Effectiveness of the MASTER Program for Teaching Special Children Multiplication and Division

Johannes E. H. Van Luit and Jack A. Naglieri

Abstract

In this study, the utility of a MAthematics Strategy Training for Educational Remediation (MASTER) program, (Van Luit, Kaskens, & Van der Krol, 1993) was examined. The effectiveness of the program, designed to encourage strategy utilization with multiplication and division problems, was investigated for 84 students with poor mathematics skills, some of whom had learning disabilities ($n = 42$) and others with mild levels of mental retardation ($n = 42$). The results showed that the use of the self-instruction program resulted in significant improvement over the general instruction program. Furthermore, far transfer was found for the children with learning disabilities in the experimental group when they used effective problem-solving strategies on nontrained tasks. The results are consistent with previous findings suggesting the importance of self-instruction in mathematics training programs.

In the past 25 years, an area of research known as cognitive arithmetic (Van Luit, 1994a) has resulted in several theories of the processes underlying arithmetic (for reviews, see Ashcraft, 1992; Clark & Campbell, 1991; Lemaire, Barrett, Fayol, & Abdi, 1994; McCloskey, Harley, & Sokol, 1991; Rickard, Healy, & Bourne, 1994). These efforts have gone beyond the basic question of "How do we do mathematics in our heads?" to related issues such as children's acquisition of arithmetic knowledge and skill (e.g., Campbell & Graham, 1985; Siegler & Jenkins, 1989) and the nature of arithmetic problems in children with learning or mathematics disabilities (e.g., Geary, 1990; Geary, Brown, & Samaranayake, 1991).

A few researchers (e.g., Geary et al., 1991) have examined specific cognitive programs for instructing special children who have mathematics disabilities to determine how they solve simple arithmetic problems. Current developmental models (e.g., Ashcraft, 1992; Fayol, 1990) have suggested that counting-based algorithms play an important role in young children's mental arithmetic. Fundamental to any mental calculation is the skill of simple arithmetic; that is, the ability to determine quickly and accurately the answers on problems such as $3 \times 6 =$ __. A basic finding of research on the cognitive processes underlying this skill (e.g., Campbell, 1987; Miller & Paredes, 1990; Rickard et al., 1994; Siegler, 1988) is that, whereas children often use explicit, consciously mediated counting algorithms in early skill acquisition (e.g., solving 3×6 by adding $6 + 6 + 6$), as skills improve there is a transition toward retrieval of arithmetic facts directly from memory.

In schools for special education, particularly for students with learning disabilities (LD) and mild levels of mental retardation (MMR), mathematics disabilities are not unusual. The difficulty starts at an early age, and children in most special education math settings will not be able to master the four basic math operations before leaving primary education. Children with MMR and LD who have mathematics difficulties need special attention so that they can acquire basic mathematics skills (Geary, 1994). Remediation generally involves programmed instructional procedures, beginning at the child's level and progressing forward at the rate at which the child is able to learn (Kirk & Gallagher, 1989). Severe arithmetic disabilities require one-to-one tutoring or small group remedial teaching to adapt to the students' rate of progress and to provide adequate reinforcement. However, some researchers (e.g., Brown & Campione, 1977, 1990; Torgeson & Goldman, 1977; Van Luit, 1989) have argued that training should (a) take into account the particular difficulties of special children, especially the lack of spontaneous use of adequate information processing and remembering strategies; and (b) follow a step-by-step process, considering the individual differences in thinking and pace, moving from concrete to abstract and from working with materials to

JOURNAL OF LEARNING DISABILITIES
VOLUME 32, NUMBER 2, MARCH/APRIL 1999, PAGES 98–107

working with mental representations, and giving task-relevant examples. Other researchers with a similar perspective (e.g., Pressley, Brown, Beard El-Dinary, & Afflerbach, 1995) developed instruction that stimulates the development of constructively responsive learning. The most important aspects of this kind of learning are as follows:

- Effective strategy instruction does not emphasize use the of single strategies. Students are taught to flexibly apply a small repertoire of strategies that reflect the processes most frequently evidenced by skilled students.
- Strategies are introduced one at a time. Students practice applying them to authentic tasks, and coordinated use of the strategies is increasingly emphasized as new ones are introduced and as children acquire facility in carrying out the individual strategies.
- Most strategy instruction occurs in small groups where students think aloud as they do mathematics and apply strategies to specific tasks.

Results of experiments that have examined cognitive learning strategies and math performance have been reported by Johnston, Whitman, and Johnson (1980), Leon and Pepe (1983), Naglieri and Gottling (1995, 1997), Van Luit (1987), and Van Luit and Van der Aalsvoort (1985). In the study by Johnston et al. (1980), three 10-year-old boys with mathematics disabilities got an individual self-instruction training. The results showed progress in the trained arithmetic tasks. Van Luit and Van der Aalsvoort (1985) studied four 12-year-old children and reported improvement. Naglieri and Gottling (1995, 1997) found that facilitating self-instruction and systematic math problem solving strategies individually or in groups resulted in improved performance for elementary school-age children with LD. Whitman and Johnston (1983) reported on three 11-year-old children for whom individual as well as group training showed similar results. Research with larger groups of children has been reported by Leon and Pepe (1983) and Van Luit (1987). In these studies, children with LD and children with MMR performed better at addition and subtraction following self-instruction training. Furthermore, the experimental self-instruction groups showed significantly better results in arithmetic than the comparison groups.

