Chapter 3. Pair of Linear Equations in Two Variable

Question-1

Find the value of k for which the system of equations have infinitely many $x - ky = 2$, $3x + 6y = -5$.

Solution:

$x - ky = 2$

$3x + 6y = -5$

$a_1 = 1$, $b_1 = -k$

$a_2 = 3$, $b_2 = 6$

Given, the system of equations have infinitely many solutions

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad = \frac{1}{3} = \frac{-k}{6}$$

$\Rightarrow k = -2$

Question-2

Draw the graph of the following equation and check whether (a) $x = 2, y = 5$ (b) $x = -1, y = 3$ are solutions of $2x + 5y = 13$.

Solution:

$5y = 13 - 2x$

$$y = \frac{13 - 2x}{5}$$

(Note: To draw a line any two points are enough. So let us consider any three points.)

$$\text{When } x = 4;\ y = \frac{13 - 2(4)}{5} = \frac{13 - 8}{5} = \frac{5}{5} = 1$$

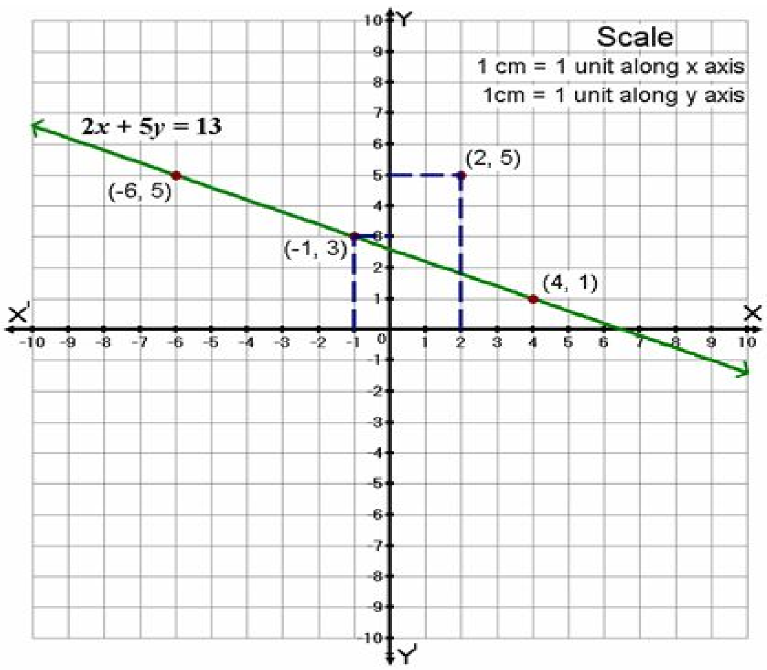
$$\text{When } x = -1;\ y = \frac{13 - 2(-1)}{5} = \frac{13 + 2}{5} = \frac{15}{5} = 3$$

$$\text{When } x = -6;\ y = \frac{13 - 2(-6)}{5} = \frac{13 + 12}{5} = \frac{25}{5} = 5$$

Table

x	4	-1	-6
y	1	3	5

Graphical Representation



x = 2, y = 5 is not the solution.

x = -1, y = 3 is the solution.

Question-3

Find graphically the vertices of the triangle whose sides have the equations. 2y - x = 8; 5y – x = 14; y – 2x = 1.

Solution:

2y - x = 8

x	0	1	2
y	4	4.5	5

5y - x = 14

x	0	1	6
y	2.8	3	4

y - 2x = 1

x	0	1	- 1
y	1	3	- 1

The vertices of the triangle are (1, 3), (2, 5) and (-4, 2).

Question-4

Draw the graph of the following equation and check whether (a) x = 2, y = 5 (b) x = -1, y = 3 are solutions of 5x + 3y = 4.

Solution:

3y = 4 - 5x

$$y = \frac{4 - 5x}{3}$$

When x = 2 ; $y = \frac{4 - 5(2)}{3} = \frac{4 - 10}{3} = \frac{-6}{3} = -2$

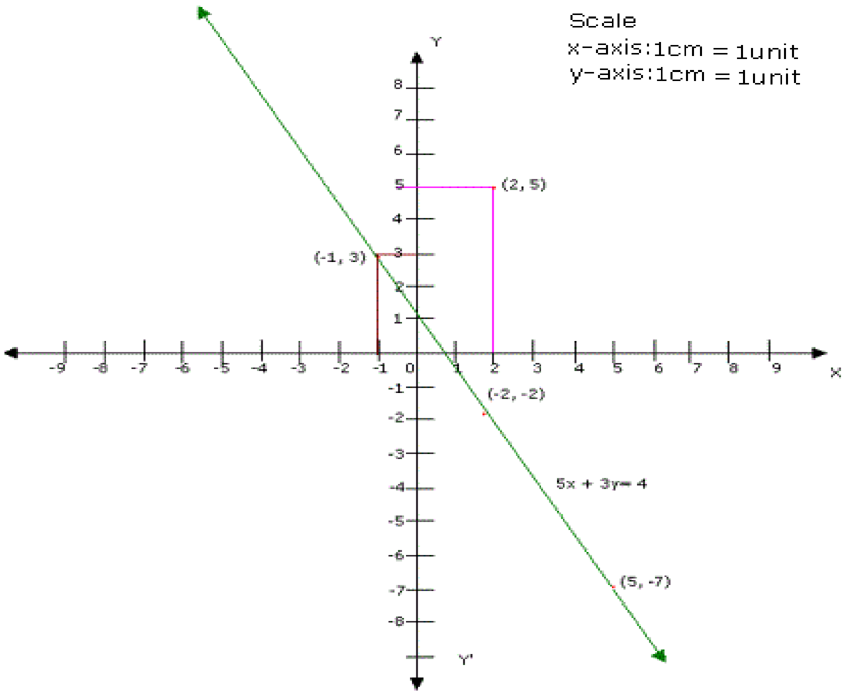
When x = 5 ; $y = \frac{4 - 5(5)}{3} = \frac{4 - 25}{3} = \frac{-21}{3} = -7$

