Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. Cards marked with numbers 1, 2, 3, ..., 25 are placed in a box and mixed thoroughly and one card is drawn at random from the box. The probability that the number on the card is a multiple of 3 and 5 is **[1]**

a) $\frac{12}{25}$ b) $\frac{4}{25}$
c) $\frac{1}{25}$ d) $\frac{8}{25}$

2. The roots of a quadratic equation $x^2 - 4px + 4p^2 - q^2 = 0$ are **[1]**

a) 2p + q, 2p - q b) p + 2q, p - 2q
c) 2p + q, 2p + q d) 2p - q, 2p - q

3. An icecream cone has hemispherical top. If the height of the cone is 9 cm and base radius is 2.5 cm, then the volume of icecream is **[1]**

a) 91.67 cm^3 b) 96.67 cm^3
c) 90.67 cm^3 d) 91.76 cm^3

4. The sum of two numbers is 17 and the sum of their reciprocals is $\frac{17}{62}$. The quadratic representation of the above situation is **[1]**

a) $\frac{1}{x} + \frac{1}{x+17} = \frac{17}{62}$ b) $\frac{1}{x(17-x)} = \frac{17}{62}$
c) $\frac{1}{x} + \frac{1}{17-x} = \frac{17}{62}$ d) $\frac{1}{x} - \frac{1}{17-x} = \frac{17}{62}$

5. The 2nd term of an AP is 13 and its 5th term is 25. What is its 17th term? **[1]**

a) 69 b) 77

c) 81 d) 73

6. The distance between the points (a, a) and $(-\sqrt{3}a, \sqrt{3}a)$ is **[1]**

a) $2\sqrt{2}$ units b) $3\sqrt{2}a$ units

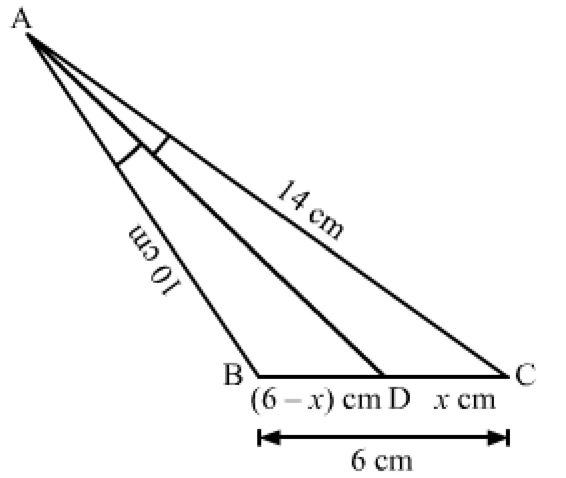
c) 2 units d) $2a\sqrt{2}$ units

7. If α and β are zeros of $x^2 + 5x + 8$, then the value of $(\alpha + \beta)$ is **[1]**

a) -8 b) 8

c) 5 d) -5

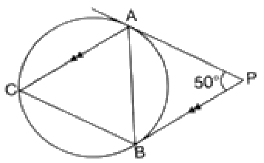
8. In a $\triangle$ABC, it is given that AD is the internal bisector of $\angle$A. If AB = 10 cm, AC = 14 cm and BC = 6 cm, the CD = ? **[1]**



a) 3.5 cm b) 7 cm

c) 4.8 cm d) 10.5 cm

9. In the given figure, PA and PB are tangents to a circle from an external point P. If $\angle APB = 50^\circ$ and AC || PB, then the measures of angles of triangle ABC are **[1]**



a) 65°, 50°, 65° b) 50°, 55°, 75°

c) 80°, 60°, 40 d) 50°, 50°, 80°

10. How many tangents can be drawn to a circle from a point on it? **[1]**

a) Two b) Zero

c) Infinite d) One

11. If secθ + tanθ = x, then tanθ = **[1]**

a) $\frac{x^2+1}{x}$ b) $\frac{x^2+1}{2x}$

c) $\frac{x^2-1}{x}$ d) $\frac{x^2-1}{2x}$

12. The prime factorisation of the number 5488 is **[1]**

a) $2^3 \times 7^3$ b) $2^4 \times 7^3$

c) d)

$2^4 \times 7^4$ | $2^3 \times 7^4$

13. If the height of a tower and the distance of the point of observation from its foot, both, are increased by 10%, then the angle of elevation of its top **[1]**

a) decreases
b) Falls
c) remains unchanged
d) increases

14. Three horses are tethered with 7-meter-long ropes at the three corners of a triangular field having sides 20 m, 34 m, and 42 m. The area of the plot which can be grazed by the horses is **[1]**

a) $77\ m^2$
b) $80\ m^2$
c) $100\ m^2$
d) $30\ m^2$

15. If the perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm, then the area of the sector is ________. **[1]**

a) $15.5\ cm^2$
b) $15.6\ cm^2$
c) $15.9\ cm^2$
d) $15.1\ cm^2$

16. The probability of guessing the correct answer to a certain test questions is $\frac{x}{12}$. If the probability of not guessing the correct answer to this question is $\frac{2}{3}$, then x = **[1]**

a) 6
b) 4
c) 2
d) 3

17. If a digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, then the probability that it is odd and is a multiple of 3 is **[1]**

a) $\frac{1}{9}$
b) $\frac{2}{9}$
c) $\frac{2}{3}$
d) $\frac{1}{3}$

18. If the arithmetic mean of x, x + 3, x + 6, x + 9 and x + 12 is 10, then x = **[1]**

a) 2
b) 1
c) 6
d) 4

19. **Assertion (A):** In a solid hemisphere of radius 10 cm, a right cone of same radius is removed out. The volume of the remaining solid is $523.33\ cm^3$ [Take π = 3.14 and $\sqrt{2}$ = 1.4] **[1]**

