# Day 2: Problem Solving, Data Analysis, and Additional Topics

# Ratios, Rates, and Proportions

Example:

Elena is conducting a study about the effects of toxins in the water on the hormones of fish. Elena surveys 350 male fish in a river and finds that 150 of the male fish have egg cells growing inside them. According to Elena's survey, what is the ratio of male fish with egg cells to male fish without egg cells in the river?

    A) 3:4
    B) 3:7
    C) 4:5
    D) 4:7

Practice:

A geneticist conducts a study to investigate the prevalence of a certain genetic marker in a particular population of United States adults. During the study, the geneticist observes the genetic code of 1,000 randomly selected US adults in that population and finds that 350 of those adults have the genetic marker. According to the results of the study, what is the ratio of observed adults with the genetic marker to observed adults without the genetic marker? Choose 1 answer:

    A) 7:13
    B) 13:7
    C) 7:20
    D) 13:20

# Percents:

Example:

If the 8 men on a construction crew make up 40% of the entire crew, how many people are in the crew?

A)   20
B)   22
C)   10
D)   28

Practice:

A gumball machine contains 23 green gumballs, 52 red gumballs, 34 blue gumballs, 61 yellow gumballs, and 30 pink gumballs. What percent of the gumballs are red?

(Free Response)

# Units:

Example:

The strength of a magnetic field is measured in teslas. One tesla is equal to one weber per square meter, as follows:

$$1\,\text{T} = 1\,\frac{\text{Wb}}{\text{m}^2}$$

Divya is building a circuit for her physics class. Point P one one of her wires has a magnetic field strength of 6 * 10^6 webers per square megameter, where one megameter is equal to 10^6 meters. What is the magnetic field strength in teslas?

A.) 6 * 10^-6 teslas

B.) 6 * 10^0 teslas

C.) 6 * 10^12 teslas

D.) 6 * 10^18 teslas

Practice:

A particular car has a fuel efficiency of 8 liters of gasoline per 100 kilometers. What is the fuel efficiency in miles per gallon? Round to the nearest integer.

(There are approximately 1.6 kilometers in 1 mile, and approximately 3.8 liters in 1 gallon.)

Free Response

# Table Data:

Example:

A college bookstore makes an order to replenish its stock of three different types of paper: college rule line paper, legal rule line paper, and graph paper. In addition, the paper is purchased bound as either spiral notebooks or paper pads. The table below shows the store's order.

**Bookstore Order of Different Types of Paper**

|  | College rule | Legal rule | Graph |
|---|---|---|---|
| Spiral notebooks | 175 | 60 | 75 |
| Paper pads | 90 | 110 | 125 |

If a graph paper item from the order is selected at random, what is the percent probability that the item is bound as a paper pad?

Practice:

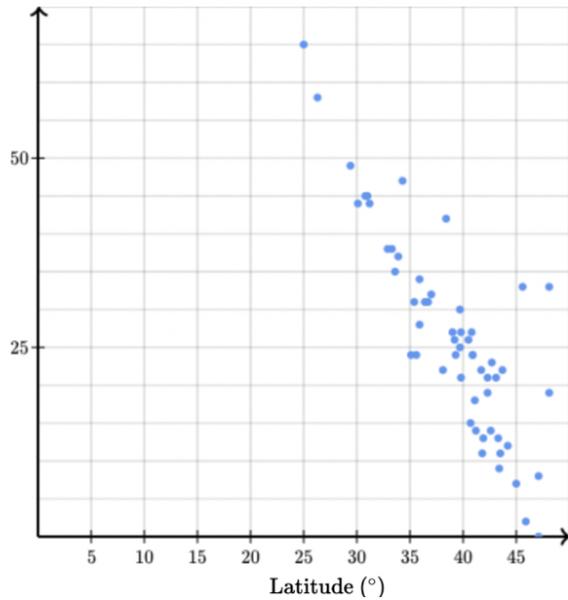The following table shows the number of U.S. coins produced by the Denver Mint from the years 2009 to 2011.

