

Chapter 5. Arithmetic Progression

Question-1

Find the sum of the following A.P. 1, 3, 5, 7,, 199.

Solution:

Given, $a = 1$, $d = 2$, $a_n = l = 199$,

$$a + (n - 1) d = 199$$

$$1 + (n - 1) 2 = 199$$

$$\Rightarrow 1 + 2n - 2 = 199$$

$$\Rightarrow 2n = 200$$

$$\therefore n = \frac{200}{2}$$

$$n = 100.$$

$$S_n = n/2 (a + l)$$

$$= 50(1 + 199)$$

$$= 50(200)$$

$$= 10000$$

Question-2

Find the A.P. whose 10th term is 5 and 18th term is 77.

Solution:

given, 10th term of an A.P = 5

$$\hookrightarrow a + (10 - 1) d = 5$$

$$\Rightarrow a + 9d = 5 \dots\dots\dots(i)$$

and 18th term = 77

$$\hookrightarrow a + (18 - 1) d = 77$$

$$\Rightarrow a + 17d = 77 \dots\dots\dots(ii)$$

$$(ii) - (i), 8d = 72$$

$$\therefore d = 9$$

Substituting the value of $d = 9$ in (i),

$$a + 81 = 5$$

$$a = 5 - 81 = -76$$

\therefore The A.P. is $-76, -67, \dots\dots\dots$

Question-3

In a certain A.P the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.

Solution:

Given, $a_{24} = 2a_{10}$

$a_{24} = a + 23d$ and $a_{10} = a + 9d$

To prove: $a_{72} = 2 a_{34}$

$a_{72} = a + 71d$

$a_{34} = a + 33d$

$a_{24} = 2a_{10}$ (Given)

$a + 23d = 2(a + 9d)$

$a + 23d = 2a + 18d$

$a - 5d = 0$

$a = 5d$(i)

$a_{72} = 2 a_{34}$

$a + 71d = 2(a + 33d)$

$a + 71d = 2a + 66d$

$a - 5d = 0$

$a = 5d$(ii)

from, (1) and (2) $a_{72} = 2 a_{34}$

Hence proved.

Question-4

a , b and c are in A.P. Prove that $b + c$, $c + a$ and $a + b$ are in A.P.

Solution:

Given, a , b and c are in A.P.

$\therefore b - a = c - b$

To prove : $b + c$, $c + a$ and $a + b$ are in A.P.

$c + a - (b + c) = a + b - (c + a)$

$\Rightarrow c + a - b - c = a + b - c - a$

$a - b = b - c$

$\Rightarrow b - a = c - b$

$\therefore a, b, c$ are in A.P.

$\therefore b + c, c + a$ and $a + b$ are in A.P.

Question-5

If 9th term of an A.P. is zero, prove that its 29th term is double the 19th term.

Solution:

$$9^{\text{th}} \text{ term} = 0$$

$$a_1 + 8d = 0$$

$$a_{29} = a_1 + 28d = a_1 + 8d + 20d = 0 + 20d = 20d$$

$$a_{19} = a_1 + 18d = a_1 + 8d + 10d = 0 + 10d = 10d$$

$$a_{29} = 2a_{19}.$$

Question-6

Determine the A.P whose third term is 16 and the difference of 5th from 7th term is 12.

Solution:

Let the A.P. be $a, a + d, a + 2d, \dots$

$$\Rightarrow \text{The third term} = a_3 = a + 2d = 16 \dots\dots\dots(i)$$

$$\text{and seventh term} = a_7 = a + 6d$$

$$\text{Given that } a_7 - a_5 = 12$$

$$\Rightarrow (a + 6d) - (a + 4d) = 12$$

$$\Rightarrow a + 6d - a - 4d = 12$$

$$\Rightarrow 2d = 12$$

$$\Rightarrow d = 6$$

Substituting the value of $d = 6$ in (i),

$$a + 12 = 16$$

$$a = 4$$

\therefore The first term of the A.P. is 4 and the common difference is 6.

\therefore The A.P. is 4, 10, 16, 22, 28, 34, ...

\therefore The fifth term $= a_5 = a + 4d$.

Question-7

The sum of the first six terms of an A.P is zero and the fourth term is 2. Find the sum of its first 30 terms.

