Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A = $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and 2A + B is a null matrix, then B is equal to: **[1]**

a) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

b) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$

c) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

d) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$

2. If x, y, z are non-zero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is **[1]**

a) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} \mathrm{x}^{-1} & 0 & 0 \\ 0 & \mathrm{y}^{-1} & 0 \\ 0 & 0 & \mathrm{z}^{-1} \end{bmatrix}$

c) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

d) $\frac{5yz}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. If A is an invertible matrix of order 3 and |A| = 5, then find |adj A|. **[1]**

a) 25

b) 5

c) -5

d) 0

4. The value of p and q for which the function f(x) = $\begin{cases} \frac{\sin(p+1)x+\sin x}{x} & , x < 0 \\ q & , x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{x^{\frac{3}{2}}} & , x > 0 \end{cases}$ is continuous for all x ∈ R, are **[1]**

a) p = $-\frac{3}{2}$, q = $\frac{1}{2}$

b) p = $-\frac{3}{2}$, q = $-\frac{1}{2}$

c) p = $\frac{5}{2}$, q = $\frac{7}{2}$

d) p = $\frac{1}{2}$, q = $\frac{3}{2}$

5. The angle between the lines 2x = 3y = -z and 6x = -y = -4z is **[1]**

a) 90^o

b) 0^o

c) 45^o

d) 30^o

6. The differential equation of the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ is called **[1]**

a) non-homogeneous differential equation

b) homogeneous differential equation

c) partial differential equation

d) linear differential equation

7. The maximum value of Z = 4x + 3y subject to constraint x + y $\leq$ 10, xy $\geq$ 0 is **[1]**

a) 40

b) 36

c) 20

d) 10

8. Range of $cosec^{-1}x$ is **[1]**

a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - {0}

c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - {1}

d) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

9. $\int_{-\pi}^{\pi} \sin^5 x dx =$? **[1]**

a) $\frac{5\pi}{16}$

b) 2π

c) 0

d) $\frac{3\pi}{4}$

10. Conisder the matrices **[1]**

$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -2 & 1 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 4 & 3 \end{bmatrix}$, C = [1 2 6]

Then, which of the following is not defined?

a) BA

b) AB

c) CB

d) CA

11. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4), and (0, 5). If the maximum value of z = ax + by, where a, b > 0 occurs at both (2, 4) and (4, 0), then: **[1]**

a) 3a = b

b) 2a = b

c) a = 2b

d) a = b

12. The two adjacent side of a triangle are represented by the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$ The area of the triangle is **[1]**

a) 41 sq units

b) 36 sq units

c) 37 sq units

d) $\frac{41}{2}$ sq units

13. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and A^2 + xI = yA then the values of x and y are **[1]**

a) x = 6, y = 6

b) x = 5, y = 8

c) x = 8, y = 8

d) x = 6, y = 8

14. If A and B are two events such that $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{2}$, then the events A and B **[1]**

are

a) Equally likely event

b) Independent

c) Dependent

d) Mutually exclusive

15. The solution of the differential equation $\left(x^2+1\right)\frac{dy}{dx}+\left(y^2+1\right)=0$, is **[1]**

a) $y=\frac{1-x}{1+x}$

b) $y=\frac{1+x}{1-x}$

c) $y = 2 + x^2$

d) Y x(x - 1)

16. The scalar product of two nonzero vectors $\vec{a}$ and $\vec{b}$ is defined as **[1]**

a) $\vec{a}.\vec{b}=|\vec{a}|\left|\vec{b}\right|\cos\theta$

b) $\vec{a}.\vec{b}=2|\vec{a}|\left|\vec{b}\right|\cos\theta$

c) $\vec{a}.\vec{b}=2|\vec{a}|\left|\vec{b}\right|\sin\theta$

d) $\vec{a}.\vec{b}=|\vec{a}|\left|\vec{b}\right|\sin\theta$

17. The point of discontinuity of the function f(x) = $\begin{cases}2x+3, & \text{if } x\le 2\\ 2x-3, & \text{if } x>2\end{cases}$ is **[1]**

a) x = 2

b) x = -1

c) x = 0

d) x = 1

18. If the points A(-1, 3, 2), B(-4, 2, -2) and C(5, 5, λ) are collinear then the value of λ is **[1]**

a) 5

b) 10

c) 8

d) 7

19. **Assertion (A):** A particle moving in a straight line covers a distance of x cm in t second, where x = $t^3 + 3t^2 - 6t + 18$. The velocity of particle at the end of 3 seconds is 39 cm/s. **[1]**

Reason (R): Velocity of the particle at the end of 3 seconds is $\frac{dx}{dt}$ at t = 3.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. Let R be any relation in the set A of human beings in a town at a particular time. **[1]**

Assertion (A): If R = {(x, y) : x is wife of y}, then R is reflexive.

Reason (R): If R = {(x, y) : x is father of y}, then R is neither reflexive nor symmetric nor transitive.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Evaluate:- $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ **[2]**

OR

$\tan^{-1}\left(\tan\frac{3\pi}{4}\right)=?$

22. Show that $f(x)=\frac{1}{1+x^2}$ is neither increasing nor decreasing on R. **[2]**

23. A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall? **[2]**

OR

Show that f(x) = $\cos^2 x$ is a decreasing function on $(0, \frac{\pi}{2})$.

24. Evaluate: $\int \frac{\left\{e^{\sin^{-1} x}\right\}^2}{\sqrt{1-x^2}} dx$ **[2]**

25. Find values of k if area of triangle is 35 square units having vertices as (2, -6), (5, 4), (k, 4). **[2]**

Section C

26. Evaluate: $\int \frac{x+2}{\sqrt{x^2+2x-1}} dx$ **[3]**

27. A factory has two machines A and B. Past records show that the machine A produced 60% of the items of output and machine B produced 40% of the items. Further 2% of the items produced by machine A were defective and 1% produced by machine B were defective. If an item is drawn at random, what is the probability that it is defective? **[3]**

28. Evaluate the integral: $\int \sqrt{\cot \theta} d\theta$ **[3]**

OR

Evaluate $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x} dx$.

