

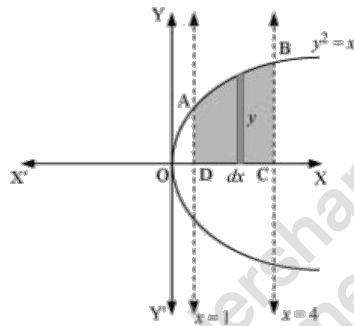
# Chapter 8 Applications of Integrals

## EXERCISE 8.1

### Question 1:

Find the area of the region bounded by the curve  $y^2 = x$  and the lines  $x = 1, x = 4$  and the  $x$ -axis in the first quadrant.

### Solution:

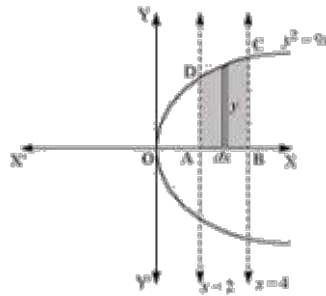


$$\begin{aligned}
 \text{ar}(ABCD) &= \int_1^4 y dx \\
 &= \int_1^4 \sqrt{x} dx \\
 &= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\
 &= \frac{2}{3} \left[ (4)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right] \\
 &= \frac{2}{3} [8 - 1] \\
 &= \frac{14}{3}
 \end{aligned}$$

### Question 2:

Find the area of the region bounded by  $y^2 = 9x, x = 2, x = 4$  and the  $x$ -axis in the first quadrant.

**Solution:**

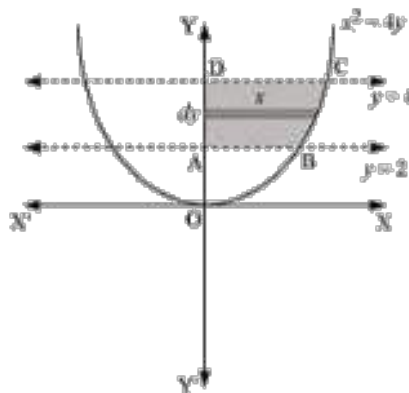


$$\begin{aligned}
 \text{ar}(ABCD) &= \int_2^4 y dx \\
 &= \int_2^4 3\sqrt{x} dx \\
 &= 3 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4 \\
 &= 2 \left[ x^{\frac{3}{2}} \right]_2^4 \\
 &= 2 \left[ (4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] \\
 &= 2 \left[ 8 - 2\sqrt{2} \right] \\
 &= (16 - 4\sqrt{2})
 \end{aligned}$$

### Question 3:

Find the area of the region bounded by  $x^2 = 4y$ ,  $y = 2$ ,  $y = 4$  and the y-axis in the first quadrant.

**Solution:**

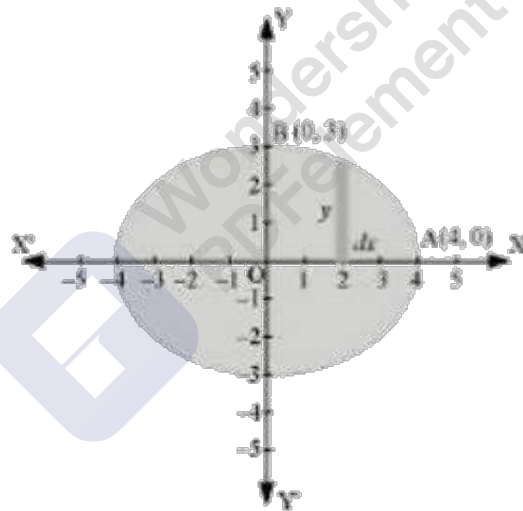


$$\begin{aligned}
 ar(ABCD) &= \int_2^4 x dy \\
 &= \int_2^4 2\sqrt{y} dy = 2 \int_2^4 \sqrt{y} dy \\
 &= 2 \left[ \frac{\frac{3}{2} y^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4 \\
 &= \frac{4}{3} \left[ (4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] = \frac{4}{3} [8 - 2\sqrt{2}] \\
 &= \left( \frac{32 - 8\sqrt{2}}{3} \right)
 \end{aligned}$$

#### Question 4:

Find the area of the region bounded by the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ .

#### Solution:



It is given that

$$\begin{aligned}
 \Rightarrow \frac{x^2}{16} + \frac{y^2}{9} &= 1 \\
 \Rightarrow \frac{y^2}{9} &= 1 - \frac{x^2}{16} \\
 \Rightarrow y &= 3\sqrt{1 - \frac{x^2}{16}}
 \end{aligned}$$

Area of ellipse =  $4 \times ar(OAB)$

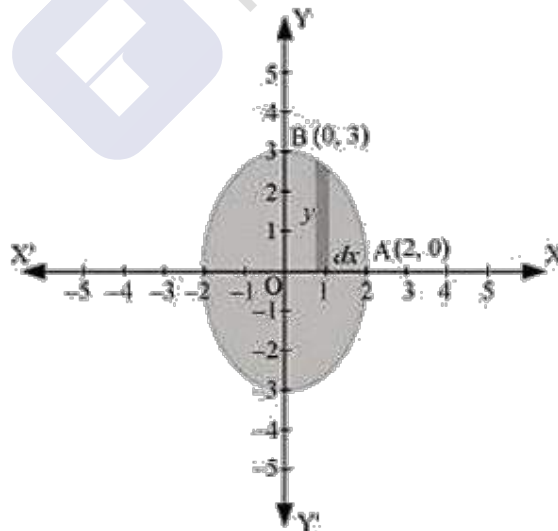
$$\begin{aligned}
 \text{ar}(OAB) &= \int_0^4 y dx \\
 &= \int_0^4 3\sqrt{1-\frac{x^2}{16}} dx \\
 &= \frac{3}{4} \int_0^4 \sqrt{16-x^2} dx \\
 &= \frac{3}{4} \left[ \frac{x}{2} \sqrt{16-x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_0^4 \\
 &= \frac{3}{4} \left[ 2\sqrt{16-16} + 8\sin^{-1}(1) - 0 - 8\sin^{-1}(0) \right] \\
 &= \frac{3}{4} \left[ \frac{8\pi}{2} \right] \\
 &= \frac{3}{4} [4\pi] \\
 &= 3\pi
 \end{aligned}$$

Area of ellipse =  $4 \times 3\pi = 12\pi$  units

### Question 5:

Find the area of the region bounded by the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$

**Solution:**



It is given that

$$\begin{aligned}
 \Rightarrow \frac{x^2}{4} + \frac{y^2}{9} &= 1 \\
 \Rightarrow y &= 3\sqrt{1-\frac{x^2}{4}}
 \end{aligned}$$

Area of ellipse  $= 4 \times ar(OAB)$

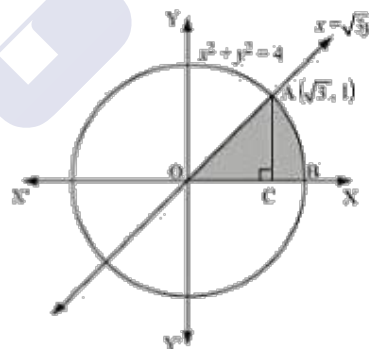
$$\begin{aligned} ar(OAB) &= \int_0^2 y dx \\ &= \int_0^2 3\sqrt{1 - \frac{x^2}{4}} dx \\ &= \frac{3}{2} \int_0^2 \sqrt{4 - x^2} dx \\ &= \frac{3}{2} \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\ &= \frac{3}{2} \left[ \frac{2\pi}{2} \right] \\ &= \frac{3\pi}{2} \end{aligned}$$

Area of ellipse  $= 4 \times \frac{3\pi}{2} = 6\pi$  units.

### Question 6:

Find the area of the region in the first quadrant enclosed by  $x$ -axis, line  $x = \sqrt{3}y$  and the circle  $x^2 + y^2 = 4$

**Solution:**



$$ar(OAB) = ar(\triangle OAC) + ar(\triangle ABC)$$

$$\begin{aligned} ar(\triangle OAC) &= \frac{1}{2} \times OC \times AC \\ &= \frac{1}{2} \times \sqrt{3} \times 1 \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned}
 ar(ABC) &= \int_{\sqrt{3}}^2 y dx \\
 &= \int_{\sqrt{3}}^2 \sqrt{4-x^2} dx \\
 &= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \left( \frac{x}{2} \right) \right]_{\sqrt{3}}^2 \\
 &= \left[ 2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} \sqrt{4-3} - 2 \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \right] \\
 &= \left[ \pi - \frac{\sqrt{3}}{2} - 2 \left( \frac{\pi}{3} \right) \right] \\
 &= \left[ \pi - \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right] \\
 &= \left[ \frac{3\pi - 2\pi}{3} - \frac{\sqrt{3}}{2} \right] \\
 &= \left[ \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right]
 \end{aligned}$$

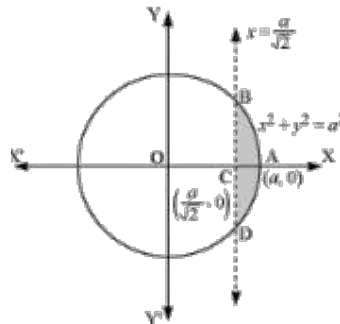
Therefore, required area enclosed  $= \frac{\sqrt{3}}{2} + \frac{\pi}{3} - \frac{\sqrt{3}}{2} = \frac{\pi}{3}$  square units.

