Year 11 Mathematical Methods

Student Workbook and Teaching Template

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Contents

1. **Imperative:** Print this pdf document or be prepared to annotate the pdf with a tablet. Some blank spaces for writing are a little small for large writing. If you cannot do either of these annotation options, then write notes on blank paper, noting the relevant position within the typed course notes. As you watch the intructional videos, write notes in the blank spaces. This step is very important.

2. The instructor should write exercises from an appropriate textbook where the text says **Exercises/Homework**.

3. **Optional but highly recommended:** Purchase and use *Mathematica* or obtain it through your institution. We will occasionally use this to display various graphics and verify calculations. All graphics shown in this document were produced with *Mathematica*. You will most likely find it very helpful with your studies. It is a symbolic computation tool which has full programming capabilities. E.g. Try writing

   ```
   Expand[(x+y)^3]
   ```

   then press Shift+Enter or

   ```
   s = 0;
   For[i = 0, i < 6, i++, s = s + i; Print[s]]
   ```

   You can call on *Wolfram alpha* from with in it by beginning a cell with = =.
   If your school has a license, to install this on your machine, visit:
   `wolfram.com/siteinfo/`
   Get *Mathematica* Desktop.
   Create a Wolfram ID, and download and install the software.

1 Year 11 Mathematical Methods

Year 11. Under Construction

1.1 Term 1

Under Construction

1.2 Term 2

Under Construction

1.3.1 Limits and the First Derivative

Let $f(x)$ be a function that is defined near a real number a. If the function $f(x)$ is the real number L near a, then we write

$$\lim_{x \longrightarrow a} f(x) = L.$$

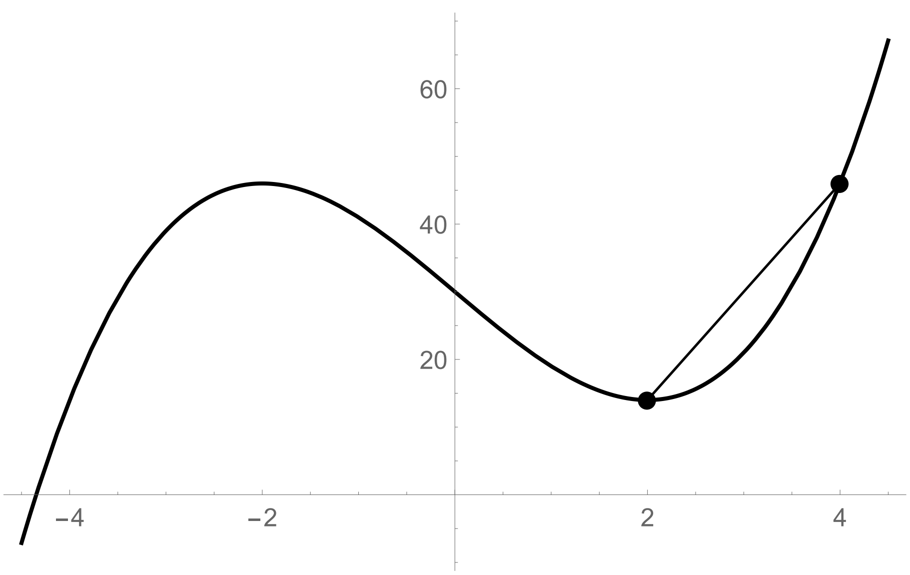
We say, the limit as x approaches a of $f(x)$ is L.

More formally, the real number L is the limit of the sequence $a_1, a_2, \ldots$ if and only if for every real number $\varepsilon > 0$, there exists a natural number N such that for all $n > N$, we have $|a_n - L| < \varepsilon$.

Example 1.1 *The function* $f(x) = \frac{x^3-1}{x-1}$ *is not defined when* $x = 1$. $f(x)$ *is defined for all real* x *except for* $x = 1$ *and hence defined for* x *near* 1. *Calculate* $\lim\limits_{x \longrightarrow 1} f(x)$.

Theorem 1 *(Properties of Limits) Let a, k_1, k_2 be particular real numbers and $f(x), g(x)$ are functions such that $\lim_{x \longrightarrow a} f(x)$ and $\lim_{x \longrightarrow a} g(x)$ exist. Then:*

- $\lim_{x \longrightarrow a} k_1 f(x) = k_1 \lim_{x \longrightarrow a} f(x)$.
- $\lim_{x \longrightarrow a} (k_1 f(x) + k_2 g(x)) = k_1 \lim_{x \longrightarrow a} f(x) + k_2 \lim_{x \longrightarrow a} g(x)$.
- $\lim_{x \longrightarrow a} (f(x) g(x)) = \left(\lim_{x \longrightarrow a} f(x) \right) \left(\lim_{x \longrightarrow a} g(x) \right)$.
- *If* $\lim_{x \longrightarrow a} g(x) \neq 0$, *then* $\lim_{x \longrightarrow a} \left(\frac{f(x)}{g(x)} \right) = \frac{\left(\lim_{x \longrightarrow a} f(x) \right)}{\left(\lim_{x \longrightarrow a} g(x) \right)}$.
- *If $f(x)$ is defined at $x = a$, then* $\lim_{x \longrightarrow a} f(x) = f(a)$.



