14. Co-ordinate Geometry

Exercise 14.1

1. Question

On which axis do the following points lie?

(i) P (5, 0) (ii) Q (0 -2)

(iii) R (- 4, 0) (iv) S (0, 5)

Answer

P on x-axis, since ordinate is zero.

Q on y-axis, since abscissa is zero.

R on x-axis, since ordinate is zero.

S on y-axis, since abscissa is zero.

2. Question

Let ABCD be a square of side 2a. Find the coordinates of the vertices of this square when

(i) A coincides with the origin and AB and AD are along OX and OY respectively.

(ii) The centre of the square is at the origin and coordinate axes are parallel to the sides AB and AD respectively.

Answer

(i) Since each side of square is 2a.

Coordinates of A are (0, 0), since it coincides with origin.

Coordinates of B are (2a, 0), for a point along x-axis ordinate is zero.

Coordinates of C are (2a, 2a), since this point is equi-distance from x-axis and y-axis.

Coordinates of D are (0, 2a), since abscissa is zero and ordinate is 2a.

(ii) Each side of square is a units.

Coordinates of A are (a, a), since this point lies in Ist coordinate.

Coordinates of B are (-a, a), since this point lies in IInd coordinate.

Coordinates of C are (-a, -a), since this point lies in IIIrd coordinate.

Coordinates of D are (a, -a), since this point lies in IVth coordinate.

3. Question

The base PQ of two equilateral triangles PQR and PQR' with side 2a lies along y-axis such that the mid-point of PQ is at the origin. Find the coordinates of the vertices R and R' of the triangles.

Answer

$R\ \left(\sqrt{3}a, 0\right)$, $R'(-\sqrt{3}a, 0)$

Since PQ is the base of two equilateral triangles with side 2a and mid-point of PQ is at origin.

Therefore point R lies on positive x-axis and point R’ lies on negative y-axis.

$OR^2 = (2a)^2 - a^2$

$OR^2 = 4a^2 - a^2$

OR = √3a

Therefore coordinates of R are (√3a, 0) and R’ (0, √3a)

Exercise 14.2

1. Question

Find the distance between the following pair of points:

(i) (-6,7) and (-1, -5)

(ii) (a + b, b + c) and (a - b, c - b)

(iii) (a sin a, - b cos a) and (-a cos a, b sin a)

(iv) (a, 0) and (0, b)

Answer

(i) (-6,7) and (-1, -5)

Distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance $=\sqrt{(-1+6)^2+(-5-7)^2}$

Distance $=\sqrt{(5)^2+(-12)^2}$

Distance $=\sqrt{25+144}$

Distance $=\sqrt{169}$ = 13 units

Thus, the distance between the points (-6,7) and (-1, -5) is 13 units

(ii) (a + b, b + c) and (a - b, c - b)

Distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance $=\sqrt{\{(a-b)-(a+b)\}^2+\{(c-b)-(b+c)\}^2}$

Distance $=\sqrt{\{a-b-a-b\}^2+\{c-b-b-c\}^2}$

Distance $=\sqrt{\{-2b\}^2+\{-2b\}^2}$

Distance $=\sqrt{8b^2}$ = $2\sqrt{2}b$ units

Thus, the distance between these points is (a + b, b + c) and (a - b, c - b) is $2\sqrt{2}b$ units.

(iii) (a sin a, - b cos a) and (-a cos a, b sin a)

Distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance $=\sqrt{(-acosa - asina)^2 + (bsina + cosa)^2}$

Distance $=\sqrt{a^2cos^2a + a^2sin^2a + 2a^2sinacosa + b^2sin^2a + b^2cos^2a + 2b^2sinacosa}$

Distance $=\sqrt{a^2(cos^2a + sin^2a) + b^2(sin^2a + cos^2a) + 2sinacosa(a^2 + b^2)}$

Because $cos^2a + sin^2a$ =1 and $2sinacosa = sin2a$, we get,

Distance $=\sqrt{a^2 + b^2 + sin2a(a^2 + b^2)}$

Or Distance = $\sqrt{a^2 + b^2(1 + sin2a)}$

Thus, the distance between points (a sin a, - b cos a) and (-a cos a, b sin a) is $\sqrt{a^2 + b^2(1 + sin2a)}$

(iv) (a, 0) and (0, b)

Distance $=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance $=\sqrt{(0 - a)^2 + (b - 0)^2}$

Distance $=\sqrt{a^2 + b^2}$Thus, the distance between points (a, 0) and (0, b) is $\sqrt{a^2 + b^2}$

2. Question

Find the value of a when the distance between the points (3, a) and (4, 1) is $\sqrt{10}$.

Answer

Distance $=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{10} = \sqrt{(4-3)^2 + (1-a)^2}$

$\sqrt{1 + (1-a)^2}$ = 10

$1 + 1 - 2a + a^2 = 10$

$a^2 - 2a - 8 = 0$

$a^2 - 4a + 2a - 8 = 0$

a(a - 4) + 2(a - 4) = 0

(a - 4) (a + 2) = 0

a = 4 and a = -2

3. Question

If the points (2, 1) and (1,-2) are equidistant from the point (x, y), show that x + 3y = 0.

Answer

Distance from point (2, 1) = Distance from point (1, -2)

$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (y+2)^2}$

Square roots are cancelled, therefore

$(x-2)^2+(y-1)^2 = (x-1)^2+(y+2)^2$

$x^2-4x+4+y^2-2y+1 = x^2-2x+1+y^2+4y+4$

$-4x+2x-2y-4y = 0$

$x+3y = 0$

4. Question

Find the values of x, y if the distances of the point (x, y) from (-3, 0) as well as from (3, 0) are 4.

Answer

Given: the distances of the point (x, y) from (-3, 0) as well as from (3, 0) are 4.

To find: the values of x, y

Solution:distances of the point (x, y) from (-3, 0) is Distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$4 = \sqrt{(-3-x)^2+(0-y)^2}$

$\sqrt{(-3-x)^2+(-y)^2} = 4$

$(-3-x)^2+(-y)^2 = 16$

$9+6x+x^2+y^2 = 16$

$6x+x^2+y^2 = 7$ .... (1)

distances of the point (x, y) from (3, 0) is

Distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$4 = \sqrt{(3-x)^2+(0-y)^2}$

$\sqrt{(3-x)^2+(-y)^2} = 4$

$(3-x)^2+(-y)^2 = 16$

$9-6x+x^2+y^2 = 16$

$-6x+x^2+y^2 = 7$ .... (2)

Subtract eq 1 from eq 2 to get,⇒ $-6x + x^2 + y^2 - (6x + x^2 + y^2) = 7 - 7$

⇒ $-6x + x^2 + y^2 - 6x - x^2 - y^2 = 0$

⇒ $-12x = 0$ ⇒ $x = 0$Putting the value of x in eq 1 we get,$6x + x^2 + y^2 = 7$

⇒ $6(0) + 0^2 + y^2 = 7$

⇒ $y^2 = 7$

⇒ $y=\pm\sqrt{7}$

Hence, X=0, $y=\pm\sqrt{7}$

5. Question

The length of a line segment is of 10 units and the coordinates of one end-point are (2,-3). If the abscissa of the other end is 10, find the ordinate of the other end.

Answer

Let the ordinate of other end is k

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$10 = \sqrt{(10-2)^2 + (k+3)^2}$

On squaring both sides, we get

$100 = (10-2)^2 + (k+3)^2$

$100 = 64 + k^2 + 6k + 9$

$k^2 + 6k - 27 = 0$

$k^2 + 9k - 3k - 27 = 0$

$k(k+9) - 3(k+9) = 0$

$(k-3)(k+9) = 0$

$k = 3; k = -9;$

Therefore ordinates are 3, -9

6. Question

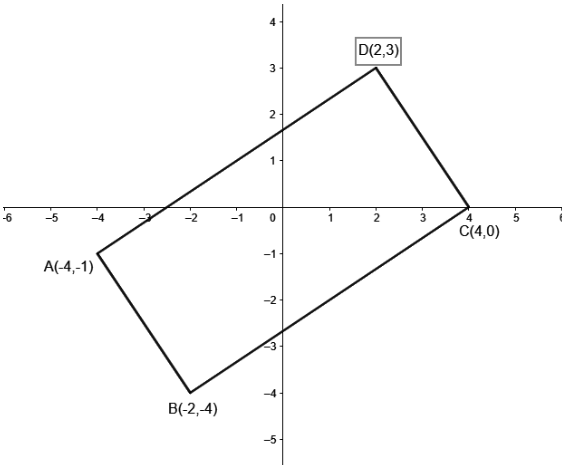
Show that the points A(- 4, -1), B(-2, - 4), C(4, 0) and D(2, 3) are the vertices points of a rectangle.

Answer

Given: the points A(- 4, -1), B(-2, - 4), C(4, 0) and D(2, 3)

To prove: the points are the vertices points of a rectangle.

Solution: Vertices of rectangle ABCD are: A(- 4, -1), B(-2, - 4), C(4, 0) and D(2, 3)



Length of sides= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(-2+4)^2 + (-4+1)^2}$ = $\sqrt{4+9}$ = $\sqrt{13}$ units

Length of side BC = $\sqrt{(4+2)^2+(0+4)^2}$ = $\sqrt{36+16}$ = $\sqrt{52}$ = $2\sqrt{13}$ units

Length of side CD = $\sqrt{(2-4)^2+(3-0)^2}$ = $\sqrt{4+9}$ = $\sqrt{13}$ units

Length of side AD = $\sqrt{(2+4)^2+(3+1)^2}$ = $\sqrt{36+16}$ = $\sqrt{52}$ = $2\sqrt{13}$ units

Length of diagonal BD = $\sqrt{(2+2)^2+(3+4)^2}$ = $\sqrt{16+49}$ = $\sqrt{65}$ units

Length of diagonal AC = $\sqrt{(4+4)^2+(0+1)^2}$ = $\sqrt{64+1}$ = $\sqrt{65}$ units

Since opposite sides are equal and diagonal are equal. Therefore given vertices are the vertices of a rectangle.

7. Question

Show that the points A (1,- 2), B (3, 6), C (5, 10) and D (3, 2) are the vertices of a parallelogram.

Answer

Vertices of a parallelogram ABCD are: A (1,- 2), B (3, 6), C (5, 10) and D (3, 2) Length of side AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Length of side AB = $\sqrt{(3-1)^2+(6+2)^2}$ = √(4+64)= √68 units

Length of side BC = $\sqrt{(5-3)^2+(10-6)^2}$ = √(4+16) = √20 units

Length of side CD = $\sqrt{(3-5)^2+(2-10)^2}$ = √(4+64)= √68 units

Length of side DA = $\sqrt{(3-1)^2+(2+2)^2}$ = √(4+16) = √20 units

Length of diagonal BD = $\sqrt{(3-3)^2+(2-6)^2}$ = √16= 4 units

Length of diagonal AC = $\sqrt{(5-1)^2+(10+2)^2}$ = √(16+144) = √160 units

Opposite sides of the quadrilateral formed by the given four points are equal i.e. (AB = CD) & (DA = BC)Also, the diagonals BD & AC are unequal.Therefore, the given points form a parallelogram.

8. Question

Prove that the points A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4) are the vertices of a square.

Answer

Vertices of a square ABCD are: A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4) Length of side AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Length of side AB = $\sqrt{(4-1)^2+(2-7)^2}$ = $\sqrt{9+25}$ = $\sqrt{34}$ units

Length of side BC = $\sqrt{(-1-4)^2+(-2-1)^2}$ = $\sqrt{25+9}$ = $\sqrt{34}$ units

Length of side CD = $\sqrt{(-4+1)^2+(4+1)^2}$ = $\sqrt{9+25}$ = $\sqrt{34}$ units

Length of side DA = $\sqrt{(-4-1)^2+(4-7)^2}$ = $\sqrt{25+9}$ = $\sqrt{34}$ units

Length of diagonal BD = $\sqrt{(-4-4)^2+(4-2)^2}$ = $\sqrt{64+4}$ = $\sqrt{68}$ units

Length of diagonal AC = $\sqrt{(-1-1)^2+(-1-7)^2}$ = $\sqrt{4+64}$ = $\sqrt{68}$ units

Since opposite sides are equal and diagonal are equal. Therefore given vertices are the vertices of a square.

9. Question

Prove that the points (3, 0), (6, 4) and (- 1, 3) are vertices of a right-angled isosceles triangle.

Answer

Vertices of a triangle ABC are: A(3, 0), B(6, 4) and C (- 1, 3)

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(6-3)^2 + (4-0)^2}$ = $\sqrt{9+16}$ = $\sqrt{25}$ units

Length of side BC = $\sqrt{(-1-6)^2 + (3-4)^2}$ = $\sqrt{49+1}$ = $\sqrt{50}$ units

Length of side AC = $\sqrt{(-1-3)^2 + (3-0)^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ units

Since AB = AC, therefore triangle is an isosceles.

$BC^2 = AB^2 + AC^2$

$(\sqrt{50})^2 = (\sqrt{25})^2 + (\sqrt{25})^2$

50 = 25 + 25

50 = 50

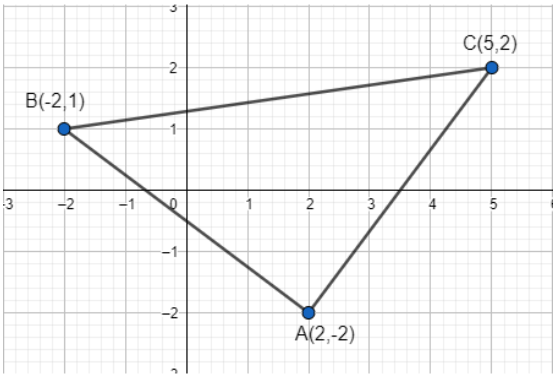
Since $BC^2 = AB^2 + AC^2$; therefore given triangle is right angled triangle.

10. Question

Prove that (2, -2), (-2, 1) and (5, 2) are the vertices of a right angled triangle. Find the area of the triangle and the length of the hypotenuse.

Answer

Solution: Vertices of a triangle ABC are: A(2, -2), B(-2, 1) and C(5, 2)



Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(-2-2)^2 + (1+2)^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ units

Length of side BC = $\sqrt{(5+2)^2 + (2-1)^2}$ = $\sqrt{49+1}$ = $\sqrt{50}$ units

Length of side AC = $\sqrt{(5-2)^2 + (2+2)^2}$ = $\sqrt{9+16}$ = $\sqrt{25}$ units

Since AB = AC, therefore triangle is an isosceles.

$BC^2 = AB^2 + AC^2$

$(\sqrt{50})^2 = (\sqrt{25})^2 + (\sqrt{25})^2$

50 = 25 + 25

50 = 50

Since $BC^2 = AB^2 + AC^2$; therefore given triangle is right angled triangle.

Area of right angled triangle = $\frac{1}{2} base \times altitude$

Area of right angled triangle = $\frac{1}{2} \times 5 \times 5 = \frac{25}{2}$ square units

Length of hypotenuse (BC) = $\sqrt{50}$ = $5\sqrt{2}$ units

11. Question

Prove that the points (2 a, 4 a), (2 a, 6 a) and $\left(2a + \sqrt{3}a, 5a\right)$ are the vertices of an equilateral triangle.

Answer

Vertices of a triangle ABC are: A(2 a, 4 a), B(2 a, 6 a) and C$\left(2a + \sqrt{3}a, 5a\right)$

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(2a - 2a)^2 + (6a - 4a)^2}$ = $\sqrt{(2a)^2}$ = 2a units

Length of side BC = $\sqrt{\left(2a + \sqrt{3}a - 2a\right)^2 + (5a - 6a)^2}$ = $\sqrt{3}$a + a units

Length of side AC = $\sqrt{\left(2a + \sqrt{3}a - 2a\right)^2 + (5a - 4a)^2}$ = $\sqrt{3}$ a + a units

The given vertices are not the vertices of an equilateral triangle

12. Question

Prove that the points (2, 3), (-4, -6) and (1, 3/2) do not form a triangle.

Answer

Let the Vertices of a triangle ABC are: A(2, 3), B(-4, -6) and C(1, 3/2)

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(-4 - 2)^2 + (-6 - 3)^2}$ = $\sqrt{36 + 81}$ = $\sqrt{117}$ units

Length of side BC = $\sqrt{(1 + 4)^2 + \left(\frac{3}{2} + 6\right)^2}$ = $\sqrt{25 + 56.25}$ = $\sqrt{81.25}$ units

Length of side AC = $\sqrt{(1 - 2)^2 + \left(\frac{3}{2} - 3\right)^2}$ = $\sqrt{1 + 2.25}$ = $\sqrt{2.25}$ units

The given vertices do not form a triangle, since sum of two sides of a triangle are not greater than third side.

13. Question

An equilateral triangle has two vertices at the points (3, 4) and (-2, 3), find the coordinates of the third vertex.

Answer

Given: An equilateral triangle has two vertices at the points (3, 4) and (-2, 3)

To find: the coordinates of the third vertex.

Solution: Let the Vertices of a triangle ABC are A(3, 4) and B (-2, 3), and C(x, y),Since it is equilateral triangle,AB=AC=BCWhere AB,AC and BC are lengths of sides of the given triangle.To find the length of a side use distance formula $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$,

Length of side AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Length of side AB = $\sqrt{(-2-3)^2+(3-4)^2}$ = $\sqrt{25+1}$ = $\sqrt{26}$ units

Length of side BC = $\sqrt{(x+2)^2+(y-3)^2}$ units

Length of side AC = $\sqrt{(x-3)^2+(y-4)^2}$ unitsNow AB=AC

$\Rightarrow(AB)^2 = (BC)^2$

$(\sqrt{26})^2 = (\sqrt{(x+2)^2+(y-3)^2}$

$(x+2)^2+(y-3)^2 = 26$

$x^2+4x+4+y^2-6y+9 = 26$

$x^2+4x+y^2-6y = 13(1)$

$(AB)^2 = (AC)^2$

$(\sqrt{26})^2 = (\sqrt{(x-3)^2+(y-4)^2}$

$(x-3)^2+(y-4)^2 = 26$

$x^2-6x+9+y^2-8y+16 = 26$

$x^2-6x+y^2-8y = 1(2)$

On subtracting eqn (2) from (1), we get

$(x^2+4x+y^2-6y)-(x^2-6x+y^2-8y) = 13-1$

$x^2+4x+y^2-6y-x^2+6x-y^2+8y = 13-1$

$4x-6y+6x-8y = 12 10x+2y=12 \quad 5x+y=6$ ...... (3)

$(AC)^2 = (BC)^2$

$(x-3)^2+(y-4)^2 = (x+2)^2+(y-3)^2$

$x^2-6x+9+y^2-8y+16 = x^2+4x+4+y^2-6y+9 -4x-2y=12$ ..... (4)

Solving equations (3) and (4), we get

$x = \frac{4}{3}$; y = $-\frac{2}{3}$

Therefore coordinates of C are ($\frac{4}{3}$, $-\frac{2}{3}$)

14. Question

Show that the quadrilateral whose vertices are (2, -1), (3, 4), (-2, 3) and (-3, -2) is a rhombus.

Answer

Let the Vertices of a quadrilateral are: A(2, -1), B(3, 4), C(-2, 3) and D(-3, -2)

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(3-2)^2 + (4+1)^2}$ = $\sqrt{1+25}$ = $\sqrt{26}$ units

Length of side BC = $\sqrt{(-2-3)^2 + (3-4)^2}$ = $\sqrt{25+1}$ = $\sqrt{26}$ units

Length of side CD = $\sqrt{(-3+2)^2 + (-2-3)^2}$ = $\sqrt{1+25}$ = $\sqrt{26}$ units

Length of side DA = $\sqrt{(-3-2)^2 + (-2+1)^2}$ = $\sqrt{25+1}$ = $\sqrt{26}$ units

Since all sides are of equal length, therefore it is a rhombus.

15. Question

Two vertices of an isosceles triangle are (2, 0) and (2, 5). Find the third vertex if the length of the equal sides is 3.

Answer

Vertices of an isosceles are: A(2, 0) and B(2, 5).

Let the third vertex is P(x, y)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side PA = $\sqrt{(x-2)^2 + (y)^2}$ units

Length of side PB = $\sqrt{(x-2)^2 + (y-5)^2}$ units

Since PA = PB

$$\sqrt{(x-2)^2 + (y)^2} = \sqrt{(x-2)^2 + (y-5)^2}$$

On squaring both sides, we get

$$(x-2)^2 + (y)^2 = (x-2)^2 + (y-5)^2$$

$$x^2 - 4x + 4 + y^2 = x^2 - 4x + 4 + y^2 - 10y + 25$$

$$y = \frac{5}{2},$$

Also, PA = 3

$$\sqrt{(x-2)^2 + (y)^2} = 3$$

On squaring both sides, we get

$$(x-2)^2 + (y)^2 = 9$$

$$x^2 - 4x + 4 + y^2 = 9$$

$$x^2 - 4x + y^2 = 5$$

On substituting $y = \frac{5}{2}$,

$$x^2 - 4x + \frac{25}{4} = 5$$

$$x^2 - 4x + \frac{25}{4} - 5 = 0$$

$$x^2 - 4x + \frac{5}{4} = 0$$

Using quadratic formula:

$$x = \frac{\{-b \pm \sqrt{b^2 - 4ac}\}}{2a}$$

$$x = \frac{\{4 \pm \sqrt{16 - 5}\}}{2}$$

$$x = \frac{\{4 \pm \sqrt{11}\}}{2}$$

Therefore coordinates of third vertex are: $\left(\frac{4+\sqrt{11}}{2}, \frac{5}{2}\right)$; $\left(\frac{4-\sqrt{11}}{2}, \frac{5}{2}\right)$

16. Question

Which point on x-axis is equidistant from (5, 9) and (- 4, 6)?

Answer

Since the point is on x-axis, therefore coordinate of y-axis is zero.

Therefore the point is P(k, 0) which is equidistance from A(5, 9) and B(- 4, 6)

PA = PB

$$\sqrt{(5-k)^2 + 9^2} = \sqrt{(-4-k)^2 + 6^2}$$

On squaring both sides

$$(5-k)^2 + 9^2 = (-4-k)^2 + 6^2$$

$$25 - 10k + k^2 + 81 = 16 - 8k + k^2 + 36$$

$$25 — 2k + 81 = 16 + 36 - 25 - 81$$

$$— 2k = 16 + 36 - 25 - 81$$

$$k = 27$$

Therefore coordinate is (27, 0)

17. Question

Prove that the points (- 2, 5), (0, 1) and (2, - 3) are collinear.

Answer

Vertices are: A(- 2, 5), B(0, 1) and C(2, - 3)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(0+2)^2 + (1-5)^2}$ = $\sqrt{4+16}$ = $\sqrt{20}$ = $2\sqrt{5}$ units

Length of side BC = $\sqrt{(2-0)^2 + (-3-1)^2}$ = $\sqrt{4+16}$ = $\sqrt{20}$ = $2\sqrt{5}$ units

Length of side AC = $\sqrt{(2+2)^2 + (-3-5)^2}$ = $\sqrt{16+64}$ = $\sqrt{80}$ = $4\sqrt{5}$ units

Since length of AB + BC = AC, therefore points are collinear.

18. Question

The coordinates of the point P are (-3, 2). Find the coordinates of the point Q which lies on the line joining P and origin such that OP = OQ.

Answer

Let the coordinates of Point Q are (x, y) and coordinates of origin O are (0, 0)

Since OP = OQ

Points are: P (-3, 2), Q(x, y) and O (0, 0)

Q is the mid point

$$0 = \frac{-3+x}{2}$$

$$x = 3$$

$$0 = \frac{2+y}{2}$$

$$y = -2$$

Therefore coordinates are (3, -2)

19. Question

Which point on y-axis is equidistant from (2, 3) and (-4, 1)?

Answer

Since the point is on y-axis, therefore coordinate of x-axis is zero.

Therefore the point is P(0, k) which is equidistance from A(2, 3) and B(- 4, 1)

PA = PB

$$\sqrt{(2-0)^2 + (3-k)^2} = \sqrt{(-4)^2 + (1-k)^2}$$

$$\sqrt{4+9+k^2-6k} = \sqrt{16+1+k^2-2k}$$

On squaring both sides, we get

$$-4k = 4$$

$$k = -1$$

Therefore coordinate is (0, -1)

20. Question

The three vertices of a parallelogram are (3, 4), (3, 8) and (9, 8). Find the fourth vertex.

Answer

Consider A(3, 4), B (3, 8) and C(9, 8).

Let the coordinates of fourth vertex are D (x, y)

In a parallelogram diagonals bisect each other

Coordinate of mid point of AC = $X = \frac{3+9}{2} = \frac{12}{2} = 6$

$$Y = \frac{4+8}{2} = \frac{12}{2} = 6$$

Therefore coordinates of mid point of AC are (6, 6)

Coordinate of mid point of BD = $X = \frac{3+x}{2}$

$$Y = \frac{y+8}{2}$$

Coordinates of point D are

$$\frac{3+x}{2} = 6$$

$$x = 12 - 3 = 9$$

$$\frac{y+8}{2} = 6$$

$$y = 12 - 8 = 4$$

Therefore coordinates of fourth vertex D are (9, 4)

21. Question

Find the circumcentre of the triangle whose vertices are (-2, -3), (- 1, 0), (7, - 6).

Answer

Vertices of triangle are A(-2, -3), B(- 1, 0), C(7, - 6)

Let the coordinates of P are (x, y)

PA = PB =PC

PA = PB

$$\sqrt{(x+2)^2 + (y+3)^2} = \sqrt{(x+1)^2 + (y-0)^2}$$

On squaring both sides, we get

$$x^2 + 4x + 4 + y^2 + 6y + 9 = x^2 + 2x + 1 + y^2$$

$$2x + 6y = -12$$

$$x + 3y = -6 \text{............} (1)$$

PA = PC

$$\sqrt{(x+2)^2+(y+3)^2}=\sqrt{(x-7)^2+(y+6)^2}$$

On squaring both sides, we get

$$x^2+4x+4+y^2+6y+9=x^2-14x+49+y^2+12y+36$$

$$18x-6y=72$$

$3x-y=12$............(2)

On solving equations (1) & (2), We get

X = 3 and y = -3

Therefore coordinates are (3, -3)

22. Question

Find the angle subtended at the origin by the line segment whose end points are (0,100) and (10, 0).

Answer

Since the abscissa of first coordinate is zero, therefore this point lies on y-axis. Ordinate of second point is zero, therefore this point lies on y-axis. We know that both the axes are perpendicular to eachother, therefore the angle between these points is 90°.

23. Question

Find the centre of the circle passing through (2, 1), (5, - 8) and (2, - 9).

Answer

Coordinates of points on a circle are A(2,1), B(5,-8) and C(2,-9).