### Question 7:

Find the area of the smaller part of the circle  $x^2 + y^2 = a^2$  cut off by the line  $x = \frac{a}{\sqrt{2}}$ .

### Solution:

The area of the smaller part of the circle,  $x^2 + y^2 = a^2$  cut off by the line,  $x = \frac{a}{\sqrt{2}}$ , is the area ABCD.



It can be observed that the area ABCD is symmetrical about x-axis.

$$ar(ABCD) = 2 \times ar(ABC)$$

$$\begin{aligned} ar(ABC) &= \int_{\frac{a}{\sqrt{2}}}^a y dx \\ &= \int_{\frac{a}{\sqrt{2}}}^a \sqrt{a^2 - x^2} dx \\ &= \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) \right]_{\frac{a}{\sqrt{2}}}^a \\ &= \left[ \frac{a^2}{2} \left( \frac{\pi}{2} \right) - \frac{a}{2\sqrt{2}} \sqrt{a^2 - \frac{a^2}{2}} - \frac{a^2}{2} \sin^{-1} \left( \frac{1}{\sqrt{2}} \right) \right] \\ &= \frac{a^2 \pi}{4} - \frac{a}{2\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - \frac{a^2}{2} \left( \frac{\pi}{4} \right) \\ &= \frac{a^2 \pi}{4} - \frac{a^2}{4} - \frac{a^2 \pi}{8} \\ &= \frac{a^2}{4} \left[ \pi - 1 - \frac{\pi}{2} \right] \\ &= \frac{a^2}{4} \left[ \frac{\pi}{2} - 1 \right] \end{aligned}$$

$$\begin{aligned} ar(ABCD) &= 2 \left[ \frac{a^2}{4} \left( \frac{\pi}{2} - 1 \right) \right] \\ &= \frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right) \end{aligned}$$

Therefore, the required area is  $\frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right)$  square units.

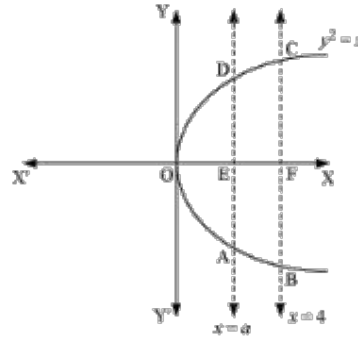
### Question 8:

The area between  $x = y^2$  and  $x = 4$  is divided into two equal parts by the line  $x = a$ , find the value of  $a$ .

### Solution:

The line  $x = a$  divides the area bounded by the parabola and  $x = 4$  into two equal parts.

Therefore,  $ar(OAD) = ar(ABCD)$



It can be observed that the given area is symmetrical about  $x$ -axis.

Hence,  $ar(OED) = ar(EFCD)$

$$\begin{aligned} ar(OED) &= \int_0^a y dx \\ &= \int_0^a \sqrt{x} dx \\ &= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^a \\ &= \frac{2}{3} a^{\frac{3}{2}} \end{aligned} \quad \dots(1)$$

$$\begin{aligned} ar(EFCD) &= \int_a^4 \sqrt{x} dx \\ &= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_a^4 \\ &= \frac{2}{3} \left[ 8 - a^{\frac{3}{2}} \right] \end{aligned} \quad \dots(2)$$

From (1) and (2), we obtain

$$\Rightarrow \frac{2}{3}(a)^{\frac{3}{2}} = \frac{2}{3} \left[ 8 - (a)^{\frac{3}{2}} \right]$$

$$\Rightarrow 2(a)^{\frac{3}{2}} = 8$$

$$\Rightarrow (a)^{\frac{3}{2}} = 4$$

$$\Rightarrow a = (4)^{\frac{2}{3}}$$

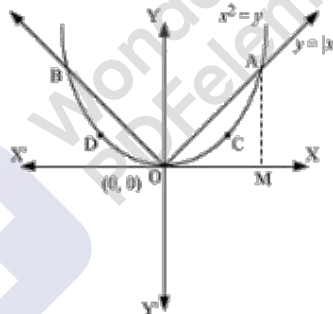
Therefore, the value of  $a = (4)^{\frac{2}{3}}$ .

### Question 9:

Find the area of the region bounded by the parabola  $y = x^2$  and the line  $y = |x|$ .

### Solution:

The area bounded by the parabola  $y = x^2$  and the line  $y = |x|$ , can be represented as



The given area is symmetrical about y-axis.

Therefore,  $ar(OACO) = ar(ODBO)$

The point of intersection of parabola  $y = x^2$  and the line  $y = |x|$ , is  $A(1,1)$ .

$$ar(OACO) = ar(\Delta OAM) - ar(OMACO)$$

$$\begin{aligned} ar(\Delta OAM) &= \frac{1}{2} \times OM \times AM \\ &= \frac{1}{2} \times 1 \times 1 \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{ar}(OMACO) &= \int_0^1 y dx \\ &= \int_0^1 x^2 dx \\ &= \left[ \frac{x^3}{3} \right]_0^1 \\ &= \frac{1}{3} \end{aligned}$$

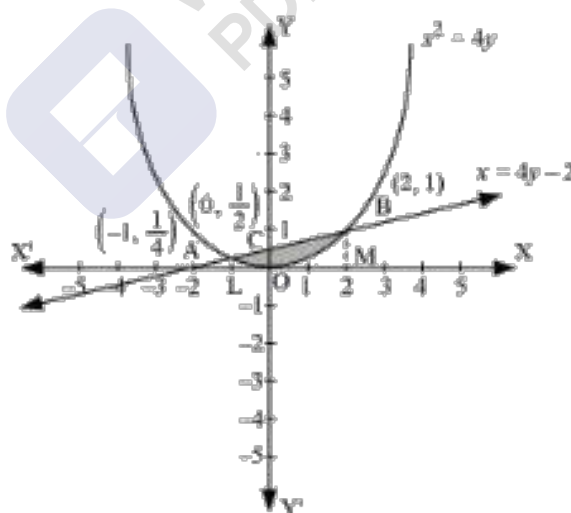
$$\begin{aligned} \text{ar}(OACO) &= \text{ar}(\triangle OAM) - \text{ar}(OMACO) \\ &= \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{6} \end{aligned}$$

Therefore, the required area  $= 2 \left[ \frac{1}{6} \right] = \frac{1}{3}$  units.

### Question 10:

Find the area bounded by the curve  $x^2 = 4y$  and the line  $x = 4y - 2$ .

**Solution:**



Coordinates of point  $A\left(-1, \frac{1}{4}\right)$ .

Coordinates of point  $B(2, 1)$ .

Draw AL and BM perpendicular to  $x$ -axis.

$$ar(OBAO) = ar(OBCO) + ar(OACO)$$

$$ar(OBCO) = ar(OMBC) - ar(OMBO)$$

$$\begin{aligned} &= \int_0^2 \frac{x+2}{4} dx - \int_0^2 \frac{x^2}{4} dx \\ &= \frac{1}{4} \left[ \frac{x^2}{2} + 2x \right]_0^2 - \frac{1}{4} \left[ \frac{x^3}{3} \right]_0^2 \\ &= \frac{1}{4} [2+4] - \frac{1}{4} \left[ \frac{8}{3} \right] \\ &= \frac{3}{2} - \frac{2}{3} \\ &= \frac{5}{6} \end{aligned}$$

$$ar(OACO) = ar(OLAC) - ar(OLAO)$$

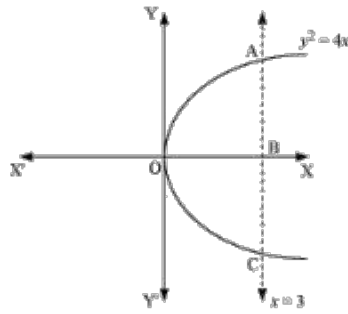
$$\begin{aligned} &= \int_{-1}^0 \frac{x+2}{4} dx - \int_{-1}^0 \frac{x^2}{4} dx \\ &= \frac{1}{4} \left[ \frac{x^2}{2} + 2x \right]_{-1}^0 - \frac{1}{4} \left[ \frac{x^3}{3} \right]_{-1}^0 \\ &= -\frac{1}{4} \left[ \frac{(-1)^2}{2} + 2(-1) \right] - \left[ -\frac{1}{4} \left( \frac{(-1)^3}{3} \right) \right] \\ &= -\frac{1}{4} \left[ \frac{1}{2} - 2 \right] - \frac{1}{12} \\ &= -\frac{1}{8} + \frac{1}{2} - \frac{1}{12} \\ &= \frac{7}{24} \end{aligned}$$

$$\text{Required area} = \left( \frac{5}{6} + \frac{7}{24} \right) = \frac{9}{8} \text{ units.}$$

### Question 11:

Find the area of the region bounded by the curve  $y^2 = 4x$  and the line  $x = 3$ .

**Solution:**



OACO is symmetrical about  $x$ -axis.

