Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 1

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A, B are square matrices of order 3, A is non-singular and AB = 0, then B is a **[1]**

a) non-singular matrix
b) null matrix
c) singular matrix
d) unit matrix

2. If A is a 2-rowed square matrix and IAI = 6 then A·adj A = ? **[1]**

a) $\begin{bmatrix} \frac{1}{6} & 0 \\ 0 & \frac{1}{6} \end{bmatrix}$
b) $\begin{bmatrix} 3 & 0 \\ 3 & 0 \end{bmatrix}$
c) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$
d) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

3. If A = $\begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$ then what is the value of A^{-1}. **[1]**

a) A^2
b) A^3
c) A
d) I

4. If $f(x) = x^2 \sin\frac{1}{x}$ where $x \neq 0$ then the value of the function f at x = 0, so that the function is continuous at x = 0, is **[1]**

a) -1
b) 1
c) 0
d) 2

5. The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is **[1]**

a) $5\sqrt{30}$
b) $\sqrt{30}$

c) $2\sqrt{30}$ d) $3\sqrt{30}$

6. What is the degree of the differential equation y = $x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^{-1}$? **[1]**

a) -1 b) 1

c) Does not exist d) 2

7. Minimize Z = 5x + 10 y subject to x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 **[1]**

a) Minimum Z = 310 at (60, 0) b) Minimum Z = 320 at (60, 0)

c) Minimum Z = 330 at (60, 0) d) Minimum Z = 300 at (60, 0)

8. Consider the vectors $\vec{a}$ = $\hat{i} - 2\hat{j} + \hat{k}$ and b = $4\hat{i} - 4\hat{j} + 7\hat{k}$. **[1]**

What is the vector perpendicular to both the vectors?

a) $-10\hat{i} + 3\hat{j} + 4\hat{k}$ b) $10\hat{i} - 3\hat{j} + 4\hat{k}$

c) $-10\hat{i} - 3\hat{j} + 4\hat{k}$ d) $10\hat{i} - 3\hat{j} - 4\hat{k}$

9. $\int$ $\sec^2(7 - 4x)$ dx = ? **[1]**

a) - 4 tan (7 – 4x) + C b) $\frac{-1}{4}\tan(7 - 4x) + C$

c) 4 tan (7 – 4x) + C d) $\frac{1}{4}\tan(7 - 4x) + C$

10. Let A = $\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$, then **[1]**

a) $A^2 = I$ b) $A^2 = 4$

c) $A^2 = A$ d) $A^2 = 0$

11. A Linear Programming Problem is as follows: **[1]**

Minimize Z = 2x + y

Subject to the constraints x $\geq$ 3, x $\leq$ 9, y $\geq$ 0

x - y $\geq$ 0, x + y $\leq$ 14

The feasible region has

a) 5 corner points including (0, 0) and (9, 5) b) 5 corner points including (7, 7) and (3, 3)

c) 5 corner points including (3, 6) and (9, 5) d) 5 corner points including (14, 0) and (9, 0)

12. What is the projection of $\vec{a} = (2\hat{i} - \hat{j} + \hat{k})$ on $\vec{b} = (\hat{i} - 2\hat{j} + \hat{k})$? **[1]**

a) $\frac{3}{\sqrt{5}}$ b) $\frac{4}{\sqrt{5}}$

c) $\frac{5}{\sqrt{6}}$ d) $\frac{2}{\sqrt{3}}$

13. If, A is square matrix of order 3 such that |A| = 3, then |adj A| is equal to ________ **[1]**

a) 3 b) 9

c) 27 d) 10

14. You are given that A and B are two events such that P(B) = $\frac{3}{5}$, P(A | B) = $\frac{1}{2}$ and P(A ∪ B) = $\frac{4}{5}$, then P(A) equals **[1]**

a) $\frac{1}{2}$ b) $\frac{1}{5}$

c) $\frac{3}{5}$ d) $\frac{3}{10}$

15. Which of the following is the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$? **[1]**

a) $y = A\cos x + B\sin x$

b) $y = (Ax + B)e^{-x}$

c) $y = Ae^{x} + Be^{-x}$

d) $y = (Ax + B)e^{x}$

16. If the position vectors of P and Q are $\hat{i} + 3\hat{j} - 7\hat{k}$ and $5\hat{i} - 2\hat{j} + 4\hat{k}$ respectively , then the cosine of the angle between $\overrightarrow{PQ}$ and y-axis is **[1]**

a) $\frac{4}{\sqrt{162}}$

b) $\frac{11}{\sqrt{162}}$

c) $\frac{5}{\sqrt{162}}$

d) $-\frac{5}{\sqrt{162}}$

17. If $f(x) = \begin{cases} \frac{1}{1+e^{1x}} & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$ then f(x) is **[1]**

a) none of these

b) differentiable but not continuous at x = 0

c) continuous but not differentiable at x = 0

d) continuous as well as differentiable at x = 0

18. Direction cosines of a line perpendicular to both x-axis and z-axis are: **[1]**

a) 0, 1, 0

b) 0, 0, 1

c) 1, 1, 1

d) 1, 0, 1

19. **Assertion (A):** $f(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing outside the interval (1, 2). **[1]**

Reason (R): $f'(x) < 0$ for $x \in (1, 2)$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The function f : R* → R* defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R* is the set of all non-zero real numbers. **[1]**

Reason (R): The function g : N → R* defined by $f(x) = \frac{1}{x}$ is one-one and onto.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Write the interval for the principal value of function and draw its graph: $\cot^{-1} x$. **[2]**

OR

Find the value of $\tan^{-1}\left(\tan \frac{2\pi}{3}\right)$.

22. A man 1.6m tall walks at the rate of 0.3 m/s away from a street light is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening? **[2]**

23. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is **[2]**

a. increasing

b. decreasing

OR

Find the intervals in which $f(x) = \frac{4x^2+1}{x}$ is increasing or decreasing.

24. Evaluate $\int_{-1}^{2}(|x+1| + |x| + |x-1|)dx$ **[2]**

25. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R. **[2]**

Section C

26. Evaluate: $\int \frac{x}{\sqrt{x+z}\sqrt{x+z}} dx$ **[3]**

27. Urn A contains 1 white, 2 black and 3 red balls; urn B contains 2 white, 1 black and 1 red ball; and urn C contains 4 white, 5 black and 3 red balls. One urn is chosen at random and two balls are drawn. These happen to be one white and one red. What is the probability that they come from urn A? **[3]**

28. Evaluate $\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right) dx.$ **[3]**

OR

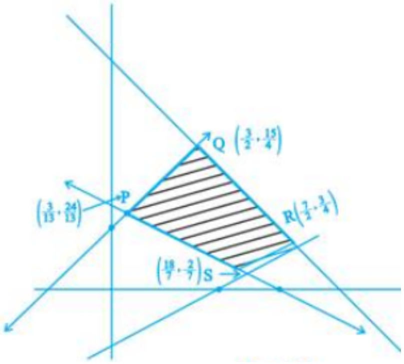
Evaluate: $\int \frac{x^2+1}{(x^2+2)(2x^2+1)} dx$

29. Find the general solution of the differential equation $\left(1+x^2\right)\frac{dy}{dx} - x = 2\tan^{-1} x$ **[3]**

OR

In the differential equation show that it is homogeneous and solve it: $x^2\frac{dy}{dx}$ = $x^2 + xy + y^2$.

