WAVE OPTICS

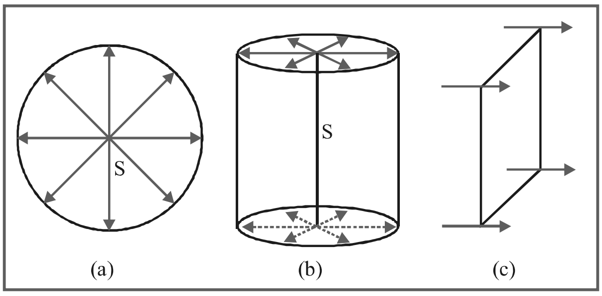
1. WAVEFRONT

A source of light sends out disturbance in all directions. In a homogeneous medium, the disturbance reaches all those particles of the medium in phase, which are located at the same distance from the source of light and hence at any instant, all such particles must be vibrating in phase with each other. *The locus of all the particles of medium, which at any instant are vibrating in the same phase, is called the* ***wavefront.***

Depending upon the shape of the source of light, wavefront can be the following types :

1.1 Spherical wavefront

A spherical wavefront is produced by a point source of light. It is because, the locus of all such points, which are equidistant from the point source, is a sphere figure (a).



1.2 Cylindrical wavefront

When the source of light is linear in shape (such as a slit), a cylindrical wavefront is produced. It is because, all the points, which are equidistant from the linear source, lie on the surface of a cylinder figure (b).

1.3 Plane wavefront

A small part of a spherical or a cylindrical wavefront originating from a distant source will appear plane and hence it is called a plane wavefront figure (c).

1.4 Ray of light

An arrow drawn normal to the wavefront and pointing in the direction of propagation of disturbance represents a ray of light. A ray of light is the path along which light travels. In figure thick arrows represent the rays of light.

Since the ray of light is normal to the wavefront, it is sometimes called as the ***wave normal.***

> ***Key points***
> The phase difference between any two points on a wavefront is zero.

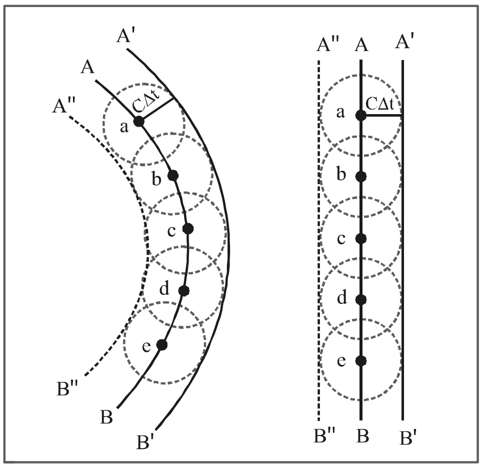
2. HUYGENS'S PRINCIPLE

Huygen's principle is a geometrical construction, which is used to determine the new position of a wavefront at a later time from its given position at any instant. In order words, the principle gives a method to know as to how light spreads out in the medium.

Huygen's principle is based on the following assumptions :

1. *Each point on the given or primary wavefront acts as a source of secondary wavelets, sending out disturbance in all directions in a similar manner as the original source of light* does.
2. *The new position of the wavefront at any instant (called secondary wavefront)* is the envelope of the secondary wavelets at that instant.

   The above two assumptions are known as ***Huygen's principle*** or ***Huygens'****construction.*



> ***Key points***
> Huygen's principle is simply a geometrical construction to find the position of wavefront at a later time.

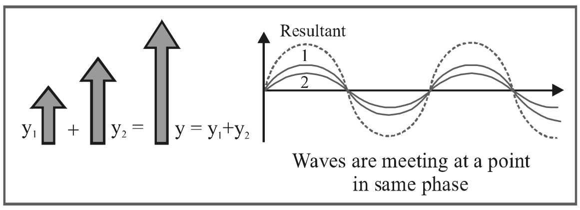
3. PRINCIPLE OF SUPER POSITION

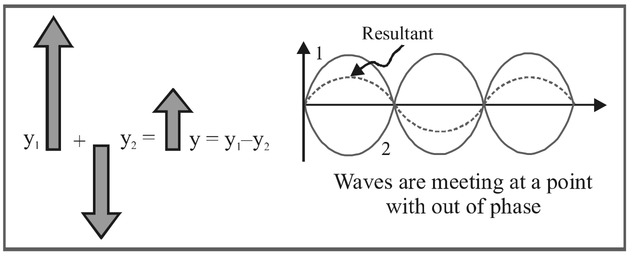
When two or more than two waves superimpose over each other at a common particle of the medium then the resultant displacement (y) of the particle is equal to the vector sum of the displacements (y_1 and y_2) produced by individual waves.