Let the coordinates of the centre of the circle be O(x, y)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Since the distance of the points A, B and C will be equal from the center, therefore

⇒ OA = OB

$$\sqrt{(x-2)^2+(y-1)^2}=\sqrt{(x-5)^2+(y+8)^2}$$

On squaring both sides, we get

⇒ $x^2 + 4 - 4x + y^2 + 1 - 2y = x^2 + 25 - 10x + y^2 + 64 + 16y$

⇒ 6x - 18y - 84 = 0

⇒ x - 3y - 14 = 0 ------------- (1)

Similarly, OC = OB

$$\sqrt{(x-2)^2+(y+9)^2}=\sqrt{(x-5)^2+(y+8)^2}$$

⇒ $x^2 + 4 - 4x + y^2 + 81 + 18y = x^2 + 25 - 10x + y^2 + 64 + 16y$

⇒ 6x - 2y - 4 = 0

⇒ 3x - y - 2 = 0 ------------- (2)

By solving equations (1) and (2), we get x = -1, y = -5

So, the coordinates of the centre of the circle is (-1, -5).

Radius of the circle = OA = $\sqrt{(-1-2)^2+(-5-1)^2}$

= $\sqrt{9+36}$

= $\sqrt{45}$

= $3\sqrt{5}$ units

24. Question

Find the value of k, if the point P (0, 2) is equidistant from (3, k) and (k, 5).

Answer

Let the point is P(0, 2) which is equidistance from A(3, k) and B(k, 5)

PA = PB

$$\sqrt{(3-0)^2+(k-2)^2} = \sqrt{(k-0)^2+(5-2)^2}$$

On squaring both sides, we get

$$9+k^2+4-4k = k^2+9$$

$$-4k = -4$$

$$k = 1$$

25. Question

If two opposite vertices of a square are (5, 4) and (1, -6), find the coordinates of its remaining two vertices.

Answer

Let ABCD is a square with A(5, 4) and C(1, -6).

Let the coordinates of B are (x, y)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ AB = BC

$$\sqrt{(x-5)^2+(y-4)^2} = \sqrt{(1-x)^2+(-6-y)^2}$$

On squaring both sides, we get

$$x^2-10x+25+y^2-8y+16 = x^2+1-2x+y^2+12y+36$$

$$-8x-20y = -4$$

$$2x-5y = 1$$

$$x = \frac{1+5y}{2}\text{..........................(1)}$$

In ΔABC, Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = 2BC^2$ [Since AB = BC]

$$\sqrt{(1-5)^2+(-6-4)^2} = 2\sqrt{(x-1)^2+(y+6)^2}$$

On squaring both sides, we get

$$16+100 = 2(x^2+1-2x+y^2+12y+36)$$

$$58 = x^2+1-2x+y^2+12y+36$$

$$21 = x^2-2x+y^2+12y$$

$x^2-2x+y^2+12y = 21$..................(2)

On substituting value of $x = \frac{1+5y}{2}$ from equation (1) in equation (2), we get

$$\left\{\frac{(1+5y)}{2}\right\}^2 - \left\{\frac{2(1+5y)}{2}\right\} + y^2 + 12y = 21$$

$$\frac{1+25y^2+10y}{4} + 1 + 5y + y^2 + 12y = 21$$

$$1+25y^2+10y+4+20y+4y^2+48y = 21$$

$$29y^2+78y = 79$$

$$29y^2+78y-79 = 0$$

On solving we get y = -3, -1

Substituting these values of y in eqn 1, we get x = 8, -2

Therefore other coordinates are B(8, -3). And D(-2,1)

26. Question

Show that the points (-3, 2), (-5, -5), (2, -3) and (4, 4) are the vertices of a rhombus. Find the area of this rhombus.

Answer

Vertices of the rhombus are: A(-3, 2), B(-5, -5), C(2, -3) and D(4, 4)

We know that diagonals of a rhombus bisect each other, therefore point of intersection of diagonals is:

Abscissa of Mid point of AC = $\frac{2-3}{2} = -\frac{1}{2}$

Ordinate of Mid point of AC = $\frac{-3+2}{2} = -\frac{1}{2}$

Abscissa of Mid point of BD = $\frac{4-5}{2} = -\frac{1}{2}$

Ordinate of Mid point of BD = $\frac{4-5}{2} = -\frac{1}{2}$

Since the diagonals AC and BD bisect each other at O, therefore it is a rhombus.

Length of diagonal AC = $\sqrt{(2+3)^2+(-3-2)^2}$ = $\sqrt{25+25}$ = $\sqrt{50}$ = $5\sqrt{2}$ units

Length of diagonal BD = $\sqrt{(4+5)^2+(4+5)^2}$ = $\sqrt{81+81}$ = $\sqrt{162}$ = $9\sqrt{2}$ units

Area of rhombus = $\frac{1}{2}\times d1 \times d2$ = $\frac{1}{2}\times 5\sqrt{2}\times 9\sqrt{2}$ = 45 sq units

Area of rhombus is 45 sq units

27. Question

Find the coordinates of the circumcentre of the triangle whose vertices are (3, 0), (-1, -6) and (4,-1). Also, find its circumradius.

Answer

Coordinates of points on a circle are A (3, 0), B(-1, -6) and C(4,-1)

Let the coordinates of the centre of the circle be O(x, y)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Since the distance of the points A, B and C will be equal from the center, therefore

⇒ OA = OC

$$\sqrt{(x-3)^2+(y-0)^2} = \sqrt{(x+1)^2+(y+6)^2}$$

On squaring both sides, we get

$(x-3)^2+(y-0)^2 = (x+1)^2+(y+6)^2$

⇒ $x^2 + 9 - 6x + y^2 = x^2 + 2x + 1 + y^2 + 36 + 12y$

⇒ -8x - 12y = 28

⇒ 2x + 3y = -7------------- (1)

Similarly, OC = OB

$$\sqrt{(x-4)^2+(y+1)^2} = \sqrt{(x+1)^2+(y+6)^2}$$

On squaring both sides, we get

$(x-4)^2+(y+1)^2 = (x+1)^2+(y+6)^2$

⇒ $x^2 + 16 - 8x + y^2 + 1 + 2y = x^2 + 1 + 2x + y^2 + 36 + 12y$

⇒ -10x - 10y = 20

⇒ x + y = -2 ------------- (2)

Solving eqn (1) and (2), we get

x = 1; y = -3

Coordinates of circum center are (1, -3)

Circum radius of the circle = OA = $\sqrt{(1-3)^2+(3)^2}$

= $\sqrt{4+13}$

= $\sqrt{13}$ units

28. Question

Find a point on the x-axis which is equidistant from the points (7, 6) and (-3, 4).

Answer

points A(7, 6) and B(-3, 4) are equidistance from point P.

Let the coordinates of point are P(x, 0)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ PA = PB

$$\sqrt{(x-7)^2 + (0-6)^2} = \sqrt{(x+3)^2 + (0-4)^2}$$

On squaring both sides, we get

$(x - 7)^2 + (0 - 6)^2 = (x + 3)^2 + (0 - 4)^2$

$x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$

$5x = 15$

$x = 3$

Therefore coordinates are (3, 0)

29 A. Question

Show that the points A(5, 6), B (1, 5), C(2, 1) and D(6, 2) are the vertices of a square.

Answer

Vertices of a quadrilateral are A(5,6), B(1,5), C(2,1) and D(6,2).

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB = $\sqrt{(1-5)^2 + (5-6)^2}$ = $\sqrt{(16+1)}$ = $\sqrt{17}$ units

BC = $\sqrt{(2-1)^2 + (1-5)^2}$ = $\sqrt{(1+16)}$ = $\sqrt{17}$ units

CD = $\sqrt{(6-2)^2 + (2-1)^2}$ = $\sqrt{(16+1)}$ = $\sqrt{17}$ units

DA = $\sqrt{(6-5)^2 + (2-6)^2}$ = $\sqrt{(1+16)}$ = $\sqrt{17}$ units

AB = BC = CD = DA

BD = $\sqrt{(6-1)^2 + (2-5)^2}$ = $\sqrt{(25+9)}$ = $\sqrt{34}$ units

AC = $\sqrt{(2-5)^2 + (1-6)^2}$ = $\sqrt{(9+25)}$ = $\sqrt{34}$ units

All the four sides of the quadrilateral are equal and diagonals are of equal length. Therefore, the given vertices form a square.

29 B. Question

Prove that the points A (2, 3), B (-2, 2), C (-1, -2), and D (3, -1) are the vertices of a square ABCD.

Answer

The Vertices of a quadrilateral are A(5,6), B(1,5), C(2,1) and D(6,2).

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB = $\sqrt{(-2-2)^2 + (2-3)^2}$ = $\sqrt{(16+1)}$ = $\sqrt{17}$ units

BC = $\sqrt{(-1+2)^2 + (-2-2)^2}$ = $\sqrt{(1+16)}$ = $\sqrt{17}$ units

CD = $\sqrt{(6-2)^2 + (2-1)^2}$ = $\sqrt{(16+1)}$ = $\sqrt{17}$ units

DA = $\sqrt{(6-5)^2 + (2-6)^2}$ = $\sqrt{(1+16)}$ = $\sqrt{17}$ units

AB = BC = CD = DA

BD = $\sqrt{(6-1)^2 + (2-5)^2}$ = $\sqrt{(25+9)}$ = $\sqrt{34}$ units

AC = $\sqrt{(2-5)^2 + (1-6)^2}$ = $\sqrt{(9+25)}$ = $\sqrt{34}$ units

All the four sides of the quadrilateral are equal and diagonals are of equal length. Therefore, the given vertices form a square.

30. Question

Find the point on x-axis which is equidistant from the points (-2, 5) and (2,-3).

Answer

Points A(-2, 5) and B(2, -3) are equidistant from point P.

Let the coordinates of point are P(x, 0)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ PA = PB

$$\sqrt{(x+2)^2 + (0-5)^2} = \sqrt{(x-2)^2 + (0+3)^2}$$

On squaring both sides, we get

$(x+2)^2 + (0-5)^2$ = $(x-2)^2 + (0+3)^2$

$x^2 + 4x + 4 + 25 = x^2 - 4x + 4 + 9$

8x = -16x = -2

Hence, coordinates are (-2, 0).

31. Question

Find the value of x such that $PQ = QR$ where the coordinates of P, Q and R are (6,-1), (1, 3) and (x, 8) respectively.

Answer

Coordinates are P(6,-1), Q(1, 3) and R(x, 8)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ PQ = QR

$$\sqrt{(1-6)^2+(3+1)^2} = \sqrt{(x-1)^2+(8-3)^2}$$

On squaring both sides, we get

$(1-6)^2+(3+1)^2 = (x-1)^2+(8-3)^2$

$25+16 = x^2-2x+1+25$

$x^2-2x-15=0$

On solving above equation, we get

$x^2-5x+3x-15=0$

$x(x-5)+3(x-5)=0$

$(x+3)(x-5)=0$

$x=-3$

$x=5$

Therefore x = -3, 5

32. Question

Prove that the points (0, 0), (5, 5) and (-5, 5) are the vertices of a right isosceles triangle.

Answer

Vertices of a quadrilateral are A (0, 0), B(5, 5) and C(-5, 5)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

AB = $\sqrt{(5-0)^2+(5-0)^2}$ = $\sqrt{(25+25)}$ = $\sqrt{50}$ units

BC = $\sqrt{(-5-5)^2+(5-5)^2}$ = $\sqrt{(100+0)}$ = $\sqrt{100}$ units

CA = $\sqrt{(-5-0)^2+(5-0)^2}$ = $\sqrt{(25+25)}$ = $\sqrt{50}$ units

Since AB = CA

Using Pythagoras theorem

$BC^2 = AC^2 + AB^2$

100 = 50 + 50

100 = 100

Therefore vertices are of right isosceles triangle.

33. Question

If the point P(x, y) is equidistant from the points A(5, 1) and B (1, 5), prove that x = y.

Answer

Coordinates are P(x, y), A(5, 1) and B (1, 5)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ PA = PB

$$\sqrt{(x-5)^2+(y-1)^2} = \sqrt{(x-1)^2+(y-5)^2}$$

On squaring both sides, we get

$(x-5)^2+(y-1)^2 = (x-1)^2+(y-5)^2$

$x^2-10x+25+y^2-2y+1 = x^2-2x+1+y^2-10y+25$

$-8x+8y = 0$

$x = y$ proved

34. Question

Q (0, 1) is equidistant from P (5, -3) and R (x, 6), find the values of x. Also, find the distances QR and PR.

Answer

Coordinates are Q(0, 1), P (5, -3) and R (x, 6),

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ QP = QR

$$\sqrt{(5-0)^2+(-3-1)^2} = \sqrt{(x-0)^2+(6-1)^2}$$

On squaring both sides, we get

$(5-0)^2+(-3-1)^2 = (x-0)^2+(6-1)^2$

$25+16 = x^2+25$

$x^2 = 16$

$x = \pm 4$ proved

QR = $\sqrt{(-4-0)^2+(6-1)^2}$ = $\sqrt{16+25}$ = $\sqrt{41}$ units

PR = $\sqrt{(4-5)^2+(6+3)^2}$ = $\sqrt{1+81}$ = $\sqrt{82}$ units

35. Question

Find the values of y for which the distance between the points P (2, -3) and Q (10, y) is 10 units.

Answer

Given: the distance between the points P (2, -3) and Q (10, y) is 10 units.

To find: The value of y.

Solution: Coordinates are P (2, -3) and Q (10, y)

We use distance formula $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ to find the distance between two points.

Since PQ = 10 unitsS0,

$\sqrt{(10-2)^2+(y+3)^2}$ = 10

On squaring both sides, we get

$(10-2)^2+(y+3)^2$ = 100

⇒ 8^2 + $(y+3)^2$ = 100

⇒ $64+y^2+6y+9$ = 100

⇒ 73 + y^2 + 6y = 100

⇒ 73 + y^2 + 6y -100 = 0

⇒ $y^2+6y-27$ = 0

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

⇒ y^2 + 9y - 3y - 27 = 0

⇒y(y + 9) - 3(y + 9) = 0

⇒(y - 3)(y + 9) = 0

⇒y = 3, -9

36. Question

Find the centre of the circle passing through (6, -6), (3, -7) and (3, 3).

Answer

Coordinates of points on a circle are A(6, -6), B(3, -7) and C(3, 3).

Let the coordinates of the centre of the circle be O(x, y)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Since the distance of the points A, B and C will be equal from the center, therefore

⇒ OA = OC

$$\sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y-3)^2}$$

On squaring both sides, we get

$(x-6)^2+(y+6)^2 = (x-3)^2+(y-3)^2$

$x^2-12x+36+y^2+12y+36 = x^2-6x+9+y^2-6y+9$

$x-3y = 9$.....................(1)

Similarly, OA = OB

$$\sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y+7)^2}$$

On squaring both sides, we get

$(x-6)^2+(y+6)^2 = (x-3)^2+(y+7)^2$

$x^2-12x+36+y^2+12y+36 = x^2-6x+9+y^2+14y+49$

$3x + y = 7$ ………….(1)

Solving eqn (1) and (2), we get

x = 3; y = -2

Coordinates of circum center are (3, -2)

37. Question

Two opposite vertices of a square are (-1, 2) and (3, 2). Find the coordinates of other two vertices.

Answer

The coordinates are A(-1, 2) and C(3, 2).

Let the coordinates of the vertex B are (x, y)

AB = BC

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(x+1)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (y-2)^2}$

On squaring both sides, we get

$(x+1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$x^2 + 2x + 1 + y^2 - 4y + 4 = x^2 - 6x + 9 + y^2 - 4y + 4$

$x = 1$

In ΔABC

$AB^2 + BC^2 = AC^2$ [Using Pythagoras theorem]

$2AB^2 = AC^2$ [Since AB = BC]

$2[(x+1)^2 + (y-2)^2] = (3+1)^2 + (2-2)^2$

$2[x^2 + 2x + 1 + y^2 - 4y + 4] = 16$

$x^2 + 2x + 1 + y^2 - 4y + 4 = 8$

$x^2 + 2x + y^2 - 4y = 3$

On substituting x = 1

$1 + 2 \times 1 + y^2 - 4y = 3$

$y^2 - 4y = 0$

$y(y-4) = 0$

$y = 0, 4$

Other coordinates are (1, 0) and (1, 4)

38. Question

Name the quadrilateral formed, if any, by the following points, and give reasons for your answers:

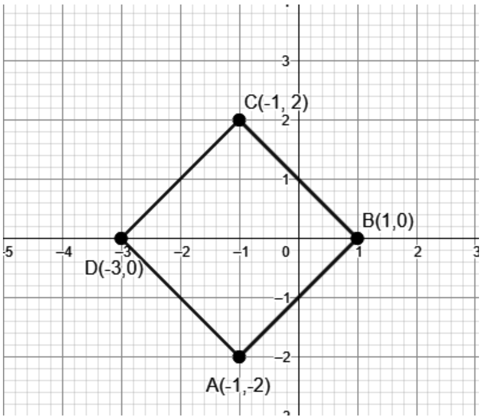
(i) A (-1, - 2), B (1, 0), C (-1, 2), D (-3, 0)

(ii) A (-3, 5), B (3, 1), C (0, 3), D (-1, - 4)

(iii) A (4, 5), B (7, 6), C (4, 3), D (1, 2)

Answer

(i) A (-1, - 2), B (1, 0), C (-1, 2), D (-3, 0)



Using distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8} \ units$$

$$BC = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} \ units$$

$$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} \ units$$

$$DA = \sqrt{(-3+1)^2 + (0+2)^2} = \sqrt{(-2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} \ units$$

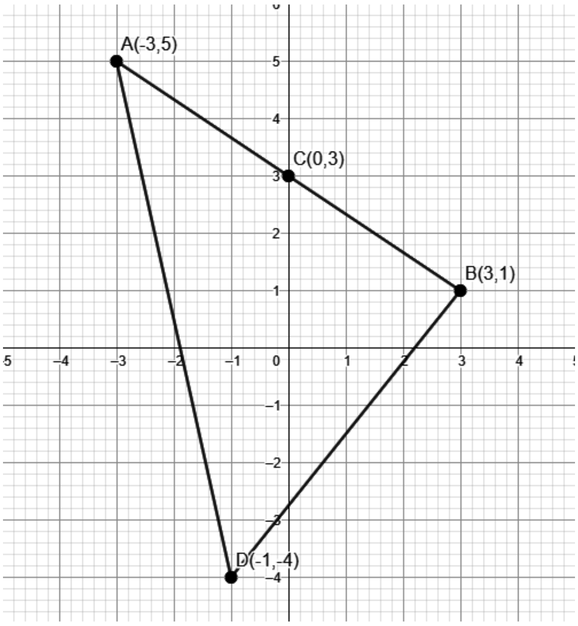
AB = BC = CD = DA

$$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{(-4)^2 + (0)^2} = \sqrt{16+0} = 4 \ units$$

$$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{(0)^2 + (4)^2} = \sqrt{0+16} = 4 \ units$$

All the four sides of the quadrilateral are equal and diagonals are of equal length. Therefore, the given vertices form a square.

(ii) A (-3, 5), B (3, 1), C (0, 3), D (-1, - 4)

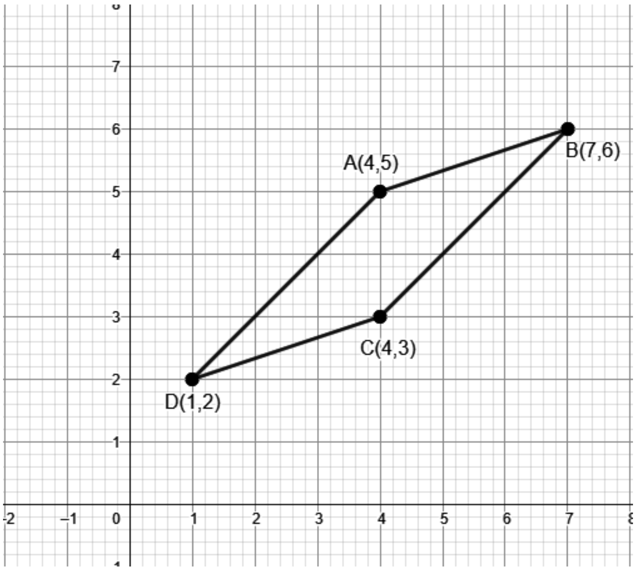


Using distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$AB = \sqrt{(3+3)^2 + (1-5)^2} = \sqrt{(6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}\,units$$
$$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{(-3)^2 + (2)^2} = \sqrt{9+4} = \sqrt{13}\ units$$
$$CD = \sqrt{(-1+0)^2 + (-4-3)^2} = \sqrt{(-1)^2 + (-7)^2} = \sqrt{1+49} = \sqrt{50}\ units$$
$$DA = \sqrt{(-1+3)^2 + (-4-5)^2} = \sqrt{(2)^2 + (-9)^2} = \sqrt{4+81} = \sqrt{85}\ units$$

Since all sides are of different length, therefore it is not a particular type of quadrilateral.

(iii) A (4, 5), B (7, 6), C (4, 3), D (1, 2)



Using distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$AB=\sqrt{(7-4)^2+(6-5)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}\ units$$
$$BC=\sqrt{(4-7)^2+(3-6)^2}=\sqrt{(3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}\ units$$
$$CD=\sqrt{(1-4)^2+(2-3)^2}=\sqrt{(-3)^2+(-1)^2}=\sqrt{9+1}=\sqrt{10}\ units$$
$$DA=\sqrt{(1-4)^2+(2-5)^2}=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}\ units$$

Coordinates of midpoint of diagonal AC X= $\frac{4+4}{2}=4$Y = $\frac{5+3}{2}=\frac{8}{2}=4$

Therefore coordinates of midpoint of AC are (4, 4)

Coordinates of midpoint of diagonal BD= X = $\frac{7+1}{2}=4$, Y = $\frac{6+2}{2}=\frac{8}{2}=4$

Therefore coordinates of midpoint of AC are (4, 4)

Since diagonals bisect each other at same point therefore quadrilateral is a parallelogram.

39. Question

Find the equation of the perpendicular bisector of the line segment joining points (7, 1) and (3, 5).

Answer

The points are A(7, 1) and B(3, 5).

Coordinates of midpoint of line AB = X = $\frac{7+3}{2}=5$, Y = $\frac{5+1}{2}=\frac{6}{2}=3$

Therefore coordinates of midpoint of AB are (5, 3)

Slope of the line = $\frac{y_2-y_1}{x_2-x_1}=\frac{5-1}{3-1}=\frac{4}{2}=2$

Negative reciprocal of slope = $-\frac{1}{2}$

Equation of line Y= mX+C

$$y=-\frac{x}{2}+C$$

$$3=-\frac{5}{2}+C$$

$$C=\frac{11}{2}$$

$$y=\ -\frac{1}{2}x+\frac{11}{2}$$

$$2y=\ -x+11$$

$$x+2y=11$$

Since diagonals bisect each other at same point therefore quadrilateral is a parallelogram.

40. Question

Prove that the points (3, 0), (4, 5), (-1, 4) and (-2, -1), taken in order, form a rhombus. Also, find its area.

Answer

Let the Vertices of a quadrilateral are: A(3, 0), B(4, 5), C(-1, 4) and D(-2, -1),

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(4-3)^2 + (5-0)^2}$ = $\sqrt{1+25}$ = $\sqrt{26}$ units

Length of side BC = $\sqrt{(-1-4)^2 + (4-5)^2}$ = $\sqrt{25+1}$ = $\sqrt{26}$ units

Length of side CD = $\sqrt{(-2+1)^2 + (-1-4)^2}$ = $\sqrt{1+25}$ = $\sqrt{26}$ units

Length of side DA = $\sqrt{(-2-3)^2 + (-1-0)^2}$ = $\sqrt{25+1}$ = $\sqrt{26}$ units

Length of diagonal AC = $\sqrt{(-1-3)^2 + (4-0)^2}$ = $\sqrt{16+16}$ = $\sqrt{32}$ units

Length of side BD = $\sqrt{(-2-4)^2 + (-1-5)^2}$ = $\sqrt{36+36}$ = $\sqrt{72}$ units

Since all sides are of equal length, therefore it is a rhombus.

Area of Rhombus = $\frac{1}{2} \times d_1 \times d_2$ = $\frac{1}{2} \times \sqrt{32} \times \sqrt{72}$ = $\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$ = 24 sq. units

41. Question

In the seating arrangement of desks in a classroom three students Rohini, Sandhya and Bina are seated at A (3, 1), B (6, 4) and C (8, 6). Do you think they are seated in a line?

Answer

Points are A (3, 1), B (6, 4) and C (8, 6)

For sitting in a line three points must be collinear i.e AB + BC = AC

Length of side AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(6-3)^2 + (4-1)^2}$ = $\sqrt{9+9}$ = $\sqrt{18}$ = $3\sqrt{2}$ units

Length of side BC = $\sqrt{(8-6)^2 + (6-4)^2}$ = $\sqrt{4+4}$ = $\sqrt{8}$ = $2\sqrt{2}$ units

Length of side AC = $\sqrt{(8-3)^2 + (6-1)^2}$ = $\sqrt{25+25}$ = $\sqrt{50}$ $= 5\sqrt{2}$ units

AB + BC = AC

$3\sqrt{2}$ + $2\sqrt{2}$ = $5\sqrt{2}$

The points are collinear.

42. Question

Find a point on y-axis which is equidistant from the points (5, - 2) and (- 3, 2).

Answer

Points A(5, -2) and B(-3, 2) are equidistance from point P.

Let the coordinates of point are P(0, y)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ PA = PB

$$\sqrt{(0-5)^2+(y+2)^2}=\sqrt{(0+3)^2+(y-2)^2}$$

On squaring both sides, we get

$(0-5)^2 + (y+2)^2 = (0+3)^2 + (y-2)^2$

$\Rightarrow (-5)^2 + (y+2)^2 = (3)^2 + (y-2)^2$

$\Rightarrow 25 + y^2 + 4 + 4y = 9 + y^2 + 4 - 4y$

$\Rightarrow 4y + 4y = 9 - 25$

$\Rightarrow 8y = -16$

$\Rightarrow y = -2$

Therefore coordinates are (0, -2).

43. Question

Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (-3, 4).

Answer

Coordinates of the points are A(3, 6) and B(-3, 4)

Let the point P(x, y) is equidistant from A and B

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\Rightarrow$ PA = PB

$$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x+3)^2+(y-4)^2}$$

On squaring both sides, we get

$(x-3)^2+(y-6)^2 = (x+3)^2+(y-4)^2$

$x^2-6x+9+y^2-12y+36=x^2+6x+9+y^2-8y+16$

$3x+y=5$

44. Question

If a point A (0, 2) is equidistant from the points B (3, p) and C (p, 5), then find the value of p.

Answer

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\Rightarrow$ AB = AC

$$\sqrt{(3-0)^2+(P-2)^2}=\sqrt{(P-0)^2+(5-2)^2}$$

On squaring both sides, we get

$(3-0)^2+(P-2)^2 = (P-0)^2+(5-2)^2$

$9+P^2-4P+4=P^2+9$

$P = 1$

45. Question

Prove that the points (7, 10), (-2, 5) and (3, -4) are the vertices of an isosceles right triangle.