30. In Fig, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of Z = x + 2y. **[3]**



OR

Solve the Linear Programming Problem graphically:

Maximize Z = 3x + 4y Subject to

$2x + 2y \leq 80$

$2x + 4y \leq 120$

31. If x = a$\left(\cos t + \log\tan\frac{t}{2}\right)$, y = a sin t, then evaluate $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$. **[3]**

Section D

32. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by sides x = 0, x = 4, y = 4 and y = 0 into three equal parts. **[5]**

33. Show that the function $f : R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, x ∈ R is one-one and onto function. **[5]**

OR

Let A = {1, 2, 3} and R = {(a, b):a, b ∈ A and $\left|a^2 - b^2\right| \leq 5$. Write R as set of ordered pairs. Mention whether R is

i. reflexive

ii. symmetric

iii. transitive

Give reason in each case.

34. Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations y + 2z = 7, x - y = 3, 2x + 3y + 4z = 17. **[5]**

35. Find the image of the point (0, 2, 3) in the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ **[5]**

OR

Show that the lines $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$ intersect and find their point of intersection.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

1. Find the probability that she gets grade A in all subjects. (1)
2. Find the probability that she gets grade A in no subjects. (1)
3. Find the probability that she gets grade A in two subjects. (2)

OR

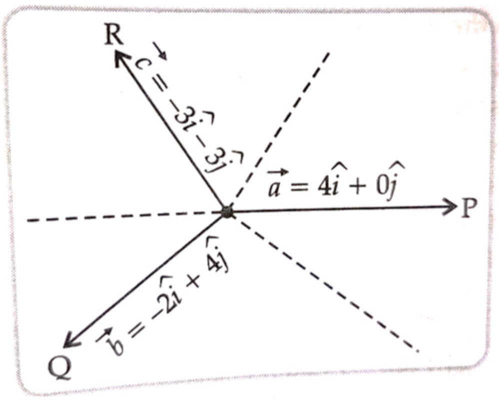
Find the probability that she gets grade A in at least one subject. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Team P, Q, R went for playing a tug of war game. Teams P, Q, R have attached a rope to a metal ring and is trying to pull the ring into their own areas (team areas when in the given figure below). Team P pulls with force $F_1 = 4\hat{i} + 0\hat{j}$ KN

Team Q pull with force $F_2 = -2\hat{i} + 4\hat{j}$ KN

Team R pulls with force $F_3 = -3\hat{i} - 3\hat{j}$ KN



i. What is the magnitude of the teams combined force? (1)

ii. Find the magnitude of Team B. (1)

iii. Which team will win the game? (2)

OR

In what direction is the ring getting pulled? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Mrs. Maya is the owner of a high-rise residential society having 50 apartments. When he set rent at ₹10000/month, all apartments are rented. If he increases rent by ₹250/ month, one fewer apartment is rented.

The maintenance cost for each occupied unit is ₹500/month.

i. If P is the rent price per apartment and N is the number of rented apartments, then find the profit. (1)

ii. If x represents the number of apartments which are not rented, then express profit as a function of x. (1)

iii. Find the number of apartments which are not rented so that profit is maximum. (2)

OR

Verify that profit is maximum at critical value of x by second derivative test. (2)

Solution

Section A

1.

(b) null matrix

Explanation: Since AB = 0

Given A is non-singular. Therefore

A^{-1} exist

Now, AB = 0

$\Rightarrow$ A $\times$(AB) = 0

$\Rightarrow$ (A $\times$ A)B = 0

$\Rightarrow$ I.B = 0

$\Rightarrow$ B = 0

Thus, B must be null matrix.

2.

(d) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

Explanation: A.(adj A) = |A|I

$= 6 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

3. **(a)** A^2

Explanation: A = $\begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$, |A| = 0 + 1 = 1

A^2 = $\begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}\begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$ = $\begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{vmatrix}$

adj(A) = $\begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{vmatrix}$

$\therefore$ A^{-1} = $\frac{adjA}{|A|}$ = $\frac{1}{1}\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ = $\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ = A^2

4.

(c) 0

Explanation: We have, $f(x) = x^2 \sin\left(\frac{1}{x}\right)$, where x $\neq$ 0

Since the function is continuous at x = 0, we have

f(0) = $\lim\limits_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$ ...(i)

Now, $-1 \leq sin\frac{1}{x} \leq 1$

$\Rightarrow -x^2 \leq x^2 sin\frac{1}{x} \leq x^2$

$\Rightarrow \lim\limits_{x \to 0}(-x^2) \leq \lim\limits_{x \to 0}(x^2 sin\frac{1}{x}) \leq \lim\limits_{x \to 0}(x^2)$

$\Rightarrow 0 \leq \lim\limits_{x \to 0}(x^2 sin\frac{1}{x}) \leq 0$

Therefore by squeeze principle, we have

f(0) = $\lim\limits_{x \to 0}(x^2 sin\frac{1}{x}) = 0$

Hence, value of the function f at x=0 so that it is continuous at x=0 is 0.

5.

(d) $3\sqrt{30}$

Explanation: Use formula for shortest distance between two skew lines.

6.

(d) 2

Explanation: Given differential equation is

y = x$\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^{-1}$ ⇒ y = x$\frac{dy}{dx}+\frac{1}{(dy/dx)}$

⇒ y$\left(\frac{dp}{dx}\right)$ = x$\left(\frac{dy}{dx}\right)^{2}$ + 1

∴ Degree = Power of highest derivative = 2

7.

(d) Minimum Z = 300 at (60, 0)

Explanation: Objective function is Z = 5x + 10 y ……………………(1).

The given constraints are : x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 .

The corner points are obtained by drawing the lines x+2y =120, x+y = 60 and x-2y = 0. The points so obtained are (60,30), (120,0), (60,0) and (40,20)

Corner points	Z = 5x + 10y
D(60 ,30)	600
A(120,0)	600
B(60,0)	300……………………..(Min.)
C(40,20)	400

Here , Z = 300 is minimum at (60, 0).

8.

(c) $-10\hat{i}-3\hat{j}+4\hat{k}$

Explanation: The vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ = $\vec{a}\times\vec{b}$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 1\\ 4 & -4 & 7\end{vmatrix}$

$=i(-14+4)-\hat{j}(7-4)+\hat{k}(-4+8)$

= $-10\hat{i}-3\hat{j}+4\hat{k}$

9.

(b) $\frac{-1}{4}\tan(7-4x)+C$

Explanation: Given integral is $\int$ $\sec^2$(7-4 x) dx=?

Let, 7 – 4x = z

⇒ -4dx = dz

So,

$\int$ $\sec^2$(7-4 x) dx=?

$=\int\sec^2 z\frac{dz}{-4}$

$=-\frac{1}{4}\int\sec^2 zdz$

$\int\sec^2(7-4x)dx$ where c is the integrating constant.

