Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

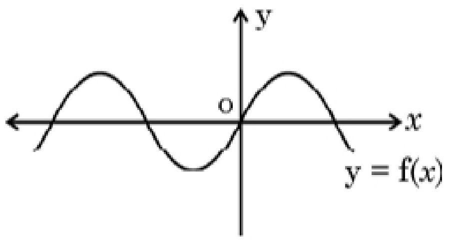
Section A

1. If a = ($2^2 \times 3^3 \times 5^4$) and b = ($2^3 \times 3^2 \times 5$) then HCF (a, b) = ? **[1]**

a) 360 b) 90

c) 180 d) 540

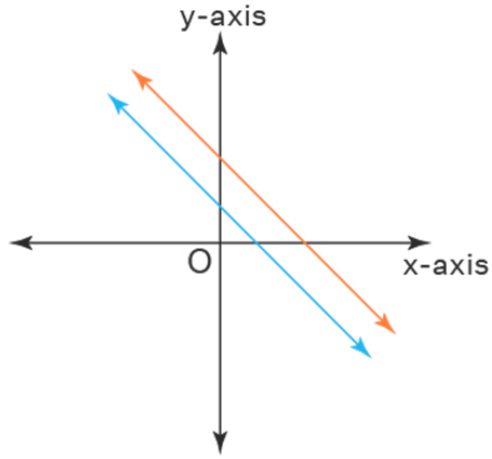
2. The graph of y = f(x) is shown in the figure for some polynomial f(x). The number of zeroes of f(x) are **[1]**



a) 2 b) 3

c) 4 d) 1

3. A system of linear equations is said to be inconsistent if it has **[1]**



a) one solution

b) at least one solution

c) two solutions

d) no solution

4. If the equation $x^2 + 5kx + 16 = 0$ has no real roots then **[1]**

a) $k > \frac{8}{5}$

b) $k < \frac{-8}{5}$

c) $\frac{-8}{5} < k < \frac{8}{5}$

d) $k > \frac{-8}{5}$

5. If the sum of n terms of an A.P. be $3n^2 + n$ and its common difference is 6 then its first term is **[1]**

a) 2

b) 1

c) 3

d) 4

6. The distance of point P(4, -5) from origin is **[1]**

a) $\sqrt{40}$ units

b) 1 unit

c) 3 units

d) $\sqrt{41}$ units

7. If (3, –6) is the mid-point of the line segment joining (0, 0) and (x, y), then the point (x, y) is: **[1]**

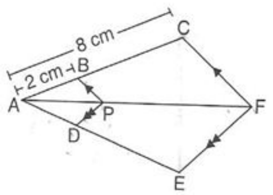
a) (6, - 6)

b) (6, -12)

c) $\left(\frac{3}{2}, -3\right)$

d) (- 3, 6)

8. In the given figure if $BP||CF$, $DP||EF$,then AD : DE is equal to **[1]**



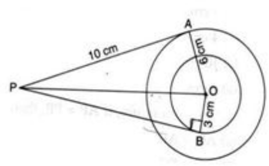
a) 1 : 3

b) 1 : 4

c) 3 : 4

d) 2 : 3

9. Two concentric circles with centre O are of radii 6 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles as shown in the figure. If AP = 10 cm, then BP is equal to **[1]**



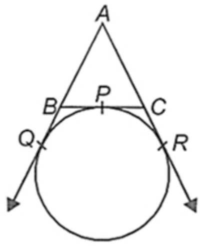
a) $\sqrt{91}$

b) $\sqrt{119}$ cm

c) $\sqrt{127}$ cm

d) $\sqrt{109}$ cm

10. In the given figure, a circle touches the side BC of $\triangle$ABC at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, then find the perimeter of $\triangle$ABC. **[1]**



a) 6 cm

b) 10 cm

c) 7 cm

d) 11 cm

11. If sinθ + cosθ = p and secθ + cosecθ = q, then q(p^2 - 1) = **[1]**

a) 2p

b) $\frac{2q}{p^2}$

c) $\frac{q}{p^2}$

d) 2

12. Which of the following is true for all values of $\theta\left(0^\circ \leq \theta \leq 90^\circ\right)$? **[1]**

a) $\text{cosec}^2\theta - \sec^2\theta = 1$

b) $\cos^2\theta - \sin^2\theta = 1$

c) $\cot^2\theta - \tan^2\theta = 1$

d) $\sec^2\theta - \tan^2\theta = 1$

13. A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60 m. Assuming that there is no slack in the string, the angle of elevation of the kite at the ground is **[1]**

a) 30°

b) 45°

c) 90°

d) 60°

14. The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 8 am and 8.25 am. **[1]**

a) 120 cm^2

b) 125.5 cm^2

c) 130.95 cm^2

d) 100 cm^2

15. If the perimeter of a sector of a circle of radius 6.5 cm is 29 cm, then its area is **[1]**

a) 56 cm^2

b) 58 cm^2

c) 52 cm^2

d) 25 cm^2

16. A card is drawn at random from a pack of 52 cards. The probability that the drawn card is not a king is **[1]**

a) $\frac{9}{13}$

b) $\frac{12}{13}$

c) $\frac{1}{13}$

d) $\frac{4}{13}$

17. What is the probability that a leap year has 52 Mondays? **[1]**

a) $\frac{5}{7}$

b) $\frac{6}{7}$

c) $\frac{2}{7}$

d) $\frac{4}{7}$

18. The most frequent value in the data is known as **[1]**

a) mean

b) mode

c) all the three | d) median

19. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 300 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + lh)

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The sum of the series with the nth term. t_n = (9 - 5n) is (465), when no. of terms n = 15. **[1]**

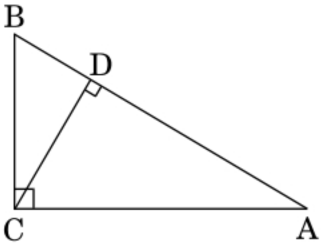
Reason (R): Given series is in A.P. and sum of n terms of an A.P. is $S_n = \frac{n}{2}[2a+(n-1)d]$

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Prove that $\frac{2}{\sqrt{7}}$ is irrational. **[2]**

22. In Figure, $\angle ACB = 90^o$ and CD $\perp$ AB. Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$. **[2]**



23. From a point P, the length of the tangent to a circle is 15 cm and distance of P from the centre of the circle is 17 cm. Then what is the radius of the circle? **[2]**

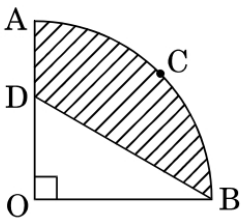
24. Prove the trigonometric identity: **[2]**

$\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$

OR

If 3tanθ = 4, evaluate $\frac{3\sin\theta+2\cos\theta}{3\sin\theta-2\cos\theta}$.

