

MATHEMATICS – Code No. 041
SAMPLE QUESTION PAPER
CLASS - XII (2025-26)

Maximum Marks: 80

Time: 3 hours

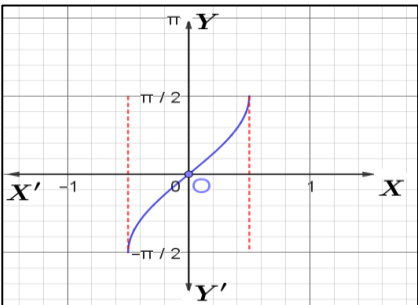
General Instructions:

Read the following instructions very carefully and strictly follow them:

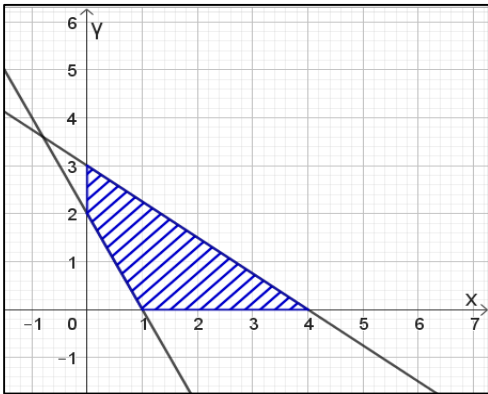
1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) with only one correct option and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculator is not allowed.

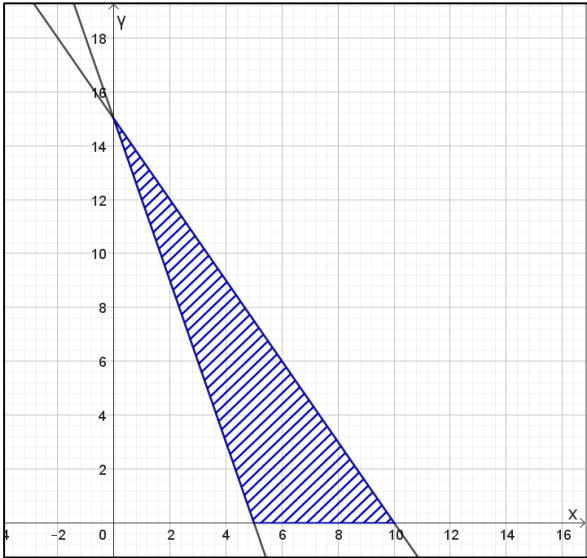
SECTION-A

**This section comprises of multiple choice questions (MCQs) of 1 mark each.
Select the correct option (Question 1 - Question 18)**

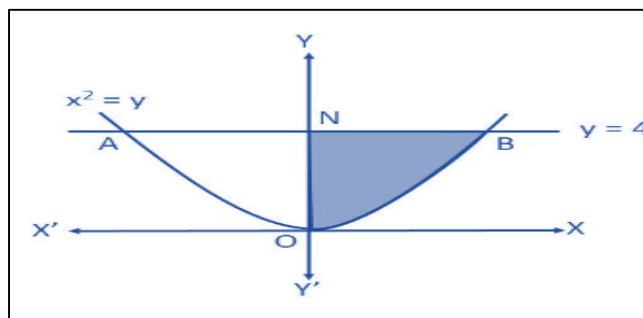
Q.No.	Questions	Marks
1.	<p>Identify the function shown in the graph</p>  <p>(A) $\sin^{-1} x$ (B) $\sin^{-1}(2x)$ (C) $\sin^{-1}\left(\frac{x}{2}\right)$ (D) $2 \sin^{-1} x$</p> <p>For Visually Impaired:</p> <p>1. Inverse Trigonometric Function, whose domain is $\left[-\frac{1}{3}, \frac{1}{3}\right]$, is ...</p> <p>(A) $\cos^{-1} x$ (B) $\cos^{-1}\left(\frac{x}{3}\right)$ (C) $\cos^{-1}(3x)$ (D) $3 \cos^{-1} x$</p>	1

2.	<p>If for three matrices $A = [a_{ij}]_{m \times 4}$, $B = [b_{ij}]_{n \times 3}$ and $C = [c_{ij}]_{p \times q}$ products AB and AC both are defined and are square matrices of same order, then value of m, n, p and q are:</p> <p>(A) $m = q = 3$ and $n = p = 4$ (B) $m = 2, q = 3$ and $n = p = 4$ (C) $m = q = 4$ and $n = p = 3$ (D) $m = 4, p = 2$ and $n = q = 3$</p>	1
3.	<p>If the matrix $A = \begin{bmatrix} 0 & r & -2 \\ 3 & p & t \\ q & -4 & 0 \end{bmatrix}$ is skew-symmetric, then value of $\frac{q+t}{p+r}$ is....</p> <p>(A) -2 (B) 0 (C) 1 (D) 2</p>	1
4.	<p>If A is a square matrix of order 4 and $adj A = 27$, then $A (adj A)$ is equal to</p> <p>(A) 3 (B) 9 (C) $3I$ (D) $9I$</p>	1
5.	<p>The inverse of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is...</p> <p>(A) $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 2 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (C) $\begin{bmatrix} -\frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{5} \end{bmatrix}$ (D) $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -5 \end{bmatrix}$</p>	1
6.	<p>Value of the determinant $\begin{vmatrix} \cos 67^\circ & \sin 67^\circ \\ \sin 23^\circ & \cos 23^\circ \end{vmatrix}$ is</p> <p>(A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1</p>	1
7.	<p>If a function defined by $f(x) = \begin{cases} kx + 1, & x \leq \pi \\ \cos x, & x > \pi \end{cases}$ is continuous at $x = \pi$, then the value of k is</p> <p>(A) π (B) $\frac{-1}{\pi}$ (C) 0 (D) $\frac{-2}{\pi}$</p>	1
8.	<p>If $f(x) = x \tan^{-1} x$, then $f'(1)$ is equal to</p> <p>(A) $\frac{\pi}{4} - \frac{1}{2}$ (B) $\frac{\pi}{4} + \frac{1}{2}$ (C) $-\frac{\pi}{4} - \frac{1}{2}$ (D) $-\frac{\pi}{4} + \frac{1}{2}$</p>	1
9.	<p>A function $f(x) = 10 - x - 2x^2$ is increasing on the interval</p> <p>(A) $(-\infty, -\frac{1}{4}]$ (B) $(-\infty, \frac{1}{4})$ (C) $[-\frac{1}{4}, \infty)$ (D) $[-\frac{1}{4}, \frac{1}{4}]$</p>	1
10.	<p>The solution of the differential equation $x dx + y dy = 0$ represents a family of</p> <p>(A) straight lines (B) parabolas (C) Circles (D) Ellipses</p>	1