Therefore,  $ar(OACO) = 2 \times ar(AOB)$

$$\begin{aligned} ar(OACO) &= 2 \left[ \int_0^3 y dx \right] \\ &= 2 \left[ \int_0^3 2\sqrt{x} dx \right] \\ &= 4 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^3 \\ &= \frac{8}{3} \left[ (3)^{\frac{3}{2}} \right] \\ &= 8\sqrt{3} \end{aligned}$$

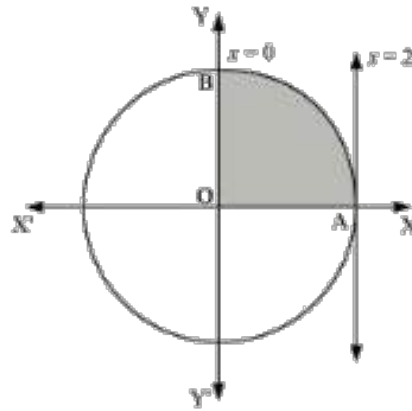
Required area is  $8\sqrt{3}$  units.

### Question 12:

Area lying in the first quadrant and bounded by the circle  $x^2 + y^2 = 4$  and the lines  $x = 0$  and  $x = 2$  is

- (A)  $\pi$                       (B)  $\frac{\pi}{2}$                       (C)  $\frac{\pi}{3}$                       (D)  $\frac{\pi}{4}$

**Solution:**



$$\begin{aligned}
 \text{ar}(OAB) &= \int_0^2 y dx \\
 &= \int_0^2 \sqrt{4-x^2} dx \\
 &= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\
 &= 2 \left( \frac{\pi}{2} \right) \\
 &= \pi
 \end{aligned}$$

Correct answer is A.



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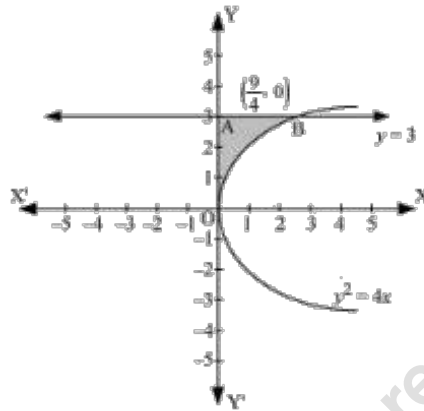
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**Question 13:**

Area of the region bounded by the curve  $y^2 = 4x$ ,  $y$ -axis and the line  $y = 3$  is

- (A) 2                      (B)  $\frac{9}{4}$                       (C)  $\frac{9}{3}$                       (D)  $\frac{9}{2}$

**Solution:**



$$\begin{aligned} \text{ar}(OAB) &= \int_0^3 x dy \\ &= \int_0^3 \frac{y^2}{4} dy \\ &= \frac{1}{4} \left[ \frac{y^3}{3} \right]_0^3 \\ &= \frac{1}{12} (27) \\ &= \frac{9}{4} \end{aligned}$$

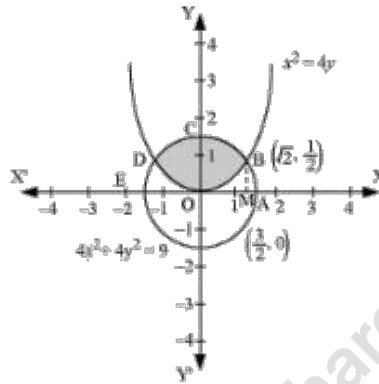
Correct answer is B.

## EXERCISE 8.2

### Question 1:

Find the area of the circle  $4x^2 + 4y^2 = 9$  which is interior to the parabola  $x^2 = 4y$ .

### Solution:



Solving  $4x^2 + 4y^2 = 9$  and  $x^2 = 4y$ , point of intersection  $B\left(\sqrt{2}, \frac{1}{2}\right)$  and  $D\left(-\sqrt{2}, \frac{1}{2}\right)$ .  
Required area is symmetrical about y-axis.

$$ar(OBCDO) = 2 \times ar(OBCO)$$

Draw BM perpendicular to OA

Coordinates of M are  $(\sqrt{2}, 0)$

$$ar(OBCO) = ar(OMBCO) - ar(OMBO)$$

$$\begin{aligned} &= \int_0^{\sqrt{2}} \sqrt{\frac{(9-4x^2)}{4}} dx - \int_0^{\sqrt{2}} \frac{x^2}{4} dx \\ &= \frac{1}{2} \int_0^{\sqrt{2}} \sqrt{9-4x^2} dx - \frac{1}{4} \int_0^{\sqrt{2}} x^2 dx \\ &= \frac{1}{4} \left[ x\sqrt{9-4x^2} + \frac{9}{2} \sin^{-1} \frac{2x}{3} \right]_0^{\sqrt{2}} - \frac{1}{4} \left[ \frac{x^3}{3} \right]_0^{\sqrt{2}} \\ &= \frac{1}{4} \left[ \sqrt{2}\sqrt{9-8} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right] - \frac{1}{12} (\sqrt{2})^3 \\ &= \frac{\sqrt{2}}{4} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6} \\ &= \frac{\sqrt{2}}{12} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} \\ &= \frac{1}{4} \left( \frac{\sqrt{2}}{3} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right) \end{aligned}$$

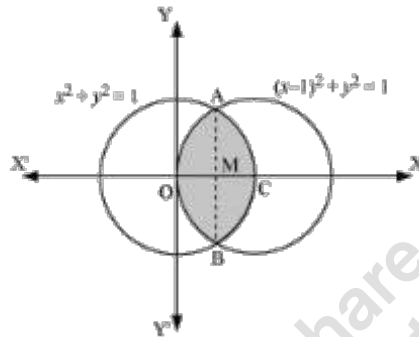
Required area OBCDO

$$= \left( 2 \times \frac{1}{4} \left[ \frac{\sqrt{2}}{3} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \right) = \left[ \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \text{ units.}$$

### Question 2:

Find the area bounded by curves  $(x-1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$ .

**Solution:**



Solving  $(x-1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$ , point of intersection  $A\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  and  $B\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$   
Required area is symmetrical about  $x$ -axis.

$$ar(OBCAO) = 2 \times ar(OCAO)$$

Join AB, intersects OC at M

AM is perpendicular to OC

Coordinates of  $M\left(\frac{1}{2}, 0\right)$

$$\begin{aligned}
 ar(OCAO) &= ar(OMAO) + ar(MCAM) \\
 &= \left[ \int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2} dx + \int_{\frac{1}{2}}^1 \sqrt{1-x^2} dx \right] \\
 &= \left[ \frac{x-1}{2} \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1}(x-1) \right]_0^{\frac{1}{2}} + \left[ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_{\frac{1}{2}}^1 \\
 &= \left[ -\frac{1}{4} \sqrt{1-\left(\frac{1}{2}\right)^2} + \frac{1}{2} \sin^{-1}\left(\frac{1}{2}-1\right) - \frac{1}{2} \sin^{-1}(-1) \right] + \left[ +\frac{1}{2} \sin^{-1}(-1) - \frac{1}{4} \sqrt{1-\left(\frac{1}{2}\right)^2} - \frac{1}{2} \sin^{-1}\left(\frac{1}{2}\right) \right] \\
 &= \left[ -\frac{\sqrt{3}}{8} + \frac{1}{2} \left(-\frac{\pi}{6}\right) - \frac{1}{2} \left(-\frac{\pi}{2}\right) \right] + \left[ \frac{1}{2} \left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} - \frac{1}{2} \left(\frac{\pi}{6}\right) \right] \\
 &= \left[ -\frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12} \right] \\
 &= \left[ -\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2} \right] \\
 &= \left[ \frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right]
 \end{aligned}$$

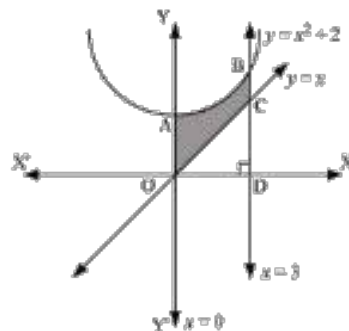
Required Area OBCAO is

$$\left( 2 \times \left[ \frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right] \right) = \left[ \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right] \text{ units.}$$

### Question 3:

Find the area of the region bounded by the curves  $y = x^2 + 2$ ,  $y = x$ ,  $x = 0$  and  $x = 3$

**Solution:**

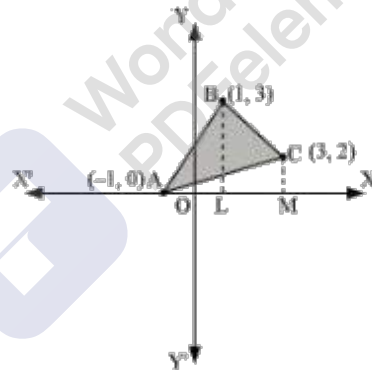


$$\begin{aligned}
 \text{ar}(OCBAO) &= \text{ar}(ODBAO) - \text{ar}(ODCO) \\
 &= \int_0^3 (x^2 + 2) dx - \int_0^3 x dx \\
 &= \left[ \frac{x^3}{3} + 2x \right]_0^3 - \left[ \frac{x^2}{2} \right]_0^3 \\
 &= [9 + 6] - \left[ \frac{9}{2} \right] \\
 &= 15 - \frac{9}{2} \\
 &= \frac{21}{2}
 \end{aligned}$$

#### Question 4:

Using integration finds the area of the region bounded by the triangle whose vertices are  $(-1, 0)$ ,  $(1, 3)$  and  $(3, 2)$ .