25. In Figure, OACB is a quadrant of a circle with centre O and radius 7 cm. If OD = 3 cm, then find the area of the shaded region. **[2]**



OR

The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 2 days. [Take π = 3.14.]

Section C

26. In order to promote reading habits among students, a school organized a Library Week. As part of the celebration, three genres of books: Biography, Mystery, and Self-help books were bought. For optimum arrangement, the organizers have stacked the books in such a way that all the books are stored topic-wise and the **[3]**

height of each stack is the same. The number of Biography books is 96, the number of Mystery books is 240 and the number of Self-help books is 336. Assuming that the books are of the same thickness, determine the number of stacks of Biography, Mystery, and Self-help books.

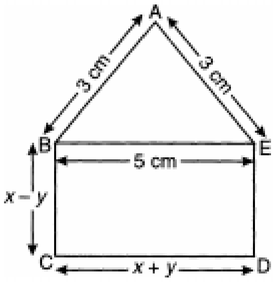
27. If α, β are the zeroes of the $x^2 + 7x + 7$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$. **[3]**

28. Solve the following system of equation by elimination method: **[3]**

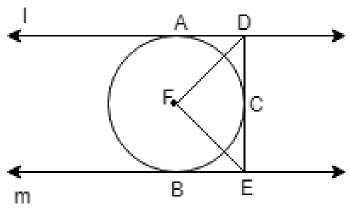
$\frac{x}{2} - \frac{y}{5} = 4$ and $\frac{x}{7} + \frac{y}{15} = 3$

OR

In the figure below ABCDE is a pentagon with BE $\parallel$ CD and BC $\parallel$ DE. BC is perpendicular to CD. If the perimeter of ABCDE is 21 cm, find the Values of x and y.



29. In Fig. l and m are two parallel tangents at A and B. The tangent at C makes an intercept DE between l and m. Prove that $\angle DFE = 90^{o}$. **[3]**



OR

Prove that the tangents drawn at the ends of a chord of a circle make equal angles with chord.

30. Prove the following identity: $\frac{\sin\theta}{1-\cos\theta} + \frac{\tan\theta}{1+\cos\theta} = sec\theta \cdot \cos ec\theta + cot\theta$ **[3]**

31. Find the median of the following data. **[3]**

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	Total
Frequency	8	16	36	34	6	100

Section D

32. Find the value of m for which the quadratic equation $(m+1)y^2 - 6(m+1)y + 3(m+9) = 0$, $m \neq -1$ has equal roots. Hence find the roots of the equation. **[5]**

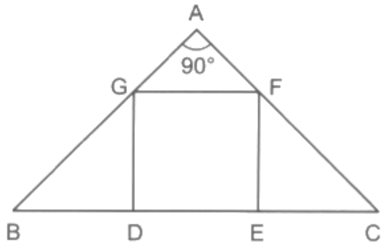
OR

The sum of ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present ages.

33. In the given figure, DEFG is a square and $\angle BAC = 90^{\circ}$. **[5]**

Prove that

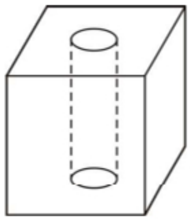
i. $\triangle AGF \sim \triangle DBG$
ii. $\triangle AGF \sim \triangle EFC$
iii. $\triangle DBG \sim \triangle EFC$
iv. $DE^2 = BD \times EC$



34. A well, whose diameter is 7m, has been dug 22.5 m deep and the earth dugout is used to form an embankment around it. If the height of the embankment is 1.5 m, find the width of the embankment. **[5]**

OR

In Figure, from a solid cube of side 7 cm, a cylinder of radius 2.1 cm and height 7 cm is scooped out. Find the total surface area of the remaining solid.



35. The median of the following data is 16. Find the missing frequencies a and b if the total of frequencies is 70. **[5]**

Class	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40
Frequency	12	a	12	15	b	6	6	4

Section E

36. **Read the text carefully and answer the questions:** **[4]**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

(a) How much distance did she cover in pacing 6 flags on either side of center point?

(b) Represent above information in Arithmetic progression

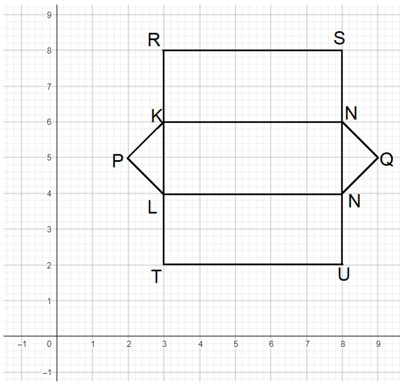
OR

How much distance did she cover in completing this job and returning to collect her books?

(c) What is the maximum distance she travelled carrying a flag?

37. **Read the text carefully and answer the questions:** **[4]**

The camping alpine tent is usually made using high-quality canvas and it is waterproof. These alpine tents are mostly used in hilly areas, as the snow will not settle on the tent and make it damp. It is easy to layout and one need not use a manual to set it up. One alpine tent is shown in the figure given below, which has two triangular faces and three rectangular faces. Also, the image of canvas on graph paper is shown in the adjacent figure.



(a) What is the distance of point Q from y-axis?

(b) What are the coordinates of U?