When x = -1 ; $y = \frac{4 - 5(-1)}{3} = \frac{4 + 5}{3} = \frac{9}{3} = 3$

Table

x	2	5	-1
y	-2	-7	3

Graphical Representation



x = 2, y = 5 is not the solution.

but x = -1, y = 3 is the solution of the given equation.

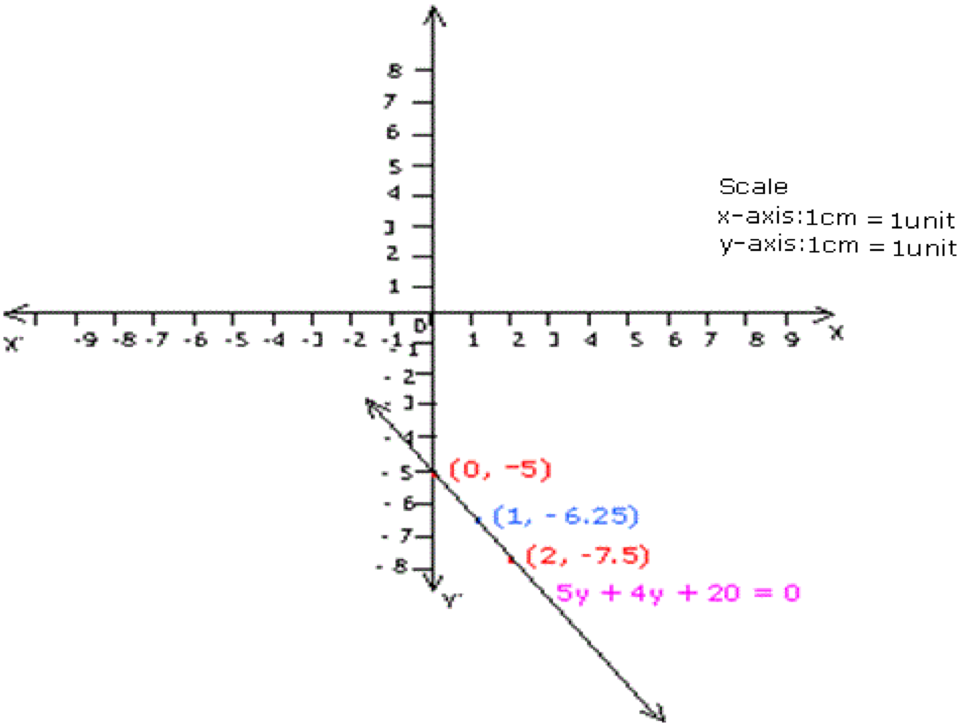
Question-5

Draw the graph of the equation 5x + 4y + 20 = 0. From the graph, find the co-ordinates of the point when i) x = 0 ii) y= - 5.

Solution:

5x + 4y + 20 = 0

x	0	1	2
y	- 5	- 6.25	-7.5



The co ordinates of the point when x = 0, y = - 5.
The co ordinates of the point when y = - 5, x = 0.

Question-6

2 audio cassettes and 3 video cassettes cost `425 and 3 audio cassettes and 2 video cassettes cost `350. What are the prices of an audio cassette and of a video cassette?.

Solution:

Let the price of an audio cassette be `x and of a video cassette be `y.

Condition 1:
2 audio cassettes and 3 video cassettes cost `425.
2x + 3y = 425........ (i)

Condition 2:
3 audio cassettes and 2 video cassettes cost `350
3x + 2y = 350.......(ii)

Multiplying (i) by 2 and (ii) by 3

$2\ (2x + 3y = 425)$

$4x + 6y = 850$ ....(iii)

$3\ (3x + 2y = 350)$

$9x + 6y = 1050$ ....(iv)

Subtracting (iv) from (iii)

$4x + 6y = 850$

$9x + 6y = 1050$

(-) (-) (-)

$-5x \quad = -200$

$x = \frac{200}{5}$

$x = 40$

Substitute x = 40 in (i)

$2\ (40) + 3y = 425$

$80 + 3y = 425$

$3y = 425 - 80$

$3y = 345$

$y = \frac{345}{3}$

$y = 115$

∴The price of an audio cassette is `40 and the price of a video cassette is `115.

