Chapter 1 Real Number

Question-1

Write the following rational numbers in decimal form:

(i) $\frac{42}{100}$

(ii) $\frac{327}{500}$

(iii) $3\frac{3}{8}$

(iv) $\frac{5}{6}$

(v) $\frac{1}{5}$

(vi) $\frac{1}{7}$

(vii) $\frac{2}{13}$

(viii) $\frac{11}{17}$

Solution:

(i) $\frac{42}{100} = 0.42$

(ii) $\frac{327}{500} = 0.654$

```
      0. 654
   |---------
 5 |  3.27
   |  30
   |---------
   |   27
   |   25
   |---------
   |     20
   |     20
   |      0
```

(iii) $3\frac{3}{8} = \frac{27}{8} = 3.375$

```
           3.375
    |-----------
 8  |  27
    |  24
    |-------
    |    30
    |    24
    |-------
    |      60
    |      56
    |-------
    |        40
    |        40
    |         0
```

(iv) $\frac{5}{6} = 0.833... = 0.8\overline{333}$

$0.8\overline{333}$

6 | 50
30
20
18
20
18
20
18
2

(v) $\frac{1}{5} = 0.2$

0.2

5 | 10
10
0

(vi) $\frac{1}{7} = 0.\overline{142857}$

$0.\overline{142857}$

7 | 10
7
30
28
20
14
60
56
40
35
50
49
1

(vii) $\frac{2}{13} = 0.\overline{153846}$

$0.\overline{153846}$

13 | 20
13
70
65
50
39
110
104
60
52
80
78
2

(viii) $\frac{11}{17} = 0.\overline{6470588235294117}$

$$
\begin{array}{r}
0.6470588235294117 \\
17\,\overline{)\,110} \\
\underline{102} \\
80 \\
\underline{68} \\
120 \\
\underline{119} \\
100 \\
\underline{85} \\
150 \\
\underline{136} \\
140 \\
\underline{136} \\
40 \\
\underline{34} \\
60 \\
\underline{51} \\
90 \\
\underline{85} \\
50 \\
\underline{34} \\
160 \\
\underline{153} \\
70 \\
\underline{68} \\
20 \\
\underline{17} \\
30 \\
\underline{17} \\
130 \\
\underline{119} \\
11
\end{array}
$$

Question-2

If a is a positive rational number and n is a positive integer greater than 1, prove that a^n is a rational number.

Solution:

We know that product of two rational number is always a rational number.

Hence if a is a rational number then

$a^2 = a \times a$ is a rational number,

$a^3 = a^2 \times a$ is a rational number,

$a^4 = a^3 \times a$ is a rational number,

...

...

$\therefore a^n = a^{n-1} \times a$ is a rational number.

Question-3

Find three rational numbers lying between 0 and 0.1. Find twenty rational numbers between 0 and 0.1. Give a method to determine any number of rational numbers between 0 and 0.1.

Solution:

The three rational numbers lying between 0 and 0.1 are 0.01, 0.05, 0.09.

The twenty rational numbers between 0 and 0.1 are 0.001, 0.002, 0.003, 0.004, ... 0.011, 0.012, ... 0.099.

To determine any number of rational numbers between 0 and 0.1 insert 0 after the decimal.

Question-4

Complete the following:

(i) Every point on the number line corresponds to a ______________ number which may be either ______________ or ______________.

(ii) The decimal form of an irrational number is neither ______________ or ______________.

(iii) The decimal representation of the rational number $\frac{8}{27}$ is ______________.

(iv) 0 is ______________ number. [Hint: a rational /an irrational]

Solution:

(i) Every point on the number line corresponds to a real number which may be either rational or irrational.

(ii) The decimal form of an irrational number is neither recurring or terminating.

(iii) The decimal representation of the rational number $\frac{8}{27}$ is $0.\overline{296}$

(iv) 0 is a rational number.

