(Chapter 6)(Electromagnetic Induction)

XII

Additional Exercises

Question 6.11:

Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 T s^{-1}. If the cut is joined and the loop has a resistance of 1.6 Ω how much power is dissipated by the loop as heat? What is the source of this power?

Answer

Sides of the rectangular loop are 8 cm and 2 cm.

Hence, area of the rectangular wire loop,

A = length × width

= 8 × 2 = 16 cm^2

= 16 × 10^{-4} m^2

Initial value of the magnetic field, $B' = 0.3$ T

Rate of decrease of the magnetic field, $\frac{dB}{dt} = 0.02$ T/s

Emf developed in the loop is given as:

$$e = \frac{d\phi}{dt}$$

Where,

$d\phi$ = Change in flux through the loop area

= AB

$$\therefore e = \frac{d(AB)}{dt} = \frac{AdB}{dt}$$

$$= 16 \times 10^{-4} \times 0.02 = 0.32 \times 10^{-4} \text{ V}$$

Resistance of the loop, R = 1.6 Ω

The current induced in the loop is given as:

$$i = \frac{e}{R}$$
$$= \frac{0.32 \times 10^{-4}}{1.6} = 2 \times 10^{-5} \text{ A}$$

Power dissipated in the loop in the form of heat is given as:

$$P = i^2 R$$
$$= \left(2 \times 10^{-5}\right)^2 \times 1.6$$
$$= 6.4 \times 10^{-10} \text{ W}$$

The source of this heat loss is an external agent, which is responsible for changing the magnetic field with time.

Question 6.12:

A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of 8 cm s^{-1} in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of 10^{-3} T cm^{-1} along the negative x-direction (that is it increases by 10^{-3} T cm^{-1} as one moves in the negative x-direction), and it is decreasing in time at the rate of 10^{-3} T s^{-1}. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.50 mΩ.

Answer

Side of the square loop, s = 12 cm = 0.12 m

Area of the square loop, A = 0.12 × 0.12 = 0.0144 m^2

Velocity of the loop, v = 8 cm/s = 0.08 m/s

Gradient of the magnetic field along negative x-direction,

$$\frac{dB}{dx} = 10^{-3} \text{ T cm}^{-1} = 10^{-1} \text{ T m}^{-1}$$

And, rate of decrease of the magnetic field,

$$\frac{dB}{dt} = 10^{-3} \text{ T s}^{-1}$$

Resistance of the loop, $R = 4.5\ \text{m}\Omega = 4.5 \times 10^{-3}\ \Omega$

Rate of change of the magnetic flux due to the motion of the loop in a non-uniform magnetic field is given as:

$$\frac{d\phi}{dt} = A \times \frac{dB}{dx} \times v$$
$$= 144 \times 10^{-4}\,\text{m}^2 \times 10^{-1} \times 0.08$$
$$= 11.52 \times 10^{-5}\ \text{T m}^2\,\text{s}^{-1}$$

Rate of change of the flux due to explicit time variation in field B is given as:

$$\frac{d\phi'}{dt} = A \times \frac{dB}{dt}$$
$$= 144 \times 10^{-4} \times 10^{-3}$$
$$= 1.44 \times 10^{-5}\ \text{T m}^2\,\text{s}^{-1}$$

Since the rate of change of the flux is the induced emf, the total induced emf in the loop can be calculated as:

$$e = 1.44 \times 10^{-5} + 11.52 \times 10^{-5}$$
$$= 12.96 \times 10^{-5}\ \text{V}$$

∴Induced current, $i = \frac{e}{R}$

$$= \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-3}}$$
$$i = 2.88 \times 10^{-2}\ \text{A}$$

Hence, the direction of the induced current is such that there is an increase in the flux through the loop along positive z-direction.

Question 6.13:

It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm^2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of magnet.