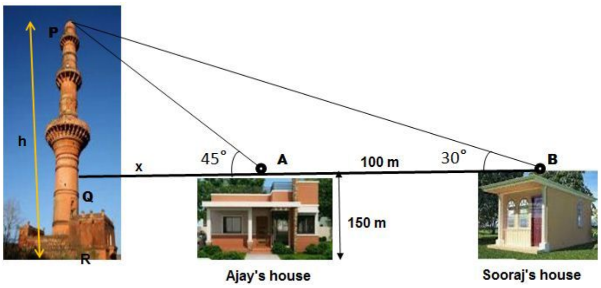
OR

What is the distance between the points P and Q?

(c) What is the Perimeter of image of a rectangular face?

38. **Read the text carefully and answer the questions:** **[4]**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45^{o} and 30^{o} respectively as shown in the figure.



(a) Find the height of the tower.

(b) What is the distance between the tower and the house of Sooraj?

OR

Find the distance between top of tower and top of Ajay's house?

(c) Find the distance between top of the tower and top of Sooraj's house?

Solution

Section A

1.

(c) 180

Explanation: It is given that: a = $(2^2 \times 3^3 \times 5^4)$ and b = $(2^3 \times 3^2 \times 5)$

$\therefore$ HCF (a, b) = Product of smallest power of each common prime factor in the numbers = $2^2 \times 3^2 \times 5$ = 180

2.

(c) 4

Explanation: f(x) intersects the x -axis at 4 points. hence , f(x) has 4 zeroes.

3.

(d) no solution

Explanation: A system of linear equations is said to be inconsistent if it has no solution means two lines are running parallel and not cutting each other at any point.

4.

(c) $\frac{-8}{5} < k < \frac{8}{5}$

Explanation: For no real roots, we must have b^2 - 4ac < 0.

$\therefore \left(25k^2 - 4 \times 16\right) < 0 \Rightarrow 25k^2 < 64 \Rightarrow k^2 < \frac{64}{25} \Rightarrow \frac{-8}{5} < k < \frac{8}{5}$.

5.

(d) 4

Explanation: Sum of n terms of an A.P = $3n^2$+n

and common difference (d) = 6

Let the first term be a, then

$S_n = \frac{n}{2}[2a + (n-1)d] = 3n^2 + n$

$\Rightarrow \frac{n}{2}[2a + (n-1)6] = 3n^2 + n$

$2a + 6n - 6 = \left(3n^2 + n\right) \times \frac{2}{n} = n\frac{(3n+1)\times 2}{n}$

$\Rightarrow 2a + 6n - 6 = (3n+1)2 = 6n + 2$

$\Rightarrow 2a = 6n + 2 - 6n + 6 = 8$

$a = \frac{8}{2} = 4$

6.

(d) $\sqrt{41}$ units

Explanation: OP = $\sqrt{(4-0)^2 + (0-(-5))^2}$

$= \sqrt{(16+25)}$

= $\sqrt{41}$ units

7.

(b) (6, -12)

Explanation: If (a, b) and (c, d) be the coordinates of any two points, then the coordinates of the mid-point joining those points be $\left(\frac{(a+c)}{2}, \frac{(b+d)}{2}\right)$.

The line segment is formed by points are (0, 0) and (x, y), whose mid-point is (3, -6).

Then,

$\frac{(0+x)}{2}$ = 3 and $\frac{(0+y)}{2}$ = -6

or, $\frac{x}{2}$ = 3 or, $\frac{y}{2}$ = -6

or, x = 6 or, y = -12

Therefore the required point is (6, -12).

8. **(a)** 1 : 3

Explanation: Since BP$\|$CF,

Then, $\frac{AP}{PF}=\frac{AB}{BC}$ [Using Thales Theorem]
$\Rightarrow \frac{AP}{PF}=\frac{2}{6}=\frac{1}{3}$
Again, since DP$\|$ EF,
Then, $\frac{AP}{PF}=\frac{AD}{DE}$ [Using Thales Theorem]
$\Rightarrow \frac{AD}{DE}=\frac{1}{3}$
$\Rightarrow$ AD : DE = 1 : 3

9.

(c) $\sqrt{127}$ cm
Explanation: Here $\angle OAP = 90^\circ$ [OA $\perp$ AP]
$\therefore$ In right angled triangle APO,
OP = $\sqrt{(10)^2+(6)^2}=\sqrt{100+36}=\sqrt{136}$ cm
Now, again, $\angle OBP = 90^\circ$ [OB $\perp$PB]
$\therefore$ In right angled triangle BPO,
PB = $\sqrt{\left(\sqrt{136}\right)^2-(9)^2}=\sqrt{136-9}=\sqrt{127}$ cm

10.

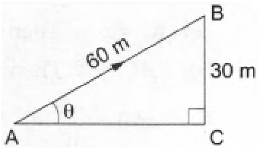
(b) 10 cm
Explanation: Perimeter of $\triangle$ABC = AB + BC + AC
= AB + (BP + PC) + AC
= (AB + BQ) + (CR + AC) [$\because$ BP = BQ, PC = CP]
= AQ + AP = 2 AQ ($\because$ AQ = AR)
= 2 $\times$ 5 = 10 cm

11. **(a)** 2p
Explanation: Given: sinθ + cosθ = p
squaring both sides we get
$\sin^2\theta + \cos^2\theta + \cos^2\theta + 2\sin\theta \cos\theta = p^2$
$1 + 2\sin\theta \cos\theta = p^2(\sin^2\theta + \cos^2\theta = 1)$
$2\sin\theta \cos\theta = p^2 - 1$ ... (i)
and also secθ + cosecθ = q (given)
$\frac{1}{cos\theta} + \frac{1}{sin\theta}$ = q
$\frac{sin\theta+cos\theta}{sin\theta cos\theta}$ = q
but sinθ + cosθ = p ... (given)
$\frac{p}{sin\theta cos\theta}$ = q ... (ii)
from (i) and (ii) we get
$q(p^2 - 1) = 2p$

12.

(d) $\sec^2\theta - \tan^2\theta = 1$
Explanation: $\because \sec^2\theta = 1 + \tan^2\theta$
$\therefore \sec^2\theta - \tan^2\theta = 1$

13. **(a)** 30°
Explanation: Let AB be the tower and B be the kite.
Let AC be the horizontal and let BC $\perp$ AC.
Let $\angle$CAB = θ.
BC = 30 m and AB = 60 m. Then,
$\frac{BC}{AB}$ = sin $\theta \Rightarrow$ sin θ = $\frac{30}{60}=\frac{1}{2} \Rightarrow$ sin θ = sin $30^o \Rightarrow \theta = 30^o$.