Checking:

(a) $2x + 3y = 2\ (40) + 3\ (115) = 80 + 345 = 425$

(b) $3x + 2y = 3\ (40) + 2\ (115) = 120 + 230 = 350$.

Question-7

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water?.

Solution:

Let the speed of the boat in still water be x km/hr
and the speed of the stream be y km/hr.

Speed of the boat while going downstream = (x+y) km/hr.
and speed of the boat while going upstream = (x-y) km/hr

Condition 1:
A boat goes 30 km upstream and 44 km downstream in 10 hrs.

Time taken to go upstream by the boat = $\frac{30}{x-y}$

and time taken to go downstream by the boat = $\frac{44}{x+y}$

$\frac{30}{x-y} + \frac{44}{x+y} = 10$ ....(i)

Condition 2:
A boat goes 40 km upstream and 55 km downstream in 13 hrs.

Time taken to go upstream by the boat = $\frac{40}{x-y}$

and time taken to go downstream by the boat = $\frac{55}{x+y}$

$\frac{40}{x-y} + \frac{55}{x+y} = 13$ ....(ii)

The L.C.M. of 44 and 55 is 440.
∴ Multiplying (i) by 5 and (ii) by 4

$\frac{150}{x-y} + \frac{220}{x+y} = 50$ ....(iii)

$\frac{160}{x-y} + \frac{220}{x+y} = 52$ ....(iv)

(-) (-) (-)

$$\frac{-10}{x-y} = -2$$

$-10 = -2(x-y)$

$5 = x-y$

$\therefore x - y = 5$ ………………(v)

Substituting x - y = 5 in (i)

$$\frac{30}{5} + \frac{44}{x+y} = 10$$

$$6 + \frac{44}{x+y} = 10$$

$$\frac{44}{x+y} = 10-6$$

$$\frac{44}{x+y} = 4$$

$44 = 4(x+y)$

$11 = x + y$

$\therefore x + y = 11$ ………………(vi)

Adding (v) and (vi)

$x - y = 5$

$\underline{x + y = 11}$

$2x \quad = 16$

$x = 8$

Substituting x = 8 in (v)

8 - y = 5

-y = 5 -8

-y = -3

y = 3

∴The speed of the stream is 3km/hr and the speed of the boat in still water is 8km /hr.

Checking:

a. $\frac{30}{x-y} + \frac{44}{x+y} = \frac{30}{8-3} + \frac{44}{8+3} = \frac{30}{5} + \frac{44}{11} = 6 + 4 = 10$

b. $\frac{40}{x-y} + \frac{55}{x+y} = \frac{40}{8-3} + \frac{55}{8+3} = \frac{40}{5} + \frac{55}{11} = 8 + 5 = 13.$

Question-8

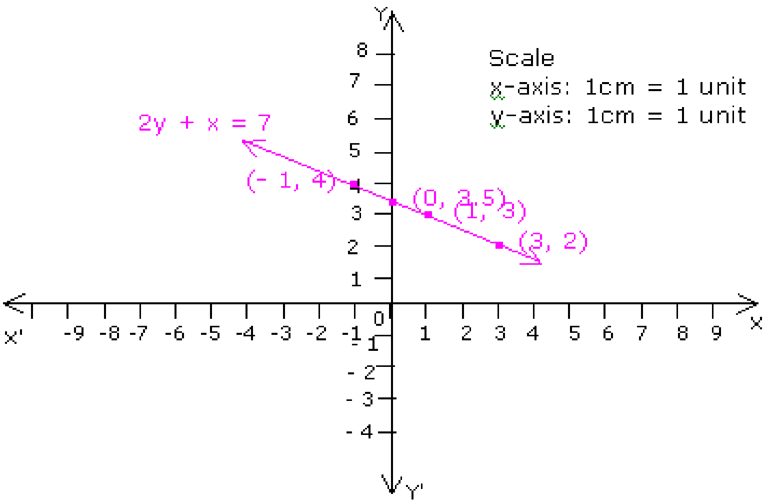
Draw the graph of the linear equation 2y + x = 7. Check whether the point (3, 2) lies on the line or not.

Solution:

2y + x = 7 ⇒ 2y = 7 - x ∴ y = $\frac{7-x}{2}$

x	1	- 1	0
y = $\frac{7-x}{2}$	3	4	3.5

x	1	- 1	0
$y = \frac{7-x}{2}$	3	4	3.5



Yes, the point (3, 2) lies on the line 2y + x = 7.

Question-9

From Delhi station if we buy 2 tickets to station A and 3 tickets to station B, the total cost is `77, but if we buy tickets to station A and 5 tickets to station B, the total cost is `124. What are the fares from Delhi to station A and to station B?.

Solution:

Let the fares from Delhi to station A and to station B be `x and `y respectively.

Condition 1:

2 tickets to station A and 3 tickets to station B costs `77.

2x + 3y = 77 .... (i)

Condition 2:

3 tickets to station A and 5 tickets to station B costs `124.