Recognizing the potential importance of self-instruction methods to increase the use of strategies in math, Van Luit, Kaskens, and Van der Krol (1993) developed a specific training program for teaching multiplication and division to children with mathematics disabilities with an emphasis on strategy utilization. This program is called MASTER (MAthematics Strategy Training for Educational Remediation) and is based on the assumption that strategy instruction within the context of mathematics can help special children improve their performance in math. In this study, we examine whether the MASTER program changes children's use of adequate problem-solving strategies in multiplication and division, and we study the effect that this change has on their performance.

Method

Participants

Eighty-four students from two school systems that provided special education to children with MMR or LD participated in this study. The students with MMR ranged in age from 11 to 14 (M = 12 years 8 months) and those with LD ranged from 9 to 11 (M = 10 years 10 months). The students were diagnosed with MMR on the basis of their scores on the Dutch version of the Wechsler Intelligence Scale for Children–Revised (WISC-R; Van Haassen et al., 1985), a Dutch intelligence test, the Revised Amsterdam Child Intelligence Test (RAKIT; Bleichrodt, Drenth, Zaal, & Resing, 1984), and additional information from their teachers and school records. Their scores on the intelligence tests were between 1.5 and 2.5 *SD*s below the normative mean (mean IQ = 68). The criteria for placement in schools for children with MMR included evidence of (a) intellectual functioning below average (IQ range = 55–80); and (b) academic functioning below the level of age-matched, average-achieving children in elementary schools. Children with MMR in the experimental group had a mean IQ of 70 and children with MMR in the comparison group had a mean IQ of 71. There were no significant differences between the experimental and comparison groups of children with MMR on the basis of age, $F(1, 40) = 0.597$, $p = .444$, gender, $F(1, 40) = 0.296$, $p = .590$, or IQ, $F(1, 40) = 1.100$, $p = .301$. The characteristics of these samples and similarities between comparison and experimental groups are provided in Table 1.

The mean IQ for the children with LD in the experimental group was 96, and the mean IQ of children with LD in the comparison sample was 101. There were no significant differences between the experimental and comparison groups of children with LD on the basis of age, $F(1, 40) = 3.384$, $p = .069$, gender $F(1, 40) = 2.179$, $p = .144$), and IQ, $F(1, 40) = 2.298$, $p = .133$. Children with LD were identified on the basis of the same test battery and inquiry sources. Their scores on the intelligence tests were less than 1.0 *SD* from the mean, and they showed low academic performance. Dutch criteria for placement of children in schools for children with LD include evidence of (a) a disorder in one or more basic psychological processes; (b) academic achievement significantly below the student's level of intellectual functioning; (c) learning problems that are not due primarily to other handicapping conditions; and (d) the ineffectiveness of general education alternatives in meeting the student's educational needs. The definition of LD for children in the Netherlands follows the

TABLE 1
Description of the Comparison and Experimental Samples of Children with LD and MMR

Measure	LD Comparison	LD Experimental	MMR Comparison	MMR Experimental
n	21	21	21	21
IQ				
Mean	100.8	96.2	70.6	70.0
SD	4.5	4.5	3.7	4.2
Minimum	90	89	60	62
Maximum	108	106	76	77
Age (months)				
Mean	130.0	130.6	152.4	152.0
SD	9.6	8.1	11.6	12.9
Minimum	119	108	128	129
Maximum	141	142	172	170
Gender				
Boys	12	14	12	10
Girls	9	7	9	11

Note. LD = learning disabilities. MMR = mild mental retardation.

definition of Kavale and Forness (1992) that is used in the United States.

All the children included in this study had serious difficulties in mathematics. Their arithmetic level was comparable to that of third-grade pupils in a primary school. The children were unable to solve most multiplication and division problems with a result below 100. Their scores on a math test involving multiplication and division varied from 18% correct for the children with MMR to 40% correct for the children with LD. The difficulty of the problems in the math test varied. Some examples of the problems in this test are "4 × 7 = __", "9 × 6 = __", "One bunch of tulips costs six guilders. What is the price of three bunches? What is the price of six bunches?" "Grandmother has 20 guilders, which she divides equally among her four grandchildren. How many guilders does each child get? If grandmother has 40 guilders, how many guilders does each child get?"

Trainers

The trainers of children in the experimental groups were remedial teachers who had least 1 to 6 years experience in special education. They all received a 1-day inservice training before starting the experiment. Before the training session they were instructed to read about the program. In the morning session, all aspects (e.g., instruction variables, materials) and questions were discussed. In the afternoon session, the trainers viewed a video of the training from a similar study; then they participated in roleplay activities in which each participant took the role of child and trainer. During the study, all trainers attended a half-day follow-up meeting once a month to address any problems with the program or to get other assistance. Furthermore, a follow-up visit to the schools was conducted at least once a month. The lessons were videotaped and discussed with the teacher in private.

The trainers for the comparison groups were also special education teachers, who had 3 to 8 years of experience in special education. They were very knowledgeable on the standard curriculum. The children in the comparison groups were given individual instruction when necessary, in the way the standard curriculum is typically provided. The instruction method that these teachers used was a common one for special education in the Netherlands.

Procedure and Materials

The sample consisted of 84 students, 42 with LD and 42 with MMR. This sample was first divided into four groups: 21 children with LD and 21 children with MMR in the experimental group, and 21 children with LD and 21 children with MMR in the control group. The experimental group received the MASTER program, including self-instruction in arithmetic training working groups of five children each. The 42 children in the comparison group, also divided into two groups of 21 in each school system, received an arithmetic training in their own classrooms from their usual teachers. These teachers used standard remedial instruction procedures and materials from the standard curriculum for special education for students with MMR and from a standard curriculum for general elementary education for the students with LD. The students with MMR and LD were randomly assigned to either treatment condition. The remedial teachers taught the children three times a week for 45 minutes. The training period was 17 weeks.

