Quadratic Equations

- **Identification of quadratic equations**

  Example: Check whether the following are quadratic equations or not.
  (i) $(2x + 3)^2 = 12x + 3$
  (ii) $x (x + 3) = (x + 1) (x – 5)$

  Solution:

  (i) $(2x + 3)^2 = 12x + 3$
  $\Rightarrow 4x^2 + 12x + y = 12x + 3$
  $\Rightarrow 4x^2 + 6 = 0$

  It is of the form $ax^2 + bx + c = 0$, where $a = 4$, $b = 0$ and $c = 6$
  Therefore, the given equation is a quadratic equation

  (ii) $x(x + 3) = (x + 1)(x - 5)$
  $\Rightarrow x^2 + 3x = x^2 + x - 5x - 5$
  $\Rightarrow 7x + 5 = 0$

  It is not of the form $ax^2 + bx + c = 0$, since the maximum power (or degree) of equation is 1.
  Therefore, the given equation is not a quadratic equation.

- **Express given situation mathematically**

  Example 1:
  An express train takes 2 hour less than a passenger train to travel a distance of 240 km. If the average speed of the express train is 20 km/h more than that of a passenger train, then form a quadratic equation to find the average speed of the express train?

  Solution:
  Let the average speed of the express train be x km/h.
  Since it is given that the speed of the express train is 20 km/h more than that of a passenger train,
  Therefore, the speed of the passenger train will be $x - 20$ km/h.

  Also we know that $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

  Time taken by the express train to cover 240 km $= \dfrac{240}{x}$

  Time taken by the passenger train to cover 240 km $= \dfrac{240}{x - 20}$

  And the express train takes 2 hour less than the passenger train. Therefore,

$$\frac{240}{x-20} - \frac{240}{x} = 2$$

$$\Rightarrow 240\left[\frac{x-(x-20)}{x(x-20)}\right] = 2$$

$$\Rightarrow 120\left(\frac{20}{x^2-20x}\right) = 1$$

$$\Rightarrow 2400 = x^2 - 20x$$

$$\Rightarrow x^2 - 20x - 2400 = 0$$

This is the required quadratic equation.

- **Solution of Quadratic Equation by Factorization Method**

  If we can factorize $ax^2 + bx + c = 0$, where $a \neq 0$, into a product of two linear factors, then the roots of this quadratic equation can be calculated by equating each factor to zero.

  Example:

  Find the roots of the equation, $2x^2 - 7\sqrt{3}x + 15 = 0$, by factorisation.

  Solution:

  $$2x^2 - 7\sqrt{3}x + 15 = 0$$

  $$\Rightarrow 2x^2 - 2\sqrt{3}x - 5\sqrt{3}x + 15 = 0$$

  $$\Rightarrow 2x\left(x - \sqrt{3}\right) - 5\sqrt{3}\left(x - \sqrt{3}\right) = 0$$

  $$\Rightarrow \left(x - \sqrt{3}\right)\left(2x - 5\sqrt{3}\right) = 0$$

  $$\Rightarrow \left(x - \sqrt{3}\right) = 0 \text{ or } \left(2x - 5\sqrt{3}\right) = 0$$

  $$\Rightarrow x = \sqrt{3} \text{ or } x = \frac{5\sqrt{3}}{2}$$

  Therefore, $\sqrt{3}$ and $\frac{5\sqrt{3}}{2}$ are the roots of the given quadratic equation.

- **Solution of Quadratic Equation by completing the square**

  A quadratic equation can also be solved by the method of completing the square.

  Example:

  Find the roots of the quadratic equation, $5x^2 + 7x - 6 = 0$, by the method of completing the square.

  Solution:

$$5x^2 + 7x - 6 = 0$$

$$\Rightarrow 5\left[x^2 + \frac{7}{5}x - \frac{6}{5}\right] = 0$$

$$\Rightarrow x^2 + 2 \times x \times \frac{7}{10} + \left(\frac{7}{10}\right)^2 - \left(\frac{7}{10}\right)^2 - \frac{6}{5} = 0$$

$$\Rightarrow \left(x + \frac{7}{10}\right)^2 - \frac{49}{100} - \frac{6}{5} = 0$$

$$\Rightarrow \left(x + \frac{7}{10}\right)^2 = \frac{169}{100}$$

$$\Rightarrow \left(x + \frac{7}{10}\right) = \pm\sqrt{\frac{169}{100}} = \pm\frac{13}{10}$$

$$\Rightarrow x + \frac{7}{10} = \frac{13}{10} \text{ or } x + \frac{7}{10} = -\frac{13}{10}$$

$$\Rightarrow x = \frac{13}{10} - \frac{7}{10} = \frac{3}{5} \text{ or } x = -\frac{13}{10} - \frac{7}{10} = -2$$

Therefore, –2 and $\frac{3}{5}$ are the roots of the given quadratic equation.

- **Quadratic Formula to find solution of quadratic equation:**

The roots of the quadratic equation, $ax^2 + bx + c = 0$, are given

by, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $b^2 - 4ac \geq 0$

Example:

Find the roots of the equation, $2x^2 - 3x - 44 = 0$, if they exist, using the quadratic formula.

Solution:

$$2x^2 - 3x - 44 = 0$$

Here, $a = 2$, $b = -3$, $c = -4$

$$\therefore b^2 - 4ac = (-3)^2 - 4 \times 2 \times (-44) = 9 + 352 = 361 > 0$$

The roots of the given equation are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{361}}{2 \times 2} = \frac{3 \pm 19}{4}$$

$$\Rightarrow x = \frac{3 + 19}{4} = \frac{11}{2} \text{ or } x = \frac{3 - 19}{4} = -4$$

The roots are –4 and $\frac{11}{2}$.

- **Nature of roots of Quadratic Equation**

  For the quadratic equation, $ax^2 + bx + c = 0$, where $a \neq 0$, the discriminant 'D' is defined as $D = b^2 - 4ac$

  The quadratic equation, $ax^2 + bx + c = 0$, where $a \neq 0$, has

1.
1.

1. two distinct real roots, if $D = b^2 - 4ac > 0$
2. two equal real roots, if $D = b^2 - 4ac = 0$
3. has no real roots, if $D = b^2 - 4ac < 0$

Example: Determine the nature of the roots of the following equations

(a) $2x^2 + 5x - 117 = 0$

(b) $3x^2 + 5x + 6 = 0$

Solution:

(a) Here, $a = 2$, $b = 5$, $c = -117$
$$\therefore D = b^2 - 4ac = 5^2 - 4 \times 2 \times (-117) = 25 + 936 = 961 > 0$$

Therefore, the roots of the given equation are real and distinct.

(b) Here, $a = 3$, $b = 5$, $c = 6$
$$\therefore D = b^2 - 4ac = 5^2 - 4 \times 3 \times 6 = 25 - 72 = -47 < 0$$

Therefore, the roots of the given equation are not real.