**Solution:**



BL and CM are perpendicular to  $x$ -axis.

$$\text{ar}(\triangle ACB) = \text{ar}(ALBA) + \text{ar}(BLMCB) - \text{ar}(AMCA)$$

Equation of AB is

$$\begin{aligned}
 y - 0 &= \frac{3 - 0}{1 - (-1)}(x + 1) \\
 y &= \frac{3}{2}(x + 1)
 \end{aligned}$$

$$\begin{aligned} \text{ar}(ALBA) &= \int_{-1}^1 \frac{3}{2}(x+1)dx \\ &= \frac{3}{2} \left[ \frac{x^2}{2} + x \right]_{-1}^1 \\ &= \frac{3}{2} \left[ \frac{1}{2} + 1 - \frac{1}{2} + 1 \right] \\ &= 3 \end{aligned}$$

Equation of BC is

$$\begin{aligned} y-3 &= \frac{2-3}{3-1}(x-1) \\ y &= \frac{1}{2}(-x+7) \end{aligned}$$

$$\begin{aligned} \text{ar}(BLMCB) &= \int_1^3 \frac{1}{2}(-x+7)dx \\ &= \frac{1}{2} \left[ -\frac{x^2}{2} + 7x \right]_1^3 \\ &= \frac{1}{2} \left[ -\frac{9}{2} + 21 + \frac{1}{2} - 7 \right] \\ &= 5 \end{aligned}$$

Equation of AC is

$$\begin{aligned} y-0 &= \frac{2-0}{3+1}(x+1) \\ y &= \frac{1}{2}(x+1) \end{aligned}$$

$$\begin{aligned} \text{ar}(AMCA) &= \frac{1}{2} \int_{-1}^3 (x+1)dx \\ &= \frac{1}{2} \left[ \frac{x^2}{2} + x \right]_{-1}^3 \\ &= \frac{1}{2} \left[ \frac{9}{2} + 3 - \frac{1}{2} + 1 \right] \\ &= 4 \end{aligned}$$

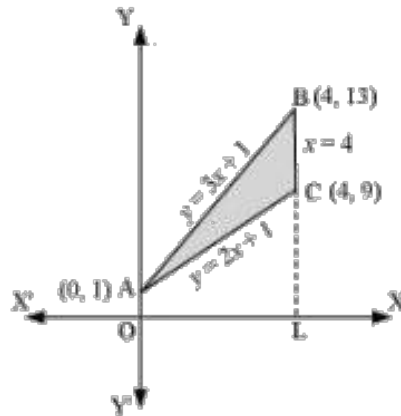
Therefore,  $\text{ar}(\triangle ABC) = (3+5-4) = 4 \text{ units}$

### Question 5:

Using integration find the area of the triangular region whose sides have the equations  $y = 2x+1$ ,  $y = 3x+1$  and  $x = 4$ .

**Solution:**

Vertices of triangle are  $A(0,1)$ ,  $B(4,13)$  and  $C(4,9)$ .



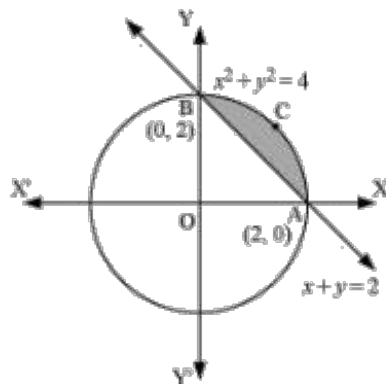
$$\begin{aligned} \text{ar}(\triangle ACB) &= \text{ar}(OLBAO) - \text{ar}(OLCAO) \\ &= \int_0^4 (3x+1) dx - \int_0^4 (2x+1) dx \\ &= \left[ \frac{3x^2}{2} + x \right]_0^4 - \left[ \frac{2x^2}{2} + x \right]_0^4 \\ &= (24+4) - (16+4) \\ &= 28 - 20 \\ &= 8 \end{aligned}$$

**Question 6:**

Smaller area enclosed by the circle  $x^2 + y^2 = 4$  and the line  $x + y = 2$  is

- (A)  $2(\pi - 2)$       (B)  $\pi - 2$       (C)  $2\pi - 1$       (D)  $2(\pi + 2)$

**Solution:**



$$\begin{aligned}
 \text{ar}(ACBA) &= \text{ar}(OACBO) - \text{ar}(\Delta OAB) \\
 &= \int_0^2 \sqrt{4-x^2} dx - \int_0^2 (2-x) dx \\
 &= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[ 2x - \frac{x^2}{2} \right]_0^2 \\
 &= \left[ 2 \times \frac{\pi}{2} \right] - [4-2] \\
 &= (\pi - 2)
 \end{aligned}$$

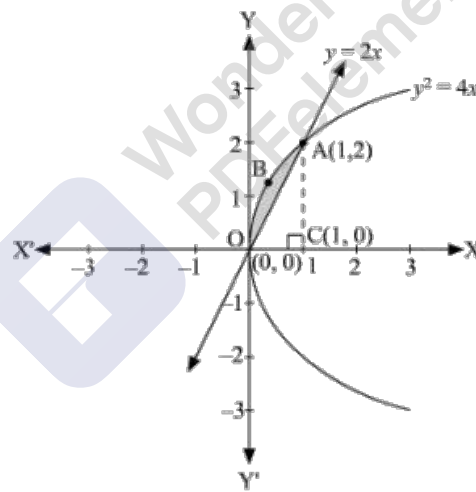
Correct answer is B.

### Question 7:

Area lying between the curve  $y^2 = 4x$  and  $y = 2x$  is

- (A)  $\frac{2}{3}$                       (B)  $\frac{1}{3}$                       (C)  $\frac{1}{4}$                       (D)  $\frac{3}{4}$

**Solution:**



Points of intersection of curve  $y^2 = 4x$  and  $y = 2x$  are  $O(0,0)$  and  $A(1,2)$ .

Draw AC perpendicular to x-axis.

Coordinates of C are (1,0)



$$\begin{aligned}ar(OBAO) &= ar(\Delta OCA) - ar(OCABO) \\&= \int_0^1 2x dx - \int_0^1 2\sqrt{x} dx \\&= 2 \left[ \frac{x^2}{2} \right]_0^1 - 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 \\&= \left| 1 - \frac{4}{3} \right| \\&= \left| -\frac{1}{3} \right| \\&= \frac{1}{3}\end{aligned}$$

Correct answer is B.



## MISCELLANEOUS EXERCISE

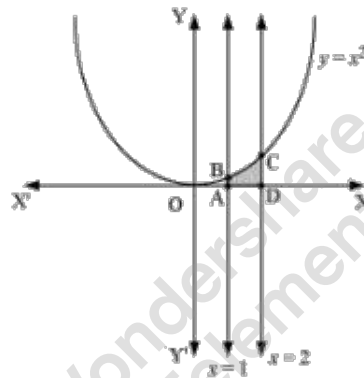
### Question 1:

Find the area under the given curves and given lines:

- (i)  $y = x^2, x = 1, x = 2$  and  $x$ -axis
- (ii)  $y = x^4, x = 1, x = 5$  and  $x$ -axis

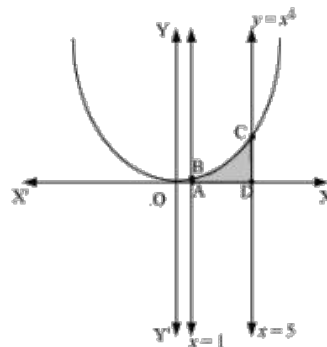
### Solution:

- (i)  $y = x^2, x = 1, x = 2$  and  $x$ -axis



$$\begin{aligned}
 \text{ar}(ADCBA) &= \int_1^2 y dx \\
 &= \int_1^2 x^2 dx \\
 &= \left[ \frac{x^3}{3} \right]_1^2 \\
 &= \frac{8}{3} - \frac{1}{3} \\
 &= \frac{7}{3}
 \end{aligned}$$

- (ii)  $y = x^4, x = 1, x = 5$  and  $x$ -axis

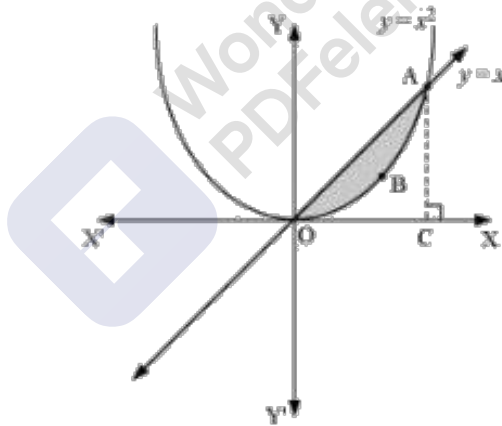


$$\begin{aligned}
 \text{ar}(ADCBA) &= \int_1^5 y dx \\
 &= \int_1^5 x^4 dx \\
 &= \left[ \frac{x^5}{5} \right]_1^5 \\
 &= \frac{(5)^5}{5} - \frac{1}{5} \\
 &= (5)^4 - \frac{1}{5} \\
 &= 625 - \frac{1}{5} \\
 &= 624.8
 \end{aligned}$$

### Question 2:

Find the area between the curves  $y = x$  and  $y = x^2$ .