The experimental groups in each school system received the training from special teachers in three groups of five children and one group of six children. During the 4-month training period, the children did not work on any other arithmetic program, nor did they receive any other arithmetic instruction. The training took place in separate classrooms three times a week for 45 minutes each session. Each training session consisted of arithmetic problems in accordance with the instructions in the MASTER training program for teaching multiplication and division to children with mathematics disabilities. The children with LD had already been taught multiplication and division in their own classrooms with a general program for mathematics education in elementary schools for at least 6 months before the start of the training. Similarly, the children with MMR had received a training period with a program for mathematics education in special schools for at least 1 year. Follow-up testing was conducted 3 months after the MASTER instructional period.

The math pre- and posttest consist of 40 items each (20 multiplication and 20 division items). The pre- and posttest are parallel versions. The 20 multiplication items and 20 division items each consist of 15 items in the form $4 \times 8 =$ __ and $64 \div 8 =$ __, and 5 in the form of a short story. The answers are scored as *right* (1) or *wrong* (0). The pretest was taken individually between 1 month and 2 weeks after the beginning of the training period. The posttest was taken individually between 3 and 6 weeks after the end of the training period.

The MASTER Training Program

The children's problem solving within and between the domains in arithmetic was taken into account in the MASTER program. Children with mathematics disabilities do not combine new information with already known information. Within the domain of multiplication, for example, they do not make the connection between an already known task ($5 \times 6 = 30$), and a new task ($6 \times 6 = ?$). Moreoever, these children do not understand the connections that can be made between domains, for example, between multiplication and division ($6 \div 3 = 2$, because $2 \times 3 = 6$). In the MASTER program (Van Luit et al., 1993) the following goals for learning were designed to enhance the child's skills:

- understanding of multiplication as repeated additions;
- understanding of the number system and the premises of some strategies like reversibility ($3 \times 8 = 8 \times 3$), associations ($9 \times 7 = 10 \times 7 - 1 \times 7$), and doubling ($8 \times 6 = 4 \times 6 + 4 \times 6$);
- importance of memorization of basic multiplication results below 100;
- understanding of division as repeated subtractions;
- understanding of the connection between division and multiplication;
- importance of memorization of all division results below 100; and
- application of multiplication and division results in real and imaginable situations.

The MASTER program consists of 23 lessons in multiplication and 19 lessons in division. The multiplication lessons consist of eight lessons on the basic procedures, 11 lessons on the multiplication tables, and four lessons on specific problems with one number below 10 and one number between 10 and 20 (e.g., $8 \times 17 =$ __). The 19 lessons in division consist of seven lessons on division, six lessons on division without remainders, five lessons on division with remainders, and one lesson with the denominator between 10 and 20 (e.g. $72 \div 12 =$ __). After the last lesson in the series there is a curriculum-based test. If a child scores 80% or more correct answers, he or she can go to the following series.

Each of the series phases involves teaching and discussing new steps in problem solving related to specific tasks. A series starts with an orientation phase, in which the child can solve the task with the help of materials. In the next phase, the connection is made with a mental solution and the child has to learn to check the solution. Finally, the cycle is ended with a phase of control, shortening, automatization, and generalization. The teacher tries to teach this complex working method using self-instruction. For children who need more assistance, suggestions or repetitions of a problem-solving strategy by their teacher become more specific. For example, the self-instruction offered in the program includes modeling. The program includes considerable explicit information about how to teach these children to address their arithmetic deficit.

The MASTER instruction program involves providing the opportunity for strategy generation and use. Children are given the opportunity to use their own solution or strategy to problems. The teacher's task is to lead the discussion in the direction of using strategies and to facilitate the discussion of the solutions provided by the students. Each student can use the strategy that he or she wishes, but the teacher assists the children in discussion and reflection about the choices made. The teacher presents training involving several strategies that become increasingly shorter. For example, an initial strategy for 7×6 could be: $1 \times 6 + 1 \times 6 + 1 \times 6 + 1 \times 6 + 1 \times 6 + 1 \times 6 + 1 \times 6$. Some children may suggest that this is too long a strategy and use: $2 \times 6 + 2 \times 6 + 2 \times 6 + 1 \times 6$, or $6 \times 6 + 1 \times 6$, or $5 \times 6 + 2 \times 6$, for example. The teacher ensures that each child understands the different solutions and encourages them to use the most efficient strategy. The main goal is to help children use simple multiplication and division results in more complex problems, such as 8×13 ($8 \times 10 + 8 \times 3$) or $64 \div 4$ ($40 \div 4 + 24 \div 4$). For more specific information about the MASTER program, see Van Luit (1994b) and Van Luit et al. (1993).

