Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. Which of the followings is an irrational number? **[1]**

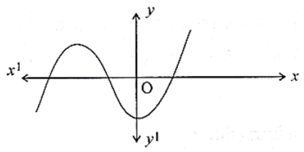
a) $(\sqrt{2}-1)^2$

b) $\left(2\sqrt{3}-\frac{1}{\sqrt{3}}\right)^2$

c) $\frac{(\sqrt{2}+5\sqrt{2})}{\sqrt{2}}$

d) $\sqrt{2}-(2+\sqrt{2})$

2. The graph of a polynomial is shown in Figure, then the number of its zeroes is: **[1]**



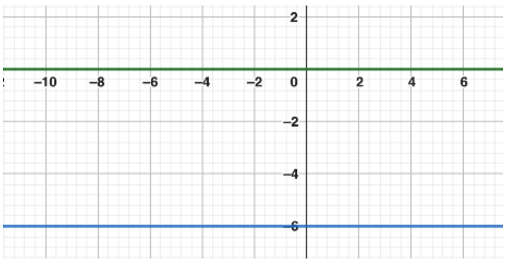
a) 4

b) 3

c) 1

d) 2

3. The pair of linear equations y = 0 and y = - 6 has: **[1]**



a) no solution

b) only solution (0, 0)

c) infinitely many solutions

d) a unique solution

4. The discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$ is: **[1]**

a) -8
b) 10
c) 8
d) $2\sqrt{2}$

5. If p - 1, p + 1 and 2p + 3 are in A.P., then the value of p is **[1]**

a) 0
b) 4
c) 2
d) -2

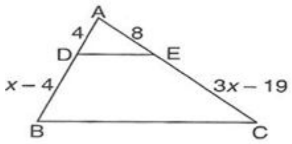
6. The points (-4, 0), (4, 0) and (0, 3) are the vertices of a: **[1]**

a) isosceles triangle
b) scalene triangle
c) equilateral triangle
d) right triangle

7. If (3, –6) is the mid-point of the line segment joining (0, 0) and (x, y), then the point (x, y) is: **[1]**

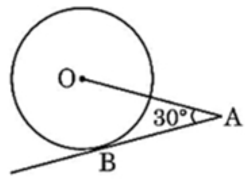
a) (6, - 6)
b) (6, -12)
c) $\left(\frac{3}{2}, -3\right)$
d) (- 3, 6)

8. In the given figure if $DE||BC$,then x is equal to:- **[1]**



a) 17.
b) 15.
c) 11.
d) 19.

9. In the given figure, AB is a tangent to the circle centered at O. If OA = 6 cm and ∠OAB = 30°, then the radius of the circle is: **[1]**



a) $3\sqrt{3}$ cm
b) 2 cm
c) $\sqrt{3}$ cm
d) 3 cm

10. A circle is of radius 3 cm. The distance between two of its parallel tangents is: **[1]**

a) 3 cm
b) 4.5 cm
c) 6 cm
d) 12 cm

11. $\left(\frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ}\right)$ is equal to: **[1]**

a) sin 60°
b) tan 60°
c) cos 30°
d) cos 60°

12. If $2\sin 2\theta = \sqrt{3}$ then θ= ? **[1]**

a) 45°
b) 90°
c) 60°
d) 30°

13. The angle of depression of a car, standing on the ground, from the top of a 75 m tower, is 30°. The distance of the car from the base of the tower (in metres) is **[1]**

a) $25\sqrt{3}$
b) $75\sqrt{3}$
c) 150
d) $50\sqrt{3}$

14. In a circle of radius 21 cm, an arc subtends an angle of 60^0 at the centre. The length of the arc is **[1]**

a) 18.16 cm
b) 23.5 cm
c) 22 cm
d) 21 cm

15. If the area of a sector of a circle is $\frac{5}{18}$ of the area of the circle, then the sector angle is equal to **[1]**

a) 100°
b) 120°
c) 90°
d) 60°

16. A coin is tossed thrice. The probability of getting at least two tails is **[1]**

a) $\frac{4}{5}$
b) $\frac{2}{3}$
c) $\frac{1}{4}$
d) $\frac{1}{2}$

17. Which of the following numbers **cannot** be the probability of an event? **[1]**

a) 5%
b) 0.5
c) $\frac{1}{0.5}$
d) $\frac{0.5}{14}$

18. Using empirical relationship, the mode of a distribution whose mean is 7.2 and the median 7.1, is: **[1]**

a) 6.5
b) 6.9
c) 6.2
d) 6.3

19. **Assertion (A):** If we join two hemispheres of same radius along their bases, then we get a sphere. **[1]**

