Chapter 2 Polynominal

Question-1

Find all the common zeroes of the polynomials: $x^3 + 5x^2 - 9x - 45$ and $x^3 + 8x^2 + 15x$.

Solution:

Let $P(x) = x^3 + 5x^2 - 9x - 45$ and $Q(x) = x^3 + 8x^2 + 15x$.

Let $P(x) = x^3 + 5x^2 - 9x - 45 = x^2(x + 5) - 9(x + 5)$

$= (x + 5)(x^2 - 9)$

$= (x + 5)(x + 3)(x - 3)$

$Q(x) = x^3 + 8x^2 + 15x = x(x^2 + 8x + 15)$

$= x(x + 5)(x + 3)$

$\therefore$ The common zeroes of the polynomials are $(x + 5)$ and $(x + 3)$.

Question-2

Determine whether the given value of x is a zero of the polynomial $3x^2 - 2x - 1$ or not; if x = 1

Solution:

Let $p(x) = 3x^2 - 2x - 1$

$\Rightarrow p(1) = 3(1)^2 - 2(1) - 1 = 3 - 2 - 1 = 0$.

Therefore the given value of x is a zero of the given polynomial.

Question-3

Determine whether the given value of x is a zero of the given polynomial or not: $2x^2 - 6x + 3$; $x = \frac{1}{2}$

Solution:

Let $p(x) = 2x^2 - 6x + 3$

$\Rightarrow p(\frac{1}{2}) = 2(\frac{1}{2})^2 - 6(\frac{1}{2}) + 3 = \frac{1}{2} - 3 + 3 = \frac{1}{2}$.

Therefore the given value of x is not a zero of the given polynomial.

Question-4

Determine whether the given value of x is a zero of the given polynomial or not: $(2x + 3)(3x - 2)$; $x = \frac{2}{3}$

Solution:

Let $p(x) = (2x + 3)(3x - 2)$

$\Rightarrow p(\frac{2}{3}) = p((2 \times \frac{2}{3} + 3)(3 \times \frac{2}{3} - 2) = (\frac{4}{3} + 3)(2 - 2) = (\frac{4}{3} + 3)\,0 = 0.$

Therefore the given value of x is a zero of the given polynomial

Question-5

Determine whether the given value of x is a zero of the given polynomial or not: $x^2 + x + 1$; $x = -1$

Solution:

Let $p(x) = x^2 + x + 1$

$\Rightarrow (-1) = (-1)^2 + (-1) + 1 = 1 - 1 + 1 = 1.$

Therefore the given value of x is not a zero of the given polynomial

Question-6

Determine whether the given values of x are zeroes of the given polynomial or not: $x^2 + 6x + 5$; $x = -1$, $x = -5$

Solution:

Let $p(x) = x^2 + 6x + 5$;

Put $x = -1$

$\Rightarrow p(-1) = (-1)^2 + 6(-1) + 5 = 1 - 6 + 5 = 0$

Put $x = -5$

$p(-5) = (-5)^2 + 6(-5) + 5 = 25 - 30 + 5 = 0$

Therefore the given value of x is a zeroes of the given polynomial.

Question-7

In the following, determine whether the given values of x are zeroes of the given polynomial or not: $6x^2 - x - 2$; $x = -\frac{1}{2}$, $x = \frac{2}{3}$

Solution:

Let $p(x) = 6x^2 - x - 2$

Put $x = -\frac{1}{2}$

$p(-\frac{1}{2}) = 6(-\frac{1}{2})^2 - (-\frac{1}{2}) - 2 = \frac{3}{2} + \frac{1}{2} - 2 = \frac{3+1-4}{2} = 0$

Put $x = \frac{2}{3}$

$p(\frac{2}{3}) = 6x^2 - x - 2 = 6(\frac{2}{3})^2 - (\frac{2}{3}) - 2 = \frac{4}{3} - \frac{2}{3} - 2 = \frac{4-2-6}{3} = \frac{-4}{3}$

Therefore the given value of $x = -\frac{1}{2}$ is a zero and $x = \frac{2}{3}$ is not the zero of the given polynomial.

Question-8

In the following, determine whether the given values of x are zeroes of the given polynomial or not:

$x^2 + \sqrt{2}x - 4$; $x = \sqrt{2}$, $x = -2\sqrt{2}$

Solution:

Let $p(x) = x^2 + \sqrt{2}x - 4$

Put $x = \sqrt{2}$

$\Rightarrow p(\sqrt{2}) = x^2 + \sqrt{2}x - 4 = (\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4 = 2 + 2 - 4 = 0$

Put $x = -2\sqrt{2}$

$p(-2\sqrt{2}) = x^2 + \sqrt{2}x - 4 = (-2\sqrt{2})^2 + \sqrt{2}(-2\sqrt{2}) - 4 = 8 - 4 - 4 = 0$

Therefore the given value of x is a zero of the given polynomial.

