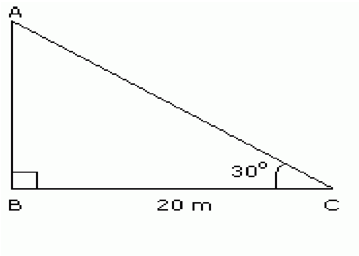
# Chapter 9. Some Applications of Trigonometry

## Question-1

**From a point 20 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.**

**Solution:**

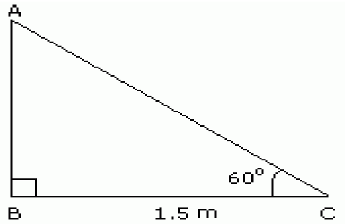



Let AB be the height of the tower and C be the point.
In rt. Δ ABC,
tan 30° = AB/BC
AB = BC tan 30°
$= \frac{20}{\sqrt{3}} = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
= 11.56 m
Therefore the height of the tower is 11.56 m.

## Question-2

**A ladder is placed against a wall such that it just reaches the top of the wall. The foot of the ladder is 1.5 m away from the wall and the ladder is inclined at an angle of 60° with the ground. Find the height of the wall.**

**Solution:**

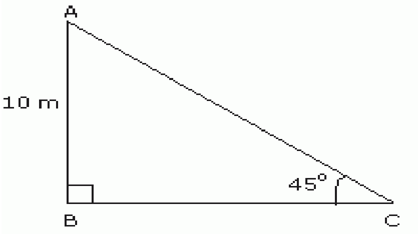



Let AC be the ladder and B be the foot of the wall.
In rt. Δ ABC,
tan 60° = AB/BC
AB = BC tan 60°
$= 1.5\sqrt{3}$ m
= 2.598 m
Therefore the height of the wall is 2.598 m.

## Question-3

**An electric pole is 10 m high. A steel wire tied to the top of the pole is affixed at a point on the ground to keep the pole upright. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.**

### Solution:




Let AB be the height of the electric pole and AC be the length of the wire.

In rt. Δ ABC,

cosec 45° = AC/AB

AC = AB cosec 45°

= $10\sqrt{2}$ m

= 10 × 1.414 m

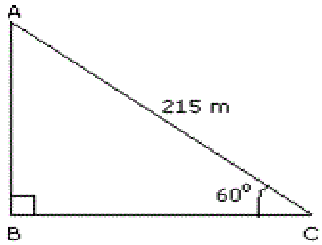
= 14.14 m

Therefore the length of the wire is 14.14 m.

## Question-4

**A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at 60° to the horizontal. Determine the height of the balloon from the ground. Assume that there is no stack in the cable.**

### Solution:




Let A be the position of the balloon.

In rt. Δ ABC,

sin 60° = AB/AC

AB = 215 sin 60°

AB = $215 \times \frac{\sqrt{3}}{2}$

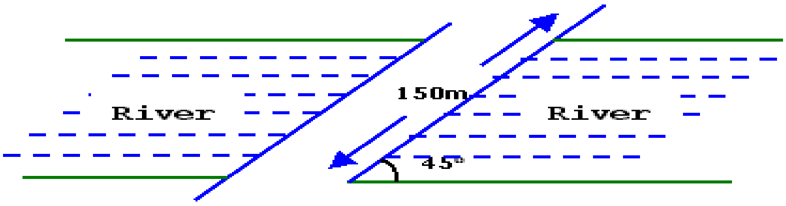
= $215 \times \frac{1.73}{2}$

= 186 m

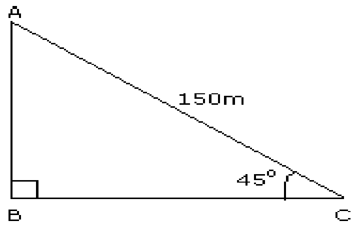
Therefore the height of the balloon from the ground is 186 m.

## Question-5

**A bridge across a river makes an angle of 45° with the river bank (fig.). If the length of the bridge across the river is 150 m, what is the width of the river?**




## Solution:




Let AB be the width of the river.

