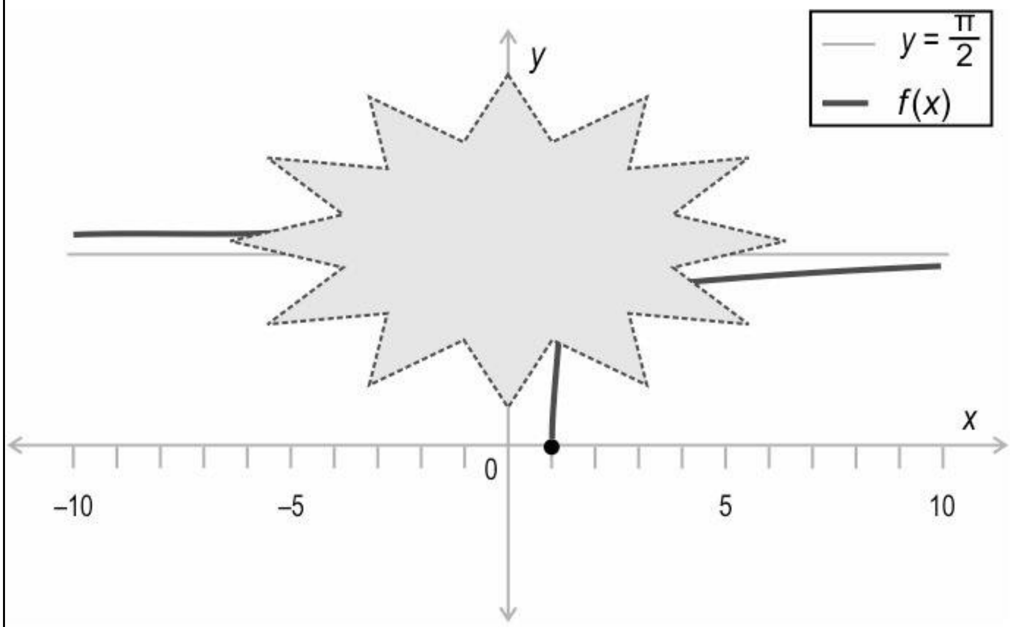
Practice Questions
Session 2022-23
Class XII
Mathematics (Code-041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

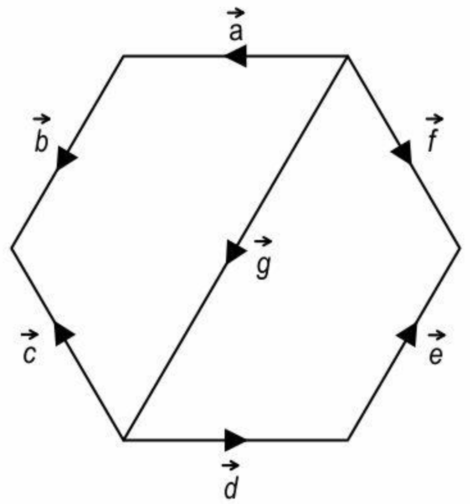
	SECTION A **(This section comprises of Multiple-choice questions (MCQ) of 1 mark each.)**	
Q.No	**Question**	**Marks**
Q.1	Shown below is the graph of a function $f(x)$ whose domain is R – (-1,1). Some portion of the graph is hidden behind the star.  Which of the following is $f(x)$? A. $\tan^{-1} x$ B. $\cot^{-1} x$ C. $\sec^{-1} x$ D. $\text{cosec}^{-1} x$	1

Q.2	P and Q are matrices such that both (P + Q) and (PQ) are defined. Which of the following is true about P and Q? A. P and Q can be any matrices but of the same order. B. P and Q must be square matrices of the same order. C. P and Q must be square matrices not necessarily of the same order. D. Order of P and Q must be of the form $m \times k$ and $k \times n$ respectively, with no condition on m and n.	1
Q.3	$\|A\| = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ Under which of the following conditions will $\|A\|$ be equal to 0? i) $a - 2p = b - 2q = c - 2r = 0$ ii) $x = y = z = 0$ iii) $a : b : c = x : y : z$ A. only ii) B. only i) and ii) C. only i) and iii) D. all - i), ii) and iii)	1
Q.4	If $abc = 2$, what is the value of the determinant below? $\begin{vmatrix} 2a & 2a & 3b+c \\ 3b & 2a+c & 3b \\ 2a+3b & c & c \end{vmatrix}$ A. -48 B. -24 C. 48 D. (cannot be found without the values of a, b and c)	1
Q.5		1

	For what value of *k* is the function *f* continuous at *x* = 0? $$f(x) = \begin{cases} \frac{\sin 2x}{8x}, & if\, x \neq 0 \\ k, & if\, x = 0 \end{cases}$$ A. 4 B. 1 C. $\frac{1}{4}$ D. $\frac{1}{8}$	
Q.6	What is the integral of the following expression? $$\frac{1}{x^2 \cos^2\left(\frac{1}{x}\right)}$$ A. $-\tan x + A$, where A is a constant. B. $-\tan\frac{1}{x} - B$, where B is a constant. C. $\frac{1}{2}\tan x + C$, where C is a constant. D. $sec\frac{1}{x}tan\frac{1}{x} - D$, where D is a constant.	1
Q.7	What is the value of the following integral? $$\int_{-1}^{1} \left|4x - x^2\right| dx$$ A. -4 B. -2 C. 0 D. 4	1
Q.8	Which of the following is CLOSEST to the area under the parabola given by $y = 4x^2$, bounded by the *x*-axis, and the lines *x* = (-1) and *x* = (-2)? A. 6 sq units B. 8 sq units C. 9 sq units D. 12 sq units	1
Q.9	Which of the following differential equation has an order of 2 and a degree of 3?	1

	A. $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^3 = 0$ B. $\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} = 0$ C. $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^2y}{dx^2} = 0$ D. $\left(\frac{d^2y}{dx^2}\right)^3 + \sin\left(\frac{d^2y}{dx^2}\right) = 0$	
Q.10	Following is a differential equation. $\frac{dy}{dx} = 4e^{3x}$ If $\mathrm{y}(0) = \frac{7}{3}$, which of the following is a particular solution of the differential equation? A. $\frac{4}{3}\, e^{3x} - \frac{4}{3}\, e^{7}$ B. $12e^{3x} - \frac{29}{3}$ C. $\frac{4}{3}\, e^{3x} + 1$ D. $4e^{3x} - \frac{5}{3}$	1
Q.11		1

Shown below is a regular hexagon whose two vertices are joined by a vector.



Which of these statement(s) is/are true?

i) $\vec{a}$ and $\vec{d}$ are equal vectors.

ii) $\vec{b}$ and $\vec{e}$ are collinear vectors.

iii) $\vec{c}$, $\vec{d}$ and $\vec{g}$ are coinitial vectors.