11.	<p>If $f(a + b - x) = f(x)$, then $\int_a^b x f(x) dx$ is equal to</p> <p>(A) $\frac{a+b}{2} \int_a^b f(b-x) dx$ (B) $\frac{a+b}{2} \int_a^b f(a-x) dx$</p> <p>(C) $\frac{b-a}{2} \int_a^b f(x) dx$ (D) $\frac{a+b}{2} \int_a^b f(x) dx$</p>	1
12.	<p>If $\int x^3 \sin^4(x^4) \cos(x^4) dx = a \sin^5(x^4) + C$, then a is equal to</p> <p>(A) $-\frac{1}{10}$ (B) $\frac{1}{20}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$</p>	1
13.	<p>A bird flies through a distance in a straight line given by the vector $\hat{i} + 2\hat{j} + \hat{k}$. A man standing beside a straight metro rail track given by $\vec{r} = (3 + \lambda)\hat{i} + (2\lambda - 1)\hat{j} + 3\lambda\hat{k}$ is observing the bird. The projected length of its flight on the metro track is</p> <p>(A) $\frac{6}{\sqrt{14}}$ units (B) $\frac{14}{\sqrt{6}}$ units (C) $\frac{8}{\sqrt{14}}$ units (D) $\frac{5}{\sqrt{6}}$ units</p>	1
14.	<p>The distance of the point with position vector $3\hat{i} + 4\hat{j} + 5\hat{k}$ from the y-axis is</p> <p>(A) 4 units (B) $\sqrt{34}$ units (C) 5 units (D) $5\sqrt{2}$ units</p>	1
15.	<p>If $\vec{a} = 3\hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 3\hat{k}$ and $\vec{c} = 6\hat{i} - \hat{j} + 2\hat{k}$ are three given vectors, then $(2\vec{a} \cdot \hat{i})\hat{i} - (\vec{b} \cdot \hat{j})\hat{j} + (\vec{c} \cdot \hat{k})\hat{k}$ is same as the vector</p> <p>(A) \vec{a} (B) $\vec{b} + \vec{c}$ (C) $\vec{a} - \vec{b}$ (D) \vec{c}</p>	1
16.	<p>A student of class XII studying Mathematics comes across an incomplete question in a book.</p> <p>Maximise $Z = 3x + 2y + 1$ Subject to the constraints $x \geq 0, y \geq 0, 3x + 4y \leq 12$,</p> <p>He/ She notices the below shown graph for the said LPP problem, and finds that a constraint is missing in it:</p> <p>Help him/her choose the required constraint from the graph.</p>  <p>The missing constraint is</p> <p>(A) $x + 2y \leq 2$ (B) $2x + y \geq 2$</p> <p>(C) $2x + y \leq 2$ (D) $x + 2y \geq 2$</p>	1

16.	<p>For Visually Impaired:</p> <p>If $Z = ax + by + c$, where $a, b, c > 0$, attains its maximum value at two of its corner points $(4,0)$ and $(0,3)$ of the feasible region determined by the system of linear inequalities, then</p> <p>(A) $4a = 3b$ (B) $3a = 4b$ (C) $4a + c = 3b$ (D) $3a + c = 4b$</p>	
17.	<p>The feasible region of a linear programming problem is bounded but the objective function attains its minimum value at more than one point. One of the points is $(5,0)$.</p>  <p>Then one of the other possible points at which the objective function attains its minimum value is</p> <p>(A) $(2,9)$ (B) $(6,6)$ (C) $(4,7)$ (D) $(0,0)$</p> <p>For Visually Impaired:</p> <p>The graph of the inequality $3x + 5y < 10$ is the</p> <p>(A) Entire XY –plane (B) Open Half plane that doesn't contain origin (C) Open Half plane that contains origin, but not the points of the line $3x + 5y = 10$ (D) Half plane that contains origin and the points of the line $3x + 5y = 10$</p>	1
18.	<p>A person observed the first 4 digits of your 6-digit PIN. What is the probability that the person can guess your PIN?</p> <p>(A) $\frac{1}{81}$ (B) $\frac{1}{100}$ (C) $\frac{1}{90}$ (D) 1</p>	1

