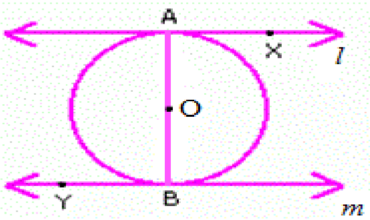
Chapter 10 Circle

Question-1

Prove that the line segment joining the points of contact of two parallel tangents to a circle is a diameter of the circle.

Solution:



Given: *l* and *m* are the tangent to a circle such that *l* || *m*, intersecting at A and B respectively.

To prove: AB is a diameter of the circle.

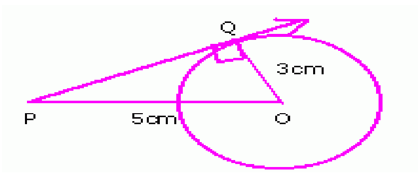
Proof:

A tangent at any point of a circle is perpendicular to the radius through the point of contact.
∴ ∠ XAO = 90°
and ∠ YBO = 90°
Since ∠ XAO + ∠ YBO = 180°
Angles on the same side of the transversal is 180°.
Hence the line AB passes through the centre and is the diameter of the circle.

Question-2

Find the length of the tangent from a point which is at a distance of 5 cm from the centre of the circle of radius 3 cm.

Solution:



Given: PQ is a tangent to the circle intersect at Q. OP = 5 cm and OQ = 3 cm.

To find: PQ

Proof:

In rt.Δ OQP, by Pythagoras theorem

PQ = $\sqrt{OP^2 - OQ^2} = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16}$ = 4 cm

Therefore the length of the tangent from a point is 4 cm.

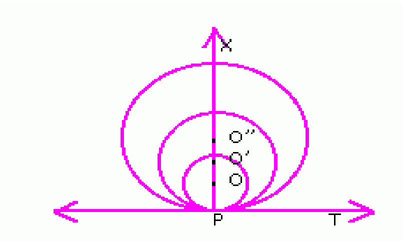
Question-3

Find the locus of centres of circles which touch a given line at a given point.

Solution:

Given: Circles with centres O, O', O" touching line T at P.

To prove: To find the locus of centres of circles which touch a given line at a given point.



Proof: As OP, O'P, O"P are the radii of the circles touching line T at P, it is perpendicular to the given line.

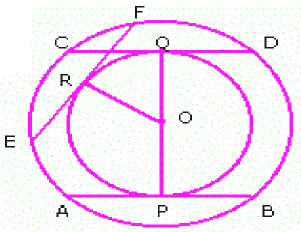
∴ OP, O'P, O"P represent the same straight line passing through P and ⊥ to PT.

Hence the locus of the centres of circles which touch a given line at a given point is a straight line to the given line at the given point.

Question-4

In two concentric circles, prove that all chords of the outer circle which touch the inner circle are of equal length.

Solution:



Given: Two concentric circles with centre O. AB, CD and EF are the chords of the outer circle.

To prove: AB = CD = EF.

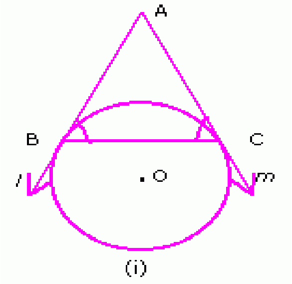
Proof:
OP, OQ and OR are the distances of the chord AB, CD and EF from the centre.
But OP = OR = OQ = radius
Since the chords are at equal distances from the centre they are equal.

Question-5

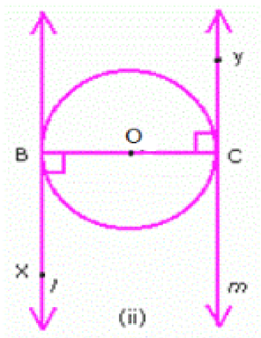
Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

Solution:
Given: l and m are tangents intersecting at A and touching the circle at B and C.