Reason (R): Expression used here to calculate volume of remaining solid = Volume of hemisphere - Volume of cone

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** Sum of natural number from 1 to 100 is 5050. **[1]**

Reason (R): Sum of n natural number is $\frac{n(n+1)}{2}$.

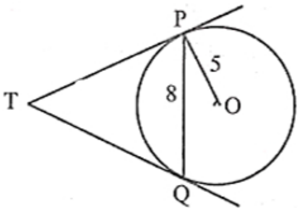
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Prove that $\frac{1}{\sqrt{2}}$ is irrational. **[2]**

22. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium. **[2]**

23. In Fig., PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP. **[2]**

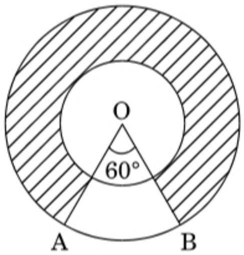


24. If $\sin\theta + \sin^2\theta = 1$, find the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$ **[2]**

OR

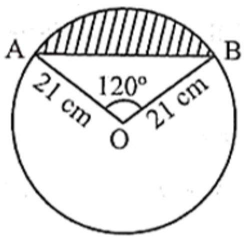
Prove the trigonometric identity: $\frac{\tan\theta}{\left(1+\tan^2\theta\right)^2} + \frac{\cot\theta}{\left(1+\cot^2\theta\right)^2} = \sin\theta\cos\theta$

25. In Figure, two concentric circles with centre O, have radii 21 cm and 42 cm. If $\angle AOB = 60^o$, find the area of the shaded region. **[2]**



OR

Find the area of the segment shown in Fig., if radius of the circle is 21 cm and $\angle AOB = 120^o$ (Use $\pi = \frac{22}{7}$)



Section C

26. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point? **[3]**

27. Point A is on x-axis, point B is on y-axis and the point P lies on line segment AB, such that P (4, - 5) and AP : PB = 5 : 3. Find the coordinates of point A and B. **[3]**

28. To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool. **[3]**

OR

A 2-digit number is four times the sum of its digits and twice the product of its digits. Find the number.

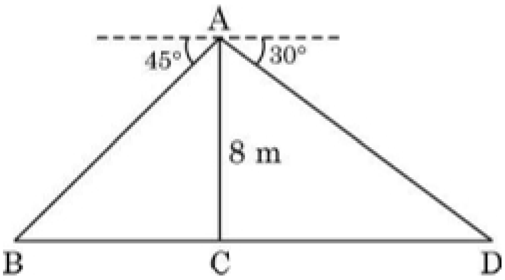
29. A point P is at a distance of 29 cm from the centre of a circle of radius 20 cm. Find the length of the tangent drawn from P to the circle. **[3]**

OR

Find the length of a tangent drawn to a circle with radius 5 cm, from a point 13 cm from the centre of the circle.

30. Prove the trigonometric identity: $\frac{\cot^2\theta(\sec\theta-1)}{(1+\sin\theta)}+\frac{\sec^2\theta(\sin\theta-1)}{(1+\sec\theta)}=0$ **[3]**

31. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°. If the bridge is at a height of 8 m from the banks, then find the width of the river. **[3]**



Section D

32. A person invested some amount at the rate of 12% simple interest and the remaining at 10%. He received yearly interest of ₹ 130 but if he had interchanged the amount invested, he would have received ₹ 4 more as the interest. How much money did he invest at different rates? **[5]**

OR

Sangeeta went to a book-seller's shop and purchased 2 textbook of IX Mathematics and 3 textbook of X mathematics for Rs.250. Her friend Meenu also bought 4 textbooks of IX Mathematics and 6 textbooks of X maths of same kind for Rs.500. Represents this situation algebraically and graphically.

33. Show that the points A(3,1), B(0, -2), C(1, 1) and D(4, 4) are the vertices of a parallelogram ABCD. **[5]**

34. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid. **[5]**

OR

A building is in the form of a cylinder surmounted by a hemispherical dome. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}\mathrm{m}^3$ of air.

35. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n^{th} term and S_n the sum of first n terms, find. n and S_n, if a = 5, d = 3 and a_n = 50. **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.

i. In the standard form of quadratic polynomial, $ax^2 + bx + c$, what are a, b and c? (1)

ii. If the roots of the quadratic polynomial are equal, what is the discriminant D? (1)

iii. If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial are $2x^2 - x + 8k$, then find the value of k? (2)

OR

What is the relation between zeros and coefficient for a quadratic polynomial? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Heart Rate: The heart rate is one of the 'vital signs' of health in the human body. It measures the number of times per minute that the heart contracts or beats. While a normal heart rate does not guarantee that a person is free of health problems, it is a useful benchmark for identifying a range of health issues.