3x + 5y = 124 ....(ii)

Multiplying (i) by 5 and (ii) by 3

5 (2x+3y = 77)

10x + 15y = 385 .... (iii)

3 (3x+5y = 124)

9x + 15y = 372.............. (iv)

Subtracting (iv) from (iii)

10x + 15y = 385

9x + 15y = 372

(-) (-) (-)

----

x = 13

----

x = 13

Substitute x = 13 in (i)

2(13) + 3y = 77

26 + 3y = 77

3y = 77 - 26

3y = 51

$y = \frac{51}{3}$

⇒ y = 17

∴ The cost of fare from Delhi to station A is `13.

and the cost of fare from Delhi to station B is `17.

Checking:

(a) 2x + 3y = 2(13) + 3 (17) = 26 + 51 = 77

(b) 3x + 5y = 3 (13) + 5 (17) = 39 + 85 = 124.

Question-10

Points A and B are 70 km apart on a high way. A car starts from A and another car starts from B at the same time. If they travel in the same

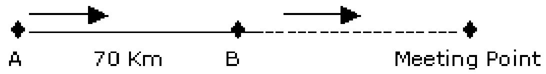
direction, they meet in 7 hours, but if they travel towards each other they meet in one hour, what are their speeds?.

Solution:

Let the speed of the car starting from A be at x km/hr and that starting from B be at y km/hr.

Condition 1:

If the cars travel in the same direction, they meet in 7 hrs.



We know that,

Distance = Speed x Time

$\therefore$ Distance covered by car starting from A in 7 hours = 7x km.

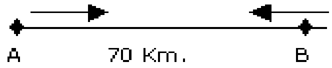
and distance covered by car starting from B in 7 hours = 7y km

$\therefore$ $7x - 7y = 70$

i.e. $x - y = 10$ (dividing by 7 throughout) .... (i)

Condition 2:

If the cars travel in opposite direction, they meet in 1 hour.



$\therefore$ Distance covered by car starting from A in 1 hour = x km.

and distance covered by car starting from B in 1 hour = y km.

$\therefore$ $x + y = 70$ .... (ii)

from (i) and (ii)

$x - y = 10$

$x + y = 70$

$2x = 80$

$x = \frac{80}{2}$

x = 40 km/hr

Substitute x = 40 in (ii)

40 + y = 70
y = 70 - 40

y = 30 km/hr.

∴ The speed of the car starting from A is 40 km/hr.
and the speed of the car starting from B is 30 km/hr.

Checking:
(a) x - y = 40 - 30 = 10
(b) x + y = 40 + 30 = 70.

Question-11

A train covered a certain distance at a uniform speed. If the train would have been 6 km/hr faster, it would have taken 4 hours less that than scheduled time and if the train were slower by 6 km/hr, it would have taken 6 hours more than the scheduled time. Find the length of the journey.

Solution:
Let the actual speed of the train be x km/hr and the actual time taken be y km/hr.
Then,Distance = Speed × Time taken = (xy) km
If the speed is increased by 6 km /hr, then time of journey is reduced by 4 hours. ⇒ When speed is (x + 6) km / hr, time of journey is (y − 4) hours. ∴ Distance = (x + 6) (y − 4) ⇒ xy = xy - 4x + 6y − 24 ⇒ - 4x + 6y + 24 = 0 ∴ - 2x + 3y - 12 = 0 ....(i)

When the speed is reduced by 6 km/hr, then the time taken for the journey is increased by 6 hours.

When the speed is (x – 6) km/hr, time of journey is (y + 6) hours ∴ Distance = (x – 6) (y + 6)

xy = (x – 6) (y + 6)

xy = xy + 6x – 6y – 36 ⇒ 6x – 6y – 36 = 0 ∴ x – y – 6 = 0..........................(ii)

Solving (i) and (ii),

Using cross multiplication, we have,

$\frac{3}{5} = \frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{-3}{-9} = \frac{c_1}{c_2} = \frac{3}{2}$ ∴ x = 30 and y = 24

Putting the values of x and y in equation (i) we get,

Distance = 30 × 24 = 720 km ∴ The length of the journey = 720 km.

Question-12

The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area is increased by 67 square units. Find the length and breadth of the rectangle.

Solution:

Let the length of the rectangle be x units and the breadth of the rectangle be y units.

Condition 1:

The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and the breadth is increased by 3 units.

(x - 5) (y + 3) = xy - 9

xy + 3x - 5y - 15 = xy - 9

3x - 5y = -9 + 15

3x - 5y = 6 .... (i)

Condition 2: If we increase the length by 3 units and breadth by 2 units, the area is increased by 67 square units.

(x+3) (y+2) = xy + 67

xy + 2x +3y + 6 = xy + 67

2x + 3y = 67-6

2x + 3y = 61 .... (ii)

Multiplying (i) by 3 and (ii) by 5

3 (3x - 5y = 6)

9x - 15y = 18 .... (iii)

5 (2x+3y = 61)

10x + 15y = 305 .... (iv)

9x - 15y = 18

10x + 15y = 305

19x = 323

$x = \frac{323}{19}$

x = 17

Substitute x = 17 in (i)

3x - 5y = 6

3(17) - 5y = 6

51 - 5y = 6

-5y = 6-51

-5y = -45

$y = \frac{-45}{-5}$

y = 9

∴ The length is 17 units and the breadth is 9 units.