**Denver Mint Circulating Coin Production (in millions)**

| Year | Pennies | Nickels | Dimes | Quarters |
|---|---|---|---|---|
| 2009 | 1,248 | 47 | 50 | 270 |
| 2010 | 2,047 | 230 | 562 | 173 |
| 2011 | 2,536 | 540 | 754 | 195 |

If a coin produced by the Denver Mint in 2010 is selected at random, what is the probability that the coin is a dime?

A.) 0.187
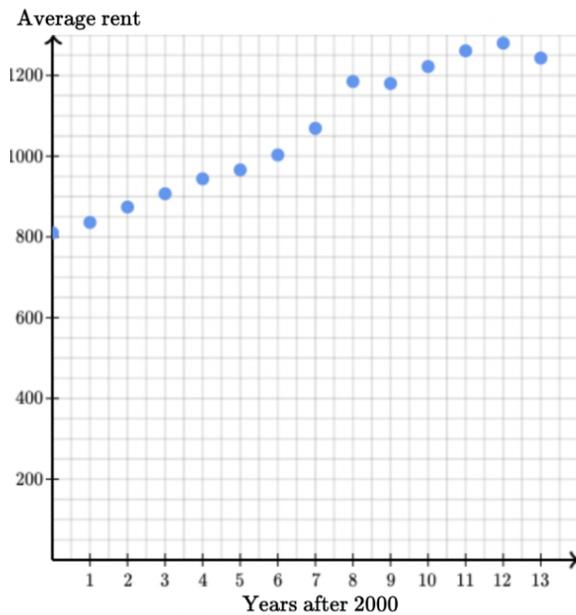
B.) 0.229

C.) 0.275

D.) 0.411

# Scatterplots:

Example:



The scatter plot shown at left plots the relationship between latitude of cities and their average January temperature. Which of the following is the best estimate of the average change in temperature associated with a 1 degrees increase in latitude?

A.) -15 degrees F

B.) -2.5 degrees F

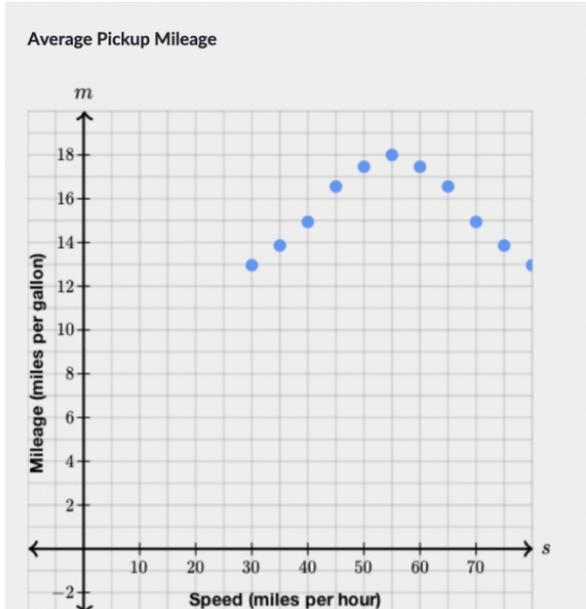C.) 2.5 degrees F

D.) 15 degrees F

Practice:



The scatter plot to the left shows the average rent (in dollars) for a 1-bedroom apartment in New York City each year between 2000 and 2013. Which of the following is the best estimate of the average change in rent each year?

