

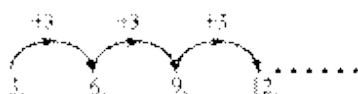
Arithmetic Progressions

- **The Concept of Arithmetic Progression**

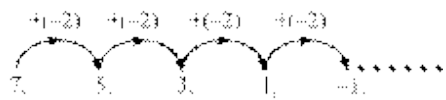
- An arithmetic progression is a list of numbers in which the difference between any two consecutive terms is equal.
- In an AP, each term, except the first term, is obtained by adding a fixed number called common difference to the preceding term.
- The common difference of an AP can be positive, negative or zero.

Example 1:

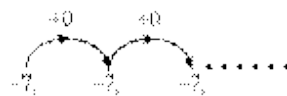
1.

1.  is an AP whose first term and common difference are 1 and 3 respectively.

2.

2.  is an AP whose first term and common difference are 7 and -2 respectively.

4.

3.  is an AP whose first term and common difference are -7 and 0 respectively.

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- The general form of an AP can be written as $a, a + d, a + 2d, a + 3d \dots$, where a is the first term and d is the common difference.
- A given list of numbers i.e., $a_1, a_2, a_3 \dots$ forms an AP if $a_{k+1} - a_k$ is the same for all values of k .

Example 2:

Which of the following lists of numbers forms an AP? If it forms an AP, then write its next three terms.

(a) -4, 0, 4, 8, ...

(b) 2, 4, 8, 16, ...

Solution:

(a) -4, 0, 4, 8, ...

$$a_2 - a_1 = 0 - (-4) = 4$$

$$a_3 - a_2 = 4 - 0 = 4$$

$$a_4 - a_3 = 8 - 4 = 4$$

$$a_{n+1} - a_n = 4; \text{ for all values of } n$$

Therefore, the given list of numbers forms an AP with 4 being its common difference.

The next three terms of the AP are $8 + 4 = 12$, $12 + 4 = 16$, $16 + 4 = 20$

Hence, AP: $-4, 0, 4, 8, 12, 16, 20 \dots$

(b) 2, 4, 8, 16, ...

$$a_2 - a_1 = 4 - 2 = 2$$

$$a_3 - a_2 = 8 - 4 = 4$$

$$a_3 - a_2 \neq a_2 - a_1$$

Therefore, the given list of numbers does not form an AP.

- **The terminology related to arithmetic progression**

- An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
- The fixed number is called the common difference (d) of the A.P. The common difference can be either positive or negative or zero.

- **The general form of an A.P.**

- $a, (a + d), (a + 2d), (a + 3d), \dots, [a + (n - 1)d], \dots$ where a is the first term and d is common difference

- **Type of AP**

- Finite AP: The APs have finite number of terms.
- Infinite AP: The APs have not finite number of terms.

- In an A.P., except the first term, all the terms can be obtained by adding the common difference to the previous term.
- In an A.P., except the last term, all the terms can be obtained by subtracting the common difference from its subsequent term.

Example:

Find the first four terms of an A.P. whose first term is 9 and the common difference is 6.

Solution:

$$a = 9, d = 6$$

$$a_2 = a + d = 9 + 6 = 15$$

$$a_3 = a + 2d = 9 + 2 \times 6 = 9 + 12 = 21$$

$$a_4 = a + 3d = 9 + 3 \times 6 = 9 + 18 = 27$$

The first four terms are 9, 15, 21, 27.

- **n^{th} term of an AP**

The n^{th} term (a_n) of an AP with first term a and common difference d is given by $a_n = a + (n - 1)d$.

Here, a_n is called the general term of the AP.

- **n^{th} term from the end of an AP**

The n^{th} term from the end of an AP with last term l and common difference d is given by $l - (n - 1)d$.

Example:

Find the 12th term of the AP 5, 9, 13 ...

Solution:

Here, $a = 5$, $d = 9 - 5 = 4$, $n = 12$

$$\begin{aligned} a_{12} &= a + (n - 1) d \\ &= 5 + (12 - 1) 4 \\ &= 5 + 11 \times 4 \\ &= 5 + 44 \\ &= 49 \end{aligned}$$

- **Sum of n terms of an AP**

- The sum of the first n terms of an AP is given by $S_n = \frac{n}{2}(2a + n - 1)d$, where a is the first term and d is the common difference.

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- If there are only n terms in an AP, then $S_n = \frac{n}{2}(a + l)$, where $l = a_n$ is the last term.

Example :

Find the value of $2 + 10 + 18 + \dots + 802$.

Solution:

2, 10, 18... 802 is an AP where $a = 2$, $d = 8$, and $l = 802$.

Let there be n terms in the series. Then,

$$a_n = 802$$

$$\Rightarrow a + (n - 1) d = 802$$

$$\Rightarrow 2 + (n - 1) 8 = 802$$

$$\Rightarrow 8(n - 1) = 800$$

$$\Rightarrow n - 1 = 100$$

$$\Rightarrow n = 101$$

$$\text{Thus, required sum} = \frac{n}{2}(a + l) = \frac{101}{2}(2 + 802) = 40602$$

- **Properties of an Arithmetic progression**

- If a constant is added or subtracted or multiplied to each term of an A.P. then the resulting sequence is also an A.P.
- If each term of an A.P. is divided by a non-zero constant then the resulting sequence is also an A.P.

- **Arithmetic mean**

- For any two numbers a and b , we can insert a number A between them such that a, A, b is an A.P. Such a number i.e., A is called the arithmetic mean (A.M) of numbers a and b and it is given by $A = \frac{a+b}{2}$.

- For any two given numbers a and b , we can insert as many numbers between them as we want such that the resulting sequence becomes an A.P.

Let $A_1, A_2 \dots A_n$ be n numbers between a and b such that $a, A_1, A_2 \dots A_n, b$ is an A.P.

Here, common difference (d) is given by $\frac{b-a}{n+1}$.

Example:

Insert three numbers between -2 and 18 such that the resulting sequence is an A.P.

Solution:

Let A_1, A_2 , and A_3 be three numbers between -2 and 18 such that $-$

$2, A_1, A_2, A_3, 18$ are in an A.P.

Here, $a = -2, b = 18, n = 5$

$$\therefore 18 = -2 + (5 - 1) d$$

$$\Rightarrow 20 = 4 d$$

$$\Rightarrow d = 5$$

$$\text{Thus, } A_1 = a + d = -2 + 5 = 3$$

$$A_2 = a + 2d = -2 + 10 = 8$$

$$A_3 = a + 3d = -2 + 15 = 13$$

Hence, the required three numbers between -2 and 18 are $3, 8$, and 13 .