14.

(c) 130.95 cm^2

Explanation: Here the angle swept is 150^o. We need to find the area of this sector which subtends 150^o at the centre.

So, area $= \pi r^2 \times \frac{\theta}{360^\circ}$

$= \frac{22}{7} \times 10^2 \times \frac{150}{360}$

= 130.95 cm^2

15.

(c) 52 cm^2

Explanation: We know that perimeter of a sector of radius, $r = 2r + \frac{\theta}{360} \times 2\pi r$ ...(1)

Therefore, substituting the corresponding values of perimeter and radius in equation (1), we get,

$29 = 2 \times 6.5 + \frac{\theta}{360} \times 2\pi \times 6.5$ ...(2)

$29 = 2 \times 6.5 \left(1 + \frac{\theta}{360} \times \pi\right)$

$\frac{29}{2 \times 6.5} = \left(1 + \frac{\theta}{360} \times \pi\right)$

$\frac{29}{2 \times 6.5} - 1 = \frac{\theta}{360} \times \pi$ ....(3)

We know that area of the sector $= \frac{\theta}{360} \times \pi r^2$

From equation (3), we get

Area of the sector $= \left(\frac{29}{2 \times 6.5} - 1\right) r^2$

Substituting r = 6.5 we get,

Area of the sector $= \left(\frac{29}{2 \times 6.5} - 1\right) 6.5^2$

$= \left(\frac{29 \times 6.5^2}{2 \times 6.5} - 6.5^2\right)$

$= \left(\frac{29 \times 6.5}{2} - 6.5^2\right)$

$= \left(\frac{29 \times 6.5}{2} - 6.5^2\right)$

$= (94.25 - 42.25)$

= 52

Therefore, area of the sector is 52 cm^2.

16.

(b) $\frac{12}{13}$

Explanation: Total number of possible outcomes = 52

Number of king cards in the pack = 4

$\therefore$ Number of cards that are not king = 52 - 4 = 48

So, favourable number of outcomes = 48

$\therefore$ Required probability = $\frac{48}{52}$ = $\frac{12}{13}$

17. **(a)** $\frac{5}{7}$

Explanation: No. of days in a leap year = 366

No. of Mondays = 52

Extra days = 366 - 52 $\times$ 7

= 366 - 364 = 2

$\therefore$ Remaining days in the week = 7 - 2 = 5

$\therefore$ Probability of being 52 Mondays in the leap year = $\frac{5}{7}$

18.

(b) mode

Explanation: The most frequent value in the data is known as the Mode. e.g let us consider the following data set: 3,5,7,5,9,5,8,4 the mode is 5 since it occurs most often in data set.

19.

(d) A is false but R is true.

Explanation: A is false but R is true.

20.

(d) A is false but R is true.

Explanation: A is false but R is true.

Section B

21. Let us assume that $\frac{2}{\sqrt{7}}$ is rational. Then, there exist positive co-primes a and b such that

$\frac{2}{\sqrt{7}} = \frac{a}{b}$

$\sqrt{7} = \frac{2b}{a}$

As 2b and a are rational numbers.

Then $\frac{2b}{a}$ is rational number.

But $\sqrt{7}$ is not a rational number.

Since a rational number cannot be equal to an irrational number . Our assumption that $\frac{2}{\sqrt{7}}$ is rational number is wrong .

Hence $\frac{2}{\sqrt{7}}$ is an irrational number

22. $\triangle \mathrm{ABC} \sim \triangle \mathrm{CBD}$ (By AA similarity)

$\frac{\mathrm{AB}}{\mathrm{CB}} = \frac{\mathrm{BC}}{\mathrm{BD}} = \frac{\mathrm{AC}}{\mathrm{CD}}$

$\Rightarrow$ $CB^2 = AB \times BD$ ...(i)

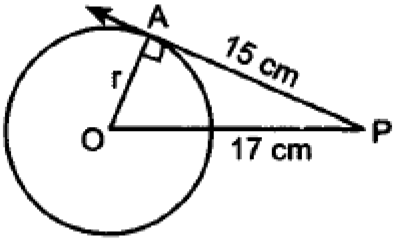
Similarly $\triangle \mathrm{ABC} \sim \triangle \mathrm{ACD}$

$\frac{\mathrm{AB}}{\mathrm{AC}} = \frac{\mathrm{BC}}{\mathrm{CD}} = \frac{\mathrm{AC}}{\mathrm{AD}}$

$\Rightarrow$ $AC^2 = AB \times AD$ ...(ii)

By (i) and (ii)

$\frac{\mathrm{CB}^2}{\mathrm{AC}^2} = \frac{\mathrm{AB} \times \mathrm{BD}}{\mathrm{AB} \times \mathrm{AD}} = \frac{\mathrm{BD}}{\mathrm{AD}}$

23. 

$\angle$OAP = 90°

in $\triangle OAP$,

By applying Pythagoras theorem , we get

$\Rightarrow 17^2 = r^2 + 15^2$

$\Rightarrow r^2 = 17^2 - 15^2 = (17-15)(17+15)$

$= 2 \times 32$

$\Rightarrow r^2 = 64 \Rightarrow r = \pm 8\mathrm{cm}$

we should not take negative value because length cannot be negative.

$\Rightarrow r = 8cm$

24. We have,

L.H.S $= \frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{1-\sin^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$ $\left[\because 1-\sin^2\theta = \cos^2\theta\right]$

$= \frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$ $[\because \frac{\cos\theta}{\sin\theta} = cot\theta]$

= R.H.S

Hence proved.