### Solution:



Point of intersection of  $y = x$  and  $y = x^2$  is A (1,1).

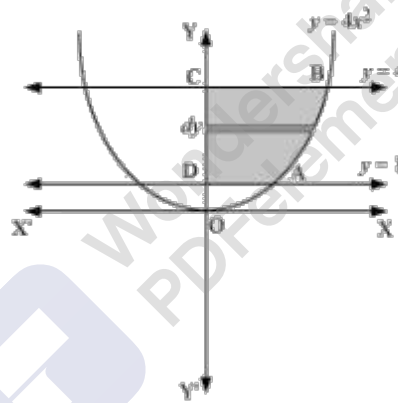
Draw AC perpendicular to x-axis.

$$\begin{aligned}
 ar(OBAO) &= ar(\Delta OCA) - ar(OCABO) \\
 &= \int_0^1 x dx - \int_0^1 x^2 dx \\
 &= \left[ \frac{x^2}{2} \right]_0^1 - \left[ \frac{x^3}{3} \right]_0^1 \\
 &= \frac{1}{2} - \frac{1}{3} \\
 &= \frac{1}{6}
 \end{aligned}$$

### Question 3:

Find the area of the region lying in the first quadrant and bounded by  $y = 4x^2$ ,  $x = 0$ ,  $y = 1$  and  $y = 4$ .

**Solution:**



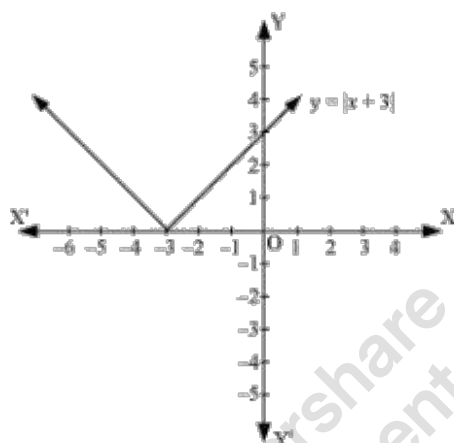
$$\begin{aligned}
 ar(ABCD) &= \int_1^4 x dy \\
 &= \int_1^4 \frac{\sqrt{y}}{2} dy \\
 &= \frac{1}{2} \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\
 &= \frac{1}{3} \left[ (4)^{\frac{3}{2}} - 1 \right] \\
 &= \frac{1}{3} [8 - 1] \\
 &= \frac{7}{3}
 \end{aligned}$$

#### Question 4:

Sketch the graph of  $y = |x+3|$  and evaluate  $\int_{-6}^0 |x+3| dx$ .

#### Solution:

|   |    |    |    |    |    |    |   |
|---|----|----|----|----|----|----|---|
| X | -6 | -5 | -4 | -3 | -2 | -1 | 0 |
| Y | 3  | 2  | 1  | 0  | 1  | 2  | 3 |



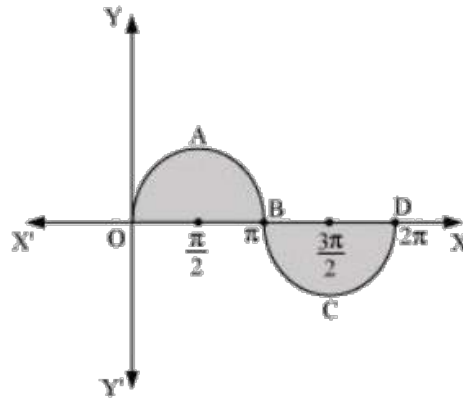
$(x+3) \leq 0$  for  $-6 \leq x \leq -3$  and  $(x+3) \geq 0$  for  $-3 \leq x \leq 0$

$$\begin{aligned}
 \int_{-6}^0 |(x+3)| dx &= -\int_{-6}^{-3} (x+3) dx + \int_{-3}^0 (x+3) dx \\
 &= -\left[ \frac{x^2}{2} + 3x \right]_{-6}^{-3} + \left[ \frac{x^2}{2} + 3x \right]_{-3}^0 \\
 &= -\left[ \left( \frac{(-3)^2}{2} + 3(-3) \right) - \left( \frac{(-6)^2}{2} + 3(-6) \right) \right] + \left[ 0 - \left( \frac{(-3)^2}{2} + 3(-3) \right) \right] \\
 &= -\left[ -\frac{9}{2} \right] - \left[ -\frac{9}{2} \right] \\
 &= 9
 \end{aligned}$$

#### Question 5:

Find the area bounded by the curve  $y = \sin x$  between  $x = 0$  and  $x = 2\pi$ .

**Solution:**



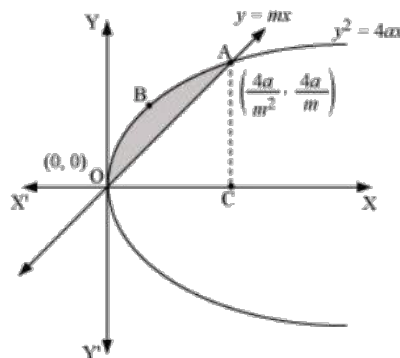
Area bounded by the curve = Area OABO + Area BCDB

$$\begin{aligned} ar(OABO) + ar(BCDB) &= \int_0^{\pi} \sin x dx + \left| \int_{\pi}^{2\pi} \sin x dx \right| \\ &= [-\cos x]_0^{\pi} + \left| [-\cos x]_{\pi}^{2\pi} \right| \\ &= [-\cos \pi + \cos 0] + |-\cos 2\pi + \cos \pi| \\ &= 1 + 1 + |(-1 - 1)| \\ &= 2 + |-2| \\ &= 2 + 2 \\ &= 4 \end{aligned}$$

### Question 6:

Find the area enclosed between the parabola  $y^2 = 4ax$  and the line  $y = mx$ .

**Solution:**



Points of intersection of curves are  $(0,0)$  and  $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$   
Draw AC perpendicular to x-axis.

$$ar(OABO) = ar(OCABO) - ar(\Delta OCA)$$

$$= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax} dx - \int_0^{\frac{4a}{m^2}} mx dx$$

$$= 2\sqrt{a} \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{4a}{m^2}} - m \left[ \frac{x^2}{2} \right]_0^{\frac{4a}{m^2}}$$

$$= \frac{4}{3} \sqrt{a} \left( \frac{4a}{m^2} \right)^{\frac{3}{2}} - \frac{m}{2} \left[ \left( \frac{4a}{m^2} \right)^2 \right]$$

$$= \frac{32a^2}{3m^3} - \frac{m}{2} \left( \frac{16a^2}{m^4} \right)$$

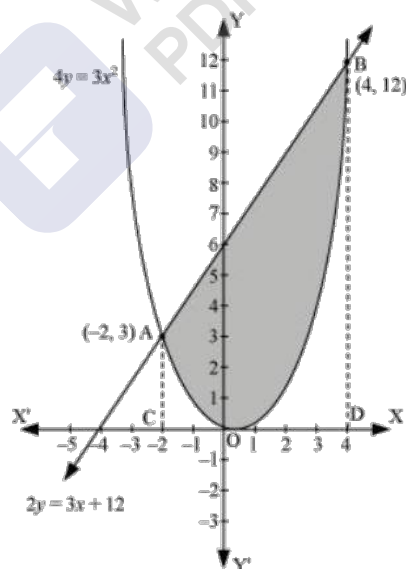
$$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3}$$

$$= \frac{8a^2}{3m^3}$$

### Question 7:

Find the area enclosed by the parabola  $4y = 3x^2$  and the line  $2y = 3x + 12$ .

**Solution:**



Points of intersection of curves are  $A(-2, 3)$  and  $B(4, 12)$ .  
Draw AC and BD perpendicular to x-axis.

$$ar(OBAO) = ar(CDBA) - ar(ODBO + OACO)$$

$$= \int_{-2}^4 \frac{1}{2}(3x+12)dx - \int_{-2}^4 \frac{3x^2}{4}dx$$

$$= \frac{1}{2} \left[ \frac{3x^2}{2} + 12x \right]_{-2}^4 - \frac{3}{4} \left[ \frac{x^3}{3} \right]_{-2}^4$$

$$= \frac{1}{2} [24 + 48 - 6 + 24] - \frac{1}{4} [64 + 8]$$

$$= \frac{1}{2} [90] - \frac{1}{4} [72]$$

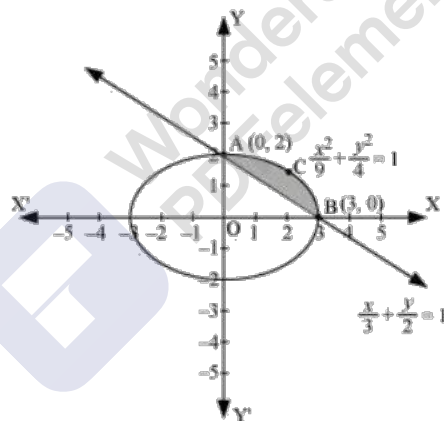
$$= 45 - 18$$

$$= 27$$

### Question 8:

Find the area of the smaller region bounded by the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the line  $\frac{x}{3} + \frac{y}{2} = 1$ .