Results

Examination of Treatment Effects

Means and standard deviations for pre- and posttest math scores and effect sizes are presented in Table 2. These data show that both the experimental and the comparison groups improved with instruction; however, the amount of improvement differed substantially. The effect size for all children in the experimental group was 3.45, whereas the comparison group improved considerably less (effect size

TABLE 2
Pre- and Posttest Score Means, Standard Deviations, and Effect Sizes for 40 Multiplication and Division Problems

Group	*n*	Pretest *M*	Pretest *SD*	Posttest *M*	Posttest *SD*	Follow-up *M*	Follow-up *SD*	Pre–Post Effect[a]	Pre–Follow-up Effect[a]
MMR and LD									
Experimental	42	11.3	6.5	31.9	5.4	31.7	5.5	3.45	3.39
Comparison	42	12.5	6.7	18.2	8.2	18.0	8.9	0.76	0.70
MMR									
Experimental	21	6.7	4.3	27.9	4.2	27.8	4.9	4.99	4.58
Comparison	21	8.0	4.9	13.4	6.0	13.1	6.9	0.99	0.85
LD									
Experimental	21	15.9	4.9	35.8	3.2	35.5	2.6	4.81	5.00
Comparison	21	17.0	5.2	23.1	7.2	22.9	8.2	0.97	0.86

Note. LD = learning disabilities. MMR = mild mental retardation.

[a]Effect Size = $\dfrac{M_1 - M_2}{\sqrt{\dfrac{(n_1 \times SD_1^2 + n_2 \times SD_2^2)}{n_1 + n_2}}}$

= 0.76). The differences between the pre- and posttest scores show that the LD experimental group, $t(1, 20) = 23.65$, $p < .001$, and the MMR experimental group, $t(1, 20) = 27.32$, $p < .001$, improved significantly, but the LD and MMR pre–post comparison groups were also significant, $t(1, 20) = 8.43$, $p < .001$), and $t(1, 20) = 10.87$, $p < .001$, respectively.

Differences between groups were tested with analysis of variance (ANOVA). Between-group results on the pretest differences between experimental and comparison groups show that the LD groups, $F(1,40) = 0.546$, $p > .10$, and the MMR groups, $F(1, 40) = 0.892$, $p > .10$, have comparable scores. These results show that the procedures used for matching the two MMR and the two LD groups yielded samples with similar pretest scores. The analyses of the experimental and comparison groups' posttest results show that the experimental LD group, $F(1, 40) = 54.444$, $p < .001$, had better scores after the training period than the LD comparison group. The LD groups' pre–post intervention mean score differences expressed as an effect size were 4.81 for the LD experimental group and 0.97 for the LD comparison group. These results indicate that the MASTER program had considerable impact on the mathematics performance of the experimental group. The results were similar for the MMR groups. The experimental MMR group, $F(1, 40) = 81.823$, $p < .001$, also showed higher scores on the posttest than the MMR comparison group. The effect sizes (4.99 for MMR experimental and 0.99 for MMR comparison) show that the children with MMR in the experimental groups earned substantially higher scores on the posttest measures than the children in the comparison group, supporting the effectiveness of the MASTER program. All children—even the poorest performers in each experimental group—showed improvement, demonstrating that the MASTER strategy was very helpful for children who began the program with little competence in multiplication and division.

The differences in scores on the pretest measures between children with MMR and children with LD were striking. The LD and MMR groups (experimental and comparison together) differed significantly at pretest, $F(1, 83) = 76.029$, $p < .001$, as well as posttest, $F(1, 83) = 21.410$, $p < .001$. The children with LD answered approximately 41% of the arithmetic problems on the pretest correctly, whereas the children with mental retardation only answered 18% of the pretest problems correctly. At the posttest, these percentages were 74% and 52% respectively. These results are also presented in Figure 1.

The eight measurements (see Table 3) that were obtained during the training period showed that the experimental group children with LD made a gradual and consistent progression but that the experimental group children with MMR did not show much progress until the third month. The performance of the comparison groups showed a slight increase during the treatment interval.

Three-Month Follow-up

The results of the 3-month follow-up (see Table 2) showed very little decrease in comparison to the posttest scores obtained at the conclusion of the training period. The differences between posttest and follow-up were nonsignificant for all four groups: LD experimental, $t(1, 20) = 0.67$, $p > .10$, MMR experimental, $t(1, 20) = 0.15$, $p > .10$, LD comparison, $t(1, 20) = 0.36$, $p > .10$, and MMR comparison, $t(1, 20) = 0.62$, $p > .10$. This means that