Thirty women were examined by doctors of AIIMS and the number of heart beats per minute were recorded and summarized as follows:

Number of heart beats per minute	Number of Women
65 - 68	2
68 - 71	4
71 - 74	3
74 - 77	8
77 - 80	7
80 - 83	4
83 - 86	2

Based on the above information, answer the following questions:

i. How many women are having heart beat in the range 68 - 77?

ii. What is the median class of heart beats per minute for these women?

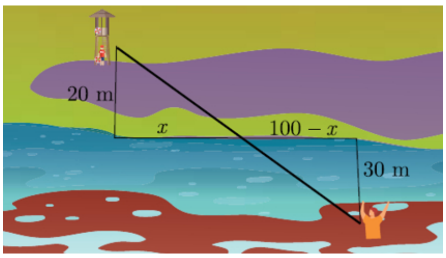
iii. a. Find the modal value of heart beats per minute for these women.

OR

b. Find the median value of heart beats per minute for these women.

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Swimmer in Distress: A lifeguard located 20 metre from the water spots a swimmer in distress. The swimmer is 30 metre from shore and 100 metre east of the lifeguard. Suppose the lifeguard runs and then swims to the swimmer in a direct line, as shown in the figure.



i. How far east from his original position will he enter the water? (Hint: Find the value of x in the sketch.) (1)

ii. Which similarity criterion of triangle is used? (1)

iii. What is the distance of swimmer from the shore? (2)

OR

What is the length of AD? (2)

Solution

Section A

1.

(c) $\frac{1}{25}$

Explanation: Multiples of 3 = 3, 6, 9, 12, 15, 18, 21, 24

Multiples of 5 = 5, 10, 15, 20, 25

Number of possible outcomes (multiple of 3 and 5) = {15} = 1

Number of Total outcomes = 25

$\therefore$ Required Probability = $\frac{1}{25}$

2. **(a)** 2p + q, 2p - q

Explanation: Given: $x^2 - 4px + 4p^2 - q^2 = 0$

$\Rightarrow (x - 2p)^2 - q^2 = 0$

Using $a^2 - b^2 = (a + b)(a - b)$,

$\Rightarrow (x - 2p + q)(x - 2p - q) = 0$

$\Rightarrow x - 2p + q = 0$ and $x - 2p - q = 0$

$\Rightarrow x = 2p - q$ and $x = 2p + q$

3. **(a)** 91.67 cm^3

Explanation: Height of ice-cream cone is 9 cm and radius of the hemispherical top is 2.5 cm.

Now, Volume of ice-cream cone = Volume of cone + volume of Hemispherical top

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times 2.5 \times 2.5(9 + 5)$

$= \frac{1}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 14$

= 91.67 cm^3

4.

(c) $\frac{1}{x} + \frac{1}{17-x} = \frac{17}{62}$

Explanation: Let one number be x, As the sum of the numbers is 17, then the other number will be (17 - x). Their reciprocals will be $\frac{1}{x}$ and $\frac{1}{17-x}$.

$\therefore$ According to question, $\frac{1}{x} + \frac{1}{17-x} = \frac{17}{62}$

5.

(d) 73

Explanation: a + d = 13 ... (i) and

a + 4d = 25 ....(ii)

From (i) and (ii), we get a = 9 and d = 4

$\Rightarrow T_{17} = (a + 16d) = (9 + 16 \times 4) = 73$

6.

(d) $2a\sqrt{2}$ units

Explanation: Let the points be A(a, a) and B$(-\sqrt{3}a, \sqrt{3}a)$

$\therefore$AB = $\sqrt{\left(-\sqrt{3}a - a\right)^2 + \left(\sqrt{3}a - a\right)^2}$

$= \sqrt{3a^2 + a^2 + 2\sqrt{3}aa + 3a^2 + a^2 - 2\sqrt{3}aa}$

$= \sqrt{6a^2 + 2a^2}$

$= \sqrt{8a^2}$

$= 2a\sqrt{2}$ units

7.

(d) -5

Explanation: x^2+5x+8

$\alpha+\beta=\frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2}$

$=\frac{-5}{1}$

= -5

8. **(a)** 3.5 cm

Explanation: By using angle bisector theore in $\triangle$ABC, we have

$\frac{\mathrm{AB}}{\mathrm{AC}}=\frac{\mathrm{BD}}{\mathrm{DC}}$

$\Rightarrow \frac{10}{14}=\frac{6-x}{x}$

$\Rightarrow 10x=84-14x$

$\Rightarrow 24x=84$

$\Rightarrow x=3.5$

Hence, the correct answer is 3.5.

9. **(a)** 65°, 50°, 65°

Explanation: Since PA = PB {Tangents from an external point to a circle]

$\therefore \angle \mathrm{PAB}=\angle \mathrm{PBA}$

Let $\angle \mathrm{PAB}=\angle \mathrm{PBA}=x$

Now, in triangle APB, x + x + 50° = 180°

$\Rightarrow$x = 65°

Since AC $\|$PB and AB is intersecting.