**Solution:**



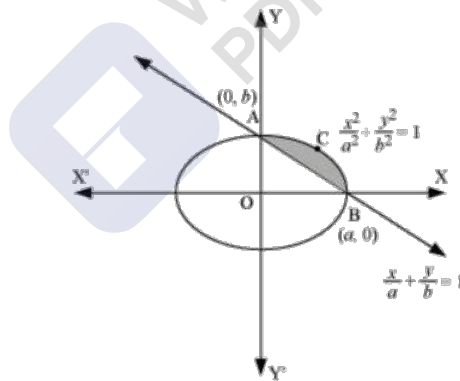


$$\begin{aligned}
 \text{ar}(BCAB) &= \text{ar}(OBCAO) - \text{ar}(OBAO) \\
 &= \int_0^3 2\sqrt{1-\frac{x^2}{9}} dx - \int_0^3 2\left(1-\frac{x}{3}\right) dx \\
 &= \frac{2}{3} \left[ \int_0^3 \sqrt{9-x^2} dx \right] - \frac{2}{3} \int_0^3 (3-x) dx \\
 &= \frac{2}{3} \left[ \frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3 - \frac{2}{3} \left[ 3x - \frac{x^2}{2} \right]_0^3 \\
 &= \frac{2}{3} \left[ \frac{9}{2} \left( \frac{\pi}{2} \right) \right] - \frac{2}{3} \left[ 9 - \frac{9}{2} \right] \\
 &= \frac{2}{3} \left[ \frac{9\pi}{4} - \frac{9}{2} \right] \\
 &= \frac{2}{3} \times \frac{9}{4} (\pi - 2) \\
 &= \frac{3}{2} (\pi - 2)
 \end{aligned}$$

### Question 9:

Find the area of the smaller region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the line  $\frac{x}{a} + \frac{y}{b} = 1$ .

**Solution:**



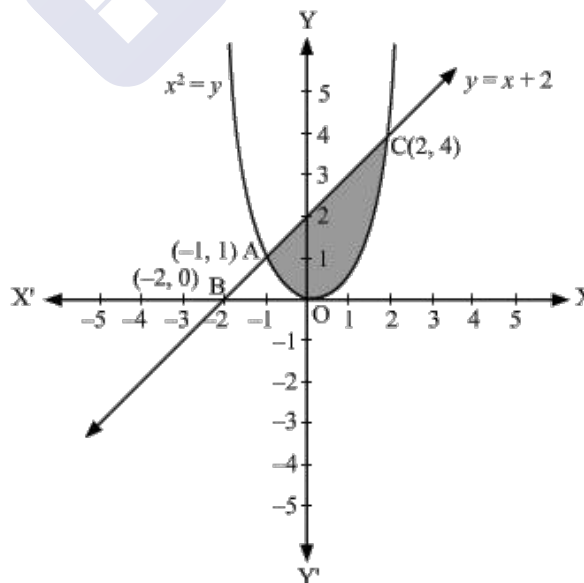
$$\begin{aligned}
 \text{ar}(CBA) &= \text{ar}(OBCAO) - \text{ar}(OBAO) \\
 &= \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx - \int_0^a b \left(1 - \frac{x}{a}\right) dx \\
 &= \frac{b}{a} \left[ \left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\}_0^a - \left\{ ax - \frac{x^2}{2} \right\}_0^a \right] \\
 &= \frac{b}{a} \left[ \left\{ \frac{a^2}{2} \left( \frac{\pi}{2} \right) \right\} - \left\{ a^2 - \frac{a^2}{2} \right\} \right] \\
 &= \frac{b}{a} \left[ \frac{a^2 \pi}{4} - \frac{a^2}{2} \right] \\
 &= \frac{ba^2}{2a} \left[ \frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{2} \left[ \frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{4} (\pi - 2)
 \end{aligned}$$

### Question 10:

Find the area of the region enclosed by the parabola  $x^2 = y$ , the line  $y = x + 2$  and x-axis.

### Solution:

Point of intersection of  $x^2 = y$  and  $y = x + 2$ , is  $A(-1, 1)$  and  $C(2, 4)$ .



Now required Area = Area of trapezium ALMB- Area of ALODBM

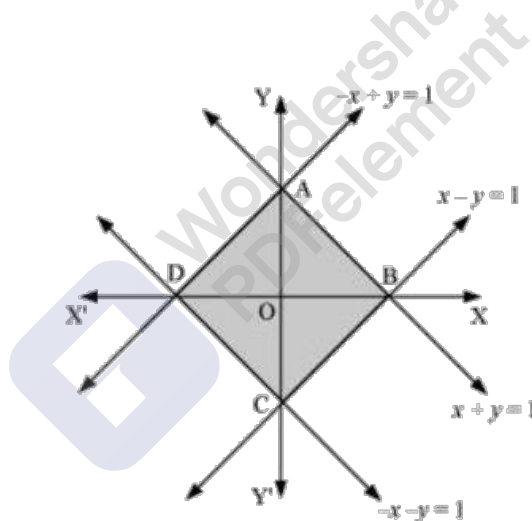
$$\begin{aligned} ar(trap.ALMB) - ar(ALODBM) &= \int_{-1}^2 (x+2) dx - \int_{-1}^2 x^2 dx \\ &= \left[ \frac{x^2}{2} + 2x \right]_{-1}^2 - \left[ \frac{x^3}{3} \right]_{-1}^2 \\ &= \left[ 2 + 4 - \frac{1}{2} + 2 \right] - \left[ \frac{8}{3} + \frac{1}{3} \right] \\ &= \frac{15}{2} - 3 \\ &= \frac{9}{2} \end{aligned}$$

### Question 11:

Using the method of integration find the area bounded by the curve  $|x| + |y| = 1$

[Hint: The required region is bounded by lines  $x + y = 1, x - y = 1, -x + y = 1$  and  $-x - y = 1$ ]

**Solution:**



Curve intersects axis at points  $A(0,1), B(1,0), C(0,-1)$  and  $D(-1,0)$ .

Curve is symmetrical about  $x$ -axis and  $y$ -axis.

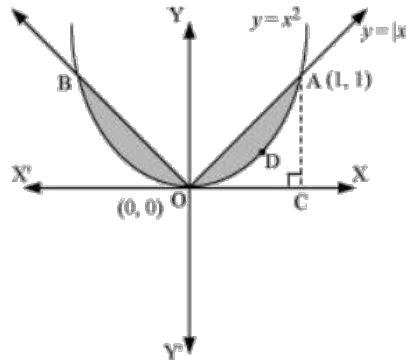
$$ar(ADCB) = 4 \times ar(OBAO)$$

$$\begin{aligned} &= 4 \int_0^1 (1-x) dx \\ &= 4 \left( x - \frac{x^2}{2} \right)_0^1 \\ &= 4 \left[ 1 - \frac{1}{2} \right] \\ &= 2 \end{aligned}$$

### Question 12:

Find the area bounded by curves  $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$

**Solution:**



Required area is symmetrical about y-axis.

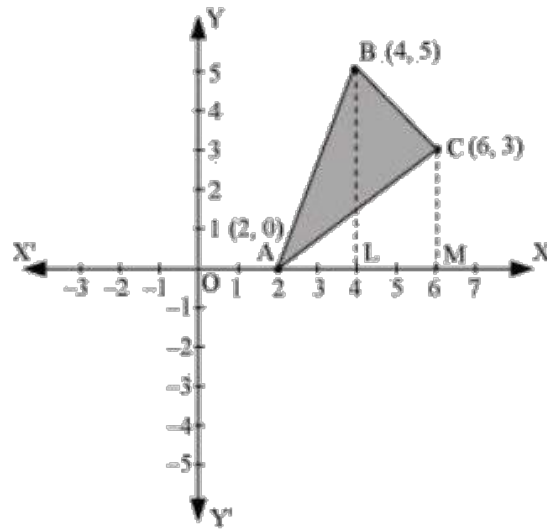
Required area =  $2[\text{Area (OCAO)} - \text{Area (OCADO)}]$

$$\begin{aligned} 2[\text{ar}(\text{OCAO}) - \text{ar}(\text{OCADO})] &= 2 \left[ \int_0^1 x dx - \int_0^1 x^2 dx \right] \\ &= 2 \left[ \left[ \frac{x^2}{2} \right]_0^1 - \left[ \frac{x^3}{3} \right]_0^1 \right] \\ &= 2 \left[ \frac{1}{2} - \frac{1}{3} \right] \\ &= 2 \left[ \frac{1}{6} \right] \\ &= \frac{1}{3} \end{aligned}$$

### Question 13:

Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are  $A(2,0)$ ,  $B(4,5)$  and  $C(6,3)$ .