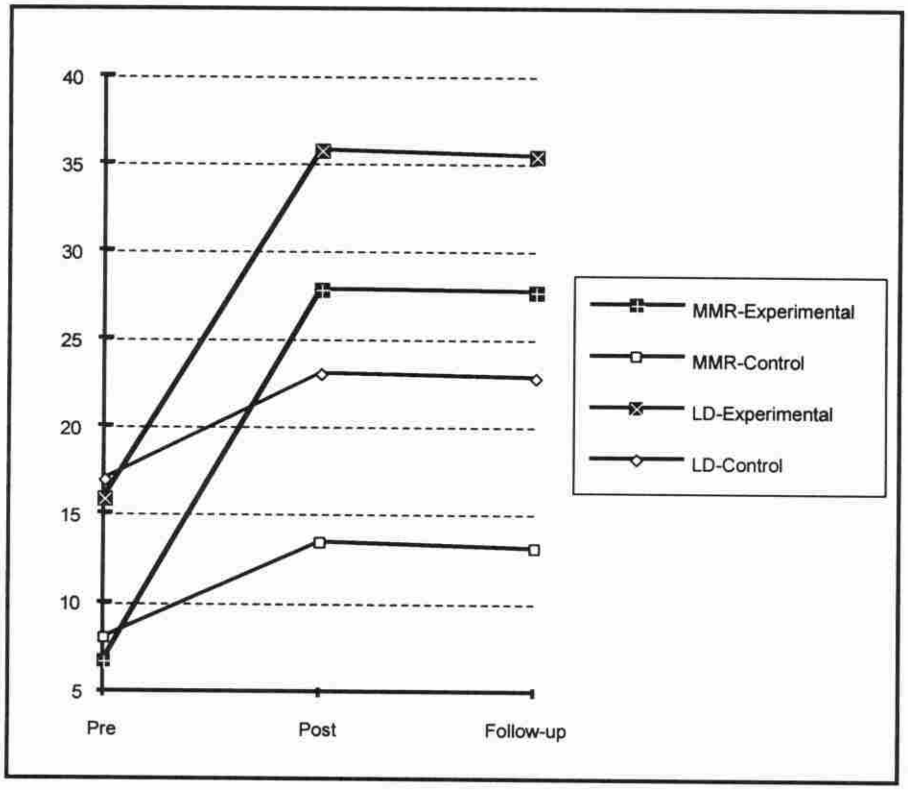

FIGURE 1. Mean number correct at preintervention, postintervention and follow-up phases. LD = learning disabilities. MMR = mild mental retardation.

TABLE 3
Percentage Correct During the MASTER Training Period at the End of 2-Week Periods of Training

	Week							
Group	2	4	6	8	10	12	14	16
MMR								
Experimental	14	18	20	25	49	65	74	77
Comparison	18	22	23	26	29	35	41	40
LD								
Experimental	43	56	62	78	82	89	91	90
Comparison	42	47	55	56	59	65	64	67

Note. LD = learning disabilities. MMR = mild mental retardation.

the progress for the students in the experimental groups was stable. After 3 months, including 6 weeks of summer holidays and return to their general classrooms, the students in the LD schools kept their high performance level. The MMR students did not show further progress; however, the pre–follow-up effect sizes suggest that the LD group maintained their level of improvement, and the MMR group showed only a minimal decline. The comparison groups' pre–follow-up effect sizes were also only slightly different from the pre–postintervention comparisons.

Examination of Transfer

Prior to starting the MASTER program, the trainers assessed each student's individual skill level. They taught the children to use specific arithmetic materials and self-instruction on the most difficult type of multiplication and division problems they had already mastered. The mastery levels were considerably easier than the types of arithmetic problems the students were working on in their own classrooms. Therefore, the arithmetic material was at a lower level than usually practiced in the classrooms. Many children—especially the children with LD—found this practice childish because they overestimated their own level of knowledge.

The trainers of the experimental groups used the first 2 weeks of the 4-month program to acclimatize the children to the newly formed groups. After this period, the students were trained with the specific treatment components. For many children, the new arithmetic program and the self-instruction procedure was very demanding—especially for the children with MMR. For example, the verbalizations of the students were often found to be limited and superfluous (see illustration in Figure 2).

After about 6 weeks of training, a change became apparent. Most children noticed the purpose of the training, and they slowly gained insight into the goals and procedures of the program. Some children quickly acquired the new procedures and came to use adequate problem-solving strategies on their own. Others consistently had difficulties with the discovery of similar rules between arithmetic problems, the adequate use of problem-solving strategies, and so on. In general, most children—especially the children with LD—did acquire the strategies that were instructed. An example of the use of such a strategy is shown in Figure 3.

The children were examined for far transfer (Brooks & Dansereau, 1987; Butterfield & Nelson, 1989) by evalu-

Problem: 2 × 7 = ?

Tr(ainer): Can you solve this problem?

Ch(ild): Yes that makes . . . eh . . . 14.

Tr: How did you do that?

Ch: Just easy, I know that two times seven makes 14.

Tr: Can you show it to me with help of the blocks?

Ch: How do you mean?

Tr: Can you solve the problem by laying down the blocks?

Ch: *lays down one group of two blocks and separately one group of seven blocks and says* Two times seven makes 14. *He is laughing softly when he says* That's not right.

Tr: How can you do it in another way?

Ch: I don't know.

Tr: That doesn't matter. We will learn that in the coming weeks.

FIGURE 2. Example of a solution by Anton (a 12-year-old boy from an MMR school) before the training.

Problem: 8 × 9 = ?

Tr(ainer): Can you solve this problem?

Ch(ild): Yes that's . . . 72.

Tr: How did you do that?

Ch: Well, first I did ten times eight—that makes 80; after that, I subtract one times eight—that makes 80 minus eight is 72.

Tr: Can you show it to me with help of the blocks?

Ch: Sure! *He lays down 10 ten-sticks, puts a piece of paper on all except the last two blocks of the ten-sticks and then puts away 1 ten-stick.* Nine times eight makes 72.

Tr: Very well done.

FIGURE 3. Example of a solution by Anton at the end of the training.

ating video recordings during specific test sessions. *Far transfer* is found if children use trained strategies even when there is a substantial difference between the tasks included in the training and the new tasks that have to be solved (Butterfield & Nelson, 1989; Gick & Holyoak, 1987). In contrast, *near transfer* occurs when a task looks like the ones from the curriculum and the child can use a well-known problem-solving strategy to solve that new task.

Three independent observers evaluated the videotapes of the children to assess for transfer. The interrater reliability was 96.7%. The students were given arithmetic problems of a type substantially more difficult than the last trained type of problem. The observers categorized problem solving into three classes: effective, potentially effective, and noneffective. An *effective* solution means a short-route problem-solving strategy with a correct answer; a *potentially effective* solution means a long solution route, with about 50% probability of a correct answer; and an *ineffective* problem-solving strategy means using different solution routes without an acceptable strategy and mostly without correct answers. Table 4 shows the division of the problem-solving strategies of the groups of children in these three classes.