To prove:
(i) ∠ ABC = ∠ ACB



(ii) ∠ XBO = ∠ OCY



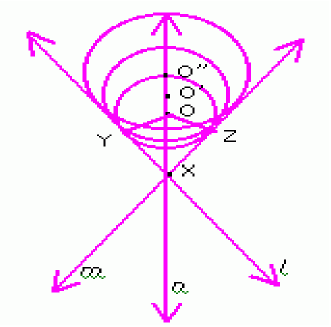
Proof:
(i) Using the theorem, the length of the tangents drawn form an external point to a circle are equal.
∠ ABC = ∠ ACB (∵ Sides opposite to equal angles are equal)

(ii) Using the theorem, the tangent at any point of a circle is perpendicular to the radius through the point of contact.
∠ XBO = 90°
∠ OCY = 90°
∠ XBO = ∠ OCY.

Question-6

Find the locus of centres of circles which touch two intersecting lines.

Solution:



In Δ YXO and Δ ZXO,

∠ OYX = ∠ OZX (= 90°)

XO = XO (Common)

OY = OZ = radius

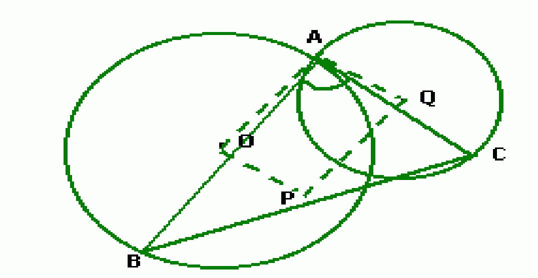
Δ YOX ≅ Δ ZOX (By SAS congruence)

∴ ∠ YXO = ∠ ZXO

∴ Locus of the centre is a straight line bisecting them between two intersecting lines.

Question-7

Let A be one point of intersection of two intersecting circles with centres O and Q. The tangents at A to the two circles meet the circles again at B and C, respectively. Let the point P be located so that AOPQ is a parallelogram. Prove that P is the circumcentre of the triangle ABC in figure.
(Hint: AQ ⊥ AB and AQ || OP. Then OP ⊥ AB and is also bisector of AB. Similarly, PQ is perpendicular bisector of AC.)



Solution:

Given: Two circles with centre O and Q intersect at A. The tangents at A to the two circles meet the circles again at B and C, respectively. AOQP is a parallelogram.

To prove: P is the circumcentre of the triangle ABC.

Proof:
AQ ⊥ AB and AQ || OP
Therefore OP ⊥ AB
Also OP bisects AB as the line drawn from the centre to the chord bisects the chord.
Hence OP is the perpendicular bisector of AB.

Similarly PQ is the perpendicular bisector of AC.

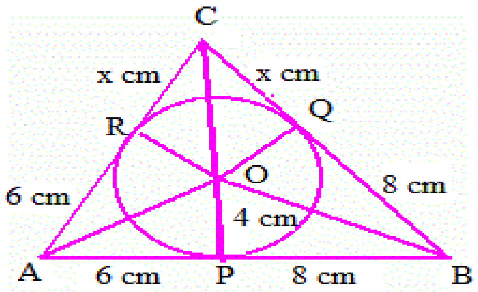
Since the perpendicular bisectors intersect at P. P is the circumcentre of the triangle ABC.

Question-8

The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.
(Hint: Equate the areas of the triangle found by using the formula √[s(s-a)(s-b)(s-c)] and also found by dividing it into three triangles.)

Solution:
Given: The radius of the in circle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm.

To find: AC and BC.



Proof:

$$S = \frac{6+x+x+8+8+6}{2}$$
$$= \frac{2x+28}{2}$$

Area of a triangle

$= \sqrt{(x+14)(x+14-x-6)(x+14-6-8)(x+14-8-x)}$

$= \sqrt{(x+14)(8)(x)(6)}$

$= \sqrt{48x(x+14)}$ ....(i)

Area of triangle = 2 area of Δ AOP + 2 area of Δ COR + 2 area of Δ PBO

$= 2 \times \frac{1}{2} \times 4 \times 6 + 2 \times \frac{1}{2} \times 4 \times 8 + 2 \times \frac{1}{2} \times 4 \times x$

$= 24 + 32 + 4x$

$= 56 + 4x$ ....(ii)

Equating (i) and (ii)

$48x(x + 14) = (56 + 4x)^2$

$48x(x + 14) = 16(x + 14)^2$

$48x = 16(x + 14)$ or $x + 14 = 0$

$48x = 16x + 224$

$32x = 224$

$x = 7$ or $x = -14$(which we ignore)

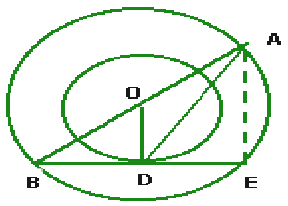
AC = 6 + 7 = 13 cm

BC = 8 + 7 = 15 cm

The other two sides of the triangles are 13 cm and 15 cm.