	<p>ASSERTION-REASON BASED QUESTIONS</p> <p>(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)</p> <p>(A) Both (A) and (R) are true and (R) is the correct explanation of (A). (B) Both (A) and (R) are true but (R) is not the correct explanation of (A). (C) (A) is true but (R) is false. (D) (A) is false but (R) is true.</p>	
19.	<p>Assertion (A): Value of the expression $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1} 1 - \sec^{-1}(\sqrt{2})$ is $\frac{\pi}{4}$.</p> <p>Reason (R): Principal value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and that of $\sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.</p>	1
20.	<p>Assertion(A): Given two non-zero vectors \vec{a} and \vec{b}. If \vec{r} is another non-zero vector such that $\vec{r} \times (\vec{a} + \vec{b}) = \vec{0}$. Then \vec{r} is perpendicular to $\vec{a} \times \vec{b}$.</p> <p>Reason (R): The vector $(\vec{a} + \vec{b})$ is perpendicular to the plane of \vec{a} and \vec{b}</p>	1
<p style="text-align: center;">SECTION B</p> <p>This section comprises of 5 very short answer (VSA) type questions of 2 marks each.</p>		
21A	Evaluate $\tan\left(\tan^{-1}(-1) + \frac{\pi}{3}\right)$	2
21B	Find the domain of $\cos^{-1}(3x - 2)$	
22	If $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, then prove that $\frac{dy}{dx} - \sec x = 0$	2
23A	Find: $\int \frac{(x-3)}{(x-1)^3} e^x dx$	2
23B	Find out the area of shaded region in the enclosed figure.	




23 B	For Visually Impaired: Find out the area of the region enclosed by the curve $y^2 = x$, $x = 3$ and x -axis in the first quadrant.	
24.	If $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$ and $f(5) = 2$, $f'(0) = 3$, then using the definition of derivatives, find $f'(5)$.	2
25.	The two vectors $\hat{i} + \hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 3\hat{k}$ represent the two sides OA and OB , respectively of a ΔOAB , where O is the origin. The point P lies on AB such that OP is a median. Find the area of the parallelogram formed by the two adjacent sides as OA and OP .	2
<p style="text-align: center;">SECTION C</p> <p style="text-align: center;">This section comprises of 6 short answer (SA) type questions of 3 marks each.</p>		
26A.	If $x^y = e^{x-y}$ prove that $\frac{dy}{dx} = \frac{\log x}{(\log(xe))^2}$ and hence find its value at $x = e$.	3
26B.	<p style="text-align: center;">OR</p> If $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ find $\frac{d^2y}{dx^2}$.	
27	A spherical ball of ice melts in such a way that the rate at which its volume decreases at any instant is directly proportional to its surface area. Prove that the radius of the ice ball decreases at a constant rate.	3
28A	Sketch the graph $y = x + 1 $. Evaluate $\int_{-4}^2 x + 1 dx$. What does the value of this integral represent on the graph?	3
28B	<p style="text-align: center;">OR</p> Using integration find the area of the region $\{(x, y) : x^2 - 4y \leq 0, y - x \leq 0\}$	
28A	Define the function $y = x + 1 $. Evaluate $\int_{-4}^2 x + 1 dx$. What does the value of this integral represent?	
28B	<p style="text-align: center;">OR</p> Using integration find the area enclosed within the curve: $25x^2 + 16y^2 = 400$	
29A	Find the distance of the point $(2, -1, 3)$ from the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \mu(3\hat{i} + 6\hat{j} + 2\hat{k})$ measured parallel to the z -axis.	3
29B	<p style="text-align: center;">OR</p> Find the point of intersection of the line $\vec{r} = (3\hat{i} + \hat{k}) + \mu(\hat{i} + \hat{j} + \hat{k})$ and the line through $(2, -1, 1)$ parallel to the z -axis. How far is this point from the z -axis?	

30.	<p>Solve graphically: Maximise $Z = 2x + y$ subject to $x + y \leq 1200$ $x + y \geq 600$ $y \leq \frac{x}{2}$ $x \geq 0, y \geq 0$.</p>	3
30	<p>For Visually Impaired:</p> <p>The objective function $Z = 3x + 2y$ of a linear programming problem under some constraints is to be maximized and minimized. The corner points of the feasible region are $A(600,0)$, $B(1200,0)$, $C(800,400)$ and $D(400,200)$. Find the point at which Z is maximum and the point at which Z is minimum. Also, find the corresponding maximum and minimum values of Z.)</p>	
31.	Two students Mehul and Rashi are seeking admission in a college. The probability that Mehul is selected is 0.4 and the probability of selection of exactly one of the them is 0.5. Chances of selection of them is independent of each other. Find the chances of selection of Rashi. Also find the probability of selection of at least one of them.	3
<p style="text-align: center;">SECTION D</p> <p style="text-align: center;">This section comprises of 4 long answer (LA) type questions of 5 marks each</p>		
32.	<p>For two matrices $A = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$, find the product AB and hence solve the system of equations:</p> $3x - 6y - z = 3$ $2x - 5y - z + 2 = 0$ $-2x + 4y + z = 5$	5
33A	Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$	5
	OR	
33B	Find $\int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta$	
34A	Solve the differential equation: $y + \frac{d}{dx}(xy) = x(\sin x + x)$	5
	OR	
34B	Find the particular solution of the differential equation: $2y e^{x/y} dx + (y - 2x e^{x/y}) dy = 0$ given that $y(0) = 1$	

35.	The two lines $\frac{x-1}{3} = -y, z + 1 = 0$ and $\frac{-x}{2} = \frac{y+1}{2} = z + 2$ intersect at a point whose y-coordinate is 1. Find the co-ordinates of their point of intersection. Find the vector equation of a line perpendicular to both the given lines and passing through this point of intersection.	5
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SECTION- E

This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each

36.	<p>Case Study -1</p> <p>A city's traffic management department is planning to optimize traffic flow by analyzing the connectivity between various traffic signals. The city has five major spots labelled A, B, C, D, and E.</p>  <p>The department has collected the following data regarding one-way traffic flow between spots:</p> <ol style="list-style-type: none"> 1. Traffic flows from A to B, A to C, and A to D. 2. Traffic flows from B to C and B to E. 3. Traffic flows from C to E. 4. Traffic flows from D to E and D to C. <p>The department wants to represent and analyze this data using relations and functions. Use the given data to answer the following questions:</p> <ol style="list-style-type: none"> I. Is the traffic flow reflexive? Justify. [1] II. Is the traffic flow transitive? Justify. [1] III A. Represent the relation describing the traffic flow as a set of ordered pairs. Also state the domain and range of the relation. <p style="text-align: center;">OR</p> <ol style="list-style-type: none"> III B. Does the traffic flow represent a function? Justify your answer- [2] 	4
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LED bulbs are energy-efficient because they use significantly less electricity than traditional bulbs while producing the same amount of light. They convert more energy into light rather than heat, reducing waste. Additionally, their long lifespan means fewer replacements, saving resources and money over time.

