

**Class XII Session 2023-24**  
**Subject - Mathematics**  
**Sample Question Paper - 2**

**Time Allowed: 3 hours**

**Maximum Marks: 80**

**General Instructions:**

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passages based/integrated units of assessment (4 marks each) with sub parts.

**Section A**

1. If  $(A - 2B) = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$  and  $(2A - 3B) = \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$  then  $B = ?$  [1]  
a)  $\begin{bmatrix} -4 & 6 \\ -3 & -3 \end{bmatrix}$  b) None of these  
c)  $\begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix}$  d)  $\begin{bmatrix} 6 & -4 \\ -3 & 3 \end{bmatrix}$
2. Let  $A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$  where  $0 \leq \theta \leq 2\pi$ . Then [1]  
a)  $\text{Det}(A) = 0$  b)  $\text{Det}(A) \in [2, 4]$   
c)  $\text{Det}(A) \in (2, 4)$  d)  $\text{Det}(A) \in (2, \infty)$
3. The system of equations  $x + 2y = 5$ ,  $4x + 8y = 20$  has [1]  
a) None of these b) no solution  
c) a unique solution d) infinitely many solutions
4. At  $x = 2$ ,  $f(x) = [x]$  is [1]  
a) Continuous but not differentiable b) None of these  
c) Continuous as well as differentiable d) Differentiable but not continuous
5. The lines  $l_1$  and  $l_2$  intersect. The shortest distance between them is [1]  
a) infinity b) negative  
c) positive d) zero

6. The degree of the differential equation  $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right)$  is [1]  
 a) not defined b) 1  
 c) 2 d) 3

7. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let  $Z = px + qy$ , where  $p, q > 0$ . Condition on  $p$  and  $q$  so that the maximum of  $Z$  occurs at both the points (15, 15) and (0, 20) is [1]  
 a)  $q = 3p$  b)  $q = 2p$   
 c)  $p = q$  d)  $p = 2q$

8. If  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$  Then [1]  
 a)  $\vec{a} \perp \vec{b}$  b) none of these  
 c)  $|\vec{a}| = |\vec{b}|$  d)  $\vec{a} \parallel \vec{b}$

9.  $\int \sqrt[3]{x} dx = ?$  [1]  
 a)  $\frac{4}{3}x^{\frac{4}{3}} + C$  b)  $\frac{3}{4}x^{\frac{4}{3}} + C$   
 c)  $\frac{3}{2}x^{\frac{2}{3}} + C$  d)  $\frac{4}{3}x^{\frac{3}{4}} + C$

10. Total number of possible matrices of order  $3 \times 3$  with each entry 2 or 0 is [1]  
 a) 27 b) 81  
 c) 9 d) 512

11. Maximize  $Z = 5x + 3y$ , subject to constraints  $x + y \leq 300$ ,  $2x + y \leq 360$ ,  $x \geq 0$ ,  $y \geq 0$ . [1]  
 a) 1020 b) 1050  
 c) 1040 d) 1030

12. If  $\vec{a} \cdot \vec{b} = 0$  and  $\vec{a} \times \vec{b} = 0$ , then which one of the following is correct? [1]  
 a)  $\vec{a}$  is parallel to  $\vec{b}$  b)  $\vec{a} = 0$  or  $\vec{b} = 0$   
 c)  $\vec{a}$  is perpendicular to  $\vec{b}$  d) None of these

13. If  $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ , then the value of  $|\text{adj } A|$  is [1]  
 a)  $a^2$  b)  $a^6$   
 c)  $a^9$  d)  $a^{27}$

14. If A and B are events such that  $P(A|B) = P(B|A)$ , then [1]  
 a)  $A \subset B$  but  $A \neq B$  b)  $A = B$   
 c)  $A \cap B = \emptyset$  d)  $P(A) = P(B)$

15. Consider the following statements in respect of the differential equation  $\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0$  [1]  
 i. The degree of the differential equation is not defined.  
 ii. The order of the differential equation is 2.

Which of the above statement(s) is/are correct?



26. Evaluate:  $\int \frac{dx}{(e^x - 1)^2}$ . [3]

27. In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII, what is the probability that a student chosen randomly studies in class XII, given that the chosen student is a girl? [3]

28. Evaluate  $\int_0^\pi e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) dx$ . [3]

OR

Evaluate:  $\frac{3x+1}{\sqrt{5-2x-x^2}} dx$

29. Solve the following differential equation: [3]

$$x dy - (y - x^3) dx = 0$$

OR

Find the particular solution of the differential equation  $(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ , given that  $y = 0$  when  $x = 1$ .

30. Solve the Linear Programming Problem graphically: [3]

Minimize  $Z = x - 5y + 20$  Subject to

$$x - y \geq 0$$

$$-x + 2y \geq 2$$

$$x \geq 3$$

$$y \leq 4$$

$$x, y \geq 0$$

OR

Solve the Linear Programming Problem graphically:

Maximize  $Z = 50x + 30y$  Subject to

$$2x + y \leq 18$$

$$3x + 2y \leq 34$$

$$x, y \geq 0$$

31. Differentiate the function with respect to  $x$ :  $\tan^{-1} \left( \frac{a+b \tan x}{b-a \tan x} \right)$ . [3]

#### Section D

32. Find the area enclosed by the parabola  $y^2 = 4ax$  and the line  $y = mx$ . [5]

33. Let  $n$  be a positive integer. Prove that the relation  $R$  on the set  $Z$  of all integers numbers defined by  $(x, y) \in R \Leftrightarrow x - y$  is divisible by  $n$ , is an equivalence relation on  $Z$ . [5]

OR

Let  $A = \mathbb{R} - \{3\}$ ,  $B = \mathbb{R} - \{1\}$ . If  $f : A \rightarrow B$  be defined by  $f(x) = \frac{x-2}{x-3} \forall x \in A$ . Then, show that  $f$  is bijective.

34. Solve the system of equations [5]

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

35. Find the perpendicular distance of the point  $(1, 0, 0)$  from the line  $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ . Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular. [5]

OR

Show that the lines  $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$  and  $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$  intersect. Also, find the equation of the plane containing them.

#### Section E

36. **Read the text carefully and answer the questions:** [4]

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

- i. 960 of the total applications were the folk genre.
  - ii. 192 of the folk applications were for the below 18 category.
  - iii. 104 of the classical applications were for the 18 and above category.
- (i) What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work.
  - (ii) An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work.
  - (iii) If  $P(A) = 0.4$ ,  $P(B) = 0.8$  and  $P(B|A) = 0.6$ , then  $P(A \cup B)$  is equal to

**OR**

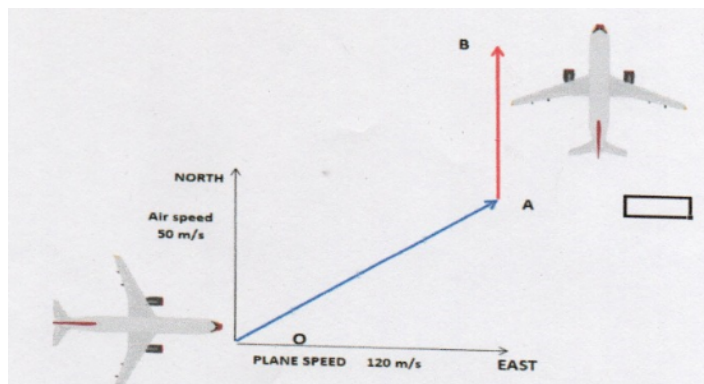
If A and B are two independent events with

$P(A) = \frac{3}{5}$  and  $P(B) = \frac{4}{9}$ , then find  $P(A' \cap B')$ .