Question-9

The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle. BD is a tangent to the smaller circle touching it at D. Find the length AD in figure. [Hint: Let line BD intersect the bigger circle at E. Join AE. AE = 2 × 8= 16 cm.
DE = BD = √(169 - 64) = √105 and ∠AED= 90°.]



Solution:

Given: Two concentric circles with radius 13cm and 8cm. AB is the diameter of the
bigger circle. BD is the tangent to the smaller circle touching it at D.

To find: AD

Proof:

In Δ BDO and Δ BEA,

∠ DBO = ∠ EBA (Common)

$\frac{BD}{BE} = \frac{BO}{BA}$ (O is the centre of the circle and OD bisects BE)

Δ BDO ~ Δ BEA

AE = 2DO = 2(8 cm) = 16 cm

BD = $\sqrt{BO^2 - DO^2} = \sqrt{13^2 - 8^2} = \sqrt{169 - 64} = \sqrt{105}$ cm

DE = BD = $\sqrt{105}$ cm

∠ AED = ∠ ODB = 90° (since Δ BDO ~ Δ BEA)

In Δ DAE,

AD = $\sqrt{AE^2 + DE^2} = \sqrt{16^2 + 105} = \sqrt{256 + 105} = \sqrt{361}$ cm.

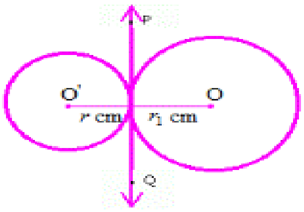
Question-10

Find the locus of the centre of a circle of constant radius (r) which touches a given circle of radius r_1 (i) externally, (ii) internally, given $r_1 > r$.

Solution:

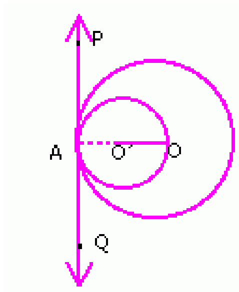
(i) $OO' = r + r_1$

The locus is a circle with centre O' and radius $r_1 + r$.



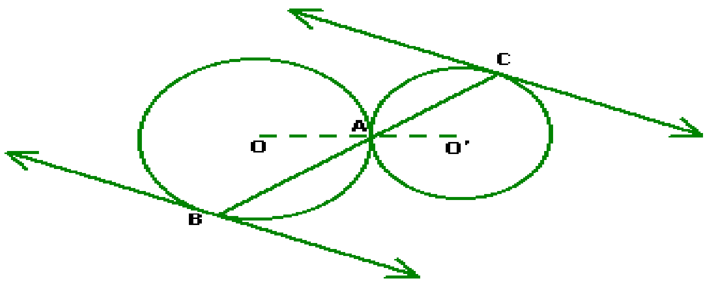
(ii) $OO' = r_1 - r$

The locus is a circle with centre O' and radius $r_1 - r$.



Question-11

In figure, two circles with centres O, O' touch externally at a point A. A line through A is drawn to intersect these circles at B and C. Prove that the tangents at B and C are parallel. [Hint: Prove that ∠ OBA = ∠ O' CA]

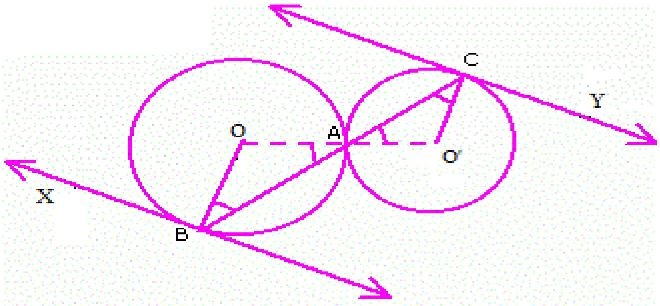


Solution:

Given: Two circles with centers O, O' touch externally at a point A. A line through A is
drawn to intersect these circles in B and C.