A company manufactures a new type of energy-efficient LED bulb. The cost of production and the revenue generated by selling x bulbs (in an hour) are modelled as

$C(x) = 0.5x^2 - 10x + 150$ and $R(x) = -0.3x^2 + 20x$ respectively, where $C(x)$ and $R(x)$ are both in ₹.



To maximize the profit, the company needs to analyze these functions using calculus. Use the given models to answer the following questions:

- I. Derive the profit function $P(x)$ [1]
- II. Find the critical points of $P(x)$. [1]
- III A. Determine whether the critical points correspond to a maximum or a minimum profit by using the second derivative test.

OR

- III B. Identify the possible practical value of x (i.e., the number of bulbs that can realistically be produced and sold) that can maximize the profit, if the resources available and the expenditure on machines allows to produce minimum 10 but not more than 18 bulbs per hour. Also calculate the maximum profit. [2]

Excessive use of screens can result in vision problems, obesity, sleep disorders, anxiety, low retention problems and can impede social and emotional comprehension and expression. It is essential to be mindful of the amount of time we spend on screens and to reduce our screen-time by taking regular breaks, setting time limits, and engaging in non-screen-based activities.



In a class of students of the age group 14 to 17, the students were categorised into three groups according to a feedback form filled by them. The first group constituted of the students who spent more than 4 hours per day on the mobile screen or the gaming screens, while the second group spent 2 to 4 hours /day on the same activities. The third group spent less than 2 hours /day on the same. The first group with the high screen time is 60% of all the students, whereas the second group with moderate screen time is 30% and the third group with low screen time is only 10% of the total number of students. It was observed that 80% students of first group faced severe anxiety and low retention issues, with 70% of second group, and 30% of third group having the same symptoms.

- I. What is the total percentage of students who suffer from anxiety and low retention issues in the class? [2]
- II. A student is selected at random, and he is found to suffer from anxiety and low retention issues. What is the probability that he/she spends screen time more than 4 hours per day? [2]

MATHEMATICS – Code No. 041
MARKING SCHEME
CLASS – XII (2025-26)

SECTION-A (MCQs of 1 mark each)		
Sol. N.	Hint / Solution	Marks
1	Clearly from the graph Domain is $\left[-\frac{1}{2}, \frac{1}{2}\right]$ So graph is of the function $\sin^{-1}(2x)$ Answer is (B) $\sin^{-1}(2x)$	1
1 (V.I.)	Domain is $\left[-\frac{1}{3}, \frac{1}{3}\right]$ So the function is $\cos^{-1}(3x)$ Answer is (C) $\cos^{-1}(3x)$	1
2	AB is defined so $n=4$ AC is defined so $p=4$ AB and AC are square matrices of same order so $m \times 3 = m \times q \Rightarrow q = 3 = m$ Answer is (A) $m = q = 3$ and $n = p = 4$	1
3	As A is skew symmetric So $p = 0, q = 2, r = -3, t = 4$ So $\frac{q+t}{p+r} = \frac{6}{-3} = -2$ Answer is (A) -2	1
4	$ adj A = 27 \Rightarrow A ^3 = 27 = 3^3 \Rightarrow A = 3$ $A(adj A) = A I = 3 I$ Answer is (C) $3 I$	1
5	Inverse of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ Answer is (B)	1
6	$\begin{vmatrix} \cos 67^\circ & \sin 67^\circ \\ \sin 23^\circ & \cos 23^\circ \end{vmatrix} = \cos 67^\circ \cos 23^\circ - \sin 67^\circ \sin 23^\circ = \cos(67^\circ + 23^\circ) = \cos 90^\circ = 0$ Answer is (A) 0	1
7	$f(x)$ is continuous at $x = \pi$ $\Rightarrow \lim_{x \rightarrow \pi^-} (kx + 1) = \lim_{x \rightarrow \pi^+} \cos x = f(\pi)$ $\Rightarrow \lim_{h \rightarrow 0} [k(\pi - h) + 1] = \lim_{h \rightarrow 0} \cos(\pi + h) = k\pi + 1$ $\Rightarrow k\pi + 1 = -1 \Rightarrow k = \frac{-2}{\pi}$ Answer is (D) $\frac{-2}{\pi}$	1