i.e. $\vec{y} = \vec{y}_1 + \vec{y}_2$

3.1 Graphical view

(i)



(ii) 

3.2 Phase/Phase difference/Path difference/Time difference

(i) Phase : The argument of sine or cosine in the expression for displacement of a wave is defined as the phase. For displacement $y = a \sin \omega t$; term ωt = phase or instantaneous phase

(ii) Phase difference (ϕ) : The difference between the phases of two waves at a point is called phase difference *i.e.* if $_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$ so phase difference = ϕ

(iii) Path difference (Δ) : The difference in path length's of two waves meeting at a point is called path difference between the waves at that point. Also $\Delta = \frac{\lambda}{2\pi} \times \phi$

(iv) Time difference (*T.D.*) : Time difference between the waves meeting at a point is $\text{T.D.} = \frac{T}{2\pi} \times \phi$

3.3 Resultant amplitude and intensity

If suppose we have two waves $y_1 = a_1 \sin \omega t$ & $y_2 = a_2 \sin(\omega t + \phi)$; where a_1, a_2 = Individual amplitudes, ϕ = Phase difference between the waves at an instant when they are meeting a point. I_1, I_2 = Intensities of individual waves

Resultant amplitude : After superimposition of the given waves resultant amplitude (or the amplitude of resultant wave) is given by $A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$

For the interfering waves $y_1 = a_1 \sin\omega t$ and $y_2 = a_2 \cos\omega t$, Phase difference between them is 90°. So resultant amplitude

$$A = \sqrt{a_1^2 + a_2^2}$$

Resultant intensity : As we know intensity ∝ (Amplitude)^2

$\Rightarrow$ $I_1 = ka_1^2$, $I_2 = ka_2^2$ and $I = kA^2$ (k is a proportionality constant). Hence from the formula of resultant amplitude, we get the following formula of resultant intensity

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

The term $2\sqrt{I_1 I_2} \cos\phi$ is called interference term. For incoherent interference this term is zero so resultant intensity $I = I_1 + I_2$.

3.4 Coherent sources

The sources of light which emits continuous light waves of the same wavelength, same frequency and in same phase or having a constant phase difference are called coherent sources.

4. INTERFERENCE OF LIGHT

When two waves of exactly same frequency (coming from two coherent sources) travels in a medium, in the same direction simultaneously then due to their superposition, at some points intensity of light is maximum while at some other points intensity is minimum. This phenomenon is called Interference of light.

4.1 Types of Interference

Constructive interference	Destructive interference
(i) When the waves meets a point with same phase, constructive interference is obtained at that point (*i.e.* maximum light)	(i) When the wave meets a point with opposite phase, destructive interference is obtained at that point (*i.e.* minimum light)
(ii) Phase difference between the waves at the point of observation $\phi = 0°$ or $2n\pi$	(ii) $\phi = 180°$ or $(2n - 1)\pi$; n = 1,2, ... or $(2n + 1)\pi$; n = 0, 1,2, ...
(iii) Path difference between the waves at the point of observation $\Delta = n\lambda$ (*i.e.* even multiple of $\lambda/2$)	(iii) $\Delta = (2n - 1)\frac{\lambda}{2}$ (*i.e.* odd multiple of $\lambda/2$)
(iv) Resultant amplitude at the point of observation will be maximum $a_1 = a_2 \Rightarrow A_{min} = 0$ If $a_1 = a_2 = a_0 \Rightarrow A_{max} = 2a_0$	(iv) Resultant amplitude at the point of observation will be minimum $A_{min} = a_1 - a_2$ If $a_1 = a_2 \Rightarrow A_{min} = 0$
(v) Resultant intensity at the point of observation will be maximum $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ $\quad I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{max} = 2I_0$	(v) Resultant intensity at the point of observation will be minimum $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ $\quad I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{min} = 0$

4.2 Resultant intensity due to two identical waves

For two coherent sources the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

For identical source $I_1 = I_2 = I_0$

$$\Rightarrow I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi = 4I_0\cos^2\frac{\phi}{2}$$

$$[1 + \cos\theta = 2\cos^2\frac{\theta}{2}]$$

Note..

⇒ In interference redistribution of energy takes place in the form of maxima and minima.