Question-5

Which of the following rational numbers have the terminating decimal representation?
(i) 3/5 (ii)7/20 (iii)2/13
(iv) 27/40 (v) 133/125 (vi) 23/7

[Making use of the result that a rational number p/q where p and q have no common factor(s) will have a terminating representation if and only if the prime factors of q are 2's or 5's or both.]

Solution:

(i) The prime factor of 5 is 5. Hence 3/5 has a terminating decimal representation.

(ii) $20 = 4 \times 5 = 2^2 \times 5$.
The prime factors of 20 are both 2's and 5's. Hence 7/20 has a terminating decimal.

(iii) The prime factor of 13 is 13. Hence 2/13 has non-terminating decimal.

(iv) $40 = 2^3 \times 5$.
The prime factors of 40 are both 2's and 5's. Hence 27/40 has a terminating decimal.

(v) $125 = 5^3$
The prime factor of 125 is 5's. Hence 13/125 has a terminating decimal.

(vi) The prime factor of 7 is 7. Hence 23/7 has a non-terminating decimal representation.

Question-6

You have seen that $\sqrt{2}$ is not a rational number. Show that $2 + \sqrt{2}$ is not a rational number.

Solution:

Let $2 + \sqrt{2}$ be a rational number say r.
Then $2 + \sqrt{2} = r$
$\sqrt{2} = r - 2$
But, $\sqrt{2}$ is an irrational number.
Therefore, $r - 2$ is also an irrational number.
=> r is an irrational number.
Hence our assumption r is a rational number is wrong.
Hence, $2 + \sqrt{2}$ is not a rational number.

Question-7

Prove that $3\sqrt{3}$ is not a rational number.

Solution:

Let $3\sqrt{3}$ be a rational number say r.

Then $3\sqrt{3}$ = r

$\sqrt{3}$ = (1/3)r

(1/3) r is a rational number because product of two rational number is a rational number.

=> $\sqrt{3}$ is a rational number but $\sqrt{3}$ is not a rational number.

Therefore our assumption that $3\sqrt{3}$ is a rational number is wrong.

Question-8

Show that $\sqrt[3]{6}$ is not a rational number.

Solution:

Let $\sqrt[3]{6}$ be a rational number, say $\frac{p}{q}$ where q ≠ 0.

Then $\sqrt[3]{6} = \frac{p}{q}$

Since $1^3 = 1$, and $2^3 = 8$, it follows that $1 < \frac{p}{q} < 2$

Then q > 1 because if q = 1 then $\frac{p}{q}$ will be an integer, and there is no integer between 1 and 2.

Now, $6 = \left(\frac{p}{q}\right)^3$

$6 = \frac{p^3}{q^3}$

$6q^2 = \frac{p^3}{q}$

q being an integer, $6q^2$ is an integer, and since q > 1 and q does not have a common factor with p and consequently with p^3.

So, $\frac{p^3}{q}$ is a fraction different from an integer.

Thus $6q^2 \neq \frac{p^3}{q}$.

This contradiction proves the result.

Question-9

Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers.

(i) $\sqrt{4}$

(ii) $3\sqrt{18}$

(iii) $\sqrt{1.44}$

(iv) $\sqrt{\frac{9}{27}}$

(v) $-\sqrt{.64}$

(vi) $\sqrt{100}$

Solution:

(i) $\sqrt{4} = 2$ is rational.

(ii) $3\sqrt{18} = 3\sqrt{9 \times 2} = 3 \times 3\sqrt{2} = 9\sqrt{2}$ is irrational.

(iii) $\sqrt{1.44} = \sqrt{\frac{144}{100}} = \frac{12}{10} = 1.2$ is rational.

(iv) $\sqrt{\frac{9}{27}} = \sqrt{\frac{9}{9 \times 3}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$ is irrational.

(v) $-\sqrt{0.64} = -0.8$ is rational.

(vi) $\sqrt{100} = 10$ is rational.

