Mathematics

(Chapter – 3) (Trigonometric Functions)

(Class – XI)

Exercise 3.1

Question 1:

Find the radian measures corresponding to the following degree measures:

(i) 25° **(ii)** – 47° 30' **(iii)** 240° **(iv)** 520°

Answer 1:

(i) 25°

We know that 180° = π radian

$$\therefore 25^\circ = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$$

(ii)–47° 30'

–47° 30' $-47\frac{1}{2}$

$= \frac{-95}{2}$ degree

Since 180° = π radian

$$\frac{-95}{2} \text{ deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian} = \left(\frac{-19}{36 \times 2}\right)\pi \text{ radian} = \frac{-19}{72}\pi \text{ radian}$$

$$\therefore -47^\circ\ 30' = \frac{-19}{72}\pi \text{ radian}$$

(iii) 240°

We know that 180° = π radian

$$\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

We know that 180° = π radian

$$\therefore 520° = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

Question 2:

Find the degree measures corresponding to the following radian measures $\left(\text{Use } \pi = \frac{22}{7}\right)$

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(i) $\frac{11}{16}$ **(ii)** – 4 **(iii)** $\frac{5\pi}{3}$ **(iv)** $\frac{7\pi}{6}$

Answer 2:

(i) $\frac{11}{16}$

We know that π radian = 180°

$$\therefore \frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree} = \frac{45 \times 11}{\pi \times 4} \text{deg ree}$$

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree} = \frac{315}{8} \text{ deg ree}$$

$$= 39\frac{3}{8} \text{ deg ree}$$

$$= 39° + \frac{3 \times 60}{8} \text{ min utes} \qquad [1° = 60']$$

$$= 39° + 22' + \frac{1}{2} \text{ min utes}$$

$$= 39°22'30" \qquad [1' = 60"]$$

(ii) – 4

We know that π radian = 180°

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ deg ree} = \frac{180 \times 7(-4)}{22} \text{ deg ree}$$

$$= \frac{-2520}{11} \text{ deg ree} = -229\frac{1}{11} \text{ deg ree}$$

$$= -229° + \frac{1 \times 60}{11} \text{ min utes} \qquad [1° = 60']$$

$$= -229° + 5' + \frac{5}{11} \text{ min utes}$$

$$= -229°5'27" \qquad [1' = 60"]$$

(iii) $\frac{5\pi}{3}$

We know that π radian = 180°

$$\therefore \frac{5\pi}{3} \text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ deg ree} = 300°$$

(iv) $\frac{7\pi}{6}$

We know that π radian = 180°

$$\therefore \frac{7\pi}{6} \text{ radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$$

Question 3:

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Answer 3:

Number of revolutions made by the wheel in 1 minute = 360

∴ Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of 6 × 2π radian, i.e., 12 π radian

Thus, in one second, the wheel turns an angle of 12π radian.

Question 4:

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm.

$\left(\text{Use } \pi = \frac{22}{7}\right)$

Answer 4:

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then

$$\theta = \frac{l}{r}$$

Therefore, forr = 100 cm, l = 22 cm, we have

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ degree} = \frac{180 \times 7 \times 22}{22 \times 100} \text{ degree}$$
$$= \frac{126}{10} \text{ degree} = 12\frac{3}{5} \text{ degree} = 12°36' \quad [1° = 60']$$

Thus, the required angle is 12°36′.

Question 5:

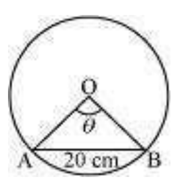
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Answer 5:

Diameter of the circle = 40 cm

∴Radius (*r*) of the circle = $\frac{40}{2} \text{ cm} = 20 \text{ cm}$

Let AB be a chord (length = 20 cm) of the circle.



In ΔOAB, OA = OB = Radius of circle = 20 cm

Also, AB = 20 cm

Thus, ΔOAB is an equilateral triangle.

∴ θ = 60° = $\frac{\pi}{3}$ radian

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ*

$$\theta = \frac{l}{r}$$

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}$ cm

Question 6:

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Answer 6:

Let the radii of the two circles be r_1 and r_2. Let an arc of length *l* subtend an angle of 60° at the centre of the circle of radius r_1, while let an arc of length *l* subtend an angle of 75° at the centre of the circle of radius r_2.

Now, 60°= $\frac{\pi}{3}$ radian and 75° = $\frac{5\pi}{12}$ radian

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ*

$$\theta = \frac{l}{r} \text{ or } l = r\theta$$

$$\therefore l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

$$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

$$\Rightarrow r_1 = \frac{r_2 5}{4}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$$

Thus, the ratio of the radii is 5:4.

Question 7:

Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm **(ii)** 15 cm **(iii)** 21 cm

Answer 7:

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$

It is given that *r* = 75 cm

(i) Here, *l* = 10 cm

$$\theta = \frac{10}{75} \text{ radian} = \frac{2}{15} \text{ radian}$$

(ii) Here, *l* = 15 cm

$$\theta = \frac{15}{75} \text{ radian} = \frac{1}{5} \text{ radian}$$

(iii) Here, *l* = 21 cm

$$\theta = \frac{21}{75} \text{ radian} = \frac{7}{25} \text{ radian}$$

Mathematics

(Chapter – 3) (Trigonometric Functions)

(Class – XI)

Exercise 3.2

Question 1:

Find the values of other five trigonometric functions if $\cos x = -\frac{1}{2}$, *x* lies in third quadrant.

Answer 1:

$$\cos x = -\frac{1}{2}$$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$
$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$
$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$
$$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$
$$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$$

Since *x* lies in the 3rd quadrant, the value of sin *x* will be negative.

$$\therefore \sin x = -\frac{\sqrt{3}}{2}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

Question 2:
Find the values of other five trigonometric functions if $\sin x = \frac{3}{5}$, *x* lies in second quadrant.

Answer 2:

$\sin x = \frac{3}{5}$

$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$

$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$

$\Rightarrow \cos^2 x = \frac{16}{25}$

$\Rightarrow \cos x = \pm\frac{4}{5}$

Since *x* lies in the 2nd quadrant, the value of cos *x* will be negative

$\therefore \cos x = -\frac{4}{5}$

$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$

$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$

Question 3:

Find the values of other five trigonometric functions if $\cot x = \frac{3}{4}$, *x* lies in third quadrant.