A. only ii)
B. only iii)
C. only i) and ii)
D. all - i), ii) and iii)

Q.12 | 1

The position vectors of the vertices P, Q and R of $\triangle PQR$ are $-\hat{i} + 2\hat{j} + 4\hat{k}$, $3\hat{i} + 6\hat{j} + 8\hat{k}$ and $4\hat{i} + \hat{j} + \hat{k}$ respectively.

Which of the following is the vector that represents the median $\overrightarrow{PS}$?

A. $\frac{7}{2}\hat{i} + \frac{9}{2}\hat{j} + \frac{9}{2}\hat{k}$
B. $2\hat{i} + 3\hat{j} + \frac{13}{3}\hat{k}$
C. $\frac{9}{2}\hat{i} + \frac{3}{2}\hat{j} + \frac{1}{2}\hat{k}$
D. $-\frac{1}{2}\hat{i} + \frac{5}{2}\hat{j} + \frac{7}{2}\hat{k}$

Q.13 | 1

<table>
<tr><td></td><td>The position vectors of the points X, Y and Z are $\hat{i} + 4\hat{j} + 3\hat{k}$, $4\hat{j} + M\hat{j} + \frac{15}{2}\hat{k}$ and $7\hat{i} - 4\hat{j} + 12\hat{k}$ respectively.

If the points X, Y and Z are collinear, which of the following could be the value of M?

A. 8
B. 4
C. 2
D. 0</td><td></td></tr>
<tr><td>Q.14</td><td>A line makes an angle of 135° with the positive direction of the x-axis, and an angle of 300° with the positive direction of the y-axis.

Which of the following could be the angle it makes with the negative direction of the z-axis?

A. 45°
B. 60°
C. (Such a line does not exist.)
D. (A unique angle made with the z-axis cannot be determined.)</td><td>1</td></tr>
<tr><td>Q.15</td><td>$\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{QR}$. The position vectors of P, Q and R are $(4\hat{i} + 7\hat{j} - \hat{k})$, $(5\hat{i} + y\hat{j} + \hat{k})$ and $(-2\hat{i} + 9\hat{j} + 4\hat{k})$ respectively.

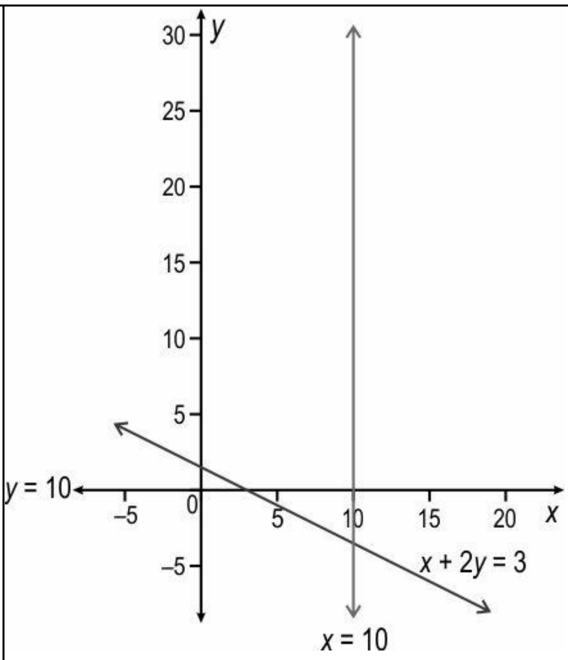
What is the value of y?

A. -9
B. -8
C. 7
D. 8</td><td>1</td></tr>
<tr><td>Q.16</td><td>A linear programming problem (LPP) along with its constraints is given below.

Minimize: $Z = 3x + 2y$

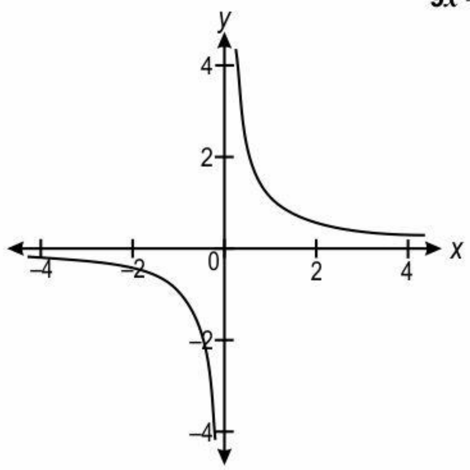
Subject to:</td><td>1</td></tr>
</table>

	$x \leq 4$ $x \geq 0,\ y \geq 0$ Which of the following is true about the above LPP? A. It has no solution. B. It has a unique solution. C. It has two distinct solutions. D. It has infinitely many solutions.	
Q.17	M and N are two events such that P(M|N) = 0.3, P(M)= 0.2 and P(N) = 0.4. Which of the following is the value of $P(M \cap N')$? A. 0.8 B. 0.12 C. 0.1 D. 0.08	1
Q.18	The constraints of a linear programming problem along with their graphs is shown below: $x + 2y \geq 3$ $x \geq 10$ $y \geq 0$	1

	 Which of the following objective functions has an optimal solution with respect to the above set of constraints? A. Minimise Z $= x + y$ B. Minimise Z $= 0.5x + y$ C. Maximise Z $= x + y$ D. Maximise Z $= 2x + y$	
Q.19		1

Shown below is the graph of the function

$f : R - \{0\} \longrightarrow R$ defined by, $f(x) = \dfrac{9 - x^2}{9x - x^3}$.



Based on the above function, two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Assertion (A): The function *f* is not onto.

Reason (R): 3 ∈ R (co-domain of *f*) has no pre-image in the domain of *f*.

A. Both A and R are true and R is the correct explanation of A.
B. Both A and R are true but R is not the correct explanation of A.
C. A is true but R is false.
D. Both A and R are false.

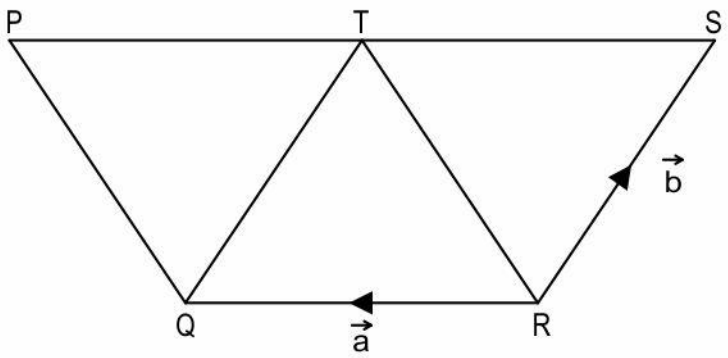
Q.20 Two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R). 1

Assertion (A): The function $f(x) = |x - 6|(\cos x)$ is differentiable in $R - \{6\}$.

