Mathematica Practical Exercise Set 4

The Mathematica Practical Exercise Sets are optional but they will help you understand Multivariate Calculus and Linear Algebra better and sooner. This exercise set covers Chapter 3 and 4 of the course notes.

Consider the maxima and minima of the function

f (x, y) = x y + 1

subject to the constraint $x^2 + y^2 = 4$. Display various contours on the surface z = f(x, y), together with the constraint $x^2 + y^2$=4.

In[964]:=
```
f = x y + 1;
g = x^2 + y^2 - 4;
Table[Subscript[S, t] = ParametricPlot3D[{u, (t - 1) / u, t}, {u, -8, 8}];, {t, -10, 10}];
A = Plot3D[f, {x, -5, 5}, {y, -5, 5}, PlotStyle → Opacity[0.5], Mesh → False];
B = ContourPlot3D[g == 0, {x, -8, 8}, {y, -8, 8}, {z, -50, 50},
   PerformanceGoal → "Quality", ContourStyle → {Green, Opacity[0.5]}, Mesh → False];
Show[Join[{A, B}, Table[Subscript[S, j], {j, -10, 10}]]]
```

Build a function div( ) which inputs a vector field F and returns div(F) provided that F has 2 or 3 components.

In[970]:=
```
div[P_] := Module[{n}, n = Length[P];
   If[n == 2, D[P[[1]], x] + D[P[[2]], y],
    If[n == 3, D[P[[1]], x] + D[P[[2]], y] + D[P[[3]], z]], "more than 3 dim"]];
div[{x^2 y, x y}]
div[{x^2 y, x y, x z}]
```

Build a function curl( ) which inputs a vector field F with 3 components and returns curl(F).

In[990]:=
```
ClearAll[F]
curl[F_] := Module[{P, Q, R}, P = F[[1]];
   Q = F[[2]];
   R = F[[3]];
   {D[R, y] - D[Q, z], D[P, z] - D[R, x], D[Q, x] - D[P, y]}];
curl[{x^2 y + z, x y + z^2, x z}]
curl[{y, -x, 1}]
curl[{x, y, 1}]
```

Build a function grad( ) which inputs a scalar function f of n variables and returns grad(f), provided that all of the variables of f are in the standard English alphabet. Recall that if f(x,y,z) has continuous 2nd partials, then

curl(∇f) = (0,0,0).

Also, if F(x,y,z) has continuous 2nd partials, then

div(curl(F)) = 0.

```
grad[F_] := Module[{A1, q1, A2, m1},
  A1 = Union[Characters[ToString[F]]];
  q1 = Length[A1];
  A2 = {"a", "b", "c", "d", "e", "f", "g", "h", "i", "j", "k",
    "l", "m", "n", "o", "p", "q", "r", "s", "t", "u", "v", "w", "x", "y", "z"};
  m1 = Sum[If[MemberQ[A2, A1[[j]]], 1, 0], {j, 1, q1}];
  vars = ToExpression[Take[A1, {q1 + 1 - m1, q1}]];
  Table[D[F, vars[[i]]], {i, 1, m1}]];

grad[x^2 y + x y - 3 y z]
grad[x^2 y + x y - 3 y z + 4 a t]
curl[grad[x^2 y + x y - 3 y z]]
div[curl[{x y + z, x z - y, z}]]
```

Plot the vector field F(x,y) = (y,-x) together with the region
R = {(x,y) : $0 \leqslant x \leqslant 2, -2 \leqslant y \leqslant x^2$ }.

```
In[1191]:= A = RegionPlot[(0 ≤ x ≤ 2) && (-2 x ≤ y ≤ x^2), {x, -1, 3}, {y, -5, 5}, PlotPoints → 100];
B = VectorPlot[{y, -x}, {x, -1, 3}, {y, -5, 5}];
Show[A, B]
```

We will study this in more detail in Chapter 6: The line integral of 1 over the region
R = {(x,y) : $x^2 + y^2 = 3^2$ }. Note that this is the circumference of the circle of radius 3.

```
In[1238]:= Integrate[1, Element[{x, y}, ImplicitRegion[x^2 + y^2 == 9, {x, y}]]]
```