Answer 3:

$$\cot x = \frac{3}{4}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{5}{3}$$

Since *x* lies in the 3rd quadrant, the value of sec *x* will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

Question 4:

Find the values of other five trigonometric functions if $\sec x = \frac{13}{5}$, *x* lies in fourth quadrant.

Answer 4:

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm\frac{12}{13}$$

Since *x* lies in the 4th quadrant, the value of sin *x* will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

Question 5:

Find the values of other five trigonometric functions if $\tan x = -\frac{5}{12}$, *x* lies in second quadrant.

Answer 5:

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{13}{12}$$

Since *x* lies in the 2nd quadrant, the value of sec *x* will be negative.

$$\therefore \sec x = -\frac{13}{12}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

$$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Question 6:

Find the value of the trigonometric function sin 765°

Answer 6:

It is known that the values of sin *x* repeat after an interval of 2π or 360°.

$$\therefore \sin 765° = \sin(2\times 360° + 45°) = \sin 45° = \frac{1}{\sqrt{2}}$$

Question 7:

Find the value of the trigonometric function cosec (–1410°)

Answer 7:

It is known that the values of cosec *x* repeat after an interval of 2π or 360°.

$$\begin{aligned}\therefore \operatorname{cosec}(-1410°) &= \operatorname{cosec}(-1410° + 4\times 360°)\\ &= \operatorname{cosec}(-1410° + 1440°)\\ &= \operatorname{cosec}30° = 2\end{aligned}$$

Question 8:

Find the value of the trigonometric function $\tan\frac{19\pi}{3}$

Answer 8:

It is known that the values of tan *x* repeat after an interval of π or 180°.

$$\therefore \tan\frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \tan 60° = \sqrt{3}$$

Question 9:

Find the value of the trigonometric function $\sin\left(-\frac{11\pi}{3}\right)$

Answer 9:

It is known that the values of sin *x* repeat after an interval of 2π or 360°.

$$\therefore \sin\left(-\frac{11\pi}{3}\right)=\sin\left(-\frac{11\pi}{3}+2\times 2\pi\right)=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

Question 10:

Find the value of the trigonometric function $\cot\left(-\frac{15\pi}{4}\right)$

Answer 10:

It is known that the values of cot *x* repeat after an interval of π or 180°.

$$\therefore \cot\left(-\frac{15\pi}{4}\right)=\cot\left(-\frac{15\pi}{4}+4\pi\right)=\cot\frac{\pi}{4}=1$$

Mathematics

(Chapter – 3) (Trigonometric Functions)

(Class – XI)

Exercise 3.3

Question 1:

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

Answer 1:

L.H.S. = $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$

$$=\frac{1}{4}+\frac{1}{4}-1=-\frac{1}{2}$$

$$=\text{R.H.S.}$$

Question 2:

Prove that $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Answer 2:

L.H.S. = $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$$=2\left(\frac{1}{2}\right)^2+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

$$=2\times\frac{1}{4}+\left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

$$=\frac{1}{2}+(-2)^2\left(\frac{1}{4}\right)$$

$$=\frac{1}{2}+\frac{4}{4}=\frac{1}{2}+1=\frac{3}{2}$$

$$=\text{R.H.S.}$$

Question 3:

Prove that $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Answer 3:

L.H.S. = $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$

$$=\left(\sqrt{3}\right)^2+\operatorname{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$=3+\operatorname{cosec}\frac{\pi}{6}+3\times\frac{1}{3}$$

$$=3+2+1=6$$

$$=\text{R.H.S}$$

Question 4:

Prove that $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Answer 4:

L.H.S = $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$$= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$

$$= 2\left\{\sin\frac{\pi}{4}\right\}^2 + 2\times\frac{1}{2} + 8$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$

$$= 1 + 1 + 8$$

$$= 10$$

$$= \text{R.H.S}$$

Question 5:

Find the value of:

(i) sin 75°

(ii) tan 15°

Answer 5:

(i) sin 75° = sin (45° + 30°)

= sin 45° cos 30° + cos 45° sin 30°

[sin (*x* + *y*) = sin *x* cos *y* + cos *x* sin *y*]

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) tan 15° = tan (45° – 30°)

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} \qquad \left[\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{\left(\sqrt{3} - 1\right)^2}{\left(\sqrt{3} + 1\right)\left(\sqrt{3} - 1\right)} = \frac{3 + 1 - 2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{4 - 2\sqrt{3}}{3 - 1} = 2 - \sqrt{3}$$

Question 6:

Prove that: $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x + y)$

Answer 6:

$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$$

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)\right]$$

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} + \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$+ \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} - \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$\left[\begin{array}{l} \because 2\cos A\cos B = \cos(A + B) + \cos(A - B) \\ -2\sin A\sin B = \cos(A + B) - \cos(A - B) \end{array}\right]$$

$$= 2 \times \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$= \cos\left[\frac{\pi}{2} - (x + y)\right]$$

$$= \sin(x + y)$$

$$= \text{R.H.S}$$

Question 7:

Prove that: $\dfrac{\tan\left(\dfrac{\pi}{4}+x\right)}{\tan\left(\dfrac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

Answer 7:

It is known that

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B} \text{ and } \tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

L.H.S. = $\dfrac{\tan\left(\dfrac{\pi}{4}+x\right)}{\tan\left(\dfrac{\pi}{4}-x\right)}=\dfrac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}=\dfrac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2=\text{R.H.S.}$

Question 8:

Prove that $$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$$

Answer 8:

$$\text{L.H.S.}=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$

$$=\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

$$=\frac{-\cos^2 x}{-\sin^2 x}$$

$$=\cot^2 x$$

$$=\text{R.H.S.}$$

Question 9:

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

Answer 9:

L.H.S. = $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$$=\sin x\cos x[\tan x+\cot x]$$

$$=\sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$

$$=(\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$$

$$=1=\text{R.H.S.}$$

Question 10:

Prove that sin (n + 1)x sin (n + 2)x + cos (n + 1)x cos (n + 2)x = cos x

Answer 10:

L.H.S. = sin (n + 1)x sin(n + 2)x + cos (n + 1)x cos(n + 2)x

$$=\frac{1}{2}\left[2\sin(n+1)x\sin(n+2)x+2\cos(n+1)x\cos(n+2)x\right]$$

$$=\frac{1}{2}\begin{bmatrix}\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}\\+\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{bmatrix}$$

$$\begin{bmatrix}\because -2\sin A\sin B=\cos(A+B)-\cos(A-B)\\2\cos A\cos B=\cos(A+B)+\cos(A-B)\end{bmatrix}$$

$$=\frac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$$

$$=\cos(-x)=\cos x=\text{R.H.S.}$$

Question 11:

Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

Answer 11:

It is known that $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$

$\therefore$ L.H.S. = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

$$=-2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$=-2\sin\frac{\pi}{4}\sin x$$

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$=-\sqrt{2}\sin x$$

$$=\text{R.H.S.}$$

.