Question-10

In the following equations, find which of the variables x, y, z etc. represent rational numbers and which represent irrational numbers:

(i) $x^2 = 5$
(ii) $y^2 = 9$
(iii) $z^2 = 0.04$
(iv) $u^2 = 17/4$
(v) $v^2 = 3$
(vi) $w^3 = 27$
(vii) $t^2 = 0.4$

Solution:

(i) $x^2 = 5$

$\therefore$ $x = \sqrt{5}$ is irrational.

(ii) $y^2 = 9$

$\therefore$ $y = 3$ is rational.

(iii) $z^2 = 0.04$

$\therefore$ $z = 0.2$ is rational.

(iv) $u^2 = \frac{17}{4}$

$\therefore$ $u = \sqrt{\frac{17}{4}}$

$= \frac{\sqrt{17}}{\sqrt{4}} = \frac{\sqrt{17}}{2}$ is irrational.

(v) $v^2 = 3$

$\therefore$ $v = \sqrt{3}$ is irrational.

(vi) $w^3 = 27$

$w = \sqrt[3]{27} = 3$ is rational.

(vii) $t^2 = 0.4$ $\therefore$ $t = \sqrt{0.4} = \sqrt{\frac{4}{10}} = \frac{2}{\sqrt{10}}$ is irrational.

Question-11

Give an example to show that the product of a rational number and an irrational number may be a rational number.

Solution:

A rational number 0 multiplied by an irrational number gives the rational number 0.

Question-12

State with reason which of the following are surds and which are not.

(i) $\sqrt{5} \times$

(ii) $\sqrt{8} \times$

(iii) $\sqrt{27} \times \sqrt{3}$

(iv) $\sqrt{16} \times \sqrt{4}$

(v) $5\sqrt{8} \times 2\sqrt{6}$

(vi) $\sqrt{125} \times \sqrt{5}$

(vii) $\sqrt{100} \times \sqrt{2}$

(viii) $6\sqrt{2} \times 9\sqrt{3}$

(ix) $\sqrt{120} \times \sqrt{45}$

(x) $\sqrt{15} \times \sqrt{6}$.

Solution:

(i) $\sqrt{5} \times \sqrt{10} = \sqrt{5} \times \sqrt{5 \times 2} = \sqrt{5} \times \sqrt{5} \times \sqrt{2} = 5\sqrt{2}$ is a surd.

(ii) $\sqrt{8} \times \sqrt{6} = \sqrt{4 \times 2} \times \sqrt{3 \times 2} = 2\times \sqrt{2} \times \sqrt{2} \times \sqrt{3} = 4\sqrt{3}$ is a surd.

(iii) $\sqrt{27} \times \sqrt{3} = \sqrt{9 \times 3} \times \sqrt{3} = 3\sqrt{3} \times \sqrt{3} = 9$ is not a surd.

(iv) $\sqrt{16} \times \sqrt{4} = 4 \times 2 = 8$ is not a surd.

(v) $5\sqrt{8} \times 2\sqrt{6} = 5\sqrt{4 \times 2} \times 2\sqrt{3 \times 2} = 5 \times 2\sqrt{2} \times 2\sqrt{2} \times \sqrt{3} = 5 \times 2 \times 2 \times 2 \times \sqrt{3} = 40\sqrt{3}$ is a surd.

(vi) $\sqrt{125} \times \sqrt{5} = \sqrt{25 \times 5} \times \sqrt{5} = 5\sqrt{5} \times \sqrt{5} = 5 \times 5 = 25$ is not a surd.

(vii) $\sqrt{100} \times \sqrt{2} = 10\sqrt{2}$ is a surd.

(viii) $6\sqrt{2} \times 9\sqrt{3} = 54\sqrt{6}$ is a surd.

(ix) $\sqrt{120} \times \sqrt{45} = \sqrt{4 \times 30} \times \sqrt{9 \times 5} = 2\sqrt{6 \times 5} \times 3\sqrt{5} = 2 \times \sqrt{6} \times \sqrt{5} \times 3 \times \sqrt{5} = 30\sqrt{6}$ is a surd.