Solution:

Let the sum of first 30 terms be S_{30} , first term be a , fourth term be a_4 and the sum of first six terms be S_6 .

Given that $S_6 = 0$ and fourth term $a_4 = 2$

$$\Rightarrow a + 3d = 2 \dots\dots\dots(i)$$

$$S_6 = 0$$

$$\frac{n}{2} (2a + 5d) = 0$$

$$\Rightarrow 2a + 5d = 0 \dots\dots\dots(ii)$$

$$(i) \times 2,$$

$$2a + 6d = 4 \dots\dots\dots(iii)$$

$$(iii) - (ii),$$

$$\therefore d = 4$$

Substituting the value of $d = 4$ in (i),

$$a + 3 \times (4) = 2$$

$$\Rightarrow a = 2 - 12 = -10$$

$$\therefore a_{30} = a + 29d$$

$$= -10 + 29 \times (4)$$

$$= -10 + 116$$

$$= 106$$

$$\therefore \text{Sum to first 30 terms} = S_{30} = \frac{n}{2}(a + l)$$

$$= \frac{30}{2}(-10 + 106)$$

$$= 15 \times 96$$

$$= 1140.$$

Question-8

An A.P consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32nd term.

Solution:

Given, $n = 60$, $a_1 = 7$,

and $a_{60} = 125$

$$\Rightarrow a_1 + 59d = 125$$

$$7 + 59d = 125$$

$$59d = 118$$

$$d = 118/59 = 2$$

$$a_{32} = a_1 + 31d = 7 + 31(2) = 7 + 62$$

$$\therefore a_{32} = 69.$$

Question-9

Find the sum of the series $51 + 50 + 49 + \dots + 21$.

Solution:

$$51 + 50 + 49 + \dots + 21$$

$$a = 51, d = -1, a_n = 21$$

$$\therefore a + (n - 1)d = a_n$$

$$51 + (n - 1)(-1) = 21$$

$$(n - 1)(-1) = 21 - 51$$

$$n - 1 = 30$$

$$\therefore n = 31$$

$$\therefore \text{Sum of the series} = \frac{31}{2}(51 + 21)$$

$$= \frac{31}{2} \times 72$$

$$= 1116$$

$$\therefore \text{The sum of the series } 51 + 50 + 49 + \dots + 21 = 1116.$$

Question-10

Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Solution:

Let the three numbers be $a - d$, a , $a + d$.

$$a - d + a + a + d = 27$$

$$3a = 27$$

$$a = 9$$

$$(a - d)(a)(a + d) = 648$$

$$a(a^2 - d^2) = 648$$

$$9(9^2 - d^2) = 648$$

$$9^2 - d^2 = 72$$

$$d^2 = 81 - 72$$

$$d^2 = 9$$

$$d = 3$$

The numbers are 6, 9, 12.

Question-11

How many terms of A.P -10, -7, -4, -1, must be added to get the sum -104?

Solution:

-10, -7, -4, -1,

$$a = -10, d = 3$$

$$S_n = \frac{1}{2}n\{2a + (n - 1)d\}$$

$$-104 = \frac{1}{2}n\{2(-10) + (n - 1)3\}$$

$$= \frac{1}{2}n(-20 + 3n - 3)$$

$$-208 = n(3n - 23)$$

$$3n^2 - 23n + 208 = 0$$

$$3n^2 - 39n + 16n + 208 = 0$$

$$3n(n - 13) + 16(n - 13) = 0$$

$$(n - 13)(3n + 16) = 0$$

$$\therefore n = 13$$

\therefore 13 terms must be added to get the sum of the A.P - 104.

Question-12

If the sum of p terms of an A.P is $3p^2 + 4p$, find its n^{th} term.

Solution:

$$S_p = 3p^2 + 4p$$

$$t_n = S_n - S_{n-1}$$

$$= (3n^2 + 4n) - [3(n - 1)^2 + 4(n - 1)]$$

$$= (3n^2 + 4n) - [3(n^2 - 2n + 1) + 4(n - 1)]$$

$$= (3n^2 + 4n) - [3n^2 - 6n + 3 + 4n - 4]$$

$$= (3n^2 + 4n) - [3n^2 - 2n - 1]$$

$$= 3n^2 + 4n - 3n^2 + 2n + 1$$

$$= 6n + 1$$

Therefore the n^{th} term is $6n + 1$.