Reason (R): A tank is made of the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and radius is 30 cm. The total surface area of the tank is 3.3 m^2.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** If S_n is the sum of the first n terms of an A.P., then its n^{th} term a_n is given by $a_n = S_n - S_{n-1}$ **[1]**

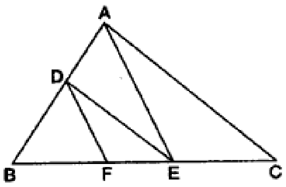
Reason (R): The 10^{th} term of the A.P. 5, 8, 11, 14, ... is 35.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

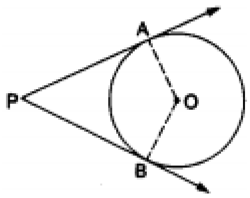
Section B

21. Find the HCF and LCM of 90 and 144 by the method of prime factorization. **[2]**

22. In figure DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$ **[2]**



23. In the given figure, O is the centre of the circle. PA and PB are tangents. Show that AOBP is a cyclic quadrilateral. **[2]**



24. Prove that: **[2]**

$\frac{1+\tan A}{2\sin A}+\frac{1+\cot A}{2\cos A}$ = cosec A + sec A

OR

If $\cot\theta=\frac{15}{8}$, then evaluate: $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$.

25. Find the area of a quadrant of a circle , whose circumference is 22 cm . **[2]**

OR

The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

Section C

26. A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft. by 8 ft. What would be the size (in inches) of the tile required that has to be cut and how many such tiles are required? **[3]**

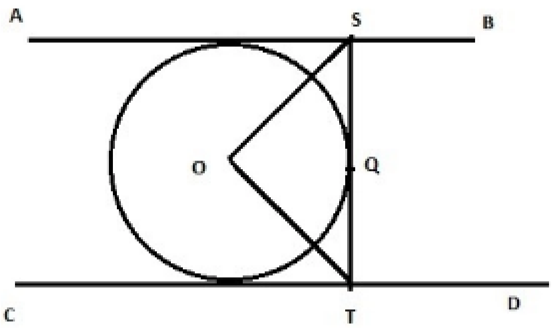
27. If α, β are zeroes of the quadratic polynomial x^2 + 3x + 2, find a quadratic polynomial whose zeroes are α + 1, β + 1. **[3]**

28. The tenth term of an A.P., is - 37 and the sum of its first six terms is - 27. Find the sum of its first eight terms. **[3]**

OR

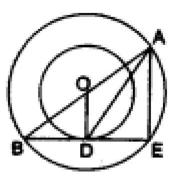
How many terms of the AP : 9, 17, 25, …. must be taken to give a sum of 636?

29. In the adjoining figure, AB and CD are two parallel tangents to a circle with centre O. ST is the tangent segment between two parallel tangents touching the circle at Q. Show that $\angle\mathrm{SOT}=90^{\circ}$ **[3]**



OR

In the given figure, the radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle and BD is a tangent to the smaller circle touching it at D. Find the length of AD.



30. Prove the trigonometric identity: **[3]**

$\sqrt{\sec^2\theta + cosec^2\theta}$ = tan θ + cot θ.

31. During the 2011 census, the records of various aspects like good health, death rate and literacy rate were recorded for all the States and Union territories of India. The Literacy rates of 40 cities are given in the following table: **[3]**

Literacy rate (in %)	35-40	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
Number of cities	1	2	3	x	y	6	8	4	2	3	2

If it is given that the mean literacy rate is 63.5, then find the missing frequencies x and y.

Section D

32. A train travels a distance of 90 km at a constant speed. Had the speed been 15 km/h more, it would have taken 30 minutes less for the journey. Find the original speed of the train. **[5]**

OR

Solve for x:

$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$; $x \neq 1, 2, 3$

33. A girl on a ship standing on a wooden platform, which is 50 m above water level, observes the angle of elevation of the top of a hill as 30^o and the angle of depression of the base of the hill as 60^o. Calculate the distance of the hill from the platform and the height of the hill. **[5]**

34. A solid is in the shape of a cone standing on a hemisphere with both their diameters being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid. [Use π = 3.14] **[5]**

OR

From a solid cylinder of height 20 cm and diameter 12 cm, a conical cavity of height 8 cm and radius 6 cm is hallowed out. Find the total surface area of the remaining solid.