To prove: Tangents B and C are parallel.

Proof:



In Δ AOB,

∠ OBA = ∠ OAB (OB = OA, opposite angles of equal sides) .... (i)

In Δ AO'C

∠ O'AC = ∠ O'CA (O'A = O'C, opposite angles of equal sides) ....(ii)

From (i), (ii) and (iii)

∠ OBA = ∠ O'CA ....(iv)

Since the line joining the centre and the tangent at the point of contact is 90°.

∠ OBX = ∠ O'CY = 90° ....(v)

Adding (iv) and (v)

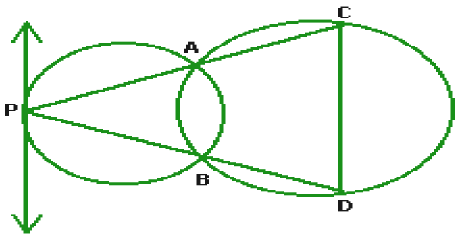
∠ OBX + ∠ OBA = ∠ O'CY + ∠ O'CA

∠ XBA = ∠ YCA

Since alternate angles are equal, tangents at B and C are parallel.

Question-12

In figure two circles intersect at two points A and B. From a point P on a circle, two line segments PAC and PBD are drawn intersecting the other circle at the points C and D. Prove that CD is parallel to the tangent at P.



Solution:

Given: Two circles intersect at A and B.

To prove: CD || tangent at P.

Proof: Join AB. Let XY be the tangent at P. Then by alternate segment theorem,

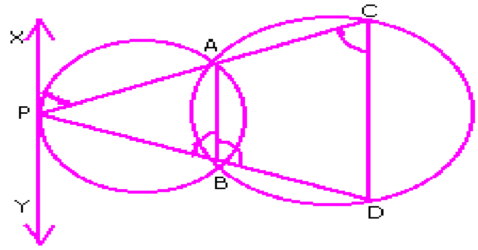
∠ APX = ∠ ABP ....(i)

Next, ABCD is a cyclic quadrilateral, therefore, by the theorem sum of the opposite angles of a quadrilateral is 180°

∠ ABD + ∠ ACD = 180°

Also ∠ ABD = ∠ ABP = 180° (Linear Pair)

∴ ∠ ACD = ∠ ABP ....(ii)



From (i) and (ii),

∠ ACD = ∠ APX

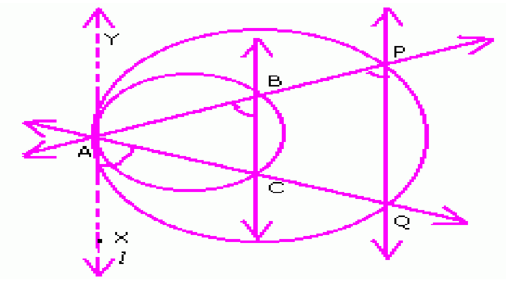
∴ XY || CD (Since alternate angles are equal).

Question-13

Two rays ABP and ACQ are intersected by two parallel lines in B, C and P, Q respectively. Prove that the circumcircles of Δ ABC and Δ APQ touch each other at A. [Hint: Draw tangent XAY to the circumcircle of triangle APQ and show that ∠ YAP = ∠ PQA = ∠ BCA.

Solution:

Given: Two rays ABP and ACQ are intersected by two parallel lines in B, C and P, Q respectively.



To prove: Circumcircles of Δ ABC and Δ APQ touch each other at A.

Construction: Draw a tangent *l* at A.

Proof:

∠ XAC = ∠ CBA (Alternate interior angles)

∠ CBA = ∠ QPB (Corresponding angles equal)

∠ XAC = ∠ QPB

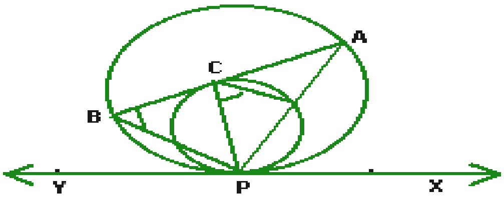
Therefore *l* || QP

Hence *l* || BC || QP

So circumcircles of Δ ABC and Δ APQ touch each other at A.