Answer

Vertices of a quadrilateral are A (7, 10), B(-2, 5) and C(3, -4)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB = $\sqrt{(-2-7)^2 + (5-10)^2}$ = $\sqrt{(81+25)}$ = $\sqrt{106}$ units

BC = $\sqrt{(3+2)^2 + (-4-5)^2}$ = $\sqrt{(25+81)}$ = $\sqrt{106}$ units

AC = $\sqrt{(3-7)^2 + (-4-10)^2}$ = $\sqrt{(16+196)}$ = $\sqrt{212}$ units

Since AB = BC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$(\sqrt{212})^2 = (\sqrt{106})^2 + (\sqrt{106})^2$

212 = 106 + 106

212 = 212

Therefore vertices are of right isosceles triangle.

46. Question

If the point P (x, 3) is equidistant from the points A (7,-1) and B (6, 8), find the value of x and find the distance AP.

Answer

Coordinates are A (7,-1) and B (6, 8)

The point P (x, 3) is equidistant.

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ PA = PB

$\sqrt{(x-7)^2 + (3+1)^2} = \sqrt{(x-6)^2 + (3-8)^2}$

On squaring both sides, we get

$(x-7)^2 + (3+1)^2 = (x-6)^2 + (3-8)^2$

$x^2 - 14x + 49 + 16 = x^2 - 12x + 36 + 25$

$x = 2$

AP = $\sqrt{(2-7)^2 + (3+1)^2}$ = $\sqrt{25+16}$ = $\sqrt{41}$ units

47. Question

If A (3, y) is equidistant from points P (8, -3) and Q (7,6) , find the value of y and find the distance AQ.

Answer

Coordinates are P(8, -3) and Q(7,6)

The point A (3, y) is equidistant.

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ PA = QA

$$\sqrt{(3-8)^2+(y+3)^2}=\sqrt{(3-7)^2+(y-6)^2}$$

On squaring both sides, we get

$(3-8)^2+(y+3)^2 = (3-7)^2+(y-6)^2$

$25+y^2+6y+9 = 16+y^2-12y+36$

$y = 1$

AQ = $\sqrt{(3-7)^2+(1-6)^2} = \sqrt{16+25} = \sqrt{41}$ units

48. Question

If (0, - 3) and (0, 3) are the two vertices of an equilateral triangle, find the coordinates of its third vertex.

Answer

Coordinates are A(0, -3) and B(0,3) and C(x, y)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ AB = AC

$$\sqrt{(0-0)^2+(3+3)^2}=\sqrt{(x-0)^2+(y+3)^2}$$

On squaring both sides, we get

$(0-0)^2+(3+3)^2 = (x-0)^2+(y+3)^2$

$36 = x^2+y^2+6y+9$

$x^2+y^2+6y=27(1)$

⇒ AB = BC

$$\sqrt{(0-0)^2+(3+3)^2}=\sqrt{(x-0)^2+(y-3)^2}$$

On squaring both sides, we get

$(0-0)^2+(3+3)^2 = (x-0)^2+(y-3)^2$

$36 = x^2+y^2-6y+9$

$x^2+y^2-6y=27(2)$

On subtracting equation (2) from (1) we get

$12y = 0$

$y = 0$

On substituting y = 0 in equation (1), we get

$x^2 + y^2 + 6y = 27$

$x^2 = 27$

$x = \pm 3\sqrt{3}$

Therefore coordinates of third vertex are $(3\sqrt{3}, 0)$, $(-3\sqrt{3}, 0)$,

49. Question

If the point P (2, 2) is equidistant from the points A (-2, k) and B (-2k, -3), find k. Also, find the length of AP.

Answer

Coordinates of points are P(2, 2) A(-2, k) and B(-2k, -3)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ PA = PB

$\sqrt{(-2-2)^2 + (k-2)^2} = \sqrt{(-2k-2)^2 + (-3-2)^2}$

On squaring both sides, we get

$(-2-2)^2 + (k-2)^2 = (-2k-2)^2 + (-3-2)^2$

$16 + k^2 - 4k + 4 = 4k^2 + 8k + 4 + 25$

$k^2 + 4k + 3 = 0$

$k^2 + 3k + k + 3 = 0$

$k(k+3) + 1(k+3) = 0$

$(k+1)(k+3) = 0$

$k = -1, -3$

AP = $\sqrt{(-2-2)^2 + (-1-2)^2} = \sqrt{16+9}$ = 5 units

50. Question

If the point A (0, 2) is equidistant from the points B (3, p) and C (p, 5) the length of AB.

Answer

Coordinates of points are A(0, 2), B(3, p) and C(p, 5)

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

⇒ AB = AC

$\sqrt{(3-0)^2 + (p-2)^2} = \sqrt{(p-0)^2 + (5-2)^2}$

On squaring both sides, we get

$(3-0)^2+(p-2)^2=(p-0)^2+(5-2)^2$

$9+p^2-4p+4=p^2+9$

$-4p=-4$

$p=1$

AB = $\sqrt{(3-0)^2+(1-2)^2}=\sqrt{9+1}=\sqrt{10}$ units

51. Question

If the point P (k -1, 2) is equidistant from the points A (3, k) and B (k,5), find the value of k.

Answer

Coordinates of points are A(3, k), B(k, 5) and P(k-1, 2)

Using distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ PA = PB

$$\sqrt{(k-1-3)^2+(2-k)^2}=\sqrt{(k-1-k)^2+(2-5)^2}$$

On squaring both sides, we get

$(k-1-3)^2+(2-k)^2=(k-1-k)^2+(2-5)^2$

$(k-4)^2+(2-k)^2=(-1)^2+(-3)^2$

$k^2-8k+16+k^2-4k+4=1+9$

$k^2-6k+5=0$

$k^2-5k-k+5=0$

$k(k-5)-1(k-5)=0$

$(k-5)(k-1)=0$

$k=1, 5$

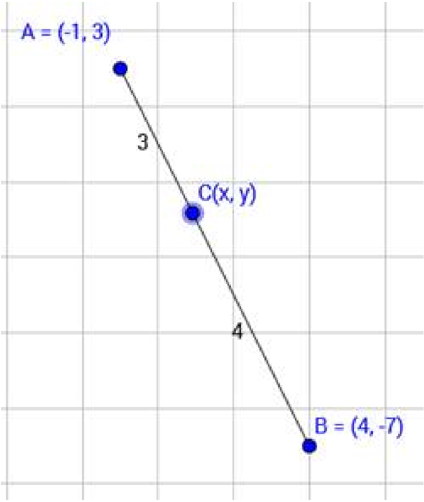
Exercise 14.3

1. Question

Find the coordinates of the point which divides the line segment joining (- 1, 3) and (4, -7) internally in the ratio 3 : 4.

Answer

Let our points be A(-1, 3) and B(4, -7) and required point be C(x, y)



Given that point divides internally in ratio of 3:4.

By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

Here, m = 3 and n = 4

$\therefore x = \frac{3 \times 4 + 4 \times (-1)}{3+4}$, $y = \frac{3 \times (-7) + 4 \times 3}{3+4}$

$\therefore x = \frac{12-4}{7}$, $y = \frac{-21+12}{7}$

$\therefore x = \frac{8}{7}$, $y = \frac{-9}{7}$

Hence, the required point is $C(\frac{8}{7}, \frac{-9}{7})$

2. Question

Find the points of trisection of the line segment joining the points:

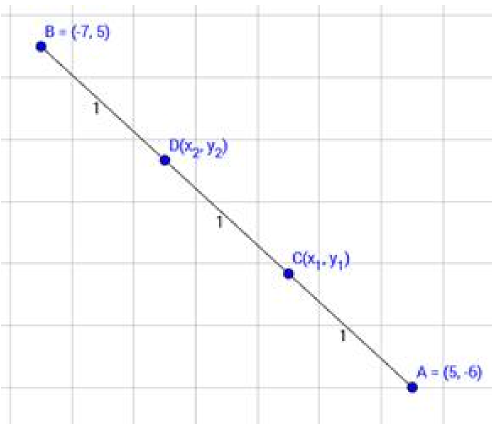
(i) (5, -6) and (- 7, 5), (ii) (3, -2) and (-3, -4), (iii) (2, -2) and (-7, 4)

Answer

(i) (5, -6) and (- 7, 5),

Let our given points be A(5,-6) and B(-7, 5) and required points be C (x_1, y_1) and D(x_2, y_2)

The points of trisection of a line are points which divide into the ratio 1:2



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(x_1 , y_1)

$x_1 = \frac{1\times(-7)+2\times5}{1+2}$,

$y_1 = \frac{1\times5+2\times(-6)}{1+2}$ ...Here m = 1 and n = 2

$\therefore x_1 = \frac{3}{3}$, $y_1 = \frac{-7}{3}$

$\therefore$ C (x_1 , y_1) ≡ (1, $\frac{-7}{3}$)

For point D(x_2 , y_2)

$X_2 = \frac{2\times(-7)+1\times5}{2+1}$, $y_2 = \frac{2\times5+1\times(-6)}{2+1}$ ...Here m = 2 and n = 1

$\therefore x_2 = \frac{-9}{3}$, $y_2 = \frac{4}{3}$

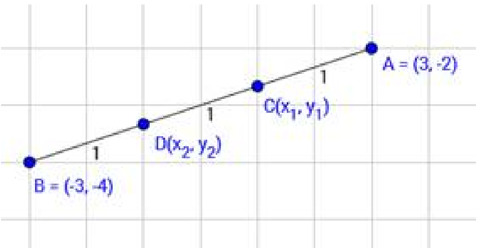
$\therefore$ D (x_2 , y_2)≡ (-3, $\frac{4}{3}$)

Hence, the points of trisection of line joining given points are (1, $\frac{-7}{3}$) and (-3, $\frac{4}{3}$)

(ii) (3, -2) and (-3, -4)

Let our given points be A(3,-2) and B(-3, -4) and required points be C (x_1 , y_1) and D(x_2 , y_2)

The points of trisection of a line are points which divide into the ratio 1:2



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(x_1 , y_1)

$x_1 = \frac{1\times(-3)+2\times3}{1+2}$, $y_1 = \frac{1\times(-4)+2\times(-2)}{1+2}$ ...Here m = 1 and n = 2

$\therefore x_1 = \frac{3}{3}$, $y_1 = \frac{-8}{3}$

$\therefore$ C (x_1 , y_1) ≡ (1 , $\frac{-8}{3}$)

For point D(x_2 , y_2)

$X_2 = \frac{2\times(-3)+1\times3}{2+1}$, $y_2 = \frac{2\times(-4)+1\times(-2)}{2+1}$ ...Here m = 2 and n = 1

$\therefore x_2 = \frac{-3}{3}$, $y_2 = \frac{-10}{3}$

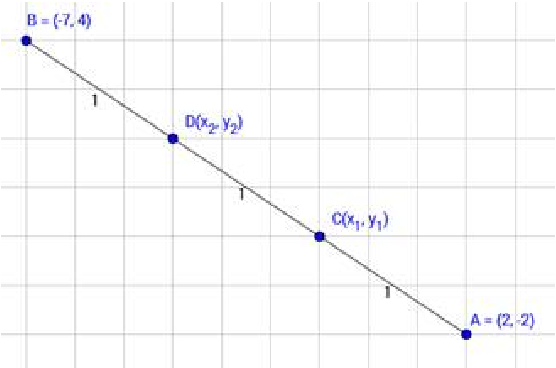
$\therefore$ D $(x_2 , y_2) \equiv (-1, \frac{-10}{3})$

Hence, the points of trisection of line joining given points are $(1 , \frac{-8}{3})$ and $(-1, \frac{-10}{3})$

(iii) (2, -2) and (-7, 4)

Let our given points be A(2,-2) and B(-7, 4) and required points be C (x_1 , y_1) and D(x_2 , y_2)

The points of trisection of a line are points which divide into the ratio 1:2



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(x_1 , y_1)

$x_1 = \frac{1 \times (-7)+2 \times 2}{1+2}$, $y_1 = \frac{1 \times 4 +2 \times (-2)}{1+2}$ ...Here m = 1 and n = 2

$\therefore x_1 = \frac{-3}{3}$, $y_1 = \frac{0}{3}$

$\therefore$ C $(x_1 , y_1) \equiv (-1 , 0)$

For point D(x_2 , y_2)

$X_2 = \frac{2 \times (-7)+1 \times 2}{2+1}$, $y_2 = \frac{2 \times 4 +1 \times (-2)}{2+1}$ ...Here m = 2 and n = 1

$\therefore x_2 = \frac{-12}{3}$, $y_2 = \frac{6}{3}$

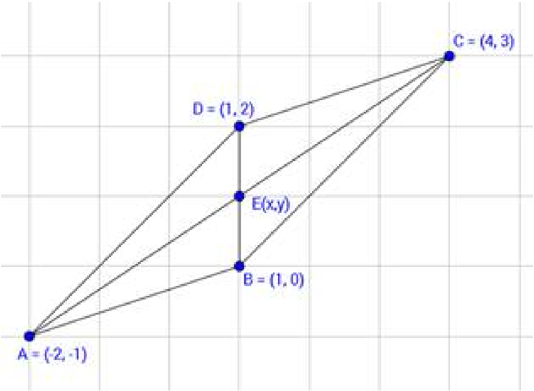
$\therefore$ D $(x_2 , y_2) \equiv (-4, 2)$

3. Question

Find the coordinates of the point where the diagonals of the parallelogram formed by joining the points (-2, -1), (1, 0), (4, 3) and (1, 2) meet.

Answer

Let our points of parallelogram be A(-2, -1), B(1, 0), C(4, 3) and D(1, 2) and mid point of diagonals be E(x,y)



We know that diagonals of parallelogram bisect each other.

Hence, we find mid point of AC.

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For point E(x, y)

$x_1 = \frac{-2+4}{2}$, $y_1 = \frac{-1+3}{2}$

$\therefore x_1 = \frac{2}{2}$, $y_1 = \frac{2}{2}$

$\therefore$ E (x, y) ≡ (1 , 1)

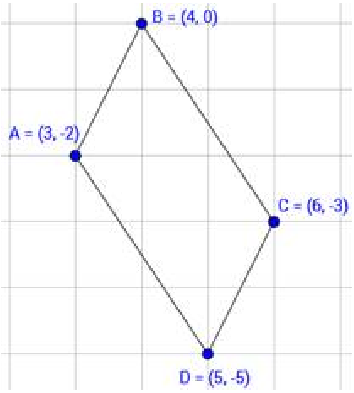
4. Question

Prove that the points (3, -2), (4, 0), (6, -3) and (5, -5) are the vertices of a parallelogram.

Answer

We know if the quadrilateral is parallelogram if opposite sides are equal.

Let our points be A(3, -2), B(4, 0), C(6, -3) and D(5, -5).



By distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For AB,

$AB = \sqrt{(4-3)^2 - (0-(-2))^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$ units

For BC,

BC = $\sqrt{(6-4)^2 - ((-3)-0)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$ units

For CD,

CD = $\sqrt{(5-6)^2 - ((-5)-(-3))^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$ units

For AD,

AD = $\sqrt{(5-3)^2 - ((-5)-(-2))^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$ units

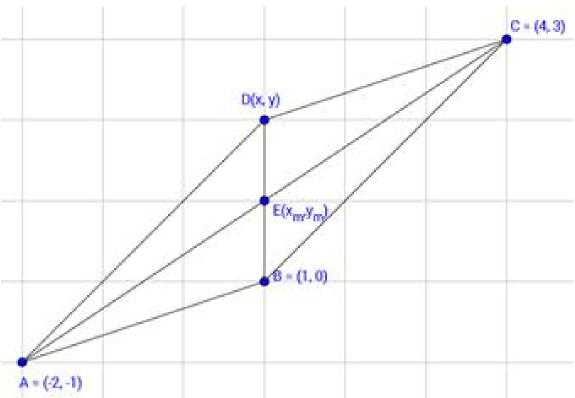
Here, we observe that AB = CD and AD = BC, which means that the quadrilateral formed by lines joining by points, is parallelogram.

5. Question

Three consecutive vertices of a parallelogram are (-2, -1), (1, 0) and (4, 3). Find the fourth vertex

Answer

Let three vertices be A(-2, -1), B(1, 0) and C(4, 3) and fourth vertex be D(x, y)



It is given that quadrilateral joining these four vertices is parallelogram.

∴□ABCD is parallelogram

We know that diagonals of parallelogram bisect each other, ie midpoint of the diagonals coincide.

Let $E(x_m , y_m)$ be the midpoint of diagonals AC and BD.

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For diagonal AC,

$x_m = \frac{-2+4}{2}$, $y_m = \frac{-1+3}{2}$

$\therefore x_m = \frac{2}{2}$, $y_m = \frac{2}{2}$

$\therefore E(x_m , y_m) \equiv (1, 1)$

For diagonal BD,

$1 = \frac{1+x}{2}$, $1 = \frac{0+y}{2}$

$\therefore x = 2 - 1$, $y = 2 - 0$

$\therefore x = 1$ and $y = 2$

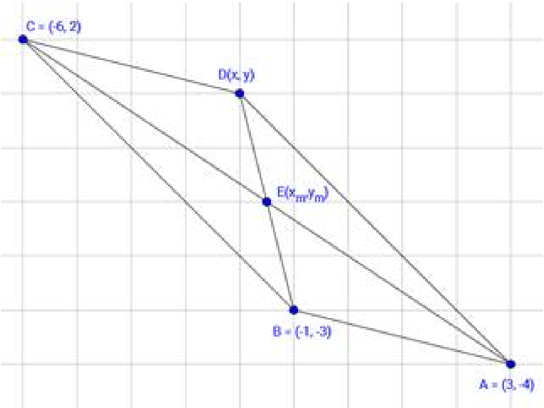
Hence, our fourth vertex is D(1, 2)

6. Question

The points (3, -4) and (-6, 2) are the extremities of a diagonal of a parallelogram. If the third vertex is (-1, -3). Find the coordinates of the fourth vertex.

Answer

Let three vertices be A(3, -4), B(-1, -3) and C(-6, 2) and fourth vertex be D(x, y)



It is given that quadrilateral joining these four vertices is parallelogram.

∴□ABCD is parallelogram

We know that diagonals of parallelogram bisect each other, ie midpoint of the diagonals coincide.

Let $E(x_m , y_m)$ be the midpoint of diagonals AC and BD.

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For diagonal AC,

$x_m = \frac{3+(-6)}{2}$, $y_m = \frac{-4+2}{2}$

$\therefore x_m = \frac{-3}{2}$, $y_m = \frac{-2}{2}$

$\therefore E(x_m , y_m) \equiv (\frac{-3}{2}, -1)$

For diagonal BD,

$\frac{-3}{2} = \frac{-1+x}{2}$, $-1 = \frac{-3+y}{2}$

∴ x = -3 + 1 , y = -2 + 3

∴ x = -2 and y = 1

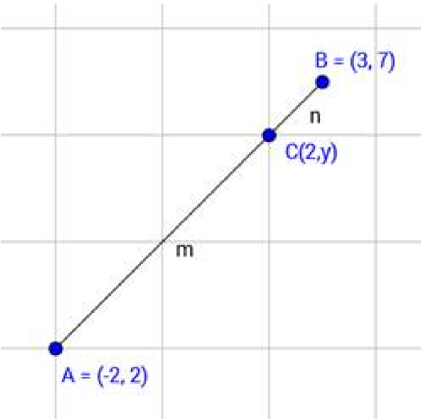
Hence, our fourth vertex is D(-2, 1)

7. Question

Find the ratio in which the point (2,y) divides the line segment joining the points A (-2, 2) and B (3, 7). Also, find the value of y.

Answer

Here, given points are A (-2, 2) and B (3, 7) and let the point dividing the line joining two points be C(2,y).



Let the ratio be m:n

By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(2,y),

$2 = \frac{m \times 3+n \times(-2)}{m+n}$ ...(1)

And $y = \frac{m \times 7+n \times 2}{m+n}$ ...(2)

Solving 1 for finding ratio between m and n,

$2 = \frac{m \times 3+n \times(-2)}{m+n}$

2(m + n) = 3m -2n

2m + 2n = 3m – 2n

∴ m = 4n

$\therefore \frac{m}{n} = \frac{4}{1}$

∴ m : n = 4 : 1

Now solving for equation 2, where m = 4 and n = 1

$y = \frac{m \times 7 + n \times 2}{m+n}$

$y = \frac{4 \times 7 + 1 \times 2}{4+1}$

∴ $y = \frac{28+2}{5}$

∴ $y = \frac{30}{5}$

∴ y =6

Hence, our point is (2, 6)

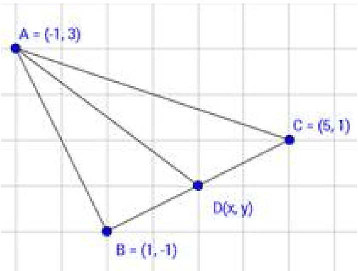
8. Question

If A (-1, 3), B (1, -1) and C (5, 1) are the vertices of a triangle ABC, find the length of the median through A.

Answer

Here given vertices of triangle are A (-1, 3), B (1, -1) and C (5, 1).

Let D, E and F be the midpoints of the sides BC, CA and AB respectively.



We need to find length of median passing through A, ie distance between AD.

Let point D ≡ (x, y)

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint D of side BC,

$x = \frac{1+5}{2}$, $y = \frac{-1+1}{2}$

∴ $x = \frac{6}{2}$, $y = \frac{0}{2}$

∴D(x , y) ≡ (3, 0)

Now, by distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For AD,

$AD = \sqrt{\left(3-(-1)\right)^2 + (0-3)^2}$

$\therefore$ AD = $\sqrt{16+9}$

AD = $\sqrt{25}$

$\therefore$AD = 5 units

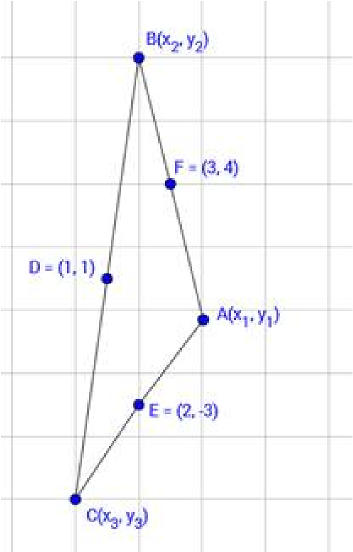
Hence, the length of the median through A is 5 units

9. Question

If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, -3) and (3, 4), find the vertices of the triangle.

Answer

Let A(x_1 , y_1), B(x_2 , y_2) and C(x_3 , y_3) be the vertices of triangle.



Let D(1, 1), E(2, -3) and F(3, 4) be the midpoints of sides BC, CA and AB respectively.

By midpoint formula.

x = $\frac{x_1+x_2}{2}$, y = $\frac{y_1+y_2}{2}$

For midpoint D(1, 1) of side BC,

1 = $\frac{x_2+x_3}{2}$, 1 = $\frac{y_2+y_3}{2}$

$\therefore x_2 + x_3$ = 2 and $y_2 + y_3$ = 2 ...(1)

For midpoint E(2, -3) of side CA,

2 = $\frac{x_1+x_3}{2}$, -3 = $\frac{y_1+y_3}{2}$

$\therefore x_1 + x_3$ = 4 and $y_1 + y_3$ = -6 ...(2)

For midpoint F(3, 4) of side AB,

3 = $\frac{x_1+x_2}{2}$, 4 = $\frac{y_1+y_2}{2}$

$\therefore x_1 + x_2$ = 6 and $y_1 + y_2$ = 8 ...(3)

Adding 1,2 and 3, we get,

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2$ = 2 + 4 + 6

And $y_2 + y_3 + y_1 + y_3 + y_1 + y_2$ = 2 – 6 +8

∴ $2(x_1 + x_2 + x_3)$ = 12 and $2(y_1 + y_2 + y_3)$ = 2

∴ $x_1 + x_2 + x_3$ = 6 and $y_1 + y_2 + y_3$ = 1

x_1 + 2 = 6 and y_1 + 2 = 2 ...from 1

∴ x_1 = 4 and y_1 = 0

Substituting above values in 3,

4 + $x_2 = 6$ and 0 + y_2 = 8

∴ x_2 = 2 and y_2 = 89

Similarly for equation 2,

4 + x_3 = 6 and 0 + y_3 = -6

∴ x_3 = 2 and y_3 = -6

Hence the vertices of triangle are A(4 , 0), B(2 ,8) and C(0 ,-6)

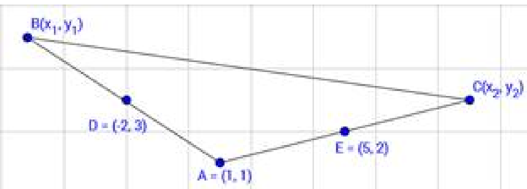
10. Question

If a vertex of a triangle be (1, 1) and the middle points of the sides through it be (-2, 3) and (5, 2), find the other vertices.

Answer

Let in ΔABC, A(1,1), B(x_1, y_1) and C(x_2, y_2).

Let D(-2, 3) and E(5, 2) be the midpoints of sides AB and AC respectively.



By midpoint formula.

x = $\frac{x_1 + x_2}{2}$, y = $\frac{y_1 + y_2}{2}$

For mid point D (-2, 3) of side AB,

-2 = $\frac{1+x_1}{2}$, 3 = $\frac{1+y_1}{2}$

1 + x_1 = -4 and 1 + y_1 = 6

x_1 = -5 and y_1 = 5

∴ B (x_1, y_1) ≡ (-5, 5)

For midpoint E(5, 2) of side AC,

5 = $\frac{1+x_2}{2}$, 2 = $\frac{1+y_2}{2}$

$1 + x_2 = 10$ and $1 + y_2 = 4$

∴ $x_2 = 9$ and $y_2 = 3$

∴ $C(x_2, y_2) \equiv (9, 3)$

Hence other two vertices are B (-5, 5) and C(9, 3)

11 A. Question

In what ratio is the line segment joining the points (-2, -3) and (3, 7) divided by the y-axis? Also, find the coordinates of the point of division.

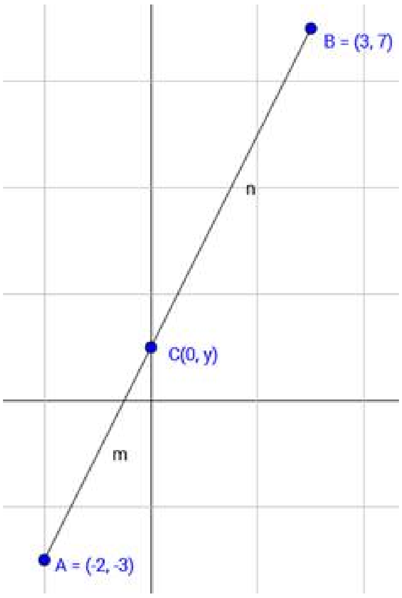
Answer

Here y axis divides our line joined by the points (say) A (-2, -3) and B (3, 7).

Let coordinate of the point be C(0, y).

Here our x- coordinate is zero, as point C lie on x-axis.