29. Find the general solution for the differential equation: $(x^2y - x^2)dx + (xy^2 - y^2)dy = 0$ **[3]**

OR

Find the particular solution of the differential equation $\left[x \sin^2\left(\frac{y}{x}\right) - y\right]$ dx + x dy = 0, given that y = $\frac{\pi}{4}$ when x = 1

30. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is || to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. **[3]**

OR

If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, find the unit vector in the direction of $2\vec{a} - \vec{b}$.

31. Show that the function f(x) defined by f(x) = $\begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$ is continuous at x = 0. **[3]**

Section D

32. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration. **[5]**

33. Let A = R - {3}, B = R - {1}. If $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$ $\forall x \in A$. Then, show that f is bijective. **[5]**

OR

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a - b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

34. Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. **[5]**

35. Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base. **[5]**

OR

Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

To hire a marketing manager, it's important to find a way to properly assess candidates who can bring radical changes and has leadership experience.

Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.

i. Find the probability that it is due to the appointment of Ajay (A). (1)

ii. Find the probability that it is due to the appointment of Ramesh (B). (1)

iii. Find the probability that it is due to the appointment of Ravi (C). (2)

OR

Find the probability that it is due to the appointment of Ramesh or Ravi. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Two motorcycles A and B are running at the speed more than allowed speed on the road along the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$, respectively.

i. Find the cartesian equation of the line along which motorcycle A is running. (1)

ii. Find the direction cosines of line along which motorcycle A is running. (1)

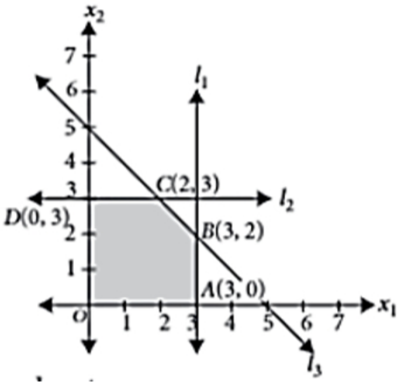
iii. Find the direction ratios of line along which motorcycle B is running. (2)

OR

Find the shortest distance between the given lines. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Corner points of the feasible region for an LPP are (0, 3), (5, 0), (6, 8), (0, 8). Let Z = 4x - 6y be the objective function.



i. At which corner point the minimum value of Z occurs? (1)

ii. At which corner point the maximum value of Z occurs? (1)

iii. What is the value of (maximum of Z - minimum of Z)? (2)

OR

The corner points of the feasible region determined by the system of linear inequalities are (2)

Solution

Section A

1.

(c) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

Explanation: $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

2.

(b) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

Explanation: Here, $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

Clearly, we can see that

$adjA = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix} \ and \ |A| = xyz$

$\therefore A^{-1} = \frac{adjA}{|A|} = \frac{1}{xyz}\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$

$= \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

3. **(a)** 25

Explanation: |A| = 5, |adj A| = $|A|^{3 - 1} = |A|^2 = 5^2 = 25$

4. **(a)** p = $-\frac{3}{2}$, q = $\frac{1}{2}$

Explanation: p = $-\frac{3}{2}$, q = $\frac{1}{2}$

5. **(a)** 90^o

Explanation: 90^o

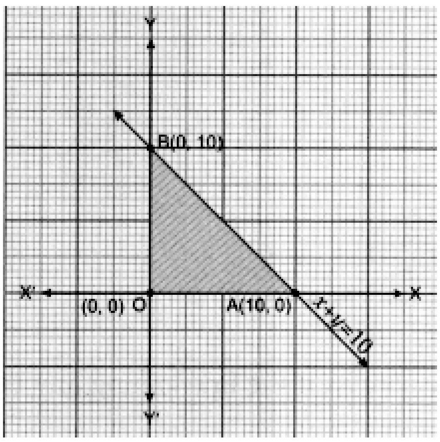
6.

(b) homogeneous differential equation

Explanation: The differential equation of the form $\frac{dy}{dx}$ = f$\left(\frac{y}{x}\right)$ or $\frac{dx}{dy}$ = g$\left(\frac{x}{y}\right)$ is called a homogeneous differential equation.

7. **(a)** 40

Explanation:



Feasible region is shaded region shown in figure with corner points 0(0, 0), A(10, 0), B(0, 10), Z(0, 0) = 0, Z (10, 0) = 40 $\longrightarrow$ maximum Z (0, 10) = 30

8.

(b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - {0}

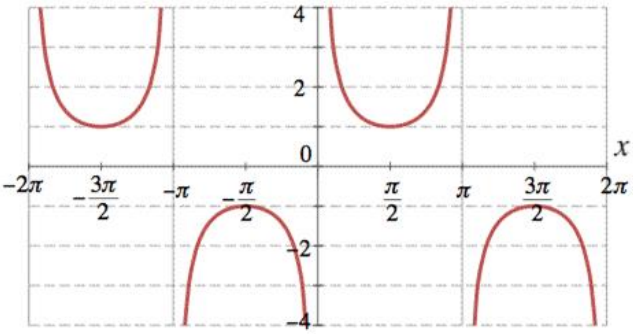
Explanation: To Find: The range of $\text{coses}^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $\text{coses}^{-1}(x)$ can be obtained from the graph of

$Y = \text{coses}^{-1}(x)$ by interchanging x and y axes.i.e, if a, b is a point on Y = cosec x then b, a is the point on the function $y = \text{coses}^{-1}(x)$

Below is the Graph of the range of $\text{coses}^{-1}(x)$



From the graph, it is clear that the range of $\text{coses}^{-1}(x)$ is restricted to interval
$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - {0}

9.

(c) 0

Explanation: If f is an odd function,
$\int_{-a}^{a} f(x)dx = 0$
as, $\int_{0}^{a} f(x)dx = -\int_{-a}^{0} f(x)dx$

$f(x) = \sin^5 x$

$f(-x) = \sin^5 (-x)$

Therefore, f(x) is odd number
$\int_{-\pi}^{\pi} \sin^5 x dx = 0$

10. **(a)** BA

Explanation: The given matrices are
$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -2 & 1 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 4 & 3 \end{bmatrix}$,and C = [1 2 6]

The order of A is 3×3, order of B is 3×2 and order of C is 1×3.