35. Find the mean and the mode of the data given below: **[5]**

Weight (in kg)	**Number of students**
40 - 45	5
45 - 50	11
50 - 55	20
55 - 60	24
60 - 65	28
65 - 70	12

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

TOWER OF PISA : To prove that objects of different weights fall at the same rate, Galileo dropped two objects with different weights from the Leaning Tower of Pisa in Italy. The objects hit the ground at the same time. An object dropped off the top of Leaning Tower of Pisa falls vertically with constant acceleration. If s is the distance of the object above the ground (in feet) t seconds after its release, then s and t are related by an equation of the form $s = a + bt^2$ where a and b are constants. Suppose the object is 180 feet above the ground 1 second after its release and 132 feet above the ground 2 seconds after its release.

i. Find the constants a and b. (1)
ii. How high is the Leaning Tower of Pisa? (1)
iii. How long does the object fall? (2)

OR

At t = 2 sec, the object is at what height? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Ashok wanted to determine the height of a tree on the corner of his block. He knew that a certain fence by the tree was 4 feet tall. At 3 PM, he measured the shadow of the fence to be 2.5 feet tall. Then he measured the tree's shadow to be 11.3 feet.

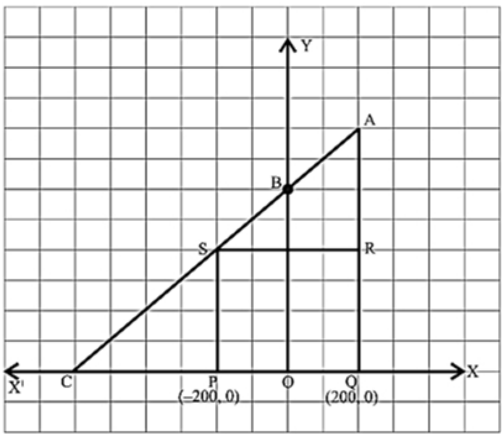
i. What is the height of the tree? (1)
ii. What will be length of shadow of tree at 12:00 pm? (1)
iii. Write the name triangle formed for this situation. (2)

OR

What will be the length of wall at 12:00 pm? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



i. Taking O as origin, coordinates of P are (-200, 0) and of Q are (200, 0). PQRS being a square, what are the coordinates of R and S? (1)

ii. What is the area of square PQRS? (1)

iii. What is the length of diagonal PR in square PQRS? (2)

OR

If S divides CA in the ratio K : 1, what is the value of K, where point A is (200, 800)? (2)

Solution

Section A

1. **(a)** $(\sqrt{2}-1)^2$
Explanation: $(\sqrt{2}-1)^2$

2.
(b) 3
Explanation: The graph of given polynomial cuts the x-axis at 3 distinct points.
therefore, No. of zeroes are 3.

3. **(a)** no solution
Explanation: Since, we have y = 0 and y = -6 are two parallel lines.
therefore, no solution exists.

4. **(a)** -8
Explanation: The given equation is of the form: $ax^2 + bx + c = 0$, where; a = 2, b = - 4 and c = 3.
Therefore, the discriminant (D) is given as $D = b^2 - 4ac$
$D = (-4)^2 - (4 \times 2 \times 3) = 16 - 24 = -8$

5. **(a)** 0
Explanation: 0

6. **(a)** isosceles triangle
Explanation: $AB^2 = (4 + 4)^2 + (0 - 0)^2 = 8^2 + 0^2 = 64 + 0 = 64$
$\Rightarrow$ AB=$\sqrt{64}$= 8 units
$BC^2 = (0-4)^2 + (3-0)^2 = (-4)^2 + 3^2 = 16 + 9 = 25$
$\Rightarrow$ BC = $\sqrt{25}$ = 5 units.
$AC^2 = (0 + 4)^2 + (3 - 0)^2 = 4^2 + 3^2 = 16 + 9 = 25$
$\Rightarrow$ A C = $\sqrt{25}$ = 5 units.
$\therefore \triangle$ABC is isosceles.

7.
(b) (6, -12)
Explanation: If (a, b) and (c, d) be the coordinates of any two points, then the coordinates of the mid-point joining those points be $\left(\frac{(a+c)}{2}, \frac{(b+d)}{2}\right)$.
The line segment is formed by points are (0, 0) and (x, y), whose mid-point is (3, -6).
Then,
$\frac{(0+x)}{2}$ = 3 and $\frac{(0+y)}{2}$ = -6
or, $\frac{x}{2}$ = 3 or, $\frac{y}{2}$ = -6
or, x = 6 or, y = -12
Therefore the required point is (6, -12).