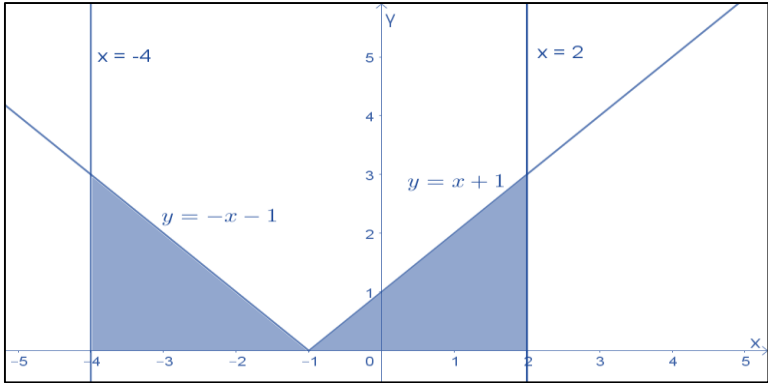
8	$f(x) = x \tan^{-1} x$ $f'(x) = 1 \cdot \tan^{-1} x + x \cdot \frac{1}{1+x^2}$ $f'(1) = 1 \cdot \tan^{-1} 1 + \frac{1}{1+1} = \frac{\pi}{4} + \frac{1}{2}$ Answer is (B) $\frac{\pi}{4} + \frac{1}{2}$	1
9	$f(x) = 10 - x - 2x^2$ $\Rightarrow f'(x) = -1 - 4x$ For increasing function $f'(x) \geq 0$ $\Rightarrow -(1 + 4x) \geq 0$ $\Rightarrow (1 + 4x) \leq 0$ $\Rightarrow x \leq -1/4$ $\Rightarrow x \in \left(-\infty, -\frac{1}{4}\right]$ Answer is (A) $\left(-\infty, -\frac{1}{4}\right]$	1
10	$xdx + ydy = 0$ $\Rightarrow \int xdx = -\int ydy$ $\Rightarrow \frac{x^2}{2} = -\frac{y^2}{2} + k$ $\Rightarrow x^2 + y^2 = 2k$ Solution is $x^2 + y^2 = 2k$, k being an arbitrary constant. Answer is (C) Circles	1
11	$I = \int_a^b x f(x) dx = \int_a^b (a + b - x) f(a + b - x) dx$ $\Rightarrow I = \int_a^b (a + b - x) f(x) dx$ (given $f(a + b - x) = f(x)$) $\Rightarrow I = \int_a^b (a + b) f(x) dx - \int_a^b x f(x) dx$ $\Rightarrow 2I = (a + b) \int_a^b f(x) dx$ $\Rightarrow I = \frac{1}{2} (a + b) \int_a^b f(x) dx$ Answer is (D) $\frac{a+b}{2} \int_a^b f(x) dx$	1
12	Let $I = \int x^3 \sin^4(x^4) \cos(x^4) dx$ Let $\sin(x^4) = t \Rightarrow 4x^3 \cos(x^4) dx = dt \Rightarrow x^3 \cos(x^4) = \frac{1}{4} dt$ Thus $I = \int t^4 \left(\frac{1}{4} dt\right) = \frac{1}{20} t^5 + C = \frac{1}{20} \sin^5(x^4) + C$ $\Rightarrow I = \frac{1}{20} \sin^5(x^4) + C = a \sin^5(x^4) + C$ So, $a = \frac{1}{20}$ Answer is (B) $\frac{1}{20}$	1
13	The projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the line $\vec{r} = (3\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ is $\frac{1 \times 1 + 2 \times 2 + 1 \times 3}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{8}{\sqrt{14}}$ units Answer is (C) $\frac{8}{\sqrt{14}}$ units	1

14	<p>The distance of the point (a, b, c) from the y-axis is $\sqrt{a^2 + c^2}$</p> <p>So, the distance is $\sqrt{3^2 + 5^2} = \sqrt{34}$ units.</p> <p>Answer is (B) $\sqrt{34}$ units</p>	1
15	<p>$(2\vec{a} \cdot \hat{i})\hat{i} - (\vec{b} \cdot \hat{j})\hat{j} + (\vec{c} \cdot \hat{k})\hat{k} = (2 \times 3)\hat{i} - (1)\hat{j} + (2)\hat{k}$</p> <p>$= 6\hat{i} - \hat{j} + 2\hat{k} = \vec{c}$</p> <p>Answer is (D) \vec{c}</p>	1
16	<p>The points (1,0) and (0,2) satisfy the equation $2x + y = 2$</p> <p>And shaded region shows that (0,0) doesn't lie in the feasible solution region</p> <p>So, the inequality is $2x + y \geq 2$</p> <p>Answer is (B) $2x + y \geq 2$</p>	1
16 (V.I.)	<p>(4,0) and (0,3) gives maximum value so</p> <p>$Z_{(4,0)} = Z_{(0,3)} \Rightarrow 4a + c = 3b + c \Rightarrow 4a = 3b$</p> <p>Answer is (A) $4a = 3b$</p>	1
17	<p>The student may read the point (2,9) from the line on the graph.</p> <p>The student may find the equation $3x + y = 15$ joining (5,0) and (0,15) and then verify the point (2,9) satisfies it.</p> <p>Answer is (A) (2,9)</p>	1
17 (V.I.)	<p>Answer is (C) Open Half plane that contains origin, but not the points of the line $3x + 5y = 10$</p>	1
18	<p>Answer is (B) $\frac{1}{100}$</p> <p>The person knows the first 4 digits. So the person has to guess the remaining two digits.</p> <p>$P(\text{guessing the PIN}) = 1 \times 1 \times 1 \times 1 \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$</p>	1
19	<p>$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1} 1 - \sec^{-1}(\sqrt{2}) = \frac{\pi}{3} + \frac{\pi}{4} - \frac{\pi}{4} = \frac{\pi}{3} \neq \frac{\pi}{4}$</p> <p>So, A is false.</p> <p>Principal Value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and that of $\sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.</p> <p>So, R is true</p> <p>Answer is (D) Assertion is false, but Reason is true</p>	1
20	<p>$C. \vec{r} \times (\vec{a} + \vec{b}) = \vec{0} \Rightarrow \vec{r}$ is parallel to $(\vec{a} + \vec{b})$ and $(\vec{a} + \vec{b})$ lies on the plane of \vec{a} and \vec{b}.</p> <p>So, \vec{r} is parallel to the plane of \vec{a} and $\vec{b} \Rightarrow \vec{r}$ is perpendicular to $(\vec{a} \times \vec{b})$.</p> <p>So, Assertion is true</p> <p>But $(\vec{a} + \vec{b})$ lies on the plane of \vec{a} and \vec{b}, so $(\vec{a} + \vec{b})$ is not perpendicular to the plane of \vec{a} and \vec{b}</p> <p>Therefore, Reason is false.</p> <p>Answer is (C) Assertion is true, but Reason is false</p>	1