Question-9

Determine whether the given values of x are zeroes of the given polynomial or not:

$9x^2-3x-2$; $x = -\frac{1}{3}$, $x = \frac{2}{3}$

Solution:

Let $p(x) = 9x^2-3x-2$

Put $x = -\frac{1}{3}$

$\Rightarrow p(-\frac{1}{3}) = 9(-\frac{1}{3})^2 - 3(-\frac{1}{3}) - 2 = 1 + 1 - 2 = 0$

Put $x = \frac{2}{3}$

$\Rightarrow p(\frac{2}{3}) = 9x^2 - 3x - 2 = 9(\frac{2}{3})^2 - 3(\frac{2}{3}) - 2 = 4 - 2 - 2 = 0$

Therefore the given value of x is a zero of the given polynomial.

Question-10

Determine whether the given values of x are zeroes of the given polynomial or not:(x+4)(x-5); x = -4, x=5

Solution:

Let p(x) = (x+4)(x-5)

Put x = -4

$\Rightarrow p(-4) = (x + 4)(x - 5) = (-4 + 4)(-4 - 5) = 0(-9) = 0$

Put x = 5

$\Rightarrow p(5) = (x + 4)(x - 5) = (5 + 4)(5 - 5) = 9(0) = 0$

Therefore the given value of x is a zero of the given polynomial.

Question-11

Determine whether the given values of x are zeroes of the given polynomial or not: (3x+8)(2x+5); $x = 2\frac{2}{3}$, $x = 2\frac{1}{2}$

Solution:

Let p(x) = (3x+8)(2x+5)

Put $x = 2\frac{2}{3} = \frac{8}{3}$

$\Rightarrow p(\frac{8}{3}) = (3 \times \frac{8}{3} + 8)(2 \times \frac{8}{3} + 5) = (8 + 8)(\frac{16}{3} + 5) = 16 \times \frac{31}{3} = \frac{496}{3}$

Put $x = 2\frac{1}{2} = \frac{5}{2}$

$\Rightarrow p(\frac{5}{2}) = (3 \times \frac{5}{2} + 8)(2 \times \frac{5}{2} + 5) = (\frac{15}{2} + 8)(5+5) = \frac{31}{2} \times 10 = 155$

Therefore the given value of x is not the zero of the given polynomial.

Question-12

Find the sum and product of the zeroes of the polynomial $x^2 - 6x + 5$;

Solution:

$x^2 - 6x + 5$

Sum of the zeroes of the polynomial = $\frac{-b}{a} = \frac{-(-6)}{1} = 6$

Product of the zeroes of the polynomial = $\frac{c}{a} = \frac{5}{1} = 5$

Question-13

Find the sum and product of the zeroes of the polynomial $px^2 + qx + pq$.

Solution:

$px^2 + qx + pq$

Sum of the zeroes of the polynomial = $\frac{-b}{a} = \frac{-q}{p}$

Product of the zeroes of the polynomial = $\frac{c}{a} = \frac{pq}{p} = q$.

Question-14

Find the sum and product of the zeroes of the polynomial $x^2 - 25$.

Solution:

$x^2 - 25$

$\therefore$ Sum of the zeroes of the polynomial $= \frac{-b}{a} = \frac{-0}{1} = 0$ $\therefore$ Product of the zeroes of the polynomial $= \frac{c}{a} = \frac{-25}{1} = -25$.

Question-15

Find the sum and product of the zeroes of the polynomial $4x^2 - 7x$.

Solution:

$4x^2 - 7x$

$\therefore$ Sum of the zeroes of the polynomial $= \frac{-b}{a} = \frac{-(-7)}{4} = \frac{7}{4}$ $\therefore$ Product of the zeroes of the polynomial $= \frac{c}{a} = \frac{0}{4} = 0$

Question-16

Form the polynomial whose zeroes are 5, 6.

Solution:

The roots are 5 and 6. $\therefore$ Sum of the zeroes $= 5 + 6 = 11$

$\therefore$ Product of the zeroes $= 5 \times 6 = 30$

The required polynomial is x^2 - (sum of the roots) x + Product of the zeroes

$\Rightarrow x^2 - 11x + 30$ is the required polynomial.

Question-17

Form the polynomial whose zeroes are 2, -2.

Solution:

The zeroes are 2 and -2.

$\therefore$ Sum of the zeroes $= 2 + (-2) = 0$

$\therefore$ Product of the zeroes $= 2 \times (-2) = -4$

The required polynomial is x^2 - (sum of the zeroes)x + Product of the zeroes

$\Rightarrow x^2 - (0)\, x + (-4)$

$\backslash\ x^2 - 4$

Question-18

Form the polynomial whose zeroes are $3 + \sqrt{3}$, $3 - \sqrt{3}$.