Let y axis divide AB in ratio of m:n.



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point C(0, y) on line joined by the points A and B,

$0 = \frac{m \times 3 + n \times (-2)}{m+n}$ ...(1)

And, $y = \frac{m \times 7 + n \times (-3)}{m+n}$ ...(2)

Solving 1,

$0(m + n) = 3m - 2n$

∴ 3m = 2n

∴m:n = 2:3

Now solving for 2, for values m = 2 and n = 3,

$y = \frac{2 \times 7 + 3 \times (-3)}{2+3}$

$y = \frac{14-6}{5}$

$\therefore y = \frac{5}{5} = 1$

$\therefore C(0, y) \equiv (0, 1)$

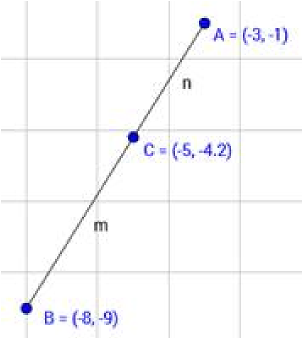
11 B. Question

In what ratio is the line segment joining (-3, -1) and (-8, -9) divided at the point (-5, -21 /5)?

Answer

Let given points be A (-3, -1) and B (-8, -9).

Let the point C(-5, -21 /5) divide AB in ratio m:n.



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point C(-5, -21 /5) on the line joined by the points A and B.

$-5 = \frac{m \times (-8) + n \times (-3)}{m+n}$ ...(1)

And, $-\frac{21}{5} = \frac{m \times (-9) + n \times (-1)}{m+n}$ ...(2)

Solving 1,

-5(m + n) = -8m - 3n

∴5m + 5n = 8m + 3n

∴ 2n = 3m

$\therefore \frac{m}{n} = \frac{2}{3}$

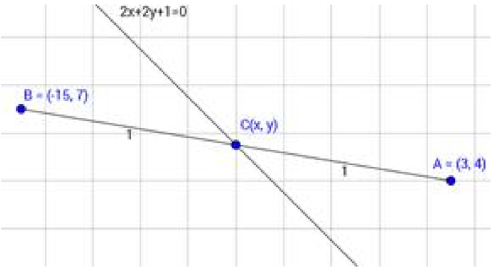
Hence, ratio is 2:3.

12. Question

If the mid-point of the line joining (3, 4) and (k, 7) is (x, y) and 2x + 2y + 1 = 0, find the value of k.

Answer

Let A(3, 4) and B(k, 7) and midpoint be C(x, y) which lies on the line 2x+2y+1 = 0



By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For point C(x, y),

$x = \frac{3+k}{2}$, $y = \frac{4+7}{2}$ ...(1)

Here, $y = \frac{11}{2}$,

Hence, substituting value of y in given equation of line,

$2x + 2 \times \frac{11}{2} + 1 = 0$

∴ 2x = -12

∴ x = -6

Now substituting value of x in equation(1), we get.

$x = \frac{3+k}{2}$

$-6 = \frac{3+k}{2}$

∴ -12 = 3 + k

∴ k = -15

Hence, the value of k is -15.

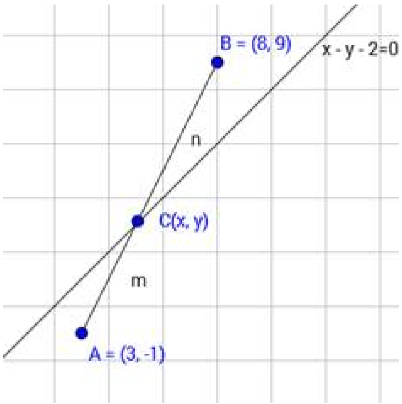
13. Question

Determine the ratio in which the straight line x - y - 2 = 0 divides the line segment joining (3, -1) and (8, 9).

Answer

Let point be A(3, -1) and B(8, 9).

Let the line divide the line joining the points A and B in the ratio m:n at any point C(x, y)



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(x, y),

$x = \frac{m\times 8+n\times 3}{m+n}$, $y = \frac{m\times 9+n\times(-1)}{m+n}$

$\therefore x = \frac{8m+3n}{m+n}$, $y = \frac{9m-n}{m+n}$

Now, substituting value of x and y in equation x - y - 2 = 0,

$\frac{8m+3n}{m+n} - \frac{9m-n}{m+n} - 2 = 0$

$\frac{8m+3n-9m+n-2m-2n}{m+n} = 0$

$\therefore -3m + 2n = 0$

$\therefore \frac{m}{n} = \frac{2}{3}$

∴ m:n = 2:3

Hence, the line divides the line segment joining A and B in the ratio 2:3 internally.

14. Question

Find the ratio in which the line segment joining (-2, -3) and (5, 6) is divided by (i) x-axis (ii) y-axis. Also, find the coordinates of the point of division in each case.

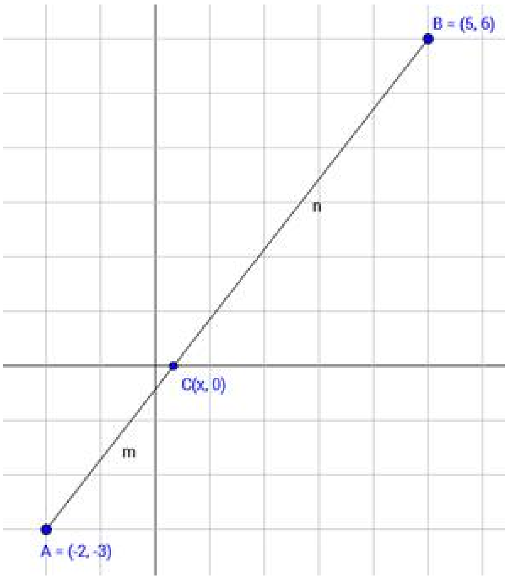
(i) x-axis

Answer

(i) x-axis

Let our points be A(-2, -3) and B(5, 6).

Let point C(x, 0) divide the line formed by joining by the points A and B in ratio of m:n.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(x, 0)

$x = \frac{m \times 5+n \times(-2)}{m+n}$, $0 = \frac{m \times 6+n \times(-3)}{m+n}$

Solving for y coordinate,

$0 = \frac{m \times 6+n \times(-3)}{m+n}$

∴ 6m -3n = 0

∴ 2m = n

∴ $\frac{m}{n} = \frac{1}{2}$

∴ m : n = 1 : 2

Now solving for x coordinate, with m = 1 and n = 2,

$x = \frac{1 \times 5+2 \times(-2)}{1+2}$

∴ $x= \frac{5-4}{3}$

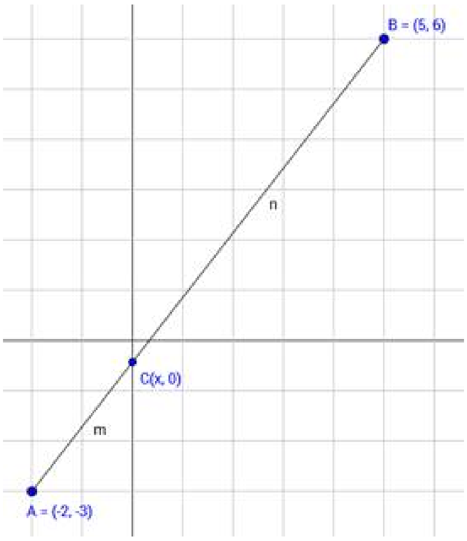
∴ $x = \frac{1}{3}$

Hence, the coordinates of required point is $C(\frac{1}{3}, 0)$

(ii) y-axis.

Let our points be A(-2, -3) and B(5, 6).

Let point C(0, y) divide the line formed by joining by the points A and B in ratio of m:n.



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point C(0, y)

$0 = \frac{m \times 5 + n \times (-2)}{m+n}$, $y = \frac{m \times 6 + n \times (-3)}{m+n}$

Solving for x coordinate,

$0 = \frac{m \times 5 + n \times (-2)}{m+n}$

$\therefore$ 5m – 2n = 0

$\therefore \frac{m}{n} = \frac{2}{5}$

$\therefore$ m : n = 2 : 5

Now solving for y coordinate, with m = 2 and n = 5,

$y = \frac{2 \times 6 + 5 \times (-3)}{2+5}$

$y = \frac{12-15}{7}$

$\therefore y = \frac{-3}{7}$

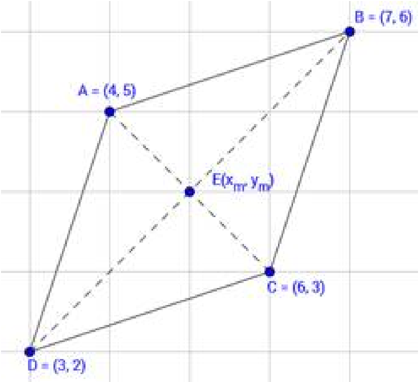
Hence, the coordinates of required point is C(0 , $\frac{-3}{7}$)

15. Question

Prove that the points (4, 5), (7, 6), (6, 3), (3, 2) are the vertices of a parallelogram. Is it a rectangle.

Answer

Let given points be A(4, 5), B(7, 6), C(6, 3), D(3, 2) and let the intersection of diagonals be E(x_m , y_m)



By midpoint formula.

x = $\frac{x_1 + x_2}{2}$, y = $\frac{y_1 + y_2}{2}$

For midpoint of diagonal AC,

X_1 = $\frac{4+6}{2}$, y_1 = $\frac{5+3}{2}$

∴x_1 = $\frac{10}{2}$ = 5 , y_1 = $\frac{8}{2}$ = 4

∴ midpoint of diagonal AC is (x_1, y_1) ≡ (5, 4) ...(1)

For midpoint of diagonal BD,

X_2 = $\frac{7+3}{2}$, y_2 = $\frac{6+2}{2}$

∴x_2 = $\frac{10}{2}$ = 5 , y_2 = $\frac{8}{2}$ = 4

∴ midpoint of diagonal BD is (x_2, y_2) ≡ (5, 4) ...(2)

Here, from 1 and 2 we say that midpoint of both the diagonals intersect at same point, ie (5, 4)

But our intersection of diagonals is at E, which means that midpoint of diagonals intersect at single point, ie E(5, 4)

We know that if midpoints of diagonals intersect at single point, then quadrilateral formed by joining the points is parallelogram.

Hence, our □ABCD is parallelogram.

Now, we shall check whether □ABCD is rectangle.

If the lengths of diagonals are same, then given quadrilateral is rectangle.

By distance formula,

XY = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For diagonal AC,

AC = $\sqrt{(6-4)^2 + (3-5)^2}$

= $\sqrt{4+4}$

= $2\sqrt{2}$ units

For diagonal BD,

AC = $\sqrt{(7-3)^2+(6-2)^2}$

= $\sqrt{16+16}$

= $4\sqrt{2}$ units.

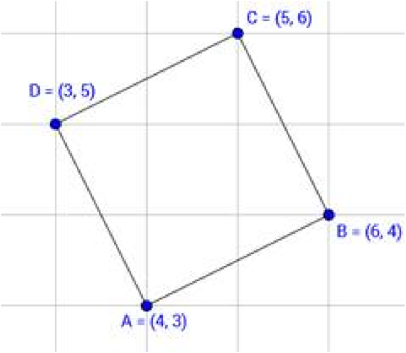
Here, AC ≠ BD, hence □ABCD is not rectangle.

16. Question

Prove that (4, 3), (6, 4), (5, 6) and (3, 5) are the angular points of a square.

Answer

Let given points be A(4, 3), B(6, 4), C(5, 6) and D(3, 5).



By distance formula,

XY = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

For AB,

AB = $\sqrt{(6-4)^2+(4-3)^2}$

= $\sqrt{4+1}$

= $\sqrt{5}$ units.

For BC,

BC = $\sqrt{(5-6)^2+(6-4)^2}$

= $\sqrt{1+4}$

=$\sqrt{5}$ units.

For CD,

CD =$\sqrt{(3-5)^2+(5-6)^2}$

= $\sqrt{4+1}$

= $\sqrt{5}$ units.

For AD,

AD = $\sqrt{(3-4)^2+(5-3)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$ units.

Here, we can observe that □ABCD is a parallelogram.

Now,

For diagonal AC,

AC = $\sqrt{(5-4)^2+(6-3)^2}$

$= \sqrt{1+9}$

$= \sqrt{10}$ units.

For diagonal BD,

BD $= \sqrt{(3-6)^2+(5-4)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$ units.

∴ AC = BD, which means diagonals are equal.

We know that quadrilateral in which all sides are equal and diagonals are equal, is a square.

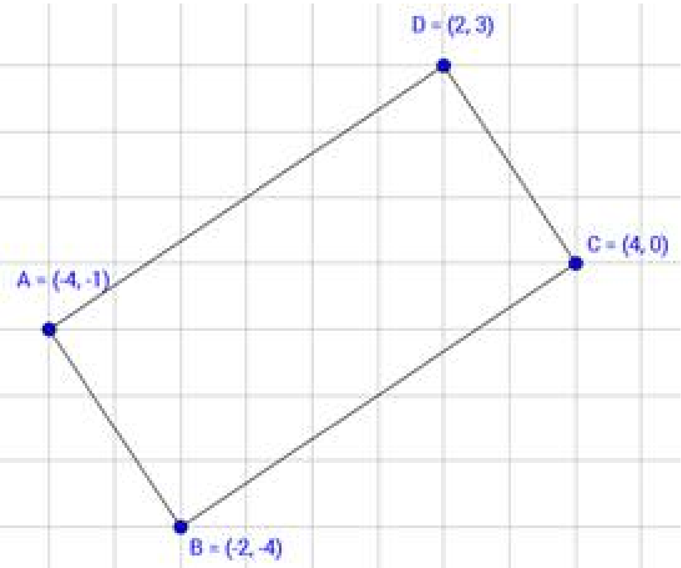
∴□ABCD is a square.

17. Question

Prove that the points (-4, -1), (-2, -4), (4, 0) and (2, 3) are the vertices of a rectangle.

Answer

Solution : Let the given points be A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3).



Use distance formula, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

For AB,

AB = $\sqrt{(-2-(-4))^2+(-4-(-1))^2}$

$$=\sqrt{(-2+4)^2+(-4+1)^2}$$

= $\sqrt{(2)^2+(-3)^2}$

= $\sqrt{4+9}$

=$\sqrt{13}$ units

For BC,

BC = $\sqrt{(4-(-2))^2+(0-(-4))^2}=\sqrt{(4+2)^2+(0+4)^2} \quad =\sqrt{6^2+4^2}$

= $\sqrt{36+\ 16}$

= $\sqrt{52}$ units

For CD,

$$CD=\sqrt{(2-4)^2+(3-0)^2} \quad =\sqrt{(-2)^2+(3)^2}$$

= $\sqrt{4+9}$

=$\sqrt{13}$ units

For AD,

AD = $\sqrt{(2-(-4))^2+(3-(-1))^2}=\sqrt{(2+4)^2+(3+1)^2} \quad =\sqrt{(6)^2+(4)^2}$

= $\sqrt{36+\ 16}$

= $\sqrt{52}$ units

Also, for diagonal AC,

AC = $\sqrt{(4-(-4))^2+(0-(-1))^2}=\sqrt{(4+4)^2+(0+1)^2} \quad =\sqrt{(8)^2+(1)^2}$
$=\sqrt{64+1}$

= $\sqrt{65}$ units

For diagonal BD,

BD = $\sqrt{(2-(-2))^2+(3-(-4))^2}=\sqrt{(2+2)^2+(3+4)^2} \quad =\sqrt{(4)^2+(7)^2}$

= $\sqrt{16+49}$

= $\sqrt{65}$ units

We can observe that AB = CD and BC = AD and also diagonal AC = BD.

We know that a quadrilateral whose opposite sides are equal and the diagonal are equal is rectangle.

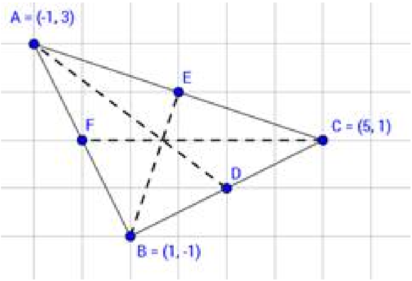
∴ ABCD is a rectangle.

18. Question

Find the lengths of the medians of a triangle whose vertices are A (-1,3), B (1,-1) and C(5,1).

Answer

Here given vertices are A(-1,3), B (1,-1) and C(5,1) and let midpoints of BC, CA and AB be D,E and F respectively.



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint D of side BC,

$x = \frac{1+5}{2}$, $y = \frac{-1+1}{2}$

$x = \frac{6}{2}$, $y = \frac{0}{2}$

∴midpoint of side BC is D(3, 0)

For midpoint E of side AB,

$x = \frac{-1+5}{2}$, $y = \frac{3+1}{2}$

$x = \frac{4}{2}$, $y = \frac{4}{2}$

∴midpoint of side AB is E(2, 2)

For midpoint F of side CA,

$x = \frac{-1+1}{2}$, $y = \frac{3-1}{2}$

$x = \frac{0}{2}$, $y = \frac{2}{2}$

∴midpoint of side CA is F(0, 1)

By distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For median AD,

$AD = \sqrt{(3-(-1))^2 + (0-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

= 5 units

For median BE,

BE = $\sqrt{(2-1)^2+(2-(-1))^2}$

= $\sqrt{1+9}$

= $\sqrt{10}$ units.

For median CF,

CF = $\sqrt{(0-5)^2+(1-1)^2}$

= $\sqrt{25}$

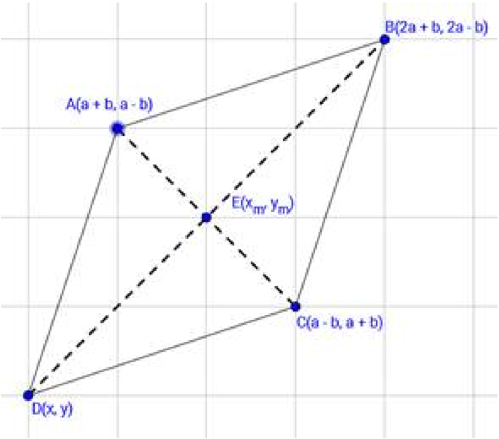
= 5 units

19. Question

Three vertices of a parallelogram are (a + b, a - b), (2a + b, 2a - b), (a - b, a + b). Find the fourth vertex.

Answer

Let A(a + b, a - b), B(2a + b, 2a - b), C(a - b, a + b) and fourth vertex be D(x, y).



It is given that ▫ABCD is parallelogram.

We know that diagonals of parallelogram bisect each other.

Let intersection of diagonals be E(x_m, y_m)

By midpoint formula.

$x_m = \frac{x_1+x_2}{2}$, $y_m = \frac{y_1+y_2}{2}$

For midpoint E of diagonal AC,

$x_m = \frac{a+b+a-b}{2}$, $y_m = \frac{a-b+a+b}{2}$

∴ x_m = a , y_m = a

∴E(x_m, y_m) ≡ (a, a)

For diagonal BD,

a = $\frac{2a+b+x}{2}$, a= $\frac{2a-b+y}{2}$

∴ 2a = 2a + b +x , 2a = 2a − b +y

∴ x = -b and y = b

Hence, the fourth vertex is D(-b, b)

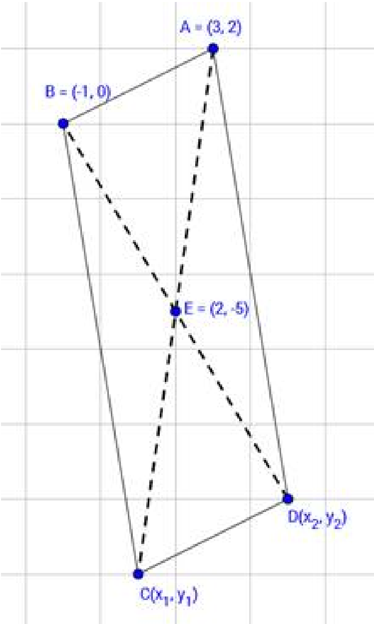
20. Question

If two vertices of a parallelogram are (3, 2), (-1, 0) and the diagonals cut at (2, -5), find the other vertices of the parallelogram.

Answer

Let the vertices be A(3, 2), B(-1, 0), C(x_1, y_1) and D(x_2, y_2).

Let diagonals cut at E(2, - 5).



We know that mid points of diagonals of parallelogram coincide.

By midpoint formula.

$x_m = \frac{x_1 + x_2}{2}$, $y_m = \frac{y_1+y_2}{2}$

For midpoint E of diagonal AC,

$2 = \frac{3+ x_1}{2}$, $-5 = \frac{2+y_1}{2}$

∴x_1 = 1 and y_1 = -12

∴ coordinates of C are (1, -12)

For midpoint E of diagonal BD,

$2 = \frac{-1+ x_2}{2}$, $-5 = \frac{0+y_2}{2}$

∴ x_2 = 5 and y_2 = -10

∴ coordinates of D are (5, -10)

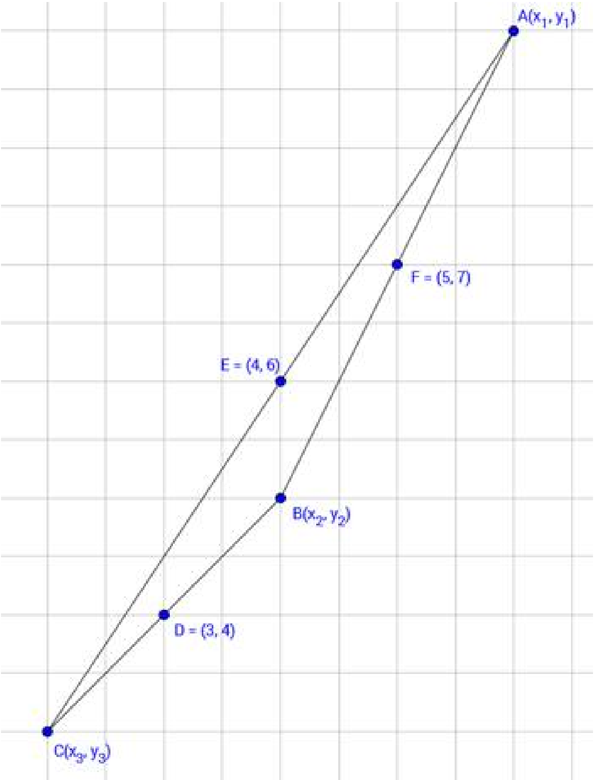
21. Question

If the coordinates of the mid-points of the sides of a triangle are (3, 4), (4, 6) and (5, 7), find its vertices.

Answer

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of triangle.

Let D(3, 4), E(4, 6) and F(5, 7) be the midpoints of sides BC, CA and AB respectively.



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint D(3, 4) of side BC,

$3 = \frac{x_2 + x_3}{2}$, $4 = \frac{y_2 + y_3}{2}$

$\therefore x_2 + x_3 = 6$ and $y_2 + y_3 = 8$ ...(1)

For midpoint E(4, 6) of side CA,

$4 = \frac{x_1 + x_3}{2}$, $6 = \frac{y_1 + y_3}{2}$

$\therefore x_1 + x_3 = 8$ and $y_1 + y_3 = 12$ ...(2)

For midpoint F(5, 7) of side AB,

$5 = \frac{x_1 + x_2}{2}$, $7 = \frac{y_1 + y_2}{2}$

$\therefore x_1 + x_2 = 10$ and $y_1 + y_2 = 14$ ...(3)

Adding 1,2 and 3, we get,

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 6 + 8 + 10$

And $y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 8 + 12 + 14$

$\therefore 2(x_1 + x_2 + x_3) = 24$ and $2(y_1 + y_2 + y_3) = 34$

$\therefore x_1 + x_2 + x_3 = 12$ and $y_1 + y_2 + y_3 = 17$

$x_1 + 6 = 12$ and $y_1 + 8 = 17$ ...from 1

$\therefore x_1 = 6$ and $y_1 = 9$

Substituting above values in 3,

$6 + x_2 = 10$ and $9 + y_2 = 14$

$\therefore x_2 = 4$ and $y_2 = 5$

Similarly for equation 2,

$6 + x_3 = 8$ and $9 + y_3 = 12$

$\therefore x_3 = 2$ and $y_3 = 3$

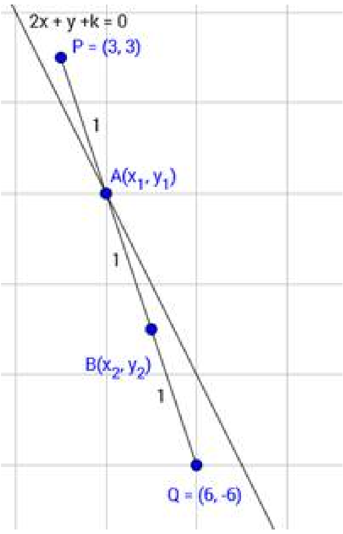
Hence the vertices of triangle are A(6 , 9), B(4 ,5) and C(2 ,3)

22. Question

The line segment joining the points P (3, 3) and Q (6, - 6) is trisected at the points A and B such that A is nearer to P. If A also lies on the line given by 2x + y + k = 0, find the value of k.

Answer

Here, given points are P (3, 3) and Q (6, - 6) which is trisected at the points(say) $A(x_1 , y_1)$ and $B(x_2 , y_2)$such that A is nearer to P.



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point $A(x_1 , y_1)$ of PQ, where m = 2 and n = 1,

$x_1 = \frac{2 \times 3 + 1 \times 6}{2+1}$, $y_1 = \frac{2 \times 3 + 1 \times (-6)}{2+1}$

$\therefore x_1 = 4$, $y_1 = 0$

∴Coordinates of A is (4,0)

It is given that point A lies on the line 2x + y + k = 0.

So, substituting value of x and y as coordinates of A,

$2 \times 4 + 0 + k = 0$

$\therefore k = -8$

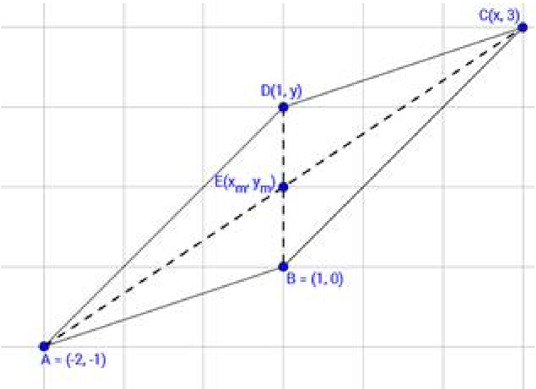
23. Question

If the points (-2 , -1), (1, 0), (x, 3) and (1, y) form a parallelogram, find the values of x and y.

Answer

Let given points be A(-2, -1), B(1, 0), C(x, 3), D(1, y) and let the intersection of diagonals be $E(x_m , y_m)$

It is given that □ABCD is a parallelogram.



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

We know that midpoint of parallelogram coincide.