$=\int\sec^2 z\frac{dz}{-4}$

$=-\frac{1}{4}\int\sec^2 zdz$

$=-\frac{1}{4}\tan z+c$

$=-\frac{1}{4}\tan(7-4\mathrm{x})+\mathrm{c}$

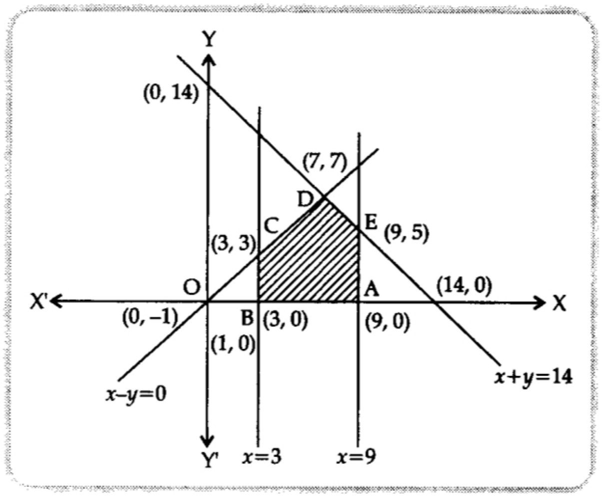
10.

(c) A^2 = A

Explanation: $A^2 = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$ = A

11.

(b) 5 corner points including (7, 7) and (3, 3)

Explanation: 

On plotting the constraints x = 3, x = 9, x = y and x + y = 14, we get the following graph. From the graph given below it, clear that feasible region is ABCDEA, including corner points A(9, 0), B(3, 0), C(3, 3), D(7, 7) and E(9, 5).

Thus feasible region has 5 corner points including (7, 7) and (3, 3).

12.

(c) $\frac{5}{\sqrt{6}}$

Explanation: Given vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}\, and\, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Now, projection $\vec{a}\ on\ \vec{b}\ is\ \vec{a}.\ \hat{b}$=$\frac{2+2+1}{\sqrt{4+1+1}} = \frac{5}{\sqrt{6}}$

13.

(b) 9

Explanation: ∵ |adj A| = $|A|^{n-1}$

Here, n = 3

∴ |adj A| = $|A|^{3-1}$

= $|A|^2 = (3)^2 = 9$

14. **(a)** $\frac{1}{2}$

Explanation: We have,

P(B) = $\frac{3}{5}$, P(A | B) and $P(A \cup B) = \frac{4}{5}$

Now, We know that

P(A|B) × P(B) = P(A ∩ B)

[Property of conditional Probability]

$\Rightarrow \frac{1}{2} \times \frac{3}{5} = P(A \cap B)$

$\Rightarrow P(A \cap B) = \frac{3}{10}$

Now,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

[Additive Law of Probability]

$\therefore \frac{4}{5} = P(A) + \frac{3}{5} - \frac{3}{10}$

$\Rightarrow P(A) = \frac{4}{5} - \frac{3}{5} + \frac{3}{10}$

$\Rightarrow P(A) = \frac{1}{5} + \frac{3}{10}$

$\Rightarrow P(A) = \frac{2+3}{10}$

$\Rightarrow P(A) = \frac{5}{10} = \frac{1}{2}$

15.

(d) $y = (Ax + B)e^x$

Explanation: For $y = (Ax + B)e^x$

$\frac{dy}{dx}$ = (Ax + B)e^x + Ae^x = (Ax + A + B)e^x

$\Rightarrow \frac{d^2y}{dx^2}$ = (Ax + A + B)e^x + Ae^x = (Ax + 2A + B)e^x

$\therefore \frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right) + y$

= (Ax + 2A + B)e^x - 2(Ax + A + B)e^x + (Ax + B)e^x

= 0

16.

(d) $-\frac{5}{\sqrt{162}}$

Explanation: Given position vectors $\overrightarrow{OP} = \hat{i} + 3\hat{j} - 7\hat{k}$ and $\overrightarrow{OQ} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

$\implies drs$ of $\overrightarrow{PQ}$ =$\overrightarrow{OQ} - \overrightarrow{OP} = 4\hat{i} - 5\hat{j} + 11\hat{k}$ and drs along with y-axis are (0,1,0) 0r $\hat{j}$

direction cosines between $\overrightarrow{PQ}$ and y-axis is $\frac{(4\hat{i}-5\hat{j}+11k).\hat{j}}{\sqrt{16+25+121}}$=$\frac{-5}{\sqrt{162}}$

17. **(a)** none of these

Explanation: Given that $f(x) = \left\{\begin{array}{l}\frac{1}{1+e^{\frac{1}{x}}}, x \neq 0 \\ 0, x = 0\end{array}\right\}$

Checking continuity at x = 0,

LHL: $\lim_{x \to 0^-} \frac{1}{1+e^{\frac{1}{x}}} = 1$

But f(x = 0) = 0

Hence, function is neither continuous nor differentiable at x = 0

18. **(a)** 0, 1, 0

Explanation: 0, 1, 0

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion: We have, f(x) = 2x^3 - 9x^2 + 12x - 3

$\Rightarrow$ f'(x) = 6x^2 - 18x + 12

For increasing function, f'(x) $\geq$ 0

$\therefore$ 6(x^2 - 3x + 2) $\geq$ 0

$\Rightarrow$ 6(x - 2)(x - 1) $\geq$ 0

$\Rightarrow$ x $\leq$ 1 and x $\geq$ 2

$\therefore$ f(x) is increasing outside the interval (1, 2), therefore it is a true statement.

Reason: Now, f'(x) < 0

$\Rightarrow$ 6(x - 2)(x - 1) < 0

$\Rightarrow$ 1 < x < 2

$\therefore$ Assertion and Reason are both true but Reason is not the correct explanation of Assertion.

20.

(c) A is true but R is false.

Explanation: Assertion: It is given that f : R* $\to$ R* is defined by

$f(x) = \frac{1}{x}$

For one-one, f(x) = f(y)

$\Rightarrow \frac{1}{x} = \frac{1}{y}$

$\Rightarrow$ x = y

Therefore, f is one-one.

For onto, it is clear that for $y \in R^*$, there exists $x = \frac{1}{y} \in R^*$ (exists as $y \neq 0$) such that

$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y$

Therefore, f is onto. Thus, the given function (f) is one-one and onto.

Reason: Now, consider function g : N $\to$ R* defined by $g(x) = \frac{1}{x}$.

We have, $g\left(x_1\right) = g\left(x_2\right) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_1 = x_2$

Therefore, g is one-one.

Further, it is clear that g is not onto as for $1 \cdot 2 \in R^*$, there does not exist any x in N such that g(x) = 1·2

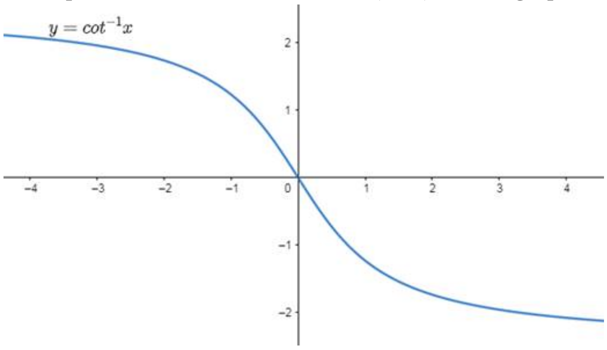
$\Rightarrow \frac{1}{x} = 1 \cdot 2$

$\Rightarrow x = \frac{1}{1 \cdot 2} \notin N$ (domain)

Hence, function g is one-one but not onto.