Answer

Area of the small flat search coil, A = 2 cm^2 = 2 × 10^{-4} m^2

Number of turns on the coil, N = 25

Total charge flowing in the coil, Q = 7.5 mC = 7.5 × 10^{-3} C

Total resistance of the coil and galvanometer, R = 0.50 Ω

Induced current in the coil,

$$I = \frac{\text{Induced emf }(e)}{R} \qquad \dots (1)$$

Induced emf is given as:

$$e = -N\frac{d\phi}{dt} \qquad \dots (2)$$

Where,

$d\phi$ = Charge in flux

Combining equations (1) and (2), we get

$$I = -\frac{N\frac{d\phi}{dt}}{R}$$

$$Idt = -\frac{N}{R}d\phi \qquad \dots (3)$$

Initial flux through the coil, ϕ_i = BA

Where,

B = Magnetic field strength

Final flux through the coil, $\phi_f = 0$

Integrating equation (3) on both sides, we have

$$\int Idt = \frac{-N}{R}\int_{\phi_i}^{\phi_f} d\phi$$

But total charge, $Q = \int Idt.$

$$\therefore Q=\frac{-N}{R}(\phi_f-\phi_i)=\frac{-N}{R}(-\phi_i)=+\frac{N\phi_i}{R}$$

$$Q=\frac{NBA}{R}$$

$$\therefore B=\frac{QR}{NA}$$

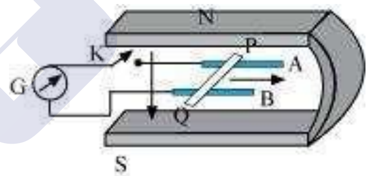
$$=\frac{7.5\times10^{-3}\times0.5}{25\times2\times10^{-4}}=0.75\text{ T}$$

Hence, the field strength of the magnet is 0.75 T.

Question 6.14:

Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T, resistance of the closed loop containing the rod = 9.0 mΩ. Assume the field to be uniform.

(a) Suppose K is open and the rod is moved with a speed of 12 cm s^{-1} in the direction shown. Give the polarity and magnitude of the induced emf.



(b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

(c) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod.

Explain.

(d) What is the retarding force on the rod when K is closed?

(e) How much power is required (by an external agent) to keep the rod moving at the same speed (=12 cm s^{-1}) when K is closed? How much power is required when K is open?

(f) How much power is dissipated as heat in the closed circuit?

What is the source of this power?

(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

Answer

Length of the rod, $l = 15$ cm $= 0.15$ m

Magnetic field strength, $B = 0.50$ T

Resistance of the closed loop, $R = 9\ m\Omega = 9 \times 10^{-3}\ \Omega$

(a) Induced emf = 9 mV; polarity of the induced emf is such that end P shows positive while end Q shows negative ends. Speed of the rod, $v = 12$ cm/s $= 0.12$ m/s

Induced emf is given as: $e = Bvl$

$= 0.5 \times 0.12 \times 0.15$

$= 9 \times 10^{-3}$ v

$= 9$ mV

The polarity of the induced emf is such that end P shows positive while end Q shows negative ends.

(b) Yes; when key K is closed, excess charge is maintained by the continuous flow of current.

When key K is open, there is excess charge built up at both ends of the rods.

When key K is closed, excess charge is maintained by the continuous flow of current. (c) Magnetic force is cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rod.

There is no net force on the electrons in rod PQ when key K is open and the rod is moving uniformly. This is because magnetic force is cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rods.

(d) Retarding force exerted on the rod, $F = IBl$

Where,

I = Current flowing through the rod

$$= \frac{e}{R} = \frac{9\times10^{-3}}{9\times10^{-3}} = 1\text{ A}$$
$$\therefore F = 1\times0.5\times0.15$$
$$= 75\times10^{-3}\text{ N}$$

(e) 9 mW; no power is expended when key K is open.

Speed of the rod, v = 12 cm/s = 0.12 m/s

Hence, power is given as:

$$P = Fv$$
$$= 75\times10^{-3}\times0.12$$
$$= 9\times10^{-3}\text{ W}$$
$$= 9\text{ mW}$$

When key K is open, no power is expended.

(f) 9 mW; power is provided by an external agent.

Power dissipated as heat = $I^2 R$

= $(1)^2 \times 9 \times 10^{-3}$

= 9 mW

The source of this power is an external agent.

(g) Zero

In this case, no emf is induced in the coil because the motion of the rod does not cut across the field lines.