37. **Read the text carefully and answer the questions:** [4]

A plane started from airport O with a velocity of 120 m/s towards east. Air is blowing at a velocity of 50 m/s towards the north As shown in the figure.

The plane travelled 1 hr in OA direction with the resultant velocity. From A and B travelled 1 hr with keeping velocity of 120 m/s and finally landed at B.



- (i) What is the resultant velocity from O to A?
- (ii) What is the direction of travel of plane O to A with east?
- (iii) What is the total displacement from O to A?

**OR**

What is the resultant velocity from A to B?

38. **Read the text carefully and answer the questions:** [4]

The temperature of a person during an intestinal illness is given by  $f(x) = -0.1x^2 + mx + 98.6$ ,  $0 \leq x < 12$ , m being a constant, where  $f(x)$  is the temperature in  $^{\circ}\text{F}$  at x days.



- (i) Is the function differentiable in the interval  $(0, 12)$ ? Justify your answer.
- (ii) If 6 is the critical point of the function, then find the value of the constant  $m$ .

## Solution

### Section A

1. (a)  $\begin{bmatrix} -4 & 6 \\ -3 & -3 \end{bmatrix}$

**Explanation:**  $(A - 2B) = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$

Multiplying equation by 2

$$2A - 4B = \begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix} \dots(i)$$

$$2A - 3B = \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix} \dots(ii)$$

(ii) - (i)

$$B = \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix} - \begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 6 \\ -3 & -3 \end{pmatrix}$$

2.

(b)  $\text{Det}(A) \in [2, 4]$

**Explanation:**  $A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$

$$|A| = 1(1 \times 1 - \sin \theta \times (-\sin \theta)) - \sin \theta(-\sin \theta + \sin \theta) + 1[(-\sin \theta) \times (-\sin \theta) - (-1) \times 1]$$

$$|A| = 1 + \sin^2 \theta + \sin^2 \theta + 1$$

$$|A| = 2 + 2 \sin^2 \theta$$

$$|A| = 2(1 + \sin^2 \theta)$$

$$\text{Now, } 0 \leq \theta \leq 2\pi$$

$$\Rightarrow \sin 0 \leq \sin \theta \leq \sin 2\pi$$

$$\Rightarrow 0 \leq \sin^2 \theta \leq 1$$

$$\Rightarrow 1 + 0 \leq 1 + \sin^2 \theta \leq 1 + 1$$

$$\Rightarrow 2 \leq 2(1 + \sin^2 \theta) \leq 4$$

$$\therefore \text{Det}(A) \in [2, 4]$$

3.

(d) infinitely many solutions

**Explanation:**  $x + 2y = 5,$

$$4x + 8y = 20$$

$$\Rightarrow A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 20 \end{bmatrix}$$

$$|A| = 8 - 8 = 0$$

$$\text{adj}A = \begin{bmatrix} 8 & -2 \\ -4 & 1 \end{bmatrix}$$

$$\text{now } (\text{adj } A)B = \begin{bmatrix} 8 & -2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 20 \end{bmatrix} = \begin{bmatrix} 40 - 40 \\ -20 + 20 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow (\text{adj } A)B = 0$$

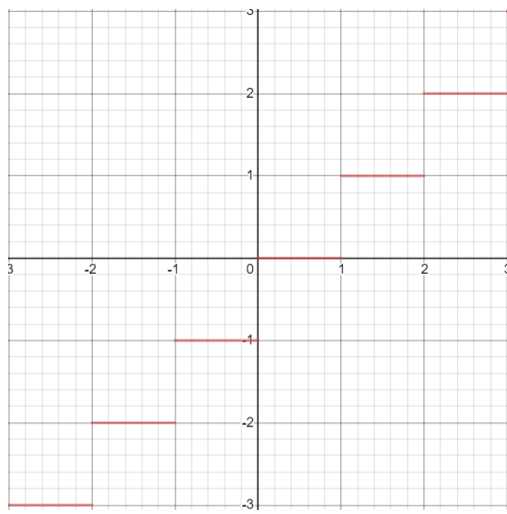
$$\text{Since, } |A|=0 \text{ and } (\text{adj}A)B=0$$

So, the pair of equation have infinitely many solutions

4.

(b) None of these

**Explanation:** Let us see that graph of the floor function, we get



We can see that  $f(x) = [x]$  is neither continuous and non differentiable at  $x = 2$ .

5.

(d) zero

**Explanation:** Since the lines intersect. Hence they have a common point in them. Hence the distance will be zero.

6. (a) not defined

**Explanation:** In general terms for a polynomial the degree is the highest power.

Degree of differential equation is defined as the highest integer power of highest order derivative in the equation

Here the differential equation is  $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right)$

Now for degree to exist the given differential equation must be a polynomial in some differentials.

Here differentials mean  $\frac{dy}{dx}$  or  $\frac{d^2y}{dx^2}$  or ....  $\frac{d^ny}{dx^n}$

The given differential equation is not polynomial because of the term  $\sin \frac{dy}{dx}$  and hence degree of such a differential equation is not defined.

7. (a)  $q = 3p$

**Explanation:** Since Z occurs maximum at (15, 15) and (0, 20), therefore,  $15p + 15q = 0p + 20q \Rightarrow q = 3p$ .

8. (a)  $\vec{a} \perp \vec{b}$

**Explanation:** Here  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$\Rightarrow |a|^2 + 2\vec{a} \cdot \vec{b} + |b|^2 = |a|^2 - 2\vec{a} \cdot \vec{b} + |b|^2$$

$$\Rightarrow 2\vec{a} \cdot \vec{b} = -2\vec{a} \cdot \vec{b}$$

$$\Rightarrow 4\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \perp \vec{b}$$

9.

(b)  $\frac{3}{4}x^{\frac{4}{3}} + C$

**Explanation:** Given integral is  $\int \sqrt[3]{x} dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sqrt[3]{x} dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c$$

$$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$$

$$= \frac{3}{4}x^{\frac{4}{3}} + c$$

10.

(d) 512



**Explanation:** Since each element  $a_{ij}$  can be filled in two ways (with either '2' or '0'), total number of possible matrices is  $8 \times 8 \times 8 = 512$

11. (a) 1020

**Explanation:** Here, Maximize  $Z = 5x + 3y$ , subject to constraints  $x + y \leq 300$ ,  $2x + y \leq 360$ ,  $x \geq 0$ ,  $y \geq 0$ .