⇒ Average intensity : $I_{av} = \frac{I_{max} + I_{min}}{2} = I_1 + I_2 = a_1^2 + a_2^2$

⇒ Ratio of maximum and minimum intensities :

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{a_1/a_2 + 1}{a_1/a_2 - 1}\right)^2$$

also $\sqrt{\frac{I_1}{I_2}} = \frac{a_1}{a_2} = \left(\frac{\sqrt{\frac{I_{max}}{I_{min}}} + 1}{\sqrt{\frac{I_{max}}{I_{min}}} - 1}\right)$

⇒ If two waves having equal intensity ($I_1 = I_2 = I_0$) meets at two locations P and Q with path difference Δ_1 and Δ_2 respectively then the ratio of resultant intensity at point P and Q will be $\frac{I_P}{I_Q} = \frac{\cos^2\frac{\phi_1}{2}}{\cos^2\frac{\phi_2}{2}} = \frac{\cos^2\left(\frac{\pi\Delta_1}{\lambda}\right)}{\cos^2\left(\frac{\pi\Delta_2}{\lambda}\right)}$

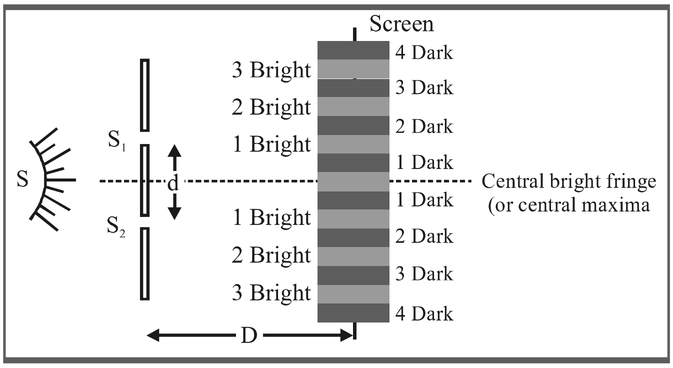
5. YOUNG'S DOUBLE SLIT EXPERIMENT (YDSE)

Monochromatic light (single wavelength) falls on two narrow slits S_1 and S_2 which are very close together acts as two coherent sources, when waves coming from two coherent sources (S_1, S_2) superimposes on each other, an interference pattern is obtained on the screen. In YDSE alternate bright and dark bands obtained on the screen. These bands are called Fringes.

d = Distance between slits

D = Distance between slits and screen

= Wavelength of monochromatic light emitted from source



(1) Central fringe is always bright, because at central position $\phi = 0°$ or $\Delta = 0$

(2) The fringe pattern obtained due to a slit is more bright than that due to a point.

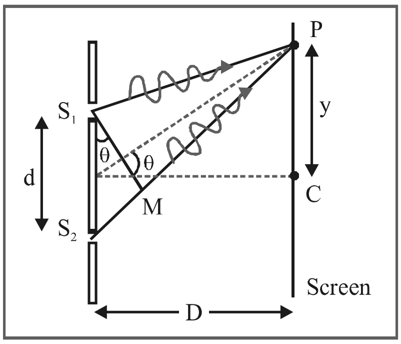
(3) If the slit widths are unequal, the minima will not be complete dark. For very large width uniform illumination occurs.

(4) If one slit is illuminated with red light and the other slit is illuminated with blue light, no interference pattern is observed on the screen.

(5) If the two coherent sources consist of object and it's reflected image, the central fringe is dark instead of bright one.

5.1 Path difference

Path difference between the interfering waves meeting at a point *P* on the screen is given by $x = \frac{yd}{D} = d\sin\theta$ where *x* is the position of point *P* from central maxima.



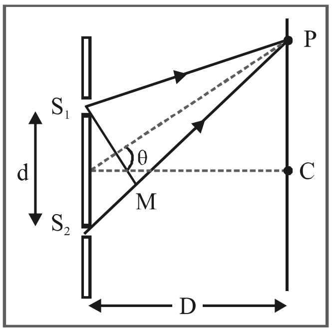
For maxima at *P* : $x = n\lambda$;

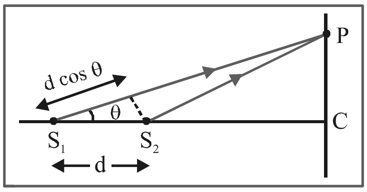
where $n = 0, \pm 1, \pm 2, \ldots\ldots$

and For minima at *P* : $x = \frac{(2n-1)\lambda}{2}$;

where $n = \pm 1, \pm 2, \ldots\ldots$

Note :- If the slits are vertical, the path difference (x) is $d \sin\theta$, so as θ increases, Δ also increases. But if slits are horizontal path difference is $d \cos\theta$, so as θ increases, x decreases.