SECTION B
(VSA type questions of 2 marks each)

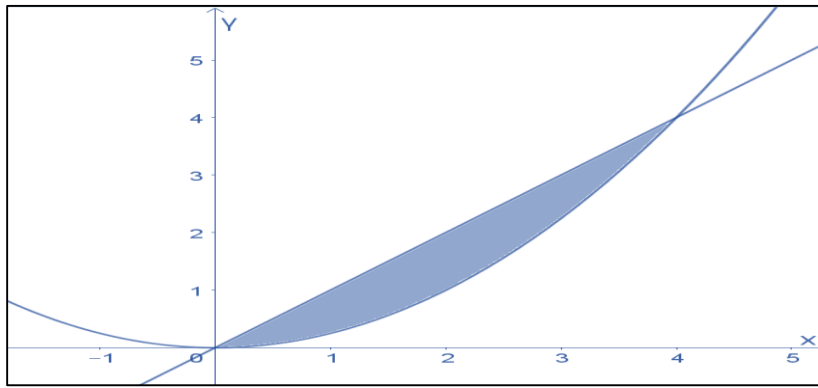
21A	$\tan\left(\tan^{-1}(-1) + \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{4} + \frac{\pi}{3}\right)$ $= \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$ $= \frac{\sqrt{3}-1}{1+\sqrt{3}} \text{ or } 2 - \sqrt{3}$ <p style="text-align: center;">OR</p>	$\frac{1}{2}$ 1 $\frac{1}{2}$ OR
21B	<p>For domain, $-1 \leq 3x - 2 \leq 1$ $\Rightarrow 1 \leq 3x \leq 3$ $\Rightarrow \frac{1}{3} \leq x \leq 1$</p> <p>So, domain of $\cos^{-1}(3x - 2)$ is $\left[\frac{1}{3}, 1\right]$</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
22	$y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ <p>Differentiating with respect to x</p> $\frac{dy}{dx} = \frac{1}{\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)} \cdot \sec^2\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot \frac{1}{2}$ $= \frac{\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)}{\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)} \cdot \frac{1}{\cos^2\left(\frac{\pi}{4} + \frac{x}{2}\right)} \cdot \frac{1}{2}$ $= \frac{1}{2 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \cos\left(\frac{\pi}{4} + \frac{x}{2}\right)} = \frac{1}{\sin\left(\frac{\pi}{2} + x\right)} = \frac{1}{\cos x}$ $\Rightarrow \frac{dy}{dx} - \sec x = 0$	$\frac{1}{2}$ 1 $\frac{1}{2}$
23A	$\int \frac{(x-3)e^x}{(x-1)^3} dx = \int \frac{(x-1-2)e^x}{(x-1)^3} dx$ $= \int \left(\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right) e^x dx = \int \left(\frac{1}{(x-1)^2} + \frac{d}{dx} \left(\frac{1}{(x-1)^2} \right) \right) e^x dx$ $= \frac{e^x}{(x-1)^2} + c \quad (\text{as } \int (f(x) + f'(x))e^x dx = e^x f(x) + c)$ <p style="text-align: center;">OR</p>	 1 1 OR
23B	$A = \int_0^4 x dy = \int_0^4 \sqrt{y} dy$ $= \frac{2}{3} \times y^{3/2} \Big _{y=0}^{y=4} = \frac{16}{3} \text{ sq. units}$	 1 1
23B	<p>For Visually Impaired:</p> $A = \int_0^3 y dx = \int_0^3 \sqrt{x} dx$ $= \frac{2}{3} \times x^{3/2} \Big _{x=0}^{x=3} = 2\sqrt{3} \text{ sq. units}$	 1 1

24	<p>Given $f(x+y) = f(x)f(y)$ $f(0+5) = f(0)f(5)$ $\Rightarrow f(0) = 1$ $f'(5) = \lim_{h \rightarrow 0} \frac{f(5+h)-f(5)}{h} = \lim_{h \rightarrow 0} \frac{f(5)f(h)-f(5)}{h} \quad [\because f(x+y) = f(x)f(y)]$ $= \lim_{h \rightarrow 0} \frac{2f(h)-2}{h} \quad [\because f(5) = 2]$ $= 2 \lim_{h \rightarrow 0} \frac{f(h)-1}{h} = 2 \lim_{h \rightarrow 0} \frac{f(h)-f(0)}{h} = 2 f'(0)$ $= 2(3) \quad [\because f'(0) = 3]$ $= 6$</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>
25	<p>The vector $\overrightarrow{OP} = \frac{1}{2}(4\hat{i} + 4\hat{k}) = 2\hat{i} + 2\hat{k}$ Area of the parallelogram formed by the two adjacent sides as OA and OP</p> $= (\overrightarrow{OA} \times \overrightarrow{OP}) = \left \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 0 & 2 \end{vmatrix} \right $ $= 2\hat{i} - 2\hat{k} $ $= 2\sqrt{2} \text{ square units.}$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
SECTION C (SA type questions of 3 marks each)		
26A	<p>$x^y = e^{x-y}$ Taking log of both sides $y \log x = (x-y) \log e$ $y \log x + y = x \quad (\text{since } \log e = 1)$ $\Rightarrow y = \frac{x}{1+\log x}$ Differentiating with respect to x $\frac{dy}{dx} = \frac{(1+\log x) \cdot 1 - x \cdot \frac{1}{x}}{(1+\log x)^2}$ $= \frac{\log x}{(\log e + \log x)^2}$ $= \frac{\log x}{(\log(xe))^2}$ Now $\left. \frac{dy}{dx} \right _{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4} \quad (\text{as } \log e = 1)$</p> <p>Alternative Solution:</p> <p>$x^y = e^{x-y}$ Taking log of both sides $y \log x = (x-y) \log e$ $y \log x + y = x \quad (\text{since } \log e = 1)$ Differentiating both sides w.r.t. x $\log x \frac{dy}{dx} + \frac{y}{x} + \frac{dy}{dx} = 1$ $\Rightarrow \frac{dy}{dx} (1 + \log x) = 1 - \frac{y}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{x-y}{x(1+\log x)} = \frac{x - \frac{x}{1+\log x}}{x(1+\log x)} = \frac{x(1+\log x) - x}{x(1+\log x)^2} = \frac{x(1+\log x - 1)}{x(\log e + \log x)^2} = \frac{\log x}{(\log(xe))^2}$ Now $\left. \frac{dy}{dx} \right _{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4} \quad (\text{as } \log e = 1)$</p>	<p>1</p> <p>1</p> <p>1</p>