Reason (R): If a function *f* is continuous at a point *c*, then it is also differentiable at that point.

A. Both (A) and (R) are true and (R) is the correct explanation for (A).
B. Both (A) and (R) are true but (R) is not the correct explanation for (A).
C. (A) is true but (R) is false.
D. (A) is false but (R) is true.

	SECTION B **(This section comprises of very short answer type-questions (VSA) of 2 marks each.)**	
Q.21	$\cot^{-1}x = \cos^{-1}(-1) - \text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ Based on the above equation, find $tan^{-1}\left(\frac{1}{x}\right)$ using the principal values of the inverse trigonometric functions. Show your work. **OR** i) Find the domain of the function below. $f(x) = \frac{1}{2}\sec^{-1}(5x - 3)$ ii) Find the range (principal value branch) of the function below. $f(x) = 3\cos^{-1}\left(\frac{1}{2x - 1}\right) - 2$ Show your work.	2
Q.22	**The matrix A =** $\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ **is the sum of a symmetric matrix B and a skew symmetric matrix C.** **Find C. Show your work.**	2
Q.23	Find $\frac{dy}{dx}$ if $y = (e^{secx} + x)^4$. Show your work.	2

Q.24	**The position vectors of the points P, Q and R are $\hat{p}$, $\hat{q}$ and $\hat{r}$ respectively. A vector $\vec{v} = k(\hat{q} + \hat{r})$ is such that $\hat{p}.\vec{v} = \hat{q}.\vec{v}$, where k is a scalar. Prove that $(\hat{p} - \hat{r}) . (\hat{p} - \hat{q}) = 0$.** **OR** In the figure below, QRST and QRTP are parallelograms.  Using the vectors shown for RQ and RS, prove that the area of QRST is equal to the area of QRTP.	2
Q.25	**The vector equation of a line AB is given by $\vec{r} = x_1(1 + \lambda)\hat{i} + y_1(1 + 2\lambda)\hat{j} + z_1(1 + 3\lambda)\hat{k}$. The coordinates of A are (x_1, y_1, z_1) and $\vec{r}$ is the position vector of a point (x, y, z) on AB.** **i) What is the equation of this line in cartesian form?** **ii) If A's coordinates are (–2, 5, –3), use the cartesian equation of the line to find the coordinates of B.** **Show your steps.**	2
	SECTION C **(This section comprises of short answer type questions (SA) of 3 marks each)**	
Q.26	Check whether the following statement is true or false. **If $u = e^{\sin^{-1}\theta}$ and $v = e^{-\cos^{-1}\theta}$, then $\frac{du}{dv}$ is a constant for any value of θ.** Show your work with valid reason. **OR**	3

	If $\frac{x^m}{y^n} = (xy)^{(m-n)}$, $(y \neq 0)$, find $\frac{dy}{dx}$. Show your work.	
Q.27	The anti-derivative of a function of the form $(3x - 1)f(x), (x \neq \frac{1}{3})$, is given by $3x^4 - \frac{13}{3}x^3 + \frac{3}{2}x^2 + C$, where C is the constant of integration. Find the value of f(6). Show your steps.	3
Q.28	Evaluate the following definite integral and show your work. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{cosec}^6 x \cot^2 x \, dx$	3
Q.29	Find the particular solution when $x = y = 0$ for the following differential equation. $dy\sqrt{1 - x^2} + (y - e^{-\sin^{-1}x})dx = 0$ Show your steps. **OR** Find the general solution of the following differential equation. $\left(x^2y + yx\sqrt{y^2 - x^2}\right)dx - x^3dy = 0$ Show your steps.	3
Q.30	Frame the below optimisation problem as a linear programming problem and determine its feasible region graphically. Bhavani Singh, a farmer, decides to raise hens and cows to make some extra money apart from his agricultural income. He wants to raise no more than 16 animals including no more than 10 hens. On an average it will cost him Rs 25 and Rs 75 per day to raise one hen and one cow respectively. He will make an average profit of Rs 12 from each hen and Rs 40 from each cow every day. He has a budget of Rs 900 per day to raise the animals. How many of each type of animals should he raise to maximise his profit?	3

Q.31 Nikhil has a bag of marbles that contains exactly 8 green marbles and 6 red marbles. He takes out three marbles successively without replacing any of the marbles. **3**

What is the probability that all three marbles taken out are green in colour? Show your steps.

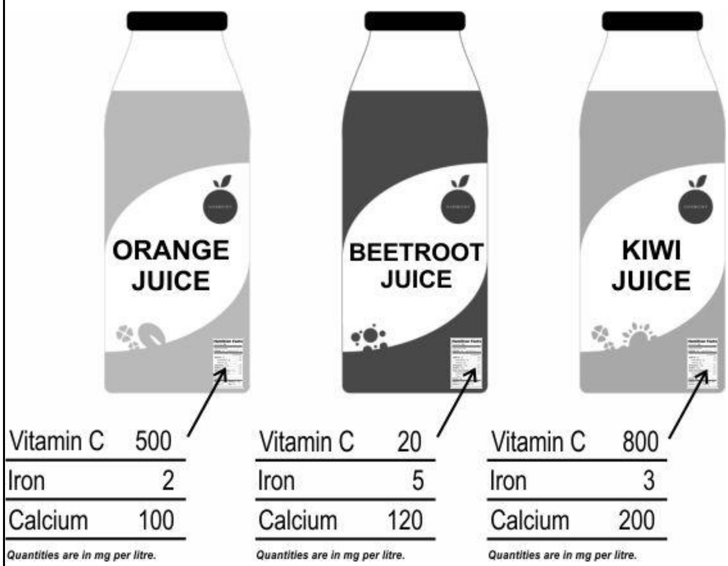
OR

In a recreational event at a school, there were 8 students, 6 parents, and 4 teachers. To play a game, two members were selected randomly one after the other.

Find the probability distribution of the number of students. Show your steps.