$\therefore \angle \mathrm{PBA}=\angle \mathrm{BAC}=65^{\circ}$ [Alternate angles]

And $\angle \mathrm{PAB}=\angle \mathrm{CBA}=65^{\circ}$ [Alternate angles]

$\therefore \angle \mathrm{ACB}=180^{\circ}-\left(65^{\circ}+65^{\circ}\right)=50^{\circ}$ [Angle sum property of a triangle]

$\therefore \angle \mathrm{BAC}=65^{\circ}, \angle \mathrm{ABC}=65^{\circ}, \angle \mathrm{ACB}=50^{\circ}$

10. **(a)** Two

Explanation: Two

11.

(d) $\frac{x^2-1}{2x}$

Explanation: Given, secθ + tanθ = x ...(i)

We know that

$\sec^2\theta-\tan^2\theta=1$

$\Rightarrow(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=1$

$\Rightarrow x(\sec\theta-\tan\theta)=1$

$\Rightarrow \sec\theta-\tan\theta=\frac{1}{x}$ ...(ii)

Subtracting (ii) from (i)

$2\tan\theta=x-\frac{1}{x}=\frac{x^2-1}{x}$

$\tan\theta=\frac{x^2-1}{2x}$

12.

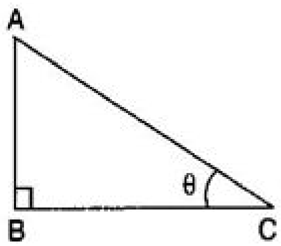
(b) $2^4\times 7^3$

Explanation: $2^4\times 7^3$

13.

(c) remains unchanged

Explanation:



Let height of the tower be $h$meters and distance of the point of observation from its foot be x meters and angle of elevation be $\theta \therefore \tan \theta=\frac{h}{x}$ ....(i)

Now, new height = h + 10% of h = $h+\frac{10}{100} h$ = $\frac{11 h}{10}$ And new distance = x + 10% of x = $x+\frac{10}{100} x$ = $\frac{11 x}{10}$ $\therefore$

$\tan \theta=\frac{\frac{11 h}{10}}{\frac{11 x}{10}}=\frac{h}{x}$ ....(ii)

From eq. (i) and (ii), it is clear that the angle of elevation is same i.e., angle of elevation remains unchanged.

14. **(a)** 77 m^2

Explanation: 77 m^2

15.

(b) 15.6 cm^2

Explanation: Perimeter of a sector of circle = $\frac{\theta}{360^{\circ}} \times 2 \pi r$ + 2r

$\Rightarrow\left(\frac{\theta}{360^{\circ}} \times 2 \pi \times 5.2\right)$ + (2 × 5.2) = 16.4

$\Rightarrow \frac{\theta}{360^{\circ}} \pi=\frac{16.4-10.4}{10.4}=\frac{6}{10.4}$

Area of sector of circle = $\frac{\theta}{360^{\circ}} \times \pi r^2$

= $\frac{6}{10.4} \times(5.2)^2$ = 15.6 cm^2

16.

(b) 4

Explanation: Probability of guessing the correct answer

$=\frac{x}{12}$

and probability of not guessing the correct

answer $=\frac{2}{3}$

$\frac{x}{12}+\frac{2}{3}=1 \because(A+\bar{A}=1)$

$\Rightarrow \frac{x}{12}=1-\frac{2}{3}=\frac{1}{3} \Rightarrow x=\frac{12}{3}=4$

$\therefore$ x = 4

17.

(b) $\frac{2}{9}$

Explanation: Total numbers of digits for 1 to 9(n) = 9

Number divisible by 3(m) = 3, 6, 9

Odd numbers out of 3, 6, 9 = 3, 9

$\therefore$ Probability $=\frac{m}{n}=\frac{2}{9}$

18.

(d) 4

Explanation: Mean of x, x + 3, x + 6, x + 9, x + 12 = 10

$\Rightarrow \frac{x+x+3+x+6+x+9+x+12}{5}=10$

$\Rightarrow \frac{5 x+30}{5}=10$

$\Rightarrow$ x + 6 = 10

$\Rightarrow$ x = 10 - 6 = 4

19.

(d) A is false but R is true.

Explanation: A is false but R is true.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. Let us assume, to the contrary, that is $\frac{1}{\sqrt{2}}$ rational.

So, we can find coprime integers a and b $(\neq 0)$ such that

$\frac{1}{\sqrt{2}}=\frac{a}{b} \Rightarrow \sqrt{2}=\frac{b}{a}$

Since, a and b are integers, $\frac{b}{a}$ is rational, and so is $\sqrt{2}$ rational.

But this contradicts the fact that is $\sqrt{2}$ irrational.

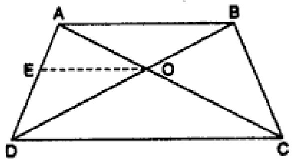
So, we conclude that is $\frac{1}{\sqrt{2}}$ irrational.

22. Given: The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$

To prove: ABCD is trapezium.

Construction: Through O draw a line OE||BA intersecting AD at E.

