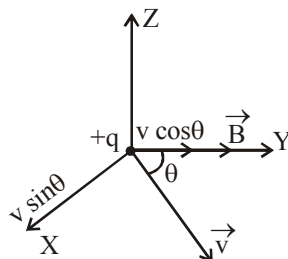


MOVING CHARGES AND MAGNETISM

GENERAL KEY CONCEPT

1. **Force on a moving charge:**— A moving charge is a source of magnetic field.



Let a positive charge q is moving in a uniform magnetic field \vec{B} with velocity \vec{v} .

$$F \propto q \Rightarrow F \propto v \sin \theta \Rightarrow F \propto B$$

$$\therefore F \propto qBv \sin \theta \Rightarrow F = kq Bv \sin \theta \quad [k = \text{constant}]$$

$k = 1$ in S.I. system.

$$\therefore F = qBv \sin \theta \text{ and } \vec{F} = q(\vec{v} \times \vec{B})$$

2. **Magnetic field strength (\vec{B}):**

In the equation $F = qBv \sin \theta$, if $q = 1$, $v = 1$,

$\sin \theta = 1$ i.e. $\theta = 90^\circ$ then $F = B$.

\therefore Magnetic field strength is defined as the force experienced by a unit charge moving with unit velocity perpendicular to the direction of magnetic field.

Special Cases:

- (1) If $\theta = 0^\circ$ or 180° , $\sin \theta = 0$

$$\therefore F = 0$$

A charged particle moving parallel to the magnetic field, will not experience any force.

- (2) If $v = 0$, $F = 0$

A charged particle at rest in a magnetic field will not experience any force.

- (3) If $\theta = 90^\circ$, $\sin \theta = 1$ then the force is maximum

$$F_{\text{max.}} = qvB$$

A charged particle moving perpendicular to magnetic field will experience maximum force.

3. **S.I. unit of magnetic field intensity.** It is called tesla (T).

$$B = \frac{F}{qv \sin \theta}$$

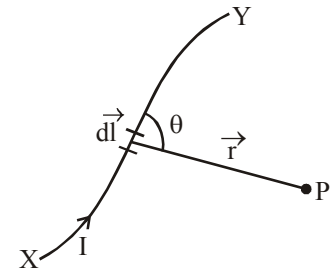
If $q = 1 \text{ C}$, $v = 1 \text{ m/s}$, $\theta = 90^\circ$ i.e. $\sin \theta = 1$ and $F = 1 \text{ N}$

Then $B = 1 \text{ T}$.

The strength of magnetic field at a point is said to be 1T if a charge of 1C while moving at right angle to a magnetic field, with a velocity of 1 m/s experiences a force of 1N at that point.

- 4. Biot-Savart's law:-** The strength of magnetic field or magnetic flux density at a point P (dB) due to current element dl depends on,

- (i) $dB \propto I$
- (ii) $dB \propto dl$
- (iii) $dB \propto \sin \theta$
- (iv) $dB \propto \frac{1}{r^2}$,



$$\text{Combining, } dB \propto \frac{Idl \sin \theta}{r^2} \Rightarrow dB = k \frac{Idl \sin \theta}{r^2} \quad [k = \text{Proportionality constant}]$$

In S.I. units, $k = \frac{\mu_0}{4\pi}$ where μ_0 is called permeability of free space.

$$\mu_0 = 4\pi \times 10^{-7} \text{ TA}^{-1}\text{m}$$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2} \text{ and } d\vec{B} = \frac{\mu_0}{4\pi} I \frac{(\vec{dl} \times \vec{r})}{r^3}$$

$d\vec{B}$ is perpendicular to the plane containing $d\vec{l}$ and \vec{r} and is directed inwards.

5. Applications of Biot-Savart's law:-

- (a) Magnetic field (B) at the Centre of a Circular Coil Carrying Current.

$$B = \frac{\mu_0 n I}{2r}$$

where n is the number of turns of the coil. I is the current in the coil and r is the radius of the coil.

- (b) Magnetic field due to a straight conductor carrying current.

$$B = \frac{\mu_0 I}{4\pi a} (\sin \phi_2 + \sin \phi_1)$$

where a is the perpendicular distance of the conductor from the point where field is to be measured.

ϕ_1 and ϕ_2 are the angles made by the two ends of the conductor with the point.