8.
(c) 11.
Explanation: Given: $DE||BC$
$\therefore \frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AE}}{\mathrm{EC}} \Rightarrow \frac{4}{x-4} = \frac{8}{3x-19}$ by using Thale's theorem
$\Rightarrow 12x - 76 = 8x - 32$
$\Rightarrow 4x = 44$
$\Rightarrow x = 11$

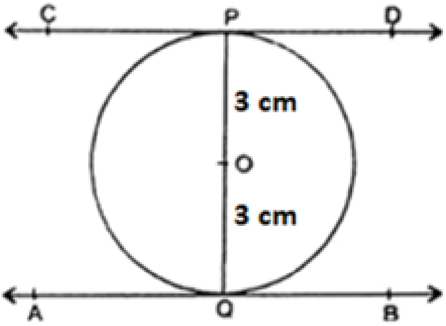
9.
(d) 3 cm
Explanation:

sin 30^o = $\frac{OB}{OA}$
$\frac{1}{2}$ = $\frac{r}{6}$
r = 3 cm

10.

(c) 6 cm

Explanation:



Distance between the two parallel tangent to a circle = diameter
= 2 $\times$ r
= 2 $\times$ 3 = 6 cm.

11.

(d) cos 60^o

Explanation: $\frac{1-\tan^2 30^o}{1+\tan^2 30^o}$ = $\frac{1-(\frac{1}{\sqrt{3}})^2}{1+(\frac{1}{\sqrt{3}})^2}$

= $\frac{1-\frac{1}{3}}{1+\frac{1}{3}}$

= $\frac{\frac{2}{3}}{\frac{4}{3}}$

= $\frac{1}{2}$

= cos 60^o

12.

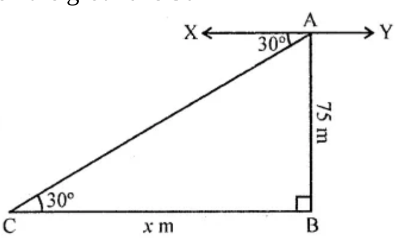
(d) 30°

Explanation: We have, $2\sin 2\theta = \sqrt{3} \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} \quad = \sin 60^\circ$
$\Rightarrow 2\theta = 60^\circ$
$\Rightarrow \theta = 30^\circ$

13.

(b) $75\sqrt{3}$

Explanation: AB is as tower and AB = 75 m
From A, the angle of depression of a car C
on the ground is 30^o



Let distane BC = x
Now in right $\triangle$ACB,
tan $\theta = \frac{AB}{BC} \Rightarrow$ tan 30^o = $\frac{75}{x}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x} \Rightarrow$ x = 75 $\sqrt{3}$ m
$\therefore$ BC = 75$\sqrt{3}$ m

14.

(c) 22 cm

Explanation: Arc length $= \frac{2\pi r\theta}{360} = \left(2 \times \frac{22}{7} \times 21 \times \frac{60}{360}\right) \text{cm} = 22\text{cm}$

15. **(a)** 100°

Explanation: We have given that area of the sector is $\frac{5}{18}$ of the area of the circle.

Therefore, area of the sector $= \frac{5}{18} \times$ area of the circle

$\Rightarrow \frac{\theta}{360} \times \pi r^2 = \frac{5}{18} \times \pi r^2$

Now we will simplify the equation as below,

$\Rightarrow \frac{\theta}{360} = \frac{5}{18}$

$\therefore \theta = \frac{5}{18} \times 360$

$\therefore \theta = 100$

Therefore, sector angle is 100°.

16.

(d) $\frac{1}{2}$

Explanation: Total outcomes = = {HHH, TTT, HHT, HTH, HTT, THH, THT, TTH} = 8

Number of possible outcomes (at least two tails) = 4

$\therefore$ Required Probability = $\frac{4}{8} = \frac{1}{2}$

17.

(c) $\frac{1}{0.5}$

Explanation: The probability of an event cannot be greater then 1.

$\therefore$ $\frac{1}{0.5}$ = 2,cannot be the possible value of probability.

18.

(b) 6.9

Explanation: 6.9

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Both A and R are true but R is not the correct explanation of A.

20.

(c) A is true but R is false.

Explanation: a_{10} = a + 9d

= 5 + 9(3) = 5 + 27 = 32

Section B

21. The prime factorization of 90 and 140 are as follows

$90 = 2 \times 3 \times 3 \times 5 \ = 2 \times 3^2 \times 5$

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

Hence HCF (90,144) = 2×3^2 = 18

and LCM (90,144) = $2^4 \times 3^2 \times 5$ = 720

22. In $\triangle ABE$, we have $DF || AE$, then

$\frac{BD}{AD} = \frac{BF}{FE}$ [By BPT] ...... (i)

In $\triangle ABC$, we have $DE || AC$, then

$\frac{BD}{AD} = \frac{BE}{EC}$ [By BPT] ...... (2)

From (i) and (2), We get

$\frac{BF}{FE} = \frac{BE}{EC}$

23. We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBP = \angle OAP = 90^\circ$

Now, In quadrilateral AOBP

$\angle APB + \angle AOB + \angle OBP + \angle OAP = 360^\circ$

$\Rightarrow \angle APB + \angle AOB + 90^\circ + 90^\circ = 360^\circ$

$\Rightarrow \angle APB + \angle AOB = 180^\circ$

Since, the sum of the opposite angles of the quadrilateral is 180°

Hence, AOBP is a cyclic quadrilateral.