In rt. Δ ABC,

sin45° = AB/AC

$AB = AC \sin 45^\circ$

$= 150 \times \frac{1}{\sqrt{2}}$

$= 150 \times \frac{\sqrt{2}}{2}$

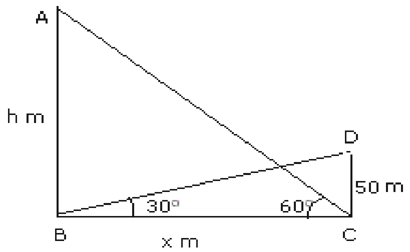
$= 106.05$ m

Therefore the width of the river is 106.05 m.

## Question-6

**The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of top of the tower from the foot of the hill is 30°. If the tower is 50 m high, what is the height of the hill ?**

## Solution:




Let h be the height of the hill and x m be the distance between the foot of the hill and foot of the tower.

In rt. Δ ABC,

cot $60^o = \frac{x}{h}$

x = h cot $60^o$ ...............(i)

In rt. Δ DBC,

cot $30^o = \frac{x}{50}$

x = 50 cot $30^o$ ...............(ii)

Equating (i) and (ii)

h cot $60^o$ = 50 cot $30^o$

h = 50 cot $30^o$ / cot $60^o$

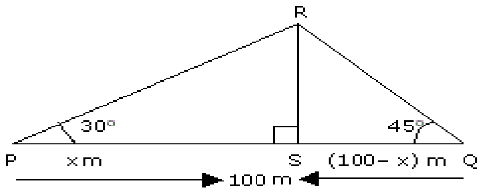
$h = 50 \times \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = 50 \times 3 = 150$ m

Therefore the height of the hill is 150 m.

## Question-7

**There is a small island in the middle of a 100 m wide river and a tall tree stands on the island. P and Q are points directly opposite each other on the two banks, and in line with the tree. If the angles of elevation of the top of the tree from P and Q are respectively 30° and 45°, find the height of the tree.**

**Solution:**




Let PQ be the width of the river and RS be the height of the tree on the island.

In rt. Δ PRS,

x = RS cot $30^o$

x = RS $\sqrt{3}$

x = $\sqrt{3}$ RS ...................(i)

In rt. Δ RSQ,

SQ = RS cot $45^o$

(100 – x) = RS

x = 100 - RS ...................(ii)

Equating (i) and (ii) we have:

$\sqrt{3}$ RS = 100 – RS

2.73 RS = 100

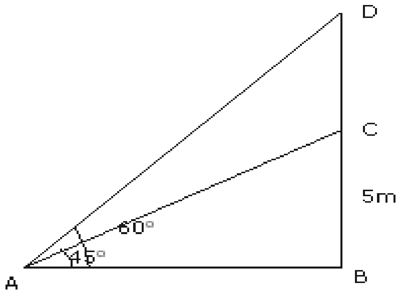
RS = 36.63 m

Therefore the height of the tree is 36.63 m.

## Question-8

**A flag-staff stands at the top of a 5 m high tower. From a point on the ground, the angle of elevation of the top of the flag-staff is 60° and from the same point, the angle of elevation of the top of the tower is 45°. Find the height of the flag-staff.**

### Solution:




Let BC be the height of the tower and DC be the height of the flag-staff.

In rt. Δ ABC,

AB = BC cot 45°

AB = 5 m ......................(i)

In rt. Δ ABD,

AB = BD cot 60°

AB = (5 + CD) $\frac{1}{\sqrt{3}}$ ......................(ii)

Equating (i) and (ii)

(5 + CD) $\frac{1}{\sqrt{3}}$ = 5

(5 + CD) = 5$\sqrt{3}$

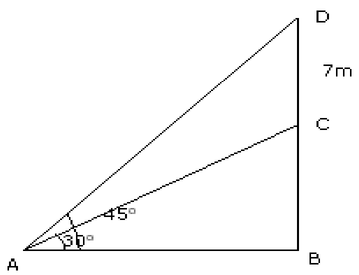
CD = 5$\sqrt{3}$ - 5 = 5(1.732 - 1) = 5 × 0.732 = 3.66 m

Therefore the height of the flag-staff is 3.66 m

## Question-9

**A vertical tower stands on a horizontal plane and is surmounted by a flag-staff of height 7 m. From a point on the plane, the angle of elevation of the bottom of the flag-staff is 30° and that of the top of the flag-staff is 45°. Find the height of the tower.**

### Solution:




Let BC be the height of the tower and DC be the height of the flag-staff.