Section B

21. Principal value branch of $\cot^{-1} x$ is (0, π) and its graph is shown below.



OR

We have, $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\tan\left(\pi - \frac{\pi}{3}\right)$

$= \tan^{-1}\left(-\tan\frac{\pi}{3}\right)$ $[\because \tan^{-1}(-x) = -\tan^{-1}x]$

$= \tan^{-1}\tan\left(\frac{-\pi}{3}\right) = -\frac{\pi}{3}$ $\left[\because \tan^{-1}(\tan x) = x,\ x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$

Note: Remember that, $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$

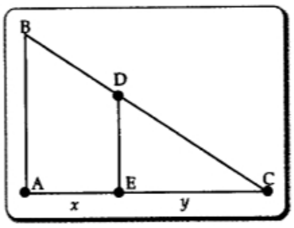
Since, $\tan^{-1}(\tan x) = x$, if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\frac{2\pi}{3} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

22. Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6 m be at a distance of x m from AB and the length of his shadow EC by y m.

Using similarity of triangles, we have

$\frac{4}{1.6} = \frac{x+y}{y}$

$\Rightarrow 3y = 2x$



Differentiating both sides w.r.t. t, we get

$3\frac{dy}{dt} = 2\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{2}{3} \times 0.3 \Rightarrow \frac{dy}{dt} = 0.2$

At any time t seconds, the tip of his shadow is at a distance of (x + y) m from AB.

The rate at which the tip of his shadow moving $= \left(\frac{dx}{dt} + \frac{dy}{dt}\right)$ m/s = 0.5 m/s

The rate at which his shadow is lengthening $= \frac{dy}{dt}$ m/s $= 0.2$ m/s

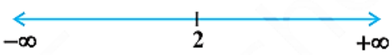
23. We have

f (x) = $x^2 - 4x + 6$

or f ′(x) = 2x – 4

Therefore, f ′(x) = 0 gives x = 2.

Now the point x = 2 divides the real line into two disjoint intervals namely, (– ∞, 2) and (2, ∞).



In the interval (– ∞, 2), f ′(x) = 2x – 4 < 0.

And in interval (2, ∞), $f'(x) = 2x - 4 > 0$

∴ (i) f is increasing in (2, ∞)

and (ii) f is decreasing in (– ∞, 2)

OR

Given: f(x) $= \frac{4x^2+1}{x}$

$\Rightarrow\ f(x) = 4x + \frac{1}{x} \Rightarrow f'(x) = 4 - \frac{1}{x^2} = \frac{4x^2-1}{x^2}$

For f(x) to be increasing, we must have

$f'(x) > 0$

$\Rightarrow\quad \frac{4x^2-1}{x^2} > 0$

$\Rightarrow\quad 4x^2 - 1 > 0$

$\Rightarrow\quad (2x-1)(2x+1) > 0$

$(x - 1/2)(x + 1/2) > 0$

$x < -1/2$ or, $x > 1/2$

$x \in (-\infty, -1/2) \cup (1/2, \infty)$

So , f(x)is increasing on $(-\infty, -1/2) \cup (1/2, \infty)$

for f(x) is to be decreasing, we must have

$\Rightarrow\quad \frac{4x^2-1}{x^2} < 0$

$\Rightarrow\quad 4x^2 - 1 < 0$

$(2x-1)(2x+1) < 0$

$-\frac{1}{2} < x < \frac{1}{2}$

$x \in (-1/2, 1/2)$

But domain f R - $\{0\}$. So, $f(x)\ (-1/2, 0) \cup (0, 1/2)$

24. Let I = $\int_{-1}^{2}(|x+1| + |x| + |x-1|)dx$, then

$I = \int_{-1}^{2}|x+1|dx + \int_{-1}^{2}|x|dx + \int_{-1}^{2}|x-1|dx$

$= \int_{-1}^{2}(x+1)dx - \int_{-1}^{0}x dx + \int_{0}^{2}x dx - \int_{-1}^{1}(x-1)dx + \int_{1}^{2}(x-1)dx$

$= \left\{\frac{x^2}{2} + x\right\}_{-1}^{2} - \left\{\frac{x^2}{2}\right\}_{-1}^{0} + \left\{\frac{x^2}{2}\right\}_{0}^{2} - \left\{\frac{x^2}{2} - x\right\}_{-1}^{1} + \left\{\frac{x^2}{2} - x\right\}_{1}^{2}$

$= \left\{(4) - \left(-\frac{1}{2}\right)\right\} - \left\{-\frac{1}{2}\right\} + \{2\} - \left\{\left(-\frac{1}{2}\right) - \left(\frac{3}{2}\right)\right\} + \left\{(0) - \left(-\frac{1}{2}\right)\right\}$

$= \left\{4 + \frac{1}{2}\right\} + \left\{\frac{1}{2}\right\} + \{2\} + \{2\} + \left\{\frac{1}{2}\right\} = \frac{19}{2}$

25. Given, f(x) = $x^3 - 3x^2 + 6x - 100$

Therefore, on differentiating both sides w.r.t. x, we get,

f'(x) = $3x^2 - 6x + 6$

= $3x^2 - 6x + 3 + 3$

= $3(x^2 - 2x + 1) + 3$

= $3(x-1)^2 + 3 > 0$

$\therefore$ f' (x) > 0

This shows that function f(x) is increasing on R.

Section C

26. Rationalize the given integrand we get

$\Rightarrow \int \frac{x}{\sqrt{x+a} - \sqrt{x-b}} \times \frac{\sqrt{x+a} + \sqrt{x-b}}{\sqrt{x+a} + \sqrt{x-b}} dx$

$\Rightarrow \int \frac{x(\sqrt{x+a} - \sqrt{x-b})}{x+a-x-b} dx$

$\Rightarrow \int \frac{x(\sqrt{x+a} - \sqrt{x-b})}{a-b} dx$

$\Rightarrow \frac{1}{a-b} \int x(\sqrt{x+a} - \sqrt{x-b}) dx$

Assume x = $\sqrt{t}$

$\Rightarrow dx = \frac{dt}{2\sqrt{t}}$

Substituting values of t, and dt,

$\Rightarrow \int \sqrt{t} \frac{(\sqrt{\sqrt{t}+a} - \sqrt{\sqrt{t}-b})}{2\sqrt{t}(a-b)} dt$

$\Rightarrow \frac{1}{2(a-b)} \int (\sqrt{\sqrt{t}+a} - \sqrt{\sqrt{t}-b}) dt$

$\Rightarrow \frac{1}{2(a-b)} \int (\sqrt{t}+a)^{1/2} dt - \int (\sqrt{t}-b)^{1/2} dt$