**Solution:**



Equation of AB is

$$\begin{aligned} y-0 &= \frac{5-0}{4-2}(x-2) \\ 2y &= 5x-10 \\ y &= \frac{5}{2}(x-2) \end{aligned}$$

Equation of BC is

$$\begin{aligned} y-5 &= \frac{3-5}{6-4}(x-4) \\ 2y-10 &= -2x+8 \\ 2y &= -2x+18 \\ y &= -x+9 \end{aligned}$$

Equation of CA is

$$\begin{aligned} y-3 &= \frac{0-3}{2-6}(x-6) \\ -4y+12 &= -3x+18 \\ 4y &= 3x-6 \\ y &= \frac{3}{4}(x-2) \end{aligned}$$

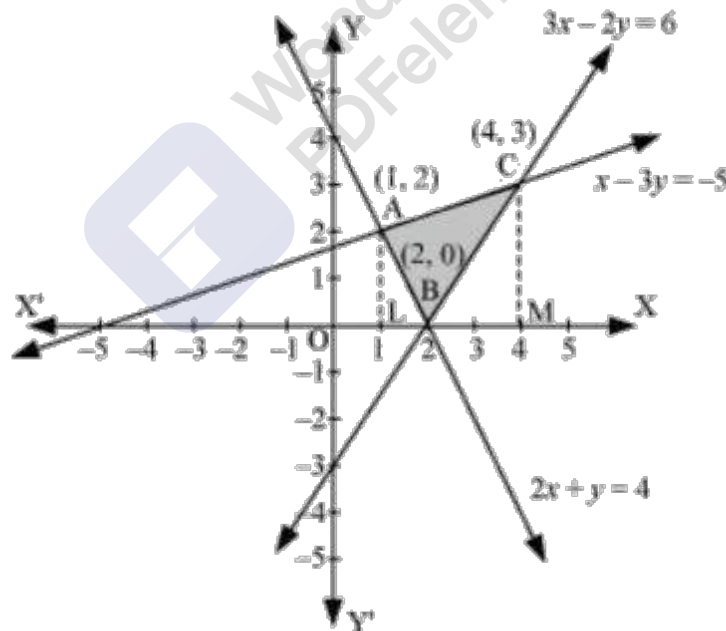
$$\begin{aligned}
 \text{ar}(\Delta ABC) &= \text{ar}(ABLA) + \text{ar}(BLMCB) - \text{ar}(ACMA) \\
 &= \int_2^4 \frac{5}{2}(x-2)dx + \int_4^6 (-x+9)dx - \int_2^6 \frac{3}{4}(x-2)dx \\
 &= \frac{5}{2} \left[ \frac{x^2}{2} - 2x \right]_2^4 + \left[ \frac{-x^2}{2} + 9x \right]_4^6 - \frac{3}{4} \left[ \frac{x^2}{2} - 2x \right]_2^6 \\
 &= \frac{5}{2} [8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4} [18 - 12 - 2 + 4] \\
 &= 5 + 8 - \frac{3}{4}(8) \\
 &= 13 - 6 \\
 &= 7 \text{ units.}
 \end{aligned}$$

#### Question 14:

Using the method of integration find the area of the region bounded by lines:

$2x + y = 4$ ,  $3x - 2y = 6$  and  $x - 3y + 5 = 0$ .

**Solution:**



AL and CM are perpendicular on x-axis.

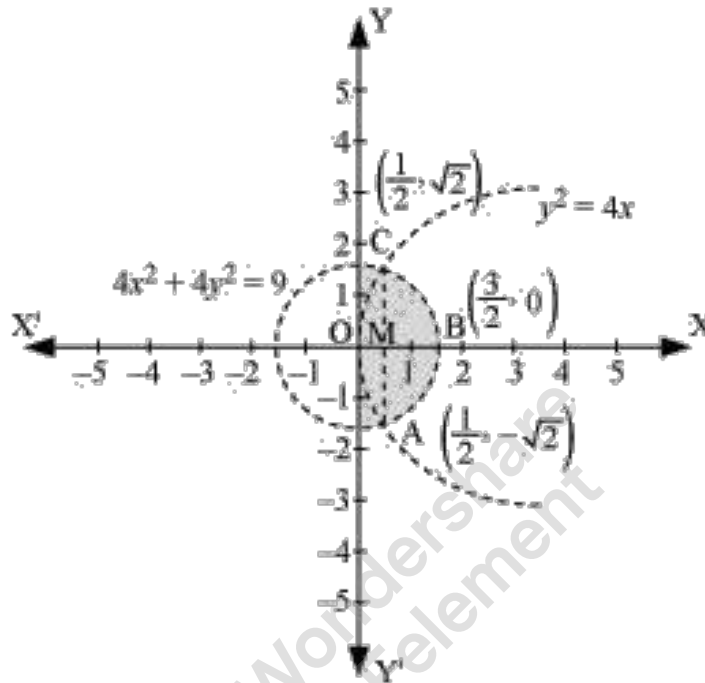
$$\begin{aligned}
 ar(\triangle ABC) &= ar(AMCA) - ar(ALB) - ar(CMB) \\
 &= \int_1^4 \left( \frac{x+5}{3} \right) dx - \int_1^2 (4-2x) dx - \int_2^4 \left( \frac{3x-6}{2} \right) dx \\
 &= \frac{1}{3} \left[ \frac{x^2}{2} + 5x \right]_1^4 - \left[ 4x - x^2 \right]_1^2 - \frac{1}{2} \left[ \frac{3x^2}{2} - 6x \right]_2^4 \\
 &= \frac{1}{3} \left[ 8 + 20 - \frac{1}{2} - 5 \right] - [8 - 4 - 4 + 1] - \frac{1}{2} [24 - 24 - 6 + 12] \\
 &= \left( \frac{1}{3} \times \frac{45}{2} \right) - (1) - \frac{1}{2} (6) \\
 &= \frac{15}{2} - 1 - 3 \\
 &= \frac{15}{2} - 4 \\
 &= \frac{15-8}{2} \\
 &= \frac{7}{2}
 \end{aligned}$$



### Question 15:

Find the area of the region  $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

**Solution:**



Points of intersection of curves are  $\left(\frac{1}{2}, \sqrt{2}\right)$  and  $\left(\frac{1}{2}, -\sqrt{2}\right)$ .

Required area is OABCO.

Area OABCO is symmetrical about x-axis.

$$\text{Area OABCO} = 2 \times \text{Area OBC}$$

$$\text{ar}(OBCO) = \text{ar}(OMC) + \text{ar}(MBC)$$

$$= \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{9 - 4x^2} dx$$

$$= \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{(3)^2 - (2x)^2} dx$$

$$\text{put } 2x = t \Rightarrow dx = \frac{dt}{2}$$

$$\text{When } x = \frac{3}{2}, t = 3 \text{ and when } x = \frac{1}{2}, t = 1$$

$$\begin{aligned} \text{ar}(OBCO) &= \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \frac{1}{4} \int_1^3 \sqrt{(3)^2 - (t)^2} dt \\ &= 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}} + \frac{1}{4} \left[ \frac{t}{2} \sqrt{9-t^2} + \frac{9}{2} \sin^{-1} \left( \frac{t}{3} \right) \right]_1^3 \\ &= 2 \left[ \frac{2}{3} \left( \frac{1}{2} \right)^{\frac{3}{2}} \right] + \frac{1}{4} \left[ \left\{ \frac{3}{2} \sqrt{9-(3)^2} + \frac{9}{2} \sin^{-1} \left( \frac{3}{3} \right) \right\} - \left\{ \frac{1}{2} \sqrt{9-(1)^2} + \frac{9}{2} \sin^{-1} \left( \frac{1}{3} \right) \right\} \right] \\ &= \frac{2}{3\sqrt{2}} + \frac{1}{4} \left[ \left\{ 0 + \frac{9}{2} \sin^{-1}(1) \right\} - \left\{ \frac{1}{2} \sqrt{8} + \frac{9}{2} \sin^{-1} \left( \frac{1}{3} \right) \right\} \right] \\ &= \frac{\sqrt{2}}{3} + \frac{1}{4} \left[ \frac{9\pi}{4} - \sqrt{2} - \frac{9}{2} \sin^{-1} \left( \frac{1}{3} \right) \right] \\ &= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \sqrt{2} - \frac{9}{8} \sin^{-1} \left( \frac{1}{3} \right) \\ &= \frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \left( \frac{1}{3} \right) + \frac{\sqrt{2}}{12} \end{aligned}$$

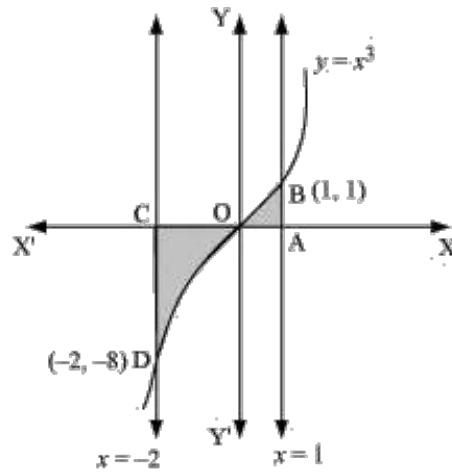
$$\begin{aligned} \text{ar}(OABCO) &= 2 \times \text{ar}(OBC) \\ &= 2 \times \frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \left( \frac{1}{3} \right) + \frac{\sqrt{2}}{12} \\ &= \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left( \frac{1}{3} \right) + \frac{\sqrt{2}}{6} \\ &= \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left( \frac{1}{3} \right) + \frac{1}{3\sqrt{2}} \end{aligned}$$