∴ Midpoint of AC = Midpoint of BD

$\therefore \left(\frac{x-2}{2}, \frac{3-1}{2}\right) = \left(\frac{1+1}{2}, \frac{y+0}{2}\right)$

$\therefore \frac{x-2}{2} = \frac{1+1}{2}$ and $\frac{3-1}{2} = \frac{y+0}{2}$

$\therefore x = 4$ and $y = 2$

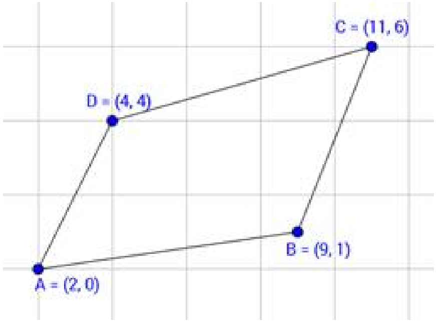
24. Question

The points A (2, 0), B (9, 1), C (11, 6) and D (4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.

Answer

Here given points are A (2, 0), B (9, 1), C (11, 6) and D (4, 4).

For a quadrilateral to be rhombus, all sides must be equal.



By distance formula,

XY = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For side AB,

AB = $\sqrt{(9-2)^2 + (1-0)^2}$

= $\sqrt{49 + 1}$

= $\sqrt{50}$ units.

For BC,

BC = $\sqrt{(11-9)^2 + (6-1)^2}$

= $\sqrt{4 + 25}$

= $\sqrt{29}$ units

CD = $\sqrt{(11-4)^2 + (6-4)^2}$

= $\sqrt{49 + 4}$

= $\sqrt{53}$ units.

AD = $\sqrt{(4-2)^2 + (4-0)^2}$

= $\sqrt{4 + 16}$

= $\sqrt{20}$ units.

Here all sides are unequal.

Hence □ABCD is not a rhombus.

25. Question

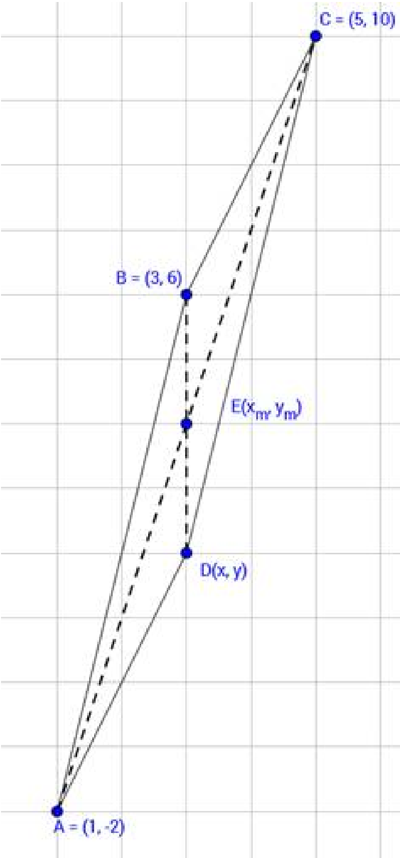
If three consecutive vertices of a parallelogram are (1, -2), (3, 6) and (5, 10), find its fourth vertex.

Answer

Let three vertices be A(1, -2), B(3, 6) and C(5, 10) and fourth vertex be D(x, y)

It is given that quadrilateral joining these four vertices is parallelogram, ie □ABCD is parallelogram.

We know that diagonals of parallelogram bisect each other, ie midpoint of the diagonals coincide.



Let $E(x_m , y_m)$ be the midpoint of diagonals AC and BD.

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For diagonal AC,

$x_m = \frac{1+5}{2}$, $y_m = \frac{-2+10}{2}$

$\therefore x_m = \frac{6}{2}$, $y_m = \frac{8}{2}$

$\therefore E(x_m , y_m) \equiv (3, 4)$

For diagonal BD,

$3 = \frac{3 + x}{2}$, $4 = \frac{6 + y}{2}$

$\therefore x = 6 - 3$, $y = 8 - 6$

$\therefore x = 3$ and $y = 2$

Hence, our fourth vertex is D(3 , 2)

26. Question

If the points A(a, -11), B(5, b), C(2, 15) and D(1, 1) are the vertices of a parallelogram ABCD, find the values of a and b.

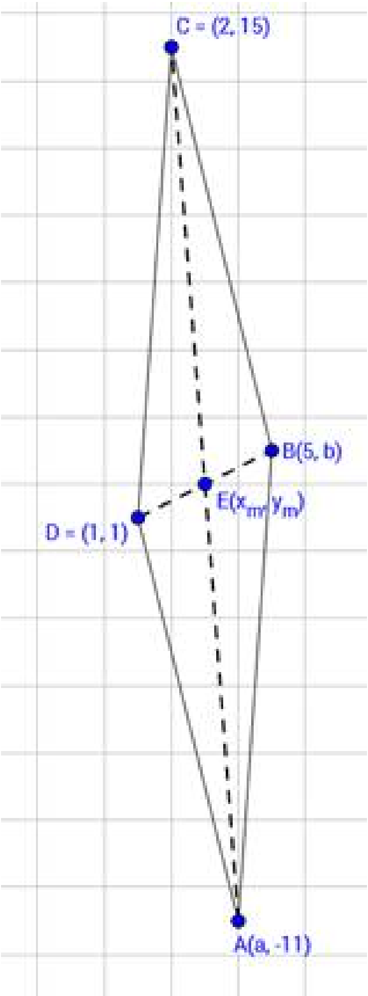
Answer

Given: the points A(a, -11), B(5, b), C(2, 15) and D(1, 1) are the vertices of a parallelogram ABCD.

To find: the values of a and b.

Solution: Given points are A(a, -11), B(5, b), C(2, 15) and D(1, 1) and let the intersection of diagonals be E(x_m , y_m)

It is given that □ABCD is a parallelogram.



By midpoint formula.

x = $\frac{x_1 + x_2}{2}$, y = $\frac{y_1 + y_2}{2}$

We know that midpoint of parallelogram coincide.

∴ Midpoint of AC = Midpoint of BD

∴ $\left(\frac{a+2}{2}, \frac{15-11}{2}\right) = \left(\frac{5+1}{2}, \frac{b+1}{2}\right)$

∴ $\frac{a+2}{2} = \frac{5+1}{2}$ and $\frac{15-11}{2} = \frac{b+1}{2}$

$$\Rightarrow \frac{a+2}{2} = \frac{6}{2}\ and\ \frac{15-11}{2} = \frac{b+1}{2}$$

$$\Rightarrow \frac{a+2}{2} = 3and\ \frac{4}{2} = \frac{b+1}{2}$$

$$\Rightarrow \frac{a+2}{2} = 3 \ and \ \ 2 = \frac{b+1}{2}$$

⇒ a + 2 = 6 and 4 = b + 1⇒ a = 6 - 2 and 4 - 1 = b

⇒ a = 4 and 3 = b

∴a = 4 and b = 3

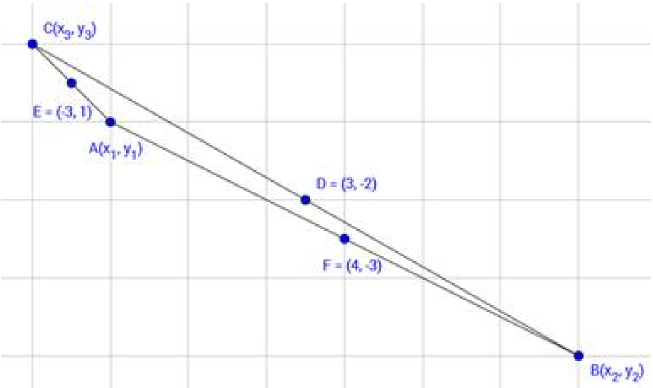
27. Question

If the coordinates of the mid-points of the sides of a triangle be (3, -2), (-3, 1) and (4, -3), then find the coordinates of its vertices.

Answer

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of triangle.

Let D(3, -2), E(-3, 1) and F(4, -3) be the midpoints of sides BC, CA and AB respectively.



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint D(3, -2) of side BC,

$3 = \frac{x_2 + x_3}{2}$, $-2 = \frac{y_2+y_3}{2}$

∴ $x_2 + x_3 = 6$ and $y_2 + y_3 = -4$ ...(1)

For midpoint E(-3, 1) of side CA,

$-3 = \frac{x_1 + x_3}{2}$, $1 = \frac{y_1+y_3}{2}$

∴ $x_1 + x_3 = -6$ and $y_1 + y_3 = 2$ ...(2)

For midpoint F(4, -3) of side AB,

$4 = \frac{x_1 + x_2}{2}$, $-3 = \frac{y_1+y_2}{2}$

∴ $x_1 + x_2 = 8$ and $y_1 + y_2 = -6$ ...(3)

Adding 1,2 and 3, we get,

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 6 - 6 + 8$

And $y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = -4 + 2 - 6$

$\therefore 2(x_1 + x_2 + x_3) = 8$ and $2(y_1 + y_2 + y_3) = -8$

$\therefore x_1 + x_2 + x_3 = 4$ and $y_1 + y_2 + y_3 = -4$

$x_1 + 6 = 4$ and $y_1 - 4 = -4$ ...from 1

$\therefore x_1 = -2$ and $y_1 = 0$

Substituting above values in 3,

$-2 + x_2 = 8$ and $0 + y_2 = -6$

$\therefore x_2 = 10$ and $y_2 = -6$

Similarly for equation 2,

$-2 + x_3 = -6$ and $0 + y_3 = 2$

$\therefore x_3 = -4$ and $y_3 = 2$

Hence the vertices of triangle are A(-2 , 0), B(10 ,-6) and C(-4 ,2)

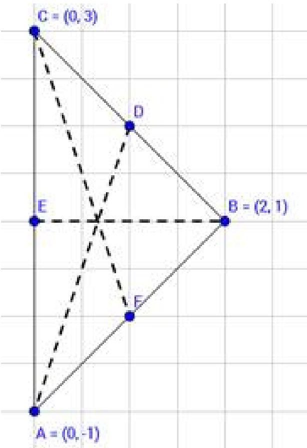
28. Question

Find the lengths of the medians of a $\triangle$ABC having vertices at A (0,-1), B (2, 1) and C (0, 3).

Answer

Here given vertices are A (0,-1), B (2, 1) and C (0, 3) and let midpoints of BC, CA and AB be D,E and F respectively.

By midpoint formula.



$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint D of side BC,

$x = \frac{2+0}{2}$, $y = \frac{1+3}{2}$

$x = \frac{2}{2}$, $y = \frac{4}{2}$

$\therefore$midpoint of side BC is D(1, 2)

For midpoint E of side AB,

$x = \frac{0+0}{2}$, $y = \frac{-1+3}{2}$

$x = \frac{0}{2}$, $y = \frac{2}{2}$

∴midpoint of side AB is E(0, 1)

For midpoint F of side CA,

$x = \frac{2+0}{2}$, $y = \frac{1-1}{2}$

$x = \frac{2}{2}$, $y = \frac{0}{2}$

∴midpoint of side CA is F(1, 0)

By distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For median AD,

$AD = \sqrt{(1-0)^2 + (2-(-1))^2}$

$= \sqrt{1+9}$

$= \sqrt{10}$ units

For median BE,

$BE = \sqrt{(0-2)^2 + (1-1)^2}$

$= \sqrt{4}$

= 2 units.

For median CF,

$CF = \sqrt{(1-0)^2 + (0-3)^2}$

$= \sqrt{1+9}$

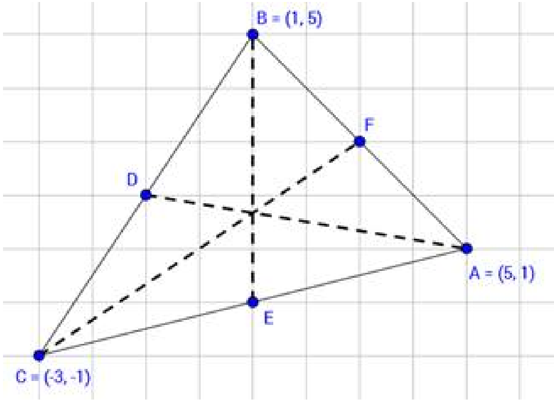
$= \sqrt{10}$ units

29. Question

Find the lengths of the medians of a $\triangle$ABC having vertices at A (5, 1), B (1, 5), and C(-3, -1).

Answer

Here given vertices are A (0,-1), B (2, 1) and C (0, 3) and let midpoints of BC, CA and AB be D,E and F respectively.



By midpoint formula.

x = $\frac{x_1 + x_2}{2}$, y = $\frac{y_1+y_2}{2}$

For midpoint D of side BC,

x = $\frac{-3+1}{2}$, y = $\frac{-1+5}{2}$

x = $\frac{-2}{2}$, y = $\frac{4}{2}$

∴midpoint of side BC is D(-1, 2)

For midpoint E of side AB,

x = $\frac{-3+5}{2}$, y = $\frac{-1+1}{2}$

x = $\frac{2}{2}$, y = $\frac{0}{2}$

∴midpoint of side AB is E(1, 0)

For midpoint F of side CA,

x = $\frac{1+5}{2}$, y = $\frac{1+5}{2}$

x = $\frac{6}{2}$, y = $\frac{6}{2}$

∴midpoint of side CA is F(3, 3)

By distance formula,

XY = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For median AD,

AD = $\sqrt{(-1-5)^2 + (2-1)^2}$

= $\sqrt{36+1}$

= $\sqrt{37}$ units

For median BE,

BE = $\sqrt{(1-1)^2 + (0-5)^2}$

$= \sqrt{25}$

= 5 units.

For median CF,

CF = $\sqrt{(-3-3)^2 + (-1-3)^2}$

$= \sqrt{36 + 16}$

$= 2\sqrt{13}$ units

30. Question

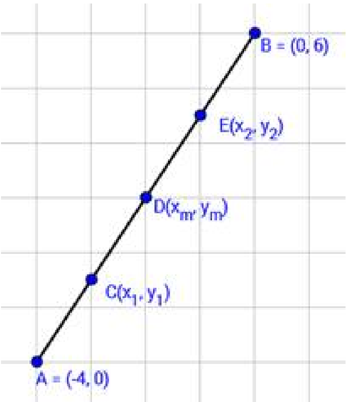
Find the coordinates of the points which divide the line segment joining the points (-4, 0) and (0, 6) in four equal parts.

Answer

Let given coordinates be A(-4, 0) and B (0, 6).

We need to divide AB into 4 equal parts, ie first we need to find midpoint of AB, which will be D and then find out midpoints of AD and DB respectively.

Let required points be C(x_1 , y_1), D(x_m , y_m) and E(x , y_2)



By midpoint formula.

x = $\frac{x_1 + x_2}{2}$, y = $\frac{y_1 + y_2}{2}$

For midpoint D of AB,

$x_m = \frac{-4+0}{2}$, $y_m = \frac{0+6}{2}$

∴ x_m = -2 and y_m = 3

∴ D(x_m , y_m) ≡ (-2 , 3)

Now, for midpoint C of AD,

$X_1 = \frac{-4-2}{2}$, $y_2 = \frac{0+3}{2}$

x_1 = -3 and y_1 = 1.5

∴C(x_1 , y_1) ≡ (-3, 1.5)

For midpoint E of DB,

$X_2 = \frac{0-2}{2}$, $y_2 = \frac{6+3}{2}$

∴ x_2 = -1 and y_2 = 4.5

∴ D(x_2 , y_2) ≡ (-1 , 4.5)

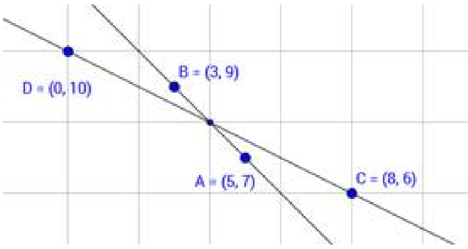
Hence the co-ordinates of the points are (-3, 1.5), (-2 , 3) and (-1 , 4.5)

31. Question

Show that the mid-point of the line segment joining the points (5, 7) and (3, 9) is also the mid-point of the line segment joining the points (8, 6) and (0, 10).

Answer

Let given points be A(5, 7) and B(3, 9) and the points of other segment line be C(8, 6) and D(0, 10)



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint of AB,

$x = \frac{5 + 3}{2}$, $y = \frac{7 +9}{2}$

x = 4 and y = 8 ...(1)

Now for midpoint of CD,

$e = \frac{8+ 0}{2}$, $d = \frac{6 +10}{2}$ ...(say)

∴ e = 4 and d = 8 ...(2)

Here from 1 and 2 we say that midpoints of AB and CD are same, ie they coincide.

32. Question

Find the distance of the point (1, 2) from the mid-point of the line segment joining the points (6, 8) and (2, 4).

Answer

Let D(x, y) be the midpoints of A(6, 8) and B(2, 4). Let our third given point be C(1, 2).

By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint D of AB,

$x = \frac{6+2}{2}$ and $y = \frac{8+4}{2}$

∴ x =4 and y = 6

∴D(x, y) ≡ (4, 6)

Now to find distance between C and D,

By distance formula,

XY = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1\,)^2}$

For CD,

CD = $\sqrt{(4-1)^2 + (6-2\,)^2}$

= $\sqrt{9+16}$

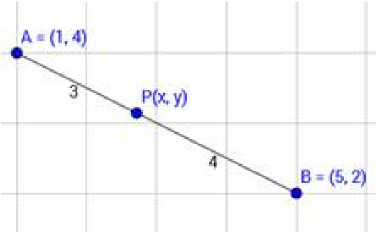
= 5 units

33. Question

If A and B are (1, 4) and (5, 2) respectively, find the coordinates of P when AP/BP = 3/4.

Answer

Given points are A(1, 4) and B(5, 2). Let P be (x, y) and given ratio is 3:4.



By section formula,

x = $\frac{mx_2+nx_1}{m+n}$, y = $\frac{my_2+ny_1}{m+n}$

For point P on AB,

x = $\frac{5\times 3+1\times 4}{3+4}$, y = $\frac{3\times 2+4\times 4}{3+4}$

x = $\frac{19}{7}$ and y = $\frac{22}{7}$

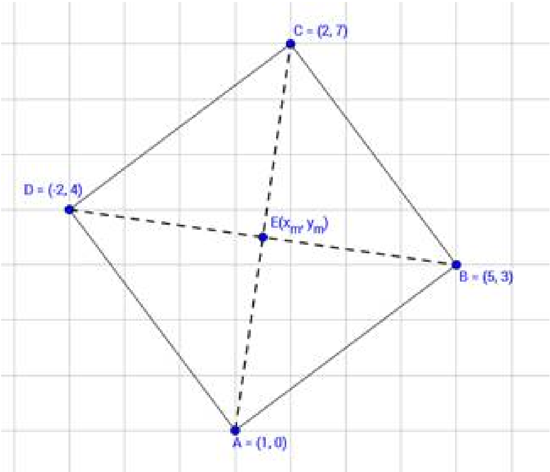
Hence, required coordinates is P($\frac{19}{7}$, $\frac{22}{7}$)

34. Question

Show that the points A (1, 0), B (5, 3), C (2, 7) and D (-2, 4) are the vertices of a parallelogram.

Answer

Let given points be A (1, 0), B (5, 3), C (2, 7) and D (-2, 4) and let the intersection of diagonals be E(x_m , y_m)



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint of diagonal AC,

$X_1 = \frac{1+2}{2}$, $y_1 = \frac{0+7}{2}$

$\therefore x_1 = \frac{3}{2}$, $y_1 = \frac{7}{2}$

$\therefore$ midpoint of diagonal AC is $(x_1, y_1) \equiv (\frac{3}{2}, \frac{7}{2})$ ...(1)

For midpoint of diagonal BD,

$X_2 = \frac{5-2}{2}$, $y_2 = \frac{3+4}{2}$

$\therefore x_2 = \frac{3}{2}$, $y_2 = \frac{7}{2}$

$\therefore$ midpoint of diagonal BD is $(x_2, y_2) \equiv (\frac{3}{2}, \frac{7}{2})$ ...(2)

Here, from 1 and 2 we say that midpoint of both the diagonals intersect at same point, ie $(\frac{3}{2}, \frac{7}{2})$

But our intersection of diagonals is at E, which means that midpoint of diagonals intersect at single point, ie E$(\frac{3}{2}, \frac{7}{2})$

We know that if midpoints of diagonals intersect at single point, then quadrilateral formed by joining the points is parallelogram.

Hence, our □ABCD is parallelogram.

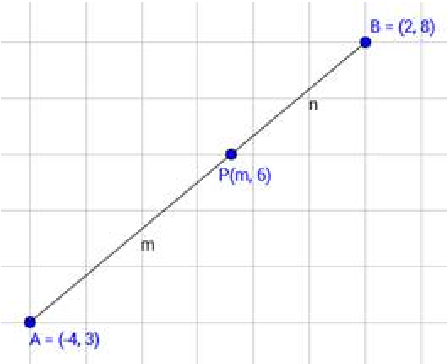
35. Question

Determine the ratio in which the point P (m, 6) divides the join of A(-4, 3) and B(2, 8). Also, find the value of m.

Answer

Here, given points are A (-4, 3) and B (2, 8) and let the point dividing the line joining two points be P(m,6).

Let the ratio be m:n



By section formula,

x = $\frac{mx_2+nx_1}{m+n}$, y = $\frac{my_2+ny_1}{m+n}$

For point P(m,6),

m = $\frac{m\times 2+n\times(-4)}{m+n}$ ...(1)

And 6 = $\frac{m\times 8+n\times 3}{m+n}$ ...(2)

Solving 2 for finding ratio between m and n,

6 = $\frac{m\times 8+n\times 3}{m+n}$

6(m + n) = 8m +3n

6m + 6n = 8m +3n

∴ 2m = 3n

∴ $\frac{m}{n}=\frac{3}{2}$

∴ m : n = 3 : 2

Now solving for equation 1, where m = 3 and n =2

m = $\frac{m\times 2+n\times(-4)}{m+n}$

∴ m = $\frac{6-8}{5}$

∴ m = $\frac{-2}{5}$

Hence, our point is ($\frac{-2}{5}$, 6)

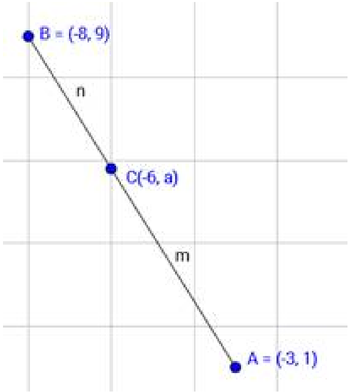
36. Question

Determine the ratio in which the point (-6, a) divides the join of A(-3, 1) and B(-8, 9). Also find the value of a.

Answer

Here, given points are A(-3, 1) and B(-8, 9) and let the point dividing the line joining two points be C(-6,a).

Let the ratio be m:n



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(-6,a),

$-6 = \frac{m\times(-8)+n\times(-3)}{m+n}$ ...(1)

And $a = \frac{m\times 9+n\times 1}{m+n}$ ...(2)

Solving 1 for finding ratio between m and n,

$-6 = \frac{m\times(-8)+n\times(-3)}{m+n}$

-6 (m + n) = -8m -3n

6m + 6n = 8m + 3n

∴ 2m = 3n

∴ $\frac{m}{n} = \frac{3}{2}$

∴ m : n = 3 : 2

Now solving for equation 2, where m = 3 and n = 2

$a = \frac{m\times 9+n\times 1}{m+n}$

$a = \frac{3\times 9+2\times 1}{3+2}$

∴ $a = \frac{27+2}{5}$

∴ $a = \frac{29}{5}$

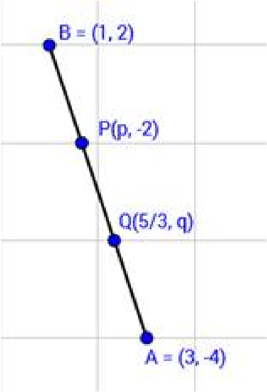
∴ value of a is $\frac{29}{5}$

37. Question

The line segment joining the points (3, -4) and (1, 2) is trisected at the points P and Q . If the coordinates of P and Q are (p, -2) and (5/3, q) respectively. Find the values of p and q.

Answer

Let given points be A(3, -4) and B(1, 2) , which is trisected at points P(p, -2) and Q(5/3, q).



By section formula,

x = $\frac{mx_2+nx_1}{m+n}$, y = $\frac{my_2+ny_1}{m+n}$As point P divides the line in 1:2 and Q divides the line in 2:1.

For point P(p, -2)of AB, where m = 1 and n = 2,

p= $\frac{1\times1+3\times2}{1+2}$, -2 = $\frac{2\times1+2\times(-4)}{1+2}$

Solving for p,

P = $\frac{7}{3}$

For point Q(5/3, q) of AB, where m = 2 and n = 1,

$\frac{5}{3} = \frac{2\times1+1\times3}{2+1}$,q = $\frac{2\times2+1\times(-4)}{2+1}$

Solving for q,

q = $\frac{4-4}{3}$

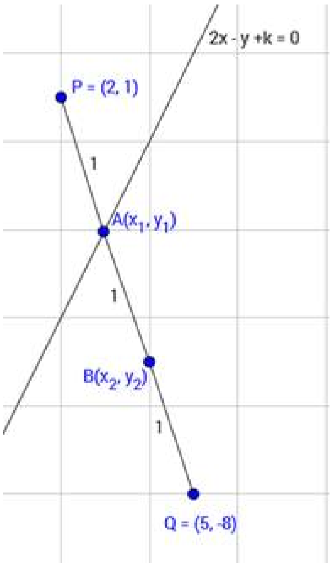
∴q = 0Hence, the value of p and q are $\frac{7}{3}$ and 0 respectively.

38. Question

The line joining the points (2,1) and (5,-8) is trisected at the points P and Q. If point P lies on the line 2x - y + k = 0. Find the value of k.

Answer

Here, given points are P (2, 1) and Q (5, - 8) which is trisected at the points(say) A(x_1 , y_1) and B(x_2 , y_2)such that A is nearer to P.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point A(x_1 , y_1) of PQ, where m = 1 and n = 2,

$x_1 = \frac{1 \times 5+2 \times 2}{1+2}$, $y_1 = \frac{1 \times (-8)+2 \times 1}{1+2}$

∴ $x_1 = 3$, $y_1 = -2$

∴Coordinates of A is (3,-2)

It is given that point A lies on the line 2x - y + k = 0.

So, substituting value of x and y as coordinates of A,

2 × 3 – (-2) + k = 0

∴ k = -8

39. Question

If A and B are two points having coordinates (-2, -2) and (2, -4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB.

Answer

Given points are A(-2, -2) and B(2, -4). Let P be (x, y)

Here given that AP = $\frac{3}{7}$ AB.

