) 66 (ii) $\frac{-3}{2}$

(b) $a^3 + a^2 + a + 5$; (i) 5 (ii) 8 (iii) 4

5. (a) $p^2 + q^2 + r^2 - pq - qr - pr$ (b) $-2x^2 - 2y^2 - 4xy + 2yz + 2zx$

(c) $5l^2 + 25ln$ (d) $-3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac$

EXERCISE 9.4

1. (i) $8x^2 + 14x - 15$ (ii) $3y^2 - 28y + 32$ (iii) $6.25l^2 - 0.25m^2$
   (iv) $ax + 5a + 3bx + 15b$ (v) $6p^2q^2 + 5pq^3 - 6q^4$ (vi) $3a^4 + 10a^2b^2 - 8b^4$
2. (i) $15 - x - 2x^2$ (ii) $7x^2 + 48xy - 7y^2$ (iii) $a^3 + a^2b^2 + ab + b^3$
   (iv) $2p^3 + p^2q - 2pq^2 - q^3$
3. (i) $x^3 + 5x^2 - 5x$ (ii) $a^2b^3 + 3a^2 + 5b^3 + 20$ (iii) $t^3 - st + s^2t^2 - s^3$
   (iv) $4ac$ (v) $3x^2 + 4xy - y^2$ (vi) $x^3 + y^3$
   (vii) $2.25x^2 - 16y^2$ (viii) $a^2 + b^2 - c^2 + 2ab$

EXERCISE 9.5

1. (i) $x^2 + 6x + 9$ (ii) $4y^2 + 20y + 25$ (iii) $4a^2 - 28a + 49$
   (iv) $9a^2 - 3a + \frac{1}{4}$ (v) $1.21m^2 - 0.16$ (vi) $b^4 - a^4$
   (vii) $36x^2 - 49$ (viii) $a^2 - 2ac + c^2$ (ix) $\frac{x^2}{4} + \frac{3xy}{4} + \frac{9y^2}{16}$
   (x) $49a^2 - 126ab + 81b^2$
2. (i) $x^2 + 10x + 21$ (ii) $16x^2 + 24x + 5$ (iii) $16x^2 - 24x + 5$
   (iv) $16x^2 + 16x - 5$ (v) $4x^2 + 16xy + 15y^2$ (vi) $4a^4 + 28a^2 + 45$
   (vii) $x^2y^2z^2 - 6xyz + 8$
3. (i) $b^2 - 14b + 49$ (ii) $x^2y^2 + 6xyz + 9z^2$ (iii) $36x^4 - 60x^2y + 25y^2$
   (iv) $\frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$ (v) $0.16p^2 - 0.4pq + 0.25q^2$ (vi) $4x^2y^2 + 20xy^2 + 25y^2$
4. (i) $a^4 - 2a^2b^2 + b^4$ (ii) $40x$ (iii) $98m^2 + 128n^2$
   (iv) $41m^2 + 80mn + 41n^2$ (v) $4p^2 - 4q^2$ (vi) $a^2b^2 + b^2c^2$ (vii) $m^4 + n^4m^2$
6. (i) 5041 (ii) 9801 (iii) 10404 (iv) 996004
   (v) 27.04 (vi) 89991 (vii) 6396 (viii) 79.21
   (ix) 99.75
7. (i) 200 (ii) 0.08 (iii) 1800 (iv) 84
8. (i) 10712 (ii) 26.52 (iii) 10094 (iv) 95.06

EXERCISE 10.1

1. (a) →(iii) →(iv) (b) →(i) →(v) (c) →(iv) →(ii)
   (d) →(v) →(iii) (e) →(ii) →(i)
2. (a) (i) →Front, (ii) →Side, (iii) →Top (b) (i) →Side, (ii) →Front, (iii) →Top
   (c) (i) →Front, (ii) →Side, (iii) →Top (d) (i) →Front, (ii) →Side, (iii) →Top
3. (a) (i) →Top, (ii) →Front, (iii) →Side (b) (i) →Side, (ii) →Front, (iii) →Top
   (c) (i) →Top, (ii) →Side, (iii) →Front (d) (i) →Side, (ii) →Front, (iii) →Top
   (e) (i) →Front, (ii) →Top, (iii) →Side

EXERCISE 10.3

1. (i) No (ii) Yes (iii) Yes
2. Possible, only if the number of faces are greater than or equal to 4
3. only (ii) and (iv)
4. (i) A prism becomes a cylinder as the number of sides of its base becomes larger and larger.
   (ii) A pyramid becomes a cone as the number of sides of its base becomes larger and larger.
5. No. It can be a cuboid also
7. Faces $\rightarrow$ 8, Vertices $\rightarrow$ 6, Edges $\rightarrow$ 30
8. No