$\therefore$ CA, AB and CB are all defined.

But BA is not defined as number of columns in B is not equal to the number of rows in A.

11.

(c) a = 2b

Explanation: The maximum value of 'z' occurs at (2, 4) and (4, 0)

$\therefore$ Value of z at (2, 4) = value of z at (4, 0)

a(2) + b(4) = a(4) + b(0)

2a + 4b = 4a + 0

4b = 4a - 2a

4b = 2a

a = 2b

12.

(d) $\frac{41}{2}$ sq units

Explanation: $\vec{a} = 3\hat{\imath} + 4\hat{\jmath}$

$\vec{b} = -5\hat{\imath} + 7\hat{\jmath}$

For area of triangle we require $\frac{1}{2}|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = 41\hat{k}$

$\frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}\sqrt{41^2} = \frac{41}{2}$

13.

(c) x = 8, y = 8

Explanation: $A^2 + xI = yA$

$\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix} + x\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$

$\begin{pmatrix} 16 & 8 \\ 56 & 32 \end{pmatrix} + x\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$

$8\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} + x\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$

Comparing L.H.S. and R.H.S.

x = 8, y = 8

14.

(b) Independent

Explanation: Given: $P(A \cup B) = \left(\frac{5}{6}\right), P(A \cap B) = \left(\frac{1}{3}\right)$ and

$P(\bar{B}) = \left(\frac{1}{2}\right), P(B) = 1 - P(\bar{B}) = 1 - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow P(B) = \frac{1}{2}$

$\Rightarrow P(A) = \frac{2}{3}$

$Now, P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{6} = P(A) + \frac{1}{2} - \frac{1}{3}$

$P(A) = \frac{5}{6} - \frac{1}{2} + \frac{1}{3} = \frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

$P(A).\,P(B) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3} = P(A \cap B)$

$\Rightarrow$ Hence, these are independent.

15. **(a)** $y = \frac{1-x}{1+x}$

Explanation: $y = \frac{1-x}{1+x}$

16. **(a)** $\vec{a}.\vec{b} = |\vec{a}|\left|\vec{b}\right|\cos\theta$

Explanation: The scalar product of two nonzero vectors $\vec{a}$ and $\vec{b}$ is defined as: $\vec{a}.\vec{b} = |\vec{a}|\left|\vec{b}\right|\cos\theta$.

17. **(a)** x = 2

Explanation: At x = 2

LHL = $\lim\limits_{x \to 2^-}$ (2x + 3) = 2 $\times$ 2 + 3 = 7

RHL = $\lim\limits_{x \to 2^+}$ (2x - 3) = 2 $\times$ 2 - 3 = 1

$\because$ LHL $\neq$ RHL

$\therefore$ Point of discontinuity of the function is x = 2.

18.

(b) 10

Explanation: Determinant of these point should be zero

$\begin{vmatrix} -1 & 3 & 2 \\ -4 & 2 & -2 \\ 5 & 5 & \lambda \end{vmatrix} = 0$

$-1(2\lambda + 10) - 3(-4\lambda + 10)$ + 2(-20 - 10) = 0

10λ = 10 +30 +60= 100

λ = 10

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: We have,

$x = t^3 + 3t^2 - 6t + 18$

Velocity, $v = \frac{dx}{dt} = 3t^2 + 6t - 6$

Thus, velocity of the particle at the end of 3 seconds is

$\left(\frac{dx}{dt}\right)_{t=3} = 3(3)^2 + 6(3) - 6$

= 27 + 18 - 6 = 39 cm/s

20.

(d) A is false but R is true.

Explanation: Assertion: Here R is not reflexive: as x cannot be wife of x.

Reason: Here, R is not reflexive; as x cannot be father of x, for any x. R is not symmetric as if x is father of y, then y cannot be father of x. R is not transitive as if x is father of y and y is father of z, then x is grandfather (not father) of z.

Section B

21. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$

$= -\frac{\pi}{6} + \frac{\pi}{3} + \tan^{-1}(-1)$

$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}$

$= -\frac{\pi}{12}$

OR

$\tan^{-1}\left(\tan\frac{3\pi}{\pi}\right) \neq \frac{3\pi}{4}$ as $\frac{3\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\because \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left[-\tan\left(\frac{\pi}{4}\right)\right]$

$= -\frac{\pi}{4}$

22. Given:

$f(x) = \frac{1}{1+x^2}$

Let $x_1 > x_2$

$\Rightarrow \quad x_1^2 > x_2^2$

$\Rightarrow \quad 1 + x_1^2 > 1 + x_2^2$

$\Rightarrow \quad \frac{1}{1+x_1^2} < \frac{1}{1+x_2^2}$

$\Rightarrow \quad f(x_1) < f(x_2)$

f(x) is decreasing on $[0, \infty)$

Case 2

$\Rightarrow \quad x_1^2 < x_2^2$

$\Rightarrow \quad 1 + x_1^2 < 1 + x_2^2$

$\Rightarrow \quad \frac{1}{1+x_1^2} > \frac{1}{1+x_2^2}$

$\Rightarrow \quad f(x_1) > f(x_2)$

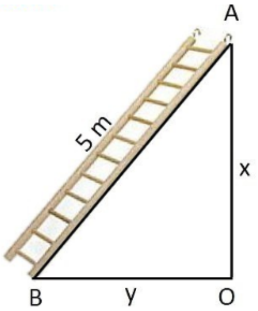
So, f(x) is increasing on $[0, \infty)$

Thus, f(x) is neither increasing nor decreasing on R.

23. Let AB be the ladder & length of ladder is 5m

i..e, AB = 5

& OB be the wall & OA be the ground.



Suppose OA = x & OB = y

Given that

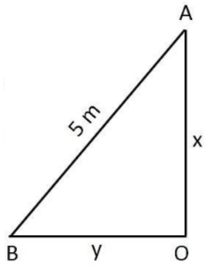
The bottom of the ladder is pulled along the ground, away the wall at the rate of 2cm/s i.e., $\frac{dx}{dt}$ = 2cm/sec ..... (i)

We need to calculate at which rate height of ladder on the wall.