Question-14

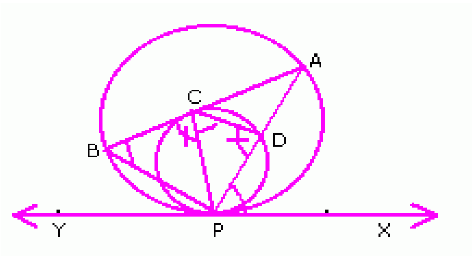
In figure, two circles touch internally at a point P. AB is a chord of the bigger circle touching the other circle at C. Prove that PC bisects the angle APB.[Hint: Draw a tangent at the point P. Joint CD, where D is the point of intersection of AP and the inner circle and prove that ∠ PBC = ∠ PCD.]



Solution:

Given: Two circles touch internally at a point P. AB is chord of the bigger circle touching the other circle at C.

To prove: ∠ BPC = ∠ CPA



Proof:

In Δ BCP and Δ CDP,

∠ XPA = ∠ PBA (Alternate segment angles)

∠ PCB = ∠ CDP (Alternate segment angles)

Δ BCP ≅ Δ CDP (By AA similarity)

∠ BPC = ∠ CPD

∴ ∠ BPC = ∠ CPA

∴ PC bisect ∠ BPA.

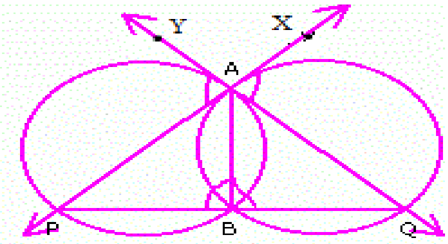
Question-15

Two circles intersect each other at two points A and B. At A, tangents AP and AQ to the two circles are drawn which intersect other circles at the points P and Q respectively. Prove that AB is the bisector of angle PBQ.

Solution:

Given: Two circles intersect each other at two points A and B. At A, tangents AP and AQ to the two circles are drawn which intersect other circles at the points P and Q respectively.

To prove: AB is the bisector of angle PBQ.



Proof:

∠ ABQ = ∠ QAX (Alternate interior angles)

∠ ABP = ∠ YAP (Alternate interior angles)

But ∠ YAP = ∠ XAQ (Vertically opposite angles)

∴ ∠ ABQ = ∠ ABP ∠ ABQ + ∠ ABP = 180^o (Linear pair)

∠ ABQ = ∠ ABP = 90^o

AB is the bisector of angle PBQ.

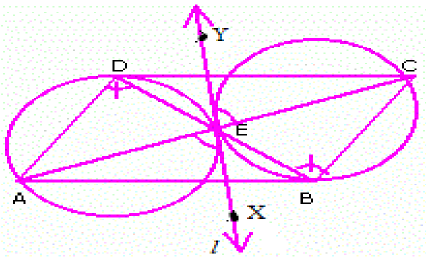
Question-16

The diagonals of a parallelogram ABCD intersect in a point E. Show that the circumcircles of Δ ADE and Δ BCE touch each other at E.

Solution:

Given: Diagonals of a parallelogram ABCD intersect in a point E.

To prove: Circumcircles of Δ ADE and Δ BCE touch each other at E.



Construction: Let *l* be a tangent to first circle.

Proof:

∠ ADB = ∠ CBD (Alternate interior angles are equal)

∠ AEX = ∠ ADB (Alternate segment theorem)

But ∠ AEX = ∠ CEY (Vertically opposite angles)

∴ ∠ CBD = ∠ CEY

Therefore by converse of alternate segment theorem,

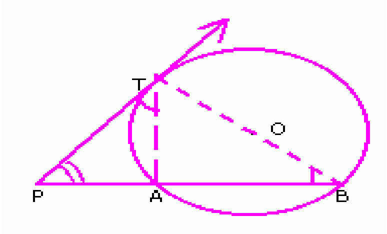
l is a tangent to second circle with point of contact at E.

Hence the circumcircles of Δ ADE and Δ BCE touch each other at E.

Question-17

If PAB is a secant to a circle intersecting it at A and B, and PT is a tangent, then PA. PB = PT^2.
(Prove using alternate segment theorem)

Solution:
Given: A secant PAB to circle with centre O intersecting it at A and B and a tangent PT to the circle.