In rt. Δ ABC,

AB = BC cot 30°

AB = BC$\sqrt{3}$ ......................(i)

In rt. Δ ABD,

AB = BD cot 45°

AB = (BC + CD) cot 45°

AB = (BC + 7) ......................(ii)

Equating (i) and (ii)

(BC + 7) = BC$\sqrt{3}$

BC($\sqrt{3}$ - 1) = 7

BC = 7/0.73 = 9.58 m

Therefore the height of the tower is 9.58 m.

## Question-10

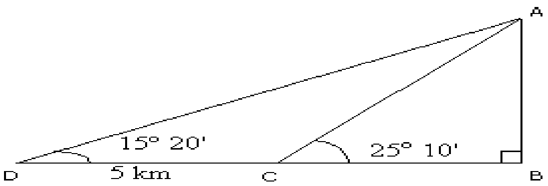
**Determine the height of a mountain if the elevation of its top at an unknown distance from the base is 25° 10' and at a distance of 5 km further off from the mountain, along the same line, the angle of elevation is 15° 20'. Give the answer in km correct to 2 decimals.**

**Solution:**

In the figure, A is the mountain top. C and D are points of observation.

In rt. Δ ABC, tan 25° 10' = $\frac{AB}{BC}$

BC tan 25°10' = AB




BC = $\frac{AB}{\tan 25^{0}10'}$

BC = $\frac{AB}{0.4699}$

In rt. Δ ABD. tan 15° 20' = $\frac{AB}{}$

In rt. Δ ABD, tan 15° 20' = $\frac{AB}{BD}$

tan 15° 20' = $\frac{AB}{BC+5}$ [BD = BC + 5]

0.2742 = $\frac{AB}{BC+5}$

Substituting BC = $\frac{AB}{0.4699}$

$\frac{AB}{\frac{AB+2.3495}{0.4699}}$ = 0.2742

0.4699 AB = 0.2742(AB + 2.3495)

0.4699 AB - 0.2742 AB = 0.644

0.1957 AB = 0.644

AB = $\frac{0.644}{0.1957}$ = 3.29 km

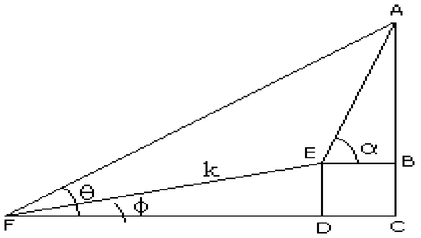
∴ The height of a mountain is 3.29 km.

## Question-11

**The angle of elevation of a cliff from a fixed point A is θ. After going up a distance of k metres towards the top of the cliff at an angle φ, it is found that the angle of elevation is α. Show that the height of the cliff in metre is $\frac{k(\cos\phi - \sin\phi \cot\alpha)}{\cot\theta - \cot\alpha}$.**

## Solution:

In rt. Δ FED,




FE = k m

sin φ = $\frac{ED}{FE}$

∴ ED = FE sin φ = k sin φ

Again, cos φ = $\frac{FD}{FE}$

∴ FD = FE cos φ = k cos f ......................(1)

In rt. Δ AFC, $\frac{FC}{AC}$ = cot θ

FC = AC cot q ...................................(2)

FC - FD = DC

But, DC = EB

∴ EB = FC - FD

Thus, EB = AC cot θ - k cos f (from(1) and (2))

Now, AB = AC - BC

But, BC = ED = k sin φ

∴ AB = AC - k sin φ

In rt. Δ AEB, cot α = $\frac{EB}{AB}$

cotα = $\frac{AC\cot\theta - k\cos\phi}{AC - k\sin\phi}$

AC cot θ - k cos φ = AC cot α - k sin φ cot α

AC cot θ - AC cosα = k cos φ - k sin φ cot α

AC (cot θ - cot α ) = k cos φ - k sin φ cotα

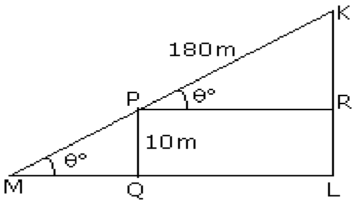
AC = $\frac{k(\cos\phi - \sin\phi \cot\alpha)}{\cot\theta - \cot\alpha}$

Hence proved the height of the cliff in metres is $\frac{k(\cos\phi - \sin\phi \cot\alpha)}{\cot\theta - \cot\alpha}$.