Corner points	$Z = 5x + 3y$
P(0, 300)	900
Q(180, 0)	900
R(60, 240)	1020.....(Max.)
S(0, 0)	0

Hence, the maximum value is 1020

12.

(b)  $\vec{a} = 0$  or  $\vec{b} = 0$

**Explanation:** Given that,  $\vec{a} \cdot \vec{b} = 0$ ,

i.e.  $\vec{a}$  and  $\vec{b}$  are perpendicular to each other and  $\vec{a} \times \vec{b} = 0$

i.e.  $\vec{a}$  and  $\vec{b}$  are parallel to each other. So, both conditions are possible iff  $\vec{a} = 0$  and  $\vec{b} = 0$

13.

(b)  $a^6$

**Explanation:**  $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

$$|A| = a^3$$

$$|\text{adj } A| = |A|^{3-1} = |A|$$

$$|\text{adj } A| = (a^3)^2 = a^6$$

14.

(d)  $P(A) = P(B)$

**Explanation:** It is given that :  $P(A | B) = P(B | A)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow \frac{1}{P(B)} = \frac{1}{P(A)} \Rightarrow P(A) = P(B)$$

15. (a) Both (i) and (ii)

**Explanation:** Both (i) and (ii)

16.

(d) None of these

**Explanation:** Given that  $\vec{a} + \vec{b}$  is collinear with  $\vec{c}$

$$\therefore \vec{a} + \vec{b} = x\vec{c} \dots(i)$$

where x is scalar and  $x \neq 0$

$\vec{b} + \vec{c}$  is collinear with  $\vec{a}$

$$\vec{b} + \vec{c} = y\vec{a} \dots(ii)$$

y is scalar and  $y \neq 0$

Subtracting (ii) from (i) we get

$$\vec{a} - \vec{c} = x\vec{c} - y\vec{a}$$

$$\vec{a} + y\vec{a} = x\vec{c} + \vec{c}$$

$$\vec{a}(1 + y) = (1 + x)\vec{c}$$

As given

$\vec{a}, \vec{c}$  are not collinear. (no two vectors are collinear)

$$\therefore 1 + y = 0 \text{ and } 1 + x = 0$$

$$y = -1 \text{ and } x = -1$$

Putting value of x in equation (i)

$$\vec{a} + \vec{b} = -\vec{c}$$

$$\vec{a} + \vec{b} + \vec{c} = 0$$

17.

(d)  $\frac{-2}{(1+x^2)}$

**Explanation:** Given that  $y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

$$\Rightarrow \sec y = \frac{x^2+1}{x^2-1}$$

Since  $\tan^2 x = \sec^2 x - 1$ , thus

$$\tan^2 y = \left(\frac{x^2+1}{x^2-1}\right)^2 - 1 = \frac{4x^2}{(x^2-1)^2}$$

$$\text{Hence, } \tan y = -\frac{2x}{1-x^2} \text{ or } y = \tan^{-1}\left(-\frac{2x}{1-x^2}\right)$$

$$\text{Let } x = \tan \theta \Rightarrow \theta = \tan^{-1} x$$

$$\text{Hence, } y = \tan^{-1}\left(-\frac{2 \tan \theta}{1-\tan^2 \theta}\right)$$

Using  $\tan 2\theta = \frac{2 \tan \theta}{1-\tan^2 \theta}$ , we obtain

$$y = \tan^{-1}(-\tan 2\theta)$$

Using  $-\tan x = \tan(-x)$ , we obtain

$$y = \tan^{-1}(\tan(-2\theta)) = -2\theta = -2\tan^{-1} x$$

Differentiating with respect to x, we obtain

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

18.

(d) coincident

**Explanation:** The equation of the given lines are

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \dots (i)$$

$$\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$$

$$= \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3} \dots (ii)$$

Thus, the two lines are parallel to the vector  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$  and pass through the points (0, 0, 0) and (1, 2, 3).

Now,

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= \vec{0} [\because \vec{a} \times \vec{a} = \vec{0}]$$

So, here the distance between the given two parallel lines is 0, the given lines are coincident.

19. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:** Let  $f(x) = x^2 - 8x + 17$

$$\therefore f'(x) = 2x - 8$$

$$\text{So, } f'(x) = 0, \text{ gives } x = 4$$

Here  $x = 4$  is the critical number

$$\text{Now, } f''(x) = 2 > 0, \forall x$$

So,  $x = 4$  is the point of local minima.

$\therefore$  Minimum value of  $f(x)$  at  $x = 4$ ,

$$f(4) = 4 \times 4 - 8 \times 4 + 17 = 1$$

Hence, we can say that both Assertion and Reason are true and Reason is the correct explanation of the Assertion.

20.

(d) A is false but R is true.

**Explanation:** Assertion is false because every function is not invertible. The function which is one-one and onto i.e. bijective functions are invertible so reason is true.

### Section B

21.  $\operatorname{cosec}^{-1}x$  represents an angle in  $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$  whose cosecant is x.

$$\text{Let } x = \operatorname{cosec}^{-1}(-2)$$

$$\Rightarrow \operatorname{cosec} x = -2 = \operatorname{cosec} \left( -\frac{\pi}{6} \right)$$

$$\Rightarrow x = -\frac{\pi}{6}$$

$\therefore$  Principal value of  $\operatorname{cosec}^{-1}(-2)$  is  $-\frac{\pi}{6}$ .

OR

Let  $\operatorname{cosec}^{-1}(-\sqrt{2}) = y$ . Then,  $\operatorname{cosec} y = -\sqrt{2} = -\operatorname{cosec} \left( \frac{\pi}{4} \right) = \operatorname{cosec} \left( -\frac{\pi}{4} \right)$ .

We know that the range of the principal value branch of

$\operatorname{cosec}^{-1}$  is  $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$  and  $\operatorname{cosec} \left( -\frac{\pi}{4} \right) = -\sqrt{2}$ .

Therefore, the principal value of  $\operatorname{cosec}^{-1}(-\sqrt{2})$  is  $-\frac{\pi}{4}$ .

22. Given:  $f(x) = 2x^3 - 24x + 107$

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 24x + 107)$$

$$\Rightarrow f'(x) = 6x^2 - 24$$

For  $f(x)$  let's find critical point, for this we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 6x^2 - 24 = 0$$

$$\Rightarrow 6(x^2 - 4) = 0$$

$$\Rightarrow (x - 2)(x + 2) = 0$$

$$\Rightarrow x = -2, 2$$

clearly,  $f'(x) > 0$  if  $x < -2$  and  $x > 2$

and  $f'(x) < 0$  if  $-2 < x < 2$

Thus, the function  $f(x)$  increases on  $(-\infty, -2) \cup (2, \infty)$  and  $f(x)$  is decreasing on interval  $x \in (-2, 2)$ .