EXERCISE 11.1

1. (a)
2. ₹ 17,875
3. Area = 129.5 m^2; Perimeter = 48 m
4. 45000 tiles
5. (b)

EXERCISE 11.2

1. 0.88 m^2
2. 7 cm
3. 660 m^2
4. 252 m^2
5. 45 cm^2
6. 24 cm^2, 6 cm
7. ₹ 810
8. 140 m
9. 119 m^2
10. Area using Jyoti's way $= 2 \times \frac{1}{2} \times \frac{15}{2} \times (30 + 15)\ m^2 = 337.5\ m^2$,

    Area using Kavita's way $= \frac{1}{2} \times 15 \times 15 + 15 \times 15 = 337.5\ m^2$
11. 80 cm^2, 96 cm^2, 80 cm^2, 96 cm^2

EXERCISE 11.3

1. (a)
2. 144 m
3. 10 cm
4. 11 m^2
5. 5 cans
6. Similarity $\rightarrow$ Both have same heights. Difference $\rightarrow$ one is a cylinder, the other is a cube. The cube has larger lateral surface area
7. 440 m^2
8. 322 cm
9. 1980 m^2
10. 704 cm^2

EXERCISE 11.4

1. (a) Volume (b) Surface area (c) Volume
2. Volume of cylinder B is greater; Surface area of cylinder B is greater.
3. 5 cm
4. 450
5. 1 m
6. 49500 L
7. (i) 4 times (ii) 8 times
8. 30 hours

EXERCISE 12.1

1. (i) $\frac{1}{9}$ (ii) $\frac{1}{16}$ (iii) 32

2. (i) $\frac{1}{(-4)^3}$ (ii) $\frac{1}{2^6}$ (iii) $(5)^4$ (iv) $\frac{1}{(3)^2}$ (v) $\frac{1}{(-14)^3}$

3. (i) 5 (ii) $\frac{1}{2}$ (iii) 29 (iv) 1 (v) $\frac{81}{16}$

4. (i) 250 (ii) $\frac{1}{60}$ 5. $m = 2$ 6. (i) –1 (ii) $\frac{512}{125}$

7. (i) $\frac{625t^4}{2}$ (ii) 5^5

EXERCISE 12.2

1. (i) 8.5×10^{-12} (ii) 9.42×10^{-12} (iii) 6.02×10^{15}
   (iv) 8.37×10^{-9} (v) 3.186×10^{10}
2. (i) 0.00000302 (ii) 45000 (iii) 0.00000003
   (iv) 1000100000 (v) 5800000000000 (vi) 3614920
3. (i) 1×10^{-6} (ii) 1.6×10^{-19} (iii) 5×10^{-7}
   (iv) 1.275×10^{-5} (v) 7×10^{-2}
4. 1.0008×10^2

EXERCISE 13.1

1. No

2.

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	96	160

3. 24 parts 4. 700 bottles 5. 10^{-4} cm; 2 cm 6. 21 m
7. (i) 2.25×10^7 crystals (ii) 5.4×10^6 crystals 8. 4 cm
9. (i) 6 m (ii) 8 m 75 cm 10. 168 km

EXERCISE 13.2

1. (i), (iv), (v) 2. 4 → 25,000; 5 → 20,000; 8 → 12,500; 10 → 10,000; 20 → 5,000
   Amount given to a winner is inversely proportional to the number of winners.
3. 8 → 45°, 10 → 36°, 12 → 30° (i) Yes (ii) 24° (iii) 9
4. 6 5. 4 6. 3 days 7. 15 boxes
8. 49 machines 9. $1\frac{1}{2}$ hours 10. (i) 6 days (ii) 6 persons 11. 40 minutes

EXERCISE 14.1

1. (i) 12 (ii) $2y$ (iii) $14pq$ (iv) 1 (v) $6ab$ (vi) $4x$
   (vii) 10 (viii) x^2y^2

2. (i) $7(x-6)$ (ii) $6(p-2q)$ (iii) $7a(a+2)$ (iv) $4z(-4+5z^2)$
(v) $10\,lm(2l+3a)$ (vi) $5xy(x-3y)$ (vii) $5(2a^2-3b^2+4c^2)$
(viii) $4a(-a+b-c)$ (ix) $xyz(x+y+z)$ (x) $xy(ax+by+cz)$

3. (i) $(x+8)(x+y)$ (ii) $(3x+1)(5y-2)$ (iii) $(a+b)(x-y)$
(iv) $(5p+3)(3q+5)$ (v) $(z-7)(1-xy)$

EXERCISE 14.2

1. (i) $(a+4)^2$ (ii) $(p-5)^2$ (iii) $(5m+3)^2$ (iv) $(7y+6z)^2$
(v) $4(x-1)^2$ (vi) $(11b-4c)^2$ (vii) $(l-m)^2$ (viii) $(a^2+b^2)^2$