## Question-12

**The length of a string between a kite and a point on the roof of the building 10 m high is 180 m. If the string makes an angle θ with the level ground such that tan θ = 4/3 how high is the kite from the ground ?**

## Solution:




In Δ KPR, ∠KPR = θ, KP = 180 m and $\tan\theta = \frac{4}{3}$

$\tan\theta = \frac{4}{3} = \frac{\text{opposite side}}{\text{adjacent side}}$ ⇒ Hypotenuse = $\sqrt{4^2 + 3^2} = 5$ ⇒ $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ $= \frac{4}{5}$ ⇒ $\sin\theta = \frac{KR}{KP} = \frac{4}{5}$ ⇒ $\frac{KR}{180} = \frac{4}{5}$ ∴ $KR = \frac{4 \times 180}{5} = 144$ m

In rectangle PQLR, PQ = RL = 10 m (Opposite sides of a rectangle) ∴ KL = KR + RL = (144 + 10) m

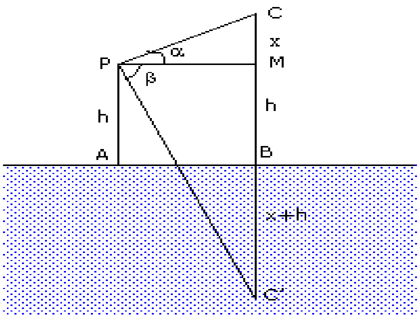
= 154 m

∴ The height of the kite from the ground is 154 m.

## Question-13

**If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β , prove that the height of the cloud is $\frac{h(\tan\alpha + \tan\beta)}{(\tan\beta - \tan\alpha)}$ .**

**Solution:**




Let AB be the surface of the lake and let P be the point of the observation such that AP = h metres. Let C be the position of the cloud and C' be its reflection in the lake.

Then, CB = C'B. Let PM be perpendicular from P on CB. Then ∠ CPM = α and ∠ MPC' = β. Let CM = x.

Then CB = CM + MB = CM + MB = CM + PA = x + h.

In Δ CPM, we have

tan α = CM /PM ⇒ tan α = x/AB [Since PM = AB] ⇒ AB = x cot α ------------(i)

In Δ PMC', we have

tan β = C'M /PM ⇒ tan β = x+2h/AB [Since C'M = C'B + BM = x + h + h] ⇒ AB = (x + 2h) cot β ---------------(ii)

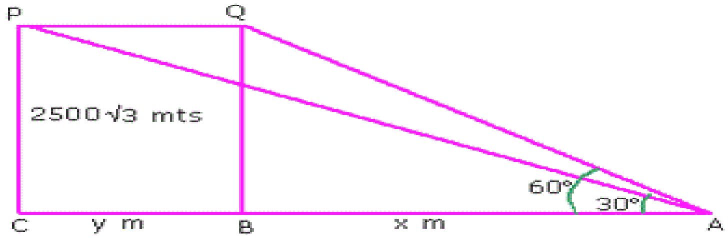
From (i) and (ii),

x cot α = (x + 2h) cot β x (cot α - cot β ) = 2h cot β ⇒ x(1/tan α - 1/tan β ) = 2h /tan β ⇒ $x = \frac{2h\tan\alpha}{(\tan\beta - \tan\alpha)}$ ∴ The height of the cloud = x + h = $\frac{2h\tan\alpha}{(\tan\beta - \tan\alpha)} + h$ $= \frac{h(\tan\alpha + \tan\beta)}{(\tan\beta - \tan\alpha)}$.

## Question-14

**An aeroplane flying horizontally at height of 2500√3 mts above that ground; is observed to be at angle of elevation 60° from the ground. After a flight of 25 seconds the angle of elevation is 30° . Find the speed of the plane in m/sec.**

**Solution:**




Let P and Q be the two positions of the aeroplane.

QB = 2500√3 mts

Let the speed of the aeroplane be s m/sec.