- (c) For an infinitely long conductor, $\phi_1 = \phi_2 = \pi/2$

$$\therefore B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}$$

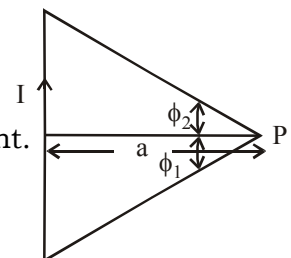
- (d) Magnetic field at a point on the axis of a Circular Coil Carrying Current.

when point P lies far away from the centre of the coil.

$$B = \frac{\mu_0}{4} \cdot \frac{2M}{x^3}$$

where $M = nIA =$ magnetic dipole moment of the coil.

x is the distance of the point where the field is to be measured, n is the number of turns, I is the current and A is the area of the coil.



6. Ampere's circuital law:-

The line integral of magnetic field \vec{B} around any closed path in vacuum is μ_0 times the total current through the closed path. i.e. $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

7. Application of Ampere's circuital law:-

(a) Magnetic field due to a current carrying solenoid, $B = \mu_0 n I$
 n is the number of turns per unit length of the solenoid.

At the edge of a short solenoid, $B = \frac{\mu_0 n I}{2}$

(b) Magnetic field due to a toroid or endless solenoid

$$B = \mu_0 n I$$

8. Motion of a charged particle in uniform electric field:-

The path of a charged particle in an electric field is a parabola.

Equation of the parabola is $x^2 = \frac{2mv^2}{qE} y$

where x is the width of the electric field.

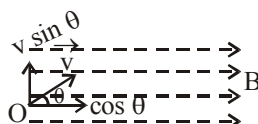
y is the displacement of the particle from its straight path.

v is the speed of the charged particle.

q is the charge of the particle

E is the electric field intensity.

m is the mass of the particle.

9. Motion of the charged particle in a magnetic field. The path of a charged particle moving in a uniform magnetic field (\vec{B}) with a velocity \vec{v} making an angle θ with \vec{B} is a helix.

The component of velocity $v \cos \theta$ will not provide a force to the charged particle, so under this velocity the particle will move forward with a constant velocity along the direction of \vec{B} . The other component $v \sin \theta$ will produce the force $F = q B v \sin \theta$, which will supply the necessary centripetal force to the charged particle in moving along a circular path of radius r .

$$\therefore \text{Centripetal force} = \frac{m(v \sin \theta)^2}{r} = B q v \sin \theta$$

$$\therefore v \sin \theta = \frac{B q r}{m}$$

$$\text{Angular velocity of rotation} = \omega = \frac{v \sin \theta}{r} = \frac{B q}{m}$$

$$\text{Frequency of rotation} = \nu = \frac{\omega}{2\pi} = \frac{B q}{2\pi m}$$

$$\text{Time period of revolution} = T = \frac{1}{\nu} = \frac{2\pi m}{B q}$$

- 10. Cyclotron:** It is a device used to accelerate and hence energies the positively charged particle. This is done by placing the particle in an oscillating electric field and a perpendicular magnetic field. The particle moves in a circular path.

\therefore Centripetal force = magnetic Lorentz force

$$\Rightarrow \frac{mv^2}{r} = Bqv \Rightarrow \frac{mv}{Bq} = r \leftarrow \text{radius of the circular path}$$

Time to travel a semicircular path = $\frac{\pi r}{v} = \frac{\pi m}{Bq} = \text{constant}$.

If v_0 be the maximum velocity of the particle and r_0 be the maximum radius of its path then

$$\frac{mv_0^2}{r_0} = Bqv_0 \Rightarrow v_0 = \frac{Bqr_0}{m}$$

$$\text{Max. K.E. of the particle} = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{Bqr_0}{m}\right)^2 \Rightarrow (\text{K.E.})_{\text{max.}} = \frac{B^2q^2r_0^2}{2m}$$

$$\text{Time period of the oscillating electric field} \Rightarrow T = \frac{2\pi m}{Bq}.$$

Time period is independent of the speed and radius.

$$\text{Cyclotron frequency} = \nu = \frac{1}{T} = \frac{Bq}{2\pi m}$$

$$\text{Cyclotron angular frequency} = \omega_0 = 2\pi\nu = \frac{Bq}{m}$$

- 11.** Force on a current carrying conductor placed in a magnetic field:

$$\vec{F} = I|\vec{\ell} \times \vec{B}| \text{ or } F = I\ell B \sin\theta$$

where I is the current through the conductor

B is the magnetic field intensity.

ℓ is the length of the conductor.

θ is the angle between the direction of current and magnetic field.

(i) When $\theta = 0^\circ$ or 180° , $\sin\theta = 0 \Rightarrow F = 0$

\therefore When a conductor is placed along the magnetic field, no force will act on the conductor.

(ii) When $\theta = 90^\circ$, $\sin\theta = 1$, F is maximum.

$$F_{\text{max}} = I\ell B$$

when the conductor is placed perpendicular to the magnetic field, it will experience maximum force.