A.) 0.025

B.) $0.17

C.) $40

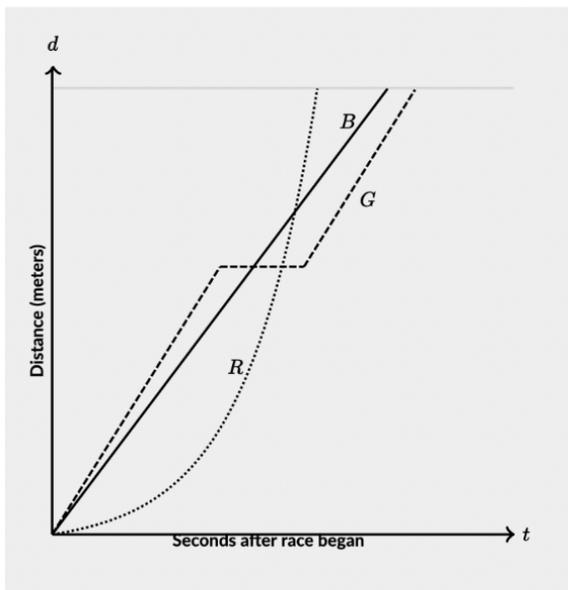D.) $500

# Key Graph Features:

Example:



**Average Pickup Mileage**

The graph at left in the sm-plane approximates the average mileage, m, in miles per gallon, that Deniz's pickup truck gets when she drives at a speed of s miles per hour. What is the best interpretation of the maximum point on the graph?

A.) Deniz's truck gets a maximum mileage of 18 miles per gallon.

B.) Deniz's truck gets a maximum mileage of 55 miles per gallon.

C.) Deniz's truck can drive at a maximum speed of 18 miles per hour.

D.) Deniz's truck can drive at a maximum speed of 55 miles per hour.

Practice:



A group of friends ran in a 200-meter race. The graph at left in the td-plane shows Ryan's distance, R, Beatriz's distance, B, and Grace's distance, G, all in meters, t seconds since the start of the race. Who stopped running temporarily before the race was over?

A.) Beatriz stopped running temporarily.

B.) Grace stopped running temporarily.

C.) Ryan stopped running temporarily.

D.) No one stopped running temporarily.

# Linear and Exponential Growth:

Example:

Which of the following equations relates y to x for the values in the table?

A.) y = 2x

B.) y = 4x

C.) y = 2^x

D.) y = ½ * 4^x

| $x$ | $y$ |
|-----|-----|
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |

Practice:

Some values of the function F are shown in the table. Which of the following

functions could be F?

A.) f(x) = 2x + 12

B.) f(x) = 12x - 8

C.) f(x) = 2^(x+1)

D.) f(x) = 4^x

| $x$ | $f(x)$ |
|-----|--------|
| 1 | 4 |
| 2 | 16 |
| 3 | 28 |
| 4 | 40 |
| 5 | 52 |

# Data Inferences:

Example:

After a three-month long motorized scooter pilot program, City A randomly surveyed residents on their feelings about the scooters . Of the 276 residents surveyed, 50 have positive feelings toward the scooters. Based on the survey results, approximately how many of the city's 500 thousand residents do NOT have positive feelings toward the scooters?

A.) 91 thousand

B.) 224 thousand
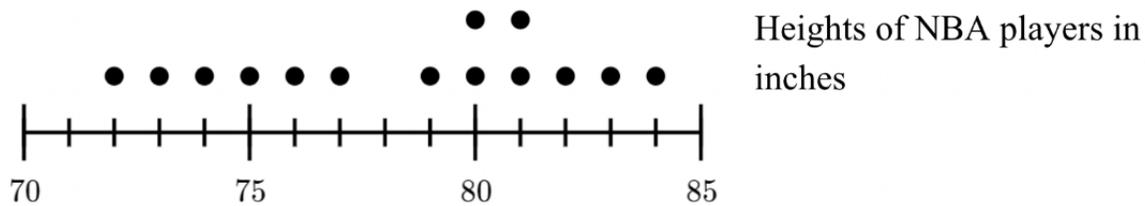
C.) 409 thousand

D.) 450 thousand

Practice:

In a random sample of 200 U.S. adults, 152 have read at least one book in the past 12 months. Based on this information, approximately how many of the 255 million U.S. adults have NOT read a book in the past 12 months?