$\tan 60° = \frac{2500\sqrt{3}}{x}$

$\Rightarrow \frac{2500\sqrt{3}}{x} = \sqrt{3}$ ∴ $x = \frac{2500\sqrt{3}}{\sqrt{3}} = 2500$ m

$\tan 30° = \frac{2500\sqrt{3}}{x+y} \Rightarrow \frac{2500\sqrt{3}}{x+y} = \frac{1}{\sqrt{3}} \Rightarrow x + y = 2500\sqrt{3} \times \sqrt{3}$

$= 2500 \times 3$

$= 7500$ ∴ $y = 7500 - 2500$

$= 5000$ m

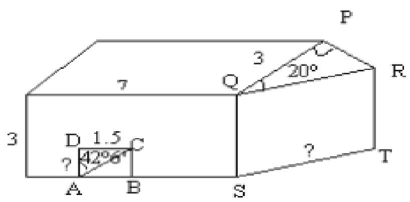
To travel 5000 m the aeroplane takes 25 seconds. ∴ The speed of the plane

$= \frac{5000}{25} = 200$ m/sec.

## Question-15

**In the figure below, some dimensions of a hut have been marked. Find the other dimensions (marked ?) of the hut correctly up to one place of decimal.**

**Solution:**




In rt, ΔPQR, ∠P = 90°

∠PQR = 20°

$\sec 20° = \frac{QR}{PQ}$

$1.0642 = \frac{QR}{3}$

QR = 1.0642 × 3 = 3.1926

QR = ST = 3.2 units

In rt. $\Delta$ ACD, $\angle CAD = 42°\ 6'$

$\cot 42°\ 6' = \frac{AD}{DC}$

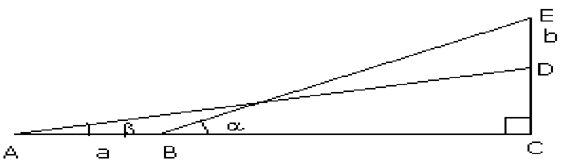
$1.1067 = \frac{AD}{1.5}$

$\therefore AD = 1.5 \times 1.1067 = 1.66$ units

## Question-16

**A ladder rests against a wall at an angle α to the horizontal. When its foot is pulled away from the wall through a distance a, it slides a distance b down the wall and makes an angle β with the horizontal. Show that a/b = (cosα - cosβ )/(sinα - sinβ ).**

### Solution:




Let the length of the ladder be l units.

In rt.$\Delta$ ACD,

$\sin\beta = \frac{DC}{AD} = \frac{DC}{l}$, $\cos\beta = \frac{a + BC}{AD} = \frac{a + BC}{l}$ .............(i)

In rt.$\Delta$ EBC,

$\sin\alpha = \frac{b + DC}{BE} = \frac{b + DC}{l}$, $\cos\alpha = \frac{BC}{BE} = \frac{BC}{l}$ .............(ii)

From (i) and (ii)

$$R.H.S = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha} = \frac{\frac{BC}{l} - \frac{a + BC}{l}}{\frac{DC}{l} - \frac{b + DC}{l}} = \frac{BC - a - BC}{DC - b - DC} = \frac{-a}{-b} = \frac{a}{b} = L.H.S.$$

## Question-17

**The line joining the top of a hill to the foot of the hill makes an angle of 30° with the horizontal through the foot of the hill. There is one temple at the top of the hill and a guest house half way from the foot to the hill . The top of the temple and the top of the guesthouse both make an elevation of 32° at the foot of the hill. If the guesthouse is 1 km away from the foot of the hill along the hill, find the heights of the guest house and the temple.**

### Solution:

In the figure, GB is the hill. AG is the temple. EF is guesthouse. C is foot of the hill.

To find EF and AG.

CE = 1 km or 1000 m

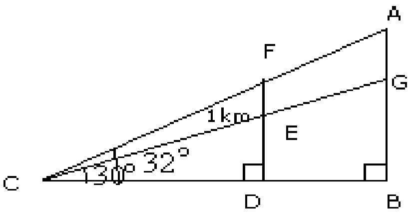
In rt. $\Delta CED$, $\frac{DC}{CE} = \cos 30°$

$\frac{CD}{1000} = \sqrt{3}/2$

$CD = 1000\sqrt{3}/2 = 866$ m

$\frac{DE}{CE} = \sin 30°$

$\frac{DE}{1000} = \frac{1}{2}$




DE = $\frac{1}{2}$ ×1000 = 500 m

In rt. ΔCFD,

$\frac{DF}{CD} = \tan 32^{o}$

$\frac{DF}{866} = 0.6249$

\ DF = 0.6249 × 866 = 541.16 m

DE = 500 m.

\ EF = 541.16 - 500 = 41.16 m

Since E is midpoint of CG. (given halfway)

∴ CG = 2000 m.