Checking:

Area of the rectangle is 17 x 9 = 153 square units

a. (x-5) (y+3) = (17-5) (9+3) = 12 × 12 = 144 = 153 - 9

b. (x+3) (y+2) = (17+3) (9+2) = 20 × 11= 220 = 153 + 67.

Question-13

In a parallelogram, one angle is 4/5 th of the adjacent angle. Determine the angles of the parallelogram.

Solution:

Let x° and y° be the two adjacent angles of the parallelogram (x>y).

According to the given condition,

y = (4/5) x --------------- (1)

But x+y =180 --------------- (2)

Putting y = (4/5)x in (2) we get x + (4/5) x = 180

or (9/5)x = 180, ⇒ x = 180 × (5/9) = 100

y = (4/5) ×100 = 80

The opposite angles of a parallelogram are equal.

∴ The angles of the parallelogram are 100°, 80°, 100° and 80°.

Question-14

A man bought 4 horses and 9 cows for `1340. He sells the horses at a profit of 10% and the cows at a profit of 20% and his whole gain is `188. What price did he pay for the horse.

Solution:

Let the cost of one horse be `x.

The cost of 4 horses and 9 cows = `1340 ⇒ The cost of one cow = Rs. 1340 − 4x

He sold the cows at a profit of 10% and the horses at a profit of 20%.

Profit when the horses are sold is 10%. ⇒ $\frac{b_1}{b_2}$ × 100 = 10% ⇒ Profit when the horses are sold = $\frac{c_1}{c_2}$ = ` $\frac{2}{3}$

Profit when the cows are sold is 20%. ⇒ $\frac{c_1}{c_2}$ × 100 = 20% ⇒ Profit when the cows are sold = $\frac{-5}{-8}$ = ` $\frac{x}{-8+10}$

The total profit he gained = ` 188 ⇒ ` $\frac{2}{3}$ + ` $\frac{x}{-8+10}$ = `188 ⇒ 2x + (1340 − 4x) = 188 × 5 ⇒ - 2x = 188 × 5 − 1340

= 940 - 1340

= - 400 ∴ x = 200 ∴ The price he paid for the horse is `200.

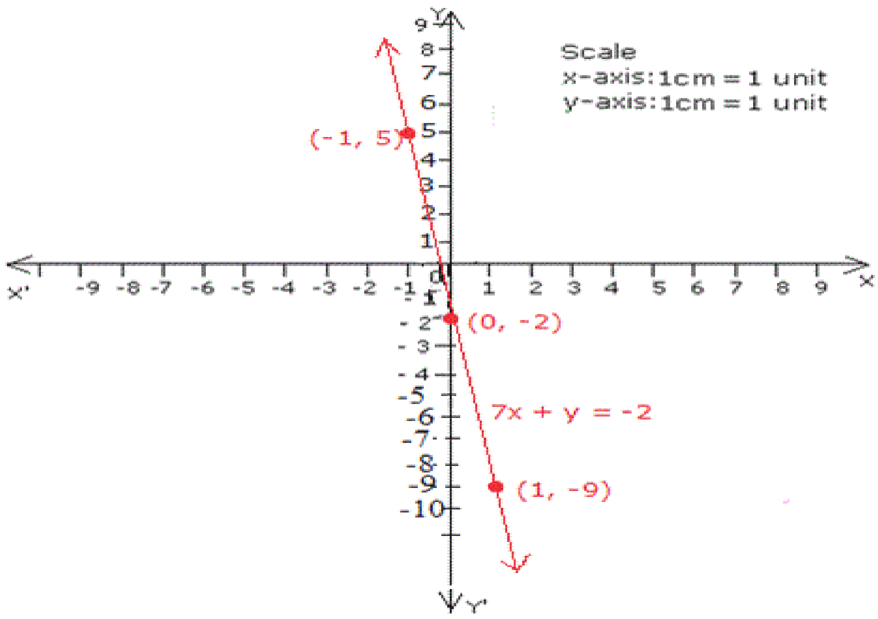
Question-15

Find the points of intersection of the line represented by the equation 7x + y = - 2 with y-axis. Check whether the point (2, 1) is a solution set of the given equation.

Solution:

7x + y = - 2 ∴ y = - 2 - 7x = - (2 + 7x)

x	0	-1	1
y = -(2 + 7x)	-2	5	-9



Question-16

A man starts his job with a certain monthly salary and earns a fixed increment every year. If his salary was `1500 after 4 years of service and `1800 after 10 years of service what was his starting salary and what is the annual increment?.

Solution:

Let the man's starting salary be `x and his annual increment be `y

Condition 1:

His salary after 4 years of service is `1500

$x + 4y = 1500$.............(i)

Condition 2:

His salary after 10 years of service is `1800

$x + 10y = 1800$..........(ii)

Subtracting (i) from (ii)

$$
\begin{array}{l}
x + 10y = 1800 \\
x + \ \ 4y = 1500 \\
(-) \quad (-) \quad (-) \\
\hline
\qquad 6y = 300 \\
\hline
\end{array}
$$

$$y = \frac{300}{6}$$

$$y = 50$$

Substitute y = 50 in (i)

$$x + 4(50) = 1500$$

$$x + 200 = 1500$$

$$x = 1500 - 200$$

$$x = 1300$$

∴ The man's starting salary is `1300 and his annual increment is `50.