Question 6.15:

An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Answer

Length of the solenoid, l = 30 cm = 0.3 m

Area of cross-section, A = 25 cm^2 = 25 × 10^{-4} m^2

Number of turns on the solenoid, N = 500

Current in the solenoid, I = 2.5 A

Current flows for time, t = 10^{-3} s

Average back emf, $e = \frac{d\phi}{dt}$ ... (1)

Where,

$d\phi$ = Change in flux

= NAB … (2)

Where,

B = Magnetic field strength

$= \mu_0 \frac{NI}{l}$ ... (3)

Where,

μ_0 = Permeability of free space = 4π × 10^{-7} T m A^{-1}

Using equations (2) and (3) in equation (1), we get

$$e = \frac{\mu_0 N^2 I A}{lt}$$

$$= \frac{4\pi \times 10^{-7} \times (500)^2 \times 2.5 \times 25 \times 10^{-4}}{0.3 \times 10^{-3}} = 6.5 \text{ V}$$

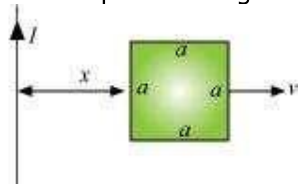
Hence, the average back emf induced in the solenoid is 6.5 V.

Question 6.16:

(a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig. 6.21.

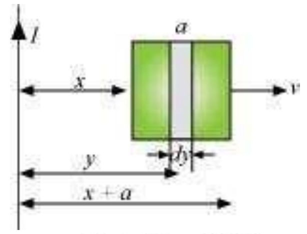
(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v = 10 m/s.

Calculate the induced emf in the loop at the instant when x = 0.2 m.
Take a = 0.1 m and assume that the loop has a large resistance.



Answer

(a) Take a small element dy in the loop at a distance y from the long straight wire (as shown in the given figure).



Magnetic flux associated with element $dy,\ d\phi = BdA$

Where, dA = Area of element dy = a dy

B = Magnetic field at distance y

$$= \frac{\mu_0 I}{2\pi y}$$

I = Current in the wire

μ_0 = Permeability of free space = 4π × 10^{-7}

$$\therefore d\phi = \frac{\mu_0 Ia}{2\pi}\frac{dy}{y}$$

$$\phi = \frac{\mu_0 Ia}{2\pi}\int\frac{dy}{y}$$

y tends from x to $a+x$.

$$\therefore \phi = \frac{\mu_0 Ia}{2\pi}\int_x^{a+x}\frac{dy}{y}$$

$$= \frac{\mu_0 Ia}{2\pi}\left[\log_e y\right]_x^{a+x}$$

$$= \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a+x}{x}\right)$$

For mutual inductance M, the flux is given as:

$\phi = MI$

$$\therefore MI = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a}{x}+1\right)$$

$$M = \frac{\mu_0 a}{2\pi}\log_e\left(\frac{a}{x}+1\right)$$

(b) Emf induced in the loop, e = B'av $= \left(\frac{\mu_0 I}{2\pi x}\right)av$

Given, I =

50 A x =

0.2 m a =

0.1 m v =

10 m/s

$$e = \frac{4\pi\times10^{-7}\times50\times0.1\times10}{2\pi\times0.2}$$

$$e = 5\times10^{-5}\ \text{V}$$

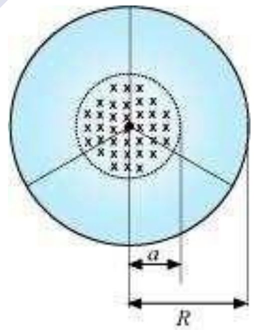
Question 6.17:

A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig. 6.22). A uniform magnetic field extends over a circular region within the rim. It is given by,

$B = -B_0\, k\ (r \le a;\ a < R)$

$= 0$ (otherwise)

What is the angular velocity of the wheel after the field is suddenly switched off?



Answer

Line charge per unit length $= \lambda = \frac{\text{Total charge}}{\text{Length}} = \frac{Q}{2\pi r}$

Where,

r = Distance of the point within the wheel

Mass of the wheel = M

Radius of the wheel = R

Magnetic field, $\vec{B} = -B_0 \hat{k}$

At distance r, the magnetic force is balanced by the centripetal force i.e.,

$$BQv = \frac{Mv^2}{r}$$

Where,

$v =$ linear velocity of the wheel

$$\therefore B2\pi r\lambda = \frac{Mv}{r}$$

$$v = \frac{B2\pi\lambda r^2}{M}$$

∴Angular velocity, $\omega = \frac{v}{R} = \frac{B2\pi\lambda r^2}{MR}$

For $r \le$ and $a < R$, we get:

$$\omega = -\frac{2\pi B_0 \ a^2\lambda}{MR}\hat{k}$$