$\frac{BG}{CG} = \sin 30^{o}$

$\frac{BG}{2000} = \frac{1}{2}$

\ BG = 1000 m

In rt. Δ CBG,

$\frac{CB}{CG} = \cos 30°$

$\frac{CB}{2000} = \frac{\sqrt{3}}{2}$

∴ CB = 1732 m

In Δ CDF and Δ CBA,

∠CDF = ∠CBA = 90°

∠ DCF = ∠ BCA (common)

∴  Δ CDF ~ Δ CBA

$\therefore \frac{CD}{CB} = \frac{DF}{AB}$

$\Rightarrow \frac{866}{1732} = \frac{541.16}{AB}$

$\Rightarrow AB = \frac{541.16 \times 1732}{866}$

∴ AB = 1082.32 m

\ AG = 1082.32 - 1000 = 82.32 m

∴ The height of guest house is 41 m and the height of temple is 82 m

## Question-18

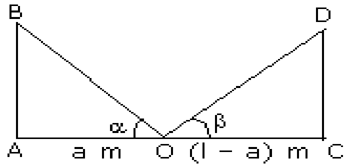
**A man standing 'a' metres behind and opposite the middle of a football goal observes that the angle of elevation of the nearer cross-bar is α and that of**

**the further crossbar is β. Show that the length of the field is, a(tanα cotβ + 1).**

## Question-18

**A man standing 'a' metres behind and opposite the middle of a football goal observes that the angle of elevation of the nearer cross-bar is α and that of the further crossbar is β. Show that the length of the field is, a(tanα cotβ + 1).**

### Solution:




Let AB and CD be the cross bars of the football goal.

Let O be a point, 'a' metres behind and opposite the middle of the football goal.

Let 'l' metres be the length of the field.

Let AB = CD = p m since AB and CD are the cross bars of the football goal.

In rt.ΔBAO,

$\frac{AB}{AO} = \tan\alpha = \tan\alpha \; a = \frac{p}{\tan\alpha}$

In rt.ΔDCO,

$\frac{CD}{CO} = \tan\beta = \tan\beta \; l - a = \frac{p}{\tan\beta}$

Length of the field = AO + OC

$= a + (l - a)$

$= \frac{p}{\tan\alpha} + \frac{p}{\tan\beta}$

By replacing p = a tan α we get,

$= \frac{a\tan\alpha}{\tan\alpha} + \frac{a\tan\alpha}{\tan\beta}$

$= a + \frac{a\tan\alpha}{\tan\beta}$

$= \frac{a(\tan\beta + \tan\alpha)}{\tan\beta}$

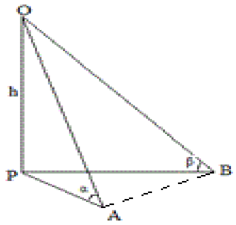
$= a(1 + \tan\alpha\cot\beta)$ m

## Question-19

**The angle of elevation of the top of a tower from a point A due south of the tower is α and from B due east of the tower is β.**

**If AB = d, show that the height of the tower is $\frac{d}{\sqrt{\cot^2\alpha + \cot^2\beta}}$.**

**Solution:**




Let OP be the tower and let A and B be two points due south and east respectively of the tower such that ∠OAP = α and ∠OBP = β. Let OP = h.

In ΔOAP, we have

$\tan\alpha = \frac{h}{OA}$

OA = h cot α ................(i)

In ΔOBP, we have

$\tan\beta = \frac{h}{OB}$

OB = h cot β ..................(ii)

Since OAB is a right angled triangle. Therefore,

$AB^2 = OA^2 + OB^2$

$d^2 = h^2 \cot^2\alpha + h^2 \cot^2\beta$

$h = \frac{d}{\sqrt{\cot^2\alpha + \cot^2\beta}}$ [Using (i) and (ii)].

## Question-20

**A tower subtends an angle α at a point A in the place of its base and the angle of depression of the foot of the tower at a point b ft. just above A is β. Prove that the height of the tower is b tanα cotβ.**

**Solution:**

Let x be the distance of the point A from the foot of the tower and h be the height of the tower.

In Δ APQ,

h = x tanα ......(i)

In Δ PRB

b = x tanβ ......(ii)

From (i) and (ii),

h = b.tanα /tanβ = b.tanα cotβ

Therefore the height of the tower is b tanα cotβ.