Decreasing when foot of the ladder is 4 m away from the wall

i.e. we need to calculate $\frac{dy}{dt}$ when x = 4 cm

Wall OB is perpendicular to the ground OA



Using Pythagoras theorem,we get

$(OB)^2 + (OA)^2 = (AB)^2$

$y^2 + x^2 = (5)^2$

$y^2 + x^2 = 24$ .... (ii)

Differentiating w.r.t. time,we get

$\frac{d(y^2+x^2)}{dt} = \frac{d(25)}{dt}$

$\frac{d(y^2)}{dt} + \frac{d(x^2)}{dt} = 0$

$\frac{d(y^2)}{dt} \times \frac{dy}{dy} + \frac{d(x^2)}{dt} \times \frac{dx}{dx} = 0$

$2y \times \frac{dy}{dt} + 2x \times \frac{dx}{dt} = 0$

$2y \times \frac{dy}{dx} + 2x \times (2) = 0$

$2y\frac{dy}{dt} + 4x = 0$

$2y\frac{dy}{dt} = -4x$

$\frac{dy}{dt} = \frac{-4x}{2y}$

We need to find $\frac{dy}{dt}$ when x = 4cm

$\frac{dy}{dt}\Big|_{x=4} = \frac{-4\times4}{2y}$

$\frac{dy}{dt}\Big|_{x=4} = \frac{-16}{2y}$ ..... (iii)

Finding value of y

From (ii)

$x^2 + y^2 = 25$

Putting x = 4

$(4)^2 + y^2 = 25$

$y^2 = 9$

y = 3

OR

Given: f(x) = $\cos^2 x$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

i. If f’(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b)
ii. If f’(x) < 0 for all ,$x \in (a, b)$ then f(x) is decreasing on (a, b)

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = $\cos^2 x$

$\Rightarrow f(x) = \frac{d}{dx}(\cos^2 x)$

= f’(x) = 3cosx(–sinx)

= f’(x) = –2sin(x)cos(x)

= f’(x) = –sin2x ; as sin2A = 2sinA cosA

Now, as given

$x \in \left(0, \frac{\pi}{2}\right)$
= 2x ∈ (0,π)
= Sin(2x)> 0
= –Sin(2x)< 0
⇒ f'(x) < 0
hence, it is the condition for f(x) to be decreasing
Thus, f(x) is decreasing on interval $\left(0, \frac{\pi}{2}\right)$.

24. Let I = $\int \frac{\left\{e^{\sin^{-1} x}\right\}^2}{\sqrt{1-x^2}} dx$ ...(i)
Also let sin x = t then, we have
d($\sin^{-1}$ x) = dt
$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}} dx = dt$
$\Rightarrow \quad dx = \sqrt{1-x^2} dt$
Putting $\sin^{-1}$x = t and dx = $\sqrt{1-x^2} dt$ in equation (i), we get
$I = \int \frac{\left(e^t\right)^2}{\sqrt{1-x^2}} \times \sqrt{1-x^2} dt$
$= \int e^{2t} dt$
$= \frac{e^2}{2} + c$
$= \frac{e^{2 \sin^{-1} x}}{2} + c$
$\therefore I = \frac{\left\{e^{\sin^{-1} x}\right\}^2}{2} + c$

25. Area of triangle=35 units
$\Rightarrow \frac{1}{2}\begin{bmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{bmatrix} = \pm 35$
Expanding along row Ist,
$\Rightarrow \frac{1}{2}$[2(4 - 4)+ 6(5 - k)+1(20 - 4k)] = ± 35
$\Rightarrow \frac{1}{2}$[30 - 6k + 20 - 4k] = ± 35
$\Rightarrow \frac{1}{2}$[50 - 10k] =± 35
⇒25 - 5k = ±35
⇒25 - 5k = 35 or 25 - 5k = -35
⇒-5k = 10 or 5k = 60
⇒k = -2 or k = 12

Section C

26. Let the given integral be,
l = $\int \frac{x+2}{\sqrt{x^2+2x-1}} dx$
Let x + 2 = $\lambda \frac{d}{dx}$ (x^2 + 2x - 1) + μ
x + 2 = λ(2x + x) + μ
x + 2 = (2λ)x + 2λ + μ
Comparing the coefficients of like powers of x,
2λ = 1 ⇒ $\lambda = \frac{1}{2}$
2λ + μ = 2
$\Rightarrow \quad 2\left(\frac{1}{2}\right) + \mu = 2$
μ = 1
So, I_1 = $\int \frac{\frac{1}{2}(2x+2)+1}{\sqrt{x^2+2x-1}} dx$
= $\frac{1}{2} \int \frac{1}{\sqrt{x^2+2x-1}}$ (2x + 2) dx + 1 $\frac{1}{\sqrt{x^2+2x+(1)^2-(1)^2-1}} dx$
I = $\frac{1}{2} \int \frac{2x+2}{\sqrt{x^2+2x-1}} dx + 1 \frac{1}{(x+1)^2-(\sqrt{2})^2} dx$
I = $\frac{1}{2} 2\sqrt{x^2+2x-1}$+ log |x + 1 + $\sqrt{(x+1)^2-(\sqrt{2})^2}| + c$ [since, $\int \frac{1}{\sqrt{x}} dx - 2\sqrt{x}, \int \frac{1}{\sqrt{x^2-a^2}}$ dx - log |x + $\sqrt{x^2-a^2}$ |+

C]

I = $\sqrt{x^2+2x-1}$ + log |x + 1 + $\sqrt{x^2+2x-1}$| + c

27. Let A, E_1 and E_2 denote the events that the item is defective, machine A is selected and machine B is selected, respectievly.Therefore,we have,

$P(E_1) = \frac{60}{100}$

$P(E_2) = \frac{40}{100}$

Now,we have,

$P\left(\frac{A}{E_1}\right) = \frac{2}{100}$

$P\left(\frac{A}{E_2}\right) = \frac{1}{100}$

Using the law of total probability, we have,

Required probability = P(A) = $P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$