$\Rightarrow \frac{1}{2(a-b)} \left(\frac{4}{3} \left(\sqrt{t} + a^2\right)^{\frac{3}{2}} - \frac{4}{3} \left(t - a^2\right)^{\frac{3}{2}} \right)$

now replacing x = $\sqrt{t}$

$\Rightarrow \frac{1}{2(a-b)}\left(\frac{2}{3}(x+a)^{\frac{3}{2}}-\frac{2}{3}(x-b)^{\frac{3}{2}}\right)$

27. Let E_1, E_2, E_3 be the events that the balls are drawn from urn A, urn B and urn C respectively, and let E be the event that the balls drawn are one white and one red. Therefore,we have,

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Since E_1, E_2, E_3 are mutually exclusive and exhaustive, therefore,we have,

$P\left(\frac{E}{E_1}\right)$probability that the balls drawn are one white and one red, given that the balls are from urn A

= $\frac{1C_1\times{}^3C_1}{{}^6C_2}=\frac{3}{15}=\frac{1}{5}$

$P\left(\frac{E}{E_2}\right)$ = probability that the balls drawn are one white and one red, given that the balls are from urn B

= $\frac{{}^2C_1\times{}^1C_1}{{}^4C_2}=\frac{2}{6}=\frac{1}{3}$

$P\left(\frac{E}{E_3}\right)$ = probability that the balls drawn are one white and one red, given that the balls are from urn C

= $\frac{{}^4C_1\times{}^3C_1}{{}^{12}C_2}=\frac{12}{66}=\frac{2}{11}$

Therefore,we have,

Probability that the balls drawn are from urn A, it being given that the balls drawn are one white and one red

$=P\left(\frac{E_1}{E}\right)$

= $\frac{P\left(\frac{E}{E_1}\right)\cdot P(E_1)}{P\left(\frac{E}{E_1}\right)\cdot P(E_1)+P\left(\frac{E}{E_2}\right)\cdot P(E_2)+P\left(\frac{E}{E_3}\right)\cdot P(E_3)}$ [by Bayes's theorem]

= $\frac{\left(\frac{1}{5}\times\frac{1}{3}\right)}{\left(\frac{1}{5}\times\frac{1}{3}\right)+\left(\frac{1}{3}\times\frac{1}{3}\right)+\left(\frac{2}{11}\times\frac{1}{3}\right)}$

= $\left(\frac{1}{15}\times\frac{495}{118}\right)=\frac{33}{118}$

Hence, the required probability is $\frac{33}{118}$

28. According to the question, $I=\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

$=\int e^{2x}\left(\frac{1-2\sin x\cos x}{2\sin^2 x}\right)dx\left[\begin{array}{l}\because 1-\cos 2x=2\sin^2 x\\ \text{and }\sin 2x=2\sin x\cos x\end{array}\right]$

$=\frac{1}{2}\int e^{2x}\left(cosec^2 x-2\cot x\right)dx$

$=\frac{1}{2}\int \underset{I}{e^{2x}}\underset{II}{\cos}ec^2 x dx-\int e^{2x}\cot x dx$

Using integration by parts for first integral :

$=\frac{1}{2}\left[e^{2x}\int\cos ec^2 x dx-\int\left\{\frac{d}{dx}\left(e^{2x}\right)\int\cos ec^2 x dx\right\}dx\right]-\int e^{2x}\cot x dx$

$=\frac{1}{2}\left[-e^{2x}\cot x+\int 2e^{2x}\cot x dx\right]+C-\int e^{2x}\cot x dx$

$=-\frac{e^{2x}}{2}\cot x+\int e^{2x}\cot x dx-\int e^{2x}\cot x dx+C$

$I=-\frac{e^{2x}}{2}\cot x+C$

OR

Let x^2 = y. Then,

$\frac{x^2+1}{(x^2+2)(2x^2+1)}$ = $\frac{y+1}{(y+2)(2y+1)}$

Using partial fractions,

$\frac{y+1}{(y+2)(2y+1)}$ = $\frac{A}{y+2}+\frac{B}{2y+1}$ ......(i)

$\Rightarrow$ y + 1 = A (2y + 1) + B (y + 2)

Putting y + 2 = 0 i.e. y = - 2 in (ii), we get

-1 = -3A $\Rightarrow$ A = $\frac{1}{3}$

Putting 2y + 1 = 0 i.e. y = -$\frac{1}{2}$ in (ii), we get

$\frac{1}{2}=B\left(\frac{3}{2}\right)\Rightarrow B=\frac{1}{3}$

Substituting the values of A and B in (i), we obtain

$\frac{y+1}{(y+2)(2y+1)}$ = $\frac{1}{3}\cdot\frac{1}{y+2}+\frac{1}{3}\frac{1}{(2y+1)}$

Replacing y by x^2, we get

$\frac{x^2+1}{(x^2+2)(2x^2+1)} = \frac{1}{3} \cdot \frac{1}{x^2+2} + \frac{1}{3(2x^2+1)}$

$\therefore$ I = $\int \frac{x^2+1}{(x^2+2)(2x^2+1)} dx = \frac{1}{3} \int \frac{1}{x^2+2} dx + \frac{1}{3} \int \frac{1}{(\sqrt{2}x)^2+1} dx$

$\Rightarrow$ I = $\frac{1}{3} \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{3\sqrt{2}} \tan^{-1}(\sqrt{2}x)$ + C

$\Rightarrow$ I = $\frac{1}{3\sqrt{2}}\left\{\tan^{-1}\frac{x}{\sqrt{2}} + \tan^{-1}\sqrt{2}x\right\}$ + C

29. The given differential equation can be rewritten as

$\frac{dy}{dx} = \frac{2\tan^{-1}x}{(1+x^2)} + \frac{x}{(1+x^2)}$

$\Rightarrow \quad dy = \left\{\frac{2\tan^{-1}x}{(1+x^2)} + \frac{x}{(1+x^2)}\right\} dx$ [separating the variables]

$\Rightarrow \int dy = \int \frac{2\tan^{-1}x}{(1+x^2)} dx + \int \frac{x}{(1+x^2)} dx + C$ where C is an arbitrary constant

$\Rightarrow y = 2\int t\, dt + \frac{1}{2} \int \frac{2x}{(1+x^2)} dx + C$

[put $\tan^{-1}$ x = t and $\frac{1}{(1+x^2)} dx = dt$ in 1st integral]

$\Rightarrow y = t^2 + \frac{1}{2}\log|1+x^2| + C$

$\Rightarrow y = \left(\tan^{-1}x\right)^2 + \frac{1}{2}\log|1+x^2| + C$

Therefore, y = $\left(\tan^{-1}x\right)^2 + \frac{1}{2}\log|1+x^2| + C$ is the required solution.

