

(Chapter 7)(Alternating Current)

XII

## Exercises

### Question 7.1:

A  $100\ \Omega$  resistor is connected to a 220 V, 50 Hz ac supply.

- (a) What is the rms value of current in the circuit?
- (b) What is the net power consumed over a full cycle?

### Answer 7.1:

Resistance of the resistor,  $R = 100\ \Omega$

Supply voltage,  $V = 220\ \text{V}$

Frequency,  $\nu = 50\ \text{Hz}$

- (a) The rms value of current in the circuit is given as

$$I = \frac{V}{R} = \frac{220}{100} = 2.20\ \text{A}$$

- (b) The net power consumed over a full cycle is given as:

$$P = VI = 220 \times 2.2 = 484\ \text{W}$$

### Question 7.2:

- (a) The peak voltage of an ac supply is 300 V. What is the rms voltage?
- (b) The rms value of current in an ac circuit is 10 A. What is the peak current?

### Answer 7.2:

- (a) Peak voltage of the ac supply,  $V_0 = 300\ \text{V}$

rms voltage is given as:

$$V = \frac{V_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212.1\ \text{V}$$

(b) The rms value of current is given as:

$$I = 10 \text{ A}$$

Now, peak current is given as:

$$I_o = \sqrt{2}I = \sqrt{2} \times 10 = 14.1 \text{ A}$$

### Question 7.3:

A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

### Answer 7.3:

Inductance of inductor,  $L = 44 \text{ mH} = 44 \times 10^{-3} \text{ H}$

Supply voltage,  $V = 220 \text{ V}$

Frequency,  $\nu = 50 \text{ Hz}$

Angular frequency,  $\omega = 2\pi\nu$

Inductive reactance,  $X_L = \omega L = 2\pi\nu L = 2\pi \times 50 \times 44 \times 10^{-3} \Omega$

rms value of current is given as:

$$I = \frac{V}{X_L} = \frac{220}{2\pi \times 50 \times 44 \times 10^{-3}} = 15.92 \text{ A}$$

Hence, the rms value of current in the circuit is 15.92 A.

### Question 7.4:

A 60  $\mu\text{F}$  capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

### Answer 7.4:

Capacitance of capacitor,  $C = 60 \mu\text{F} = 60 \times 10^{-6} \text{ F}$

Supply voltage,  $V = 110 \text{ V}$

Frequency,  $\nu = 60 \text{ Hz}$

Angular frequency,  $\omega = 2\pi\nu$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2\pi \times 60 \times 60 \times 10^{-6}} \Omega$$

rms value of current is given as:

$$I = \frac{V}{X_C} = \frac{220}{2\pi \times 60 \times 60 \times 10^{-6}} = 2.49 \text{ A}$$

Hence, the rms value of current is 2.49 A.

### Question 7.5:

In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.

### Answer 7.5:

In the inductive circuit,

Rms value of current,  $I = 15.92 \text{ A}$

Rms value of voltage,  $V = 220 \text{ V}$

Hence, the net power absorbed can be obtained by the relation,

$$P = VI \cos \Phi$$

Where,

$\Phi$  = Phase difference between  $V$  and  $I$ .

For a pure inductive circuit, the phase difference between alternating voltage and current is  $90^\circ$  i.e.,  $\Phi = 90^\circ$ .

Hence,  $P = 0$  i.e., the net power is zero.

In the capacitive circuit,

rms value of current,  $I = 2.49 \text{ A}$

rms value of voltage,  $V = 110 \text{ V}$

Hence, the net power absorbed can be obtained as:

$$P = VI \cos \Phi$$

For a pure capacitive circuit, the phase difference between alternating voltage and current is  $90^\circ$  i.e.,  $\Phi = 90^\circ$ .

Hence,  $P = 0$  i.e., the net power is zero.

### Question 7.6:

Obtain the resonant frequency  $\omega_r$  of a series LCR circuit with  $L = 2.0 \text{ H}$ ,  $C = 32 \mu\text{F}$  and  $R = 10 \Omega$ . What is the Q-value of this circuit?