5.2 More about fringe

(i) All fringes are of equal width. Width of each fringe is

$$\beta = \frac{\lambda D}{d} \text{ and angular fringe width } \theta = \frac{\lambda}{d}$$

(ii) If the whole YDSE set up is taken in another medium then changes so β changes

e.g. in water $\lambda_w = \frac{\lambda_a}{\mu_w} \Rightarrow \beta_w = \frac{\beta_a}{\mu_w} = \frac{3}{4}\beta_a$

(iii) Fringe width $\beta \propto \frac{1}{d}$ i.e. with increase in separation between the sources, β decreases.

(iv) Position of n^{th} bright fringe from central maxima

$$x_n = \frac{n\lambda D}{d} = n\beta \; ; n = 0, 1, 2, ...$$

(v) Position of n^{th} dark fringe from central maxima

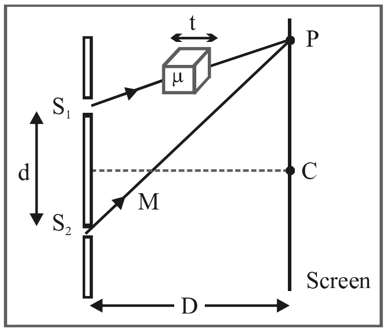
$$x_n = \frac{(2n-1)\lambda D}{2d} = \frac{(2n-1)\beta}{2} \; ; n = 1, 2, 3....$$

(vi) In YDSE, if n_1 fringes are visible in a field of view with light of wavelength λ_1, while n_2 with light of wavelength $_2$ in the same field, then $n_1\lambda_1 = n_2\lambda_2$.

5.3 Shifting of fringe pattern in YDSE

If a transparent thin film of mica or glass is put in the path of one of the waves, then the whole fringe pattern gets shifted.

If film is put in the path of upper wave, fringe pattern shifts upward and if film is placed in the path of lower wave, pattern shift downward.



Fringe shift = $\frac{D}{d}(\mu-1)\,t = \frac{\beta}{\lambda}(\mu-1)t$

$\Rightarrow$ Additional path difference = $(\mu - 1)t$

$\Rightarrow$ If shift is equivalent to n fringes then $n = \frac{(\mu-1)t}{\lambda}$

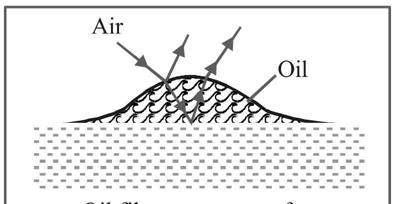
or $t = \frac{n\lambda}{(\mu-1)}$

$\Rightarrow$ Shift is independent of the order of fringe (*i.e.* shift of zero order maxima = shift of n^{th} order maxima.

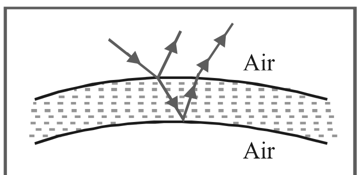
$\Rightarrow$ Shift is independent of wavelength.

6. ILLUSTRATIONS OF INTERFERENCE

Interference effects are commonly observed in thin films when their thickness is comparable to wavelength of incident light (If it is too thin as compared to wavelength of light it appears dark and if it is too thick, this will result in uniform illumination of film). Thin layer of oil on water surface and soap bubbles shows various colours in white light due to interference of waves reflected from the two surfaces of the film.



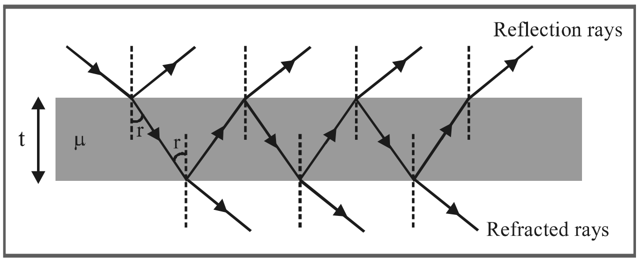
Oil film on water surface



Soap bubble in air

6.1 Thin films

In thin films interference takes place between the waves reflected from it's two surfaces and waves refracted through it.