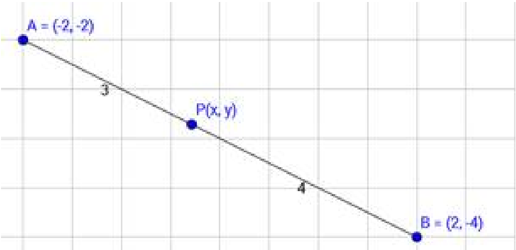
But AB = AP + BP

∴7AP = 3AB

7AP = 3(AP + BP)

∴4AP = 3BP

∴ $\frac{AP}{BP} = \frac{3}{4}$



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point P on AB, where m = 3 and n = 4

$x = \frac{3 \times 2 + 4 \times (-2)}{3+4}$, $y = \frac{3 \times (-4) + 4 \times (-2)}{3+4}$

$x = \frac{-2}{7}$ and $y = \frac{-20}{7}$

Hence, required coordinates is $P(\frac{-2}{7}, \frac{-20}{7})$

40. Question

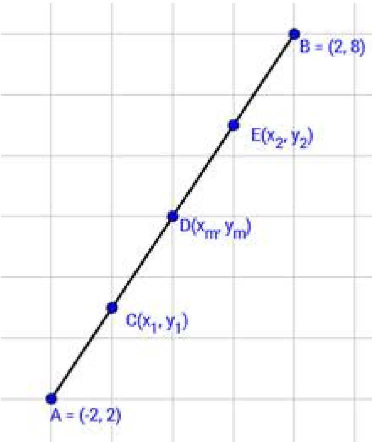
Find the coordinates of the points which divide the line segment joining A (-2, 2) and B (2, 8) into four equal parts.

Answer

Let given coordinates be A(-2, 2) and B (2, 8).

We need to divide AB into 4 equal parts, ie first we need to find midpoint of AB, which will be D and then find out midpoints of AD and DB respectively.

Let required points be $C(x_1, y_1)$, $D(x_m, y_m)$ and $E(x, y_2)$



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint D of AB,

$x_m = \frac{-2+2}{2}$, $y_m = \frac{2+8}{2}$

$\therefore x_m = 0$ and $y_m = 5$

$\therefore D(x_m, y_m) \equiv (0, 5)$

Now, for midpoint C of AD,

$X_1 = \frac{-2+0}{2}$, $y_2 = \frac{2+5}{2}$

$x_1 = -1$ and $y_1 = \frac{7}{2}$

$\therefore C(x_1 , y_1) \equiv (-1, \frac{7}{2})$

For midpoint E of DB,

$X_2 = \frac{2+0}{2}$, $y_2 = \frac{8+5}{2}$

$\therefore x_2 = 1$ and $y_2 = \frac{13}{2}$

$\therefore E(x_2 , y_2) \equiv (1 , \frac{13}{2})$

Hence the co-ordinates of the points are $(-1, \frac{7}{2})$, (0 , 5) and $(1 , \frac{13}{2})$

41. Question

A (4, 2), B (6, 5) and C (1, 4) are the vertices of $\triangle$ABC.

(i) The median from A meets BC in D. Find the coordinates of the point D.

(ii) Find the coordinates of point P on AD such that AP : PD = 2 :1.

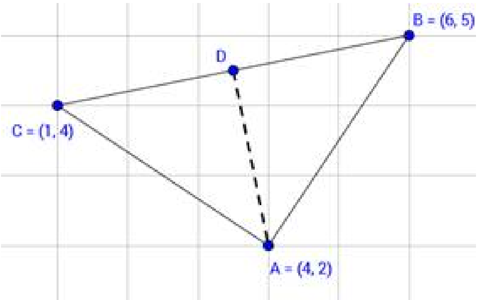
(iii) Find the coordinates of the points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

(iv) What do you observe?

Answer

(i) The median from A meets BC in D. Find the coordinates of the point D.

Here given vertices are A (4, 2), B (6, 5) and C (1, 4).



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1+y_2}{2}$

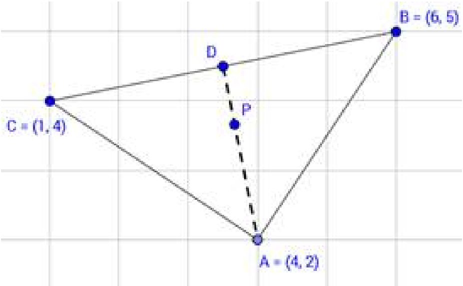
For midpoint D of side BC,

$x = \frac{6+1}{2}$, $y = \frac{5+4}{2}$

$x = \frac{7}{2}$, $y = \frac{9}{2}$

Hence, the coordinates of D are $(\frac{7}{2}, \frac{9}{2})$

(ii) Find the coordinates of point P on AD such that AP : PD = 2 :1.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point P on AD, where m = 2 and n = 1

$x = \frac{2\times\frac{7}{2}+1\times4}{2+1}$, $y = \frac{2\times\frac{9}{2}+1\times2}{2+1}$

$\therefore x = \frac{11}{3}$ and $y = \frac{11}{3}$

(iii) Find the coordinates of the points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

By midpoint formula.

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint E of side AC,

$x = \frac{1+4}{2}$, $y = \frac{4+2}{2}$

$x = \frac{5}{2}$, $y = \frac{6}{2}$

Hence, the coordinates of E are $(\frac{5}{2}, 3)$

For midpoint F of side AB,

$x = \frac{6+4}{2}$, $y = \frac{5+2}{2}$

$x = \frac{10}{2}$, $y = \frac{7}{2}$

Hence, the coordinates of F are $(5, \frac{7}{2})$

By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point Q on BE, where m = 2 and n = 1

$x = \frac{2\times\frac{5}{2}+1\times6}{2+1}$, $y = \frac{2\times3+1\times5}{2+1}$

$\therefore$ x = $\frac{11}{3}$ and y = $\frac{11}{3}$

For point R on CF, where m = 2 and n = 1

x = $\frac{2\times5+1\times1}{2+1}$, y = $\frac{2\times\frac{7}{2}+1\times4}{2+1}$

$\therefore$ x = $\frac{11}{3}$ and y = $\frac{11}{3}$

(iv) What do you observe?

We observe that the point P,Q and R coincides with the centroid.

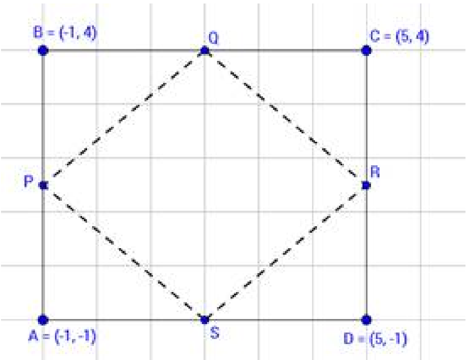
This also shows that centroid divides the median in the ratio 2:1

42. Question

ABCD is a rectangle formed by joining the points A (-1, -1), B (-1, 4), C (5, 4) and D (5,-1). P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

Answer

Here given that A (-1, -1), B (-1, 4), C (5, 4) and D (5,-1).Also P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively.



By midpoint formula.

x = $\frac{x_1+x_2}{2}$, y = $\frac{y_1+y_2}{2}$

For midpoint P of side AB,

x = $\frac{-1-1}{2}$, y = $\frac{-1+4}{2}$

x = -1 , y = $\frac{3}{2}$

Hence, the coordinates of P are (-1 , $\frac{3}{2}$)

For midpoint Q of side BC,

x = $\frac{-1+5}{2}$, y = $\frac{4+4}{2}$

x = 2 , y = 4

Hence, the coordinates of Q are (2 ,4)

For midpoint R of side CD,

$x = \frac{5+5}{2}$, $y = \frac{-1+4}{2}$

$x = 5$, $y = \frac{3}{2}$

Hence, the coordinates of R are $(5 , \frac{3}{2})$

For midpoint S of side AD,

$x = \frac{-1+5}{2}$, $y = \frac{-1-1}{2}$

x = 2 , y = -1

Hence, the coordinates of S are (2 ,-1)

Now we find length of the length of the □PQRS,

By distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For PQ,

$PQ = \sqrt{(2-(-1))^2 + \left(4 - \frac{3}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{2}}$

$= \sqrt{\frac{61}{2}}$ units

For QR,

$QR = \sqrt{(5-2)^2 + \left(\frac{3}{2} - 4\right)^2}$

$= \sqrt{9 + \frac{25}{2}}$

$= \sqrt{\frac{61}{2}}$ units

For RS,

$RS = \sqrt{(2-5)^2 + \left(-1 - \frac{3}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{2}}$

$= \sqrt{\frac{61}{2}}$ units

For PS,

$PS = \sqrt{(2-(-1))^2 + \left(-1 - \frac{3}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{2}}$

$= \sqrt{\frac{61}{2}}$ units

Here we can observe that all lengths of ▫PQRS are equal.

Now for diagonal PR,

PR = $\sqrt{(5-(-1))^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2}$

$= \sqrt{36+0}$

$= 6$ units

Now for diagonal QS,

QS = $\sqrt{(2-2)^2 + (-1\ - 4)^2}$

$= \sqrt{0+25}$

= 5 units

Here in ▫PQRS, diagonals are unequal.

We know that a quadrilateral whose all sides are equal and diagonals are unequal, it is a rhombus.

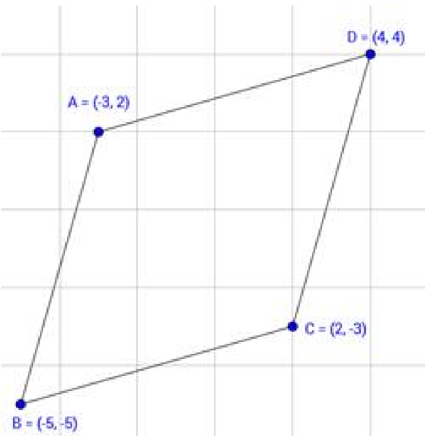
Hence, our ▫PQRS is rhombus .

43. Question

Show that A(-3, 2), B (-5, -5), C (2, -3) and D(4, 4) are the vertices of a rhombus.

Answer

solution: Given points are A(-3, 2), B (-5, -5), C (2, -3) and D(4, 4)



Use distance formula

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1\ )^2}$

For AB ,

AB = $\sqrt{(-5-(-3))^2 + (-5-2\ )^2}$

$$=\sqrt{(-5+3)^2+(-5-2)^2}$$

$$=\sqrt{(-2)^2+(-7)^2}$$

= $\sqrt{4+49}$

= $\sqrt{53}$ units

For BC ,

BC = $\sqrt{(2-(-5))^2+(-3-(-5)\ )^2}$

$$=\sqrt{(2+5)^2+(-3+5)^2}$$

$$=\sqrt{7^2+2^2}$$

= $\sqrt{49+4}$

= $\sqrt{53}$ units

For CD,

CD = $\sqrt{(4-2)^2+(4-(-3)\ )^2}$

$$=\sqrt{(4-2)^2+(4+3)^2}$$

$$=\sqrt{(2)^2+(7)^2}$$

= $\sqrt{4+49}$

= $\sqrt{53}$ units

For AD ,

AD= $\sqrt{(4-(-3))^2+(4-2\ )^2}$

$$=\sqrt{(4+3)^2+(4-2)^2}$$

$$=\sqrt{(7)^2+(2)^2}$$

= $\sqrt{49+4}$

= $\sqrt{53}$ units

Here we can observe that all lengths of □PQRS are equal.

Now for diagonal AC,

AC = $\sqrt{(2-(-3))^2+(-3\ -2)^2}$

$$=\sqrt{(2+3)^2+(-3-2)^2}$$

$$=\sqrt{(5)^2+(-5)^2}$$

= $\sqrt{25+25}$

= $\sqrt{50}$units

Now for diagonal BD,

BD = $\sqrt{(4-(-5))^2+(4-(-5)\,)^2}$

$$=\sqrt{(4+5)^2+(4+5)^2}$$

$$=\sqrt{(9)^2+(9)^2}$$

= $\sqrt{81+81}$

= $\sqrt{162}$ unitsAB = BC = CD = AD

And AC ≠ BD

Here in ABCD, diagonals are unequal.

We know that a quadrilateral whose all sides are equal and diagonals are unequal, it is a rhombus.

Hence, ABCD is rhombus .

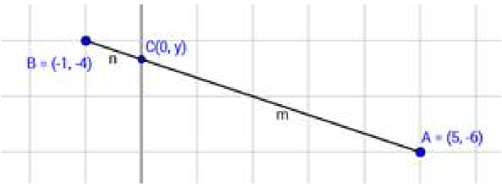
44. Question

Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also, find the coordinates of the point of division.

Answer

Let our points be A(5, -6) and B(-1, -4).

Let point C(0, y) divide the line formed by joining by the points A and B in ratio of m:n.



By section formula,

x = $\frac{mx_2+nx_1}{m+n}$, y = $\frac{my_2+ny_1}{m+n}$

For point C(0, y)

0 = $\frac{m\times(-1)+n\times5}{m+n}$, y = $\frac{m\times(-4)+n\times(-6)}{m+n}$

Solving for x coordinate,

0 = $\frac{m\times(-1)+n\times5}{m+n}$

∴ m = 5n

∴ $\frac{m}{n} = \frac{5}{1}$

∴ m : n = 5 : 1

Now solving for y coordinate, with m = 5 and n = 1,

$y = \frac{my_2+ny_1}{m+n}$

$y = \frac{-4\times5-6}{6}$

∴ $y = \frac{-13}{6}$

There is no need to solve for x, as our point lies on y-axis

Hence, the coordinates of required point is $C(0\ ,\ \frac{-13}{6})$

45. Question

If the points A (6, 1), B (8, 2), C (9, 4) and D (k, p) are the vertices of a parallelogram taken in order, then find the values of k and p.

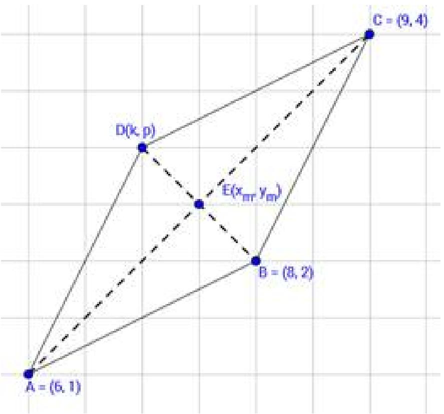
Answer

Our given vertices are A(1, -2), B(3, 6) and C(5, 10) and fourth vertex be D(k, p)

It is given that quadrilateral joining these four vertices is parallelogram, ie ▫ABCD is parallelogram.

We know that diagonals of parallelogram bisect each other, ie midpoint of the diagonals coincide.

Let $E(x_m\ ,\ y_m)$ be the midpoint of diagonals AC and BD.



By midpoint formula,

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For diagonal AC,

$x_m = \frac{6+9}{2}$, $y_m = \frac{4+1}{2}$

∴ $x_m = \frac{15}{2}$, $y_m = \frac{5}{2}$

∴ $E(x_m\ ,\ y_m) \equiv (\frac{15}{2}, \frac{5}{2})$

For diagonal BD,

$\frac{15}{2} = \frac{8+k}{2}, \frac{5}{2} = \frac{p+2}{2}$

∴ k = 15 – 8 , y = 5 – 2

∴ k = 7 and p = 3

Hence, our fourth vertex is D(7 , 3)

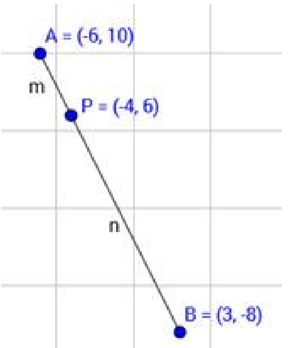
46. Question

In what ratio does the point (-4, 6) divide the line segment joining the points A (-6, 10) and B(3, -8)?

Answer

Given points are A (-6, 10) and B(3, -8)

Let the point C(-4, 6) divide AB in ratio m:n.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point C(-4, 6) on the line joined by the points A and B.

$-4 = \frac{m\times3+n\times(-6)}{m+n}$ ...(1)

And, $6 = \frac{m\times(-8)+n\times10}{m+n}$ ...(2)

Solving 1,

-4(m + n) = 3m - 6n

∴4m + 4n = -3m + 6n

∴ 7m = 2n

$\therefore \frac{m}{n} = \frac{2}{7}$

Hence, ratio is 2:7.

47. Question

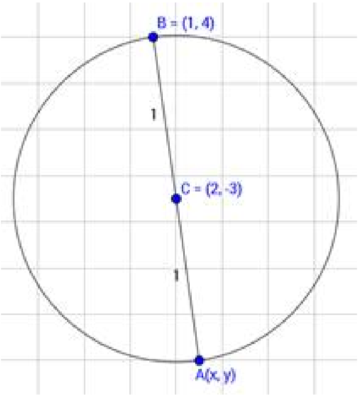
Find the coordinates of a point A, where AB is a diameter of the circle whose centre is (2, -3) and B is (1, 4).

Answer

Here given that AB is a diameter of the circle whose centre is (say) C(2, -3) and B is (1, 4)

Let A be (x, y)

We know that as C is center, AC = CB or C is midpoint of AB.



By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For Center C,

$2 = \frac{x + 1}{2}$ and $-3 = \frac{y + 4}{2}$

∴ x = 4 – 1 and y = -6 – 4

∴ x = 3 and y = -10

Hence, coordinates of A are (3, -10)

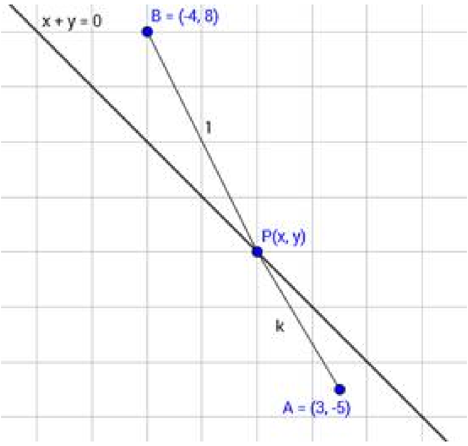
48. Question

A point P divides the line segment joining the points A (3, -5) and B (-4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line x + y = 0, then find the value of k.

Answer

Here given points are A (3, -5) and B (-4, 8) .

Let point P be (x, y) which divides AB in ratio of k:1, also point P lies on line x + y = 0



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point P on the line joined by the points A and B.

$x = \frac{k \times (-4) + 1 \times 3}{k+1}$, $y = \frac{k \times 8 + 1 \times (-5)}{k+1}$

Putting in given equation,

$\frac{k \times (-4) + 1 \times 3}{k+1}, \frac{k \times 8 + 1 \times (-5)}{k+1}$ = (x, y)

$$x = \frac{-4k + 3}{k + 1} \text{ and } y = \frac{8k - 8}{k + 1}$$

$$Now\ (x, y)\ lies\ on\ the\ line\ x + y = 0$$

$$Therefore,\ the\ points\ will\ satisfy\ the\ equation.$$

$$Hence,$$

$$\frac{-4k + 3}{k + 1} + \frac{8k - 8}{k + 1} = 0$$

- 4 k + 3 + 8 k - 8 = 04k - 5 = 0

∴ 4k = 5

∴ k = 5/2

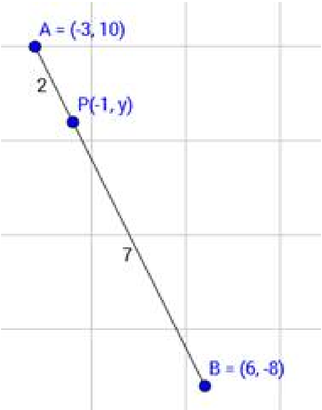
49. Question

Find the ratio in which the point P(-1, y) line segment joining A (-3,10) and B(6, -8) divides it. Also find the value of y.

Answer

Here, given points are A (-3,10) and B(6, -8) and the point dividing the line joining two points is P(-1,y).

Let the ratio be m:n



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

For point P(-1,a),

$-1 = \frac{m \times 6 + n \times (-3)}{m+n}$ ...(1)

And y = $\frac{m \times (-8) + n \times 10}{m+n}$ ...(2)

Solving 1 for finding ratio between m and n,

-1 = $\frac{m \times 6 + n \times (-3)}{m+n}$

- (m + n) = 6m -3n

m + n = -6m + 3n

∴ 7m = 2n

∴ $\frac{m}{n} = \frac{2}{7}$

∴ m : n = 2 : 7

Now solving for equation 2, where m = 2 and n = 7

y = $\frac{m \times (-8) + n \times 10}{m+n}$

y = $\frac{2 \times (-8) + 7 \times 10}{2+7}$

∴ y = $\frac{-16+70}{9}$

∴ y = $\frac{54}{9}$

∴ value of y is 6

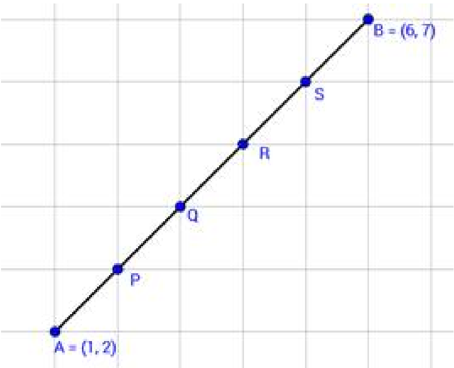
50. Question

Points p, Q, R and S divide the segment joining the points A (1, 2) and B (6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.

Answer

Here given points are A (1, 2) and B (6, 7) which is divided into 5 equal parts by points P, Q, R and S

∴ AP = PQ= QR = RS = SB



The point P divides the line segment AB in the ratio 1:4.

By section formula,

x = $\frac{mx_2 + nx_1}{m+n}$, y = $\frac{my_2 + ny_1}{m+n}$

For point P,

$x = \frac{1 \times 6+4 \times 1}{1+4}$, $y = \frac{1 \times 7+4 \times 1}{1+4}$

x = 2 and y = 3

∴ Coordinate of P is (2 ,3)

The point Q divides the line segment AB in the ratio of 2:3.

For point Q,

$x = \frac{2 \times 6+3 \times 1}{2+3}$, $y = \frac{2 \times 7+3 \times 1}{2+3}$

x = 3 and y = 4

∴ Coordinate of Q is (3 ,4)

The point R divides the line segment AB in the ratio of 3:2.

For point R,

$x = \frac{3 \times 6+2 \times 1}{3+2}$, $y = \frac{3 \times 7+2 \times 1}{3+2}$

x = 4 and y = 5

∴ Coordinate of R is (4 , 5)

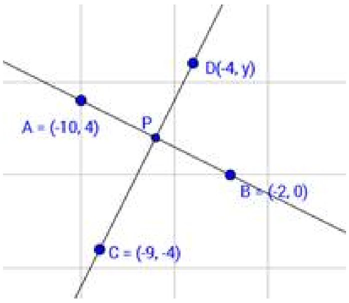
51. Question

The mid-point P of the line segment joining the points A (- 10, 4) and B (- 2, 0) lies on the line segment joining the points C (- 9,-4) and D (- 4, y). Find the ratio in which P divides CD. Also, find the value of y.

Answer

Here given points are A (- 10, 4) and B (- 2, 0)and the points of other segment line are C (- 9,-4) and D (- 4, y)

Let the point of intersection between AB and CD be P



By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1+y_2}{2}$

For midpoint of AB,

$e = \frac{-10-2}{2}$, $d = \frac{4+0}{2}$ ...(say)

e = -6 and d = 2 ...(1)

By section formula,

$x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

For point P on CD, where ratio is m:n,

-6 = $\frac{m\times(-4)+n\times(-9)}{m+n}$ and 2 = $\frac{m\times y+n\times(-4)}{m+n}$

Solving for m and n,

-6 = $\frac{m\times(-4)+n\times(-9)}{m+n}$

∴-6(m + n) = -4m -9n

6m + 6n = 4m + 9n

2m = 3n

∴$\frac{m}{n}$ = $\frac{3}{2}$

∴ Ratio is 3:2

Now solving for y, where m = 3 and n = 2,

2 = $\frac{3\times y+2\times(-4)}{3+2}$

∴ 3y -8 = 10

∴ 3y = 18

∴y = 6

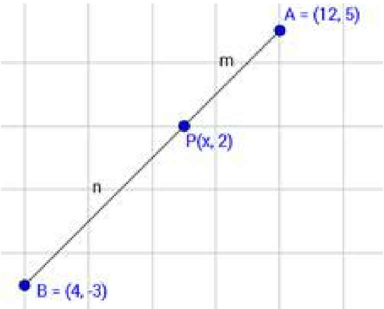
52. Question

Find the ratio in which the point P (x, 2) divides the line segment joining the points A (12,5) and B (4, -3). Also, find the value of x.

Answer

Here, given points are A (12,5) and B (4, -3) and let the point dividing the line joining two points be P(x,2)

Let the ratio be m:n



By section formula,

x = $\frac{mx_2+nx_1}{m+n}$, y = $\frac{my_2+ny_1}{m+n}$

For point P(x,2),

x = $\frac{m\times 4+n\times 12}{m+n}$ ...(1)

And 2 = $\frac{m\times(-3)+n\times 5}{m+n}$ ...(2)

Solving 2 for finding ratio between m and n,

2 = $\frac{m\times(-3)+n\times 5}{m+n}$

2(m + n) = -3m +5n

2m + 2n = -3m +5n

∴ 5m = 3n

∴ $\frac{m}{n} = \frac{3}{5}$

∴ m : n = 3 : 5

Now solving for equation 1, where m = 3 and n =5

x = $\frac{3 \times 4 + 5 \times 12}{3+5}$

∴ x = $\frac{12+60}{8}$

∴ x = 9

Hence, our point is (9, 2)

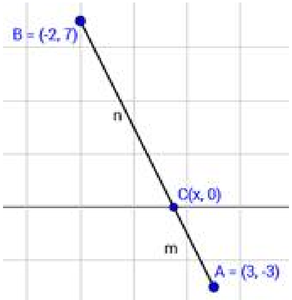
53. Question

Find the ratio in which the line segment joining the points A (3,-3) and B (-2, 7) is divided by x-axis. Also, find the coordinates of the point of division.

Answer

Our points are A (3,-3) and B (-2, 7)

Let point C(x, 0) divide the line formed by joining by the points A and B in ratio of m:n.



By section formula,

x = $\frac{mx_2 + nx_1}{m+n}$, y = $\frac{my_2 + ny_1}{m+n}$

For point C(x, 0)

x = $\frac{m \times (-2) + n \times 3}{m+n}$, 0 = $\frac{m \times 7 + n \times (-3)}{m+n}$

Solving for y coordinate,

0 = $\frac{m \times 7 + n \times (-3)}{m+n}$

∴ 7m -3n = 0

∴ 7m = 3n

∴ $\frac{m}{n} = \frac{3}{7}$

∴ m : n = 3 : 7

Now solving for x coordinate, with m = 3 and n = 7,

$$x = \frac{3 \times (-2) + 7 \times 3}{3+7}$$

$$\therefore x = \frac{-6+21}{10}$$

$$\therefore x = \frac{15}{10} = \frac{3}{2}$$

Hence, the coordinates of required point is $C(\frac{3}{2}, 0)$

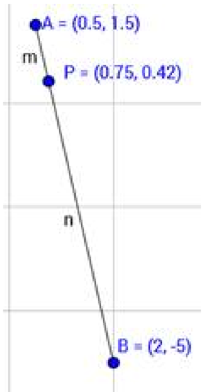
54. Question

Find the ratio in which the points P (3/4, 5/12) divides the line segments joining the points A(1/2 , 3/2) and B(2, -5).