To prove: PA. PB = PT^2

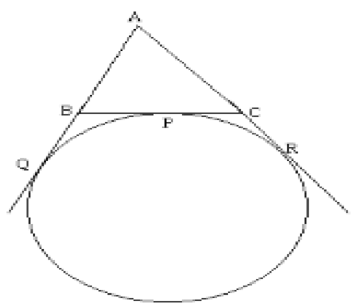
Construction: Join TA and TB.
Proof:

In Δ PBT and Δ PTA
∠ BPT = ∠ APT (Common)
∠ PBT = ∠ PTA (Alternative segment theorem)
∴ Δ PBT ~ Δ PTA (AA similarity)
$\frac{PA}{PT} = \frac{PT}{PB}$
PA.PB = PT^2
Hence proved.

Question-18

A circle touches the side BC of a triangle ABC at P and touches AB and AC when produced at Q and R respectively. Show that AQ = ½ (perimeter of triangle ABC).

Solution:
Given: A circle touches the side BC of Δ ABC at P and AB, AC produced at Q and R respectively.



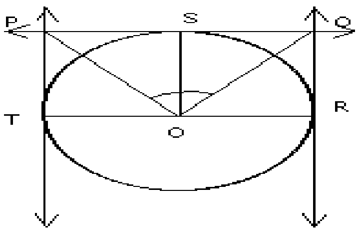
To Prove: AQ is half the perimeter of Δ ABC.

Proof:
Lengths of two tangents from an external points are equal.
∴ AQ = AR, BQ = BP and CP = CR

Perimeter of Δ ABC = AB + BC + AC
= AB + (BP + PC) + (AR - CR)
= (AB + QB) + PC + AQ - PC (∵ AQ = AR, BQ = BP and CP = CR)
= AQ + AQ
= 2AQ
AQ is half the perimeter of Δ ABC.

Question-19

Prove that the intercept of a tangent between 2 parallel tangents to a circle subtends a right angle at the centre.



Solution:
Given: A circle with centre 'O' has PT and QR as two tangents parallel to each other touching the circles at T and R. PQ is the intercept between 2 tangents touching the circle at S.

To prove : ∠ POQ = 90°

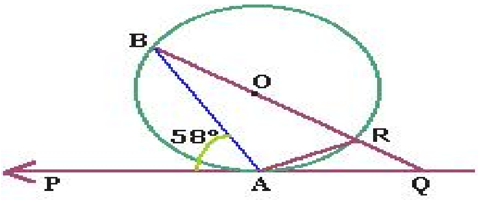
Construction: Join OS and OT

Proof: Consider D POS and D POT
As OS is the radius at the point of contact where the tangent intercept touches the circle.
∠ OSP = 90°,
Similarly ∠OTP = 90°
i.e. ∠OSP = ∠OTP
Hypotenuse OP is common
OS = OT (radii of the same circle)
\ D POT @ D SOP (RHS)
\ POT = SOP (cpct)
Similarly D SOQ, D ROQ are congruent
\ ∠SOQ = ∠ROQ
As TOR is a straight line
∠POT + ∠SOP + ∠SOQ + ∠ROQ = 180°
But ∠POT = ∠SOP and ∠SOQ = ∠ ROQ (already proved)
∠SOP + ∠SOP + ∠SOQ + ∠SOQ = 180°
\ 2 (∠SOP + ∠SOQ) - 180°

$\angle SOP + \angle SOQ = \frac{180°}{2} = 90°$.

Question-20

In the given figure, O is the centre of the circle. PQ is a tangent to the circle at A. If ∠ PAB = 58°, find ∠ ABQ and ∠ AQB.



Solution:

Given: O is the centre of the circle. PQ is a tangent to the circle at A. ∠ PAB = 58°.

To find: ∠ ABQ and ∠ AQB.

∠ BAR = 90° (Angle in a semicircle)

∠ ARB = ∠ PAB = 58° (Alternate segment theorem)

∠ ABQ = 180° - (∠ BAR + ∠ ARB) (Angle sum property of a triangle)

= 180° - (90° + 58°)

= 180° − 148° = 32°

∠ QAR = ∠ ABR = 32° (Alternate segment theorem)

and ∠ AQB = 180° − (∠ ABQ + ∠ BAQ) (Angle sum property of a triangle)

= 180° − (32° + 90° + 32°)

= 180° − 154° = 26°.