Question 12:

Prove that $\sin^2 6x-\sin^2 4x=\sin 2x\sin 10x$

Answer 12:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴ L.H.S. = $\sin^2 6x - \sin^2 4x$

= (sin 6x + sin 4x) (sin 6x − sin 4x)

$$=\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

= (2 sin 5x cos x) (2 cos 5x

sin x) = (2 sin 5x cos 5x) (2

sin x cos x)

= sin 10x sin 2x

= R.H.S.

Question 13:

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Answer 13:

It is known that

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴ L.H.S. = $\cos^2 2x - \cos^2 6x$

= (cos 2x + cos 6x) (cos 2x − 6x)

$$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

$$=\left[2\cos 4x\cos(-2x)\right]\left[-2\sin 4x\sin(-2x)\right]$$

= [2 cos 4x cos 2x] [−2 sin 4x (−sin 2x)]

$= (2 \sin 4x \cos 4x)(2 \sin 2x \cos 2x)$

$= \sin 8x \sin 4x =$ R.H.S.

Question 14:

Prove that $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Answer 14:

L.H.S. $= \sin 2x + 2 \sin 4x + \sin 6x$

$= [\sin 2x + \sin 6x] + 2 \sin 4x$

$$= \left[2\sin\left(\frac{2x+6x}{2}\right)\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$= 2 \sin 4x \cos (-2x) + 2 \sin 4x$

$= 2 \sin 4x \cos 2x + 2 \sin 4x$

$= 2 \sin 4x (\cos 2x + 1)$

$= 2 \sin 4x (2 \cos^2 x - 1 + 1)$

$= 2 \sin 4x (2 \cos^2 x)$

$= 4\cos^2 x \sin$

$4x =$ R.H.S.

Question 15:

Prove that $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

Answer 15:

L.H.S = cot 4x (sin 5x + sin 3x)

$$= \frac{\cot 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \left(\frac{\cos 4x}{\sin 4x}\right)[2\sin 4x \cos x]$$

= 2 cos 4x cos x

R.H.S. = cot x (sin 5x – sin 3x)

$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{\cos x}{\sin x}[2\cos 4x \sin x]$$

= 2 cos 4x. cos x

L.H.S. = R.H.S.

Question 16:

Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Answer 16:

It is known that

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴ L.H.S = $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

$$= \frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

$$= -\frac{\sin 2x}{\cos 10x}$$

$$= \text{R.H.S.}$$

Question 17:

Prove that : $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Answer 17:

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

$$= \frac{\sin 4x}{\cos 4x}$$

$$= \tan 4x = \text{R.H.S.}$$

Question 18:

Prove that $\dfrac{\sin x - \sin y}{\cos x + \cos y} = \tan\dfrac{x-y}{2}$

Answer 18:

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$\therefore$L.H.S. = $\dfrac{\sin x - \sin y}{\cos x + \cos y}$

$$= \frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

Question 19:

Prove that $\dfrac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Answer 19:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin x+\sin 3x}{\cos x+\cos 3x}$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$$=\frac{\sin 2x}{\cos 2x}$$

$=\tan 2x$

$=\text{R.H.S}$

Question 20:

Prove that $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

Answer 20:

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \cos^2 A - \sin^2 A = \cos 2A$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

$$= -2\times(-\sin x)$$

$$= 2\sin x = \text{R.H.S.}$$

Question 21:

Prove that $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Answer 21:

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$\left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$

$$= \cot 3x = \text{R.H.S.}$$

Question 22:

Prove that cot x cot $2x$ – cot $2x$ cot $3x$ – cot $3x$ cot x = 1

Answer 22:

L.H.S. = cot x cot $2x$ – cot $2x$ cot $3x$ – cot $3x$ cot x

= cot x cot $2x$ – cot $3x$ (cot $2x$ + cot x)

= cot x cot $2x$ – cot ($2x$ + x) (cot $2x$ + cot x)

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right](\cot 2x + \cot x)$$

$$\left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$$

$= \cot x \cot 2x - (\cot 2x \cot x - 1) = 1 = $ R.H.S.

Question 23:

Prove that $\tan 4x = \dfrac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$

Answer 23:

It is known that. $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\therefore$ L.H.S. = tan 4x = tan 2(2x)

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

$$= \frac{2\left(\dfrac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\dfrac{4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$= \frac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[\dfrac{\left(1-\tan^2 x\right)^2-4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{\left(1-\tan^2 x\right)^2-4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$

Question 24:

Prove that: $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

Answer 24:

L.H.S. = $\cos 4x$

$= \cos 2(2x)$

$= 1 - 2\sin^2 2x$ [$\cos 2A = 1 - 2\sin^2 A$]

$= 1 - 2(2\sin x \cos x)^2$ [$\sin 2A = 2\sin A \cos A$]

$= 1 - 8\sin^2 x$

$\cos^2 x$ = R.H.S.

Question 25:

Prove that: $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Answer 25:

L.H.S. = $\cos 6x$

$= \cos 3(2x)$

$= 4\cos^3 2x - 3\cos 2x$ [$\cos 3A = 4\cos^3 A - 3\cos A$]

$= 4[(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)$ [$\cos 2x = 2\cos^2 x - 1$]

$= 4[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x)] - 6\cos^2 x + 3$

$= 4[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x] - 6\cos^2 x + 3$

$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18$

$\cos^2 x - 1$ = R.H.S.