Answer

Given points are A(1/2 , 3/2) and B(2, -5)

Let the point P(3/4, 5/12) divide AB in ratio m:n.



By section formula,

$$x = \frac{mx_2 + nx_1}{m+n}, \; y = \frac{my_2 + ny_1}{m+n}$$

For point P on the line joined by the points A and B.

$$\frac{3}{4} = \frac{m \times 2 + n \times \frac{1}{2}}{m+n} \; ...(1)$$

$$\text{And, } \frac{5}{12} = \frac{m \times (-5) + n \times \frac{3}{2}}{m+n} \; ...(2)$$

Solving 1,

3(m + n) = 8m + 2n

∴3m + 3n = 8m + 2n

∴ 5m = n

$$\therefore \frac{m}{n} = \frac{1}{5}$$

Hence, ratio is 1:5.

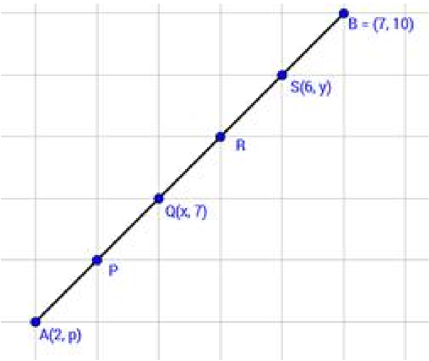
55. Question

If the points P, Q(x, 7), R, S(6, y) in this order divide the line segment joining A(2, p) and B (7, 10) in 5 equal parts, find x, y and p.

Answer

Here given points are A (2, p) and B (7, 10) which is divided into 5 equal parts by points P, Q(x, 7), R and S(6, y)

∴ AP = PQ= QR = RS = SB



By section formula,

$x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

The point Q divides the line segment AB in the ratio of 2:3.

For point Q,

$x = \frac{2 \times 7 + 3 \times 2}{2+3}$, $7 = \frac{2 \times 10 + 3 \times p}{2+3}$

Solving above equations, we get,

x = 4 and p = 5

For point P, divides the line segment AB in the ratio 4:1.

$6 = \frac{4 \times 7 + 1 \times 2}{4+1}$, $y = \frac{4 \times 10 + 1 \times p}{4+1}$

Solving for y and substituting value of p,

$y = \frac{40+5}{5}$

∴ y = 9

Hence, values are x = 4, y = 9 and p = 5

Exercise 14.4

1. Question

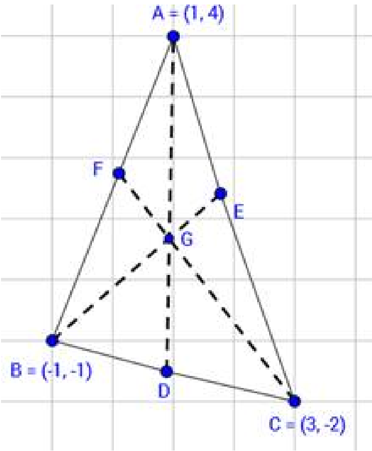
Find the centroid of the triangle whose vertices are:

(i) (1, 4), (-1, -1), (3, -2)

(ii) (- 2, 3), (2, -1), (4, 0)

Answer

(i) (1, 4), (-1, -1), (3, -2)



We know that centroid of a triangle for the vertices (x_1 , y_1), (x_2 , y_2) and (x_3 , y_3) is

G(x, y) = ($\frac{x_1+x_2+x_3}{3}$, $\frac{y_1+y_2+y_3}{3}$)

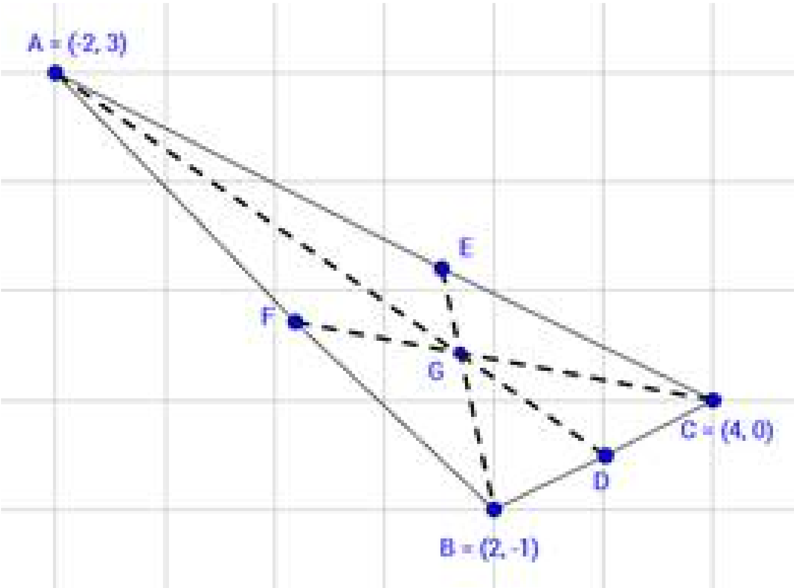
∴For coordinates (1, 4), (-1, -1), (3, -2),

Centroid of triangle = ($\frac{1-1+3}{3}$, $\frac{4-1-2}{3}$)

= (1, $\frac{1}{3}$)

Hence, centroid of triangle is (1, $\frac{1}{3}$)

(ii) (- 2, 3), (2, -1), (4, 0)



We know that centroid of a triangle for (x_1 , y_1), (x_2 , y_2) and (x_3 , y_3) is

G(x, y) = ($\frac{x_1+x_2+x_3}{3}$, $\frac{y_1+y_2+y_3}{3}$)

∴For coordinates (- 2, 3), (2, -1), (4, 0)

Centroid of triangle = ($\frac{-2+2+4}{3}$, $\frac{3-1+0}{3}$)

= ($\frac{4}{3}$, $\frac{2}{3}$)

Hence, centroid of triangle is ($\frac{4}{3}, \frac{2}{3}$)

2. Question

Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the coordinates of the third vertex.

Answer

Let the vertex of the triangle be A(1, 2), B(3, 5) and C(x, y)

Let the centroid be D(0, 0), as it is given that centroid is given at origin.

We know that centroid of a triangle for (x_1 , y_1), (x_2 , y_2) and (x_3 , y_3) is

C(x, y) = ($\frac{x_1+x_2+x_3}{3}$, $\frac{y_1+y_2+y_3}{3}$)

For given coordinates A(1, 2), B(3, 5) and C(x, y), centroid is,

(0, 0) = ($\frac{1+3+x}{3}$, $\frac{2+5+y}{3}$)

Solving for x and y,

1 + 3 +x =0 and $2 + 5 + y$ = 0

∴ x = -4 and y = -7

Hence, the coordinate of third vertex is C(-4, -7)

3. Question

Prove analytically that the line segment joining the middle points of two sides of a triangle is equal to half of the third side.

Answer

Let ΔABC be any triangle such that O is the origin.

∴Let coordinates be A(0, 0), B(x_1 , y_1), C(x_2 , y_2).

Let D and E are the mid-points of the sides AB and AC respectively.

We have to prove that line joining the mid-point of any two sides of a triangle is equal to half of the third side which means,

DE = $\frac{1}{2}$ BC

By midpoint formula,

x = $\frac{x_1+x_2}{2}$, y = $\frac{y_1+y_2}{2}$

For midpoint D on AB,

x =$\frac{x_1+0}{2}$, y = $\frac{y_1+0}{2}$

∴ x = $\frac{x_1}{2}$ and y = $\frac{y_1}{2}$

∴ Coordinate of D is ($\frac{x_1}{2}$, $\frac{y_1}{2}$)

For midpoint E on AC,

$x = \frac{x_2 + 0}{2}$, $y = \frac{y_2 + 0}{2}$

$\therefore x = \frac{x_2}{2}$ and $y = \frac{y_2}{2}$

$\therefore$ Coordinate of E is $\left(\frac{x_2}{2}, \frac{y_2}{2}\right)$

By distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For BC,

$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For DE,

$DE = \sqrt{\left(\frac{x_2}{2} - \frac{x_1}{2}\right)^2 + \left(\frac{y_2}{2} - \frac{y_1}{2}\right)^2}$

$= \frac{1}{2}\left(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right)$

$= \frac{1}{2} BC$

$\therefore DE = \frac{1}{2} BC$

Hence, we proved that line joining the mid-point of any two sides of a triangle is equal to half of the third side.

4. Question

Prove that the lines joining the middle points of the opposite sides of a quadrilateral and the join of the middle points of its diagonals meet in a point and bisect one another.

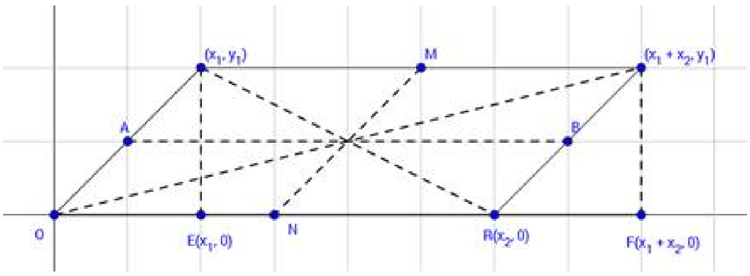
Answer

Let us consider a Cartesian plane having a parallelogram OABC in which O is the origin.

We have to prove that middle point of the opposite sides of a quadrilateral and the join of the mid-points of its diagonals meet in a point and bisect each other.

Let coordinates be A(0, 0).

So other coordinates will be $B(x_1 + x_2, y_1)$, $C(x_2, 0)$ ... refer figure.



Let P, Q, R and S be the mid-points of the sides AB, BC, CD, DA respectively.

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint P on AB,

x $=\frac{x_1+x_2+x_1}{2}$, y = $\frac{y_1+y_1}{2}$

∴ x = $\frac{2x_1+x_2}{2}$, y = $\frac{2y_1}{2}$

∴ Coordinate of P is ($\frac{2x_1+x_2}{2}$, y_1)

For midpoint Q on BC,

x $=\frac{x_1+x_2+x_2}{2}$, y = $\frac{y_1+0}{2}$

∴ x = $\frac{x_1+2x_2}{2}$, y = $\frac{y_1}{2}$

∴ Coordinate of Q is ($\frac{x_1+2x_2}{2}$, $\frac{y_1}{2}$)

For R, we can observe that, R lies on x axis.

∴ Coordinate of R is ($\frac{x_2}{2}$, 0)

For midpoint S on OA,

x $=\frac{x_1+0}{2}$, y = $\frac{y_1+0}{2}$

∴ x = $\frac{x_1}{2}$, y = $\frac{y_1}{2}$

∴ Coordinate of S is ($\frac{x_1}{2}$, $\frac{y_1}{2}$)

For midpoint of PR,

x = $\frac{\frac{2x_1+x_2}{2}+\frac{x_2}{2}}{2}$, y = $\frac{y_1+0}{2}$

∴ x = $\frac{x_1+x_2}{2}$, y = $\frac{y_1}{2}$

∴ Midpoint of PR is ($\frac{x_1+x_2}{2}$, $\frac{y_1}{2}$)

Similarly midpoint of QS is ($\frac{x_1+x_2}{2}$, $\frac{y_1}{2}$)

Also, similarly midpoint of AC and OA is ($\frac{x_1+x_2}{2}$, $\frac{y_1}{2}$)

Hence, midpoints of PR, QS, AC and OA coincide

∴We say that middle point of the opposite sides of a quadrilateral and the join of the mid-points of its diagonals meet in a point and bisect each other.

5. Question

If G be the centroid of a triangle ABC and P be any other point in the plane, prove that $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3\ GP^2$.

Answer

we will solve it by taking the coordinates $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$

Let the co ordinates of the centroid be G(u, v).

$G(u, v) = \left(\frac{x_1+x_2+x_3}{3} , \frac{y_1+y_2+y_3}{3} \right)$

let the coordinates of P(h, k).

now we will find L.H.S and R.H.S. separately.

$PA^2+PB^2+PC^2$

$= (h - x_1)^2 +(k - y_1)^2 +(h - x_2)^2+ (k - y_2)^2 +(h - x_3)^2 +(k - y_3)^2$ ...by distance formula.

$= 3(h^2+k^2)+(x_1^2+x_2^2+x_3^2)+(y_1^2+y_2^2+y_3^2)-2h(x_1+x_2+x_3)-2k(y_1+y_2+y_3)$

$= 3(h^2+k^2)+(x_1^2+x_2^2+x_3^2)+(y_1^2+y_2^2+y_3^2)-2h(3u)-2k(3v)$

$GA^2+GB^2+GC^2+3GP^2$

$= (u-x_1)^2+(v-y_1)^2+(u-x_2)^2+(v-y_2)^2+(u-x_3)^2+(v-y_3)^2+3[(u-h)^2+(v-k)^2]$ ......by distance formula.

$= 3(u^2+v^2)+(x_1^2+y_1^2+x_2^2+y_2^2+x_3^2+y_3^2) - 2u(x_1+x_2+x_3) - 2v(y_1+y_2+y_3) + 3[u^2+h^2-2uh+v^2+k^2-2vk]$

$= 6(u^2+v^2)+(x_1^2+y_1^2+x_2^2+y_2^2+x_3^2+y_3^2)-2u(3u)-2v(3v) +3(h^2+k^2) -6uh-6vk$

$= (x_1^2+x_2^2+x_3^2)+(y_1^2+y_2^2+y_3^2)+3(h^2+k^2)-6uh-6vk$

Hence LHS = RHS

(The above relation is known as Leibniz Relation)

Hence Proved.

6. Question

If G be the centroid of a triangle ABC, prove that:

$AB^2 + BC^2 + CA^2 = 3 (GA^2 + GB^2 + GC^2)$

Answer

We know that centroid of a triangle for (x_1 , y_1), (x_2 , y_2) and (x_3 , y_3) is

$G(x, y) = \left(\frac{x_1+x_2+x_3}{3} , \frac{y_1+y_2+y_3}{3} \right)$

We assume centroid of ΔABC at origin.

For x=0 and y=0

$\frac{x_1+x_2+x_3}{3} = 0$ and $\frac{y_1+y_2+y_3}{3} = 0$

$\therefore x_1 + x_2 + x_3 = 0$ and $y_1 + y_2 + y_3 = 0$

Squaring on both sides, we get

$x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_3x_1 = 0$ and $y_1^2 + y_2^2 + y_3^2 + 2y_1y_2 + 2y_2y_3 + 2y_3y_1 = 0$ ... (1)

$AB^2 + BC^2 + CA^2$

$= [(x_2 - x_1)^2 + (y_2 - y_1)^2] + [(x_3 - x_2)^2 + (y_3 - y_2)^2] + [(x_1 - x_3)^2 + (y_1 - y_3)^2]$

$= (x_1^2 + x_2^2 - 2x_1x_2 + y_1^2 + y_2^2 - 2y_1y_2)+(x_2^2 + x_3^2 - 2x_2x_3 + y_2^2 + y_3^2 - 2y_2y_3)+(x_1^2 + x_3^2 - 2x_1x_3 + y_1^2 + y_3^2 - 2y_1y_3)$

$= (2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3 - 2x_1x_3) + (2y_1^2 + 2y_2^2 + 2y_3^2 - 2y_1y_2 - 2y_2y_3 - 2y_1y_3)$

$= (3x_1^2 + 3x_2^2 + 3x_3^2) + (3y_1^2 + 3y_2^2 + 3y_3^2)$ …from 1

$= 3(x_1^2 + x_2^2 + x_3^2) + 3(y_1^2 + y_2^2 + y_3^2)$ … (2)

$3(GA^2 + GB^2 + GC^2)$

$= 3 [(x_1 - 0)^2 + (y_1 - 0)^2 + (x_2 - 0)^2 + (y_2 - 0)^2 + (x_3 - 0)^2 + (y_3 - 0)^2]$

$= 3 (x_1^2 + y_1^2 + x_2^2 + y_2^2 + x_3^2 + y_3^2)$

$= 3 (x_1^2 + x_2^2 + x_3^2) + 3(y_1^2 + y_2^2 + y_3^2)$ … (3)

From (2) and (3), we get

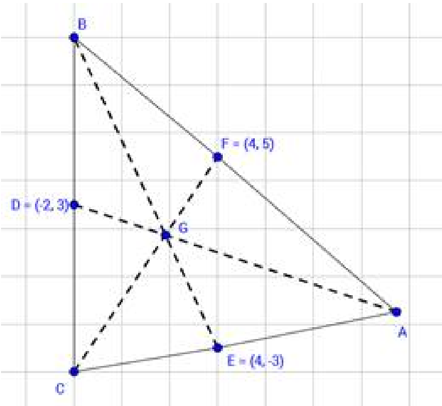
$AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$

7. Question

If (-2, 3), (4, -3) and (4, 5) are the mid-points of the sides of a triangle, find the coordinates of its centroid.

Answer

We know that centroid of ΔDEF will be the same that of ΔABC as ΔDEF is formed by midpoints of ΔABC.



∴ We know that centroid of a triangle for (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$G(x, y) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3} \right)$

$\therefore G(x, y) = \left(\frac{4+4-2}{3}, \frac{5-3+3}{3} \right)$

$\therefore G(x, y) = \left(2, \frac{5}{3} \right)$

Hence the centroid is $\left(2, \frac{5}{3} \right)$

8. Question

In Fig. 14.40, a right triangle BOA is given. C is the mid-point of the hypotenuse AB. Show that it is equidistant from the vertices 0, A and B.

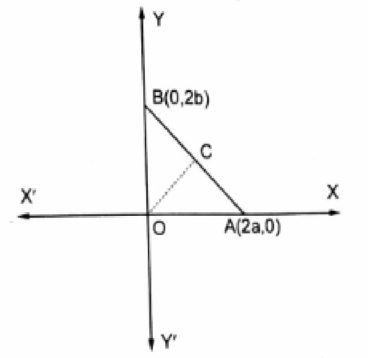


Fig. 14.40

Answer

Given that ΔBOA is right angled triangle

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

For midpoint C on AB,

$x = \frac{2a + 0}{2}$, $y = \frac{0 + 2b}{2}$

∴ x = a and y = b

∴ Coordinates of C are (a, b)

It is given that C is the midpoint of AB.

By distance formula,

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For OC,

$OC = \sqrt{(a - 0)^2 + (b - 0)^2}$

$= \sqrt{a^2 + b^2}$ ...(1)

For AC,

$AC = \sqrt{(2a - a)^2 + (0 - b)^2}$

$= \sqrt{a^2 + b^2}$

As C is midpoint, AC = CB. ...(2)

Hence from 1 and 2, we say that is point C is equidistant from the vertices 0, A and B.

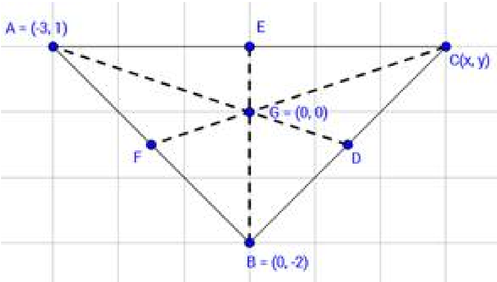
9. Question

Find the third vertex of a triangle, if two of its vertices are at (-3, 1) and (0, -2) and the centroid is at the origin.

Answer

Let the vertex of the triangle be A(1, 2), B(3, 5) and C(x, y)

Let the centroid be G(0, 0), as it is given that centroid is given at origin.



We know that centroid of a triangle for (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$G(x, y) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

For given coordinates A(1, 2), B(3, 5) and C(x, y), centroid is,

$$(0, 0) = \left(\frac{-3+0+x}{3}, \frac{1-2+y}{3}\right)$$

Solving for x and y,

$-3 + x = 0$ and $-1 + y = 0$

$\therefore x = 3$ and $y = 1$

Hence, the coordinate of third vertex is C(3, 1).

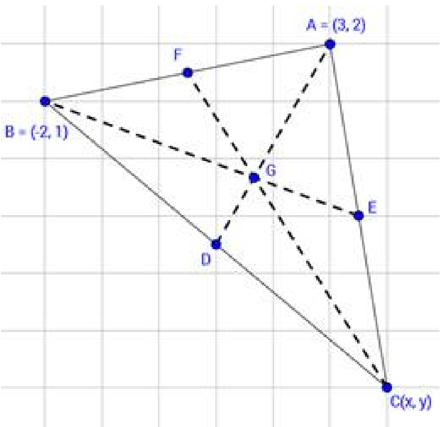
10. Question

A (3, 2) and B (-2, 1) are two vertices of a triangle ABC whose centroid G has the coordinates (5/3, - 1/3). Find the coordinates of the third vertex C of the triangle.

Answer

Let the vertex of the triangle be A(3, 2), B(-2, 1) and C(x, y)

Let the centroid be $G(\frac{5}{3}, \frac{-1}{3})$, as it is given that centroid is given at origin.



We know that centroid of a triangle for (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$G(x, y) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

For given coordinates A(3, 2), B(-2, 1) and C(x, y)

$$\left(\frac{5}{3}, \frac{-1}{3}\right) = \left(\frac{3-2+x}{3}, \frac{2+1+y}{3}\right)$$

Solving for x and y,

$-3 + 2 + x = 5$ and $2 + 1 + y = -1$

$\therefore$ x = 6 and y = -4

Hence, the coordinate of third vertex is C(6, -4).

Exercise 14.5

1. Question

Find the area of a triangle whose vertices are

(i) (6, 3), (-3, 5) and (4, - 2)

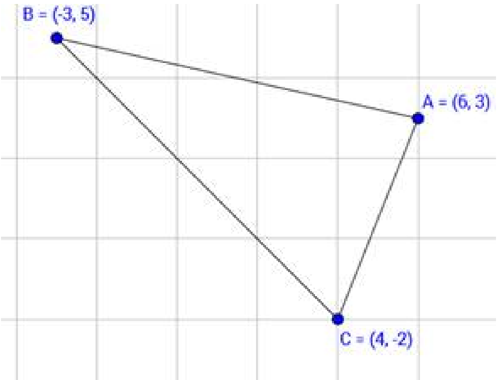
(ii) $\left(at_1^2,\ 2at_1\right),\left(at_2^2,\ 2at_2\right)$ *and* $\left(at_3^2,\ 2at_3\right)$

(iii) (a, c + a), (a, c) and (-a, c - a)

Answer

(i) (6, 3), (-3, 5) and (4, - 2)

Let $A \equiv (x_1, y_1) \equiv (6, 3)$, $B \equiv (x_2, y_2) \equiv (-3, 5)$ and $C \equiv (x_3, y_3) \equiv (4, -2)$



Area of ΔABC = $\frac{1}{2}|x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)|$ sq. units

$\therefore$ Area of ΔABC = $\frac{1}{2}$| { 6 (5 – (-2)) – 3 (-2 – 3) + 4 (3 – 5)} |

$=\frac{1}{2}$ | { 6 × 7 + 15 – 8 } |

$= \frac{1}{2}$ | 57 – 8|

$= \frac{49}{2}$ sq. units

(ii) $\left(at_1^2,\ 2at_1\right),\left(at_2^2,\ 2at_2\right)$ *and* $\left(at_3^2,\ 2at_3\right)$

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Here, $(x_1, y_1)=(at_1^2,2at_1)$, $(x_2,y_2)=(at_2^2,2at_2)$, $(x_3,y_3)=(at_3^2,2at_3)$

$\therefore$, area= $\frac{1}{2}|at_1^2(2at_2-2at_3)+at_2^2(2at_3-2at_1)+at_3^2(2at_1-2at_2)|$

$=\frac{1}{2}|2a^2t_1^2t_2-2a^2t_1^2t_3+2a^2t_2^2t_3-2a^2t_2^2t_1+2a^2t_3^2t_1-2a^2t_3^2t_2|$

$=\frac{1}{2} \times 2a^2|t_1^2t_2-t_1^2t_3+t_2^2t_3-t_2^2t_1+t_3^2t_1-t_3^2t_2|$

$=a^2|t_1^2t_2-t_1^2t_3+t_2^2t_3-t_2^2t_1+t_3^2t_1-t_3^2t_2|$

$=a^2|t_1^2(t_2-t_3)+t_2t_3(t_2-t_3)-t_1(t_2^2-t_3^2)|$

$=a^2|t_1^2(t_2-t_3)+t_2t_3(t_2-t_3)-t_1(t_2+t_3)(t_2-t_3)|$

$=a^2|(t_2-t_3)(t_1^2+t_2t_3-t_1t_2-t_1t_3)|$

$=a^2|(t_2-t_3)\{t_1(t_1-t_2)-t_3(t_1-t_2)\}|$

$=a^2|(t_2-t_3)(t_1-t_2)(t_1-t_3)|$

∴ Area is $a^2|(t_2-t_3)(t_1-t_2)(t_1-t_3)|$ sq. units

(iii) (a, c + a), (a, c) and (-a, c - a)

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area $= \frac{1}{2}|\, a\,(c - c + a) + a(\, c - a - c - a) - a\,(c + a - c)|$

$= \frac{1}{2}|\, a\,(a\,) + a(-2a) - a(\,a)|$

$= \frac{1}{2}|\,-2a^2\,|$

$= a^2$

∴ Area is a^2 sq. units

2. Question

Find the area of the quadrilaterals, the coordinates of whose vertices are

(i) (-3, 2), (5, 4), (7, - 6) and (-5, - 4)

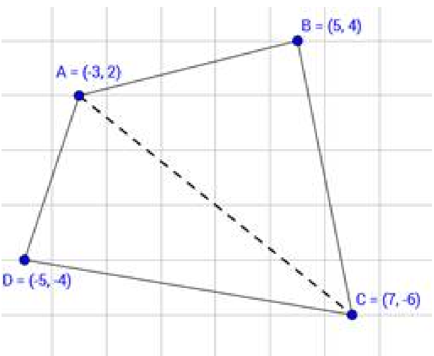
(ii) (1, 2), (6, 2), (5, 3) and (3, 4)

(iii) (-4, - 2), (-3, - 5), (3, - 2), (2, 3)

Answer

(i) (-3, 2), (5, 4), (7, - 6) and (-5, - 4)

Let the vertices of the quadrilateral be A (−3, 2), B (5, 4), C (7, −6), and D (−5, −4). Join AC to form two triangles ΔABC and ΔACD.