Interference in reflected light	**Interference in refracted light**
Condition of constructive interference (maximum intensity) $\Delta = 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$ For normal incidence $r = 0$ so $2\mu t = (2n \pm 1)\frac{\lambda}{2}$	Condition of constructive interference (maximum intensity) $\Delta = 2\mu t \cos r = (2n)\frac{\lambda}{2}$ For normal incidence $2\mu t = n\lambda$
Condition of destructive interference (minimum intensity) $\Delta = 2\mu t \cos r = (2n)\frac{\lambda}{2}$ For normal incidence $2\mu t = n\lambda$	Condition of destructive interference (minimum intensity) $\Delta = 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$ For normal incidence $2\mu t = (2n \pm 1)\frac{\lambda}{2}$

Note...

The Thickness of the film for interference in visible light is of the order of 10,000 Å.

7. DOPPLER'S EFFECT IN LIGHT

The phenomenon of apparent change in frequency (or wavelength) of the light due to relative motion between the source of light and the observer is called Doppler's effect.

According to special theory of relativity

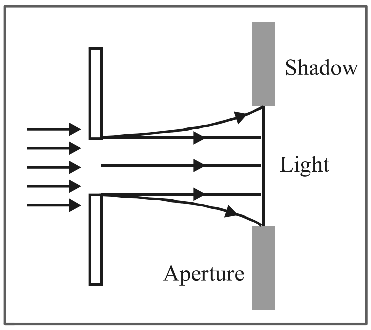
$$\frac{v'}{v} = \frac{1 \pm v/c}{\sqrt{1 - v^2/c^2}}$$

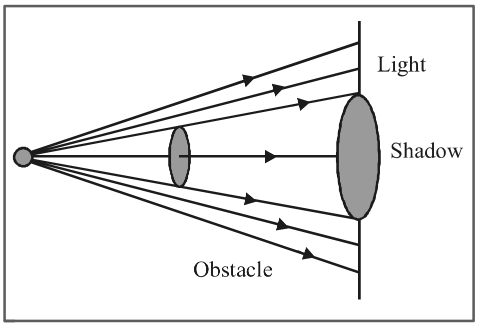
If v = actual frequency, v' =Apparent frequency, v = speed of source *w.r.t* stationary observer, c = speed of light

Source of light moves towards the stationary observer ($v \ll c$)	**Source of light moves away from the stationary observer ($v \ll c$)**
(i) Apparent frequency $v' = v\left(1 + \frac{v}{c}\right)$ and Apparent wavelength $\lambda' = \lambda\left(1 - \frac{v}{c}\right)$	(i) Apparent frequency $v' = v\left(1 - \frac{v}{c}\right)$ and Apparent wavelength $\lambda' = \lambda\left(1 + \frac{v}{c}\right)$
(ii) Doppler's shift : Apparent wavelength < actual wavelength, So spectrum of the radiation from the source of light shifts towards the red end of spectrum. This is called Red shift Doppler's shit $\Delta\lambda = \lambda.\frac{v}{c}$	(ii) Doppler's shift : Apparent wavelength > actual wavelength, So spectrum of the radiation from the source of light shifts towards the violet end of spectrum. This is called Violet shift Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$

8. DIFFRACTION OF LIGHT

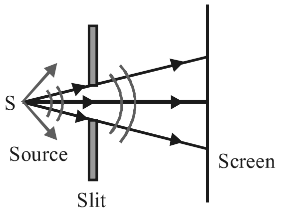
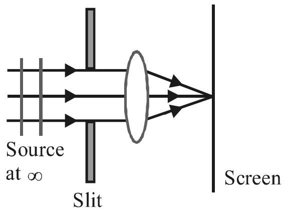
It is the phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wavelength of light.





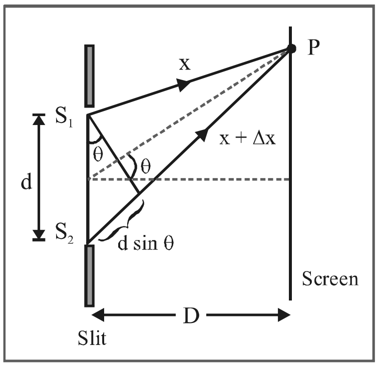
8.1 Types of diffraction

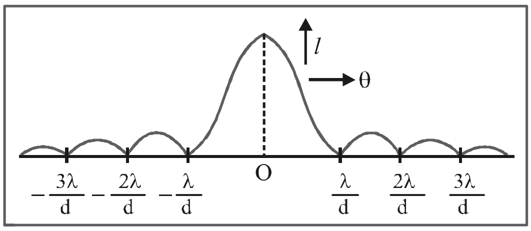
The diffraction phenomenon is divided into two types

Fresnel diffraction	Fraunhofer diffraction
(i) If either source or screen or both are at finite distance from the diffracting device (obstacle or aperture), the diffraction is called Fresnel type.	(i) In this case both source and screen are effectively at infinite distance from the diffracting device.
(ii) Common examples : Diffraction at a straight edge narrow wire or small opaque disc etc.	(ii) Common examples : Diffraction at single slit, double slit and diffraction grating.
	