The **first derivative** $f'(x)$ or **derivative** for short, is defined by the limit

$$f'(x) = \lim_{h \longrightarrow 0} \frac{1}{h} (f(x+h) - f(x)).$$

This gives a function representing the slope of the function $f(x)$ at any particular x value in the domain of f. To **differentiate** is to find $f'(x)$.

Example 1.2 *Use the definition of the derivative to find* $f'(x)$ *for the function* $f(x) = x^2$.

Example 1.3 *Use the definition of the derivative to find* $f'(x)$ *for the function* $f(x) = x^3 - 4x + 3$.

Exercises/Homework:

Consider the polynomial function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0.$$

We learn how to use rules to differentiate polynomial functions to obtain the polynomial function $f'(x)$.

Let $f(x) = kx^n$, where k is a particular real number and n is a non-negative integer. Using the limit definition of $f'(x)$,

$$f'(x) = \lim_{h \longrightarrow 0} \frac{1}{h}(f(x+h) - f(x))$$

$$= .$$

$$=$$

$$=$$

$$=$$

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$$=$$

Rule: To differentiate $f(x) = kx^n$, bring down n and subtract 1 from the exponent so that $(kx^n)' = knx^{n-1}$.

Note: Since $\lim_{x \longrightarrow a} (p(x)+q(x)) = \lim_{x \longrightarrow a} p(x) + \lim_{x \longrightarrow a} q(x)$ and the derivative of a constant is 0, the derivative of

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

is

$$f'(x) = n a_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + 2a_2 x + a_1.$$

Notation: $f'(x) = (f(x))' = \frac{d}{dx}(f(x))$.

Example 1.4 *Let $f(x) = x^3 + 3x - 7$. Calculate $f'(x)$ using both the limit definition and the rules for differentiating polynomials.*

Example 1.5 *Let $f(x) = -3x^5 + 2x^3 - 12x^2 + 14x - 32$. Calculate $f'(x)$ using rules for differentiating polynomials.*

Exercises/Homework:

Let $f(x) = kx^{-n}$, where n is a positive integer and k is a particular non-zero real number. Using the limit definition,

$$
\begin{aligned}
f'(x) &= \lim_{h \longrightarrow} \frac{1}{h}\left(f(x+h) - f(x)\right), \\
&= . \qquad . \\
&= \\
&= \\
&= \\
&= \\
&= \\
&=
\end{aligned}
$$

Theorem 2 *Let $f(x) = kx^n$, where k and n are real numbers.*

- *If $n = 0$, then $f'(x) = 0$. (The derivative of a constant is zero.)*
- *If $n \neq 0$, then $f'(x) = nkx^{n-1}$.*

Example 1.6 *Differentiate the function* $f(x) = 2x^3 - \frac{5}{x} + 14x - 8$.

Example 1.7 *Differentiate the function* $f(x) = -12x^5 + 3x^2 + \frac{2}{x^2} - \frac{1}{x}$.

Exercises/Homework:

We plot $y = f'(x)$ for functions $f(x)$ and answer questions on the sign of the first derivative of a function.

Example 1.8 *Let* $f(x) = 2x^3 - 4x + 5$. *Plot* $y = f'(x)$ *and determine where* $f'(x)$ *is positive, negative, and zero. Plot* $y = f(x)$ *also and consider any turning points in the context of the sign of* $f'(x)$.

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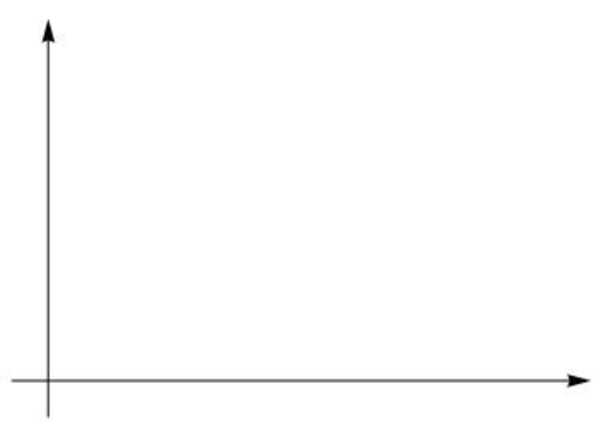

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Example 1.9 *Let* $f(x) = x^2 + 3x - 4$*. Plot* $y = f(x)$ *and* $y = f'(x)$ *on the same graph. State where* $f'(x) > 0$ *and* $f'(x) < 0$*.*

Let a and b be real numbers. We say that $f(x)$ is **increasing** on the interval (a,b) if for all x: $a < x < b$, $f'(x) > 0$. $f(x)$ is **increasing** on the interval $[a,b]$ if for all x: $a \le x \le b$, $f'(x) > 0$.

We say that $f(x)$ is **decreasing** on the interval (a,b) if for all x: $a < x < b$, $f'(x) < 0$. $f(x)$ is **decreasing** on the interval $[a,b]$ if for all x: $a \le x \le b$, $f'(x) < 0$.

Note: If $f(x)$ is increasing of $[a,b]$, then $f(a) < f(b)$. If $f(x)$ is decreasing of $[a,b]$, then $f(a) > f(b)$.

Exercises/Homework:

Under Construction

References

[1] D. Greenwood, S. Woolley, J. Goodman, J. Vaughan, S. Palmer, Essential Mathematics for the Australian Curriculum, 4th Ed. Cambridge, 2024.

[2] A. J., Washington. Basic Technical Mathematics with Calculus, SI Version.