Proof: In $\triangle DBA \because OE||BA$



$\therefore \frac{DO}{BO} = \frac{DE}{AE} \Rightarrow \frac{CO}{AO} = \frac{DE}{AE}$

$\because \frac{AO}{BO} = \frac{CO}{DO} [Given]$

$\Rightarrow \frac{DO}{BO} = \frac{CO}{AO} \Rightarrow \frac{AO}{CO} = \frac{AE}{DE}$ ....[Taking reciprocals]

$\therefore$In $\triangle ADC$

OE $\parallel$ CD ....[By converse basic proportionality theorem]

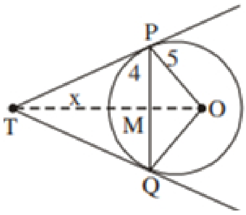
But OE $\parallel$ BA

BA $\parallel$ CD........[By construction]

The quadrilateral ABCD is a Trapezium.

23. Join OT and OQ.

TP = TQ



$\therefore$ TM $\perp$ PQ and bisects PQ

Hence PM = 4 cm

Therefore OM = $\sqrt{25-16}$ = $\sqrt{9}$ = 3cm.

Let TM = x

From $\triangle$PMT, $PT^2 = x^2 + 16$

From $\triangle$POT, $PT^2 = (x + 3)^2 - 25$

Hence $x^2 + 16 = x^2 + 9 + 6x - 25$

$\Rightarrow 6x = 32 \Rightarrow x = \frac{16}{3}$

Hence $PT^2 = \frac{256}{9} + 16 = \frac{400}{9}$

$\therefore PT = \frac{20}{3}$ cm.

24. We have,

$\sin\theta + \sin^2\theta = 1 \Rightarrow \sin\theta = 1 - \sin^2\theta \Rightarrow \sin\theta = \cos^2\theta$

$\therefore \cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$

$= (\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta) + 2(\cos^4\theta + \cos^2\theta - 1)$

$= (\cos^4\theta + \cos^2\theta)^3 + 2(\cos^4\theta + \cos^2\theta - 1)$

$= (\sin^2\theta + \cos^2\theta)^3 + 2(\sin^2\theta + \cos^2\theta - 1)$ $[\because \cos^2\theta = \sin\theta \because \cos^4\theta = \sin^2\theta]$

$= 1 + 2(1 - 1) = 1$

OR

$\frac{\tan\theta}{(1+\tan^2\theta)^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$

$= \frac{\tan\theta}{(\sec^2\theta)^2} + \frac{\cot\theta}{(\cos ec^2\theta)^2}$

$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sec^4\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{1}{cosec^4\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \cos^4\theta + \frac{\cos\theta}{\sin\theta} \times \sin^4\theta$

$= \sin\theta\cos^3\theta + \cos\theta\sin^3\theta$

$= \sin\theta\cos\theta(\cos^2\theta + \sin^2\theta)$

$= \sin\theta\cos\theta$ = RHS

Hence proved.

25. Area of shaded region

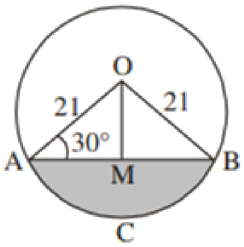
$= \left[\pi(42)^2 - \pi(21)^2\right] \frac{300^\circ}{360^\circ}$

$= \frac{22}{7} \times 63 \times 21 \times \frac{5}{6}$.

= 3465 cm^2

OR

Draw OM ⊥ AB



$\angle OAB = \angle OBA = 30^o$

$\sin 30^o = \frac{1}{2} = \frac{OM}{21} \Rightarrow OM = \frac{21}{2}$

$\cos 30^o = \frac{\sqrt{3}}{2} = \frac{AM}{21} \Rightarrow AM = \frac{21}{2}\sqrt{3}$

Area of $\triangle OAB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$

$= \frac{441}{4}\sqrt{3}\ cm^2$

∴ Area of shaded region = Area (sector OACB) - Area ($\triangle$OAB)

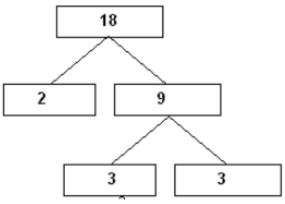
$= \frac{22}{7} \times 21 \times 21 \times \frac{120}{360} - \frac{441}{4}\sqrt{3}$

$= \left(462 - 441\frac{\sqrt{3}}{4}\right) cm^2$ or 271.3 cm^2 (approx.)

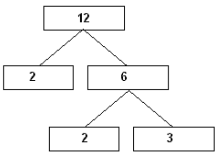
Section C

26. By taking LCM of time taken (in minutes) by Sonia and Ravi, We can get the actual number of minutes after which they meet again at the starting point after both start at the same point and at the same time, and go in the same direction.

$18 = 2 \times 3 \times 3 = 2 \times 3^2$



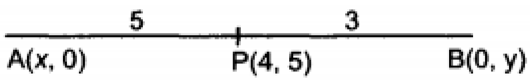
$12 = 2 \times 2 \times 3 = 2^2 \times 3$



LCM $(18, 12) = 2^2 \times 3^2 = 36$

Therefore, both Sonia and Ravi will meet again at the starting point after 36 minutes.

