Noname manuscript No.
(will be inserted by the editor)

Demonstration of Quantum Information Splitting Using a Five-qubit Cluster State: An IBM Quantum Experience

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Received: date / Accepted: date

Abstract Quantum Information Splitting (QIS) is an important technique widely used in quantum teleportation, where a sender sends quantum information (state) to particular recipients by doing some operations on their qubits. Only all of the receivers can collectively work out to recover the content which was sent by the sender. Here, we provide a new efficient scheme for splitting up of quantum information using five-qubit cluster states. We demonstrate our work by performing experiments on the IBM quantum computer and conclude that a two-qubit arbitrary state can be split by using a five-qubit cluster state. We properly implement the schemes on the quantum computer by designing appropriate quantum circuits and collect experimental results with good fidelity.

Keywords Quantum Teleportation, Cluster States, Quantum Information Splitting, IBM Quantum Experience

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1 Introduction

In quantum physics, entanglement is one of the remarkable properties which precisely explains non-intuitive quantum correlations created between two or more particles [1]. Secret sharing [2,3], various quantum information processing tasks like teleportation [4], superdense coding [5], quantum cryptography [6], information delay protocol [7], one way quantum computation [8] can be performed using the property of entanglement. In quantum information splitting (QIS), entanglement is one of the main source [1] as well as an important feature of QIS. In quantum communication, quantum information splitting is a prime technique. Quantum teleportation is a method for transferring information between parties, using a distributed entangled state and a classical communication channel. A secret message can be shared between multiple parties, where one or more members can receive the desired message, with the concurrence of the sender and other members, one can easily send a state by using this tool. This problem has several consequences in the area of banking and intelligence sharing. Quantum information splitting (QIS) has brought in a new way with security and with a technique to detect eavesdropping. In 1999, Hillery [2] first proposed a technique, how a GHZ state can be handled for quantum information splitting (QIS). For arbitrary two-qubit states, QIS can be explained by using Bell pairs [9]. In the QIS process, researchers are still using different entanglement channel, but in this context, we are considering a specific channel, the five-qubit cluster state [10,11,12]. QIS of $|\psi_i\rangle$ was experimentally shown by using single photon sources [12]. In 2009, Muralidharan *et al.* tried to show QIS by using cluster state [13]. However, Here we make the quantum circuit by using IBM quantum processor, which is a new quantum circuit that can be used to split a quantum information. By using this circuit, one can use to send any arbitrary two-qubit state to another party without any fear of eavesdropping. Some special entangled multi-qubit states have been useful to split any arbitrary two-qubit states [14] mentioned that the GHZ and asymmetric W state can't be used for QIS of arbitrary two-qubit state. However, an arbitrary two-qubit state can be split by two GHZ states.

Since 2016, IBM quantum experience has given access to the prototypes of quantum computers for testing and simulating quantum algorithms. IBM Q offers a cloud-based quantum computing platform, allowing the users to design quantum circuits using a interactive graphical user interface and test those circuits, both on a classical computer and on actual quantum processors. Several researchers have been benefited from this unique quantum experience provided by IBM. IBM provides the composer on its website, which is a cloud-based quantum computing platform [15]. Any user can give a quantum circuit on the five-qubit, and sixteen-qubit devices for a real run or simulation. IBM Q Experience has now been used to perform many real experiments on quantum chips. The real experiments include quantum simulation [16,17,18,19,20,21,22,23,24], developing quantum algorithms [25,26,27,28,29,30,31,32], testing of quantum information theoretical tasks [19,25,33,34,35], quantum cryptography [36,37,38,39], quantum error correction [40,41,42,43], quantum applications [20,22,36,43,44,45,46,47] to name a few.

The composition of the paper is as follows. Section 2 discusses the proposed scheme for quantum information splitting. Then the experimental realization of the scheme is demonstrated in Section 3. In Section 4, the experimental results are illustrated in detail. Finally, we conclude in Section 5 with discussions on future research works along this direction.