8.2 Diffraction of light at a single slit

In case of diffraction at a single slit, we get a central bright band with alternate bright (maxima) and dark (minima) bands of decreasing intensity as shown





(i) Width of central maxima $\beta_0 = \frac{2\lambda D}{d}$ and angular width $= \frac{2\lambda}{d}$

(ii) Minima occurs at a point on either side of the central maxima, such that the path difference between the waves from the two ends of the aperture is given by $\Delta = n\lambda$; where n = 1, 2, 3 ..... *i.e.* $d \sin\theta = n\lambda$;

$$\Rightarrow \quad \sin\theta = \frac{n\lambda}{d}$$

(iii) The secondary maxima occurs, where the path difference between the waves from the two ends of the aperture is given by $\Delta = (2n+1)\frac{\lambda}{2}$; where n = 1, 2, 3 .....

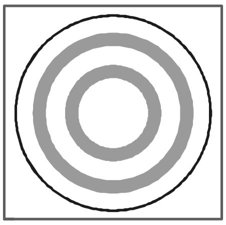
$$i.e.\ \ d\sin\theta = (2n+1)\frac{\lambda}{2} \quad \Rightarrow \quad \sin\theta = \frac{(2n+1)\lambda}{2d}$$

8.3 Comparison between interference and diffraction

Interference	Diffraction
Results due to the superposition of waves from two coherent source.	Results due to the superposition of wavelets from different parts of same wave front. (single coherent source)
All fringes are of same width $\beta = \frac{\lambda D}{d}$	All secondary fringes are of same width but the central maximum is of double the width $\beta_0 = 2\beta = 2\frac{\lambda D}{d}$
All fringes are of same intensity	Intensity decreases as the order of maximum increases.
Intensity of all minimum may be zero. Positions of *n*th maxima and minima $X_{n(\text{Bright})} = \frac{n\lambda D}{d}$, $X_{n(\text{Dark})} = (2n-1)\frac{\lambda D}{d}$	Intensity of minima is not zero. Positions of *n*th secondary maxima and $X_{n(\text{Bright})} = (2n+1)\frac{\lambda D}{d}$, $X_{n(\text{Dark})} = \frac{n\lambda D}{d}$
Path difference for *n*th maxima $\Delta = n\lambda$	for *n*th secondary maxima $\Delta = (2n+1)\frac{\lambda}{2}$
Path difference for *n*th minima $\Delta = (2n-1)\lambda$	Path difference for *n*th minima $\Delta = n\lambda$

8.4 Diffraction and optical instruments

The objective lens of optical instrument like telescope or microscope etc. acts like a circular aperture. Due to diffraction of light at a circular aperture, a converging lens cannot form a point image of an object rather it produces a brighter disc known as Airy disc surrounded by alternate dark and bright concentric rings.



The angular half width of Airy disc = $\theta = \frac{1.22\lambda}{D}$ (where D = aperture of lens)

The lateral width of the image = $f\theta$ (where f = focal length of the lens)

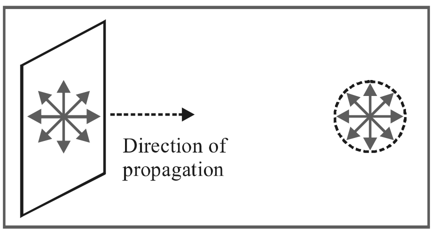
> *Note...* Diffraction of light limits the ability of optical instruments to form clear images of objects when they are close to each other.

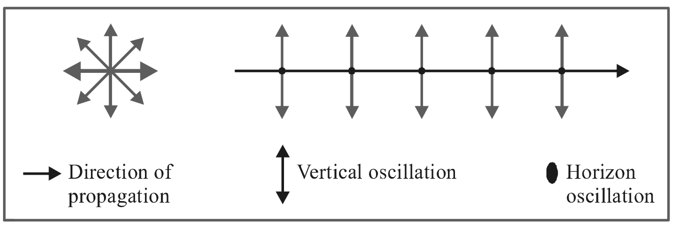
9. POLARISATION OF LIGHT

Light propagates as transverse EM waves. The magnitude of electric field is much larger as compared to magnitude of magnetic field. We generally prefer to describe light as electric field oscillations.

9.1 Unpolarised light

The light having electric field oscillations in all directions in the plane perpendicular to the direction of propagation is called Unpolarised light. The oscillation may be resolved into horizontal and vertical component.