Table 4 shows the far transfer of the problem-solving strategies of children with MMR and LD. Of the children with LD in the experimental group, 100% used an effective or potentially effective way of problem solving on the far transfer tasks. Only 24% of the children in the experimental group of children with MMR were able to solve the nontrained tasks in an effective or potentially effective way. For the comparison groups of children with LD and MMR, these percentages were 52% and 5%, respectively. The achievement of the children in the experimental groups appears to be more effective than the achievement of the children in the comparison groups. Training of children with LD in mathematics by the MASTER program apparently encouraged the spontaneous application of problem solving to nontrained arithmetic tasks. Only a few children from the MMR group were capable of solving these tasks in a similar manner.

Discussion

The results of this study demonstrate that the MASTER arithmetic training program can be effectively employed in teaching multiplication and division to children with MMR and LD who are poor at mathematics. The children received training to develop problem-solving strategies on the basis of discussion and reflection. Then they were given the opportunity to choose their own strategy, and the teacher used discussion, logical arguments, and reflection to show the child alternatives. The results of this method are consistent with other research (e.g., Leon & Pepe, 1983; Naglieri & Gottling, 1995, 1997; Van Luit, 1989) that has involved mathematics strategy instruction training. The effectiveness of this training can be explained in terms of successful integration of parts of the arithmetic curriculum and self-instruction/self-monitoring. The children in the experimental groups achieve significantly better on arithmetic tasks than the children in the comparison groups because more effective strategies for the solution of mathematics were employed. These results suggest that the children with LD and MMR benefited from the specific training program (Van Luit et al., 1993), but only the children with LD were able to solve far transfer tasks adequately after the experimental training.

TABLE 4
Methods of Problem Solving on Far Transfer Tasks

Group	*n*	Ways of problem solving		
		Effective	Potentially effective	Noneffective
MMR				
Experimental	21	1	4	16
Comparison	21	0	1	20
LD				
Experimental	21	17	4	0
Comparison	21	6	5	10

Note. LD = learning disabilities. MMR = mild mental retardation.

These results, like those of Perry (1991), suggest that children with LD may improve in mathematics if they are provided a basic approach to problem solving and are given the opportunity to generate their own procedures for solving mathematical problems, and that this may result in transfer of learning. In this experiment, transfer was possible for most of the children with LD, but not for the children with MMR. Geary (1994) noted the complexity of difficulties found for children with MMR, and suggested that the most likely consequence of these difficulties is the lack of transfer. Day and Hall (1988) and Campione, Brown, and Ferrara (1982) also noted that children with MMR only show modest near transfer if the transfer training takes long enough. Campione, Brown, Ferrara, Jones, and Steinberg (1985) suggested that children with MMR need considerable help to understand and solve near transfer problems. One important reason for the lack of transfer is the observation that children who show transfer are good at strategy use (Pressley, Snyder, & Cariglia-Bull, 1987). The literature on children with MMR has shown that these children are poor at strategy use; therefore, it is understandable that we do not find transfer in children with MMR.

The present results for Dutch children could be generalized to similar children in other countries. The children in the Netherlands are like those in the United States in some important ways. For example, Dockrell and McShane (1993) indicated that approximately 12% of children in the United States receive special education services—the same percentage as in the Netherlands. Of these children in the age range 4 to 12, about 30,000 (2.1% of the whole elementary school population) attend a school for children with LD, and 24,000 (1.7%) attend a school for children with MMR. About 6% of all Dutch children in the age range 4 to 12 years receive education in a special school. The other 6% with difficulties receive remedial teaching in general education. The characteristics of the children in special schools in the Netherlands and in the United States are identical with respect to IQ. But there are big differences from state to state in the United States regarding the placement of children in special schools or special classes (e.g., Danielson & Bellamy, 1989; O'Reilly, 1995). In the Netherlands, about 15% of the population in elementary education are children of immigrants (especially Turkish, Moroccan and Surinam children). These children have additional problems because they receive most of their education in their second language, in which they are typically less proficient than in their native language. At the end of elementary school, immigrant children are about 2 years behind in comparison to Dutch children of the same age. The results of this investigation, therefore, can be generalized with caution to similar children in the United States.

Among mathematics educators and researchers who are interested in the process of learning and understanding, there is a consensus that to learn mathematics means to construct mathematics. With the adoption of realistic mathematics education in primary schools (Schoenfeld, 1989; Siegler & Jenkins, 1989; Skemp, 1989), the care for children with poor achievement has increased. This is not surprising because the poorer achieving student will drop out sooner, given his or her limited contribution to the learning process (Geary, 1994). Realistic mathematics education involves learning to use several problem-solving strategies (apply heuristics in problem solving) by top-down processing. This means that

- Children should discover different strategies for themselves by solving many different mathematics problems. Furthermore, they should discover the quickest problem-solving strategy for a given situation.
- A short group instruction for all children, followed by discussions in small groups of children about their (individual) solutions of a problem, is warranted. The teacher does not provide solutions but leads the discussions about the different strategies that are suggested by some children.

- Most mathematics problems for group discussions are presented in meaningful and rich contexts, using a variety of mutually linked models, schemes and graphs. There is also time for individual exercise.