$= \frac{60}{100} \times \frac{2}{100} + \frac{40}{100} \times \frac{1}{100}$

$= \frac{120}{10000} + \frac{40}{10000}$

$= \frac{120+40}{10000} = \frac{160}{10000}$ = 0.016

28. I = $\int \sqrt{\cot\theta} d\theta$

Let cot θ = x^2

$\Rightarrow$ -cosec2 θ $d\theta$ = 2x dx

$\Rightarrow d\theta = \frac{-2x}{\cos ec^2\theta} dx$

$= \frac{-2x}{1+\cot^2\theta} dx$

$= \frac{-2x}{1+x^4} dx$

$\therefore$ I = $-\int \frac{2x^2}{1+x^4} dx$

$= -\int \frac{2}{\frac{1}{x^2}+x^2} dx$

Dividing numerator and denominator by x^2

$= -\int \frac{1+\frac{1}{x^2}+1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx$

$= -\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2}$ - $\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2-2}$

Let x - $\frac{1}{x}$ = t $\Rightarrow$ $\left(1+\frac{1}{x^2}\right)$dx = dt

and x + $\frac{1}{x}$ = z $\Rightarrow$ $\left(1-\frac{1}{x^2}\right) dx$ = dz

$\Rightarrow$ I = $-\int \frac{dt}{t^2+2}$ - $\int \frac{dz}{z^2-2}$

$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)$ - $\frac{1}{2\sqrt{2}}\log\left|\frac{z-\sqrt{2}}{z+\sqrt{2}}\right|$ + C

$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)$ - $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right|$ + C

I = $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\cot\theta-1}{\sqrt{2\cot\theta}}\right)$ - $\frac{1}{2\sqrt{2}}\log\left|\frac{\cot\theta+1-\sqrt{2\cot\theta}}{\cot\theta+1-\sqrt{2\cot\theta}}\right|$ + C

OR

Given I = $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x} dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} dx$

$\left[\begin{array}{c} \because \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} \\ \text{and } 1+\cos x = 2\cos^2\frac{x}{2} \end{array}\right]$

$\Rightarrow I = \frac{1}{2}\int_0^{\pi/2} \underset{I}{x} \sec\underset{II}{^2}\frac{x}{2}dx + \int_0^{\pi/2}\tan\frac{x}{2}dx$

$\Rightarrow I = \frac{1}{2}\left\{\left[x\int\sec^2\frac{x}{2}dx\right]_0^{\pi/2} - \int_0^{\pi/2}\left[\frac{d}{dx}(x)\int\left(\sec^2\frac{x}{2}dx\right)\right]dx\right\} + \int_0^{\pi/2}\tan\frac{x}{2}dx$

$\Rightarrow I = \frac{1}{2}\left\{\left[x\cdot\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi/2} - \int_0^{\pi/2}\frac{\tan\frac{x}{2}}{\frac{1}{2}}dx\right\}$

$+ \int_0^{\pi/2}\tan\frac{x}{2}dx$

[Integration by parts]

$= \left[x \cdot \tan\frac{x}{2}\right]_0^{\pi/2} - \int_0^{\pi/2}\tan\frac{x}{2}dx + \int_0^{\pi/2}\tan\frac{x}{2}dx$

$= \frac{\pi}{2}\cdot\tan\frac{\pi}{4} - 0$

$\therefore I = \frac{\pi}{2}\ \left[\because \tan\frac{\pi}{4} = 1\right]$

29. The given differential equation is,

$x^2 (y - 1) dx + y^2(x - 1) dy = 0$

$\frac{x^2}{x-1}dx + \frac{y^2}{y-1}dy = 0$

Add and subtract 1 in numerators ,we have,

$\frac{x^2-1+1}{(x-1)}dx + \frac{y^2-1+1}{(y-1)}dy$=0

By the identity $(a^2 - b^2) = (a + b).(a - b)$

$\frac{(x+1)(x-1)+1}{(x-1)}dx+\frac{(y+1)(y-1)+1}{(y-1)}dy$=0

Splitting the terms,

$(x+1)dx + \frac{1}{(x-1)}dx$ + (y + 1)dy $+\frac{1}{(y-1)}dy$=0

Integrating,we get,

$\int(x+1)dx + \int\frac{1}{(x-1)}dx+\int(y+1)dy + \int\frac{1}{(y-1)}dy = C$

$\frac{x^2}{2} + x + \log|x-1|+\frac{y^2}{2} + y + \log|y-1|$=C

$\frac{1}{2}.(x^2 + y^2) + (x + y) + \log|(x - 1)(y - 1)|$=C

This is the required solution.

OR

We can rewrite the given differential equation as,

$\frac{dy}{dx} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right)$

This is of the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ So, it is homogeneous.

Putting y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (i), we get

$v + x\frac{dv}{dx}$ = v $-\sin^2$ v

$\Rightarrow\quad x\frac{dv}{dx}$ = $-\sin^2$ v

$\Rightarrow -cosec^2 v dv = \frac{1}{x}dx$

$\Rightarrow \int\left(-cosec^2 v\right)dv = \int\frac{1}{x}dx$ [on integrating both sides]

$\Rightarrow$ cot v = log |x| + C, where C is an arbitrary constant

$\Rightarrow$ cot $\frac{y}{x}$ = log |x| + C ...(ii) $\left[\because v = \frac{y}{x}\right]$

Putting x = 1 and y = $\frac{\pi}{4}$ in (ii), we get C = 1.

$\therefore$ cot $\frac{y}{x}$ = log |x| + 1 is the desired solution.

30. Let $\vec{\beta}_1 = \lambda\vec{\alpha}\ \left[\because \vec{\beta}_1 || to\ \vec{\alpha}\right]$

$\vec{\beta}_1 = \lambda\left(3\hat{i} - \hat{j}\right)$

$= 3\lambda\hat{i} - \lambda\hat{j}$

$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$

$= \left(2\hat{i} + \hat{j} - 3\hat{k}\right) - \left(3\lambda\hat{i} - \lambda\hat{j}\right)$

$= (2 - 3\lambda)\hat{i} + (1 + \lambda)\hat{j} - 3\hat{k}$

$\vec{\alpha}.\vec{\beta}_2 = 0\ \ \left[\because \vec{\beta}_2 \perp \vec{\alpha}\right]$

$3(2 - 3\lambda) - (1 + \lambda) = 0$

$\lambda = \frac{1}{2}$

$\vec{\beta}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$

$\vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

OR

We need to find the unit vector in the direction of $2\vec{a} - \vec{b}$.