OR

$3tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{3}$

Given,

$= \frac{3\sin\theta+2\cos\theta}{3\sin\theta-2\cos\theta}$

$= \frac{3\tan\theta+2}{3\tan\theta-2}$ [Dividing numerator and denominator by cos θ]

$= \frac{\left(3\times\frac{4}{3}+2\right)}{\left(3\times\frac{4}{3}-2\right)} = \frac{6}{2} = 3$

25. Area of quadrant = $\frac{1}{4}\pi(7)^2 = \frac{49}{4}\pi$ cm^2

Area of triangle = $\frac{1}{2}$ $\times$ 7 $\times$ 3 = $\frac{21}{2}$ cm^2

Area of shaded region = $\frac{49}{4}\pi - \frac{21}{2}$

= $\frac{7}{2}\left(\frac{7}{2}\pi - 3\right)$ cm^2 or 28 cm^2

OR

The hour hand covers 4 complete circles in 2 days (48 hours)

Distance = $2 \times \frac{22}{7} \times 4 \times 4$

= 100.57 cm

The minute hand covers = 48 Circles in 2 days (Each hour = 1 circle)

Distance =$2 \times \frac{22}{7} \times 6 \times 48$

= 1810.23 cm

Total distance = 100.57 + 1810.23

= 1910.8 cm

Section C

26. In order to arrange the books as required, we have to find the largest number that divides 96,240 and 336 exactly. Clearly, such a number is their HCF.

We have,

96=2^5×3,

240=2^4×3×5

336=2^4×3×7

$\therefore$ HCF of 96,240 and 336 is 2^4×3=48

So, there must be 48 books in each stack.

$\therefore$Number of stacks of Biography books=$\frac{96}{48}$=2

Number of stacks of Mystery books=$\frac{240}{48}$=5

Number of stacks of Self-help books=$\frac{336}{48}$=7

27. Let the given polynomial is p(x) =x^2 + 7x + 7

Here,a = 1, b = 7, c = 7

$\therefore \alpha, \beta$ are both zeroes of p(x)

$\therefore \quad \alpha + \beta = \frac{-b}{a}$ = -7.........(i)

$\alpha\beta = \frac{c}{a} = 7$ ....(ii)

Now,

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta$

$= \frac{-7}{7} - 2 \times 7$

= -1 - 14

= -15

Hence the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ is - 15.

28. The given system of equation is

$\frac{x}{2} - \frac{y}{5} = 4$ ...(1)

$\frac{x}{7} + \frac{y}{15} = 3$ ...(2)

Multiplying equation (2) by 3, we get

$\frac{3x}{7} + \frac{y}{5} = 9$ ...(3)

Adding equation (1) and equation (3), we get

$\frac{x}{2} + \frac{3x}{7} = 13$

$\Rightarrow \frac{13}{14}x = 13 \Rightarrow x = \frac{13\times14}{13} = 14$

Substituting this value of x in equation (2), we get

$\frac{14}{7} + \frac{y}{15} = 3$

$\Rightarrow 2+\frac{y}{15}=3 \Rightarrow \frac{y}{15}=3-2$

$\Rightarrow \frac{y}{15}=1 \Rightarrow y=15$

So, the solution of the given system of equations is

x = 14, y = 15

Verification ; Substituting x = 14, y = 15.

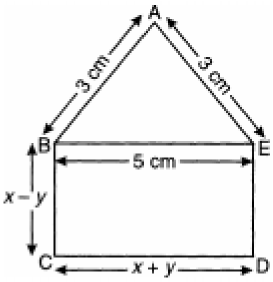
We find that both the equations (1) and (2) are satisfied as shown below;

$\frac{x}{2}-\frac{y}{5}=\frac{14}{2}-\frac{15}{5}=7-3=4$

$\frac{x}{7}+\frac{y}{15}=\frac{14}{7}+\frac{15}{15}=2+1=3$

Hence, the solution is correct.

OR



Since BC | | DE and BE | | CD with BC $\perp$ CD, BCDE is a rectangle.

Since, BE = CD

$\therefore$ x + y = 5 ..(i)

Also, DE = BC = x - y

Since, perimeter of ABCDE is 21

$\therefore$AB + BC + CD+ DE + EA = 21

$\Rightarrow$3 + x - y + x + y + x - y + 3 = 21

$\Rightarrow$6 + 3x - y = 21

$\Rightarrow$3x - y = 15.....(ii)

Adding eqns. (i) and (ii), we get

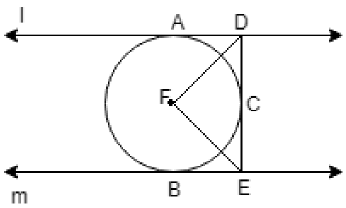
4x = 20

$\Rightarrow$x = 5

On substituting the value of x in (i), we get

y = 0

$\therefore$x = 5 and y = 0.

29. 

Since tangents drawn from an external point to a circle are equal. Therefore, DA = DC.

Thus, in triangles ADF and DFC, we have

DA = DC

DF = DF Common]

AF = CF (radii of the circle)

So, by SSS-criterion of congruence, we obtain

$\Delta ADF \cong \Delta DFC$

$\Rightarrow \quad \angle ADF=\angle CDF$

$\Rightarrow \quad \angle ADC=2\angle CDF$ ...(i)

Similarly, we can prove that

$\angle BEF=\angle CEF$

$\Rightarrow \quad \angle CEB=2\angle CEF$ .....(ii)

Now, $\angle ADC+\angle CEB$ = 180° (Sum of the interior angles on the same side of transversal is 180^{o})

$\Rightarrow 2\angle CDF+2\angle CEF$ = 180° [Using equations (i) and (ii)]

$\Rightarrow \angle CDF+\angle CEF$ = 90°

$\Rightarrow 180^\circ - \angle DFE = 90^\circ$ $\left[\begin{array}{l}\because \angle CDF, \angle CEF \text{ and } \angle DFE \text{ are angles of a triangle} \\ \therefore \angle CDF + \angle CEF + \angle DFE = 180^\circ\end{array}\right.$

$\Rightarrow \quad \angle DFE = 90^\circ$

OR

Let NM be chord of circle with centre C.

Let tangents at M and N meet at the point O.

Since OM is a tangent

∴ MO ⊥ CM i.e. ∠OMC = 90°

∵ ON is a tangent

∴ ON ⊥ CN i.e. ∠ONC = 90°

Again in ΔCMN , CM = CN = r

∴ ∠CMN = ∠CNM

∴ ∠OMC – ∠CMN = ∠ONC – ∠CNM

⇒ ∠OML ≅ ∠ONL

Thus, tangents make equal angle with the chord.