23. We know that Volume of right circular cone =  $\frac{\pi r^2 h}{3}$

$$\frac{\partial V}{\partial t} = \frac{\pi}{3} \left( 2rh \frac{\partial r}{\partial t} + r^2 \frac{\partial h}{\partial t} \right)$$

$$\frac{\partial v}{\partial t} = \frac{\pi}{3} (108 \times -3 + 81 \times 2)$$

$$\frac{\partial v}{\partial t} = \frac{\pi}{3} (-162) = -54\pi \text{ cm}^2/\text{min}$$

Therefore Volume is decreasing at rate  $54\pi \text{ cm}^2/\text{min}$ .

OR

Given curve is,

$$6y = x^3 + 2$$

$$\Rightarrow 6 \frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt} \dots (i)$$

$$\text{Given: } \frac{dy}{dt} = 2 \cdot \frac{dx}{dt} \dots (ii)$$

$$\text{from (i) and (ii), } 2 \left( 2 \frac{dx}{dt} \right) = x^2 \cdot \frac{dx}{dt}$$

$$\Rightarrow x = \pm 2$$

$$\text{when } x = 2, y = \frac{5}{3}; \text{ when } x = -2, y = -1$$

Therefore, Points are  $\left( 2, \frac{5}{3} \right)$  and  $(-2, -1)$

24. Let  $I = \int_0^1 \log(1+x) dx$ , then

$$I = \int_0^1 \log(1+x) \times 1 dx$$

$$= [\log(1+x)x]_0^1 - \int_0^1 \frac{x}{1+x} dx$$

$$= [\log(1+x)x]_0^1 - \int_0^1 \left( 1 - \frac{1}{1+x} \right) dx$$

$$= [x \log(1+x)]_0^1 - [x - \log(1+x)]_0^1$$

$$= \log 2 - 1 + \log 2$$

$$= 2 \log 2 - 1$$

$$= \log 4 - \log e$$

$$= \log \frac{4}{e}$$

25. we have,  $f(x) = kx^3 - 9x^2 + 9x + 3$

$$\Rightarrow f'(x) = 3kx^2 - 18x + 9$$

Since  $f(x)$  is increasing on  $\mathbb{R}$ , therefore,  $f'(x) > 0 \forall x \in \mathbb{R}$

$$\Rightarrow 3kx^2 - 18x + 9 > 0, \forall x \in R$$

$$\Rightarrow kx^2 - 6x + 3 > 0, \forall x \in R$$

$$\Rightarrow k > 0 \text{ and } 36 - 12k < 0 \quad [\because ax^2 + bx + c > 0, \forall x \in R \Rightarrow a > 0 \text{ and discriminant} < 0]$$

$$\Rightarrow k > 3$$

Hence,  $f(x)$  is increasing on  $R$ , if  $k > 3$ .

### Section C

26. Putting  $t = e^x - 1$

$$e^x = t + 1$$

$$dt = e^x dx$$

$$\frac{dt}{e^x} = dx$$

$$\frac{dt}{t+1} = dx$$

Putting above by have by partial fractions.  $\frac{1}{(1+t)t^2} = \frac{A}{t+1} + \frac{Bt+C}{t^2} \dots (1)$

$$A(t^2) + (Bt + C)(t + 1) = 1$$

$$\text{Put } t + 1 = 0$$

$$t = -1$$

$$A = 1$$

Equating coefficients

$$A + B = 0$$

$$1 + B = 0$$

$$B = -1$$

$$C = 1$$

From equation (1), we get,

$$\frac{1}{(1+t)t^2} = \frac{1}{t+1} + \frac{-t+1}{t^2}$$

$$\int \frac{1}{(1+t)t^2} dt = \int \frac{1}{t+1} dt - \int \frac{t}{t^2} dt + \int \frac{1}{t^2} dt$$

$$= \log|t+1| - \int \frac{1}{t} dt + \int \frac{1}{t^2} dt$$

$$= \log|t+1| - \log|t| - \frac{1}{t} + c$$

$$\int \frac{1}{(e^x-1)^2} dx = \log|e^x| - \log|e^x-1| - \frac{1}{e^x-1} + c$$

27. Let 'A' be the event that the chosen student studies in class XII and B be the event that the chosen student is a girl.

There are 430 girls out of 1000 students

$$\text{So, } P(B) = P(\text{Chosen student is girl}) = \frac{430}{1000} = \frac{43}{100}$$

Since, 10% of the girls studies in class XII

So, total number of girls studies in class XII

$$= \frac{10}{100} \times 430 = 43$$

Then,  $P(A \cap B) = P(\text{Chosen student is a girl of class XII})$

$$= \frac{43}{1000}$$

$\therefore$  Required probability =  $P(A/B)$

$$= \frac{P(A \cap B)}{P(B)} \quad \left[ \because P(A/B) = \frac{P(A \cap B)}{P(B)} \right]$$

$$= \frac{43/1000}{43/100} = \frac{1}{10}$$

28. According to the question,  $I = \int_0^\pi e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) dx \dots (i)$

$$\text{Consider, } I_1 = \int_I e^{2x} \sin\left(\frac{\pi}{4} + x\right) dx \dots (ii)$$

By using integration by parts, we get

$$= \sin\left(\frac{\pi}{4} + x\right) \int e^{2x} dx - \int \left\{ \frac{d}{dx} \sin\left(\frac{\pi}{4} + x\right) \int e^{2x} dx \right\} dx$$

$$= \sin\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} - \int \cos\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} dx$$

$$= \frac{e^{2x}}{2} \sin\left(\frac{\pi}{4} + x\right) - \frac{1}{2} \int_I e^{2x} \cos\left(\frac{\pi}{4} + x\right) dx$$

By using integration by parts for second integral, we get

$$= \frac{e^{2x}}{2} \sin\left(\frac{\pi}{4} + x\right) - \frac{1}{2} \left[ \cos\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} - \int -\sin\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} dx \right]$$

$$\begin{aligned}
&= \frac{e^{2x}}{2} \sin\left(\frac{\pi}{4} + x\right) - \frac{e^{2x}}{4} \cos\left(\frac{\pi}{4} + x\right) - \frac{1}{4} \int e^{2x} \sin\left(\frac{\pi}{4} + x\right) dx \\
\Rightarrow I_1 &= \frac{e^{2x}}{4} \left\{ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right\} - \frac{1}{4} I_1 \quad [\text{From eq.(ii)}] \\
\Rightarrow I_1 + \frac{1}{4} I_1 &= \frac{e^{2x}}{4} \left\{ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right\} \\
\Rightarrow \frac{5}{4} I_1 &= \frac{e^{2x}}{4} \left\{ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right\} \\
\Rightarrow I_1 &= \frac{e^{2x}}{5} \left\{ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right\}
\end{aligned}$$