First, let us calculate $2\vec{a} - \vec{b}$.

As we have,

$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ ...(a)
$\vec{\mathrm{b}} = 2\hat{i} + \hat{j} - 2\hat{\mathrm{k}}$ ...(b)
Then multiply equation (a) by 2 on both sides,
$2\vec{\mathrm{a}} = 2(\hat{\mathrm{i}} + \hat{\mathrm{j}} + 2\hat{\mathrm{k}})$
We can easily multiply vector by a scalar by multiplying similar components, that is, vector's magnitude by the scalar's magnitude.
$\Rightarrow 2\vec{\mathrm{a}} = 2\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 4\hat{\mathrm{k}}$
Subtract (b) from (c). We get,
$2\vec{\mathrm{a}} - \vec{\mathrm{b}} = (2\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 4\hat{\mathrm{k}}) - (2\hat{\mathrm{i}} + \hat{\mathrm{j}} - 2\hat{\mathrm{k}})$
$\Rightarrow 2\vec{\mathrm{a}} - \vec{\mathrm{b}} = 2\hat{i} - 2\hat{\mathrm{i}} + 2\hat{\mathrm{j}} - \hat{\mathrm{j}} + 4\hat{\mathrm{k}} + 2\vec{k}$
$\Rightarrow 2\vec{\mathrm{a}} - \vec{\mathrm{b}} = \hat{\mathrm{j}} + 6\hat{\mathrm{k}}$
For finding unit vector, we have the formula:
$2\hat{a} - \hat{b} = \frac{2\vec{a} - \vec{b}}{|2\vec{a} - \vec{b}|}$
Now we know the value of $2\vec{a} - \vec{b}$, so we just need to substitute in the above equation.
$\Rightarrow 2\hat{a} - \hat{b} = \frac{\hat{j} + 6\hat{k}}{|\hat{j} + 6\hat{k}|}$
Here, $|\hat{j} + 6\hat{\mathrm{k}}| = \sqrt{1^2 + 6^2}$
$\Rightarrow 2\hat{a} - \hat{b} = \frac{\hat{j} + 6\hat{k}}{\sqrt{1^2 + 6^2}}$
$\Rightarrow 2\hat{\mathrm{a}} - \hat{\mathrm{b}} = \frac{\hat{\mathrm{j}} + 6\hat{\mathrm{k}}}{\sqrt{1 + 36}}$
$\Rightarrow 2\hat{a} - \hat{b} = \frac{\hat{j} + 6\hat{k}}{\sqrt{37}}$
Thus, unit vector in the direction of $2\vec{a} - \vec{b}$ is $\frac{\hat{j} + 6\hat{k}}{\sqrt{37}}$.

31. To show that the given function is continuous at x = 0, we show that
$(\text{LHL})_{x = 0} = (\text{RHL})_{x = 0} = f(0)$ ....(i)
Here, we have f(x) = $\begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1 - \sqrt{1 - x})}{x}, & x < 0 \end{cases}$
Now, LHL = $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{4(1 - \sqrt{1 - x})}{x}$
$= \lim_{h \to 0} \frac{4[1 - \sqrt{1 - (0 - h)}]}{0 - h}$
$= \lim_{h \to 0} \frac{4[1 - \sqrt{1 + h}]}{-h}$
$= \lim_{h \to 0} \frac{4[1 - \sqrt{1 + h}]}{-h} \times \frac{1 + \sqrt{1 + h}}{1 + \sqrt{1 + h}}$
$= \lim_{h \to 0} \frac{4\left[(1)^2 - (\sqrt{1 + h})^2\right]}{-h[1 + \sqrt{1 + h}]}$
$= \lim_{h \to 0} \frac{4[1 - (1 + h)]}{-h[1 + \sqrt{1 + h}]}$
$= \lim_{h \to 0} \frac{-h \times 4}{-h[1 + \sqrt{1 + h}]}$
$= \lim_{h \to 0} \frac{4}{1 + \sqrt{1 + h}}$
$= \frac{4}{1 + \sqrt{1}} = \frac{4}{2}$ = 2
and RHL = $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(\frac{\sin x}{x} + \cos x\right)$
$\Rightarrow \quad \text{RHL} = \lim_{h \to 0} \left(\frac{\sin h}{h} + \cos h\right)$
$= \lim_{h \to 0} \frac{\sin h}{h} + \lim_{h \to 0} \cos h$
= 1 + cos 0
= 1 + 1
= 2
Also , given that x = 0, f(x) = 2 $\Rightarrow$f(0) = 2

Since, $(LHL)_{x=0} = (RHL)_{x=0} = f(0) = 2$

Therefore, f(x) is continuous at x = 0.

Section D

32. According to the question ,

Given equation of circle is $x^2 + y^2 = 16$ ...(i)

Equation of line given is ,

$\sqrt{3}y = x$ ...(ii)

$\Rightarrow y = \frac{1}{\sqrt{3}}x$ represents a line passing through the origin.