	OR	OR
26B	$\frac{dx}{d\theta} = a(1 - \cos \theta), \frac{dy}{d\theta} = a(0 + \sin \theta),$ $\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \sin \theta}{a(1 - \cos \theta)}$ $= \frac{2 \sin(\frac{\theta}{2}) \cos(\frac{\theta}{2})}{2 \sin^2(\frac{\theta}{2})} = \cot \frac{\theta}{2}$ $\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2} \operatorname{cosec}^2\left(\frac{\theta}{2}\right) \frac{d\theta}{dx}$ $= -\frac{1}{2a} \operatorname{cosec}^2\left(\frac{\theta}{2}\right) \frac{1}{2 \sin^2(\frac{\theta}{2})}$ $= -\frac{1}{4a} \operatorname{cosec}^4\left(\frac{\theta}{2}\right)$	<p>1</p> <p>1</p> <p>1</p>
27	<p>Let r be the radius of ice ball at time t.</p> $V = \frac{4}{3} \pi r^3 \dots\dots\dots (1)$ $S = 4\pi r^2 \dots\dots\dots (2)$ <p>Given $\frac{dV}{dt} \propto S$</p> $\Rightarrow \frac{dV}{dt} = -k S \text{ (where } k \text{ is some positive constant) } \dots\dots\dots (3)$ <p>Differentiating (1) w.r.t. t, we get</p> $\frac{dV}{dt} = \frac{4}{3} \pi \cdot (3 r^2) \frac{dr}{dt}$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \dots\dots\dots (4)$ $\Rightarrow -k S = 4\pi r^2 \frac{dr}{dt} \quad (\text{from (3) and (4)})$ $\Rightarrow -k S = S \frac{dr}{dt} \quad (\text{using (2)})$ $\Rightarrow \frac{dr}{dt} = -k$ <p>\Rightarrow radius of the ice-ball decreases at a constant rate</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
28A	 $\int_{-4}^2 x + 1 dx = \int_{-4}^{-1} (-x - 1) dx + \int_{-1}^2 (x + 1) dx$ $= -\left[\frac{(x+1)^2}{2}\right]_{-4}^{-1} + \left[\frac{(x+1)^2}{2}\right]_{-1}^2$ $= -\left(0 - \frac{9}{2}\right) + \left(\frac{9}{2} - 0\right) = 9$ <p>It represent the area of shaded region bounded by the curve $y = x + 1$, x - axis and the lines $x = -4$ and $x = 2$</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>

28B

OR



$$\begin{aligned}
 \text{Required Area} &= \int_0^4 x \, dx - \int_0^4 \frac{x^2}{4} \, dx \\
 &= \left[\frac{x^2}{2} \right]_0^4 - \frac{1}{12} [x^3]_0^4 \\
 &= \frac{1}{2} (16 - 0) - \frac{1}{12} (64 - 0) = 8 - \frac{16}{3} = \frac{8}{3} \text{ sq. units}
 \end{aligned}$$

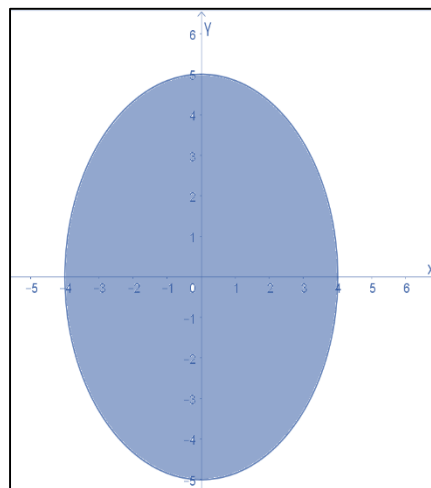
For Visually Impaired:

$$y = |x + 1| = f(x) = \begin{cases} -x - 1, & x < -1 \\ x + 1, & x \geq -1 \end{cases}$$

$$\begin{aligned}
 \int_{-4}^2 |x + 1| \, dx &= \int_{-4}^{-1} (-x - 1) \, dx + \int_{-1}^2 (x + 1) \, dx \\
 &= -\left[\frac{(x+1)^2}{2} \right]_{-4}^{-1} + \left[\frac{(x+1)^2}{2} \right]_{-1}^2 \\
 &= -\left(0 - \frac{9}{2} \right) + \left(\frac{9}{2} - 0 \right) = 9
 \end{aligned}$$

It represent the area of shaded region bounded by the curve $y = |x + 1|$, x - axis and the lines $x = -4$ and $x = 2$

OR



[illegible]