24. LHS = $\frac{1+\tan A}{2\sin A}+\frac{1+\cot A}{2\cos A}$

$=\frac{\cos A+\sin A}{2\sin A\cos A}+\frac{\sin A+\cos A}{2\cos A\sin A}$

$=\frac{2(\cos A+\sin A)}{2\sin A\cos A}$

= cosec A + sec A

= RHS

OR

Given $\cot\theta=\frac{15}{8}$

To evaluate: $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

$=\frac{2(1+\sin\theta)(1-\sin\theta)}{2(1+\cos\theta)(1-\cos\theta)}$

$=\frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}=\frac{\cos^2\theta}{\sin^2\theta}=\cot^2\theta$

$=(\cot\theta)^2=\left(\frac{15}{8}\right)^2=\frac{225}{64}$

Hence, the value of the given expression is $\frac{225}{64}$.

25. Given , Circumference = 22 cm

$\Rightarrow 2\pi r=22$

$\Rightarrow r=\frac{7}{2}=3.5cm$

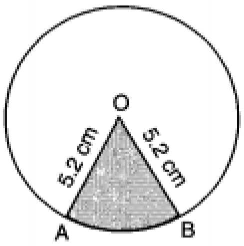
Area of Circle = πr^2 = $\frac{22}{7}\times(3.5)^2=38.5cm^2$

Area of quadrant of circle = $\frac{Area\,of\,circle}{4}$

$=\frac{38.5}{4}=9.625cm^2$

$\therefore$Area of the quadrant of cicle = 9.625 cm^2

OR



Let OAB be the given sector.

It is given that Perimeter of sector OAB = 16.4 cm

$\Rightarrow$OA + OB + arc AB = 16.4 cm

$\Rightarrow$5.2 + 5.2 + arc AB = 16.4

$\Rightarrow$arc AB = 6 cm

$\Rightarrow l=6cm$

$\therefore$Area of sector OAB =$\frac{1}{2}lr=\frac{1}{2}\times6\times5.2$cm^2 = 15.6 cm^2

Section C

26. **Given:** Size of bathroom = 10 ft by 8 ft.

= (10 $\times$ 12) inch by (8 × 12) inch

= 120 inch by 96 inch

Area of bathroom = 120 inch by 96 inch

To find the largest size of tile required , we find HCF of 120 and 96.

By applying Euclid's division lemma

120 = 96 $\times$ 1 + 24

96 = 24 $\times$ 4 + 0

Therefore, HCF = 24

Therefore, Largest size of tile required = 24 inches

no.of tiles required $=\frac{\text{area of bathroom}}{\text{area of a tile}}=\frac{120\times96}{24\times24}=5\times4$ = 20 tiles

Hence number of tiles required is 20 and size of tiles is 24 inches.

27. p(x) = x^2 + 3x + 2

α,β are its zeroes

$\therefore\alpha+\beta$ = -3, $\alpha\beta$ = 2

Now,

$(\alpha + 1) + (\beta + 1) = \alpha + \beta + 2 = -3 + 2 = -1$

$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1 = + 2 - 3 + 1 = 0$

$\therefore$ Required Polynomial is $k(x^2 + x)$ or $x^2 + x$

28. Let the first term be a and the common difference be d.

$a_n = a + (n - 1)d$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

As per given condition

$a_{10} = -37$

a + 9d = - 37 .,.(i)

Sum of first 6 term is - 27

$S_6 = \frac{6}{2}[2a + (6 - 1)d]$

3(2a + 5d) = -27

or, 2a + 5d = - 9 ...(ii)

Multiplying (i) by 2 and subtracting (ii) from it, we get

(2a + 18d) - (2a + 5d) = - 74 + 9

2a + 18d - 2a - 5d = -65

13d = - 65

d = -65/13

d = - 5

Putting d = -5 in (i) we get

a + 9d = - 37

$a + 9 \times - 5 = -37$

a = - 37 + 45

a = 8

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{8}{2}[2 \times 8 + (8 - 1)(-5)]$

$= 4[16 + (7)(-5)]$

$= 4[16 - 35]$

$= 4 \times -19$

$= -76$

Hence, $S_n = -76$

OR

The given AP is 9, 17, 25,...