A.) 48 million

B.) 61 million

C.) 103 million

D.) 194 million

# Center, Spread, and Shape of Distributions:
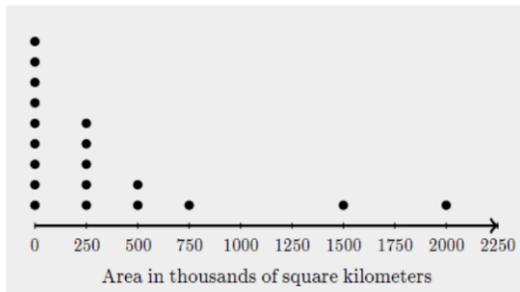
Example:



Heights of NBA players in inches

The dot plot above depicts the heights in inches of players on a professional basketball team. What would happen to the standard deviation of the data set if the lowest and highest heights were removed?

A)   It would increase
B)   It would decrease
C)   It would remain the same
D)   There is not enough information to determine the impact on the standard deviation of the

Practice:



Area in thousands of square kilometers

The area in thousands of square kilometers of 19 countries in East and Southeast Asia are shown in the dot plot at left. Each value is rounded to the nearest 250 thousand square kilometers. China, which has an area of 9,597 thousand square kilometers, was excluded from the data. If China were included, then which of the following changes would <u>NOT</u> occur?

A)   The mean area would increase
B)   The median area would increase
C)   The range of the areas would increase.
D)   The standard deviation of the areas would increase.

# Data Collection and Conclusions:

Example:

A company wants to determine the effectiveness of its Spanish language learning app. The company studied a large random sample of first-year Spanish majors and found that, on average, students who used the app received higher grades in their Spanish classes than students who did not use the app. Based on the design and results of the study, which of the following is an appropriate conclusion?

A)   Using the app will improve the grades of any college student.
B)   Using the app will improve the grades of any Spanish major.
C)   Using the app is likely to improve the grades of first-year Spanish majors.
D)   There is an association between using the app and higher grades in Spanish classes for first-year Spanish majors, but the effect of using the app is unclear.
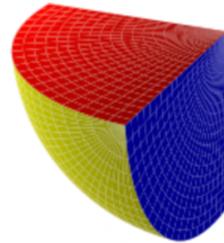
Practice:

A research study was conducted to determine if a gene therapy is successful in treating color blindness in mice. From a large population of mice with color blindness, 200 were selected at random. Half of the mice were randomly assigned to receive the therapy, and the other half did not. The results showed that 80% of the mice that received therapy had some of their ability to see color restored. Based on the design and results of the study, which of the following is an appropriate conclusion?

A)   The gene therapy is better at treating color blindness than other available treatments.
B)   The gene therapy is likely to fully restore the ability to see color for mice with color blindness
C)   The gene therapy is likely to improve the ability to see color for mice with color blindness.
D)   The gene therapy will improve the ability to see color for humans with color blindness.
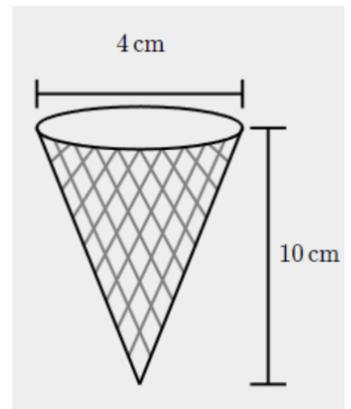
# Volume Word Problems:

Example:

A sphere of radius 2 inches is cut by three planes passing through its center. This partitions the solid into 8 equal parts, one which is shown above. The volume of each part is $t\pi$ inches. What is the value of $t$? This is a free response question.



Practice:

An ice cream cone maker wants to build a more stable cone by increasing the diameter and decreasing the height of the cone. The cone currently has diameter **4** centimeters (cm) and height **10 cm**, as shown to the left. They want the height of the new cone to be **8 cm**. Which of the following is closest to the smallest radius the new cone can have so that the volume is at least the volume of the old cone?