9.2 Polarised light

The light having oscillations only in one plane is called Polarised or plane polarised light.

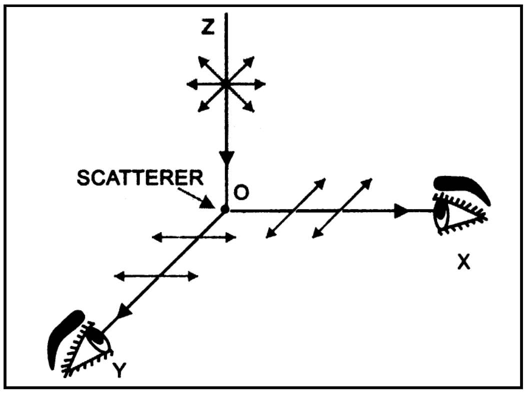
(i) The plane in which oscillation occurs in the polarised light is called plane of oscillation.

(ii) The plane perpendicular to the plane of oscillation is called plane of polarisation.

(iii) Light can be polarised by transmitting through certain crystals such as tourmaline or polaroids.

9. 3 Polarization by Scattering

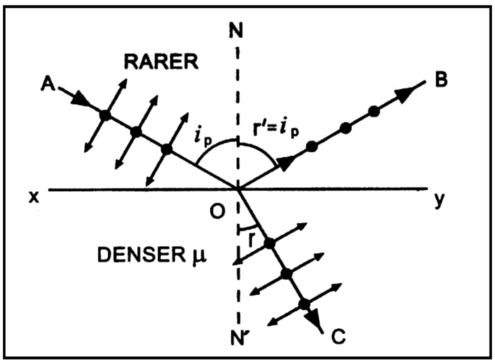
When a beam of white light is passed through a medium containing particles whose size is of the order of wavelength of light, then the beam gets scattered. When the scattred light is seen in a direction perpendicular to the direction of incidence, it is found to be plane polarized (as detected by the analyser). The phenomenon is called polarization by scattering.



9.4 Polarization of Light by Reflection

When unpolarized light is reflected from a surface, the reflected light may be completely polarised, partially polarized or unpolarized. This would depend on the angle of incidence.

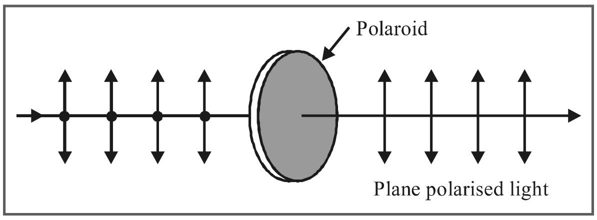
The angle of incidence at which the reflected light is completely plane polarized is called polarizing angle or Brewster's angle. (i_p)



9.5 Polaroids

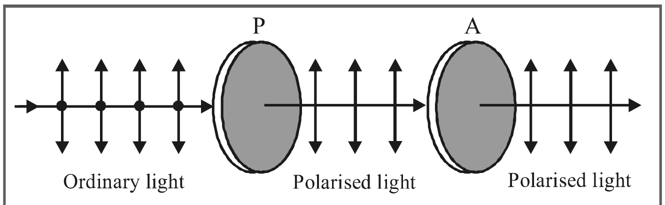
It is a device used to produce the plane polarised light. It is based on the principle of selective absorption and is more effective than the tourmaline crystal. **or**

It is a thin film of ultramicroscopic crystals of quinine idosulphate with their optic axis parallel to each other.

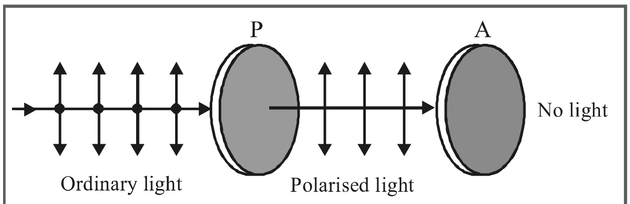


(i) Polaroids allow the light oscillations parallel to the transmission axis pass through them.

(ii) The crystal or polaroid on which unpolarised light is incident is called polariser. Crystal or polaroid on which polarised light is incident is called analyser.