OR

The given differential equation is,

$x^2\frac{dy}{dx}$ = $x^2 + xy + y^2$

$\Rightarrow \frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$

$\Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$

$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$

$\Rightarrow$the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put y = vx

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{v} + \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}$

$v + x\frac{dv}{dx} = 1 + \frac{vx}{x} + \left(\frac{vx}{x}\right)^2$ = 1 + v + $(v)^2$

$\Rightarrow x\frac{dv}{dx}$ = 1 + v + $(v)^2$ - v = 1 + $(v)^2$

$\Rightarrow \frac{dv}{1+(v)^2} = \frac{dx}{x}$

Integrating both the sides we get:

$\Rightarrow \int \frac{dv}{1+(v)^2} = \int \frac{dx}{x} + c$

$\Rightarrow$ $\tan^{-1}v$ = ln |x| + c

Resubstituting the value of y = vx, we get,

$\Rightarrow tan^{-}(\frac{y}{x})$= ln |x| + c

30. From the shaded bounded region, it is clear that the coordinates of corner points are $\left(\frac{3}{13}, \frac{24}{13}\right)$, $\left(\frac{18}{7}, \frac{2}{7}\right)$, $\left(\frac{7}{2}, \frac{3}{4}\right)$ and $\left(\frac{3}{2}, \frac{15}{4}\right)$

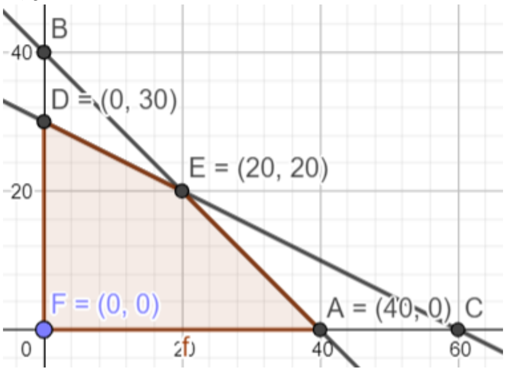
Also, we have to determine maximum and minimum value of Z = x + 2y.

Corner Points	**Corresponding value of Z**
$\left(\frac{3}{13}, \frac{24}{13}\right)$	$\frac{3}{13} + \frac{48}{13} = \frac{51}{13} = 3\frac{12}{13}$
$\left(\frac{18}{7}, \frac{2}{7}\right)$	$\frac{18}{7} + \frac{4}{7} = \frac{22}{7} = 3\frac{1}{7}$ (Minimum)
$\left(\frac{7}{2}, \frac{3}{4}\right)$	$\frac{7}{2} + \frac{6}{4} = \frac{20}{4} = 5$
$\left(\frac{3}{2}, \frac{15}{4}\right)$	$\frac{3}{2} + \frac{30}{4} = \frac{36}{4} = 9$ (Maximum)

Hence, the maximum and minimum value of are 9 and $3\frac{1}{7}$ respectively.

OR

We have to maximize z = 3x + 4y First, we will convert the given inequations into equations, we obtain the following equations:
2x + 2y = 80,2x + 4y = 120
Region represented by 2x + 2y $\leq$ 80 :
The line 2x + 2y = 80 meets the coordinate axes at A(40,0) and B(0,40) respectively. By joining these points we obtain the line 2x + 2y = 80 Clearly (0,0) satisfies the inequation 2x + 2y $\leq$ 80 . So, the region containing the origin represents the solution set of the inequation 2x + 2y $\leq$ 80
Region represented by 2x + 4y $\leq$ 120 :
The line 2x + 4y = 120 meets the coordinate axes at
C(60,0) and D(0,30) respectively. By joining these points we obtain the line 2x + 4y $\leq$ 120
Clearly (0,0) satisfies the inequation 2x + 4y $\leq$ 120 . So, the region containing the origin represents the solution set of the inequation 2x + 4y $\leq$ 120
The feasible region determined by subject to the constraints are, 2x + 2y $\leq$ 80,2x + 4y $\leq$ 120 and the non-negative restrictions $x, y \geq 0$ are as follows:



The corner points of the feasible region are O(0,0), A(40,0) E(20,20) and D(0,30)
The values of objective function at the corner points are as follows:

Corner point	Z = 3x + 4y
O(0, 0)	$3 \times 0 + 4 \times 0 = 0$
A(40, 0)	$3 \times 40 + 4 \times 0 = 120$
F(20, 20)	$3 \times 20 + 4 \times 20 = 140$
D(0, 30)	$10 \times 0 + 4 \times 30 = 120$

We see that the maximum value of the objective function Z is 140 which is at E(20,20) that means at x = 20 and y = 20
Thus, the optimal value of objective function z is 140.

31. Given, x = a$\left(\cos t + \log \tan \frac{t}{2}\right)$ ....(i)
and y = a sin t..........(ii)
Therefore, on differentiating both sides w.r.t t, we get,

$$\frac{dx}{dt} = a\left[\frac{d}{dt}(\cos t) + \frac{d}{dt}\log \tan \frac{t}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{\tan \frac{t}{2}} \frac{d}{dt}\left(\tan \frac{t}{2}\right)\right]$$ [by using chain rule of derivative]

$$= a\left[-\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \sec^2 \frac{t}{2} \frac{d}{dt}\left(\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{1}{\tan \frac{t}{2}} \times \sec^2 \frac{t}{2} \times \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{\frac{\sin t/2}{\cos t/2}} \times \frac{1}{\cos^2 \frac{t}{2}} \times \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{2 \sin \frac{t}{2} \cdot \cos \frac{t}{2}}\right]$$

$$= a\left[-\sin t + \frac{1}{\sin t}\right] \quad [\because \sin 2\theta = 2 \sin \theta \cos \theta]$$

$$= a\left(\frac{1-\sin^2 t}{\sin t}\right)$$

$\Rightarrow \quad \frac{dx}{dt} = a\left(\frac{\cos^2 t}{\sin t}\right) \left[\because 1 - \sin^2\theta = \cos^2\theta\right]$ ....(iii)

Again,on differentiating both sides of (ii) w.r.t t, we get,

$\frac{dy}{dt}$ = a cos t...............(iv)

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\left(\frac{\cos^2 t}{\sin t}\right)}$ [from Eqs(iii) and (iv)]

$= \frac{a\cos t}{a\cos^2 t} \times \sin t$ = tan t

Therefore,on differentiating both sides of above equation w.r.t x, we get,

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan t)$

$= \frac{d}{dt}(\tan t)\frac{dt}{dx}\left[\because \frac{d}{dx}f(t) = \frac{d}{dt}f(t) \cdot \frac{dt}{dx}\right]$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \sec^2 t \times \frac{\sin t}{a\cos^2 t}$ [From Eq.(iii)]

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\sin t\sec^4 t}{a}$

Therefore, on putting t = $\frac{\pi}{3}$,we get,

$\left[\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{3}} = \frac{\sin\frac{\pi}{3}\times\sec^4\frac{\pi}{3}}{a} = \frac{\frac{\sqrt{3}}{2}\times(2)^4}{a}$

$= \frac{8\sqrt{3}}{a}$

Section D

32. The given curves are $y^2 = 4x$ and $x^2 = 4y$

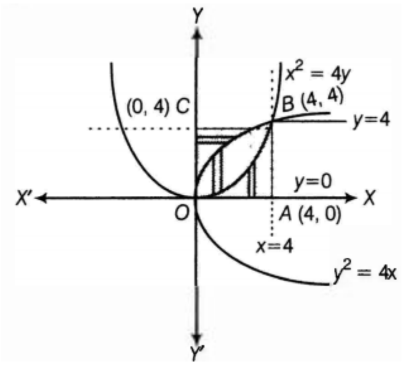
Let OABC be the square whose sides are represented by following equations

Equation of OA is y = 0

Equation of AB is x = 4

Equation of BC is y = 4

Equation of CO is x = 0



On solving equations $y^2 = 4x$ and $x^2 = 4y$, we get A(0, 0) and B(4, 4) as their points of intersection.