33A	<p>Let $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$</p> $I = \int_0^{\frac{\pi}{4}} \frac{\log(1+\tan\theta)}{1+\tan^2\theta} \cdot \sec^2 \theta d\theta$ $I = \int_0^{\frac{\pi}{4}} \log(1+\tan\theta) d\theta = \int_0^{\frac{\pi}{4}} \log \left[1 + \tan \left(\frac{\pi}{4} - \theta \right) \right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log \left[1 + \frac{1-\tan\theta}{1+\tan\theta} \right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log \left[\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta} \right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log \left[\frac{2}{1+\tan\theta} \right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log 2 d\theta - \int_0^{\frac{\pi}{4}} \log[1+\tan\theta] d\theta$ $= \log 2 \times x \Big _0^{\frac{\pi}{4}} - I$ $\Rightarrow 2I = \frac{\pi}{4} \log 2$ $\Rightarrow I = \frac{\pi}{8} \log 2$	<p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p>
33B	<p style="text-align: center;">OR</p> $I = \int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta = \int \frac{(3 \sin \theta - 2) \cos \theta}{5 - (1 - \sin^2 \theta) - 4 \sin \theta} d\theta$ <p>Let $\sin \theta = t \Rightarrow \cos \theta d\theta = dt$</p> $I = \int \frac{(3t-2)}{5-(1-t^2)-4t} dt$ $= \int \frac{(3t-2)}{t^2-4t+4} dt = \int \frac{3t-2}{(t-2)^2} dt$ <p>Let $\frac{3t-2}{(t-2)^2} = \frac{A}{(t-2)} + \frac{B}{(t-2)^2}$</p> $3t - 2 = A(t - 2) + B$ <p>Comparing the coefficients of t and constant terms on both sides</p> $A = 3, -2A + B = -2, B = 4$ $\int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta = \int \frac{3}{t-2} dt + \int \frac{4}{(t-2)^2} dt$ $= 3 \log t-2 - \frac{4}{t-2} + C$ $= 3 \log \sin \theta - 2 - \frac{4}{\sin \theta - 2} + C$	<p>OR</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$+ \frac{1}{2}$</p> <p>1+1</p> <p>$\frac{1}{2}$</p>
34A	$y + \frac{d}{dx}(xy) = x(\sin x + x)$ $\Rightarrow y + \left(x \frac{dy}{dx} + y\right) = x(\sin x + x)$ $\Rightarrow 2y + x \frac{dy}{dx} = x(\sin x + x)$ $\Rightarrow \frac{dy}{dx} + \frac{2y}{x} = (\sin x + x)$ <p>This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$</p> <p>$P = \frac{2}{x}, Q = (\sin x + x)$</p> $\text{I.F} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$ <p>Solution will be $y \cdot \text{I.F} = \int Q \cdot \text{I.F} dx$</p> $yx^2 = \int (\sin x + x) x^2 dx$ $yx^2 = \int \sin x \cdot x^2 dx + \int x^3 dx$	<p>1</p> <p>1</p> <p>1</p>

SECTION- E (3 case-study/passage-based questions of 4 marks each)		
36	<p>I. Traffic flow is not reflexive as $(A, A) \notin R$ (or no major spot is connected with itself)</p> <p>II. Traffic flow is not transitive as $(A, B) \in R$ and $(B, E) \in R$, but $(A, E) \notin R$</p> <p>III A. $R = \{(A, B), (A, C), (A, D), (B, C), (B, E), (C, E), (D, E), (D, C)\}$ Domain = $\{A, B, C, D\}$ Range = $\{B, C, D, E\}$</p> <p style="text-align: center;">OR</p> <p>III B. No, the traffic flow doesn't represent a function as A has three images.</p>	<p>1</p> <p>1</p> <p>1</p> <p>$\frac{1}{2} + \frac{1}{2}$</p> <p>1+1</p>
37	<p>I. $P(x) = R(x) - C(x) = -0.3x^2 + 20x - (0.5x^2 - 10x + 150)$ $= -0.8x^2 + 30x - 150$</p> <p>II. For critical points $P'(x) = 0 \Rightarrow -1.6x + 30 = 0$ $\Rightarrow x = \frac{30}{1.6} = \frac{300}{16} = 18.75$</p> <p>III A. Now $P''(x) = -1.6$ In particular $P''(18.75) = -1.6 < 0$ So, critical value $x = 18.75$ corresponds to a maximum profit.</p> <p style="text-align: center;">OR</p> <p>III B. As x is the number of bulbs, so practically 18 bulbs correspond to a maximum profit. Maximum profit is $P(18) = -0.8 \times 18^2 + 30 \times 18 - 150$ $= -259.2 + 540 - 150$ $= 540 - 409.2 = ₹130.80$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
38	<p>Let the events be E_1: the student is in the first group (time spent on screen is more than 4 hours) E_2: the student is in the second group (time spent on screen is 2 to 4 hours) E_3: the student is in third group (time spent on screen is less than 2 hours) A: the event of the student showing symptoms of anxiety and low retention</p> <p>$P(E_1) = \frac{60}{100}$ $P(E_2) = \frac{30}{100}$ and $P(E_3) = \frac{10}{100}$ $P(A E_1) = \frac{80}{100}$ $P(A E_2) = \frac{70}{100}$ and $P(A E_3) = \frac{30}{100}$</p> <p>I. $P(A) = P(E_1) \times P(A E_1) + P(E_2) \times P(A E_2) + P(E_3) \times P(A E_3)$ $= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{70}{100} + \frac{10}{100} \times \frac{30}{100} = \frac{72}{100} = 72\%$</p> <p>II. $P(E_1 A) = \frac{P(E_1 \cap A)}{P(A)}$ $= \frac{\left(\frac{60}{100} \times \frac{80}{100}\right)}{\left(\frac{72}{100}\right)} = \frac{48}{72} = \frac{2}{3}$</p>	<p>2</p> <p>2</p>