2. (i) $(2p-3q)(2p+3q)$ (ii) $7(3a-4b)(3a+4b)$ (iii) $(7x-6)(7x+6)$
(iv) $16x^3(x-3)(x+3)$ (v) $4lm$ (vi) $(3xy-4)(3xy+4)$
(vii) $(x-y-z)(x-y+z)$ (viii) $(5a-2b+7c)(5a+2b-7c)$

3. (i) $x(ax+b)$ (ii) $7(p^2+3q^2)$ (iii) $2x(x^2+y^2+z^2)$
(iv) $(m^2+n^2)(a+b)$ (v) $(l+1)(m+1)$ (vi) $(y+9)(y+z)$
(vii) $(5y+2z)(y-4)$ (viii) $(2a+1)(5b+2)$ (ix) $(3x-2)(2y-3)$

4. (i) $(a-b)(a+b)(a^2+b^2)$ (ii) $(p-3)(p+3)(p^2+9)$
(iii) $(x-y-z)(x+y+z)[x^2+(y+z)^2]$ (iv) $z(2x-z)(2x^2-2xz+z^2)$
(v) $(a-b)^2(a+b)^2$

5. (i) $(p+2)(p+4)$ (ii) $(q-3)(q-7)$ (iii) $(p+8)(p-2)$

EXERCISE 14.3

1. (i) $\frac{x^3}{2}$ (ii) $-4y$ (iii) $6pqr$ (iv) $\frac{2}{3}x^2y$ (v) $-2a^2b^4$

2. (i) $\frac{1}{3}(5x-6)$ (ii) $3y^4-4y^2+5$ (iii) $2(x+y+z)$
(iv) $\frac{1}{2}(x^2+2x+3)$ (v) q^3-p^3

3. (i) $2x-5$ (ii) 5 (iii) $6y$ (iv) xy (v) $10abc$

4. (i) $5(3x+5)$ (ii) $2y(x+5)$ (iii) $\frac{1}{2}r(p+q)$ (iv) $4(y^2+5y+3)$
(v) $(x+2)(x+3)$

5. (i) $y+2$ (ii) $m-16$ (iii) $5(p-4)$ (iv) $2z(z-2)$ (v) $\frac{5}{2}q(p-q)$
(vi) $3(3x-4y)$ (vii) $3y(5y-7)$

EXERCISE 14.4

1. $4(x-5)=4x-20$ **2.** $x(3x+2)=3x^2+2x$ **3.** $2x+3y=2x+3y$
4. $x+2x+3x=6x$ **5.** $5y+2y+y-7y=y$ **6.** $3x+2x=5x$

7. $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$ **8.** $(2x)^2 + 5x = 4x^2 + 5x$

9. $(3x + 2)^2 = 9x^2 + 12x + 4$

10. (a) $(-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = -2$ (b) $(-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28$

(c) $(-3)^2 + 5(-3) = 9 - 15 = -6$

11. $(y - 3)^2 = y^2 - 6y + 9$ **12.** $(z + 5)^2 = z^2 + 10z + 25$

13. $(2a + 3b)(a - b) = 2a^2 + ab - 3b^2$ **14.** $(a + 4)(a + 2) = a^2 + 6a + 8$

15. $(a - 4)(a - 2) = a^2 - 6a + 8$ **16.** $\frac{3x^2}{3x^2} = 1$

17. $\frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2} = 1 + \frac{1}{3x^2}$ **18.** $\frac{3x}{3x+2} = \frac{3x}{3x+2}$

19. $\frac{3}{4x+3} = \frac{3}{4x+3}$ **20.** $\frac{4x+5}{4x} = \frac{4x}{4x} + \frac{5}{4x} = 1 + \frac{5}{4x}$

21. $\frac{7x+5}{5} = \frac{7x}{5} + \frac{5}{5} = \frac{7x}{5} + 1$

EXERCISE 15.1

1. (a) 36.5° C (b) 12 noon (c) 1 p.m, 2 p.m.

(d) 36.5° C; The point between 1 p.m. and 2 p.m. on the *x*-axis is equidistant from the two points showing 1 p.m. and 2 p.m., so it will represent 1.30 p.m. Similarly, the point on the *y*-axis, between 36° C and 37° C will represent 36.5° C.

(e) 9 a.m. to 10 a.m., 10 a.m. to 11 a.m., 2 p.m. to 3 p.m.

2. (a) (i) ₹ 4 crore (ii) ₹ 8 crore

(b) (i) ₹ 7 crore (ii) ₹ 8.5 crore (approx.)