27. Let coordinates of A are (x, 0) and coordinates of B are (0, y)



Using section formula, we get

$4 = \frac{5 \times 0 + 3 \times x}{5 + 3}$

$\Rightarrow$ 32 = 3x

$\Rightarrow x = \frac{32}{3}$

Similarly, $5 = \frac{5 \times y + 3 \times 0}{5 + 3}$

$\Rightarrow$ 40 = 5y

$\Rightarrow$ y = 8

$\therefore$ Coordinate of A are $\left(\frac{32}{3}, 0\right)$ and coordinates of B are (0, 8).

28. According to question,two pipes are used to fill a swimming pool.

Pipe with larger diameter is used for 4 hours and pipe with smaller diameter is used for 9 hours.

Let x hours be the total time taken by the larger pipe to fill the tank

so in 1 hour it would fill $\frac{1}{x}$ part of the tank.

Similarly, y hours are needed for the smaller pipe,

then in 1 hour it would fill $\frac{1}{y}$ part.

So, y=10+x ...(1)

$\frac{4}{x} + \frac{9}{y}$ = 1/2

using (1), $\frac{4}{x} + \frac{9}{10+x}$ =1/2 ...(2)

$\Rightarrow x^2 - 16x - 80 = 0$

$\Rightarrow$ (x-20)(x+4)=0

Since value of x cannot be negative

Therefore, x=20 and y=30

Hence, Larger diameter pipe fills in 20 hours, and Smaller diameter pipe fills in 30 hours.

OR

Let the ten's place digit be y and unit's place be x.

Therefore, number is 10y + x.

According to given condition,

10y + x = 4(x + y) and 10y + x = 2xy

$\Rightarrow$ x = 2y and 10y + x = 2xy

Putting x = 2y in 10y + x = 2xy

10y + 2y = 2.2y.y

$12y = 4y^2$

$4y^2 - 12y = 0 \Rightarrow 4y(y - 3) = 0$

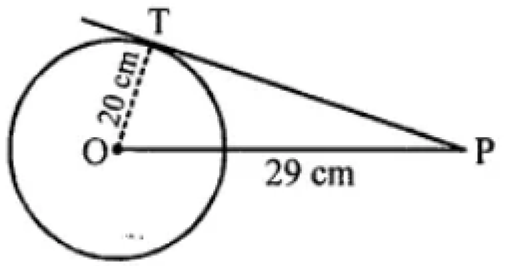
$\Rightarrow$ y - 3 = 0 or y = 3

Hence, the ten's place digit is 3 and units digit is 6 (2y = x)

Hence the required number is 36.

29. PT is the tangent to the circle with centre O and radius OT = 20 cm.

P is a point 29 cm away from O.



OP = 29 cm, OT = 20 cm

OT is radius and PT is the tangent

OT $\perp$ PT

Now, in right $\triangle$OPT,

$OP^2 = OT^2 + PT^2$ (Pythagoras Theorem)

$\Rightarrow (29)^2 = (20)^2 + PT^2$

$\Rightarrow 841 = 400 + PT^2$

$\Rightarrow PT^2 = 841 - 400$

$\Rightarrow PT^2 = 441 = (21)^2$

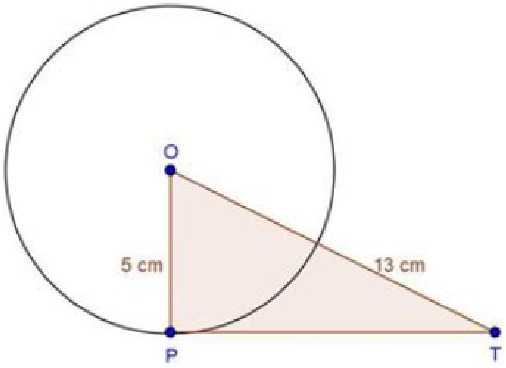
$\Rightarrow$ PT = 21

Length of tangent, PT = 21 cm

OR

According to question draw a circle with centre O and radius 5cm also given that T is any point outside of the circle.

Now,Since tangent at a point on the circle is perpendicular to the radius through the point.



Therefore, OP is perpendicular to PT.

In right triangle OPT, we have

$OT^2 = OP^2 + PT^2$

$\Rightarrow (13)^2 = (5)^2 + PT^2$

$\Rightarrow PT^2 = 13^2 - 5^2$

$\Rightarrow PT^2 = 169 - 25$

$\Rightarrow PT^2 = 144$

$\Rightarrow PT^2 = 12^2$

$\Rightarrow PT = 12$ cm

Hence, the length of a tangent is 12 cm.

30. LHS = $\frac{\cot^2\theta(\sec\theta-1)}{(1+\sin\theta)} + \frac{\sec^2\theta(\sin\theta-1)}{(1+\sec\theta)}$

$= \frac{\cot^2\theta(\sec\theta-1)(1+\sec\theta)+\sec^2\theta(\sin\theta-1)(1+\sin\theta)}{(1+\sin\theta)(1+\sec\theta)}$

$= \frac{\cot^2\theta\left(\sec^2\theta-1\right)+\sec^2\theta\left(\sin^2\theta-1\right)}{(1+\sin\theta)(1+\sec\theta)}$

$= \frac{\cot^2\theta\tan^2\theta+\sec^2\theta\left(-\cos^2\theta\right)}{(1+\sin\theta)(1+\sec\theta)}$