Mathematics

(Chapter – 3) (Trigonometric Functions)

(Class – XI)

Exercise 3.4

Question 1:

Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

Answer 1:

$\tan x = \sqrt{3}$

It is known that $\tan\frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$.

Now, $\tan x = \tan\frac{\pi}{3}$

$\Rightarrow x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Question 2:

Find the principal and general solutions of the equation $\sec x = 2$

Answer 2:

$\sec x = 2$

It is known that $\sec\frac{\pi}{3} = 2$ and $\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$.

Now, $\sec x = \sec\frac{\pi}{3}$

$\Rightarrow \cos x = \cos\frac{\pi}{3}$ $\qquad \left[\sec x = \frac{1}{\cos x}\right]$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in \mathbf{Z}$

Question 3:

Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

Answer 3:

$\cot x = -\sqrt{3}$

It is known that $\cot\frac{\pi}{6} = \sqrt{3}$

$\therefore \cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$

i.e., $\cot\frac{5\pi}{6} = -\sqrt{3}$ and $\cot\frac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are $x = \frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

Now, $\cot x = \cot\frac{5\pi}{6}$

$\Rightarrow \tan x = \tan\frac{5\pi}{6}$ $\quad\left[\cot x = \frac{1}{\tan x}\right]$

$\Rightarrow x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

Question 4:

Find the general solution of cosec $x = -2$

Answer 4:

cosec x = −2

It is known that

$$\operatorname{cosec}\frac{\pi}{6} = 2$$

$$\therefore \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2 \text{ and } \operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$$

$$\text{i.e., } \operatorname{cosec}\frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec}\frac{11\pi}{6} = -2$$

Therefore, the principal solutions are $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

$$\text{Now, } \operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$$

$$\Rightarrow \sin x = \sin\frac{7\pi}{6} \qquad \left[\operatorname{cosec} x = \frac{1}{\sin x}\right]$$

$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

Question 5:

Find the general solution of the equation $\cos 4x = \cos 2x$

Answer 5:

$\cos 4x = \cos 2x$

$\Rightarrow \cos 4x - \cos 2x = 0$

$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$

$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \sin 3x \sin x = 0$

$\Rightarrow \sin 3x = 0 \quad \text{or} \quad \sin x = 0$

$\therefore 3x = n\pi \quad \text{or} \quad x = n\pi, \text{ where } n \in Z$

$\Rightarrow x = \frac{n\pi}{3} \quad \text{or} \quad x = n\pi, \text{ where } n \in Z$

Question 6:

Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$

Answer 6:

$\cos 3x + \cos x - \cos 2x = 0$

$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x(2\cos x - 1) = 0$

$\Rightarrow \cos 2x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$

$\Rightarrow \cos 2x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$

$\therefore 2x = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x = \cos\frac{\pi}{3}, \text{ where } n \in Z$

$\Rightarrow x = (2n+1)\frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$

Question 7:

Find the general solution of the equation $\sin 2x + \cos x = 0$

Answer 7:

$$\sin 2x + \cos x = 0$$
$$\Rightarrow 2\sin x \cos x + \cos x = 0$$
$$\Rightarrow \cos x (2\sin x + 1) = 0$$
$$\Rightarrow \cos x = 0 \quad \text{or} \quad 2\sin x + 1 = 0$$
$$\text{Now, } \cos x = 0 \Rightarrow \cos x = (2n+1)\frac{\pi}{2}, \text{ where } n \in Z$$
$$2\sin x + 1 = 0$$
$$\Rightarrow \sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6}$$
$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n \frac{7\pi}{6}, n \in Z$

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Question 8:

Find the general solution of the equation $\sec^2 2x = 1 - \tan 2x$

Answer 8:

$$\sec^2 2x = 1 - \tan 2x$$
$$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$$
$$\Rightarrow \tan^2 2x + \tan 2x = 0$$
$$\Rightarrow \tan 2x(\tan 2x + 1) = 0$$
$$\Rightarrow \tan 2x = 0 \quad \text{or} \quad \tan 2x + 1 = 0$$

$$\text{Now, } \tan 2x = 0$$
$$\Rightarrow \tan 2x = \tan 0$$
$$\Rightarrow 2x = n\pi + 0, \text{ where } n \in Z$$
$$\Rightarrow x = \frac{n\pi}{2}, \text{ where } n \in Z$$
$$\tan 2x + 1 = 0$$
$$\Rightarrow \tan 2x = -1 = -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$$
$$\Rightarrow 2x = n\pi + \frac{3\pi}{4}, \text{ where } n \in Z$$
$$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}, \text{ where } n \in Z$$

Therefore, the general solution is $\frac{n\pi}{2}$ or $\frac{n\pi}{2} + \frac{3\pi}{8}, n \in Z$

Question 9:

Find the general solution of the equation $\sin x+\sin 3x+\sin 5x=0$

Answer 9:

$$\sin x+\sin 3x+\sin 5x=0$$

$$(\sin x+\sin 5x)+\sin 3x=0$$

$$\Rightarrow\left[2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right]+\sin 3x=0 \qquad \left[\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow 2\sin 3x\cos(-2x)+\sin 3x=0$$

$$\Rightarrow 2\sin 3x\cos 2x+\sin 3x=0$$

$$\Rightarrow \sin 3x(2\cos 2x+1)=0$$

$$\Rightarrow \sin 3x=0 \quad \text{or} \quad 2\cos 2x+1=0$$

Now, $\sin 3x=0\Rightarrow 3x=n\pi$, where $n\in Z$

i.e., $x=\frac{n\pi}{3}$, where $n\in Z$

$$2\cos 2x+1=0$$

$$\Rightarrow \cos 2x=\frac{-1}{2}=-\cos\frac{\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)$$

$$\Rightarrow \cos 2x=\cos\frac{2\pi}{3}$$

$$\Rightarrow 2x=2n\pi\pm\frac{2\pi}{3}, \text{ where } n\in Z$$

$$\Rightarrow x=n\pi\pm\frac{\pi}{3}, \text{ where } n\in Z$$

Therefore, the general solution is $\frac{n\pi}{3}$ or $n\pi\pm\frac{\pi}{3}$, $n\in Z$

Mathematics

(Chapter – 3) (Trigonometric Functions)

(Class – XI)

Miscellaneous Exercise on chapter 3

Question 1:

Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Answer 1:

L.H.S.