A)   2.0 cm
B)   2.3 cm
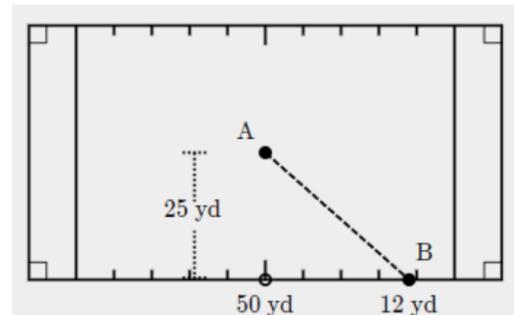C)   4.5 cm
D)   5.0 cm

# Right Triangle Word Problems:

Example:

Two football players are located at points A and B in a rectangular football field as shown at left. Point A is located 50 yards (yd) from the east edge and 25 yd from the south edge; Point B is located 12 yd from the east edge and 0 yd from the south edge. What is the distance, in yards, between the two players? This is a free response question.
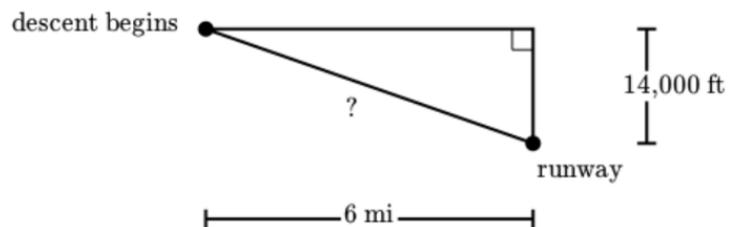
(Round your answer to the nearest tenth of a yard)



Practice:

Saul's plane is 6 miles (mi) from the airport runway when he begins his descent from 14,000 feet (ft): see the figure above. There are 5,280 feet in 1 mile. How far, in miles, does the plane actually travel in air during its descent? This is a free response question.

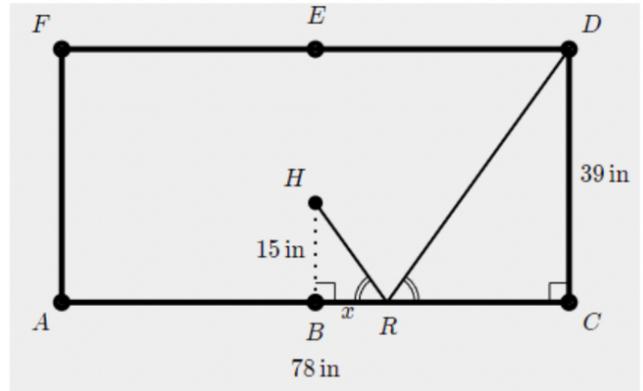(Round your answer to the nearest mile)

# Congruence and Similarity:

Example:

The diagram at left shows a rectangular billiard table with a width of 39 inches (in) and a length of 78 in. A player is trying to shoot the red ball located at point H, into the corner pocket D by bouncing the ball off of a point R that is x inches from point B. The player knows that when the ball bounces off point R, ∠HRB will be congruent to ∠DRC. If the point B is the midpoint of line AC and the red ball is 15 in above B, approximately what is the value of x?
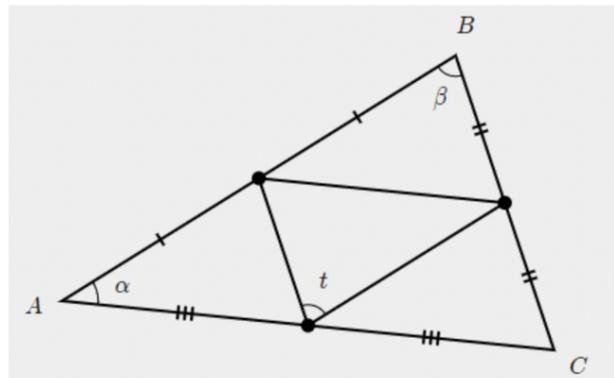
A)   7 in
B)   9 in
C)   11 in
D)   13 in



Practice:

In triangle ABC shown at left, tick marks of equal number represent sides of equal length. Therefore, the point along line segment AB is the midpoint of AB. What is the value of β in terms of α and t?
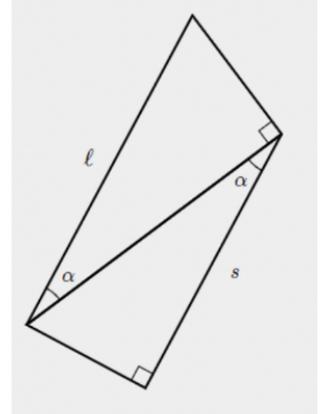
A)   α
B)   t
C)   π/2-α
D)   π/2-t

# Right Triangle Trigonometry:

Example:

In the figure to the left, two right triangles share a side and the angle α. The measure of α is unknown. Which of the following is equivalent to the ratio $\ell$ : s ?
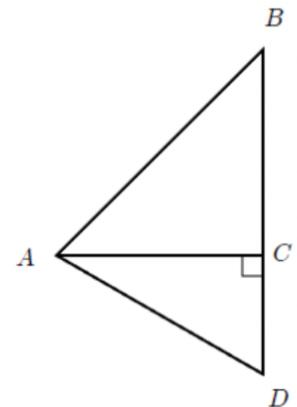
A)   $\sin(\alpha) : 1$
B)   $1 : \cos(\alpha)$
C)   $\sin^2(\alpha) : 1$
D)   $1 : \cos^2(\alpha)$



Practice:

In the figure to the right, $\cos(\angle BAC) = \sqrt{2}/2$ and $\sin(\angle CAD) = \frac{1}{2}$. What is the measure of $\angle BAD$?

A)   45°
B)   60°
C)   75°
D)   105°

# Angles, Arc Lengths, and Trig Functions:

Example:

Which of the following radian measures is equal to 135°?

A.) $(\pi/4)$ radians

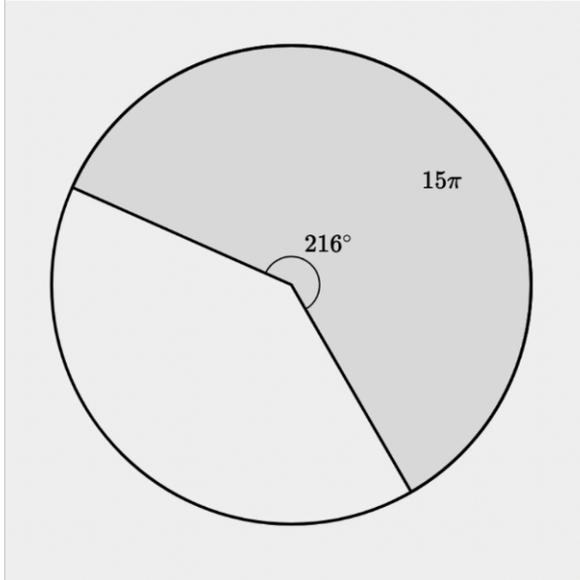B.) $(\pi/2)$ radians

C.) $(3\pi/4)$ radians

D.) $\pi$ radians

Practice:

If $\theta = (4\pi/9)$ radians, what is the value of $\theta$ in degrees?

A.) 20°

B.) 36°

C.) 80°

D.) 720°

# Circle Theorems:

Example:



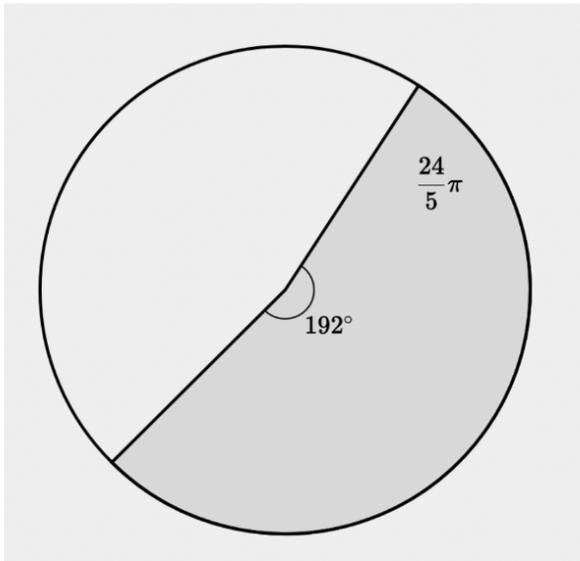The circle shown to the left has a sector with area $15\pi$ and central angle of 216°. What is the area of the circle?