### Answer 7.6:

Inductance,  $L = 2.0 \text{ H}$

Capacitance,  $C = 32 \mu\text{F} = 32 \times 10^{-6} \text{ F}$

Resistance,  $R = 10 \Omega$

Resonant frequency is given by the relation,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}} = \frac{1}{8 \times 10^{-3}} = 125 \text{ rad/s.}$$

Now, Q-value of the circuit is given as:

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{10} \sqrt{\frac{2}{32 \times 10^{-6}}} = \frac{1}{10 \times 4 \times 10^{-3}} = 25$$

Hence, the Q-Value of this circuit is 25.

### Question 7.7:

A charged 30  $\mu\text{F}$  capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

### Answer 7.7:

Capacitance,  $C = 30\mu\text{F} = 30 \times 10^{-6} \text{ F}$  Inductance,  $L = 27 \text{ mH} = 27 \times 10^{-3} \text{ H}$

Angular frequency is given as:

$$\begin{aligned} \omega_r &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}} = \frac{1}{9 \times 10^{-4}} = 1.11 \times 10^3 \text{ rad/s} \end{aligned}$$

Hence, the angular frequency of free oscillations of the circuit is  $1.11 \times 10^3 \text{ rad/s}$ .

### Question 7.8:

Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at later time?

### Answer 7.8:

Capacitance of the capacitor,  $C = 30 \mu\text{F} = 30 \times 10^{-6} \text{ F}$

Inductance of the inductor,  $L = 27 \text{ mH} = 27 \times 10^{-3} \text{ H}$

Charge on the capacitor,  $Q = 6 \text{ mC} = 6 \times 10^{-3} \text{ C}$

Total energy stored in the capacitor can be calculated as:

$$E = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{(6 \times 10^{-3})^2}{30 \times 10^{-6}} = \frac{6}{10} = 0.6 \text{ J}$$

Total energy at a later time will remain the same because energy is shared between the capacitor and the inductor.

### Question 7.9:

A series LCR circuit with  $R = 20 \Omega$ ,  $L = 1.5 \text{ H}$  and  $C = 35 \mu\text{F}$  is connected to a variable frequency  $200 \text{ V}$  ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

### Answer 7.9:

At resonance, the frequency of the supply power equals the natural frequency of the given LCR circuit.

Resistance,  $R = 20 \Omega$

Inductance,  $L = 1.5 \text{ H}$

Capacitance,  $C = 35 \mu\text{F} = 35 \times 10^{-6} \text{ F}$

AC supply voltage to the LCR circuit,  $V = 200 \text{ V}$

Impedance of the circuit is given by the relation,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance,  $X_L = X_C$

$$\therefore Z = R = 20 \Omega$$

Current in the circuit can be calculated as:

$$I = \frac{V}{Z} = \frac{200}{20} = 10 A$$

Hence, the average power transferred to the circuit in one complete cycle:

$$VI = 200 \times 10 = 2000 W.$$

### Question 7.10:

A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200  $\mu\text{H}$ , what must be the range of its variable capacitor?

[Hint: For tuning, the natural frequency i.e., the frequency of free oscillations of the LC circuit should be equal to the frequency of the radio wave.]

### Answer 7.10:

The range of frequency ( $\nu$ ) of a radio is 800 kHz to 1200 kHz.

Lower tuning frequency,  $\nu_1 = 800 \text{ kHz} = 800 \times 10^3 \text{ Hz}$

Upper tuning frequency,  $\nu_2 = 1200 \text{ kHz} = 1200 \times 10^3 \text{ Hz}$

Effective inductance of circuit  $L = 200 \mu\text{H} = 200 \times 10^{-6} \text{ H}$