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)\quad\left[\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$=2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

$$=2\cos\frac{\pi}{13}\times 2\times 0\times\cos\frac{5\pi}{26}$$

= 0 = R.H.S

Question 2:

Prove that: (sin 3*x* + sin *x*) sin *x* + (cos 3*x* − cos *x*) cos *x* = 0

Answer 2:

L.H.S.

= (sin 3*x* + sin *x*) sin *x* + (cos 3*x* − cos *x*) cos *x*

$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$

$= \cos 3x \cos x + \sin 3x \sin x - (\cos^2 x - \sin^2 x)$

$= \cos(3x - x) - \cos 2x \qquad [\cos(A-B) = \cos A \cos B + \sin A \sin B]$

$= \cos 2x - \cos 2x$

$= 0$

= RH.S.

Question 3:

Prove that: $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \frac{x+y}{2}$

Answer 3:

L.H.S. = $(\cos x + \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$

$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y - \sin x \sin y)$

$= 1 + 1 + 2\cos(x+y) \qquad [\cos(A+B) = (\cos A \cos B - \sin A \sin B)]$

$= 2 + 2\cos(x+y)$

$= 2[1 + \cos(x+y)]$

$= 2\left[1 + 2\cos^2\left(\frac{x+y}{2}\right) - 1\right] \qquad [\cos 2A = 2\cos^2 A - 1]$

$= 4\cos^2\left(\frac{x+y}{2}\right) = \text{R.H.S.}$

Question 4:

Prove that: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x-y}{2}$

Answer 4:

L.H.S.

$$= (\cos x - \cos y)^2 + (\sin x - \sin y)^2$$
$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$
$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2[\cos x \cos y + \sin x \sin y]$$
$$= 1 + 1 - 2[\cos(x-y)] \quad [\cos(A-B) = \cos A \cos B + \sin A \sin B]$$
$$= 2[1 - \cos(x-y)]$$
$$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x-y}{2}\right)\right\}\right] \quad [\cos 2A = 1 - 2\sin^2 A]$$
$$= 4\sin^2\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

Question 5:

Prove that: $\sin x+\sin 3x+\sin 5x+\sin 7x=4\cos x\cos 2x\sin 4x$

Answer 5:

It is known that $\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$.

L.H.S. = $\sin x+\sin 3x+\sin 5x+\sin 7x$

$=(\sin x+\sin 5x)+(\sin 3x+\sin 7x)$

$=2\sin\left(\frac{x+5x}{2}\right)\cdot\cos\left(\frac{x-5x}{2}\right)+2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)$

$=2\sin 3x\cos(-2x)+2\sin 5x\ \cos(-2x)$

$=2\sin 3x\cos 2x+2\sin 5x\cos 2x$

$=2\cos 2x[\sin 3x+\sin 5x]$

$=2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right)\cdot\cos\left(\frac{3x-5x}{2}\right)\right]$

$=2\cos 2x[2\sin 4x\cdot\cos(-x)]$

$=4\cos 2x\sin 4x\cos x$ = R.H.S.

Question 6:

Prove that: $\dfrac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}=\tan 6x$

Answer 6:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right),\ \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$$

L.H.S. = $\dfrac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$

$$=\frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\sin\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\cos\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}$$

$$=\frac{[2\sin 6x\cdot\cos x]+[2\sin 6x\cdot\cos 3x]}{[2\cos 6x\cdot\cos x]+[2\cos 6x\cdot\cos 3x]}$$

$$=\frac{2\sin 6x[\cos x+\cos 3x]}{2\cos 6x[\cos x+\cos 3x]}$$

= tan 6*x*

= R.H.S.

Question 7:

Prove that: $\sin 3x+\sin 2x-\sin x=4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Answer 7:

L.H.S. = $\sin 3x+\sin 2x-\sin x$

$=\sin 3x+(\sin 2x-\sin x)$

$=\sin 3x+\left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right]$ $\quad\left[\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$=\sin 3x+\left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)\right]$

$=\sin 3x+2\cos\frac{3x}{2}\sin\frac{x}{2}$

$=2\sin\frac{3x}{2}\cdot\cos\frac{3x}{2}+2\cos\frac{3x}{2}\sin\frac{x}{2}$ $\quad[\sin 2A=2\sin A\cdot\cos B]$

$=2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right)+\sin\left(\frac{x}{2}\right)\right]$

$=2\cos\left(\frac{3x}{2}\right)\left[2\sin\left\{\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right\}\cos\left\{\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right\}\right]$ $\quad\left[\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$=2\cos\left(\frac{3x}{2}\right).2\sin x\cos\left(\frac{x}{2}\right)$

$=4\sin x\cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right)=\text{R.H.S.}$

Question 8:

Find $sin\ x/2, cos\ x/2\ \ and\ tan\ x/2$, if $tanx = -\frac{4}{3}$, x in quadrant II

Answer 8:

Here, *x* is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

Therefore, $\sin\frac{x}{2},\ \cos\frac{x}{2}$ and $\tan\frac{x}{2}$

are lies in first quadrant.

It is given that $\tan x = -\frac{4}{3}$.

$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{-4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$\therefore \cos^2 x = \frac{9}{25}$

$\Rightarrow \cos x = \pm\frac{3}{5}$

As *x* is in quadrant II, cos*x* is negative.

$$\cos x = \frac{-3}{5}$$

Now, $\cos x = 2\cos^2\frac{x}{2} - 1$

$$\Rightarrow \frac{-3}{5} = 2\cos^2\frac{x}{2} - 1$$

$$\Rightarrow 2\cos^2\frac{x}{2} = 1 - \frac{3}{5}$$

$$\Rightarrow 2\cos^2\frac{x}{2} = \frac{2}{5}$$

$$\Rightarrow \cos^2\frac{x}{2} = \frac{1}{5}$$

$$\Rightarrow \cos\frac{x}{2} = \frac{1}{\sqrt{5}} \qquad \left[\because \cos\frac{x}{2} \text{ is positive}\right]$$

$$\therefore \cos\frac{x}{2} = \frac{\sqrt{5}}{5}$$

$$\sin^2\frac{x}{2} + \cos^2\frac{x}{2} = 1$$

$$\Rightarrow \sin^2\frac{x}{2} + \left(\frac{1}{\sqrt{5}}\right)^2 = 1$$

$$\Rightarrow \sin^2\frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\Rightarrow \sin\frac{x}{2} = \frac{2}{\sqrt{5}} \qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$$

$$\therefore \sin\frac{x}{2} = \frac{2\sqrt{5}}{5}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)} = 2$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{2\sqrt{5}}{5}$, $\frac{\sqrt{5}}{5}$, and 2.