To find the point of intersection of circle and line ,

substitute eq. (ii) in eq.(i) , we get

$x^2 + \frac{x^2}{3} = 16$

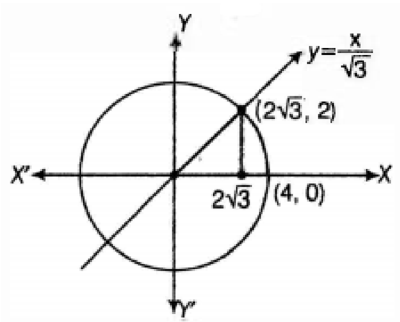
$\frac{3x^2+x^2}{3} = 16$

$\Rightarrow 4x^2 = 48$

$\Rightarrow x^2 = 12$

$\Rightarrow$ x= $\pm 2\sqrt{3}$

When x= $2\sqrt{3}$, then $y = \frac{2\sqrt{3}}{\sqrt{3}} = 2$



Required area (In first quadrant) = (Area under the line y = $\frac{1}{\sqrt{3}}$x from x = 0 to $2\sqrt{3}$) + (Area under the circle from x = $2\sqrt{3}$ to x=4)

$= \int_0^{2\sqrt{3}} \frac{1}{\sqrt{3}} x dx + \int_{2\sqrt{3}}^{4} \sqrt{16-x^2} dx$

$= \frac{1}{\sqrt{3}} \left[\frac{x^2}{2}\right]_0^{2\sqrt{3}}$ + $\left[\frac{x}{2}\sqrt{16-x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_{2\sqrt{3}}^{4}$

$= \frac{1}{2\sqrt{3}}\left[(2\sqrt{3})^2 - 0\right] + \left[0 + 8\sin^{-1}(1) - \frac{2\sqrt{3}}{2}\sqrt{16-12} - 8\sin^{-1}\left(\frac{2\sqrt{3}}{4}\right)\right]$

$= 2\sqrt{3} + 8\left(\frac{\pi}{2}\right) - \frac{2\sqrt{3}}{2} \times 2 - 8\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 2\sqrt{3} + 4\pi - 2\sqrt{3} - 8\left(\frac{\pi}{3}\right)$

$= 4\pi - \frac{8\pi}{3}$

$= \frac{12\pi - 8\pi}{3}$

$= \frac{4\pi}{3}$ sq units.

33. Given that, A = R - {3}, B = R - {1}.

$f : A \rightarrow B$ is defined by $f(x) = \frac{x-2}{x-3}\ \forall x \in A$

For injectivity

Let $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$

$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

So, f(x) is an injective function

For surjectivity

Let $y = \frac{x-2}{x-3} \Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = 2 - 3y $\Rightarrow x = \frac{2-3y}{1-y}$

$\Rightarrow x = \frac{3y-2}{y-1} \in A,\ \forall y \in B$ [codomain]

So, f(x) is surjective function.

Hence, f(x) is a bijective function.

OR

A = {1, 2, 3, 4, 5} and R = {(a, b) : |a - b| is even}, then R = {(1, 3), (1, 5), (3, 5), (2, 4)}

1. For (a, a), |a - a| = 0 which is even. ∴ R is reflexive.
   If |a - b| is even, then |b - a| is also even. ∴ R is symmetric.
   Now, if |a - b| and |b - c| is even then |a - b + b - c| is even
   ⇒ |a - c| is also even. ∴ R is transitive.
   Therefore, R is an equivalence relation.
2. Elements of {1, 3, 5} are related to each other.
   Since |1 - 3| = 2, |3 - 5| = 2, |1 - 5| = 4 all are even numbers
   ⇒ Elements of {1, 3, 5} are related to each other.
   Similarly elements of (2, 4) are related to each other.
   Since |2 - 4| = 2 an even number, then no element of the set {1, 3, 5} is related to any element of (2, 4).
   Hence no element of {1, 3, 5} is related to any element of {2, 4}.

34. $B' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$

Let $P = \frac{1}{2}(B + B') = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix}$

$P' = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} = P$

Thus $P = \frac{1}{2}(B + B')$ is a symmetric matrix

Let $Q = \frac{1}{3}(B - B') = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{5}{2} \\ \frac{-1}{2} & 0 & -3 \\ \frac{-5}{2} & 3 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$

Q' = -Q

Thus $Q = \frac{1}{2}(B - B')$ is a skew symmetric matrix

$P + Q = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$

35. Let r be the radius, h be the height, V be the volume and S be the total surface area of a right circular cylinder which is open at the top .

Now, given that $V = \pi r^2 h$

$\Rightarrow \quad h = \frac{V}{\pi r^2}$

We know that, total surface area S is given by

$S = 2\pi rh + \pi r^2$

[∵Cylinder is open at the top, therefore S= curved surface area of cylinder+area of base]

$\Rightarrow \quad S = 2\pi r\left(\frac{V}{\pi r^2}\right) + \pi r^2$

$\left[\text{put } h=\frac{V}{\pi r^2}, \text{ from Eq. (i)}\right]$

$\Rightarrow \quad S=\frac{2V}{r}+\pi r^2$

On differentiating both sides w.r.t.r, we get

$\frac{dS}{dr}=-\frac{\mathbf{2V}}{r^2}+2\pi r$

For maxima or minima, put $\frac{dS}{dr}=0$

$\Rightarrow \quad -\frac{2V}{r^2}+2\pi r=0 \Rightarrow V=\pi r^3$

$\Rightarrow \quad \pi r^2 h=\pi r^3 \quad [\because V=\pi r^2 h]$

$\Rightarrow \quad h=r$

Also, $\frac{d^2S}{dr^2}=\frac{d}{dr}\left(\frac{dS}{dr}\right)=\frac{d}{dr}\left(\frac{-\mathbf{2V}}{r^2}+2\pi r\right)$

$\Rightarrow \quad \frac{d^2S}{dr^2}=\frac{4V}{r^3}+2\pi$

On putting r=h, we get

$\left[\begin{matrix} d^2S \\ dr^2 \end{matrix}\right]_{r=h}=\frac{4V}{h^3}+2\pi>0 \text{ as } h>0$

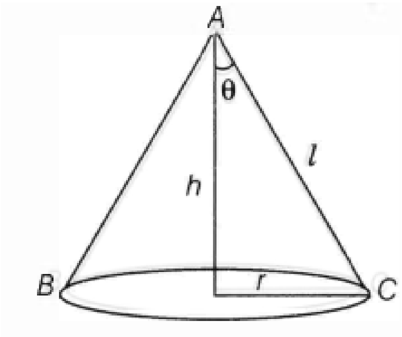
Then, $\frac{d^2S}{dr^2}>0$

Thus,S is minimum.

Hence, S is minimum, when h = r, i.e. when height of cylinder is equal to radius of the base.

OR

Let r be the radius of the base, h be the height, V be the volume, S be the surface area of the cone,slant height= AC = l and θ be the semi-vertical angle.