Capacitance of variable capacitor for  $\nu_1$  is given as:

$$C_1 = \frac{1}{\omega_1^2 L}$$

Where,

$\omega_1 =$  Angular frequency for capacitor  $C_1$

$$= 2\pi\nu_1$$

$$= 2\pi \times 800 \times 10^3 \text{ rad/s}$$

$$\begin{aligned}\therefore C_1 &= \frac{1}{(2\pi \times 800 \times 10^3)^2 \times 200 \times 10^{-6}} \\ &= 1.9809 \times 10^{-10} \text{ F} = 198 \text{ pF}\end{aligned}$$

Capacitance of variable capacitor for  $v_2$  is given as:

$$C_2 = \frac{1}{\omega_2^2 L}$$

Where,

$\omega_2$  = Angular frequency for capacitor  $C_2$

$$= 2\pi\nu_2$$

$$= 2\pi \times 1200 \times 10^3 \text{ rad/s}$$

$$\begin{aligned}\therefore C_2 &= \frac{1}{(2\pi \times 1200 \times 10^3)^2 \times 200 \times 10^{-6}} \\ &= 0.8804 \times 10^{-10} \text{ F} = 88 \text{ pF}\end{aligned}$$

Hence, the range of the variable capacitor is from 88.04 pF to 198.1 pF.

### Question 7.11:

Figure 7.21 shows a series LCR circuit connected to a variable frequency 230 V source.  $L = 5.0 \text{ H}$ ,  $C = 80\mu\text{F}$ ,  $R = 40 \Omega$

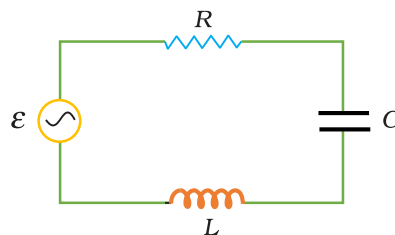


FIGURE 7.21



- (a) Determine the source frequency which drives the circuit in resonance.
- (b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.
- (c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

**Answer 7.11:**

Inductance of the inductor,  $L = 5.0 \text{ H}$

Capacitance of the capacitor,  $C = 80 \text{ } \mu\text{H} = 80 \times 10^{-6} \text{ F}$

Resistance of the resistor,  $R = 40 \text{ } \Omega$

Potential of the variable voltage source,  $V = 230 \text{ V}$

- (a) Resonance angular frequency is given as:

$$\begin{aligned}\omega_r &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \frac{10^3}{20} = 50 \text{ rad/s}\end{aligned}$$

Hence, the circuit will come in resonance for a source frequency of 50 rad/s.

- (b) Impedance of the circuit is given by the relation:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance,  $X_L = X_C \Rightarrow Z = R = 40 \text{ } \Omega$

Amplitude of the current at the resonating frequency is given as:  $I_o = \frac{V_o}{Z}$

Where,

$$V_o = \text{Peak voltage} = \sqrt{2} V$$

$$\therefore I_o = \frac{\sqrt{2} V}{Z} = \frac{\sqrt{2} \times 230}{40} = 8.13 A$$

Hence, at resonance, the impedance of the circuit is  $40 \Omega$  and the amplitude of the current is  $8.13 A$ .

(c) rms potential drop across the inductor,

$$(V_L)_{rms} = I \times \omega_r L$$

Where,

$$I_{rms} = \frac{I_o}{\sqrt{2}} = \frac{\sqrt{2} V}{\sqrt{2} Z} = \frac{230}{40} = \frac{23}{4} A$$

$$\therefore (V_L)_{rms} = \frac{23}{4} \times 50 \times 5 = 1437.5 V$$

Potential drop across the capacitor:

$$\therefore (V_C)_{rms} = I \times \frac{1}{\omega_r C} = \frac{23}{4} \times \frac{1}{50 \times 80 \times 10^{-6}} = 1437.5 V$$

Potential drop across the resistor:

$$(V_R)_{rms} = IR = \frac{23}{4} \times 40 = 230 V$$

Potential drop across the LC combination:

$$V_{LC} = I(X_L - X_C)$$

At resonance,  $X_L = X_C \Rightarrow V_{LC} = 0$

Hence, it is proved that the potential drop across the LC combination is zero at resonating frequency.