- 12.** Force between two parallel conductors carrying current:-

(a) When the current is in same direction the two conductors will attract each other with a force

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r} \text{ per unit length of the conductor}$$

(b) When the current is in opposite direction the two conductors will repel each other with the same force.

(c) S.I. unit of current is 1 ampere. (A).

1A is the current which on flowing through each of the two parallel uniform linear conductor placed in free space at a distance of 1 m from each other produces a force of 2×10^{-7} N/m along their lengths.

13. Torque on a current carrying coil placed in a magnetic field:-

$\vec{\tau} = \vec{M} \times \vec{B} \Rightarrow \tau = MB \sin \alpha = nIBA \sin \alpha$ where M is the magnetic dipole moment of the coil.

$$M = nIA$$

where n is the number of turns of the coil.

I is the current through the coil.

B is the magnetic field intensity.

A is the area of the coil.

α is the angle between the magnetic field (\vec{B}) and the perpendicular to the plane of the coil.

Special Cases:

(i) If the coil is placed parallel to magnetic field $\theta = 0^\circ$, $\cos \theta = 1$ then torque is maximum.

$$\tau_{\max.} = nIBA$$

(ii) If the coil is placed perpendicular to magnetic field, $\theta = 90^\circ$, $\cos \theta = 0$

$$\therefore \tau = 0$$

14. Moving coil galvanometer:- This is based on the principle that when a current carrying coil is placed in a magnetic field it experiences a torque. There is a restoring torque due to the phosphor bronze strip which brings back the coil to its normal position.

In equilibrium, Deflecting torque = Restoring torque

$$nIBA = k\theta \quad [k = \text{restoring torque/unit twist of the phosphor bronze strip}]$$

$$I = \frac{k}{nBA} \theta = G\theta \quad \text{where } G = \frac{k}{nBA} = \text{Galvanometer constant}$$

$$\therefore I \propto \theta$$

Current sensitivity of the galvanometer is the deflection produced when unit current is passed through the galvanometer.

$$I_s = \frac{\theta}{I} = \frac{nBA}{k}$$

Voltage sensitivity is defined as the deflection produced when unit potential difference is applied across the galvanometer.

$$V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{nBA}{kR} \quad [R = \text{Resistance of the galvanometer}]$$

15. Condition for the maximum sensitivity of the galvanometer:-

The galvanometer is said to be sensitive if a small current produces a large deflection.

$$\therefore \theta = \frac{nBA}{k} I$$

$\therefore \theta$ will be large if (i) n is large, (ii) B is large (iii) A is large and (iv) k is small.

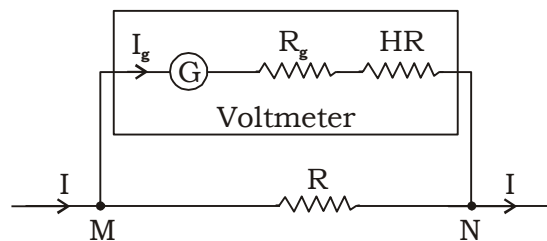
16. Conversion of galvanometer into voltmeter and ammeter

(a) A galvanometer is converted to voltmeter by putting a high resistance in series with it.

Total resistance of voltmeter = $R_g + R$ where R_g is the galvanometer resistance. R is the resistance added in series.

$$\text{Current through the galvanometer} = I_g = \frac{V}{R_g + R}$$

where V is the potential difference across the voltmeter.

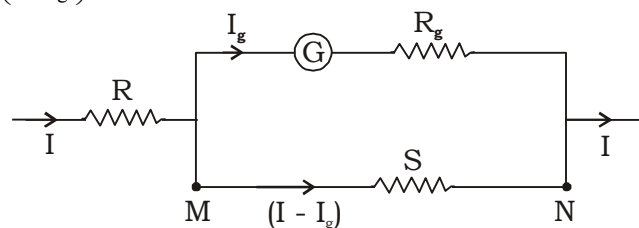


$$\therefore R = \frac{V}{I_g} - R_g$$

Range of the voltmeter: $0 - V$ volt.

(b) A galvanometer is converted into an ammeter by connecting a low resistance in parallel with it (shunt)

Shunt = $S = \left(\frac{I_g}{I - I_g} \right) R_g$ where R_g is the galvanometer resistance.



I is the total current through the ammeter.

I_g is the current through the ammeter. Effective resistance of the ammeter

$$R = \frac{R_g}{R_g + S}$$

The range of the ammeter is $0 - I$ A. An ideal ammeter has zero resistance.