Surface Areas and Volumes

- **Curve Surface area of combination of solids:**
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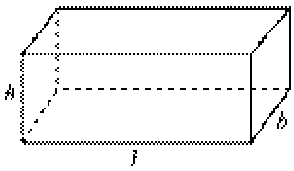
    - Curved surface area of a cuboid = $2h(l + b)$



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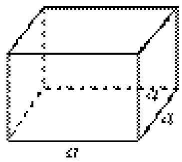
    - Curved surface area of a cube = $4a^2$



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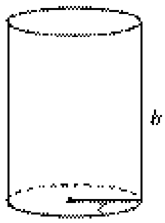
    - Curved surface area (CSA) of a cylinder = $2\pi rh$



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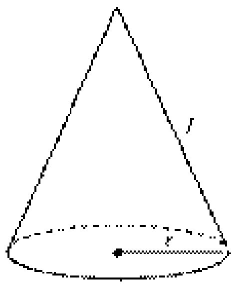
    - Slant height of the cone, $l = \sqrt{r^2 + h^2}$, where h is the height of the cone

    Curved surface area (CSA) of a cone = πrl



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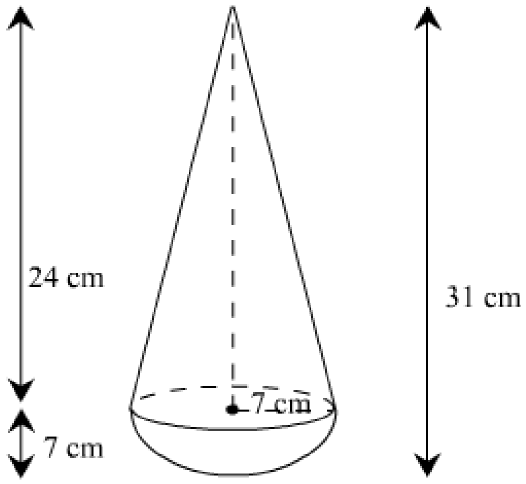
    - Curved surface area (CSA) of a hemisphere = $2\pi r^2$

Note: Height of the hemisphere = Radius of the hemisphere

Example:

A toy is in the form of a hemisphere mounted by a cone. The diameter of the hemisphere is 14 cm and height of the whole toy is 31 cm. If the surface of the toy is painted at the rate of Rs 1 per 6 cm^2, then find the cost required to paint the entire toy.

Solution:



It is given that the diameter of the hemisphere is 14 cm.

∴ Radius of the hemisphere, r = 7 cm

Radius of the base of the cone, r = 7 cm

From the figure, height of cone, h = 31 – 7 = 24 cm

Now, $l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 7^2} = \sqrt{625} = 25 cm$

∴ Surface area of the toy = C.S.A. of hemisphere + C.S.A. of cone

$= 2\pi r^2 + \pi rl$

$= \pi r(2r + l)$

$= \frac{22}{7} \times 7 \times (2 \times 7 + 25)$

= 858 cm^2

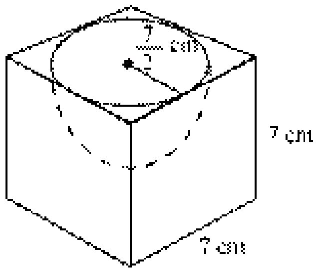
∴ Cost required for painting = 858 × $\frac{1}{6}$ = Rs 143.

- **Total Surface area of combination of solids**

Example:

A hemispherical depression of the greatest possible diameter is cut out from one face of a cubical wooden block. If the edge of the cube is 7 cm long, then find the surface area of the remaining solid.

Solution:



It is clear that the greatest possible diameter of the hemisphere is 7 cm.

Radius of hemisphere = $\frac{7}{2}$ cm

Total Surface area of the remaining solid = TSA of the cube + CSA of the hemisphere – Area of the base of the hemisphere

TSA of the cube $= 6\times(7\text{ cm})^2 = 6\times 49\text{ cm}^2 = 294\text{ cm}^2$

CSA of the hemisphere = $2\times\pi\times\left(\frac{7}{2}\right)^2\text{ cm}^2 = 2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\text{ cm}^2 = 77\text{ cm}^2$

Area of the base of the hemisphere $= \pi\times\left(\frac{7}{2}\right)^2\text{ cm}^2 = \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\text{ cm}^2 = 38.5\text{ cm}^2$

Therefore, area of the remaining solid = (294 + 77 – 38.5) cm^2 = 332.5 cm^2

- **Volume of combination of solids**

1. Volume of a cuboid = $l \times b \times h$, where l, b, h are respectively length, breadth and height of the cuboid.
2. Volume of a cube = a^3, where a is the edge of the cube.
3. Volume of a cylinder = $\pi r^2 h$, where r is the radius and h is the height of the cylinder.
4. Volume of a cone $= \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height of the cone.
5. Volume of a sphere of radius $r = \frac{4}{3}\pi r^3$.
6. Volume of a hemisphere of radius $r = \frac{2}{3}\pi r^3$.