The Area bounded by these curves

$= \int_0^4 \left[y_{(\text{parabola } y^2=4x)} - y_{(\text{parabola } x^2=4y)}\right] dx$

$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$

$= \left[2 \cdot \frac{2}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$

$= \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$

$= \frac{4}{3} \cdot (4)^{3/2} - \frac{64}{12}$

$= \frac{4}{3} \cdot \left(2^2\right)^{3/2} - \frac{64}{12}$

$= \frac{4}{3} \cdot (2)^3 - \frac{64}{12}$

$= \frac{32}{3} - \frac{16}{3}$

$= \frac{16}{3}$ sq units

Hence, area bounded by curves $y^2 = 4x$ and x = 4y is $\frac{16}{3}$sq units ......(i)

Area bounded by curve $x^2 = 4y$ and the lines x = 0, x = 4 and X-axis

$= \int_0^4 y_{(\text{parabola } x^2=4y)}\, dx$

$= \int_0^4 \frac{x^2}{4} dx$
$= \left[\frac{x^3}{12}\right]_0^4$
$= \frac{64}{12}$
$=\frac{16}{3}$ sq units ....(ii)

The area bounded by curve y^2 = 4x, the lies y = 0, y = 4 and Y-axis

$= \int_0^4 x_{(\text{parabola } y^2=4x)} dy$
$= \int_0^4 \frac{y^2}{4} dy$
$= \left[\frac{y^3}{12}\right]_0^4$
$= \frac{64}{12}$
$= \frac{16}{3}$ sq units ....(iii)

From Equations. (i), (ii) and (iii), area bounded by the parabolas y^2 = 4x and x^2 = 4y divides the area of square into three equal parts.

33. f is one-one: For any x, y ∈ R - {+1}, we have $f(x) = f(y)$

$\Rightarrow \frac{x}{1+|x|} = \frac{y}{|y|+1}$
⇒ xy + x = xy + y
⇒ x = y

Therefore, f is one-one function.

If f is one-one, let y = R - {1}, then f(x) = y

$\Rightarrow \frac{x}{x+1} = y$
$\Rightarrow x = \frac{y}{1-y}$

It is cleat that x ∈ R for all y = R - {1}, also x = ≠ -1

Because x = -1

$\Rightarrow \frac{y}{1-y} = -1$
⇒ y = -1 + y

which is not possible.

Thus for each R - {1} there exists $x = \frac{y}{1-y} \in$ R - {1} such that

$f(x) = \frac{x}{x+1} = \frac{\frac{y}{1-y}}{\frac{y}{1-y}+1} = y$

Therefore f is onto function.

OR

Given that

Let A = {1, 2, 3} and R = {(a, b) : a, b ∈ A and $\left|a^2 - b^2\right| \leq 5$

Put a = 1, b = 1 $\left|1^2 - 1^2\right| \leq 5, (1,1)$ is an ordered pair.
Put a = 1, b = 2 $\left|1^2 - 2^2\right| \leq 5, (1,2)$ is an ordered pair.
Put a = 1, b = 3 $\left|1^2 - 3^2\right| > 5, (1,3)$ is not an ordered pair.
Put a = 2, b = 1 $\left|2^2 - 1^2\right| \leq 5, (2,1)$ is an ordered pair.
Put a = 2, b = 2 $\left|2^2 - 2^2\right| \leq 5, (2,2)$ is an ordered pair.
Put a = 2, b = 3 $\left|2^2 - 3^2\right| \leq 5, (2,3)$ is an ordered pair.
Put a = 3, b = 1 $\left|3^2 - 1^2\right| > 5, (3,1)$ is not an ordered pair.
Put a = 3, b = 2 $\left|3^2 - 2^2\right| \leq 5, (3,2)$ is an ordered pair.
Put a = 3, b = 3 $\left|3^2 - 3^2\right| \leq 5, (3,3)$ is an ordered pair.

R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

i. For (a, a) ∈ R
   $\left|a^2 - a^2\right| = 0 \leq 5$. Thus, it is reflexive.
ii. Let (a, b) ∈ R
   $(a,b) \in R, \left|a^2 - b^2\right| \leq 5$
   $\left|b^2 - a^2\right| \leq 5$
   (b, a) ∈ R
   Hence, it is symmetric

iii. Put a = 1, b = 2, c = 3

$|1^2 - 2^2| \leq 5$

$|2^2 - 3^2| \leq 5$

But $|1^2 - 3^2| > 5$

Thus, it is not transitive

34. We have, A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$\therefore$ BA = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ = 6I

$\therefore$ $B^{-1} = \frac{A}{6} = \frac{1}{6}$ A = $\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Also, x - y = 3, 2x + 3y + 4z = 17 and y + 2z = 7

$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ [using Eq. (i)]

$= \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

$\therefore$ x = 2, y = -1 and z = 4

35. We have,

$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$

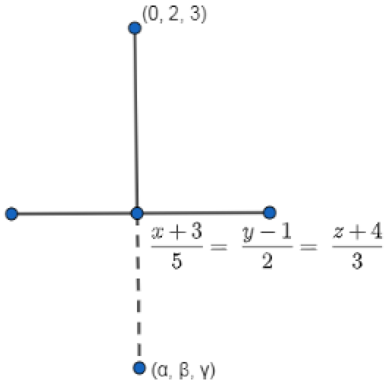
Therefore, the foot of the perpendicular is $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

The direction ratios of the perpendicular is

$(5\lambda - 3 - 0) : (2\lambda + 1 - 2) : (3\lambda - 4 - 3)$

$\Rightarrow (5\lambda - 3) : (2\lambda - 1) : (3\lambda - 7)$

Direction ratio of the line is 5 : 2 : 3



From the direction ratio of the line and direction ratio of its perpendicular, we have

$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$\Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$

$\Rightarrow 38\lambda = 38$

$\Rightarrow \lambda = 1$

Therefore, the foot of the perpendicular is (2, 3, -1)

The foot of the perpendicular is the mid-point of the line joining (0, 2, 3) and (a, β, y)

Therefore, we have

$\frac{\alpha+0}{2} = 2 \Rightarrow \alpha = 4$

$\frac{\beta+2}{2}=3 \Rightarrow \beta=4$
$\frac{\gamma+3}{2}=-1 \Rightarrow \gamma=-5$
Thus, the imgae is (4, 4, -5)_

OR

Given Cartesian equations of lines
$L_1=\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$
Line L1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)
Thus, vector equation of line L1 is
$\vec{r}=(\hat{i}-\hat{j}+\hat{k})+\lambda(3\hat{i}+2\hat{j}+5\hat{k})$
And
$L_2: \frac{x-2}{2}=\frac{y-1}{3}=\frac{z+1}{-2}$
Line L_2 is passing through point (2, -1, 1) and has direction ratios (2, 3, -2)