2 Scheme of quantum information splitting by using five-qubit cluster state

The general form of N-cubit cluster state can be represented as follow,

$$|C_N\rangle = \frac{1}{2^{\frac{N}{2}}} \otimes (|0\rangle_a \sigma_z^{a+1} + |1\rangle_a) \tag{1}$$

The five-qubit cluster state can be given as, $|C_5\rangle$= $(|00000\rangle + |11100\rangle + |11011\rangle + |00111\rangle)$. We can generate this five-qubit cluster state as shown in Fig. 1.

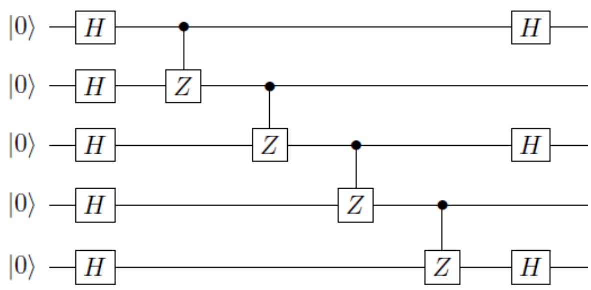

Fig. 1: Quantum circuit generating the five-qubit cluster state, $|C_5\rangle$.

Cluster state can be used for teleportation of arbitrary single and two-qubit states. We can use this cluster state as QIS of an arbitrary two-qubit state. In this section, we demonstrate a scheme for QIS of an arbitrary two-qubit state using the following five-qubit cluster state, $\psi_{ab} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$ where, $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Suppose Alice has an arbitrary two-qubit state. She wants to share this state between Charlie (receiver) and Bob (controller). Charlie will get the original state ψ_{ab}, sent by Alice when Bob cooperates. For these purpose, five-qubit cluster state is shared by Alice, Bob, and Charlie. We let Alice possess the qubit 1 and 2 along with ψ_{ab} which is to be split among the two parties, Charlie and Bob. We let Bob possess the qubit 3 and Charlie to possess the qubits 4 and 5 of cluster state $|C_5\rangle$. Alice first combines the state ψ_{AB} with $|C_5\rangle$ and measures each of her four-qubit individually in the basis ($|0\rangle$, $|1\rangle$) and conveys the outcome of her measurement to Charlie by four classical bit of information. Each of four-partite measurement basis can also be taken into Bell basis and single particle measurement for experimental realization. At this state, Charlie will not be able to decode ψ_{AB} with Alice measurement alone. To decode original state ψ_{AB}, Bob measures in Hadamard basis on his qubit and sends the result of his measurement to Charlie via one classical bit. After obtaining the outcomes of both Alice and Bob, Charlie performs controlled unitary operations on his qubits to get state ψ_{AB}. This is the complete protocol for QIS of arbitrary two-qubit states ψ_{AB}.

3 Experimental Realization in IBM QE

To achieve QIS in IBM quantum processor, we use the equivalent quantum circuit given in Fig. 2. This can also be realized in QISKit platform, which is based on python.

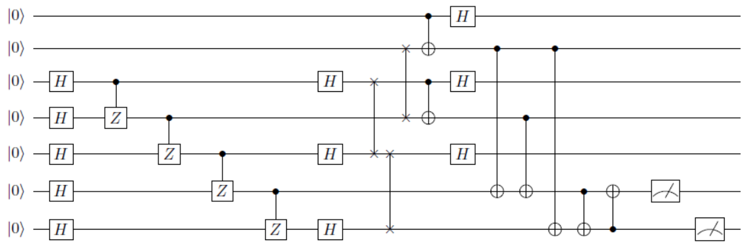

Fig. 2: A generalized circuit for QIS using five cubit cluster state

Fig. 2 shows an actual quantum circuit used for the implementation of QIS in IBM QE processor. Initially, all qubits are in $|0\rangle$ states. We select quantum gates from the toolbox and put an appropriate location according to Fig. 2. First, two-qubit represents the ψ_{AB}, and the last five-qubit represents the cluster state which is distributed among Alice, Bob, and Charlie. First two qubits represent of Alice, the third one is Bob, and the last two qubits represent Charlie of the five-qubit cluster states. Cluster state is generated using quantum gate as can be seen in Fig. 1. Three swapping operations are used for effective qubit location. A pair of CNOT and Hadamard gate used for Bell basis measurement by Alice on her qubits. After then, Hadamard operation is applied to measure the Bob's qubit. Finally, based on these two measurement outcomes, we use proper controlled unitary operations (CNOT operation) on Charlie's qubit to achieve ψ_{AB}.