Here, a = 9

d = 17 - 9 = 8

Let n terms of the AP must be taken

Then, $S_n = 636$

$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = 636$

$\Rightarrow \frac{n}{2}[2(9) + (n - 1)8] = 636$

$\Rightarrow n[9 + (n - 1)4] = 636$

$\Rightarrow n[9 + 4n - 4] = 636$

$\Rightarrow n[(4n + 5)] = 636$

$\Rightarrow 4n^2 + 5n - 636 = 0$

$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$

$\Rightarrow n(4n + 53) - 12(4n + 53) = 0$

$\Rightarrow (4n + 53)(n - 12) = 0$

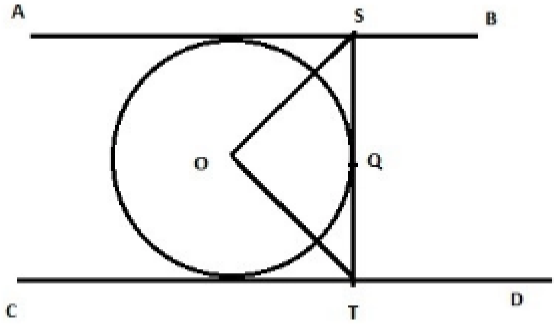
$\Rightarrow 4n + 53 = 0$ or $n - 12 = 0$

$\Rightarrow n = -\frac{53}{4}$ or n = 12

$n = -\frac{53}{4}$ is in admissible as n, being the number of terms, is a natural number

$\therefore$ n = 12

Hence, 12 terms of the AP must be taken.

29. 

From the given figure we have, AB⊥ST, then ∠ASQ = 90° and CD⊥TS then ∠CTQ = 90°

$\angle ASO = \angle QSO = \frac{90}{2}$ = 45°

Similarly, $\angle OTQ$ = 45°

To find $\angle SOT$

Consider △SOT

$\angle OTS$ = 45° and $\angle OST$ = 45°

$\angle SOT + \angle OTS + \angle OST$ = 180° (by angle sum property of a triangle)

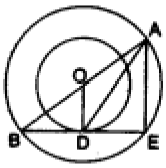
$\angle SOT$ = 180° - ($\angle OTS + \angle OST$) = 180° - (45° + 45°) = 180° - 90° = 90°

Therefore, $\angle SOT$ = 90°

Hence proved

OR

Given, the radii of two concentric circles are 13 cm and 8 cm.



We have $\angle AEB$ = 90° [angle in a semicircle].

Also, $OD \perp BE$ and OD bisects BE.

In right $\triangle OBD$, we have

$OB^2 = OD^2 + BD^2$ [by Pythagoras' theorem]

$\Rightarrow$ BD = $\sqrt{OB^2 - OD^2}$

$= \sqrt{13^2 - 8^2}\text{cm}$

$= \sqrt{105}\text{cm}$

BE = 2BD = $2\sqrt{105}\text{cm}$ [$\because$ D is the midpoint of BE]

In right $\triangle AEB$, we have

$AB^2 = AE^2 + BE^2$ [by Pythagoras' theorem]

$\Rightarrow \quad AE = \sqrt{AB^2 - BE^2}$

$= \sqrt{26^2 - (2\sqrt{105})^2}\text{cm}$

$= \sqrt{256}\text{cm}$

= 16 cm.

In right $\triangle AED$, we have

$AD^2 = AE^2 + DE^2$ [by Pythagoras' theorem]

$\Rightarrow AD = \sqrt{AE^2 + DE^2}$

$= \sqrt{16^2 + (\sqrt{105})^2}\text{cm}$

= 19 cm.

30. We have to prove :-

$\sqrt{\sec^2\theta + cosec^2\theta}$ = tan θ + cot θ.

Now, take LHS $= \sqrt{\sec^2\theta + cosec^2\theta}$

$= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}} = \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}}$

$= \sqrt{\frac{1}{\sin^2\theta \cdot \cos^2\theta}} = \frac{1}{\sin\theta\cos\theta}$

= cosecθ secθ.....(1)

Now, take RHS = tanθ + cotθ

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta \cdot \cos\theta}$

= cosec θ · sec θ .....(2)

Hence, from (1) & (2)

LHS=RHS , Proved.

31.