Transmission axes of the polariser and analyser are parallel to each other, so whole of the polarised light passes through analyser



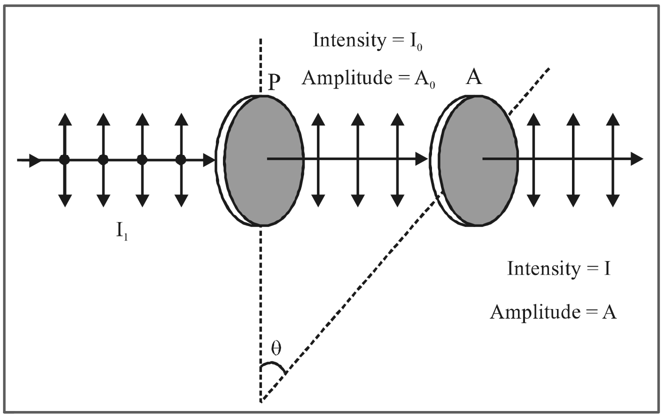
Transmission axis of the analyser is perpendicular to the polariser, hence no light passes through the analyser

Note... When unpolarised light is incident on the polariser, the intensity of the transmitted polarised light is half the intensity of unpolarised light.

(iii) Main uses of polaroids are in wind shields of automobiles, sun glassess etc. They reduce head light glare of cares and improve colour contrast in old paintings. They are also used in three dimensional motion pictures and in optical stress analysis.

9.6 Malus law

This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.



$I = I_0 \cos^2\theta$ and and $A^2 = A_0^2 \cos^2\theta \Rightarrow A = A_0 \cos\theta$

If $\theta = 0°$, $I = I_0$, $A = A_0$,

If $\theta = 45°$, $I = I_0/2$, $A = A_0/\sqrt{2}$

If $\theta = 90°$, $I = 0$, $A = 0$

(ii) If I_i = Intensity of unpolarised light.

So $I_0 = \dfrac{I_i}{2}$ *i.e.* if an unpolarised light is converted into plane polarised light (say by passing it through a polaroid or a Nicol-prism), its intensity becomes half

and $I = \dfrac{I_i}{2}\cos^2\theta$

Note...

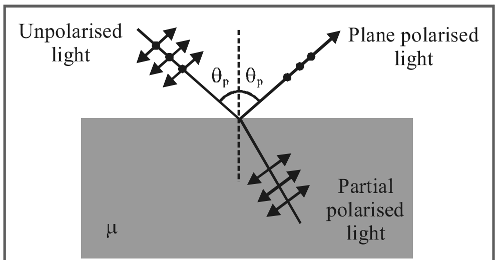
Percentage of polarisation $= \dfrac{(I_{max} - I_{min})}{(I_{max} + I_{min})} \times 100$

9.7 Brewster's law

Brewster discovered that when a beam of unpolarised light is reflected from a transparent medium (refractive index = μ), the reflected light is completely plane polarised at a certain angle of incidence (called the angle of polarisation θ_p).

Also $\mu = \tan\theta_p$ Brewster's law

(i) For $i < \theta_p$ or $i > \theta_p$



Polarisation by reflection

Both reflected and refracted rays becomes partially polarised

(ii) For glass $\theta_P \approx 57°$, for water $\theta_P \approx 53°$

10. VALIDITY OF RAY-OPTICS

When a parallel beam of light travels upto distances as large as few metres it broadens by diffraction of light travels.

10.1 Fresnel Distance

Fresnel distance is the minimum distance a beam of light can travel before its deviation from straight line path becomes significant/noticeable.

$$Z_F = \frac{a^2}{\lambda}$$

Since wavelength of light is very small deviation is very small and light can be assumed as travelling in a straight line.

Hence we can ignore broadening of beam by diffraction upto distances as large as a few meters, i.e., we can assume that light travels along straight lines. Hence ray optics can be taken as a limiting case of wave optics.

Hence Ray optics can be taken as a limiting case of waveoptics.

11. RESOLVING POWER

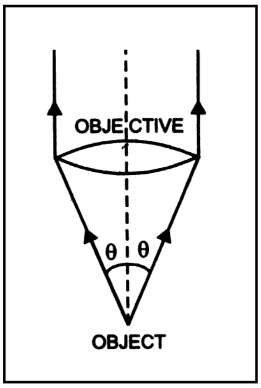
When two point objects are close to each other their images diffraction patterns are also close and overlap each other.

The minimum distance between two objects which can be seen seperately by the object instrument is called limit of resolution of the instrument.

$$\text{Resolving Power (R.P.)} = \frac{1}{\text{Limit of Revolution}}$$

11.1 Resolving power of Microscope

$$\text{R. P. of microscope} = \frac{2\mu \sin\theta}{\lambda}$$



11.2 Resolving power of Telescope

$$\text{R.P. of telescope} = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

where D is aperture of telescope.