$= \frac{\cot^2\theta\tan^2\theta-\sec^2\theta\cos^2\theta}{(1+\sin\theta)(1+\sec\theta)}$

$= \frac{\cot^2\theta\times\frac{1}{\cot^2\theta}-\sec^2\theta\times\frac{1}{\sec^2\theta}}{(1+\sin\theta)(1+\sec\theta)}$

$= \frac{1-1}{(1+\sin\theta)(1+\sec\theta)}$

= 0

= RHS

31. If the line through A is the bridge

In $\triangle$ACB, $\angle B = 45^{o}$

AC = 8

BC = AC, $\sqrt{3} = 8\sqrt{3}$

In $\triangle$ACD, $\angle D = 30^{o}$

CD = AC = 8

Hence width of the river 8 + $8\sqrt{3}$ = 8(1 + $\sqrt{3}$) m

Section D

32. Suppose that he invested ₹ x at the rate of 12% simple interest and ₹ y at the rate 10% simple interest.

Then, according to the question, $\frac{12x}{100} + \frac{10y}{100} = 130$

$\Rightarrow$ 12x+10y=13000 ....Dividing throughout by 2

$\Rightarrow$ 6x+5y=6500 ....(1)

and, $\frac{12y}{100} + \frac{10x}{100} = 134$

$\Rightarrow$ 12y+10x=13400

$\Rightarrow$ 6y+5x=6700 ....Dividing throughout by 2

$\Rightarrow$ 5x+6y=6700 ....(2)

Multiplying equation (1) by 6 and equation (2) by 5, we get

36x+30y=39000 ....(3)

25x+30y=33500 ....(4)

⇒subtracting (3) and (4) we get x = 500

Substituting this value of x in equation (1), we get 6(500)+5y=6500

⇒ 3000+5y=6500

⇒ 5y=6500-3000

⇒ 5y=3500

$\Rightarrow y=\frac{3500}{5}=700$

So, the solution of the equation (1) and (2) is x=500 and y=700

Hence, he invested ₹ 500 at the rate of 12% simple interest and ₹ 700 at the rate of 10% simple interest.

verification.Substituting x=500, y=700,

We find that both the equation (1) and (2) are satisfied as shown below:

6x + 5y=6(500)+5(700)=3000+3500=6500

5x+6y=5(500)+6(700)=2500+4200=6700

This verifies the solution.

OR

Let the cost of a IX Maths textbook be Rs.x and the cost of a X Maths textbook be Rs.y.

Then the algebraic representation is given by the following equations

2x + 3y = 250 ...(1)

and 4x + 6y = 500 ...(2)

To represent these equations graphically, we find two

solutions for each equation.

These solution are given below:

For equations (1) 2x + 3y = 250

$\Rightarrow y=\frac{250-2x}{3}$

Table 1 of solutions

x	50	125
y	50	0

For equation (2)

4x + 6y = 500

⇒ 6y = 500 - 4x

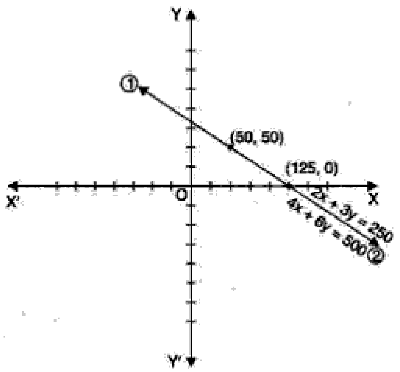
$\Rightarrow y=\frac{500-4x}{6}$

Table 2 of solutions

x	50	125
y	50	0

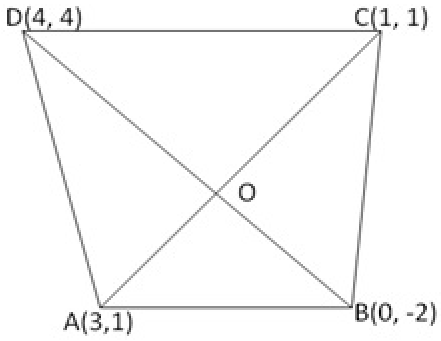
We plot these points on a graph paper, we find that both the lines coincide.

This is so, because, both the equations are equivalent, i.e., one can be derived from the other.



33. Let A(3, 1), B(0, -2), C(1, 1) and D(4, 4) be the vertices of quadrilateral

Join AC, BD. AC and BD, intersect other at the point O.



We know that the diagonals of a parallelogram bisect each other

Therefore, O is midpoint of AC as well as that of BD

Now midpoint of AC is $\left(\frac{3+1}{2}, \frac{1+1}{2}\right)$ ie., $(2,1)$

And midpoint of BD is $\left(\frac{0+4}{2}, \frac{-2+4}{2}\right)$ ie., $(2,1)$

Mid point of AC is the same as midpoint of BD

Hence, A, B, C, D, are the vertices of a parallelogram ABCD

34. According to the question,a hemispherical depression is cut from one face of the cubical block such that the diameter l of the hemisphere is equal to the edge of the cube.

Let the radius of hemisphere = r

$\therefore r=\frac{l}{2}$

Now, the required surface area = Surface area of cubical block - Area of base of hemisphere + Curved surface area of hemisphere.