SECTION D
(This section comprises of long answer-type questions (LA) of 5 marks each)

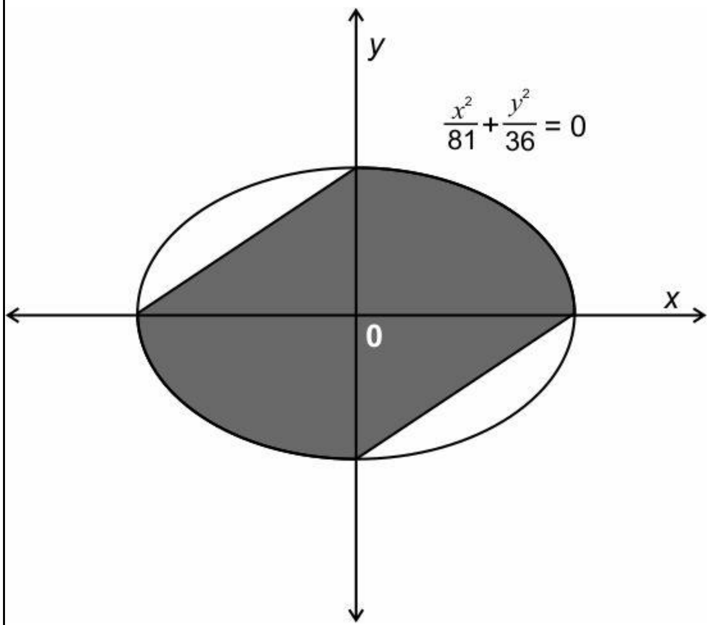
Q.32 Sravan is a nutritionist. He wants to create a mixture of orange juice, beetroot juice and kiwi juice that can provide 1860 mg of vitamin C, 22 mg of iron and 760 mg of calcium. The quantity of each nutrient per litre of juice is shown below. **5**



Using the matrix method, find how many litres of each juice Sravan should add into the mixture. Show your work.

<table>
<tr>
<td>Q.33</td>
<td>

Shown below is an ellipse whose equation is $\frac{x^2}{81} + \frac{y^2}{36} = 1.$



Find the area of the shaded region in terms of π. Show your steps.

</td>
<td>5</td>
</tr>
<tr>
<td>Q.34</td>
<td>

In a realistic city model, a metro track and a road above it are straight lines represented by $\hat{i} - 2\hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $2\hat{i} - 2\hat{j} + 3\hat{k} + \mu(-\hat{i} + \hat{j} + 2\hat{k})$ respectively.

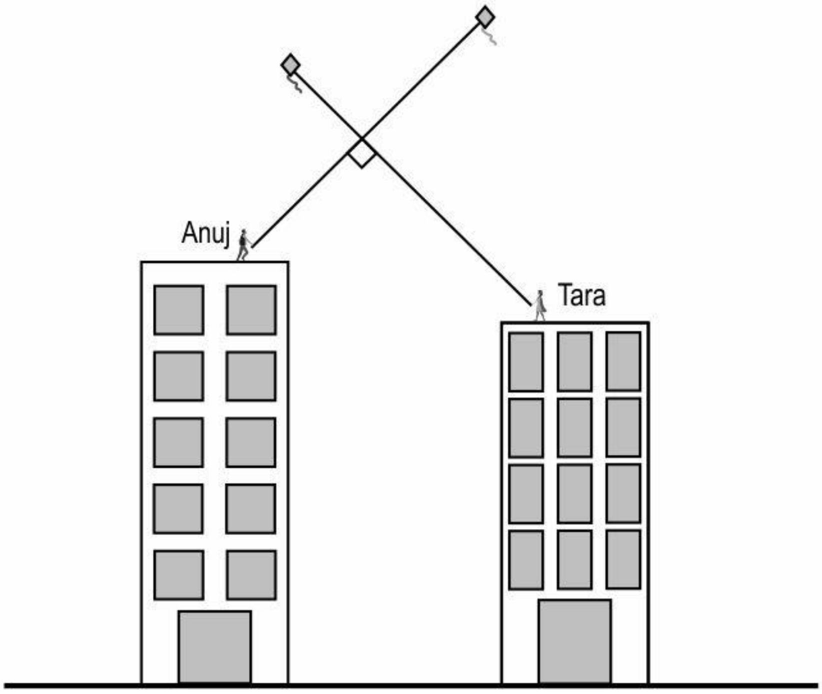
Find the shortest distance between the metro track and the road, in the model. Show your work.

</td>
<td>5</td>
</tr>
</table>

OR

Anuj and Tara are flying kites from their rooftops. Anuj's kite's string is represented by a straight line, given by $\frac{x-4}{1} = \frac{y-2}{3} = \frac{z-1}{2}$. Tara's position is at $(2\hat{i} - 2\hat{j} + \hat{k})$, and her kite's string is perpendicular to Anuj's.

Find out how far Tara is from where the kite strings intersect. Show your work.



Q.35 Integrate the given function and show your steps. 5

$\int x^7 \sin(2x^4)\, dx$

OR

Evaluate the integral and show your steps.

$\int \frac{1}{8 - x^3}\, dx$

SECTION E

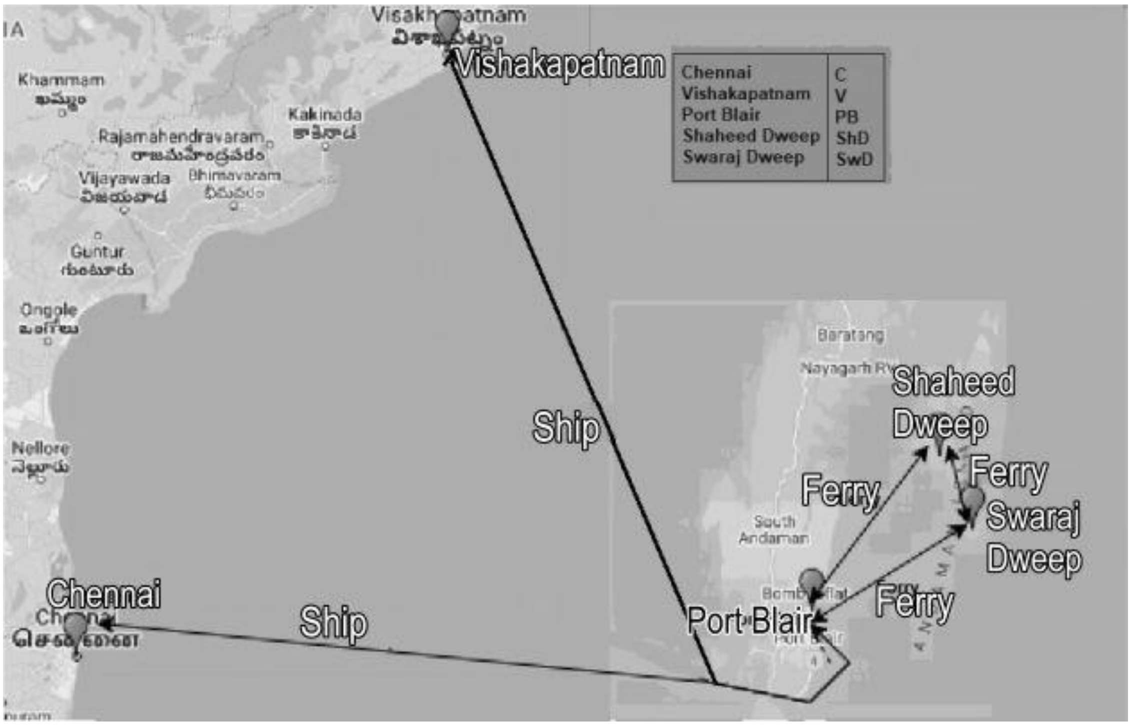
(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-questions. First two case study questions have three sub questions of marks 1, 1, 2 respectively. The third case study question has two sub questions of 2 marks each.)