From (i) and (ii), we get

$$\begin{aligned}
\therefore I &= [I_1]_0^\pi \\
&= \left[ \frac{e^{2x}}{5} \left\{ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right\} \right]_0^\pi \\
&= \frac{1}{5} \left[ e^{2\pi} \left\{ 2 \sin\left(\frac{\pi}{4} + \pi\right) - \cos\left(\frac{\pi}{4} + \pi\right) \right\} - e^0 \left\{ 2 \sin\left(\frac{\pi}{4} + 0\right) - \cos\left(\frac{\pi}{4} + 0\right) \right\} \right] \\
&= \frac{1}{5} \left[ e^{2\pi} \left\{ -2 \sin \frac{\pi}{4} + \cos \frac{\pi}{4} \right\} - e^0 \left\{ 2 \sin \frac{\pi}{4} - \cos \frac{\pi}{4} \right\} \right] \\
&= \frac{1}{5} \left[ e^{2\pi} \left\{ -2 \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right\} - 1 \left\{ 2 \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right\} \right] \\
&= \frac{1}{5} \left[ e^{2\pi} \left\{ -\frac{1}{\sqrt{2}} \right\} - \frac{1}{\sqrt{2}} \right] \\
&= -\frac{1}{5\sqrt{2}} [e^{2\pi} + 1] \\
\therefore I &= -\frac{1}{5\sqrt{2}} [e^{2\pi} + 1] \text{ sq units.}
\end{aligned}$$

OR

Let the given integral be,

$$I = \frac{3x+1}{\sqrt{5-2x-x^2}} dx$$

$$\text{Let } 3x+1 = \lambda \frac{d}{dx} (5-2x+x^2) + \mu$$

$$= \lambda(-2-2x) + \mu$$

$$3x+1 = (-2\lambda)x + 2\lambda + \mu$$

Comparing the coefficients of like powers of x,

$$-2\lambda = 3 \Rightarrow \lambda = -\frac{3}{2}$$

$$-2\lambda + \mu = 1$$

$$\Rightarrow -2 \left( -\frac{3}{2} \right) + \mu = 1$$

$$\mu = -2$$

$$\text{So, } I = \int \frac{-\frac{3}{2}(-2-2x)-2}{\sqrt{5-2x-x^2}} dx$$

$$= -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{-(x^2+2x-5)}} dx$$

$$I = -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{-(x^2+2x+(1)^2-(1)^2-5)}} dx$$

$$I = -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{-(x+1)^2-(\sqrt{6})^2}} dx$$

$$I = -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{(\sqrt{6})^2-(x+1)^2}} dx$$

$$I = -\frac{3}{2} \times 2\sqrt{5-2x-x^2} - 2 \sin^{-1} \left( \frac{x+1}{\sqrt{6}} \right) + c \quad [\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \left( \frac{x}{a} \right) + c]$$

$$I = -3\sqrt{5-2x-x^2} - 2 \sin^{-1} \left( \frac{x+1}{\sqrt{6}} \right) + c$$

$$29. xdy - (y - x^3)dx = 0$$

This can be written as

$$xdy = (y - x^3)dx$$

Divide throughout by x,

$$\frac{dy}{dx} = \frac{y}{x} - x^2$$

$$\frac{dy}{dx} - \frac{y}{x} = -x^2$$

This is a linear differential equation of the form,

$$\frac{dy}{dx} + Py = Q$$

The integrating factor I.F is

$$e^{\int P dx} = e^{\int \frac{-1}{x} dx} = e^{-\log x} = e^{\log(\frac{1}{x})} = \frac{1}{x}$$

The required solution is

$$y e^{\int P dx} = \int Q e^{\int P dx} \cdot dx + c$$

$$y \cdot \left(\frac{1}{x}\right) = - \int x^2 \times \frac{1}{x} dx + c$$

$$\frac{y}{x} = - \int x dx + c$$

$$\frac{y}{x} = \frac{-x^2}{2} + c$$

$$\frac{y}{x} + \frac{x^2}{2} = c$$

$$2y + x^3 = 2cx$$

$\Rightarrow x^3 - 2cx + 2y = 0$  is the required solution.

OR

$$(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$$

Divide both sides by  $1 + x^2$

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2) \cdot (1+x^2)}$$

$$\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right) y = \frac{1}{(1+x^2)^2}$$

Comparing with  $\frac{dy}{dx} + Py = Q$ ,

$$P = \frac{2x}{1+x^2} \text{ \& } Q = \frac{1}{(1+x^2)^2}$$

Finding Integrating factor:

$$IF = e^{\int P dx}$$

$$IF = e^{\int \frac{2x}{1+x^2} dx}$$

$$\text{Let } 1 + x^2 = t$$

Diff. w.r.t. x

$$2x = \frac{dt}{dx}$$

$$dx = \frac{dt}{2x}$$

$$\text{Thus, } IF = e^{\int \frac{2x}{t} \cdot \frac{dt}{2x}}$$

$$IF = e^{\int \frac{dt}{t}}$$

$$IF = e^{\log |t|}$$

$$IF = t$$

$$IF = 1 + x^2$$

Solution of the differential equation:

$$y \times I.F. = \int Q \times I.F. dx$$

Putting values,

$$y \times (1 + x^2) = \int \frac{1}{(1+x^2)^2} (1 + x^2) dx$$

$$y (1 + x^2) = \int \frac{1}{(1+x^2)} dx$$

$$y (1 + x^2) = \tan^{-1} x + C \dots (1)$$

Putting that  $y = 0$  and  $x = 1$ ,

$$0(1 + 1^2) = \tan^{-1}(1) + C$$

$$0 = \frac{\pi}{4} + C$$

$$C = -\frac{\pi}{4}$$

Putting value of C in eq(1),

$$y(1 + x^2) = \tan^{-1} x + C$$

$$y(1 + x^2) = \tan^{-1} x - \frac{\pi}{4}$$

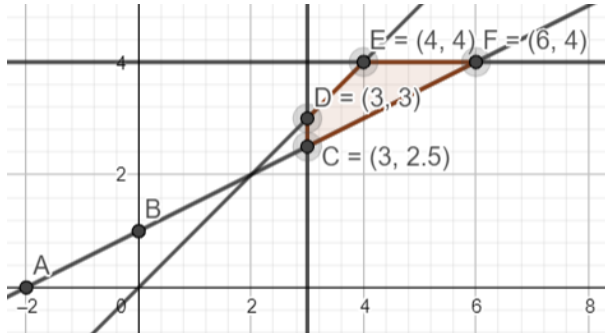
30. First, we will convert the given inequations into equations, we obtain the following equations:

$$x - y = 0, -x + 2y = 2, x = 3, y = 4, x = 0 \text{ and } y = 0$$

Region represented by  $x - y \geq 0$  or  $x \geq y$  The line  $x - y = 0$  or  $x = y$  passes through the origin. The region to the right of line  $x = y$  will satisfy the given inequation. Check by taking an example like if we take a point (4,3) to the right of the line  $x = y$ . Here  $x \geq y$ . So, it satisfies the given inequation. Take a point (4,5) to the left of the line  $x = y$ . Here,  $x \leq y$ . That means it does not satisfy the

given inequation. Region represented by  $-x + 2y \geq 2$  The line  $-x + 2y = 2$  meets the coordinate axes at A(-2,0) and B(0,1) respectively. By joining these points we obtain the line  $-x + 2y = 2$ . Clearly (0,0) does not satisfy the inequation  $-x + 2y \geq 2$ . So, the region in xy plane which does not contain the origin represents the solution set of the inequation  $-x + 2y \geq 2$  The line  $x = 3$  is the line that passes through the point (3,0) and is parallel to Y-axis.  $x \geq 3$  is the region to the right of line  $x = 3$  The line  $y = 4$  is the line that passes through the point (0,4) and is parallel to X-axis.  $y \leq 4$  is the region below the line  $y = 4$  Region represented by  $x \geq 0$  and  $y \geq 0$  :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations  $x \geq 0$  and  $y \geq 0$  The feasible region determined by subject to the constraints are  $x - y \geq 0$ ,  $-x + 2y \geq 2$ ,  $x \geq 3$ ,  $y \leq 4$ , non negative,  $x \geq 0$  and  $y \geq 0$  are as follows.