Checking:

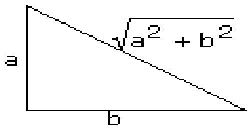
(a) Man's salary after 4 years = 1300 + 4(50) = 1300 + 200 = `1500

(b) Man's salary after 10 years = 1300 + 10(50) = 1300 + 500 = `1800.

Question-17

The perimeter of a right-angled triangle is five times the length of its shortest side. The numerical value of the area of the triangle is 15 times the numerical value of the length of the shortest side. Find the lengths of the three sides of the triangle.

Solution:



Let the two sides be a cm and b cm respectively. Let a cm be the shortest side. $\Rightarrow a + b + (a^2+b^2)^{1/2} = 5a$ ....(i) $\Rightarrow 1/2ab = 15a$ ....(ii) $\Rightarrow ab = 30a \therefore b = 30$ (i)

$a + b + (a^2+b^2)^{1/2} = 5a$

$(a^2+b^2)^{1/2} = 5a - a - b$

$= 4a - b$

Squaring both sides,

$a^2 + b^2 = (4a - b)^2$

$\Rightarrow a^2 + b^2 = 16a^2 + b^2 - 8ab$

$\Rightarrow 15a^2 = 8ab \therefore a = 16$. $\therefore$ The sides of the triangle are 16 cm, 30 cm and 34 cm.

Question-18

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water?.

Solution:

Let the speed of the boat in still water be x km/hr
and the speed of the stream be y km/hr.

Speed of the boat while going downstream = (x + y) km/hr.
and speed of the boat while going upstream = (x - y) km/hr

Condition 1:

A boat goes 30 km upstream and 44 km downstream in 10 hrs.

Time taken to go upstream by the boat = $\frac{30}{x-y}$

and time taken to go downstream by the boat = $\frac{44}{x+y}$

$\frac{30}{x-y} + \frac{44}{x+y} = 10$ ....(i)

Condition 2:

A boat goes 40 km upstream and 55 km downstream in 13 hrs.

Time taken to go upstream by the boat = $\frac{40}{x-y}$

and time taken to go downstream by the boat = $\frac{55}{x+y}$

$\frac{40}{x-y} + \frac{55}{x+y} = 13$ ....(ii)

The L.C.M. of 44 and 55 is 440.
∴ Multiplying (i) by 5 and (ii) by 4

$\frac{150}{x-y} + \frac{220}{x+y} = 50$ ....(iii)

$\frac{160}{x-y} + \frac{220}{x+y} = 52$ ....(iv)

(-) (-) (-)

$\frac{-10}{x-y} = -2$

$-10 = -2(x - y)$

$5 = x - y$

∴ $x - y = 5$ ....(v)

Substituting $x - y = 5$ in (i)

$30/5 + \frac{44}{x+y} = 10$

$6 + \frac{44}{x+y} = 10$

$\frac{44}{x+y} = 10 - 6$

$\frac{44}{x+y} = 4$

$44 = 4(x + y)$

$11 = x + y$

∴ $x + y = 11$..................(vi)

Adding (v) and (vi)

$x - y = 5$

$x + y = 11$

$2x = 16$

$x = 8$

Substituting $x = 8$ in (v)

$8 - y = 5$

$-y = 5 - 8$

$-y = -3$

$y = 3$

∴The speed of the stream is 3km/hr and the speed of the boat in still water is 8km /hr.

Checking:

a. $\frac{30}{x-y} + \frac{44}{x+y} = \frac{30}{8-3} + \frac{44}{8+3} = \frac{30}{5} + \frac{44}{11} = 6 + 4 = 10$

b. $\frac{40}{x-y} + \frac{55}{x+y} = \frac{40}{8-3} + \frac{55}{8+3} = \frac{40}{5} + \frac{55}{11} = 8 + 5 = 13.$

Question-19

Solve the following system of equations:

$$\frac{x+y-8}{2} = \frac{x+2y-14}{8} = \frac{3x+y-12}{11}$$

Solution:

$$\frac{x+y-8}{2} = \frac{x+2y-14}{8}$$

$8(x+y-8) = 2(x+2y-14)$

$6x + 4y - 36 = 0$

$3x + 2y - 18 = 0$ --------(i)

$$\frac{x+2y-14}{8} = \frac{3x+y-12}{11}$$

$11(x + 2y - 14) = 8(3x + y - 12)$

$13x - 14y + 58 = 0$ ------(ii)

Multiplying (i) by 7 and subtracting from (ii)

$34x - 68 = 0$ or $x = 2$

Substituting $x = 2$ in (i),

$3(2) + 2y - 18 = 0$

$6 + 2y - 18 = 0$

$2y - 12 = 0$

$y = 6$

The solution set is {2, 6}.