Q.36 **Answer the questions based on the given information.**

Port Blair, the capital city of Andaman and Nicobar Islands is directly connected to Chennai and Vishakapatnam via ship route. The ships sail from Chennai/Vishakapatnam to Port Blair and vice versa.

Swaraj Dweep and Shaheed Dweep are two popular tourist islands in Andaman Islands. One has to take a ferry from Port Blair to reach these islands. There are ferries that sail frequently between the three islands - Port Blair (PB), Swaraj Dweep (SwD) and Shaheed Dweep (ShD).

Shown below is a schematic representation of the ship routes and ferry routes.



(Note: The image is for representation purpose only.)

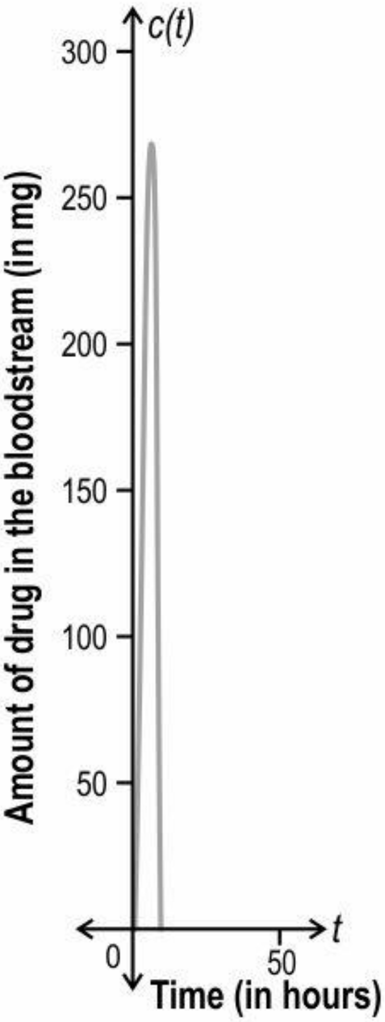
X is the set of all 5 places and Y is the set of 3 places in Andaman Islands.

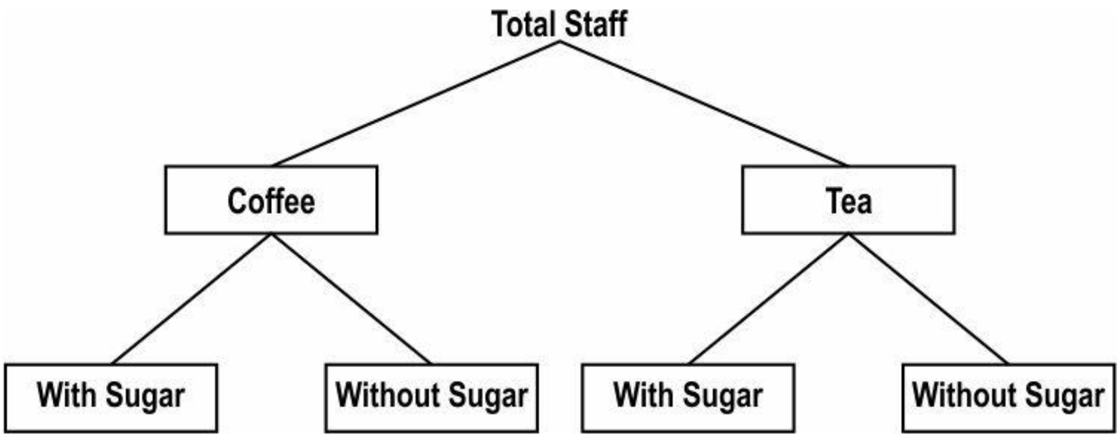
That is, X = {C, V, PB, SwD, ShD} and Y = {PB, SwD, ShD}.

A relation R defined on the set X is given by, R = {$(x_1,\ x_2)$: there is a direct ship or direct ferry from x_1 to x_2}.

A function f: Y $\rightarrow$ X is defined by, $f(PB) = V, f(SwD) = PB, f(ShD) = SwD$.

	i) List all the elements of R.	1
	ii) Is the relation R symmetric? Give a valid reason.	1
	iii) Is the relation R transitive? Give a valid reason.	2
	OR	
	Check whether the function *f* is one-one and onto. Give valid reasons.	
Q.37	**Answer the questions based on the given information**. A medicinal drug administered into a human body requires some time to produce its effect on the body. The amount (in mg) of a certain medicinal drug in the bloodstream at *t* hours after administering the drug to an individual is given by the function: $C(t) = -t^3 + 4.5t^2 + 54t, \quad 0 \leq t \leq 10$ Shown below is the graph of $C(t)$ in the interval [0, 10].	



	i) Find the rate at which the amount of drug is changing in the bloodstream at 5 hours after administering the drug. Show your work. ii) Show that the function $C(t)$ is strictly increasing in the interval (3, 4). **OR** The amount of the drug in the bloodstream at which the action of the drug is maximum is denoted by C_{max}. How long after administering the drug is C_{max} attained? Show your work and give valid reasons. iii) Find the amount of drug in the bloodstream at the time when the effect of the drug is maximum. Show your work.	1 2 1
Q.38	**Answer the questions based on the given information.** A school conducted a survey of their school staff to find their beverage preferences. Each of them picked either tea or coffee as their first preference and then with sugar or without sugar as their second preference as shown in the below tree diagram.  Some of the insights from the survey are given below. ♦ 60% percent of the staff prefer coffee. ♦ 90% of those who prefer coffee prefer it with sugar. ♦ 20% of those who prefer tea prefer it without sugar. i) What is the probability that a person selected randomly from the staff prefers a beverage with sugar? Show your steps. ii) What is the probability that a person from the staff selected at random prefers coffee given that it is without sugar? Show your steps.	 2 2

Practice Questions - Marking Scheme
Session 2022-23
Class XII
Mathematics (Code – 041)