30. LHS

$= \frac{\sin\theta}{1-\cos\theta} + \frac{\tan\theta}{1+\cos\theta}$

$= \frac{\sin\theta}{1-\cos\theta} + \frac{\frac{\sin\theta}{\cos\theta}}{1+\cos\theta}$

$= \frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{\cos\theta(1+\cos\theta)}$

$= \frac{\sin\theta\cdot\cos\theta(1+\cos\theta)+\sin\theta(1-\cos\theta)}{(1-\cos\theta)\cos\theta(1+\cos\theta)}$ [taking LCM]

$= \frac{\sin\theta\cdot\cos\theta+\sin\theta\cdot\cos^2\theta+\sin\theta-\sin\theta\cdot\cos\theta}{\cos\theta(1-\cos^2\theta)}$ [Since,(a-b)(a+b) = a^2 - b^2]

$= \frac{\sin\theta\cdot\cos^2\theta+\sin\theta}{\cos\theta\cdot\sin^2\theta}$. [Since, $\text{Sin}^2A + \text{Cos}^2A = 1$]

$= \frac{\sin\theta\cdot\cos^2\theta}{\cos\theta\cdot\sin^2\theta} + \frac{\sin\theta}{\cos\theta\cdot\sin^2\theta}$

$= \frac{\cos\theta}{\sin\theta} + \frac{1}{\cos\theta\cdot\sin\theta}$

= cot θ + sec θ . cosec θ

= sec θ . cosec θ + cot θ

= RHS.

Hence, Proved.

31.

Class Interval	**Frequency**	**Cumulative Frequency**
0 - 10	8	8
10 - 20	16	24
20 - 30	36	60
30 - 40	34	94
40 - 50	6	100

Here, N = 100 $\Rightarrow \frac{N}{2} = 50$

The cumulative frequency just greater than 50 is 60.

Hence, median class is 20 - 30.

∴ l = 20, h = 10, f = 36, cf = cf of preceding class is 24

Now, Median = $l + \left\{\text{h} \times \frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

$= 20 + \left\{10 \times \frac{(50-24)}{36}\right\}$

$= 20 + \left\{10 \times \frac{26}{36}\right\}$

= 20 + 7.22

= 27.2

Thus, the median of the data is 27.2.

Section D

32. In equation $(m+1)y^2 - 6(m+1)y + 3(m+9) = 0$

A = m + 1, B = - 6(m + 1), C = 3(m + 9)

For equal roots, $D = B^2 - 4AC = 0$

$36(m+1)^2 - 4(m+1) \times 3(m+9) = 0$

$\Rightarrow 3\left(m^2 + 2m + 1\right) - (m+1)(m+9) = 0$

$\Rightarrow 2m^2 - 4m - 6 = 0$

$\Rightarrow m^2 - 2m - 3 = 0$

$\Rightarrow m^2 - 3m + m - 3 = 0$

$\Rightarrow m(m-3) + 1(m-3) = 0$

$\Rightarrow (m-3)(m+1) = 0$

$\therefore$ m = -1, 3

Neglecting $m \neq -1$

$\therefore m = 3$

$\therefore$ the equation becomes $4y^2 - 24y + 36 = 0$

$\Rightarrow y^2 - 6y + 9 = 0$

$\Rightarrow (y-3)(y-3) = 0$

$\Rightarrow (y-3) = 0 \quad and \quad (y-3) = 0$

$\therefore$ roots are y = 3,3

OR

Let the present age of father be x years.

Son's present age = (45 - x) years.

Five years ago:

Father's age = (x - 5) years

Son's age = (45 - x - 5) years = (40 - x) years.

According to question,

$\therefore$ (x - 5) (40 - x) = 124

$\Rightarrow$ 40x - x^2 - 200 + 5x = 124

$\Rightarrow$ x^2 - 45x + 324 = 0

Spilting the middle term,

$\Rightarrow$ x^2 - 36x - 9 x + 324 = 0

$\Rightarrow$ x(x - 36) - 9(x - 36) = 0

$\Rightarrow$ (x - 9)(x - 36) = 0

$\Rightarrow$ x = 9, or 36

We can't take father age as 9 years

So, x = 36, we have

Father's present age = 36 years

Son's present age = 9 years

Hence, Father's present age = 36 years and Son's present age = 9 years.

33. Given A $\triangle ABC$ in which $\angle BAC = 90^\circ$ and DEFG is a square.

Proof

i. In $\triangle AGF$ and $\triangle DBG$, we have

$\angle GAF = \angle BDG = 90^\circ$

$\angle AGF = \angle DBG$ [corresponidng angles]

[$\because GF \| BC$ and AB is the transversal]

$\therefore \quad \triangle AGF \sim \triangle DBG$

ii. In $\triangle AGF$ and $\triangle EFC$, we have

$\angle FAG = \angle CEF = 90^\circ$

$\angle GFA = \angle FCE$ [corresponding angles]

[$\because GF \| BC$ and AC is the transversal]

$\therefore \quad \triangle AGF \sim \triangle EFC$

iii. $\triangle DBG \sim \triangle AGF$ and $\triangle AGF \sim \triangle EFC$

$\Rightarrow \triangle DBG \sim \triangle EFC$

iv. $\triangle DBG \sim \triangle EFC$

$\Rightarrow \frac{BD}{FE} = \frac{DG}{EC}$

$\Rightarrow \frac{BD}{DE} = \frac{DE}{EC}$ [$\because$ DG = DE and FE = DE).

Hence, $DE^2 = BD \times EC$.

34. According to question

Diameterdiameter of the well = 7m

Radius of the well (r) = $\frac{7}{2}$ m = 3.5m and, height of the well (h) = 22.5 m

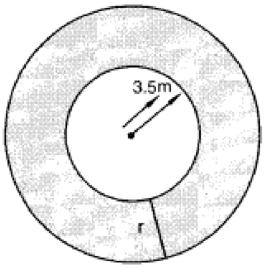
$\therefore$ Volume of the earth dugout = $\pi \times (3.5)^2 \times 22.5$ m^3 = $\pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{45}{2}$ m^3

Let the width of the embankment be r metres. Clearly, embankment forms a cylindrical shell whose inner and outer radii are 3.5 m and (r + 3.5) m respectively and height 1.5 m.