The corner points of the feasible region are

$C(3, \frac{5}{2})$ ,  $D(3, 3)$ ,  $E(4, 4)$  and  $F(6, 4)$

The values of objective function at the corner points are as follows:

Corner point:  $z = x - 5y + 20$

$C(3, \frac{5}{2}) : 3 - 5 \times \frac{5}{2} + 20 = \frac{21}{2}$

$D(3, 3) : 3 - 5 \times 3 + 20 = 8$

$E(4, 4) : 4 - 5 \times 4 + 20 = 4$

$F(6, 4) : 6 - 5 \times 4 + 20 = 6$

Therefore, the minimum value of objective function  $Z$  is 4 at the point  $E(4, 4)$ . Hence,  $x = 4$  and  $y = 4$  is the optimal solution of the given LPP.

Thus, the optimal value of objective function  $Z$  is 4.

OR

First, we will convert the given inequations into equations, we obtain the following equations:

$2x + y = 18$ ,  $3x + 2y = 34$

Region represented by  $2x + y \geq 18$  :

The line  $2x + y = 18$  meets the coordinate axes at A(9,0) and B(0,18) respectively. By joining these points we obtain the line  $2x + y = 18$  Clearly (0,0) does not satisfy the inequation  $2x + y \geq 18$ . So, the region in xy plane which does not contain the origin represents the solution set of the inequation  $2x + y \geq 18$ .

Region represented by  $3x + 2y \leq 34$  :

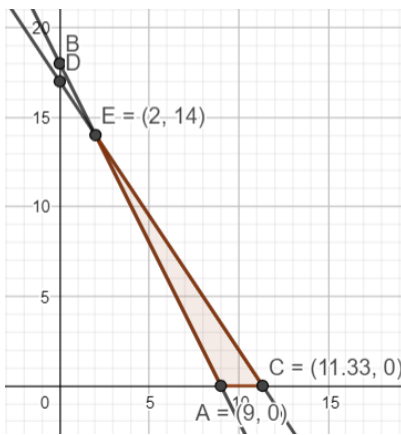
The line  $3x + 2y = 34$  meets the coordinate axes at

$C(\frac{34}{3}, 0)$  and  $D(0, 17)$  respectively.

By joining these points we obtain the line  $3x + 2y = 34$  Clearly (0,0) satisfies the inequation  $3x + 2y \leq 34$ . So, the region containing the origin represents the solution set of the inequation  $3x + 2y \leq 34$

The corner points of the feasible region are A(9,0)

$C(\frac{34}{3}, 0)$  and  $E(2, 14)$  and feasible region is bounded



The values of Z objective function at these corner points are as follows.

Corner point	$Z = 50x + 30y$
A(9, 0)	$50 \times 9 + 3 \times 0 = 450$
$C\left(\frac{34}{3}, 0\right)$	$50 \times \frac{34}{3} + 30 \times 0 = \frac{1700}{3}$
E(2, 14)	$50 \times 2 + 30 \times 14 = 520$

Therefore, the maximum value of objective function Z is

$\frac{1700}{3}$  at the point  $\left(\frac{34}{3}, 0\right)$  Hence,  $x = \frac{34}{3}$  and  $y = 0$  is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is  $\frac{1700}{3}$ .

$$\begin{aligned}
 31. \text{ Let, } y &= \tan^{-1} \left[ \frac{a+b \tan x}{b-a \tan x} \right] \\
 \Rightarrow y &= \tan^{-1} \left[ \frac{\frac{a+b \tan x}{b}}{\frac{b-a \tan x}{b}} \right] \\
 \Rightarrow y &= \tan^{-1} \left[ \frac{\frac{a}{b} + \tan x}{1 - \frac{a}{b} \tan x} \right] \\
 \Rightarrow y &= \tan^{-1} \left[ \frac{\tan\left(\tan^{-1} \frac{a}{b}\right) + \tan x}{1 - \tan\left(\tan^{-1} \frac{a}{b}\right) \times \tan x} \right] \\
 \Rightarrow y &= \tan^{-1} \left[ \tan\left(\tan^{-1} \frac{a}{b} + x\right) \right] \\
 \Rightarrow y &= \tan^{-1} \left( \frac{a}{b} \right) + x
 \end{aligned}$$

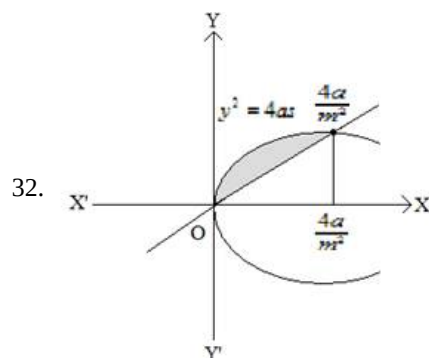
Differentiate it with respect to x,

$$\frac{dy}{dx} = 0 + 1$$

$$\therefore \frac{dy}{dx} = 1$$

Hence the derivative is equal to 1 for the given function .

#### Section D



32.

$$y^2 = 4ax \dots\dots(1)$$

$$y = mx \dots\dots(2)$$

Using (2) in (1), we get,

$$(mx)^2 = 4ax$$

$$\Rightarrow m^2 x^2 = 4ax$$

$$x(m^2 x - 4a) = 0$$



$$\Rightarrow x = 0, \frac{4a}{m^2}$$

From (2),

When  $x = 0, y = m(0) = 0$

$$\text{When } x = \frac{4a}{m^2}, y = m \times \frac{4a}{m^2} = \frac{4a}{m}$$

$\therefore$  points of intersection are  $(0, 0)$  and  $(\frac{4a}{m^2}, \frac{4a}{m})$

$$\begin{aligned} \text{Area} &= \int_0^{4a/m^2} \sqrt{4ax} dx - \int_0^{4a/m^2} mx dx \\ &= \sqrt{4a} \int_0^{4a/m^2} \sqrt{x} dx - m \int_0^{4a/m^2} x dx \\ &= \sqrt{4a} \left[ \frac{2}{3} x^{3/2} \right]_0^{4a/m^2} - m \left[ \frac{x^2}{2} \right]_0^{4a/m^2} \\ &= \sqrt{4a} \left[ \frac{2}{3} \left( \frac{4a}{m^2} \right)^{3/2} - 0 \right] - \frac{m}{2} \left[ \left( \frac{4a}{m^2} \right)^2 - 0 \right] \\ &= \frac{2}{3m^3} (4a)^2 - \frac{1}{2m^3} (4a)^2 \\ &= \frac{(4a)^2}{m^3} \left[ \frac{2}{3} - \frac{1}{2} \right] \\ &= \frac{8a^2}{3m^3} \text{ sq unit.} \end{aligned}$$