	SECTION A - Multiple Choice Questions - 1 Mark each	
Q.No.	**Answer/Solution**	**Marks**
Q.1	C. $\sec^{-1} x$	1
Q.2	B. P and Q must be square matrices of the same order.	1
Q.3	D. all - i), ii) and iii)	1
Q.4	A. -48	1
Q.5	C. $\frac{1}{4}$	1
Q.6	B. $-tan\frac{1}{x}-$ B, where B is a constant.	1
Q.7	D. 4	1
Q.8	C. 9 sq units	1
Q.9	B. $\left(\frac{d^2y}{dx^2}\right)^3+\frac{dy}{dx}=0$	1
Q.10	C. $\frac{4}{3}e^{3x}+1$	1
Q.11	A. only ii)	1
Q.12	C. $\frac{9}{2}\hat{i}+\frac{3}{2}\hat{j}+\frac{1}{2}\hat{k}$	1
Q.13	D. 0	1
Q.14	B. 60°	1
Q.15	D. 8	1
Q.16	B. It has a unique solution.	1
Q.17	D. 0.08	1
Q.18	A. Minimise $Z = x + y$	1
Q.19	C. (A) is true but (R) is false.	1
Q.20	C. (A) is true but (R) is false.	1
	SECTION B - VSA questions of 2 marks each	
Q.21	Solves the RHS to obtain $\frac{2\pi}{3}$ as follows: $\cos^{-1}(-1)-\mathrm{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ $=\pi-\frac{\pi}{3}$ $=\frac{2\pi}{3}$ Equates the LHS to obtain $x=-\frac{1}{\sqrt{3}}$ as follows:	0.5 1

	$\cot^{-1}(x) = \frac{2\pi}{3}$ $x = \cot\left(\frac{2\pi}{3}\right)$ $x = -\left(\frac{1}{\sqrt{3}}\right)$ Finds $tan^{-1}\left(\frac{1}{x}\right)$ as $-\frac{\pi}{3}$ as follows: $\tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$ **OR** i) Finds the domain as $\left(-\infty, \frac{2}{5}\right] \cup \left[\frac{4}{5}, \infty\right)$as follows: $5x - 3 \leq -1$ or $5x - 3 \geq 1$ $\therefore x \leq \frac{2}{5}$ or $x \geq \frac{4}{5}$ ii) Finds the range as $[-2,\ 3\pi - 2]$ as follows: $0 \leq \cos^{-1}\left(\frac{1}{2x-1}\right) \leq \pi$ $3(0) - 2 \leq 3\cos^{-1}\left(\frac{1}{2x-1}\right) - 2 \leq 3(\pi) - 2$ $3(0) - 2 \leq y \leq 3(\pi) - 2$ $-2 \leq y \leq 3\pi - 2$	 0.5 1 1
Q.22	Writes the expression for C as $\frac{1}{2}(A - A')$. Finds A' as: $A' = \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}$	0.5 0.5

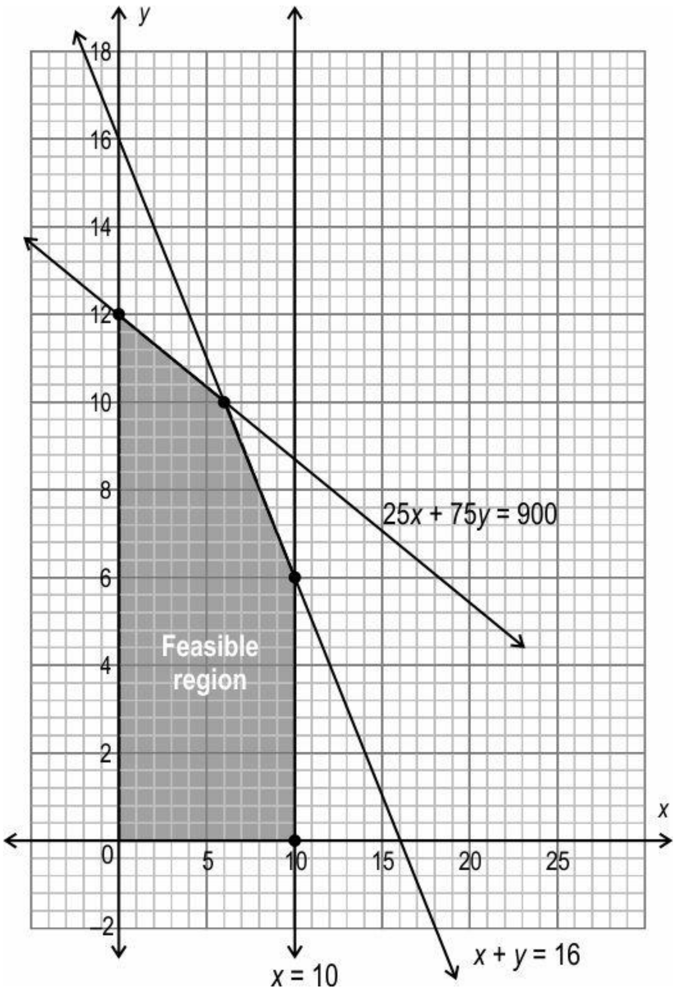
	Finds C as: $C = \frac{1}{2}\left(\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}\right)$ $= \begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & -2 \\ 2 & 2 & 0 \end{bmatrix}$	1
Q.23	Differentiates y with respect to x using chain rule as: $\frac{dy}{dx} = 4(e^{\sec x} + x)^3 \frac{d}{dx}(e^{\sec x} + x)$ Simplifies the above differential as: $\frac{dy}{dx} = 4(e^{\sec x} + x)^3(e^{\sec x} secxtanx + 1)$	1 1
Q.24	Substitutes $\vec{v} = k(\hat{q} + \hat{r})$ in $\hat{p}.\vec{v} = \hat{q}.\vec{v}$ to get: $\hat{p}.\hat{q} + \hat{p}.\hat{r} = \hat{q}.\hat{q} + \hat{q}.\hat{r}$ $\hat{p}.\hat{q} + \hat{p}.\hat{r} = 1 + \hat{q}.\hat{r}$ Rearranges the terms of the above equation as: $1 - \hat{p}.\hat{q} + \hat{q}.\hat{r} - \hat{p}.\hat{r} = 0$ Simplifies the above equation as: $\implies \quad \hat{p}(\hat{p} - \hat{q}) - \hat{r}(\hat{p} - \hat{q}) = 0$ $\implies \quad (\hat{p} - \hat{q}) \,.\, (\hat{p} - \hat{r}) = 0$ **OR** Uses the cross-product of vectors and writes: Area of QRST = $\lvert\overrightarrow{RQ} \times \overrightarrow{RS}\rvert = \lvert\vec{a} \times \vec{b}\rvert$ Uses the cross-product of vectors and writes: Area of QRTP = $\lvert\overrightarrow{RQ} \times \overrightarrow{RT}\rvert$	0.5 0.5 1 0.5 0.5