Question 9:

Find , $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\cos x = -\frac{1}{3}$, *x* in quadrant III

Answer 9:

Here, *x* is in quadrant III.

$$\text{i.e., } \pi < x < \frac{3\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Therefore, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are negative, where $\sin\frac{x}{2}$ as is positive.

It is given that $\cos x = -\frac{1}{3}$.

$$\cos x = 1 - 2\sin^2\frac{x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{1-\left(-\frac{1}{3}\right)}{2} = \frac{\left(1+\frac{1}{3}\right)}{2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

$$\Rightarrow \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$$

$$\therefore \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Now

$$\cos x = 2\cos^2\frac{x}{2} - 1$$

$$\Rightarrow \cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3-1}{3}\right)}{2} = \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

$$\Rightarrow \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \qquad \left[\because \cos\frac{x}{2} \text{ is negative}\right]$$

$$\therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)} = -\sqrt{2}$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{6}}{3}$, $\frac{-\sqrt{3}}{3}$, and $-\sqrt{2}$

.

Question 10:

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\sin x = \frac{1}{4}$, *x* in quadrant II

Answer 10:

Here, *x* is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin\frac{x}{2}, \cos\frac{x}{2}$, $\tan\frac{x}{2}$ are all positive.

It is given that $\sin x = \frac{1}{4}$.

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\Rightarrow \cos x = -\frac{\sqrt{15}}{4}$$

[cos x is negative in quadrant II]

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2} = \frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4+\sqrt{15}}{8}$$

$$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}} \qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4+\sqrt{15}}{8}\times\frac{2}{2}}$$

$$= \sqrt{\frac{8+2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8+2\sqrt{15}}}{4}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4-\sqrt{15}}{8}$$

$$\Rightarrow \cos\frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}} \qquad \left[\because \cos\frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4-\sqrt{15}}{8}\times\frac{2}{2}}$$

$$= \sqrt{\frac{8-2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8-2\sqrt{15}}}{4}$$

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8-2\sqrt{15}}}{4}\right)}=\frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

$$=\sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}}\times\frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$$

$$=\sqrt{\frac{\left(8+2\sqrt{15}\right)^2}{64-60}}=\frac{8+2\sqrt{15}}{2}=4+\sqrt{15}$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$

are $\frac{\sqrt{8+2\sqrt{15}}}{4}$, $\frac{\sqrt{8-2\sqrt{15}}}{4}$,

and $4+\sqrt{15}$.

Mathematics

(Chapter – 3) (Trigonometric Functions)(Supplementary Exercise)
(Class 11)

Exercise 3.5

Question 1:

In any triangle ABC, if a = 18, b = 24, $c = 30$, find: $\cos A$, $\cos B$, $\cos C$.

Answer 1:

Using cosine formula $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, we have

$$\cos A = \frac{24^2 + 30^2 - 18^2}{2 \times 24 \times 30} = \frac{576 + 900 - 324}{1440} = \frac{1152}{1440} = \frac{4}{5}$$

Similarly, using $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$, we have

$$\cos B = \frac{30^2 + 18^2 - 24^2}{2 \times 30 \times 18} = \frac{900 + 324 - 576}{1080} = \frac{648}{1080} = \frac{3}{5}$$

and using $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$, we get

$$\cos C = \frac{18^2 + 24^2 - 30^2}{2 \times 18 \times 24} = \frac{324 + 576 - 900}{864} = \frac{0}{864} = 0$$

Question 2:

In any triangle ABC, if a = 18, b = 24, $c = 30$, find: $\cos A$, $\cos B$, $\cos C$.

Answer 2:

Using cosine formula $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, we have

$$\cos A = \frac{24^2 + 30^2 - 18^2}{2 \times 24 \times 30} = \frac{576 + 900 - 324}{1440} = \frac{1152}{1440} = \frac{4}{5}$$

$$\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

Similarly, using $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$, we have

$$\cos B = \frac{30^2 + 18^2 - 24^2}{2 \times 30 \times 18} = \frac{900 + 324 - 576}{1080} = \frac{648}{1080} = \frac{3}{5}$$

$$\sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

and using $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$, we get

$$\cos C = \frac{18^2 + 24^2 - 30^2}{2 \times 18 \times 24} = \frac{324 + 576 - 900}{864} = \frac{0}{864} = 0$$

$$\sin C = \sqrt{1 - \cos^2 C} = \sqrt{1 - (0)^2} = \sqrt{1 - 0} = \sqrt{1} = 1$$

Question 3:

For any triangle ABC, prove that: $\frac{a + b}{c} = \frac{\cos\left(\frac{A - B}{2}\right)}{\sin\frac{C}{2}}$

Answer 3:

$$\text{LHS} = \frac{a+b}{c}$$

$$= \frac{k\sin A + k\sin B}{k\sin C} \qquad \left[\because \text{Using } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k\right]$$

$$= \frac{k(\sin A + \sin B)}{k\sin C} = \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{\sin C} \qquad \left[\because \sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right]$$

$$= \frac{2\sin\left(90 - \frac{C}{2}\right)\cos\frac{A-B}{2}}{2\sin\frac{C}{2}\cos\frac{C}{2}} \qquad \left[\because \frac{A+B}{2} = 90° - \frac{C}{2} \text{ and } \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$$

$$= \frac{\cos\frac{C}{2}\cos\frac{A-B}{2}}{\sin\frac{C}{2}\cos\frac{C}{2}} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}} = \text{RHS}$$

Question 4:

For any triangle ABC, prove that: $\frac{a-b}{c} = \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\frac{C}{2}}$

Answer 4:

$$\text{LHS} = \frac{a-b}{c}$$

$$= \frac{k\sin A - k\sin B}{k\sin C} \qquad \left[\because \text{Using } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k\right]$$

$$= \frac{k(\sin A - \sin B)}{k\sin C} = \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{\sin C} \qquad \left[\because \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right]$$

$$= \frac{2\cos\left(90 - \frac{C}{2}\right)\sin\frac{A-B}{2}}{2\sin\frac{C}{2}\cos\frac{C}{2}} \qquad \left[\because \frac{A+B}{2} = 90° - \frac{C}{2} \text{ and } \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$$

$$= \frac{\sin\frac{C}{2}\sin\frac{A-B}{2}}{\sin\frac{C}{2}\cos\frac{C}{2}} = \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\frac{C}{2}} = \text{RHS}$$

Question 5:

For any triangle ABC, prove that: $\sin\frac{B-C}{2} = \frac{b-c}{a}\cos\frac{A}{2}$

Answer 5:

$$\text{RHS} = \frac{b-c}{a}\cos\frac{A}{2}$$

$$= \frac{k\sin B - k\sin C}{k\sin A}\cos\frac{A}{2} \qquad \left[\because \text{Using } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k\right]$$

$$= \frac{k(\sin B - \sin C)}{k\sin A}\cos\frac{A}{2} = \frac{2\cos\frac{B+C}{2}\sin\frac{B-C}{2}}{\sin A}\cos\frac{A}{2} \qquad \left[\because \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right]$$

$$= \frac{2\cos\left(90 - \frac{A}{2}\right)\sin\frac{B-C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}}\cos\frac{A}{2} \qquad \left[\because \frac{A+B}{2} = 90° - \frac{C}{2} \text{ and } \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$$

$$= \frac{\sin\frac{A}{2}\sin\frac{B-C}{2}}{\sin\frac{A}{2}} = \sin\frac{B-C}{2} = \text{LHS}$$

Question 6:

For any triangle ABC, prove that: $a(b\cos C - c\cos B) = b^2 - c^2$

Answer 6:

LHS $= a(b\cos C - c\cos B)$

$$= a\left[b\left(\frac{a^2+b^2-c^2}{2ab}\right) - c\left(\frac{a^2+c^2-b^2}{2ac}\right)\right] \qquad \left[\because \text{Using } \cos A = \frac{b^2+c^2-a^2}{2bc}\right]$$

$$= a\left[\frac{a^2+b^2-c^2}{2a} - \frac{a^2+c^2-b^2}{2a}\right] = a\left[\frac{a^2+b^2-c^2-a^2-c^2+b^2}{2a}\right]$$

$$= a\left[\frac{2(b^2-c^2)}{2a}\right] = b^2 - c^2 = \text{RHS}$$

Question 7:

For any triangle ABC, prove that: $a(\cos C - \cos B) = 2(b-c)\cos^2\frac{A}{2}$

Answer 7:

LHS $= a(\cos C - \cos B)$

$$= a\left[\left(\frac{a^2+b^2-c^2}{2ab}\right) - \left(\frac{a^2+c^2-b^2}{2ac}\right)\right] \qquad \left[\because \text{Using } \cos A = \frac{b^2+c^2-a^2}{2bc}\right]$$

$$= a\left[\frac{ca^2+cb^2-c^3-ba^2-bc^2+b^3}{2abc}\right]$$

$$= \frac{b^3-c^3+b^2c-bc^2+a^2c-a^2b}{2bc} = \frac{(b-c)(b^2+bc+c^2)+bc(b-c)-a^2(b-c)}{2bc}$$

$$= (b-c)\left[\frac{b^2+bc+c^2+bc-a^2}{2bc}\right] = (b-c)\left[\frac{2bc}{2bc} + \frac{b^2+c^2-a^2}{2bc}\right]$$

$$= (b-c)[1+\cos A] \qquad \left[\because \text{Using } \cos A = \frac{b^2+c^2-a^2}{2bc}\right]$$

$$= 2(b-c)\cos^2\frac{A}{2} = \text{RHS} \qquad \left[\because 1+\cos A = 2\cos^2\frac{A}{2}\right]$$

Question 8:

For any triangle ABC, prove that: $\frac{\sin(B-C)}{\sin(B+C)} = \frac{b^2-c^2}{a^2}$

Answer 8:

$$\text{LHS} = \frac{\sin(B-C)}{\sin(B+C)} = \frac{\sin B\cos C - \cos B\sin C}{\sin B\cos C + \cos B\sin C}$$

$$= \frac{kb\left(\frac{a^2+b^2-c^2}{2ab}\right) - \left(\frac{a^2+c^2-b^2}{2ac}\right)kc}{kb\left(\frac{a^2+b^2-c^2}{2ab}\right) + \left(\frac{a^2+c^2-b^2}{2ac}\right)kc} \qquad \left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \text{ and } \cos A = \frac{b^2+c^2-a^2}{2bc}\right]$$

$$= \frac{\left(\frac{a^2+b^2-c^2}{2a} - \frac{a^2+c^2-b^2}{2a}\right)}{\left(\frac{a^2+b^2-c^2}{2a} + \frac{a^2+c^2-b^2}{2ac}\right)} = \frac{\left(\frac{a^2+b^2-c^2-a^2-c^2+b^2}{2a}\right)}{\left(\frac{a^2+b^2-c^2+a^2+c^2-b^2}{2a}\right)} = \frac{2(b^2-c^2)}{2(a^2)}$$

$$= \frac{b^2-c^2}{a^2} = \text{RHS}$$

Question 9:

For any triangle ABC, prove that: $(b+c)\cos\frac{B+C}{2} = a\cos\frac{B-C}{2}$

Answer 9:

$\text{LHS} = (b+c)\cos\frac{B+C}{2}$

$= (k\sin B + k\sin C)\cos\frac{B+C}{2}$ $\left[\because \text{Using } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k\right]$

$= k(\sin B + \sin C)\cos\frac{B+C}{2}$

$= k\left(2\sin\frac{B+C}{2}\cos\frac{B-C}{2}\right)\cos\frac{B+C}{2}$ $\left[\because \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right]$