Solution:

The zeroes are $3 + \sqrt{3}$ and $3 - \sqrt{3}$. $\therefore$ Sum of the zeroes = $(3 + \sqrt{3}) + (3 - \sqrt{3}) = 6$

$\therefore$ Product of the zeroes = $(3 + \sqrt{3})(3 - \sqrt{3}) = 9 - (\sqrt{3})^2$

$= 9 - 3 = 6$

\ The required polynomial is

x^2 - (sum of the zeroes) x + Product of the zeroes $\Rightarrow x^2 - (6)\,x + (6)$

$\therefore x^2 - 6x + 6$

Question-19

Form the polynomial whose zeroes are $\frac{4+\sqrt{2}}{2}$, $\frac{4-\sqrt{2}}{2}$.

Solution:

The zeroes are $\frac{4+\sqrt{2}}{2}$ and $\frac{4-\sqrt{2}}{2}$. $\therefore$ Sum of the zeroes = $\frac{4+\sqrt{2}}{2} + \frac{4-\sqrt{2}}{2}$

$= \frac{4+\sqrt{2}+4-\sqrt{2}}{2}$

$= \frac{8}{2} = 4$

$\therefore$ Product of the zeroes = $\frac{4+\sqrt{2}}{2} \times \frac{4-\sqrt{2}}{2}$

$= \frac{(4+\sqrt{2})(4-\sqrt{2})}{2\times 2}$

$= \frac{16-(\sqrt{2})^2}{4}$

$= \frac{16-2}{4}$

$= \frac{14}{4}$

$= \frac{7}{2}$

$\therefore$ The required polynomial is x^2 - (sum of the zeroes) x + Product of the zeroes

$\Rightarrow x^2 - (4)\,x + (\frac{7}{2}) \text{ Þ } 2x^2 - 8x + 7$

Question-20

If α and β are the roots of the polynomial $ax^2 + bx + c$, then find the value of $\alpha^2 + \beta^2$

Solution:

The polynomial is $ax^2 + bx + c$ whose zeroes are α and β .

The sum of the zeroes = $\alpha + \beta = \frac{-b}{a}$

The product of the zeroes = $\alpha\,\beta = \frac{c}{a}$

$$\therefore \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a}$$

$$= \frac{b^2}{a^2} - \frac{2c}{a}$$

$$= \frac{b^2 - 2ac}{a^2}$$

Question-21

Form the polynomial whose zeroes are $3 + \sqrt{3}$, $3 - \sqrt{3}$.

Solution:

The zeroes are $3 + \sqrt{3}$, $3 - \sqrt{3}$. $\therefore$ Sum of the zeroes $= (3 + \sqrt{3}) + (3 - \sqrt{3}) = 6$ $\therefore$ Product of the zeroes $= (3 + \sqrt{3})(3 - \sqrt{3})$

$= 9 - (\sqrt{3})^2$

$= 9 - 3$

$= 6$

$\therefore$ The required polynomial is

x^2 - (sum of the zeroes) x + Product of the zeroes $\Rightarrow x^2 - (6)x + (6) \text{ Þ } x^2 - 6x + 6$

Question-22

Form the polynomial whose zeroes are $2 + \frac{1}{\sqrt{2}}, 2 - \frac{1}{\sqrt{2}}$.

Solution:

The zeroes are $2 + \frac{1}{\sqrt{2}}$ and $2 - \frac{1}{\sqrt{2}}$

$$\text{Sum of the zeroes} = \left(2 + \frac{1}{\sqrt{2}}\right) + \left(2 - \frac{1}{\sqrt{2}}\right)$$

$$= 2 + \frac{1}{\sqrt{2}} + 2 - \frac{1}{\sqrt{2}}$$

$$= 2 + 2 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

$$= 4$$

$$\therefore \text{Product of the zeroes} = \left(2 + \frac{1}{\sqrt{2}}\right) \times \left(2 - \frac{1}{\sqrt{2}}\right)$$

$$= 2\left(2 - \frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(2 - \frac{1}{\sqrt{2}}\right)$$

$$= 2\,(2) - 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}(2) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)$$

$$= 4 - \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} - \frac{1}{2}$$

$$= 4 - \frac{1}{2}$$

$$= \frac{8-1}{2}$$

$$= \frac{7}{2}$$

The required polynomial $x^2 - (\text{sum of the zeroes})\ x + \text{Product of the zeroes}$

$\Rightarrow x^2 - 4x + \frac{7}{2}$

$\therefore 2x^2 - 8x + 7$