4 Results

Our designed circuit can be used to split any arbitrary two-qubit state between two parties without any fear of eavesdropping. Here we explicate a part of the results. As a part of our result first, we consider verifying our results in "IBM qasm simulator". From Fig. 3 initially, we consider that Alice has a Bell state. From Table 2, the probability distribution shows that Charlie has got the exact Bell state that Alice had with the probability of 0.503 and 0.497 for $|00\rangle$ and $|11\rangle$ states respectively. In Table 1, we have shown the results for a Bell state considering different shots. We have seen that probability distribution of state $|11\rangle$ and $|00\rangle$ varies as we change the number of shots. Again, we can run our proposed circuits by using a real device "IBM-16-Melbourne". The Table 2 is for Bell states done in "IBM-16-Melbourne" real device and for "qasm simulator". Here, the error is due to a large number of CNOT gates present in our proposed circuit. From table 2, we have got 71.2 % fidelity according to Bhattacharyya coefficient, $F = \sum \sqrt{p_i q_i}$ that compares the probability distribution obtained when measuring in the real device with the qasm simulator.

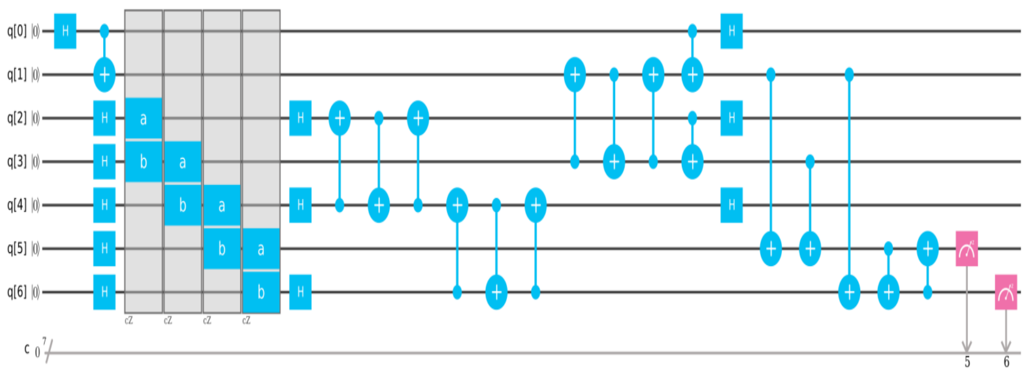

Fig. 3: QIS scheme where Alice initially prepared Bell state.

No. of shots	Prob. of $\|00\rangle$	Prob. of $\|11\rangle$
10	0.600	0.400
50	0.540	0.460
500	0.516	0.484
8192	0.495	0.505

Table 1: Probability distribution for different shots

Table 2: Probability distribution for 8192 shots

States	Prob. in IBM-16-Melbourne (p_i)	Prob. in IBM qsam simulator (q_i)
00	0.249	0.503
01	0.208	-
10	0.285	-
11	0.258	0.497

5 Conclusion

Cluster states are created through appropriate circuit diagrams involving only quantum gates which have been experimentally realized here. We have successfully implemented quantum information splitting using the IBM Quantum computer. Splitting of any arbitrary two-qubit quantum state among parties has been verified in quantum simulator and QISKit (IBM Quantum Processor) taking different numbers of shots. The probability distribution of each state has been observed. Here we have shown only for Bell state. In the qasm simulator, Alice can split Bell state to Charlie without any error, but in the real device, we were getting an error due to a large number of CNOT gates present in our circuit. Currently available open quantum computing platform, IBM has high gate errors, which prevent explicit simulation of the proposed circuit with error-correction. This proposed circuit can be run on a quantum processor (real device) in the future without any error.

Acknowledgments

A.B. and S.S. acknowledge Indian Institute of Science Education and Research, Kolkata for accommodating hospitality. B.K.B. acknowledges the support of Institute fellowship provided by IISER Kolkata. We are notably grateful to IBM quantum experience project. The discussions and opinions developed in this paper are only those of the authors and do not reflect the opinions of IBM or any of its employees.

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