C.I.	x_i	u_i	f_i	f_iu_i
35-40	37.5	- 5	1	- 5
40-45	42.5	- 4	2	- 8
45-50	47.5	- 3	3	- 9
50-55	52.5	- 2	x	- 2x
55-60	57.5	- 1	y	- y
60-65	62.5 = A	0	6	0
65-70	67.5	1	8	8
70-75	72.5	2	4	8
75-80	77.5	3	2	6
80-85	82.5	4	3	12
85-90	87.5	5	2	10
Total			$\Sigma f_i = 31 + x + y$	$\Sigma f_iu_i = 22 - 2x - y$

Let Assumed Mean, A = 62.5

Here, $\Sigma f_i = 31 + x + y = 40$

$\Rightarrow$ x + y = 9..........(i)

$\Sigma f_iu_i = 22 - 2x - y$

Now, Mean = A + $\frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$\Rightarrow$ 63.5 = 62.5 + $\frac{(22-2x-y)}{40} \times 5$

$\Rightarrow$ 2x + y = 14...........(ii)

Solving eqns (i) and (ii), x = 5 and y = 4.

Section D

32. Let the original speed of the train be x km/hr.

We know that time taken to cover 'd' km with speed 's' km/h $= \frac{d}{s}$ $\therefore$time taken to cover 90 km $= \frac{90}{x}$ hours

&, Time taken to cover 90 km when the speed is increased by 15 km/hr $= \frac{90}{x+15}$ hours

According to the question ;

$\frac{90}{x} - \frac{90}{x+15} = \frac{30}{60}$ (time reduced by 30 minutes with increased speed)

$\Rightarrow \frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$

$\Rightarrow \frac{90x + 1350 - 90x}{x^2 + 15x} = \frac{1}{2}$

$\Rightarrow \frac{1350}{x^2 + 15x} = \frac{1}{2}$

$\Rightarrow 2700 = x^2 + 15x$

$\Rightarrow x^2 + 15x - 2700 = 0$

$\Rightarrow x^2 + 60x - 45x - 2700 = 0$

$\Rightarrow$ x(x + 60) - 45(x + 60) = 0

$\Rightarrow$ (x + 60)(x - 45) = 0

$\Rightarrow$ x + 60 = 0 or x - 45 = 0

$\Rightarrow$ x = -60 or x = 45

Since the speed cannot be negative, $x \neq -60$.

$\Rightarrow$ x = 45

Thus, the original speed of the train is 45 km/hr.

OR

Given, $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$

$\frac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$

$\frac{x-3+x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3}$

$\frac{2x-4}{(x-1)(x-2)(x-3)} = \frac{2}{3}$

$\frac{2(x-2)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$

$\frac{2}{(x-1)(x-3)} = \frac{2}{3}$

(x - 1) (x - 3) = 3

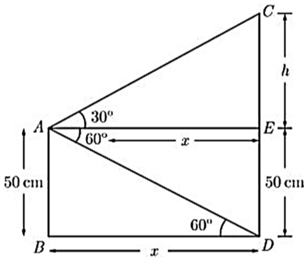
x^2 - 4x + 3 = 3

x^2 -4x = 0

x(x - 4) = 0

x = 0 , x - 4= 0

x = 0, x = 4

33. 

Here, CD = CE + ED = h + 50

Now, In $\triangle$ABD

$\tan 60^\circ = \frac{AB}{BD}$

$\sqrt{3} = \frac{50}{x}$

$x = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3}$ m

In $\triangle$CEA

$\tan 30^\circ = \frac{CE}{AE}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$h = \frac{x}{\sqrt{3}} = \frac{50\sqrt{3}}{3\times\sqrt{3}}$

$h = \frac{50}{3}$ m

CD = h + 50

$CD = \frac{50}{3} + 50 = \frac{50+150}{3}$

CD = 66.66 m

34. Clearly r = $\frac{1}{2}$, h = $\frac{1}{2}$

Volume of solid = Volume of Cone + Volume of Hemisphere

= $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

= $\frac{1}{3} \times 3.14 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{2}{3} \times 3.14 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

= $\frac{1}{3} \times 3.14 \times \frac{1}{2} \times \frac{1}{2} \times \left[\frac{1}{2} + 2\left(\frac{1}{2}\right)\right]$

= $\frac{1}{3} \times \frac{3.14}{4} \times \frac{3}{2}$

= $\frac{1.57}{4} = \frac{157}{400}$ cm^3 or 0.3925 cm^3

OR

Given, Height of cylinder h_1 = 20 cm

Radius of cylinder = $\frac{12}{2}$ = 6 cm

Height of the cone (h_2) = 8 cm

Radius of the cone r = 6 cm

Total surface area of remaining solid = Curved surface area of cylinder + Curved surface area of cone + Area of the top face of the cylinder