Then, $V=\frac{1}{3}\pi r^2 h$

$\Rightarrow \quad 3V=\pi r^2 h$

$\Rightarrow \quad 9V^2=\pi^2 r^4 h^2 \quad [\text{on squaring both sides}]$

$\Rightarrow \quad h^2=\frac{9V^2}{\pi^2 r^4}$ .....(i)

and curved surface area, $S=\pi r l$

$\Rightarrow \quad S=\pi r\sqrt{r^2+h^2} \quad \left[\because l=\sqrt{h^2+r^2}\right]$

$\Rightarrow \quad S^2=\pi^2 r^2(r^2+h^2)[\text{ on squaring both sides}]$

$\Rightarrow \quad S^2=\pi^2 r^2\left(\frac{9V^2}{\pi^2 r^4}+r^2\right)$ [from Eq. (i)]

$\Rightarrow \quad S^2=\frac{9V^2}{r^2}+\pi^2 r^4$ ....(ii)

When S is least, then S^2 is also least.

Now, $\frac{d}{dr}(S^2)=-\frac{18V^2}{r^3}+4\pi^2 r^3$ ....(iii)

For maxima or minima, put $\frac{d}{dr}(S^2)=0$

$\Rightarrow \quad -\frac{18V^2}{r^3}+4\pi^2 r^3=0$

$\Rightarrow \quad 18V^2=4\pi^2 r^6$

$\Rightarrow \quad 9V^2=2\pi^2 r^6$ ....(iv)

Again, on differentiating Eq. (iii) w.r.t.r, we get

$\frac{d^2}{dr^2}(S^2)=\frac{54V^2}{r^4}+12\pi^2 r^2>0$

At $r=\left(\frac{9V^2}{2\pi^2}\right)^{1/6}, \frac{d^2}{dr^2}(S^2)>0$

So, S^2 or S is minimum, when

$V^2=2\pi^2 r^6/9$

On putting $V^2 = 2\pi^2 r^6/9$ in Eq. (i) we get

$2\pi^2 r^6 = \pi^2 r^4 h^2$

$\Rightarrow \quad 2r^2 = h^2$

$\Rightarrow \quad h = \sqrt{2}r$

$\Rightarrow \quad \frac{h}{r} = \sqrt{2}$

$\Rightarrow \quad \cot\theta = \sqrt{2} \quad \left[\text{from the figure, } \cot\theta = \frac{h}{r}\right]$

$\therefore \quad \theta = \cot^{-1}\sqrt{2}$

Hence, the semi-vertical angle of the right circular cone of given volume and least cured surface area is $\cot^{-1}\sqrt{2}$.

Section E

36. i. Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{4}{7}\times 0.3}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{1.2}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{1.2}{7}}{\frac{3}{7}}$

$= \frac{1.2}{3} = \frac{12}{30} = \frac{2}{5}$

ii. Let E_1: Ajay(A) is selected, E_2: Ramesh(B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_2/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{1}{7}\times 0.8}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{0.8}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{0.8}{7}}{\frac{3}{7}}$

$= \frac{0.8}{3} = \frac{8}{30} = \frac{4}{15}$

iii. Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_3/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{2}{7}\times 0.5}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{1}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{1}{3}$

OR

Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

Ramesh or Ravi

$\Rightarrow P(E_2/A) + P(E_3/A) = \frac{4}{15} + \frac{1}{3} = \frac{9}{15} = \frac{3}{5}$

37. i. The line along which motorcycle A is running, $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$, which can be rewritten as

$(x\hat{i} + y\hat{j} + z\hat{k}) = \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$

$\Rightarrow x = \lambda$, $y = 2\lambda$, $z = -\lambda \Rightarrow \frac{x}{1} = \lambda$, $\frac{y}{2} = \lambda$, $\frac{z}{-1} = \lambda$

Thus, the required cartesian equation is $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

ii. Clearly, D.R.'s of the required line are < 1, 2, -1 >

$\therefore$ D.C.'s are

$(\frac{1}{\sqrt{1^2+2^2+(-1)^2}}, \frac{2}{\sqrt{1^2+2^2+(-1)^2}}, \frac{-1}{\sqrt{1^2+2^2+(-1)^2}})$

i.e., $(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}})$

iii. The line along which motorcycle B is running, is $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$, which is parallel to the vector $2\hat{i} + \hat{j} + \hat{k}$.

$\therefore$ D.R.'s of the required line are (2, 1, 1).

OR

Here, $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}, \vec{a}_2 = 3\hat{i} + 3\hat{j}, \vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix}$ = $3\hat{i}$ - $3\hat{j}$ - $3\hat{k}$

Now, $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j}) \cdot (3\hat{i} - 3\hat{j} - 3\hat{k})$

= 9 - 9 = 0

Hence, shortest distance between the given lines is 0.

38. i.

Corner Points	**Value of Z = 4x - 6y**
(0, 3)	4 × 0 - 6 × 3 = - 18
(5, 0)	4 × 5 - 6 × 0 = 20
(6, 8)	4 × 6 - 6 × 8 = - 24
(0, 8)	4 × 0 - 6 × 8= - 48

Minimum value of Z is - 48 which occurs at (0, 8).

ii.

Corner Points	**Value of Z = 4x - 6y**
(0, 3)	4 × 0 - 6 × 3 = - 18
(5, 0)	4 × 5 - 6 × 0 = 20
(6, 8)	4 × 6 - 6 × 8 = - 24
(0, 8)	4 × 0 - 6 × 8 = - 48

Maximum value of Z is 20, which occurs at (5, 0).

iii.

Corner Points	**Value of Z = 4x - 6y**
(0, 3)	4 × 0 - 6 × 3 = - 18
(5, 0)	4 × 5 - 6 × 0 = 20
(6, 8)	4 × 6 - 6 × 8 = - 24
(0, 8)	4 × 0 - 6 × 8 = - 48

Maximum of Z - Minimum of Z = 20 - (-48) = 20 + 48 = 68

OR

The corner points of the feasible region are O(0, 0), A(3, 0), B(3, 2), C(2, 3), D(0, 3).