Question-20

The ratio of two numbers is 2 : 3. If two is subtracted from the first number and 8 from the second, the ratio becomes the reciprocal of the original ratio. Find the numbers.

Solution:

Let the two numbers be 2x and 3x.

If two is subtracted from the first number and 8 from the second, the ratio becomes the reciprocal of the original ratio. $\Rightarrow \frac{1}{2} = \frac{2x-2}{3x-8} = \frac{3}{2}$ $\Rightarrow$ 2 (2x − 2) = 3 (3x − 8) ⇒ 4x − 4 = 9x − 24 ⇒ 9x − 4x = 24 − 4 ⇒ 5x = 20 ∴ x = 4

Substituting the value of x = 4, the two numbers are 8 and 12.

Question-21

Solve the following system of equations:

$\frac{x+y}{xy} = 2; \ \frac{x-y}{xy} = 6.$

Solution:

$\frac{x+y}{xy} = 2; \ \frac{x-y}{xy} = 6$

Putting 1/x = u and 1/y = v, we get

v + u = 2 -----(i)

v - u = 6 ------(ii)

Subtracting (i) and (ii),

2u = -4 or u = -2;

Hence x = $-\frac{1}{2}$.

Substituting u = -2 in (i),

v = 4.

Hence y = $\frac{1}{4}$.

The solution set is {$-\frac{1}{2}$, $\frac{1}{4}$}.

Question-22

Determine graphically, the vertices of the triangle, the equations of whose sides are given below:

2x − y + 1 = 0, x − 5y + 14 = 0 and x − 2y + 8 = 0.

Solution:

2x − y + 1 = 0

y = 2x + 1

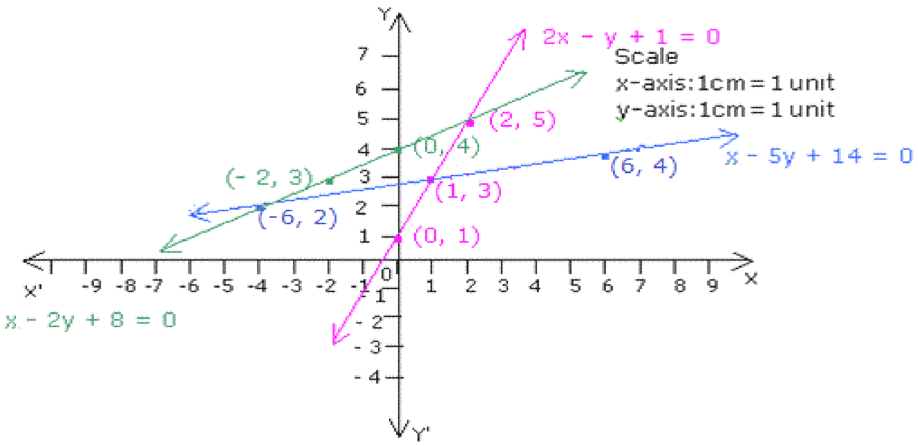
x	0	1	2
y = 2x + 1	1	3	5

x − 5y + 14 = 0 ⇒ 5y = x + 14 ∴ y = $\frac{x+14}{5}$

x	1	-4	6
y = $\frac{x+14}{5}$	3	2	4

$x - 2y + 8 = 0 \Rightarrow 2y = x + 8 \therefore y = \frac{x+8}{2}$

x	0	2	- 2
$y = \frac{x+8}{2}$	4	5	3



The vertices of the triangle formed by the three lines are (2, 5), (1, 3) and (-4, 2).

Question-23

Solve the following system of equations :

$(a + c)x - (a - c)y = 2ab$;

$(a + b)x - (a - b)y = 2ab$.

Solution:

$(a+c)x - (a-c)y = 2ab$ ------(i)

$(a+b)x - (a-b)y = 2ab$ -----(ii)

Multiplying (ii) by (a+c) and (i) by (a+b), and subtracting,

$(a+b)(a+c)x - (a+b)(a-c)y = 2ab(a+b)$..........(iii)

$(a+b)(a+c)x - (a-b)(a+c)y = 2ab(a+c)$..........(iv)

Subtracting (iii) and (iv)

$-(a+b)(a-c)y + (a-b)(a+c)y = 2ab(a + b) - 2ab(a + c)$

$(-a^2 - ab + ac + bc + a^2 + ac - ab - bc)y = 2ab(a + b - a - c)$

$-2a(b - c)y = 2ab(b - c)$

$\Rightarrow y = -b$;

Substituting $y = -b$ in (i),

$(a + c)x - (a-c)(-b) = 2ab$;

$(a + c)x = 2ab - ab + bc$

$= ab + bc$

= b(a +c)

Þ x = b

∴ The solution set is {b, -b}.

Question-24

Find three consecutive numbers such that seven times the smallest number may be equal to three times the sum of the other two.

Solution:

Let the smallest number = x.
Then the other two numbers are x + 1 and x + 2.
Also 7 times smallest number = 7x.
3 times the sums of other two numbers = 3(x + 1 + x + 2).
According to the condition of the problem
7x = 3(x + 1+ x + 2) or 7x = 6x + 9
x = 9
Hence the numbers are 9, 10, 11.