(c) ₹ 4 crore (d) 2005

3. (a) (i) 7 cm (ii) 9 cm

(b) (i) 7 cm (ii) 10 cm

(c) 2 cm (d) 3 cm (e) Second week (f) First week

(g) At the end of the 2nd week

4. (a) Tue, Fri, Sun (b) 35° C (c) 15° C (d) Thurs

6. (a) 4 units = 1 hour (b) $3\frac{1}{2}$ hours (c) 22 km

(d) Yes; This is indicated by the horizontal part of the graph (10 a.m. - 10.30 a.m.)

(e) Between 8 a.m. and 9 a.m.

7. (iii) is not possible

EXERCISE 15.2

1. Points in (a) and (b) lie on a line; Points in (c) do not lie on a line

2. The line will cut *x*-axis at (5, 0) and *y*-axis at (0, 5)

3. O(0, 0), A(2, 0), B(2, 3), C(0, 3), P(4, 3), Q(6, 1), R(6, 5), S(4, 7), K(10, 5), L(7, 7), M(10, 8)
4. (i) True (ii) False (iii) True

EXERCISE 15.3

1. (b) (i) 20 km (ii) 7.30 a.m. (c) (i) Yes (ii) ₹ 200 (iii) ₹ 3500
2. (a) Yes (b) No

EXERCISE 16.1

1. A = 7, B = 6
2. A = 5, B = 4, C = 1
3. A = 6
4. A = 2, B = 5
5. A = 5, B = 0, C = 1
6. A = 5, B = 0, C = 2
7. A = 7, B = 4
8. A = 7, B = 9
9. A = 4, B = 7
10. A = 8, B = 1

EXERCISE 16.2

1. $y = 1$
2. $z = 0$ or 9
3. $z = 0, 3, 6$ or 9
4. 0, 3, 6 or 9

JUST FOR FUN

1. **More about Pythagorean triplets**

   We have seen one way of writing pythagorean triplets as $2m, m^2 - 1, m^2 + 1$.

   A pythagorean triplet a, b, c means $a^2 + b^2 = c^2$. If we use two natural numbers m and $n (m > n)$, and take $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$, then we can see that $c^2 = a^2 + b^2$.

   Thus for different values of m and n with $m > n$ we can generate natural numbers a, b, c such that they form Pythagorean triplets.

   For example: Take, $m = 2, n = 1$.

   Then, $a = m^2 - n^2 = 3$, $b = 2mn = 4$, $c = m^2 + n^2 = 5$, is a Pythagorean triplet. (Check it!)

   For, $m = 3, n = 2$, we get,

   $a = 5, b = 12, c = 13$ which is again a Pythagorean triplet.

   Take some more values for m and n and generate more such triplets.

2. When water freezes its volume increases by 4%. What volume of water is required to make 221 cm^3 of ice?

3. If price of tea increased by 20%, by what per cent must the consumption be reduced to keep the expense the same?

4. Ceremony Awards began in 1958. There were 28 categories to win an award. In 1993, there were 81 categories.

(i) The awards given in 1958 is what per cent of the awards given in 1993?

(ii) The awards given in 1993 is what per cent of the awards given in 1958?

5. Out of a swarm of bees, one fifth settled on a blossom of *Kadamba*, one third on a flower of *Silindhiri,* and three times the difference between these two numbers flew to the bloom of *Kutaja*. Only ten bees were then left from the swarm. What was the number of bees in the swarm? (Note, *Kadamba, Silindhiri* and *Kutaja* are flowering trees. The problem is from the ancient Indian text on algebra.)

6. In computing the area of a square, Shekhar used the formula for area of a square, while his friend Maroof used the formula for the perimeter of a square. Interestingly their answers were numerically same. Tell me the number of units of the side of the square they worked on.

7. The area of a square is numerically less than six times its side. List some squares in which this happens.

8. Is it possible to have a right circular cylinder to have volume numerically equal to its curved surface area? If yes state when.

9. Leela invited some friends for tea on her birthday. Her mother placed some plates and some *puris* on a table to be served. If Leela places 4 *puris* in each plate 1 plate would be left empty. But if she places 3 *puris* in each plate 1 *puri* would be left. Find the number of plates and number of *puris* on the table.

10. Is there a number which is equal to its cube but not equal to its square? If yes find it.

11. Arrange the numbers from 1 to 20 in a row such that the sum of any two adjacent numbers is a perfect square.

Answers

2. $212\frac{1}{2}\text{ cm}^3$

3. $16\frac{2}{3}\%$

4. (i) 34.5% (ii) 289%

5. 150

6. 4 units

7. Sides = 1, 2, 3, 4, 5 units

8. Yes, when radius = 2 units

9. Number of *puris* = 16, number of plates = 5

10. – 1

11. One of the ways is, 1, 3, 6, 19, 17, 8 (1 + 3 = 4, 3 + 6 = 9 etc.). Try some other ways.