	Simplifies RHS of the above equation as: $= \lvert\overrightarrow{RQ} \times (\overrightarrow{RQ} + \overrightarrow{QT})\rvert$ $= \lvert\vec{a} \times (\vec{a} + \vec{b})\rvert$ $= \lvert(\vec{a} \times \vec{a}) + (\vec{a} \times \vec{b})\rvert$ $= \lvert 0 + \vec{a} \times \vec{b}\rvert = \lvert\vec{a} \times \vec{b}\rvert$ Concludes that the area of parallelogram QRST is equal to the area of parallelogram QRTP.	1
Q.25	i) Expands the vector form to get the following: $x\hat{i} + y\hat{j} + z\hat{k} = (x_1 + \lambda x_1)\hat{i} + (y_1 + 2\lambda y_1)\hat{j} + (z_1 + 3\lambda z_1)\hat{k}$ Eliminates λ by equating the like coefficients of the position vectors of the *x*, *y* and *z* axes to get the cartesian equation as follows: $\frac{x - x_1}{x_1} = \frac{y - y_1}{2y_1} = \frac{z - z_1}{3z_1}$ ii) Assumes the coordinates of B as (x_2, y_2, z_2) and compares the cartesian form of the equation from step 2 with the regular form of the cartesian equation to find: $x_2 = 2x_1$, $y_2 = 3y_1$ and $z_2 = 4z_1$ Substitutes values x_1 = (-2), y_1 = 5 and z_1 = (-3) in the equations from step 3 to get coordinates of B as (-4, 15, -12).	0.5 0.5 0.5 0.5

	SECTION C - Short Answer Questions of 3 Marks each	
Q.26	Finds $\frac{du}{d\theta}$ as: $\frac{du}{d\theta} = e^{\sin^{-1}\theta} \times \frac{1}{\sqrt{1-\theta^2}}$ Finds $\frac{dv}{d\theta}$ as: $\frac{dv}{d\theta} = e^{-\cos^{-1}\theta} \times \frac{1}{\sqrt{1-\theta^2}}$ Uses parametric differentiation and finds $\frac{du}{dv}$ as: $\frac{du}{dv} = \frac{e^{\sin^{-1}\theta}}{e^{-\cos^{-1}\theta}} = e^{\sin^{-1}\theta + \cos^{-1}\theta} = e^{\frac{\pi}{2}}$ Concludes that the given statement is true as $e^{\frac{\pi}{2}}$ is a constant. **OR** Rewrites the given equation by taking logarithm on both sides as: $\mathrm{m}(\log \mathrm{x}) - \mathrm{n}(\log \mathrm{y}) = (\mathrm{m} - \mathrm{n})(\log \mathrm{x} + \log \mathrm{y})$ Differentiates the above equation as: $\frac{m}{x}\, dx - \frac{n}{y}\, dy = (m-n)\left(\frac{1}{x}\, dx + \frac{1}{y}\, dy\right)$ Rearranges the above equation to get: $\frac{n}{x}\, dx = \frac{m}{y}\, dy$ Finds $\frac{dy}{dx}$ to be $\frac{ny}{mx}$.	1 1 0.5 0.5 1 1 0.5 0.5
Q.27	Interprets the question statement and writes it as: $(3x-1)\,f(x) = \frac{d}{dx}(3x^4 - \frac{13}{3}x^3 + \frac{3}{2}x^2 + C)$ Finds the derivative in the above step as: $(3\mathrm{x} - 1)\mathrm{f}(\mathrm{x}) = 12x^3 - 13x^2 + 3\mathrm{x}$	0.5 1

	Factorises the above cubic polynomial as: $(3x - 1)f(x) = (3x - 1)(4x^2 - 3x)$ and determines the value of $f(x)$ as $(4x^2 - 3x)$. Substitutes $x = 6$ in $f(x)$ and evaluates $f(6)$ as 126.	1 0.5
Q.28	Rewrites the integral using the identity $\text{cosec}^2 x = 1 + \cot^2 x$ as: $\int \cot^2 x\,(1 + \cot^2 x)^2\, \text{cosec}^2 x\, dx$ Substitutes $\cot x = u$ and hence $\text{cosec}^2 x\, dx = - du$ in the above step and rewrites the integral as: $-\int u^2 (1 + u^2)^2\, du$ Integrates the above expression as: $-\left[\frac{u^3}{3} + \frac{u^7}{7} + \frac{2u^5}{5}\right]$ Substitutes $\cot x$ in place of u in the above expression to get: $-\left[\frac{\cot^3 x}{3} + \frac{\cot^7 x}{7} + \frac{2\cot^5 x}{5}\right]$ Substitutes the limits in the above expression to get $\frac{92}{105}$.	0.5 0.5 0.5 0.5 1
Q.29	Rearranges the given differential equation as: $\frac{dy}{dx} + \frac{y}{\sqrt{1-x^2}} = \frac{e^{-\sin^{-1} x}}{\sqrt{1-x^2}}$ Finds the integrating factor as follows as the equation obtained in the above step is of the form $\frac{dy}{dx} + y\,P(x) = Q(x)$. Integrating factor = $e^{\int P(x)\, dx}$ $= e^{\int \frac{1}{\sqrt{1-x^2}} dx} = e^{\sin^{-1} x}$	0.5 0.5

	Finds the solution as: $ye^{\int P(x)\,dx} = \int Q(x) \times e^{\int P(x)\,dx} dx + C$ $\Rightarrow ye^{\sin^{-1}x} = \int \frac{e^{-\sin^{-1}x}}{\sqrt{1-x^2}} \times e^{-\sin^{-1}x} dx + C$ $\Rightarrow ye^{\sin^{-1}x} = \sin^{-1}x + C$ Where C is the constant of integration.	1.5
	Substitutes $x = y = 0$ in the above equation and finds the value of C as 0. Writes the particular solution as: $ye^{\sin^{-1}x} = \sin^{-1}x$	0.5
	OR	
	Rearranges the given equation in terms of $\frac{y}{x}$ as: $\frac{dy}{dx} = \frac{y}{x} + \frac{y}{x^2}\sqrt{y^2-x^2}$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{y}{x}\sqrt{\frac{y^2}{x^2} - 1}$	0.5
	Considers $y = vx$ and finds $\frac{dy}{dx}$ in terms of v as: $\frac{dy}{dx} = v + x\frac{dy}{dx}$	0.5
	Equates the RHS obtained in steps 1 and 2 to get: $x\frac{dv}{dx} = v\sqrt{v^2-1}$	0.5
	Rearranges the terms using the variable separable method as: $\frac{dv}{v\sqrt{v^2-1}} = \frac{dx}{x}$	0.5
	Integrates on both sides to find the general solution as: $\sec^{-1}v = \log\|x\| + C$ or $\sec^{-1}\frac{y}{x} = \log\|x\| + C$, where C is the constant of integration.	1