(x) $\sqrt{15} \times \sqrt{6} = \sqrt{5 \times 3} \times \sqrt{2 \times 3} = \sqrt{5} \times \sqrt{3} \times \sqrt{2} \times \sqrt{3} = 3 \times \sqrt{10}$ is a surd.

Question-13

Give two examples to show that the product of two irrational numbers may be a rational number.

Solution:

Take a = $(2+\sqrt{3})$ and b =$(2-\sqrt{3})$; a and b are irrational numbers, but their product

$(2+\sqrt{3})(2-\sqrt{3}) = 4 - 3 = 1$ is a rational number.

Take c = $\sqrt{3}$ and d = $-\sqrt{3}$; c and d are irrational numbers, but their product = -3,

is a rational number.

Question-14

Find the value of $\sqrt{5}$ correct to two places of $\sqrt{5}$ decimal.

Solution:

We know that $2^2 = 4 < 5 < 9 = 3^2$
Taking positive square roots we get
$2 < \sqrt{5} < 3$.

Next, $(2.2)^2 = 4.84 < 5 < 5.29 = (2.3)^2$
Taking positive square roots, we have
$2.2 < \sqrt{5} < 2.3$

Again, $(2.23)^2 = 4.9729 < 5 < 5.0176 = (2.24)^2$
Taking positive square roots, we obtain
$2.23 < \sqrt{5} < 2.24$
Hence the required approximation is 2.24 as $(2.24)^2$ is nearest to 5 than $(2.23)^2$.

Question-15

Prove that $\sqrt{3} - \sqrt{2}$ is irrational.

Solution:

Let $\sqrt{3} - \sqrt{2}$ be a rational number, say r
Then $\sqrt{3} - \sqrt{2} = r$
On squaring both sides we have

$$(\sqrt{3} - \sqrt{2})^2 = r^2$$
$$3 - 2\sqrt{6} + 2 = r^2$$
$$5 - 2\sqrt{6} = r^2$$
$$-2\sqrt{6} = r^2 - 5$$
$$\sqrt{6} = -(r^2 - 5)/2$$

Now $-(r^2 - 5)/2$ is a rational number and $\sqrt{6}$ is an irrational number.
Since a rational number cannot be equal to an irrational number. Our assumption that
$\sqrt{3} - \sqrt{2}$ is rational is wrong.

Question-16

Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

Solution:

Let $\sqrt{3}$ + $\sqrt{5}$ be a rational number, say r

Then $\sqrt{3}$ + $\sqrt{5}$ = r

On squaring both sides,

$(\sqrt{3} + \sqrt{5})^2 = r^2$

$3 + 2\sqrt{15} + 5 = r^2$

$8 + 2\sqrt{15} = r^2$

$2\sqrt{15} = r^2 - 8$

$\sqrt{15} = (r^2 - 8)/2$

Now $(r^2 - 8)/2$ is a rational number and $\sqrt{15}$ is an irrational number.

Since a rational number cannot be equal to an irrational number. Our assumption that

$\sqrt{3}$ + $\sqrt{5}$ is rational is wrong.

Question-17

Examine, whether the following numbers are rational or irrational:

(i) $(\sqrt{2} + 2)^2$

(ii) $(2 - \sqrt{2}) \times (2 + \sqrt{2})$

(iii) $(\sqrt{2} + \sqrt{3})^2$

(iv) $\frac{6}{3\sqrt{2}}$

Solution:

(i) $(\sqrt{2} + 2)^2 = (\sqrt{2})^2 + 2\sqrt{2}x2 + (2)^2 = 2 + 4\sqrt{2} + 4 = 6 + 4\sqrt{2}$.

\ It is an irrational number.

(ii) $(2 - \sqrt{2}) \times (2 + \sqrt{2}) = (2)^2 - (\sqrt{2})^2 = 4 - 2 = 2$.

\ It is a rational number.