$= 2k\sin\left(90 - \frac{A}{2}\right)\cos\frac{B-C}{2}\cos\left(90 - \frac{A}{2}\right)$ $\left[\because \frac{A+B}{2} = 90° - \frac{C}{2}\right]$

$= 2k\cos\frac{A}{2}\cos\frac{B-C}{2}\sin\frac{A}{2}$

$= k\sin A\cos\frac{B-C}{2}$ $\left[\because 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sin\theta\right]$

$= a\cos\frac{B-C}{2} = \text{RHS}$

Question 10:

For any triangle ABC, prove that: $a\cos A + b\cos B + c\cos C = 2a\sin B\sin C$

Answer 10:

$\text{LHS} = a\cos A + b\cos B + c\cos C$

$= k\sin A\cos A + k\sin B\cos B + k\sin C\cos C$ $\left[\because \text{Using } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k\right]$

$= \frac{k}{2}(2\sin A\cos A + 2\sin B\cos B + 2\sin C\cos C)$

$= \frac{k}{2}(\sin 2A + \sin 2B + 2\sin C\cos C)$

$= \frac{k}{2}[2\sin(A+B)\cos(A-B) + 2\sin C\cos C]$ $\left[\because \sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right]$

$= \frac{k}{2}[2\sin(180 - C)\cos(A-B) + 2\sin C\cos C]$ $[\because A+B = 180° - C]$

$= \frac{k}{2}[2\sin C\cos(A-B) + 2\sin C\cos C] = k\sin C[\cos(A-B) + \cos C]$

$= k\sin C[\cos(A-B) + \cos\{180 - (A+B)\}]$ $[\because A+B+C = 180°]$

$= k\sin C[\cos(A-B) - \cos(A+B)]$

$= k\sin C[2\sin A\sin B]$ $[\because \cos(A-B) - \cos(A+B) = 2\sin A\sin B]$

$= 2a\sin B\sin C = \text{RHS}$ $[\because k\sin A = a]$

Question 11:

For any triangle ABC, prove that: $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^2+b^2+c^2}{2abc}$

Answer 11:

$\text{LHS}=\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}$

$=\frac{1}{a}\left(\frac{b^2+c^2-a^2}{2bc}\right)+\frac{1}{b}\left(\frac{a^2+c^2-b^2}{2ac}\right)+\frac{1}{c}\left(\frac{a^2+b^2-c^2}{2ab}\right)\quad\left[\because \text{Using } \cos A=\frac{b^2+c^2-a^2}{2bc}\right]$

$=\frac{b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2}{2abc}=\frac{a^2+b^2+c^2}{2abc}=\text{RHS}$

Question 12:

For any triangle ABC, prove that: $(b^2-c^2)\cot A+(c^2-a^2)\cot B+(a^2-b^2)\cot C=0$

Answer 12:

$\text{LHS}=(b^2-c^2)\cot A+(c^2-a^2)\cot B+(a^2-b^2)\cot C$

$=(b^2-c^2)\frac{\cos A}{\sin A}+(c^2-a^2)\frac{\cos B}{\sin B}+(a^2-b^2)\frac{\cos C}{\sin C}$

$=(b^2-c^2)\left[\frac{1}{ka}\left(\frac{b^2+c^2-a^2}{2bc}\right)\right]+(c^2-a^2)\left[\frac{1}{kb}\left(\frac{a^2+c^2-b^2}{2ac}\right)\right]+(a^2-b^2)\left[\frac{1}{kc}\left(\frac{a^2+b^2-c^2}{2ab}\right)\right]$

$\left[\because \text{Using } \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=k \text{ and } \cos A=\frac{b^2+c^2-a^2}{2bc}\right]$

$=\frac{1}{kabc}[(b^2-c^2)(b^2+c^2-a^2)+(c^2-a^2)(a^2+c^2-b^2)+(a^2-b^2)(a^2+b^2-c^2)]$

$=\frac{1}{kabc}[(b^4+b^2c^2-a^2b^2-b^2c^2-c^4+c^2a^2)+(c^2a^2+c^4-b^2c^2-a^4-c^2a^2+a^2b^2)+(a^4+a^2b^2-c^2a^2-a^2b^2-b^4+b^2c^2)]$

$=\frac{1}{kabc}(0)=0=\text{RHS}$

Question 13:

For any triangle ABC, prove that: $\frac{b^2-c^2}{a^2}\sin 2A+\frac{c^2-a^2}{b^2}\sin 2B+\frac{a^2-b^2}{c^2}\sin 2C=0$

Answer 13:

$\text{LHS}=\frac{b^2-c^2}{a^2}\sin 2A+\frac{c^2-a^2}{b^2}\sin 2B+\frac{a^2-b^2}{c^2}\sin 2C$

$=\frac{b^2-c^2}{a^2}2\sin A\cos A+\frac{c^2-a^2}{b^2}2\sin B\cos B+\frac{a^2-b^2}{c^2}2\sin C\cos C\quad[\because \sin 2A=2\sin A\cos A]$

$=\left(\frac{b^2-c^2}{a^2}\right)\left[ka\left(\frac{b^2+c^2-a^2}{2bc}\right)\right]+\left(\frac{c^2-a^2}{b^2}\right)\left[kb\left(\frac{a^2+c^2-b^2}{2ac}\right)\right]+\left(\frac{a^2-b^2}{c^2}\right)\left[kc\left(\frac{a^2+b^2-c^2}{2ab}\right)\right]$

$\left[\because \text{Using } \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=k \text{ and } \cos A=\frac{b^2+c^2-a^2}{2bc}\right]$

$=\frac{k}{abc}[(b^2-c^2)(b^2+c^2-a^2)+(c^2-a^2)(a^2+c^2-b^2)+(a^2-b^2)(a^2+b^2-c^2)]$

$=\frac{k}{abc}[(b^4+b^2c^2-a^2b^2-b^2c^2-c^4+c^2a^2)+(c^2a^2+c^4-b^2c^2-a^4-c^2a^2+a^2b^2)+(a^4+a^2b^2-c^2a^2-a^2b^2-b^4+b^2c^2)]$

$=\frac{k}{abc}(0)=0=\text{RHS}$