33. We observe the following properties of relation R.

Reflexivity: For any  $a \in \mathbb{N}$

$$a - a = 0 = 0 \times n$$

$\Rightarrow a - a$  is divisible by  $n$

$$\Rightarrow (a, a) \in R$$

Thus,  $(a, a) \in R$  for all  $a \in \mathbb{Z}$ . So, R is reflexive on  $\mathbb{Z}$

Symmetry: Let  $(a, b) \in R$ . Then,

$$(a, b) \in R$$

$\Rightarrow (a - b)$  is divisible by  $n$

$$\Rightarrow (a - b) = np \text{ for some } p \in \mathbb{Z}$$

$$\Rightarrow b - a = n(-p)$$

$$\Rightarrow b - a \text{ is divisible by } n \left[ \because p \in \mathbb{Z} \Rightarrow -p \in \mathbb{Z} \right]$$

$$\Rightarrow (b, a) \in R$$

Thus,  $(a, b) \in R \Rightarrow (b, a) \in R$  for all  $a, b \in \mathbb{Z}$ .

So, R is symmetric on  $\mathbb{Z}$ .

Transitivity: Let  $a, b, c \in \mathbb{Z}$  such that  $(a, b) \in R$  and  $(b, c) \in R$ . Then,

$$(a, b) \in R$$

$\Rightarrow (a - b)$  is divisible by  $n$

$$\Rightarrow a - b = np \text{ for some } p \in \mathbb{Z}$$

and,  $(b, c) \in R$

$\Rightarrow (b - c)$  is divisible by  $n$

$$\Rightarrow b - c = nq \text{ for some } q \in \mathbb{Z}$$

$\therefore (a, b) \in R$  and  $(b, c) \in R$

$$\Rightarrow a - b = np \text{ and } b - c = nq$$

$$\Rightarrow (a - b) + (b - c) = np + nq$$

$$\Rightarrow a - c = n(p + q)$$

$$\Rightarrow a - c \text{ is divisible by } n \left[ \because p, q \in \mathbb{Z} \Rightarrow p + q \in \mathbb{Z} \right]$$

$$\Rightarrow (a, c) \in R$$

Thus,  $(a, b) \in R$  and  $(b, c) \in R \Rightarrow (a, c) \in R$  for all  $a, b, c \in \mathbb{Z}$ .

OR

Given that,  $A = \mathbb{R} - \{3\}, B = \mathbb{R} - \{1\}$ .

$$f: A \rightarrow B \text{ is defined by } f(x) = \frac{x-2}{x-3} \quad \forall x \in A$$

For injectivity

$$\text{Let } f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$$

$$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$$

$$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$$

So,  $f(x)$  is an injective function

For surjectivity

$$\text{Let } y = \frac{x-2}{x-3} \Rightarrow x-2 = xy-3y$$

$$\Rightarrow x(1-y) = 2-3y \Rightarrow x = \frac{2-3y}{1-y}$$

$$\Rightarrow x = \frac{3y-2}{y-1} \in A, \forall y \in B \text{ [codomain]}$$

So,  $f(x)$  is surjective function.

Hence,  $f(x)$  is a bijective function.

$$34. \text{ Let } \frac{1}{x} = u, \frac{1}{y} = v \text{ and } \frac{1}{z} = w$$

$$2u + 3v + 10w = 4$$

$$4u - 6v + 5w = 1$$

$$6u + 9v - 20w = 2$$

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} u \\ v \\ w \end{bmatrix} B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$$= 2[120 - 45] - 3[-80 - 30] + 10[36 + 36]$$

$$= 150 + 330 + 720 = 1200 \neq 0$$

$\Rightarrow A$  is non-singular and hence  $A^{-1}$  exists.

$$\text{Now, } A_{11} = 75, A_{12} = 110, A_{13} = 72$$

$$A_{21} = 150, A_{22} = -100, A_{23} = 0$$

$$A_{31} = 75, A_{32} = 30, A_{33} = -2$$

$$\therefore \text{adj}A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A) = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\begin{bmatrix} y \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5}$$

$$\frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$$

$$x = 2, y = 3, z = 5$$

35. Suppose the point  $(1, 0, 0)$  be  $P$  and the point through which the line passes be  $Q(1, -1, -10)$ . The line is parallel to the vector

$$\vec{b} = 2\hat{i} - 3\hat{j} + 8\hat{k}$$

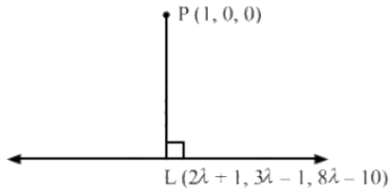
Now,

$$\vec{PQ} = 0\hat{i} - \hat{j} - 10\hat{k}$$

$$\therefore \vec{b} \times \vec{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 8 \\ 0 & -1 & -10 \end{vmatrix}$$

$$\begin{aligned}
&= 38\hat{i} + 20\hat{j} - 2\hat{k} \\
&\Rightarrow |\vec{b} \times \vec{PQ}| = \sqrt{38^2 + 20^2 + 2^2} \\
&= \sqrt{1444 + 400 + 4} \\
&= \sqrt{1848} \\
d &= \frac{|\vec{b} \times \vec{PQ}|}{|\vec{b}|} \\
&= \frac{\sqrt{1848}}{\sqrt{77}} \\
&= \sqrt{24} \\
&= 2\sqrt{6}
\end{aligned}$$

Suppose L be the foot of the perpendicular drawn from the point P(1,0,0) to the given line-



The coordinates of a general point on the line

$$\begin{aligned}
\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} \text{ are given by} \\
\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda \\
\Rightarrow x = 2\lambda + 1
\end{aligned}$$

$$y = -3\lambda - 1$$

$$z = 8\lambda - 10$$

Suppose the coordinates of L be

$$(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$$

Since, The direction ratios of PL are proportional to,

$$2\lambda + 1 - 1, -3\lambda - 1 - 0, 8\lambda - 10 - 0, \text{ i.e., } 2\lambda, -3\lambda - 1, 8\lambda - 10$$

Since, The direction ratios of the given line are proportional to 2, -3, 8, but PL is perpendicular to the given line.

$$\therefore 2(2\lambda) - 3(-3\lambda - 1) + 8(8\lambda - 10) = 0$$

$\Rightarrow \lambda = 1$  Substituting  $\lambda = 1$  in  $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$  we get the coordinates of L as (3, -4, -2). Equation of the line PL is given by

$$\begin{aligned}
\frac{x-1}{3-1} = \frac{y-0}{-4-0} = \frac{z-0}{-2-0} \\
= \frac{x-1}{2} = \frac{y}{-4} = \frac{z}{-2} \\
\Rightarrow \vec{r} = \hat{i} + \lambda(\hat{i} - 2\hat{j} - \hat{k})
\end{aligned}$$