Area of □ ABCD = Area of ΔABC + Area of ΔACD

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC

$= \frac{1}{2}|-3(4 - (-6))+5(-6 - 2)+7(2 - 4)|$

$= \frac{1}{2}|-30 - 40 -14|$

= 42 sq. units

Area of ΔACD

$= \frac{1}{2}|-3(-6 - 4)+7(-4 - 2) - 5(2 + 6)|$

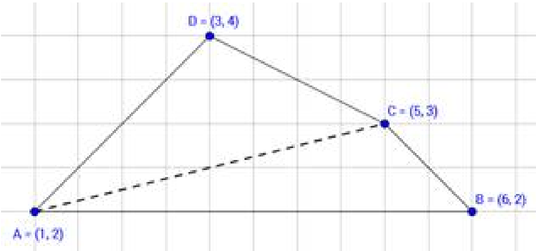
$= \frac{1}{2}|6 - 42 - 40|$

= 38 sq. units

Area of □ ABCD = 42 + 38 = 80 sq. units

(ii) (1, 2), (6, 2), (5, 3) and (3, 4)

Let the vertices of the quadrilateral be A (1, 2), B (6, 2), C (5, 3), and D (3, 4). Join AC to form two triangles ΔABC and ΔACD.



Area of □ ABCD = Area of ΔABC + Area of ΔACD

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC

$= \frac{1}{2}|1(2 - 3)+6(3 - 2)+5(2 - 2)|$

$= \frac{1}{2}|-1 + 6|$

$= \frac{5}{2}$ sq. units

Area of ΔACD

$= \frac{1}{2}|1(3 - 4)+5(4 - 2) + 3(2 - 3)|$

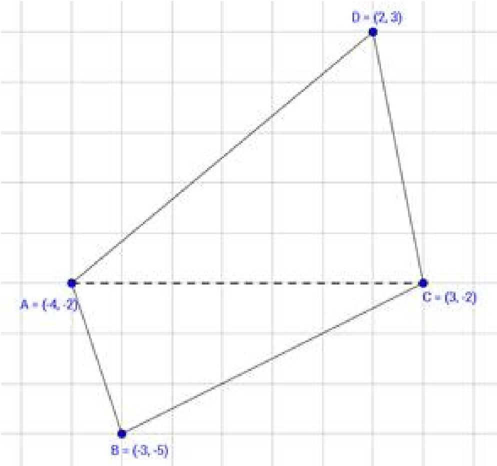
$= \frac{1}{2}|-1 + 10 -3|$

= 3 sq. units

Area of □ ABCD = $\frac{5}{2}$ + 3 = $\frac{11}{2}$ sq. units

(iii) (-4, - 2), (-3, - 5), (3, - 2), (2, 3)

Let the vertices of the quadrilateral be A (−4, −2), B (−3, −5), C (3, −2), and D (2, 3). Join AC to form two triangles ΔABC and ΔACD



Area of □ ABCD = Area of ΔABC + Area of ΔACD

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC

$= \frac{1}{2}|-4(-5 - (-2)) - 3(-2 - (-2)) + 3(-2 - (-5))|$

$= \frac{1}{2}|12 + 0 + 9|$

$= \frac{21}{2}$ sq. units

Area of ΔACD

$= \frac{1}{2}|-4(-2 - 3) - 3(3 - (-2)) + 2(-2 - (-2))|$

$= \frac{1}{2}|20 + 15 + 0|$

$= \frac{35}{2}$ sq. units

Area of □ ABCD = $\frac{21}{2} + \frac{35}{2}$ = 28 sq. units

3. Question

The four vertices of a quadrilateral are (1, 2), (-5, 6), (7, -4) and (k, -2) taken in order. If the area of the quadrilateral is zero, find the value of k.

Answer

Let four vertices of quadrilateral be A (1, 2) and B (−5, 6) and C (7, −4) and D (*k*, −2)

Area of □ ABCD = Area of ΔABC + Area of ΔACD = 0 sq. unit

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC

$= \frac{1}{2} |1(6 - (-4)) - 5(-4 -2) + 7(2 - 6)|$

$= \frac{1}{2} |10 + 30 -28|$

= 6 sq. units

Area of ΔACD

$= \frac{1}{2} |1(-2 - (-4)) + k(-4 -2) + 7(2 - (-2))|$

$= \frac{1}{2} |2 - 6k + 30|$

= (3k -15) sq. units

Area of ΔABC + Area of ΔACD = 0 sq. unit

∴ 6 + 3k -15 =0

3k -9 =0

∴ k =3

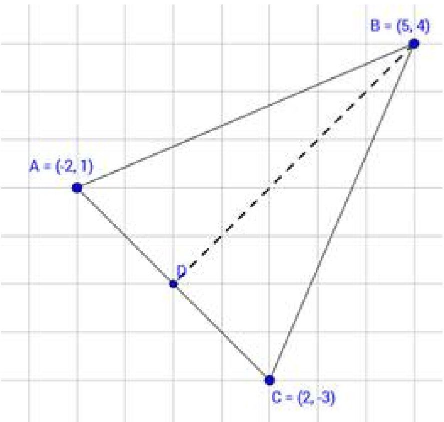
Hence, the value of k is 3

4. Question

The vertices of Δ ABC are (-2, 1), (5, 4) and (2, -3) respectively. Find the area of the triangle and the length of the altitude through A.

Answer

Let three vertices be A (−2, 1) and B (5, 4) and C(2, −3)



Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC

$= \frac{1}{2} |-2(4 - (-3)) + 5(-3 -1) + 2(1 - 4)|$

$= \frac{1}{2}$ |-14 - 20 - 6|

= 20 sq. units

Now to find length of BC,

By distance formula,

XY = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For BC,

BC = $\sqrt{(2-5)^2 + (-3-4)^2}$

$= \sqrt{9+49}$

$= \sqrt{58}$ sq. units

Area of ΔABC = $\frac{1}{2}$ × Base × Altitude

∴ 20 = $\frac{1}{2} \times \sqrt{58}$ × Altitude

∴ Altitude = $\frac{40}{\sqrt{58}}$ units

Hence, the length of altitude through A is $\frac{40}{\sqrt{58}}$ units.

5. Question

Show that the following sets of points are collinear.

(a) (2, 5), (4, 6) and (8, 8)

(b) (1, -1), (2, 1) and (4, 5).

Answer

(a) Let three given points be *A*(2*,* 5)*, B*(4*,* 6) and *C*(8*,* 8).

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC

$= \frac{1}{2}$ |2(6 – 8) + 4(8 -5) + 8(5 – 6)|

$= \frac{1}{2}$ |-4 + 12 - 8|

= 0 sq. units

We know that if area enclosed by three points is zero, then points are collinear.

Hence, given three points are collinear.

(b) Let three given points be *A*(1*,* −1)*, B*(2*,* 1) and *C*(4*,* 5)

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC

$= \frac{1}{2}|1(1-5)+2(5+1)+4(-1-1)|$

$= \frac{1}{2}|-4+12-8|$

= 0 sq. units

We know that if area enclosed by three points is zero, then points are collinear.

Hence, given three points are collinear.

6. Question

Prove that the points (a, 0), (0, b) and (1, 1) are collinear if, $\frac{1}{a}+\frac{1}{b}=1$

Answer

Let three given points be $A(a,0)$, $B(0,b)$ and $C(1,1)$.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC

$= \frac{1}{2}|a(b-1)+1(0-b)|$

$= \frac{1}{2}|ab-a-b|$

Here given that $\frac{1}{a}+\frac{1}{b}=1$

$\therefore \frac{a+b}{ab}=1$

$\therefore a+b=ab$

Now,

Area of ΔABC

$= \frac{1}{2}|ab-(a+b)|$

$= \frac{1}{2}|ab-ab|$

$= \frac{1}{2}|0|$

= 0 sq. units

We know that if area enclosed by three points is zero, then points are collinear.

Hence, given three points are collinear.

7. Question

The point A divides the join of P (-5, 1) and Q (3, 5) in the ratio k : 1. Find the two values of k for which the area of a ABC where B is (1, 5) and C (7, -2) is equal to 2 units.

Answer

coordinates A can be given by using section formula for internal division,

A = ($\frac{-5+3k}{k+1}$, $\frac{1+5k}{k+1}$)

and B (1,5), C (7,−2)

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC

$= \frac{1}{2} | \frac{-5+3k}{k+1}(7) + 1(-2 - \frac{1+5k}{k+1}) + 7(\frac{1+5k}{k+1} - 5) |$

But Area of ΔABC = 2

$\therefore \frac{1}{2} | \frac{-5+3k}{k+1}(7) + 1(-2 - \frac{1+5k}{k+1}) + 7(\frac{1+5k}{k+1} - 5) | = 2$

Solving above we get,

$| \frac{14k-66}{k+1} | = 4$

Taking positive sign, 14k−66=4k+4

10k = 70

k=7

Taking negative sign we get,

14k−66=−4k−4

18k = 62

$k = \frac{62}{18} = \frac{31}{9}$

8. Question

The area of a triangle is 5. Two of its vertices are (2, 1) and (3, -2). The third vertex lies on y = x + 3. Find the third vertex.

Answer

Let ABC be a triangle with A(a, b),B(2,1) and C(3,-2).

A lies on the line y=x+3 means,

b=a+3 ...(1).

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC = 5

Substituting the values of A, B and C in formula, we, get,

$5 = \frac{1}{2} | 3a + b - 7 |$

Taking positive value for | 3a + b − 7 |,

3a+b=17 ....(2)

Solving 1 and 2 simultaneously,

a = $\frac{7}{2}$ and b = $\frac{13}{2}$

Hence coordinates of the vertex A are ($\frac{7}{2}$, $\frac{13}{2}$).

Taking negative value for | 3a + b – 7 |,

$\frac{1}{2}$(3a+b−7) = − 5

3a+b=−3 ...(3)

Solving 1 and 2 simultaneously,

A = $\frac{-3}{2}$ and b = $\frac{3}{2}$ and the vertex A is ($\frac{-3}{2}$, $\frac{3}{2}$)

Hence the coordinates of third vertex are ($\frac{7}{2}$, $\frac{13}{2}$) or ($\frac{-3}{2}$, $\frac{3}{2}$).

9. Question

If $a \neq b \neq c$, prove that the points (a, a^2), (b,b^2),(c, c^2) can never be collinear.

Answer

Area of the triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$Area\ of\ \triangle = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

For points to be collinear, the Area enclosed by them should be equal to 0

∴ For given points,

$$Area = \frac{1}{2}[a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)]$$

Area = 1/2 |(b – c)(a - b)(c – a)|

Area ≠ 0

Also it is given that $a \neq b \neq c$.

Hence area of triangle made by these points is never zero. Hence given points are never collinear.

10. Question

Four points A (6, 3), B (-3, 5), C (4, - 2) and D (x, 3x) are given in such a way that $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$, find x.

Answer

Four points A (6, 3), B (−3, 5) C (4, −2) and D(*x*, 3*x*)

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

= $\frac{1}{2}$ $|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC

= $\frac{1}{2}$ |6(5 – (- 2)) - 3(-2 -3) + 4(3 – 5)|

$= \frac{1}{2}$ |42 + 15 -8|

$= \frac{49}{2}$ sq. units

Area of ΔDBC

$= \frac{1}{2}$ |x(5 − (-2)) + 3(-2 -3x) + 4(3x - 5))|

$= \frac{1}{2}$ |7x +6 + 9x + 12x - 20|

$= \frac{1}{2}$ | 28x -14|

= ± 7(2x -1)

It is given that $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$

∴2 × ΔDBC = ΔABC

2 × (± 7(2x -1)) = $\frac{49}{2}$

∴ ± 4(2x -1) = 7

∴ 4(2x -1) = 7 or -4(2x -1) = 7

∴ 8x − 4 =7 or -8x + 4 =7

∴ 8x =11 or -8x =3

∴x= $\frac{11}{8}$ or x = $\frac{-3}{8}$

Hence, the value of x is $\frac{11}{8}$ or $\frac{-3}{8}$

11. Question

For what value of a the point (a, 1),(1, -1) and(11, 4) are collinear?

Answer

The three given points are *A*(*a,* 1)*, B*(1*,* −1) and *C*(11*,* 4).

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Given that area of ΔABC = 0

∴ 0 = $\frac{1}{2}$ |a(-1 − 4) + 1(4 - 1) + 11(1 − (-1))|

∴ 0 = $\frac{1}{2}$ |-5a + 3 + 22|

∴ -5a + 3 + 22 = 0

a = 5

Hence the value of a is 5

12. Question

Prove that the points (a, b),(a_1,b_1) and (a - a_1, b - b_1) are collinear if $ab_1 = a_1b$

Answer

Consider the following points A(a,b), B(a_1,b_1), C(a−a_1,b−b_1)

Since the given points are collinear, we have area(△ABC)=0

First find the area of area(△ABC) as follows:

area(△ABC)=$_{1/2}$ $|x_1(y_1-y3)+x_1(y3-y_1)+x3(y_1-y_1)|$

$=\frac{1}{2}|a(b_1-(b-b_1))+a_1((b-b_1)-b)+(a-a_1)(b-b_1)|$

$=\frac{1}{2}|a(b_1-b+b_1)+a_1(b-b_1-b)+a(b-b_1)-a_1(b-b_1)|$

$=\frac{1}{2}|-ab-a_1b_1+ab-ab_1+a_1b+a_1b_1|$

$=\frac{1}{2}|-(ab_1-a_1b)|$

$= (ab_1-a_1b)$

This gives, $ab_1-a_1b=0$

∴ $ab_1 = a_1b$

13. Question

If three points (x_1, y_1), (x_2, y_2), (x_3, y_3) lie on the same line, prove that $\frac{y_2-y_3}{x_2x_3}+\frac{y_3-y_1}{x_3x_1}+\frac{y_1-y_2}{x_1x_2}=0$

Answer

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Given that all points are collinear.

∴ area = 0

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2) = 0$

Dividing by $x_1\ x_2\ x_3$,

$\therefore \frac{x_1(y_2-y_3)}{x_1\ x_2\ x_3}+\frac{x_2(y_3-y_1)}{x_1\ x_2\ x_3}+\frac{x_3(y_1-y_2)}{x_1\ x_2\ x_3}=0$

$\frac{y_2-y_3}{x_2x_3}+\frac{y_3-y_1}{x_3x_1}+\frac{y_1-y_2}{x_1x_2}=0$

Hence proved.

14. Question

If (x, y) be on the line joining the two points (1, -3) and (-4, 2), prove that x+y+2=0.

Answer

Given: The point (x, y) is on the line joining the two points (1, -3) and (-4, 2).

To Prove: x+y+2=0

Proof: When the points line on the same line they are called collinear points.As the point (x, y) lies on the line joining the points (1, −3) and (−4, 2), it means that the three points are collinear.If the points are in same straight line they cannot form a triangle which implies that area of triangle becomes zero.If the vertices of the triangle are given in the form of (a,b)where a and b are the coordinates of a given point in the direction of x and y axis respectively.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) *and* (x_3,y_3) is given as:

$$Area(\triangle)=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)| \quad .... (1)$$

Now, for the three points to be collinear,

$$Area(\triangle)=0$$

Now if the points (x, y), (1, −3) and (−4, 2) are collinear,the area of the triangle formed by these points is zero.

Substitute the given values in equation (1)0,

So,$\frac{1}{2}|x(-3-2)+1(2-y)-4(y+3)|=0$

$-5x + 2 - y - 4y - 12 = 0$

$-5x - 5y - 10 = 0$

Taking "-5" common from the equation we get,

⇒ $-5(x+y+2)=0$

⇒ $(x+y+2)=0$

Hence proved, $(x+y+2)=0$

Conclusion: If (x, y) be on the line joining the two points (1, -3) and (-4, 2), then x+y+2=0.

15. Question

Find the value of k if points (k, 3), (6, - 2) and (-3, 4) are collinear.

Answer

The three given points are $A(k, 3)$, $B(6, -2)$ and $C(-3, 4)$. It is also said that they are collinear and hence the area enclosed by them should be 0.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Given that area of ΔABC = 0

$\therefore 0 = \frac{1}{2} |k(-2 - 4) + 6(4 - 3) - 3(3 - (-2))|$

$\therefore 0 = \frac{1}{2} |-6k + 6 - 15|$

$\therefore -\frac{1}{2} |-6k + 9| = 0$

$6k + 9 = 0$

$\therefore k = \frac{-3}{2}$

Hence, the value of k is $\frac{-3}{2}$

16. Question

Find the value of k, if the points A (7, -2), B (5, 1) and C (3, 2k) are collinear.

Answer

The three given points are *A*(7*,* −2)*, B*(5*,* 1) and *C*(3*,* 2*k*). It is also said that they are collinear and hence the area enclosed by them should be 0.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Given that area of ΔABC = 0

$\therefore 0 = \frac{1}{2} |7(1 - 2k) + 5(2k - (-2)) + 3(-2 - 1)|$

$\therefore 0 = \frac{1}{2} |7 - 14k + 10k +10 -6 -3|$

$\therefore -\frac{1}{2} |8 - 4k|= 0$

$8 - 4k = 0$

$-4k = -8$

$\therefore k = 2$

17. Question

If the point P (m, 3) lies on the line segment joining the points A$\left(-\frac{2}{5},6\right)$ and B (2, 8), find the value of m.

Answer

It is said that the point *P*(*m,*3) lies on the line segment joining the points A$\left(-\frac{2}{5},6\right)$ and B (2, 8).

Hence we understand that these three points are collinear. So the area enclosed by them should be 0.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Given that area of ΔABP = 0

$\therefore 0 = \frac{1}{2} |m(6 - 8) - \frac{2}{5}(8 - 3) + 2(3 - 6)|$

$\therefore -2m - 2 -6 = 0$

$-2m = 8$

$m = -4$

Hence the value of m = -4

18. Question

If R (x, y) is a point on the line segment joining the points P (a, b) and Q (b, a), then prove that x + y = a + b.

Answer

Given : R (x, y) is a point on the line segment joining the points P (a, b) and Q (b, a).

To prove: x + y = a + b

Proof: It is said that the point *R*(*x, y*) lies on the line segment joining the points *P*(*a, b*) and *Q*(*b, a*). Thus, these three points are collinear.

So the area enclosed by them should be 0.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3) is:

$$Area(\triangle) = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

Given that area of ΔPQR = 0

$\therefore \frac{1}{2}\,|x(b - a) + a(a - y) + b(y - b)| = 0$

$\therefore bx - ax + a^2 - ay + by - b^2 = 0$

$\therefore ax + ay - bx - by - a^2 - b^2 = 0$

$\therefore ax + ay - bx - by = a^2 + b^2$

$(a - b)(x + y) = (a - b)(a + b)$

$\therefore x + y = a + b$

Hence proved.

19. Question

Find the value of k, if the points A (8, 1), B (3, - 4) and C (2, k) are collinear.

Answer

Given points are *A*(8,1),*B*(3,−4) and *C*(2,*k*).It is also said that they are collinear and hence the area enclosed by them should be 0.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2}\,|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Given that area of ΔABC = 0

$\therefore 0 = \frac{1}{2}\,|8(-4 - k) + 3(k - 1) + 2(1 - (-4))|$

$\therefore 0 = \frac{1}{2}\,|-32 - 8k + 3k - 3 + 10|$

$\therefore 5k + 25 = 0$

$\therefore k = -5$

Hence, the value of k is -5.

20. Question

Find the value of a for which the area of the triangle formed by the points A (a, 2a), B (-2, 6) and C (3, 1) is 10 square units.

Answer

Given points are *A*(*a*,2*a*), *B*(−2,6) and *C*(3,1). It is also said that the area enclosed by them is 10 square units.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Given that area of ΔABC = 10

$\therefore 10 = \frac{1}{2} |a(6 - 1) -2 (1 - 2a) + 3(2a - 6)|$

$\therefore 20 = |5a - 2 + 4a + 6a - 18|$

$\therefore 20 = | 15a - 20|$

$\therefore 15a - 20 = \pm 20$

Taking positive sign,

$15a - 20 = 20$

$a = \frac{8}{3}$

Taking negative sign,

$15a - 20 = -20$

$a = 0$

Hence, the value of a are 0 and $\frac{8}{3}$

21. Question

If the vertices of a triangle are (1,-3), (4, p) and (-9, 7) and its area is 15 sq. units, find the value(s) of p.

Answer

Let A(1, −3), B(4, *p*) and C(−9, 7) be the vertices of the ΔABC.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Given that area of ΔABC = 15

$\therefore 15 = \frac{1}{2} |1(p - 7) +4 (7 - (-3)) - 9(-3 - p)|$

$\therefore 30 = |p - 7 + 40 + 27 + 9p|$

$\therefore 30 = | 10p + 60|$

$\therefore 10p + 60 = \pm 30$

Taking positive sign,

$10p + 60 = 30$

$p = -3$

Taking negative sign,

$10p + 60 = - 30$

$p = -9$

Hence, the value of p are -3 and -9

22. Question

Find the area of a parallelogram ABCD if three of its vertices are A(2, 4), B $(2 + \sqrt{3}, 5)$ and C(2, 6).

Answer

It is given that A(2, 4), B$(2 + \sqrt{3}, 5)$ and C(2, 6) are the vertices of the parallelogram ABCD.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of □ABCD = 2 × Area of ΔABC

Area of ΔABC $= \frac{1}{2} |2(5 - 6)+ (2 + \sqrt{3})(6 - 4)+2(4 - 5)|$

$= \frac{1}{2} |-2 + 4 + 2\sqrt{3} - 2|$

$= \frac{1}{2} \times 2\sqrt{3}= \sqrt{3}$ sq. units

∴ Area of □ABCD $= 2 \times \sqrt{3} = 2\sqrt{3}$ sq. units

Hence, the area of given parallelogram is $2\sqrt{3}$ sq. units

23. Question

Find the value (s) of k for which the points (3k - 1, k - 2), (k, k - 7) and (k - 1,-k - 2) are collinear.

Answer

Let A (3*k* − 1, *k* − 2) , B (*k*, *k* − 7) and C (*k* − 1, −*k* − 2) be the given points.For points to be collinear area of triangle formed by the vertices must be zero.

Area of the triangle having vertices (x_1,y_1) , (x_2,y_2) and (x_3,y_3) $= \frac{1}{2} |x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) |$

area of ΔABC = 0

⇒ (3k−1) [(k−7) − (−k−2)] + k [(−k−2) − (k−2)] + (k−1) [(k−2) − (k−7)] =0⇒ (3k−1) [k−7 + k + 2] + k [−k−2 − k+ 2] + (k−1) [k−2 − k + 7] =0

⇒ (3k−1) (2k−5) + k (−2k) + 5 (k −1) =0⇒ $6k^2$ - 15k -2k + 5 - $2k^2$ + 5k - 5 = 0

⇒ $6k^2−17k + 5−2k^2 + 5k−5=0$

⇒ $4k^2−12k=0$

⇒ 4k (k−3) =0

⇒ k=0 or k−3=0

⇒ k=0 or k=3

Hence, the value of *k* is 0 or 3.

24. Question

If the points A (-1,-4), B (b,c) and C (5,-1) are collinear and 2b + c = 4, find the values of b and c.

Answer

The given points A(−1, −4), B(*b*, *c*) and C(5, −1) are collinear.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Given that area of ΔABC = 0

∴ −1[c − (− 1)]+b[− 1 − (− 4)] + 5(− 4 − c) = 0

∴ − c − 1 + 3b − 20 − 5c = 0

3b − 6c = 21

∴b − 2c = 7 ...(1)

Also it is given that 2b + c = 4 ...(2)

Solving 1 and 2 simultaneously, we get,

2(7 + 2c) + c = 4

14 + 4c + c = 4

5c = − 10

c = − 2

∴ b = 3

Hence, value of b and c are 3 and -2 respectively

25. Question

If the points A (-2,1), B (a, b) and C (4,-1) are collinear and a – b = 1, find the values of a and b.

Answer

The given points A(−2, 1), B(*a*, *b*) and C(4, −1) are collinear.

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Given that area of ΔABC = 0

∴ −2[b − (− 1)] + a(− 1 − 1) + 4 (1 −b) = 0

-2b – 2 − 2a + 4 − 4b = 0

− 2a − 6b = − 2

a + 3b = 1 ...(1)

Also it is given that a – b = 1 ...(2)

Solving 1 and 2 simultaneously,

B + 1 + 3b = 1

4b = 0

∴ b = 0

∴ a =1

Hence, the values of *a* and *b* are 1 and 0.

26. Question

If A (-3, 5), B (-2,-7), C (1,-8) and D (6, 3) are the vertices of a quadrilateral ABCD, find its area.

Answer

Given vertices of a quadrilateral ABCD are A(−3, 5), B(−2, −7), C(1, −8) and D(6, 3)

Area of the quadrilateral ABCD = Area of ΔABC + Area of ΔACD

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔABC $= \frac{1}{2} | -3[-7-(-8)] + (-2)(-8-5) + 1[5-(-7)] |$

$= \frac{1}{2} | -3 + 26 + 12 |$

$= \frac{35}{2}$ sq. units

Area of ΔACD $= \frac{1}{2} | -3(-8-3) + 1(3-5) + 6[5-(-8)] |$

$= \frac{1}{2} | 33 - 2 + 78 |$

$= \frac{109}{2}$ sq. units

Area of the quadrilateral ABCD $= \frac{35}{2} + \frac{109}{2} = 72$ sq. units

∴Hence, the area of the quadrilateral is 72 sq. units.

27. Question

If P (-5, - 3), Q (-4, -6), R (2,-3) and S (1, 2) are the vertices of a quadrilateral PQRS, find its area.

Answer

Let P(−5,−3); Q(−4,−6); R(2,−3) and S(1,2) be the vertices of quadrilateral PQRS.

Area of the quadrilateral PQRS = Area of ΔPQR + Area of ΔPSR

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔPQR $= \frac{1}{2} | -5(-6+3) - 4(-3+3) + 2(-3+6) |$

$= \frac{1}{2} |15 + 0 + 6|$

$= \frac{21}{2}$ sq. units

Area of ΔPSR $= \frac{1}{2} | -5(2+3) + 1(-3+3) + 2(-3-2) |$

$= \frac{1}{2} | -25 + 0 - 10 |$

$= \frac{35}{2}$ sq. units

Area of the quadrilateral PQRS $= \frac{21}{2} + \frac{35}{2} = 28$ sq. units

∴Hence, the area of the quadrilateral is 28 sq. units.

(given answer is wrong, its not 13, it is 28)

28. Question

Find the area of the triangle PQR with Q (3, 2) and the mid-points of the sides through Q being (2, -1) and (1, 2).

Answer

Let the co-ordinates of P and R be (a,b) and (c,d) and coordinates of Q are (3, 2)

By midpoint formula.

$x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

(2 , - 1) is the mid-point of PQ.

$\therefore 2 = \frac{3+a}{2}$ and $-1 = \frac{2+b}{2}$

∴ a = 1 and b = -4

∴ Coordinates of P are (1, -4)

(1 , 2) is the mid-point of QR.

$\therefore 1 = \frac{3+c}{2}$ and $2 = \frac{2+d}{2}$

∴ c = -1 and d = 2

∴ Coordinates of P are (-1, 2)

Area of the triangle having vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$= \frac{1}{2} |x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of ΔPQR $= \frac{1}{2} |\ 3(-4-2) + 2(-1-1) + 1(2-4)\ |$

$= \frac{1}{2} |-18-4-2|$

= 12 sq. units

Hence the area of ΔPQR is 12 sq. units