A.) $(1/9)\,\pi$

B.) $9\,\pi$

C.) $(1/25)\,\pi$

D.) $25\,\pi$

Practice:



The circle shown to the left has a sector with area $(24/5)\,\pi$ and central angle of 192°. What is the area of the circle?

A.) $(1/9)\,\pi$

B.) $9\,\pi$

C.) $(75/192)\,\pi$

D.) $(192/75)\,\pi$

# Circle Equations:

Example:

A circle in the *xy*-plane has a center at (-12, 15) and a radius that is 9 units long. Which of the following is an equation of the circle?

A.) $(x - 12)^2 + (y + 15)^2 = 9$

B.) $(x + 12)^2 + (y - 15)^2 = 9$

C.) $(x - 12)^2 + (y + 15)^2 = 81$

D.) $(x + 12)^2 + (y - 15)^2 = 81$

Practice:

A circle in the *xy*-plane has its center at (-⅔,-3/4 ) and radius 5. Which of the following is an equation of the circle?

A.) $(x + ⅔)^2 + (y + ¾)^2 = 5$

B.) $(x - ⅔)^2 + (y + ¾)^2 = 25$

C.) $(x + ⅔)^2 + (y - ¾)^2 = 5$

D.) $(x + ⅔)^2 + (y + ¾)^2 = 25$

# Complex Numbers:

Example:

What is the sum of the complex numbers $2 + 4i$ and $3 - 7i$, where $i = \sqrt{-1}$?

A.) $5 - 3i$

B.) $5 + 11i$

C.) $6 - 28i$

D.) $6 - 3i$

Practice:

Which of the following is equal to $(7 + 3i) - (4 + i)$ ?

(Note: $i = \sqrt{-1}$)

A.) 5

B.) 5i

C.) $3 + 2i$

D.) $3 + 4i$

# Additional Practice

1.)
Find the solution to the system of equations:
7x+5y =20
2x+y=4

   A) x = 5/7, y = 3
   B) x = 0, y = 4
   C) x = 2, y = 0
   D) x = -2, y = 0

2.)
A line in the xy-plane passes through the point (4, -1) and is perpendicular to the line with equation y = x + 5. Which of the following is an equation of the line?

   A) y = x + 3
   B) y = x − 3
   C) y = −x + 3
   D) y = −x − 3

3.) (Free Response)
$C = 0.25(L\text{-}1200)$
The cost, C, in dollars, to the holder of a liability insurance account given a total liability of $L$ dollars is given by the equation above as long as $C$ is positive. By how many dollars does the liability increase when the cost to the holder increases by 1 dollar?

4.)
Find the solution to the system of equations:
5x+2y= -9
3x+4y=1/5

   A) x = -13/5, y = 2
   B) x = 0, y = -9/2
   C) x = -2/5, y = -7/2
   D) x = -4/3, y = 21/20

5.)
Which of the following is an equation of the line A graphed in the xy-plane that passes through the point $(-1, 3.5)$ and is perpendicular to the line B whose equation is $x + 4.5 = 0$?

    A) $x = -1$
    B) $x = 3.5$
    C) $y = 3.5$
    D) $y = 4.5$

6.)
An unloaded semi-truck, with the driver aboard, weighs 30,000 pounds. When fully loaded, the semi-truck holds 26 pallets of cargo, and each of the 18 tires of the fully loaded semi-truck bears approximately 3,330 pounds. What is the approximate average weight of one pallet of cargo?

    A) 1,131 pounds
    B) 1,280 pounds
    C) 1,633 pounds
    D) 1,665 pounds

7.)
A 400-seat public theater collects $5 per theater goer, but does not collect any money from accompanying children under the age of five. If children under five still require seats, which of the following functions best models how many dollars, $d$, the theater earns during a sold-out show, with $c$ children under the age of five in attendance?