Thus, vector equation of line L2 is
$\vec{\mathbf{r}}=(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})+\mu(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}})$
Now, to calculate distance between the lines,
$\vec{r}=(\hat{\imath}-\hat{j}+\hat{k})+\lambda(3\hat{i}+2\hat{j}+5\hat{k})$
$\vec{\mathbf{r}}=(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})+\mu(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}})$
Here,we have
$\overrightarrow{a_1}=1-j+\hat{k}$
$\overrightarrow{b_1}=3\hat{i}+2\hat{j}+5\hat{k}$
$\overrightarrow{a_2}=2\hat{\imath}+\hat{\jmath}-\hat{k}$
$\overrightarrow{b_2}=2\hat{i}+3\hat{j}-2\hat{k}$
Thus,
$\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 3 & 2 & 5\\ 2 & 3 & -2\end{vmatrix}$
$=\hat{\imath}(-4-15)-\hat{\jmath}(-6-10)+\hat{k}(9-4)$
$\Rightarrow \overrightarrow{b_1}\times\overrightarrow{b_2}=-19\hat{i}+16\hat{j}+5\hat{k}$
$\Rightarrow |\overrightarrow{b_1}\times\overrightarrow{b_2}|=\sqrt{(-19)^2+16^2+5^2}$
$=\sqrt{361+256+25}$
$=\sqrt{642}$
$\overrightarrow{a_2}-\overrightarrow{a_1}=(2-1)\hat{\imath}+(1+1)\hat{\jmath}+(-1-1)\hat{k}$
$\therefore \overrightarrow{a_2}-\overrightarrow{a_1}=1+2\hat{\jmath}-2\hat{k}$
Now,
$(\overrightarrow{b_1}\times\overrightarrow{b_2})\cdot(\overrightarrow{a_2}-\overrightarrow{a_1})=(-19\hat{\imath}+16\hat{\jmath}+5\hat{k})\cdot(\hat{i}+2\hat{j}-2\hat{k})$
$=((-19)\times 1)+(16\times 2)+(5\times(-2))$
= -19 + 32 - 10
= 3
Thus, the shortest distance between the given lines is
$d=\left|\frac{(\overrightarrow{b_1}\times\overrightarrow{b_2})\cdot(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$
$\Rightarrow d=\left|\frac{3}{\sqrt{642}}\right|$
$\therefore d=\frac{3}{\sqrt{642}}$ units
As $d\neq 0$
Hence, given lines do not intersect each other.

Section E

36. i. P (Grade A in Maths) = P(M) = 0.2
P (Grade A in Physics) = P(P) = 0.3
P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in all subjects) = P(M $\cap P \cap$ C)

= P(M) $\times$ P(P) $\times$ P(C)

= 0.2 $\times$ 0.3 $\times$ 0.5 = 0.03

ii. P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in on subjects) = $P(\overline{M} \cap \overline{P} \cap \overline{C})$

= $P(\overline{M}) \times P(\overline{P}) \times P(\overline{C})$

= 0.8 $\times$ 0.7 $\times$ 0.5 = 0.280

iii. P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in 2 subjects)

$\Rightarrow$ P(grade A in M and P not in C) + P(grade A in P & C not in M) + P(grade A in M & C not in P)

$\Rightarrow$ P$\left(M \cap P \cap \overline{C}\right)$ + P$\left(P \cap C \cap \overline{M}\right)$ + P$(M \cap C \cap \overline{P})$

$\Rightarrow$ 0.2 $\times 0.3 \times$ 0.5 + 0.3 $\times 0.5 \times$ 0.8 + 0.2 $\times 0.5 \times$ 0.7 = 0.03 + 0.12 + 0.07

P(getting grade A in 2 subjects) = 0.22

OR

P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in 1 subjects)

$\Rightarrow$ P(grade A in M not in P and C) + P(grade A in P not in M and C) + P(grade A in C not in P and M)

$\Rightarrow$ P$\left(M \cap \overline{P} \cap \overline{C}\right)$ + P$\left(P \cap \overline{C} \cap \overline{M}\right)$ + P$(C \cap \overline{M} \cap \overline{P})$

$\Rightarrow$ 0.2 $\times 0.7 \times$ 0.5 + 0.3 $\times 0.5 \times$ 0.8 + 0.5 $\times 0.8 \times$ 0.7 = 0.07 + 0.12 + 0.028

P(getting grade A in 1 subjects) = 0.47

37. i. Let F be the combined force,

$\therefore \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

= $(4\hat{i} + 0\hat{j}) + (-2\hat{i} + 4\hat{j}) + (-3\hat{i} - 3\hat{j})$

= $(4 - 2 - 3)\hat{i} + (0 + 4 - 3)\hat{j}$

= - $\hat{i} + \hat{j}$

$\therefore \vec{F}| = \left|\sqrt{(-1)^2 + 1^2}\right|$

= $|\sqrt{2}|$ KN

ii. Magnitude of force of Team B =

$\left|\vec{F}_2\right| = \left|\sqrt{(-2)^2 + 4^2}\right| = \sqrt{20}$ KN

= $2\sqrt{5}$ KN

iii. We have,

$\left|\vec{F}_1\right| = \left|\sqrt{(4)^2 + 0^2}\right|$ = 4 KN

$\left|\vec{F}_2\right| = \left|\sqrt{(-2)^2+4^2}\right|$ = $\sqrt{20}$ KN

and $\left|\vec{F}_3\right| = \left|\sqrt{(-3)^2+(-3)^2}\right|$ = $\sqrt{18}$ KN

Here, the magnitude of force F_2 is greater, therefore team Q will win the game.

OR

We have,

Combined force, $\vec{F} = -\hat{i} + \hat{j}$

$\therefore \theta$ $\tan^{-1}\left(\frac{F_y}{F_x}\right)$ = $\tan^{-1}\left(\frac{1}{-1}\right)$

= $\tan^{-1}(1)$

= $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

= $\frac{3\pi}{4}$ radians

38. i. If P is the rent price per apartment and N is the number of rented apartments, the profit is given by NP - 500 N = N(P - 500)
[$\because$ ₹500/month is the maintenance charge for each occupied unit]

ii. Let R be the rent price per apartment and N is the number of rented apartments.
Now, if x be the number of non-rented apartments, then N(x) = 50 - x and R(x) = 10000 + 250x
Thus, profit = P(x) = NR = (50 - x) (10000 + 250 x - 500)
= (50 - x) (9500 + 250 x) = 250(50 - x) (38 + x)

iii. We have, P(x) = 250(50 - x) (38 + x)
Now, P'(x) = 250[50 - x - (38 + x)] = 250[12 - 2x]
For maxima/minima, put P'(x) = 0
$\Rightarrow$ 12 - 2x = 0 $\Rightarrow$ x = 6
Number of apartments are 6.
OR
P′(x) = 250(12 - 2x)
P′′(x) = - 500 < 0
$\Rightarrow$ P(x) is maximum at x = 6