A realistic method thus presumes interactive education with the entire group. However, from those group assignments alone, the poorer mathematics students pick up too little information and are often not sufficiently involved in the lesson to master mathematics (Wilkinson, Martino, & Camilli, 1994). About 60% of the Dutch schools for children with LD use such a realistic curriculum, but none of the schools for children with MMR do (e.g., Van Luit, 1994b). However, for children with severe difficulties, realistic math education is not feasible. The children in both the experimental and the comparison groups in this study had received mathematics (remedial) education on the basis of traditional curricula. For them, direct instruction by the remedial teacher was common; however, this does not appear to have been the most effective method of instruction.

We conclude that the MASTER training program of strategy instruction, applied in the teaching of certain arithmetic domains, had positive influences on the achievement of children with LD because adequate problem-solving strategies were facilitated. The results of this study suggest that the MASTER program may provide a method for general and remedial teachers to help children with disabilities improve their math performance. The results are consistent with previous findings suggesting the importance of implementing strategy instruction in arithmetic training programs (Naglieri & Gottling, 1995, 1997; Whitman, 1987). Further research with programs developed for other arithmetic domains that include self-instruction of strategy use should be considered, especially for children with severe mathematics disabilities.

ABOUT THE AUTHORS

Johannes E. H. Van Luit, *PhD, is an associate professor of Special Education at Utrecht University in The Netherlands. His current interests include exceptional children, early mathematics education, program and test development, and cognitive problem solving.* ***Jack A. Naglieri,*** *PhD, is a professor of School Psychology and Psychology at Ohio State University. His current interests include the PASS Theory, Cognitive Assessment System, assessment of personality, exceptional children, and cognitive interventions. Address: Johannes E. H. Van Luit, Utrecht University, Department of Special Education, Heidelberglaan 1, NL 3584 CS Utrecht, The Netherlands.*

AUTHORS' NOTE

The authors recognize the helpful comments provided by Vladimir Sloutsky on an earlier draft of this article

REFERENCES

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition, 44,* 75–106.

Bleichrodt, N., Drenth, P. J. D., Zaal, J. N., & Resing, W. C. M. (1984). Revisie Amsterdamse Kinder Intelligentie Test [Revised Amsterdam Child Intelligence Test]. Lisse, The Netherlands: Swets & Zeitlinger.

Brooks, L. W., & Dansereau, D. F. (1987). Transfer of information: an instructional perspective. In S. M. Cormier & J. D. Hagman (Eds.), *Transfer of learning: Contemporary research and applications* (pp. 121–151). San Diego: Academic Press.

Brown, A. L., & Campione, C. (1977). Training strategic study time apportionment in educable retarded children. *Intelligence, 1,* 94–107.

Brown, A. L., & Campione, C. (1990). Communities of learning and thinking: A context by any other name. *Contributions to Human Development, 21,* 108–126.

Butterfield, E. C., & Nelson, G. D. (1989). Theory and practice of teaching for transfer. *Educational Technological Research and Development, 37,* 5–38.

Campbell, J. I. D. (1987). Production, verification, and priming of multiplication facts. *Memory and Cognition, 15,* 348–364.

Campbell, J. I. D., & Graham, D. J. (1985). Mental multiplication skill: Structure, process, and acquisition. *Canadian Journal of Psychology, 39,* 338–366.

Campione, J. C., Brown, A. ,L., & Ferrara, R. A. (1982). Mental retardation and intelligence. In R. J. Sternberg (Ed.), Handbook of human intelligence (pp. 392–490). New York: Cambridge University Press.

Campione, J. C., Brown, A. L., Ferrara, R. A., Jones, R. S., & Steinberg, E. (1985). Breakdowns in flexible use of information: Intelligence-related differences in transfer following equivalent learning performance. *Intelligence, 9,* 297–315.

Clark, J. M., & Campbell, J. I. D. (1991). Integrated versus modular theories of number skills and acalculia. *Brain and Cognition, 17,* 204–239.

Danielson, L. C., & Bellamy, G. T. (1989). State variation in placement of children with handicaps in segregated environments. *Exceptional Children, 55,* 448–455.

Day, J. D., & Hall, L. K. (1988). Intelligence-related differences in learning and transfer and enhancement of transfer among mentally retarded persons. *American Journal on Mental Retardation, 93,* 125–137.

Dockrell, J., & McShane, J. (1993). *Children's learning difficulties. A cognitive approach.* Oxford, England: Blackwell.

Fayol, M. (1990). *L'enfant et le nombre.* [The child and the number]. Neuchatel, Switzerland: Delachaux & Niestle.

Geary, D. C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology, 49,* 363–383.

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications.* Washington: American Psychological Association.

Geary, D. C., Brown, S. C., & Samaranayake, V. A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology, 27,* 787–797.

Gick, M. L., & Holyoak, K. J. (1987). The cognitive basis of transfer. In S. M. Cormier & J. D. Hagman (Eds.), *Transfer of learning: Contemporary research and applications* (pp. 9–46). San Diego: Academic Press.

Hall, R. J. (1980). Cognitive behavior modification and information processing skills of exceptional children. *Exceptional Education Quarterly, 1,* 9–15.

Johnston, M. B., Whitman, T. L., & Johnson, M. (1980). Teaching addition and subtraction to mentally retarded children: A self-instruction program. *Applied Research in Mental Retardation, 1,* 141–160.

Kavale, K. A., & Forness, S. R. (1992). History, definition, and diagnosis. In N. N. Singh, & I. L. Beale (Eds.), *Learning disabilities. Nature, theory, and treatment* (pp. 3–43). New York: Springer-Verlag.