Slant height of the cone(l) = $\sqrt{h_2^2 + r^2}$

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36}$

= 10 cm

$\therefore$ Curved surface area of cone = πrl

$= \frac{22}{7} \times 6 \times 10$

$= \frac{1320}{7}$ cm^2

Curved surface area of cylinder = $2\pi rh$

= 2 $\times$ $\frac{22}{7} \times 6 \times 20$

$= \frac{5280}{7}$ cm^2

Area of the top face of the cylinder = πr^2

$= \frac{22}{7} \times 6 \times 6$

$= \frac{792}{7}$ cm^2

$\therefore$ Total surface area of the remaining solid

$= \frac{1320}{7} + \frac{5280}{7} + \frac{792}{7}$

$= \frac{7392}{7}$

= 1056 cm^2

35.

Class	**x_i**	**f_i**	**$u_i = \frac{x_i - 57 \cdot 5}{5}$**	**f_iu_i**
40 - 45	42·5	5	-3	-15
45 - 50	47·5	11	-2	-22
50 - 55	52·5	20	-1	-20
55 - 60	57·5=a	24	0	0
60 - 65	62·5	28	1	28
65 - 70	67·5	12	2	24
		100		**-5**

Mean = $a + \frac{\sum f_i u_i}{\sum f_i} \times h$

= $57.5 + \frac{-5}{100} \times 5$ = 57.25

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

= $60 + \frac{28-24}{2(28)-24-12} \times 5$ = 61

Section E

36. i. S = a + bt^2

At t = 1 sec

180 = a + b ...(i)

At t = 2 sec

132 = a + 4b ...(ii)

from (i) and (ii)

180 - 132 = -3b

48 = -3b

b = -16

Put b = -16, in equation (i)

180 = a + (-16)

a = 196

ii. At t = 0

s = a + b(0)

s = a

s = 196

i.e., The height of Tower of Pisa = 196 feet

iii. s = a + bt^2

0 = 196 - $16t^2$

-196 = $-16t^2$

196 $\div$ 16 = t

t = $\frac{14}{4}$

t = 3.5 sec

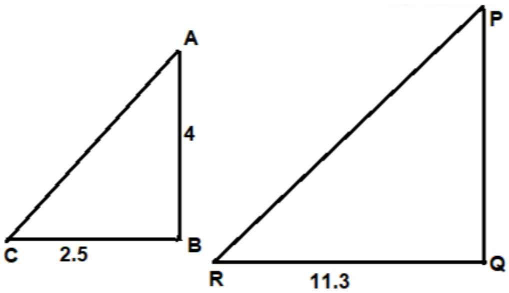
OR

s = a + bt^2

s = 196 + (-16) $(2)^2$

s = 196 - 64

s = 132 feet

37. i. 

Let AB be a wall and PQ is a tree

BC and QR are their shadow respectively at 3 p.m.

$\therefore \triangle$ABC $\sim \triangle$PQR

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$

$\frac{4}{PQ} = \frac{2.5}{11.3}$

2.5 $\times$ PQ = 4 $\times$ 11.3

PQ = 18.08

$\therefore$ height of tree = 18.08 feet

ii. 0

iii. Right triangle

OR

Zero

38. i. Since, PQRS is a square

$\therefore$ PQ = QR = RS = PS

Length of PQ = 200 - (-200) = 400

$\therefore$ The coordinates of R = (200, 400)

and coordinates of S = (-200, 400)

ii. Area of square PQRS = $(\text{side})^2$

= $(PQ)^2$

= $(400)^2$

= 1,60,000 sq. units

iii. By Pythagoras theorem

$(PR)^2 = (PQ)^2 + (QR)^2$

= 1,60,000 + 1,60,000

= 3,20,000

$\Rightarrow$ PR = $\sqrt{3,20,000}$

= 400 $\times \sqrt{2}$ units

OR

Since, point S divides CA in the ratio K : 1

$\therefore \left(\frac{Kx_2+x_1}{K+1}, \frac{Ky_2+y_1}{K+1}\right)$ = (-200, 400)

$\Rightarrow \left(\frac{K(200)+(-600)}{K+1}, \frac{K(800)+0}{K+1}\right)$ = (-200, 400)

$\Rightarrow \left(\frac{200K-600}{K+1}, \frac{800K}{K+1}\right)$ = (-200, 400)

$\therefore \frac{800K}{K+1}$ = 400

$\Rightarrow$ 800K = 400K + 400

$\Rightarrow$ 400K = 400

$\Rightarrow$ K = 1