### Question 16:

Area bounded by the curve  $y = x^3$ , the  $x$ -axis and the coordinates  $x = -2$  and  $x = 1$  is

- (A)  $-9$                       (B)  $-\frac{15}{4}$                       (C)  $\frac{15}{4}$                       (D)  $\frac{17}{4}$

**Solution:**



$$\begin{aligned}
 \text{required area} &= \int_{-2}^0 y dx + \int_0^1 y dx \\
 &= \int_{-2}^0 x^3 dx + \int_0^1 x^3 dx \\
 &= \left[ \frac{x^4}{4} \right]_{-2}^0 + \left[ \frac{x^4}{4} \right]_0^1 \\
 &= \left[ \frac{(-2)^4}{4} + \frac{1}{4} \right] \\
 &= \left( 4 + \frac{1}{4} \right) = \frac{17}{4}
 \end{aligned}$$

Correct answer is D

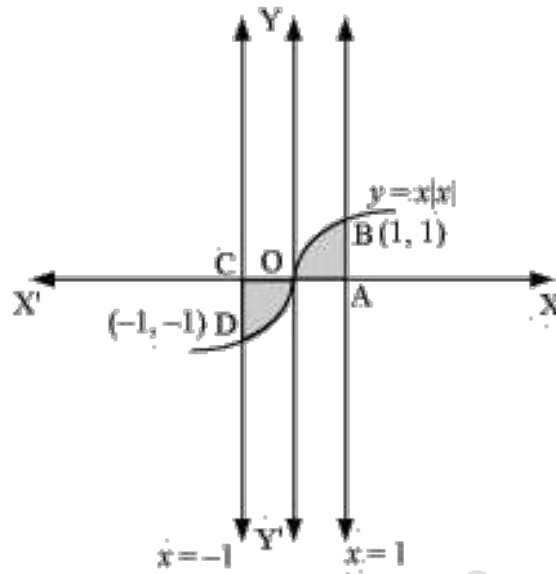
### Question17:

The area bounded by the curve  $y = x|x|$ ,  $x$ -axis and the coordinates  $x = -1$  and  $x = 1$  is given by

[Hint:  $y = x^2$  if  $x > 0$  and  $y = -x^2$  if  $x < 0$ ]

- (A) 0                      (B)  $\frac{1}{3}$                       (C)  $\frac{2}{3}$                       (D)  $\frac{4}{3}$

**Solution:**



$$\begin{aligned}
 \text{required area} &= \int_{-1}^1 y dx \\
 &= \int_{-1}^1 x|x| dx \\
 &= \int_{-1}^0 x^2 dx + \int_0^1 x^2 dx \\
 &= \left[ \frac{x^3}{3} \right]_{-1}^0 + \left[ \frac{x^3}{3} \right]_0^1 \\
 &= -\left( -\frac{1}{3} \right) + \frac{1}{3} \\
 &= \frac{2}{3}
 \end{aligned}$$

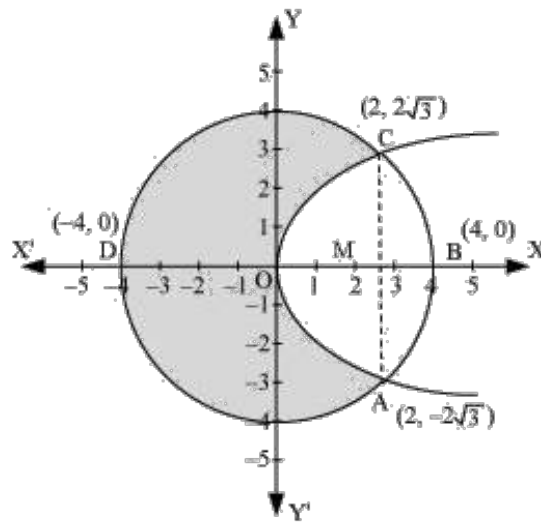
Correct answer is C.

### Question 18:

The area of the circle  $x^2 + y^2 = 16$  exterior to the parabola  $y^2 = 6x$ .

- (A)  $\frac{4}{3}(4\pi - \sqrt{3})$       (B)  $\frac{4}{3}(4\pi + \sqrt{3})$       (C)  $\frac{4}{3}(8\pi - \sqrt{3})$       (D)  $\frac{4}{3}(8\pi + \sqrt{3})$

**Solution:**



Required area = 2[ Area (OADO) + Area (ADBA)]

$$\begin{aligned}
 2[ar(OADO) + ar(ADBA)] &= 2\left[\int_0^2 \sqrt{6x} dx + \int_2^4 \sqrt{16-x^2} dx\right] \\
 &= 2\int_0^2 \sqrt{6x} dx + 2\int_2^4 \sqrt{16-x^2} dx \\
 &= 2\sqrt{6}\int_0^2 \sqrt{x} dx + 2\int_2^4 \sqrt{16-x^2} dx \\
 &= 2\sqrt{6} \times \frac{2}{3} \left[x^{\frac{3}{2}}\right]_0^2 + 2\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_2^4 \\
 &= \frac{4\sqrt{6}}{3}(2\sqrt{2}-0) + 2\left[\left\{0+8\sin^{-1}(1)\right\}-\left\{2\sqrt{3}+8\sin^{-1}\left(\frac{1}{2}\right)\right\}\right] \\
 &= \frac{16\sqrt{3}}{3} + 2\left[8 \times \frac{\pi}{2} - 2\sqrt{3} - 8 \times \frac{\pi}{6}\right] \\
 &= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8\pi}{3} \\
 &= \frac{16\sqrt{3} + 24\pi - 12\sqrt{3} - 8\pi}{3} \\
 &= \frac{4\sqrt{3} + 16\pi}{3} \\
 &= \frac{4}{3}[4\pi + \sqrt{3}] \text{ units}
 \end{aligned}$$

$$\text{Area of circle} = \pi (r)^2$$

$$= \pi (4)^2$$

$$= 16\pi$$

$$\text{required area} = 16\pi - \frac{4}{3} [4\pi + \sqrt{3}]$$

$$= \frac{4}{3} [4 \times 3\pi - 4\pi - \sqrt{3}]$$

$$= \frac{4}{3} (8\pi - \sqrt{3})$$

Correct answer is C.

### Question 19:

The area bounded by the  $y$ -axis,  $y = \cos x$  and  $y = \sin x$  when  $0 \leq x \leq \frac{\pi}{2}$ .

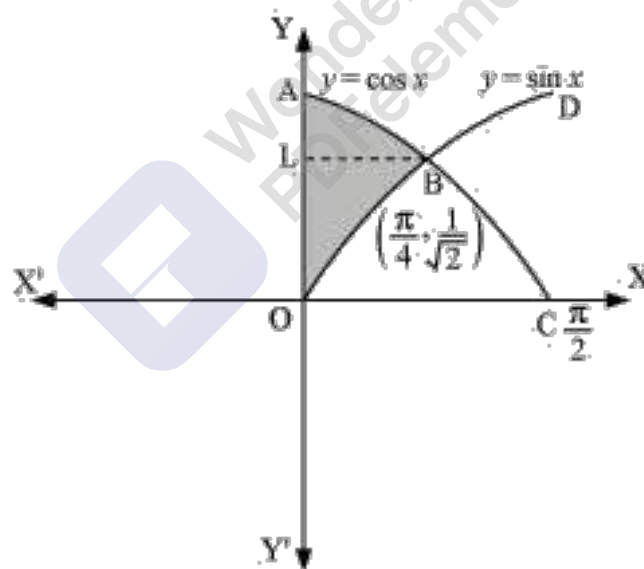
(A)  $2(\sqrt{2}-1)$

(B)  $\sqrt{2}-1$

(C)  $\sqrt{2}+1$

(D)  $\sqrt{2}$

**Solution:**



Required area = Area (ABLA) + Area (OBLO)

$$\begin{aligned}
 ar(ABLA) + ar(OBLO) &= \int_{\frac{1}{\sqrt{2}}}^1 x dy + \int_0^{\frac{1}{\sqrt{2}}} x dy \\
 &= \int_{\frac{1}{\sqrt{2}}}^1 \cos^{-1} y dy + \int_0^{\frac{1}{\sqrt{2}}} \sin^{-1} x dy \\
 &= \left[ y \cos^{-1} y - \sqrt{1-y^2} \right]_{\frac{1}{\sqrt{2}}}^1 + \left[ x \sin^{-1} x + \sqrt{1-x^2} \right]_0^{\frac{1}{\sqrt{2}}} \\
 &= \left[ \cos^{-1}(1) - \frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} \right] + \left[ \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} - 1 \right] \\
 &= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \\
 &= \frac{2}{\sqrt{2}} - 1 \\
 &= \sqrt{2} - 1
 \end{aligned}$$

Correct answer is B.