$\therefore$ Volume of the embankment = Area of ring at top $\times$ height of the embankment

= $\pi\{(r + 3.5)^2 - (3.5)^2\} \times 1.5$ m^3 = $\pi(r + 7)\, r \times \frac{3}{2}$ m^3

But, Volume of the embankment = Volume of the well



$\Rightarrow \pi r(r+7) \times \frac{3}{2} = \pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{45}{2}$

$\Rightarrow$ r (r + 7) = $\frac{49}{4} \times 15$

$\Rightarrow 4r^2 + 28r = 735$

$\Rightarrow 4r^2 + 28r - 735 = 0$

$4r^2 + 70x - 42x - 735 = 0$

$\Rightarrow$ 2r(2r + 35) - 21(2r + 35) = 0

$\Rightarrow$ (2r + 35)(2r - 21) = 0

$\Rightarrow$ 2r + 35 = 0 or 2r - 21 = 0

$\Rightarrow$ r = $\frac{-35}{2}$ or x = $\frac{21}{2}$

$\frac{-35}{2}$ is negative, hence neglect this value

$\Rightarrow x = \frac{21}{2}$ = 10.5 m

Hence, the width of the embankment is 10.5 m

OR

We have;

A Cube,

Cube's $\frac{length}{Edge}$, a = 7 cm

A Cylinder:

Cylinder's Radius, r = 2.1 cm or r = $\frac{21}{10}$cm

Cylinder's Height, h = 7 cm

$\because$ A cylinder is scooped out from a cube,

$\therefore$ TSA of the resulting cuboid:

= TSA of whole Cube - 2 $\times$ (Area of upper circle or Area of lower circle) + CSA of the scooped out Cylinder

= $6a^2 + 2\pi rh - 2 \times (\pi r^2)$

= $6 \times (7)^2 + 2 \times (22 \div 7 \times 2.1 \times 7) - 2 \times [22 \div 7 \times (2.1)^2]$

= $6 \times 49 + (44 \div 7 \times 14.7) - (44 \div 7 \times 4.41)$

= 294 + 92.4 - 27.72

= 294 + 64.68

= 358.68 cm^2

Hence, the total surface area of the remaining solid is 358.68 cm^2

35. Let the missing frequencies are a and b.

Class Interval	Frequency f_i	Cumulative frequency
0 - 5	12	12
5 - 10	a	12 + a
10 - 15	12	24 + a
15 - 20	15	39 + a
20 - 25	b	39 + a + b
25 - 30	6	45 + a + b
30 - 35	6	51 + a + b
35 - 40	4	55 + a + b = 70

Then, 55 + a + b = 70

a + b = 15 ......(1)

Median is 16, which lies in 15 - 20

So, The median class is 15 - 20

Therefore, l = 15, h = 5, N = 70, f = 15 and cf = 24 + a

Median is 16, which lies in the class 15 - 20. Hence, median class is 15 - 20.

$\therefore l = 15, h = 5, f = 15, c.f. = 24 + a$

Now, Median = $l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

$\therefore 16 = 15 + \left\{5 \times \frac{(35-24-a)}{15}\right\}$

$\Rightarrow 16 = 15 + \left\{\frac{11-a}{3}\right\}$

$\Rightarrow 1 = \frac{11-a}{3}$

$\Rightarrow 3 = 11 - a$

$\Rightarrow a = 8$

Now, $55 + a + b = 70$

$\Rightarrow 55 + 8 + b = 70$

$\Rightarrow 63 + b = 70$

$\Rightarrow b = 7$

Hence, the missing frequencies are a = 8 and b = 7.

Section E

36. **Read the text carefully and answer the questions:**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

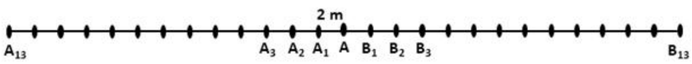
(i) Distance covered in placing 6 flags on either side of center point is 84 + 84 = 168 m

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_6 = \frac{6}{2}[2 \times 4 + (6 - 1) \times 4]$

$\Rightarrow S_6 = 3[8 + 20]$

$\Rightarrow S_6 = 84$

(ii) 

Let A be the position of the middle-most flag.

Now, there are 13 flags (A_1, A_2 ... A_{12}) to the left of A and 13 flags (B_1, B_2, B_3 ... B_{13}) to the right of A.

Distance covered in fixing flag to A_1 = 2 + 2 = 4 m

Distance covered in fixing flag to A_2 = 4 + 4 = 8 m

Distance covered in fixing flag to A_3 = 6 + 6 = 12 m

...

Distance covered in fixing flag to A_{13} = 26 + 26 = 52 m

This forms an A.P. with,

First term, a = 4

Common difference, d = 4

and n = 13

OR

$\therefore$ Distance covered in fixing 13 flags to the left of A = S_{13}

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{13} = \frac{13}{2}[2 \times 4 + 12 \times 4]$

$= \frac{13}{2} \times [8 + 48]$

$= \frac{13}{2} \times 56$

= 364

Similarly, distance covered in fixing 13 flags to the right of A = 364

Total distance covered by Ruchi in completing the task

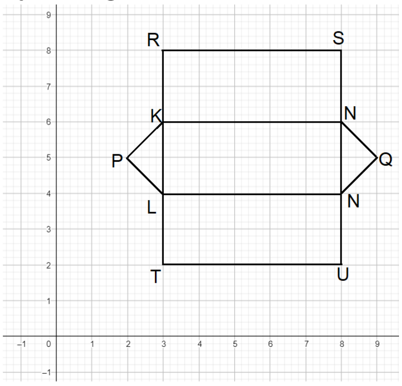
= 364 + 364 = 728 m

(iii) Maximum distance travelled by Ruchi in carrying a flag

= Distance from A_{13} to A or B_{13} to A = 26 m

37. **Read the text carefully and answer the questions:**

The camping alpine tent is usually made using high-quality canvas and it is waterproof. These alpine tents are mostly used in hilly areas, as the snow will not settle on the tent and make it damp. It is easy to layout and one need not use a manual to set it up. One alpine tent is shown in the figure given below, which has two triangular faces and three rectangular faces. Also, the image of canvas on graph paper is shown in the adjacent figure.