OR

$$\text{Given lines are } \vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$$

$$\text{and } \vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

On comparing both equations of lines with

$$\vec{r} = \vec{a} + \lambda\vec{b} \text{ respectively, we get ,}$$

$$\vec{a}_1 = \hat{i} + \hat{j} - \hat{k}, \vec{b}_1 = 3\hat{i} - \hat{j}$$

$$\text{and } \vec{a}_2 = 4\hat{i} - \hat{k}, \vec{b}_2 = 2\hat{i} + 3\hat{k}$$

$$\begin{aligned}
\text{Now } \vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} \\
&= \hat{i}(-3 - 0) - \hat{j}(9 - 0) + \hat{k}(0 + 2) \\
&= -3\hat{i} - 9\hat{j} + 2\hat{k}
\end{aligned}$$

$$\text{and } \vec{a}_2 - \vec{a}_1 = (4\hat{i} - \hat{k}) - (\hat{i} + \hat{j} - \hat{k}) = 3\hat{i} - \hat{j}$$

$$\begin{aligned}
\text{Now, } (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (3\hat{i} - \hat{j}) \cdot (-3\hat{i} - 9\hat{j} + 2\hat{k}) \\
&= -9 + 9 = 0
\end{aligned}$$

Hence, given lines are coplanar.

Now, cartesian equations of given lines are

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$$

$$\text{and } \frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3}$$

Then, equation of plane containing them is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z+1 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 0$$

$$(x-1)(-3-0) - (y-1)(9-0) + (z+1)(0+2) = 0$$

$$-3x + 3 - 9y + 9 + 2z + 2 = 0$$

$$3x + 9y - 2z = 14$$

### Section E

#### 36. Read the text carefully and answer the questions:

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres- folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

- 960 of the total applications were the folk genre.
- 192 of the folk applications were for the below 18 category.
- 104 of the classical applications were for the 18 and above category.

(i) According to given information, we construct the following table.

Given, total applications = 2000

	Folk Genre	Classical Genre
	960 (given)	2000 - 960 = 1040
Below 18	192 (given)	1040 - 104 = 936
18 or Above 18	960 - 192 = 768	104 (given)

Let  $E_1$  = Event that application for folk genre

$E_2$  = Event that application for classical genre

A = Event that application for below 18

B = Event that application for 18 or above 18

$$\therefore P(E_2) = \frac{1040}{2000}$$

$$\text{and } P(B \cap E_2) = \frac{104}{2000}$$

$$\text{Required Probability} = \frac{P(B \cap E_2)}{P(E_2)}$$

$$= \frac{\frac{104}{2000}}{\frac{1040}{2000}} = \frac{1}{10}$$

(ii) Required probability =  $P\left(\frac{\text{folk}}{\text{below 18}}\right)$

$$= P\left(\frac{E_1}{A}\right)$$

$$= \frac{P(E_1 \cap A)}{P(A)}$$

$$\text{Now, } P(E_1 \cap A) = \frac{192}{2000}$$

$$\text{and } P(A) = \frac{192+936}{2000} = \frac{1128}{2000}$$

$$\therefore \text{Required probability} = \frac{\frac{192}{2000}}{\frac{1128}{2000}} = \frac{192}{1128} = \frac{8}{47}$$

(iii) Here,

$$P(A) = 0.4, P(B) = 0.8 \text{ and } P(B|A) = 0.6$$

$$\therefore P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow P(B \cap A) = P(B|A) \cdot P(A)$$

$$= 0.6 \times 0.4 = 0.24$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.4 + 0.8 - 0.24$$

$$= 1.2 - 0.24 = 0.96$$

OR

Since, A and B are independent events, A' and B' are also independent. Therefore,

$$P(A' \cap B') = P(A') \cdot P(B')$$

$$= (1 - P(A))(1 - P(B))$$

$$= \left(1 - \frac{3}{5}\right) \left(1 - \frac{4}{9}\right)$$

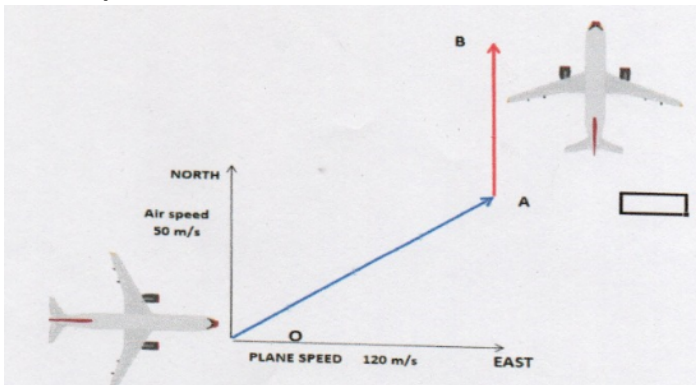
$$= \frac{2}{5} \cdot \frac{5}{9}$$

$$= \frac{2}{9}$$

**37. Read the text carefully and answer the questions:**

A plane started from airport O with a velocity of 120 m/s towards east. Air is blowing at a velocity of 50 m/s towards the north As shown in the figure.

The plane travelled 1 hr in OA direction with the resultant velocity. From A and B travelled 1 hr with keeping velocity of 120 m/s and finally landed at B.



(i) Resultant velocity from O to A

$$= \sqrt{(V_{\text{Plane}})^2 + (V_{\text{wind}})^2}$$

$$= \sqrt{(120)^2 + (50)^2}$$

$$= \sqrt{14400 + 2500}$$

$$= \sqrt{16900}$$

$$= 130 \text{ m/s}$$

(ii)  $\tan \theta = \frac{V_{\text{wind}}}{V_{\text{aeroplane}}}$

$$\tan \theta = \frac{50}{120}$$

$$\tan \theta = \frac{5}{12}$$

$$\theta = \tan^{-1} \left( \frac{5}{12} \right)$$

(iii) Displacement from O to A = Resultant velocity  $\times$  time

$$|\vec{OA}| = |\vec{V}| \times t$$

$$= 130 \times \frac{18}{5} \times 1$$

$$= 468 \text{ km}$$

OR

Since, from A to B both Aeroplane and wind have velocity in North direction.

So,

$$\vec{V}_{\text{plane, AtoB}} = 120 + 50$$

$$= 170 \text{ m/s}$$

**38. Read the text carefully and answer the questions:**

The temperature of a person during an intestinal illness is given by  $f(x) = -0.1x^2 + mx + 98.6$ ,  $0 \leq x < 12$ , m being a constant, where  $f(x)$  is the temperature in  $^{\circ}\text{F}$  at x days.



(i)  $f(x) = -0.1x^2 + mx + 98.6$ , being a polynomial function, is differentiable everywhere, hence, differentiable in  $(0, 12)$ .

(ii)  $f(x) = -0.2x + m$

At Critical point

$$0 = -0.2 \times 6 + m$$

$$m = 1.2$$