Question-25

A man covers a distance of 14 km in an hour, partly on foot at the rate of 4km/hr and partly on scooter at 44 km/hr. Find the distance travelled by him on foot.

Solution:

Let the distance travelled on foot be x km.
Then, distance covered on scooter = (14 - x) km.

Therefore time taken to cover the distance on foot = $\frac{x}{4}$ hrs.
Time taken to cover the distance on scooter = $\frac{14-x}{44}$ hrs.
$\frac{x}{4} + \frac{14-x}{44} = 1$

11x + 14 − x = 44
10x = 30
x = 3 km.

Hence, the distance covered on foot is 3 km.

Question-26

4 men and 4 boys can do a piece of work in 3 days, while 2 men and 5 boys can finish it in 4 days. How long would it take 1 man alone to do it?.

Solution:

Suppose 1 man alone can finish it in x days and 1 boy alone can finish it in y days. Then,

1 man's 1 day's work = 1/x.
1 boy's 1 day's work = 1/y.
Therefore $4 \times \frac{1}{x} + 4 \times \frac{1}{y} = \frac{1}{3}$

$\frac{1}{x} + \frac{1}{y} = \frac{1}{12}$ ....(i)

and $2 \times \frac{1}{x} + 5 \times \frac{1}{y} = \frac{1}{4}$

$\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$ ....(ii)

Putting $\frac{1}{x}$ = u and $\frac{1}{y}$ = v, these equations become:

$u + v = \frac{1}{12}$........................(iii)

$2u + 5v = \frac{1}{4}$........................(iv)

Multiplying (iii) by 2, we get:

$2u + 2v = \frac{1}{6}$........................(v)

Subtracting (v) from (iv), we get :

$3v = \frac{1}{12} \Rightarrow v = \frac{1}{36}$

Substituting $v = \frac{1}{36}$ in (iii), we get:

$u + \frac{1}{36} = \frac{1}{12}$

$\Rightarrow u = \frac{1}{12} - \frac{1}{36} = \frac{2}{36} = \frac{1}{18}$.

Now, $u = \frac{1}{18} \Rightarrow \frac{1}{x} = \frac{1}{18} \Rightarrow x = 18$.

$v = \frac{1}{36} \Rightarrow \frac{1}{y} = \frac{1}{36} \Rightarrow y = 36$.

Therefore 1 man alone can finish the work in 18 days.

Question-27

A man walks a certain distance at a certain speed. Had he walked (1/2) km/hr faster, he would have taken 3hours longer. Find the distance.

Solution:

Let the original speed be x km/hr and time taken be y hrs.
Then, distance covered = xy km.

Speed = $\left(x + \frac{1}{2}\right)$km/hr,

Time taken = (y - 1)hrs.

Distance = $\left(x + \frac{1}{2}\right)(y - 1)$ km.

Therefore $xy = \left(x + \frac{1}{2}\right)(y - 1)$

$xy = xy - x + \frac{1}{2}y - \frac{1}{2}$

$-x + \frac{1}{2}y - \frac{1}{2} = 0$

$y - 2x - 1 = 0$

$y - 2x = 1$ ....(i)

New speed = (x - 1) km/hr, time taken = (y + 3) hrs.
Therefore distance = (x - 1)(y + 3) km.
Therefore xy = (x - 1)(y + 3)

$3x - y = 3$ ....(ii)

Adding (i) and (ii), we get x = 4
Substituting x = 4 in (i), we get: y =9

Therefore speed = 4 km/hr, time taken = 9 hrs
Hence, distance= 4 × 9 km = 36 km.

Question-28

By selling a table and a chair for `1896, a trader gains 25% on the table and 10% on the chair. If he sells them for `1770, he makes a profit of 10% on the table and 25% on the chair. Find the cost price of each.

Solution:
Let C.P of the table be `x and C.P. of the chair be `y.
Then, $\frac{125}{100}x + \frac{110}{100}y = 1896$
25x + 22y = 37920 ....(i)
$\frac{110}{100}x + \frac{125}{100}y = 1770$
22x + 25y = 35400 ....(ii)

Adding (i) and (ii), we get:
47(x + y) = 73320
x + y = 1560 ....(iii)
Subtracting x = 1200 in (iii), we get:
1200 + y = 1560
y = 360.
Therefore C.P. of 1 table = `1200
Therefore C.P. of 1 chair = `360.

Question-29

If three times the larger of the two numbers is divided by the smaller one, we get 4 as quotient and 3 as the remainder. Also if seven times the smaller number is divided by the larger one, we get 5 as quotient and 1 as remainder. Find the numbers.

Solution:
Let the larger number be x and smaller one be y.
We know that
Dividend = (Divisor × Quotient) + Remainder
3x = 4y + 3
3x − 4y − 3 = 0 ....(ii)

7y = 5x + 1
5x − 7y + 1 = 0 ....(iii)
Solving (ii) and (iii) by cross − multiplication, we get
$\frac{x}{-4-21} = \frac{-y}{3+15} = \frac{1}{-21+20}$
x = 25 and y = 18.