Kirk, S. A., & Gallagher, J. J. (1989). *Educating exceptional children.* Boston: Houghton Mifflin.

Lemaire, P., Barrett, S. E., Fayol, M., & Abdi, H. (1994). Automatic activation of addition and multiplication facts in elementary school children. *Journal of Experimental Child Psychology, 57,* 224–258.

Leon, J. A., & Pepe, H. J. (1983). Self-instructional training: Cognitive behavior modification for remediating arithmetic deficits. *Exceptional Children, 50,* 54–60.

McCloskey, M., Harley, W., & Sokol, S. M. (1991). Models of arithmetic fact retrieval: An evaluation in light of findings from normal and brain-damaged subjects. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 17,* 377–397.

Miller, K. F., & Paredes, D.R. (1990). Starting to add worse: Effects of learning to multiply on children's addition. *Cognition, 37,* 213–242.

Naglieri, J. A., & Gottling, S. H. (1995). A cognitive education approach to math instruction for the learning disabled: An individual study. *Psychological Reports, 76,* 1343–1354.

Naglieri, J. A., & Gottling, S. H. (1997). Mathematics instruction and PASS cognitive processes: An intervention study. *Journal of Learning Disabilities, 30,* 513–520.

O'Reilly, F. E. (1995). *State special education funding formulas and the use of separate placements for students with disabilities: Exploring linkages.* Palo Alto, CA: CSEF.

Perry, M. (1991). Learning and transfer: Instructional conditions and conceptual change. *Cognitive Development, 6,* 449–468.

Pressley, M., Brown, R., Beard El-Dinary, P., & Afflerbach, P. (1995). The comprehension instruction that students need: Instruction fostering constructively responsive reading. *Learning Disabilities Research & Practice, 10,* 215–224.

Pressley, M., Snyder, B. L., & Cariglia-Bull, T. (1987). How can good strategy use be taught to children? Evaluation of six alternative approaches. In S. M. Cormier & J. D. Hagman (Eds.), *Transfer of learning: Contemporary research and applications* (pp. 81–120). San Diego: Academic Press.

Rickard, T. C., Healy, A. F., & Bourne, L. E. (1994). On the cognitive structure of basic arithmetic skills: Operation, order, and symbol transfer effects. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 20,* 1139–1153.

Schoenfeld, A. H. (1989). Teaching mathematical thinking and problem solving. In L. B. Resnick & B. L. Klopfer (Eds.), *Toward the thinking curriculum: Current cognitive research.* Washington, DC: ASCD.

Siegler, R. S. (1988). Strategy choice procedures and the development of multiplication skills. *Journal of Experimental Psychology: General, 117,* 258–275.

Siegler, R. S., & Jenkins, E. A. (1989). *How children discover new strategies.* Hillsdale, NJ: Erlbaum.

Skemp, R. R. (1989). *Mathematics in primary school.* London: Routledge.

Torgeson, J. K., & Goldman, T. (1997). Verbal rehearsal and short-term memory in reading-disabled children. *Child Development, 48,* 56–60.

Van Haassen, P. P., De Bruyn, E. E. J., Pijl, Y. J., Poortinga, Y. H., Lutje Spelberg, H. C., Vander Steener, G., Coetsier, P., Spoeldens-Claes, R., & Stinissen, J. (1985). *WISC-R, Nederlandse uitgave* [WISC-R, Dutch edition]. Lisse, The Netherlands: Swets & Zeitlinger.

Van Luit, J. E. H. (1987). Teaching impulsive children with arithmetic deficits in special education: A self-instructional training program. *European Journal of Special Needs Education, 2,* 237–246.

Van Luit, J. E. H. (1989). Self-instructional training or remediating arithmetic deficits in MMR and LD children. In J. J. Dumont & H. Nakken (Eds.), *Learning disabilities. Vol. 2: Cognitive, social and remedial aspects.* Amsterdam: Swets & Zeitlinger.

Van Luit, J. E. H. (1994a). Current trends in research on learning and instruction of mathematics. In J. E. H. Van Luit (Ed.), *Research on learning and instruction of mathematics in kindergarten and primary school* (pp. 13–21). Doetinchem, The Netherlands: Graviant.

Van Luit, J. E. H. (1994b). The effectiveness of structural and realistic arithmetic curricula in children with special needs. *European Journal of Special Needs Education, 9,* 16–26.

Van Luit, J. E. H., Kaskens, J., & Van de Krol, R. (1993). *Speciaal rekenhulpprogramma vermenigvuldigen en verdelen.* [MASTER strategy training program]. Doetinchem, The Netherlands: Graviant.

Van Luit, J. E. H., & Van der Aalsvoort, G. M. (1985). Learning subtraction in a special school: A self-instructional training strategy for educable mentally retarded children with arithmetic deficits. *Instructional Science, 14,* 179–189.

Whitman, T. L. (1987). Self-instruction, individual differences, and mental retardation. *American Journal of Mental Deficiency, 92,* 213–223.

Whitman, T. L., & Johnston, M. B. (1983). Teaching addition and subtraction with regrouping to educable mentally retarded children: A group self-instructional training program. *Behavior Therapy, 14,* 127–143.

Wilkinson, L. C., Martino, A., & Camilli, G. (1994). Groups that work: Social factors in elementary students' mathematics problem solving (pp. 75–105). In J. E. H. Van Luit (Ed.), *Research on learning and instruction of mathematics in kindergarten and primary school.* Doetinchem, The Netherlands: Graviant.