$=6(\text{ side })^2-\pi r^2+2\pi r^2$

$=6l^2-\pi\left(\frac{l}{2}\right)^2+2\pi\left(\frac{l}{2}\right)^2$

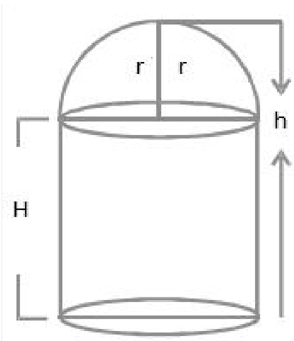
$=6l^2-\frac{\pi l^2}{4}+\frac{\pi}{2}l^2$

$=6l^2+\frac{\pi l^2}{4}$

Surface area $=\frac{1}{4}(24+\pi)l^2$ units.

$=\frac{1}{4}\left(24+\frac{22}{7}\right)l^2$

OR



Let the radius of the hemispherical dome be r and the total height of the building be h.

Since, the base diameter of the dome is equal to $\frac{2}{3}$ of the total height

$2r=\frac{2}{3}h$

$\Rightarrow r=\frac{h}{3}$

Let H be the height of the cylindrical position.

$\Rightarrow H=h-r=h-\frac{h}{3}=\frac{2h}{3}$

Volume of air inside the building = Volume of air inside the dome + Volume of air inside the cylinder

$\Rightarrow 67\frac{1}{21}=\frac{2}{3}\pi r^3+\pi r^2 H$

$\Rightarrow \frac{1408}{21}=\pi r^2\left(\frac{2}{3}r+H\right)$

$\Rightarrow \frac{1408}{21}=\frac{22}{7}\times\left(\frac{h}{3}\right)^2\left(\frac{2}{3}\times\frac{h}{3}+\frac{2h}{3}\right)$

$\Rightarrow \frac{1408\times 7}{22\times 21}=\frac{h^2}{9}\times\left(\frac{2h}{9}+\frac{2h}{3}\right)$

$\Rightarrow \frac{64}{3}=\frac{h^2}{9}\times\left(\frac{8h}{9}\right)$

$\Rightarrow \frac{64\times 9\times 9}{3\times 8}=h^3$

$\Rightarrow h^3 = 8 \times 27$

$\Rightarrow h = 6$

Thus, the height of the building is 6 m.

35. Given,

First term(a) = 5

Common difference(d) = 3

and, nth term (a_n) = 50

$\Rightarrow$a + (n - 1)d = 50

$\Rightarrow$5 + (n - 1)(3) = 50

$\Rightarrow$5 + 3n - 3 = 50

$\Rightarrow$3n = 50 - 5 + 3

$\Rightarrow$3n = 48

$\Rightarrow \quad n = \frac{48}{3} = 16$

Therefore, $S_n = \frac{n}{2}[a + a_n]$

$= \frac{16}{2}[5 + 50]$

$= 8 \times 55$

= 440

Section E

36. i. a is a non zero real number and b and c are any real numbers.

ii. D = 0

iii. $2x^2 - x + 8k$

$\alpha \times \frac{1}{\alpha} = \frac{8k}{2}$

1 = 4k

k = $\frac{1}{4}$

OR

$\alpha + \beta = \frac{-b}{a}$ i.e., $\left(\frac{-\text{coefficient of x}}{\text{coefficient of x}^2}\right)$

$\alpha\beta = \frac{c}{a}$ i.e., $\left(\frac{\text{constant term}}{\text{coeff of } x^2}\right)$

37. i. Women having heart beat in range 68 - 77

= 4 + 3 + 8 = 15

ii. Median class = 74 - 77

iii. a. Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$l = 74, f_1 = 8, f_0 = 3, f_2 = 7,\ h = 3$

$\therefore$ Modal value = $74 + \left(\frac{8-3}{16-3-7}\right) \times 3$

= 76.5

OR

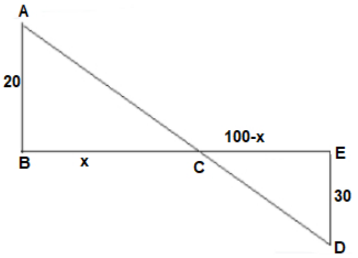
b.

No. of heart beats	f	cf
65 - 68	2	2
68 - 71	4	6
71 - 74	3	9
74 - 77	8	17
77 - 80	7	24
80 - 83	4	28
83 - 86	2	30

Median = $I + \frac{\frac{N}{2} - Cf}{f} \times h$

$= 74 + \frac{(15-9)}{8} \times 3$

= 76.25

38. i. 

$\triangle ABC \sim \triangle DEC$

$\frac{20}{30} = \frac{x}{100-x}$

2000 - 20x = 30x

2000 = 50x

x = 40 m

ii. AA

iii. 60 metres

OR

AD = AC + CD

$= \sqrt{20^2 + 40^2} + \sqrt{60^2 + 30}^2$

$= \sqrt{400 + 1600} + \sqrt{3600 + 900}$

$= \sqrt{2000} + \sqrt{4500}$

$\Rightarrow 20\sqrt{5}$ + $30\sqrt{5}$

$\Rightarrow 50\sqrt{5}$ m