(i) Coordinates of Q are (9, 5).

$\therefore$ Distance of point Q from y-axis = 9 units

(ii) Coordinates of point U are (8, 2).

OR

We have, P(2, 5) and Q (9, 5)

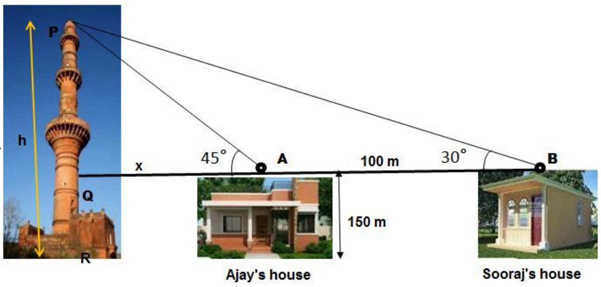
$\therefore$ PQ = $\sqrt{(2-9)^2 + (5-5)^2}$ = $\sqrt{49 + 0}$ = 7 units

(iii) Length of TU = 5 units and of TL = 2 units

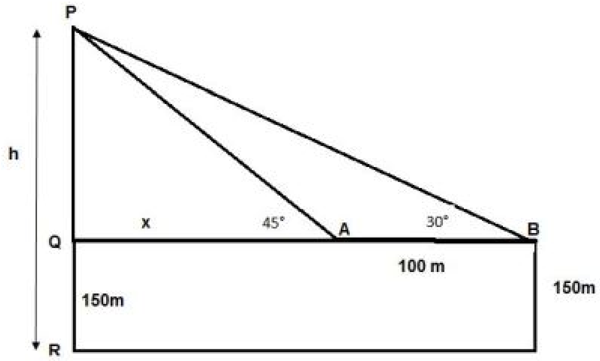
$\therefore$ Perimeter of image of a rectangular face = 2(5 + 2) = 14 units

38. **Read the text carefully and answer the questions:**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45^{o} and 30^{o} respectively as shown in the figure.



(i) The above figure can be redrawn as shown below:



Let PQ = y

In △PQA,

tan 45 = $\frac{PQ}{QA} = \frac{y}{x}$

$1 = \frac{y}{x}$

x = y ...(i)

In △PQB,

tan 30 = $\frac{PQ}{QB} = \frac{PQ}{x+100} = \frac{y}{x+100} = \frac{x}{x+100}$

$\frac{1}{\sqrt{3}} = \frac{x}{x+100}$

$x\sqrt{3} = x + 100$

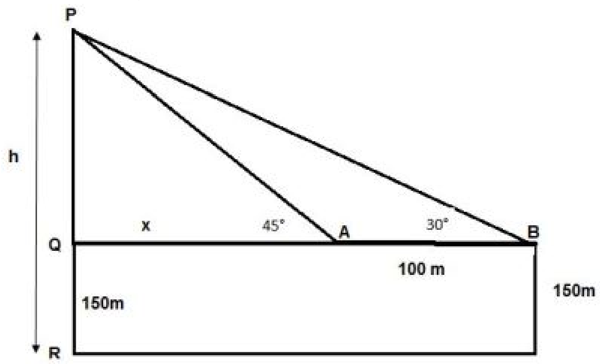
$x = \frac{100}{\sqrt{3}-1} = 136.61$ m

From the figure, height of tower h = PQ + QR

= x + 150 = 136.61 + 150

h = 286.61 m

(ii) The above figure can be redrawn as shown below:

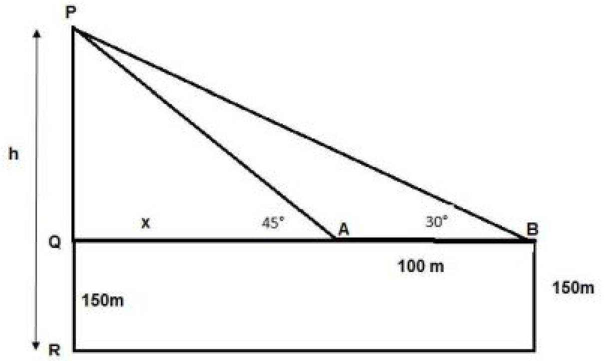


Distance of Sooraj's house from tower = QA + AB

= x + 100 = 136.61 + 100 = 236.61 m

OR

The above figure can be redrawn as shown below:



Distance between top of the tower and top of Ajay's house is PA

In $\triangle$PQA

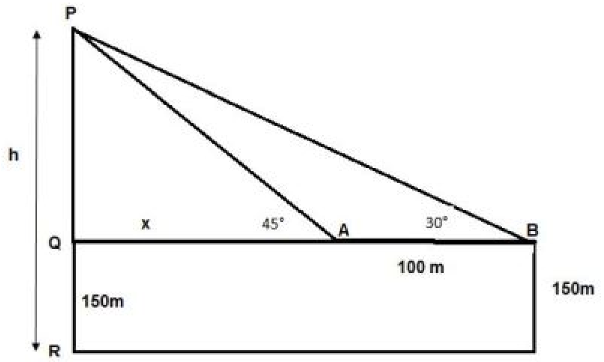
sin 45^{o} = $\frac{PQ}{PA}$

$\Rightarrow PA = \frac{PQ}{\sin 45^{o}}$

$\Rightarrow$ PA = $\frac{y}{\frac{1}{\sqrt{2}}} = \sqrt{2} \times 136.61$

$\Rightarrow$ PA = 193.20 m

(iii)The above figure can be redrawn as shown below:



Distance between top of tower and Top of Sooraj's house is PB

In $\triangle$PQB

sin 30^{o} = $\frac{PQ}{PB}$

$\Rightarrow PB = \frac{PQ}{\sin 30^{o}}$

$\Rightarrow$ PB = $\frac{y}{\frac{1}{2}} = 2 \times 136.61$

$\Rightarrow$ PB = 273.20 m