    A) $d(c) = 5(400 + c)$
    B) $d(c) = 5(400 - c)$
    C) $d(c) = 400(5 + c)$
    D) $d(c) = 400(5 - c)$

8.)
Over the course of 4 years of training for the 100-meter dash, Erica's best time at each end-of-year track meet improved linearly by 0.3 seconds per year. Her best time at her first end-of-year track meet was 13 seconds. Which of the following equations shows Erica's best time, $b$, after $y$ years of training for $1 \leq y \leq 4$?

    A) $b = 13 - 0.3(y - 1)$
    B) $b = 13 - 0.3y$
    C) $b = (13/0.3)(y - 1)$
    D) $b = (13/0.3)y$

9.)
There are currently about 2.4 million square miles of tropical rainforest, but most experts agree that about 125 square miles of tropical rainforest are being lost per day. If this rate of depletion continues, which of the following inequalities best describes the number of years from now, $y$,when the rainforest will be depleted to 2 million square miles or less? Assume 1 year = 365 days.

   A) $2{,}400{,}000 - 45{,}625y \leq 2{,}000{,}000$
   B) $2{,}400{,}000 + 45{,}625y \leq 2{,}000{,}000$
   C) $2{,}400{,}000 - 125y > 2{,}000{,}000$
   D) $2{,}400{,}000 + 45{,}625y > 2{,}000{,}000$

10.)
Joanne and Richard volunteer at a hospital. Joanne volunteers 4 hours more per week than Richard does. In a given week, they do not volunteer for more than a combined total of 16 hours. If x is the number of hours that Richard volunteers, which inequality best models this situation?

   A) $x + 4 \leq 16$
   B) $2x + 4 \leq 16$
   C) $2x + 8 \leq 16$
   D) $2x - 4 \leq 16$

11.)
43 = 8c - 5

What is the value of c?

   A) 5
   B) 6
   C) 4
   D) -1

12.)
41 = 12d - 7

What is the value of d?

   A) 4
   B) 6
   C) 3
   D) -2

13.)
$6 = 2(y + 2)$

What is the value of y?

   A) 2
   B) -1
   C) 1
   D) 0


14.)

Each cup of cooked brown rice has 216 calories and 5 grams of protein. Each cup of kidney beans has 220 calories and 16 grams of protein. Which of the following systems of equations could be used to determine the amount of rice, $r$, in cups, and the amount of beans, $b$, in cups, that should be eaten in order to consume 435 calories and 18.25 grams of protein?

   A) $216r + 5b = 435$
          $220r + 16b = 18.25$
   B) $216b + 5r = 435$
          $220b + 16r = 18.25$
   C) $216r + 220b = 435$
          $5r + 16b = 18.25$
   D) $216b + 220r = 435$
          $5b + 16r = 18.25$

15.)
A concert venue sold two types of tickets for an upcoming concert: reserved tickets and general admission tickets. Reserved tickets sold for $50.00 each and general admission tickets sold for $34.00 each. If 1,520 tickets were sold for a total of $64,000, which of the following systems could be used to find the number of reserved tickets, $r$, and the number of general admission tickets, $g$, that were sold?

   A) $50g + 34r = 64,000$
          $r + g = 1,520$
   B) $50r + 34g = 64,000$
          $r + g = 1,520$
   C) $50g + 34r = 1,520$
          $r + g = 64,000$
   D) $50r + 34g = 1,520$
          $r + g = 64,000$

16.)

Vanessa has a $900 travel and lodging budget for her vacation. She found round-trip plane tickets for $x$ dollars total, a hotel for $y$ dollars per night, and free shuttle service between the airport and the hotel. If she plans to stay at the hotel for 5 nights, and she spends less than what she budgeted on travel and lodging, which of the following inequalities best describes the scenario?

    A) $x + 5y > 900$
    B) $5x + y > 900$
    C) $x + 5y < 900$
    D) $5x + y < 900$