(iii) $(\sqrt{2} + \sqrt{3})^2 = (\sqrt{2})^2 + 2\sqrt{2} \times \sqrt{3} + (\sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$

∴ It is an irrational number.

(iv) $\frac{6}{3\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

\ It is an irrational number.

Question-18

Prove that

(a) $2 + \sqrt{3}$ is not a rational number and

(b) $\sqrt[3]{7}$ is not a rational number.

Solution:

(a) If possible, let $2 + \sqrt{3} = a$, where a is rational.

Then, $(2 + \sqrt{3})^2 = a^2$

$7 + 4\sqrt{3} = a^2$

$\sqrt{3} = \frac{a^2 - 7}{4}$ --------(i)

Now, a is rational $\Rightarrow \frac{a^2 - 7}{4}$ is rational.

$\sqrt{3}$ is rational [from (i)]

This is a contradiction.

Hence, $2 + \sqrt{3}$ is not a rational number.

(b) If possible, let $\sqrt[3]{7} = \frac{p}{q}$, where p and q are integers, having no common factors and $q \neq 0$.

Then, $(\sqrt[3]{7})^3 = (\frac{p}{q})^3$

$\Rightarrow 7q^3 = p^3$ ------(i)

$\Rightarrow p^3$ is a multiple of 7

$\Rightarrow$ p is multiple of 7.

Let p = 7m, where m is an integer.

Then, $p^3 = 343\ m^3$ ------(ii)

$\Rightarrow 7q^3 = 343\ m^3$ [from (i) and (ii)]

$\Rightarrow q^3 = 49\ m^3 \Rightarrow q^3$ is a multiple of 7.

$\Rightarrow$ q is a multiple of 7.

Thus, p and q are both multiples of 7, or 7 is a factor of p and q.

This contradicts our assumption that p and q have no common factors.

Hence $\sqrt[3]{7}$ is not a rational number.

Question-19

Examine whether the following numbers are rational or irrational:

(i) $(3 + \sqrt{2})^2$

(ii) $(3 - \sqrt{3})(3 + \sqrt{3})$

(iii) $\frac{6}{2\sqrt{3}}$

Solution:

(i) $(3 + \sqrt{2})^2 = 9 + 2 + 6\sqrt{2} = 11 + 6\sqrt{2}$, which is irrational.

(ii) $(3 - \sqrt{3})(3 + \sqrt{3}) = (3)^2 - (\sqrt{3})^2 = 9 - 3 = 6$, which is rational.

(iii) $\frac{6}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6 \times \sqrt{3}}{6} = \sqrt{3}$, which is irrational.

Question-20

Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Solution:

Irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$ is $\sqrt{\sqrt{2} \times \sqrt{3}}$,i.e $\sqrt{\sqrt{6}} = 6^{(1/4)}$

Irrational numbers lying between $\sqrt{2}$ and $6^{(1/4)}$ is $\sqrt{\sqrt{2} \times 6^{\frac{1}{4}}} = 2^{(1/4)} \times 6^{(1/8)}$

Hence two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$ are $6^{(1/4)}$ and $2^{(1/4)} \times 6^{(1/8)}$.

Question-21

Express $\frac{7}{64}$ as a decimal fraction.

Solution:

```
     0.109375
64 | 7.000
     64
     ---
     600
     576
     ---
     240
     192
     ---
     480
     448
     ---
     320
     320
     ---
     0
```

Therefore $\frac{7}{64} = 0.109375$

Question-22

Express $\frac{12}{125}$ as a decimal fraction.

Solution:

```
      0.096
125)12.000
    1125
     750
     750
       0
```

Therefore $\frac{12}{125} = 0.096.$

Question-23

Express $\frac{451}{13}$ as a decimal fraction.

Solution:

```
      34.692307
13 | 451·0000000
     39
      61
      52
       90
       78
       120
        117
          30
          26
           40
           39
            100
             91
              9
```

Therefore **34.692307**