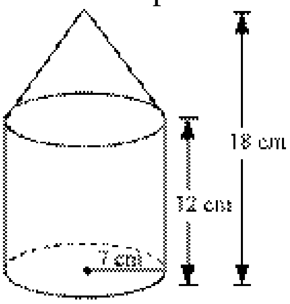
Note: Volume of the combination of solids is the sum of the volumes of the individual solids.

Example:

A solid is in the shape of a cylinder surmounted by a cone. The diameter of the base of the solid is 14 cm and the height of the solid is 18 cm. If the length of the cylindrical part in 12 cm, then find the volume of the solid.

Solution:

Volume of the solid = Volume of the cylindrical part + Volume of the conical part



Diameter of the base = 14 cm

Radius of the base $= \frac{14}{2} \text{ cm} = 7 \text{ cm}$

Radius of the conical part = Radius of the cylindrical part = 7 cm

Height of the solid = 18 cm

Height of the cylindrical part = 12 cm

Height of the conical part = (18 – 12) cm = 6 cm

Volume of the cylindrical part $= \pi \times (\text{Radius})^2 \times \text{Height} = \frac{22}{7} \times 7^2 \times 12 \text{ cm}^3 = 1848 \text{ cm}^3$

Volume of the conical part $= \frac{1}{3} \pi \times (\text{Radius})^2 \times \text{Height} = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 6 \text{ cm}^3 = 308 \text{ cm}^3$

Thus, volume of the solid = (1848 + 308) cm^3 = 2156 $cm^{3.}$

- **Conversion of a solid from one shape into another**

  When a solid is converted into another solid of a different shape, the volume of the solid does not change.

Example:

A metallic block of dimensions 8.4 cm × 6 cm × 4.4 cm is melted and recast into the shape of a cone of radius 4.2 cm. Find the height of the cone.

Solution:

Volume of the cuboidal block = Length × Breadth × Height

= 8.4 × 6 × 4.4 cm^3

Let h be the height of the cone.
Radius of the cone = 4.2 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 4.2^2 \times h$

Since the cuboidal block is converted into the shape of a cone
Volume of the cuboid = Volume of the cone

$$\Rightarrow (8.4 \times 6 \times 4.4)\ \text{cm}^3 = \frac{1}{3}\pi \times (4.2\ \text{cm})^2 \times h$$

$$\Rightarrow h = \frac{8.4 \times 6 \times 4.4 \times 3 \times 7}{4.2 \times 4.2 \times 22}\ \text{cm} = 12\ \text{cm}$$

Thus, the height of the cone is 12 cm.

- **Frustum of a cone**
  - CSA of the frustum of a cone $= \pi(r_1 + r_2)l$
  - TSA of the frustum of a cone

$$= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$$

where r_1 and r_2 are the radii of the $(r_1 > r_2)$ ends of the frustum of a cone, and $l = \sqrt{h^2 + ($

Example:

A bucket, made up of aluminium sheet, is opened from the top and is in the shape of the frustum of a cone of slant height 32 cm, with the radii of its lower and upper ends being 7 cm and 14 cm respectively. Find the area of the aluminium sheet used for making it.

Solution:

The area of the aluminium sheet used for making the bucket
= CSA of the frustum of the cone + Area of the lower base

Now, the CSA of the frustum of the cone $= \pi(r_1 + r_2)l$, where r_1 and r_2 are radii of the lower and upper ends and l is the slant height of the bucket

CSA of the frustum of the cone

$$= \pi(7 + 44) \times 32\ \text{cm}^2 = \frac{22}{7} \times 21 \times 32\ \text{cm}^2 = 2112\ \text{cm}^2$$

Area of the lower base $= \frac{22}{7} \times 7 \times 7\ \text{cm}^2 = 154\ \text{cm}^2$

Thus, the area of the aluminium sheet used = (2112 + 154) cm^2 = 2266 cm^2

- **Frustum of a cone**

Volume of the frustum of a cone

$$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1.r_2)$$

where r_1 and r_2 are the radii of the ends ($r_1 > r_2$) of the frustum of a cone and h is the height of the frustum.

Example:

If the radii of the circular ends of a conical bucket which is 42 cm high, are 21 cm and 14 cm. If the bucket is filled will milk. Find the quantity of milk (in litres) in the bucket.

Solution:

Clearly, bucket forms a frustum of a cone such that the radii of its circular ends are r_1 = 21 cm, r_2 = 14 cm and height h = 42 cm. Therefore,

Capacity of milk in bucket = volume of the frustum

$$= \frac{1}{3}\pi h\left(r_1^2 + t_2^2 + r_1 \cdot r_2\right)$$

$$= \frac{1}{3}\times\frac{22}{7}\times 42(21^2 + 14^2 + 21\times 14)$$

$$= 44(441 + 196 + 294)$$

$$= 40964 \text{ cm}^3$$

We know

1 litre = 1000 cm3

$$\therefore 1\,\text{cm}^3 = \frac{1}{1000}\text{Litres}$$

Therefore, quantity of milk in bucket = $40964\times\frac{1}{1000}\text{litres} = 40.964 \text{ Litres}$