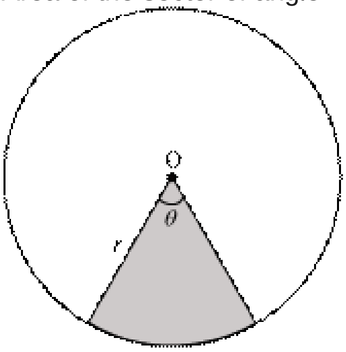
Areas Related to Circles

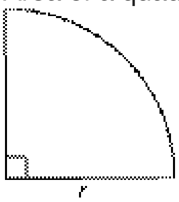
- **Area of sector:**

Area of the sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$, where *r* is the radius of the circle.



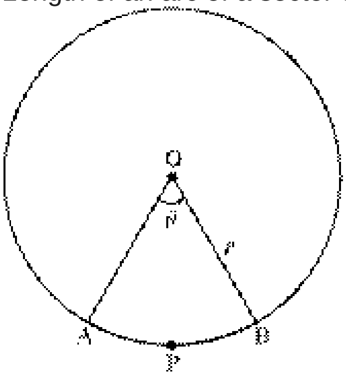
- **Area of quadrant:**

Area of a quadrant of a circle with radius *r* = $\frac{\pi r^2}{4}$ $\left[\theta = 90° \Rightarrow \frac{\theta}{360°} = \frac{90°}{360°} = \frac{1}{4}\right]$



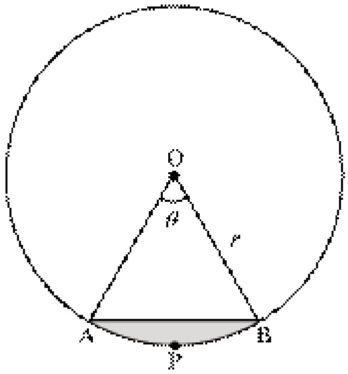
- **Area of a semicircle** $= \frac{180°}{360°} \times \pi r^2 = \frac{1}{2}\pi r^2$
- **Length of an arc:**

Length of an arc of a sector of angle $\theta = \frac{\theta}{360°} \times 2\pi r$, where *r* is the radius of the circle



Perimeter of a Sector = l+2r

- **Area of the segment of a circle**



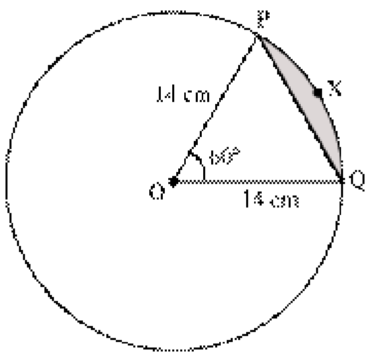
Area of segment APB

$$= \textbf{Area of sector OAPB} - \textbf{Area of } \Delta\textbf{OAB}$$

$$= \frac{\theta}{360} \times \pi r^2 - \textbf{area of } \Delta\textbf{OAB}$$

Example:

In the given figure, the radius of the circle is 14 cm, and ÐPOQ = 60º. Find the area of the segment P XQ.



Solution:

Area of segment PXQ = Area of sector OPXQ − Area of ΔOPQ — (1)

Area of sector OPXQ

$= \frac{60°}{360°} \times \pi \times 14 \times 14 \text{ cm}^2$ $\left[\text{Area of sector of angle } \theta \text{ and radius } r = \frac{\theta}{360} \times \pi r^2\right]$

$= \frac{1}{6} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$= \frac{22}{3} \times 14 = \frac{308}{3} \text{ cm}^2$

In ΔOPQ, we have

OP = OQ **[radii of the same circle]**

$\Rightarrow \angle OPQ = \angle OQP = \frac{1}{2}(180 - 60°) = 60°$

ΔOPQ is an equilateral triangle.

$\text{Area of } \Delta OPQ = \frac{\sqrt{3}}{4} \times (14)^2 \text{cm}^2$

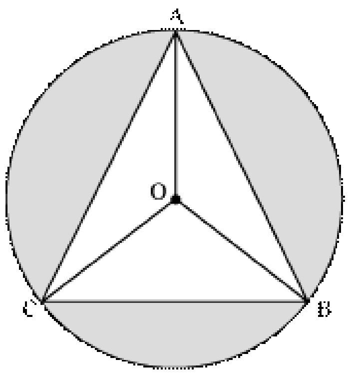
$= 49\sqrt{3} \text{ cm}^2$

$\therefore \text{From (1), Area of segment PXQ} = \left(\frac{308}{3} - 49\sqrt{3}\right)\text{cm}^2$

- **Areas of Combination of Plane Figures**

Example:

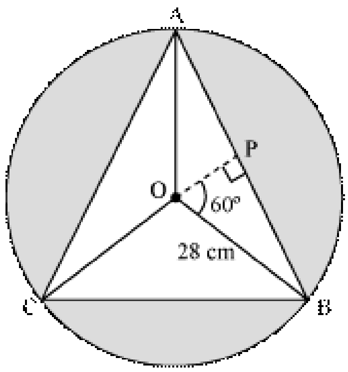
In the given figure A, B, and C are points on the circle with centre O, such that
∠AOB = ∠AOC = ∠BOC. If the radius of the circle is 28 cm. Find the area of the shaded region.



Solution:

Area of the shaded region = Area of the circle – Area of ΔABC

$\text{Area of the circle} = \pi \times (\text{Radius})^2 = \frac{22}{7} \times 28 \times 28 \text{ cm}^2 = 22 \times 4 \times 28 \text{ cm}^2$



Since $\angle AOB = \angle BOC = \angle AOC$

$\angle AOB + \angle BOC + \angle AOC = 360^\circ$

$\Rightarrow \angle AOB = \angle BOC = \angle AOC = \frac{1}{3} \times 360^\circ = 120^\circ$

It can be easily shown that

$\Delta AOB \cong \Delta AOC \cong \Delta BOC$

$\Rightarrow AB = BC = CA$

$\therefore \Delta ABC$ is an equilateral triangle

Draw $OP \perp AB$

Then, in ΔOAP and ΔOBP, we have

$\angle OPA = \angle OPB = 90^\circ$

$OA = OB$ [radii of the same circle]

$OP = OP$ [common]

$\therefore \Delta OAP \cong \Delta OBP$ [by RHS congruency criterion]

$\Rightarrow \angle AOP = \angle BOP$ [C.P.C.T]

$\therefore \angle BOP = \frac{1}{2} \times 120^\circ = 60^\circ$

Now, $\frac{PB}{OB} = \text{Sin } 60^\circ = \frac{\sqrt{3}}{2}$

$\therefore PB = \frac{\sqrt{3}}{2} \times 28 = 14\sqrt{3}$

$\therefore AB = AP + PB = 2PB = 2 \times 14\sqrt{3} = 28\sqrt{3}$ cm

$\therefore$ Area of $\Delta ABC = \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times 28\sqrt{3} \times 28\sqrt{3} \text{ cm}^2 = 7 \times 28 \times 3\sqrt{3} \text{ cm}^2$

Thus, area of the shaded region

$= \left(22 \times 4 \times 28 - 7 \times 28 \times 3\sqrt{3}\right) \text{ cm}^2$

$= 28\left(88 - 21\sqrt{3}\right) \text{ cm}^2$