Q.30	Takes the number of hens and cows to be x and y respectively and formulates the linear programming problem as follows: Maximise $Z = 12x + 40y$ subject to constraints, $25x + 75y \leq 900$ $x + y \leq 16$ $x \leq 10$ $x \geq 0$ $y \geq 0$ Graphs the constraints and marks the feasible region as:  	1.5 1.5

Q.31	Takes E, F and G to be the events of taking out green marbles in the first, second and third draws respectively, and writes: $P(E) = P(green\ marble\ in\ first\ draw) = \frac{8}{14}$	0.5
	Finds the probability that the second marble taken out is green provided first is also green as: $P(F\|E) = P(green\ marble\ in\ the\ second\ draw) = \frac{7}{13}$	0.5
	Finds the probability that the third marble taken out is green provided first two are also green as: $P(G\|EF) = P(green\ marble\ in\ third\ draw) = \frac{6}{12}$	0.5
	Finds the probability that all three marbles taken out are green in colour as: $P(E) \times P(F\|E) \times P(G\|EF)$ $= \frac{8}{14} \times \frac{5}{13} \times \frac{6}{12}$ $= \frac{10}{91}$	1.5
	OR	
	Assumes the number of students as a random variable X and writes that it can take values of 0, 1 and 2.	
	Finds P(X = 0) as: P(non-student and non-student) $= \frac{10}{18} \times \frac{9}{17}$ $= \frac{90}{306}$	0.5
	Finds P(X = 1) as: P(student and non-student) or P(non-student and student) $= \frac{8}{18} \times \frac{10}{17} \times \frac{10}{18} \times \frac{8}{17}$	1.5

	$= \frac{160}{306}$ Finds $P(X = 2)$ as: P(student and student) $= \frac{8}{18} \times \frac{7}{17}$ $= \frac{56}{306}$ Writes the required probability distribution as: <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{90}{306}$</td><td>$\frac{160}{306}$</td><td>$\frac{56}{306}$</td></tr></table>	0.5 0.5
	SECTION D - Long answer type questions (LA) of 5 marks each	
Q.32	Assumes the number of litres of orange juice, beetroot juice and kiwi juice as x, y and z, respectively to frame equations as follows: $500x + 20y + 800z = 1860$ $2x + 5y + 3z = 22$ $100x + 120y + 200z = 760$ Writes the above system of equations in the matrix form using AX = B as $\begin{bmatrix} 500 & 20 & 800 \\ 2 & 5 & 3 \\ 100 & 120 & 200 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1860 \\ 22 \\ 760 \end{bmatrix}$ Finds $\lvert A \rvert = 110000 \neq 0$ and hence writes that A is non-singular and has a unique solution. Finds *adj* A as: $\begin{bmatrix} 640 & 92000 & -3940 \\ -100 & 20000 & 100 \\ -260 & -58000 & 2460 \end{bmatrix}$	0.5 0.5 0.5 1

	Finds A^{-1} using $\|A\|$ and *adj* A as: $A^{-1} = \frac{1}{\|A\|} \times adj\,A$ $= \frac{1}{5500}\begin{bmatrix} 32 & 4600 & -197 \\ -5 & 1000 & 5 \\ -13 & -2900 & 123 \end{bmatrix}$	1
	Writes that $X = A^{-1}B$ and finds X as $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$	1
	Concludes that 2 litres of orange juice, 3 litres of beetroot juice and 1 litre of kiwi juice should go into the mixture.	0.5
Q.33	Writes the endpoints of the ellipse as $(-9, 0), (9, 0),\ (0, 6)$ and $(0, -6)$ respectively.	1
	Expresses *y* in terms of *x* as: $y = \pm\frac{6}{9}\sqrt{9^2 - x^2}$	
	Sets up the equation for the area of the shaded region as: $\text{Shaded Area} = \left\| \int_{-9}^{0} \frac{6}{9}\sqrt{9^2 - x^2}\,dx \right\| + \text{Area of 2 triangles} + \int_{0}^{9} \frac{6}{9}\sqrt{9^2 - x^2}\,dx$ $= 2\int_{0}^{9} \frac{6}{9}\sqrt{9^2 - x^2}\,dx + \text{Area of 2 triangles}$	1
	Evaluates the 1st part of the above equation as: $2\int_{0}^{9} \frac{6}{9}\sqrt{9^2 - x^2}\,dx$ $= 2 \times \frac{6}{9}\left[\frac{x}{2}\sqrt{81 - x^2} + \frac{81}{9}\sin^{-1}\frac{x}{9}\right]_0^9$	1

	Applies the upper and the lower limit and finds the value of the integral as: $= 2 \times \frac{6}{9}\left\{\left[\frac{9}{2}\sqrt{81-81} + \frac{81}{2}\sin^{-1}\frac{9}{9}\right] - \left[\frac{0}{2}\sqrt{81-0} + \frac{81}{2}\sin^{-1}\frac{0}{9}\right]\right\}$ $= 2 \times \frac{6}{9} \times \frac{81}{2} \times \frac{\pi}{2} = 27\pi$ Evaluates the 2[nd] part of the equation from step 2 as: Area of 2 triangles $= 2 \times \frac{1}{2} \times 9 \times 6 = 54$ sq units Adds the area obtained in step 3 and 4 to find the area of the shaded region in terms of π as: $(27\pi + 54)$ sq units or $27(\pi + 2)$ sq units.	1 0.5 0.5
Q.34	Compares $\vec{r_1} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r_2} = \vec{a_2} + \mu\vec{b_2}$ with the given equations to get $\vec{a_1} = \hat{i} - 2\hat{j} + 2\hat{k}$, $\vec{a_2} = 2\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b_1} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b_2} = -\hat{i} + \hat{j} + 2\hat{k}$. Notes that the lines are skewed and writes the formula to find the shortest distance between the lines (d) as follows: $d = \left\lvert \frac{(\vec{b_1} \times \vec{b_2}) . (\vec{a_2} - \vec{a_1})}{\lvert\vec{b_1} \times \vec{b_2}\rvert} \right\rvert$ Solves $(\vec{b_1} \times \vec{b_2})$ as $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ -1 & 1 & 2 \end{vmatrix} = 7\hat{i} + \hat{j} + 3\hat{k}$. Finds $\lvert\vec{b_1} \times \vec{b_2}\rvert = \sqrt{49 + 1 + 9} = \sqrt{59}$, and $(\vec{a_2} - \vec{a_1}) = \hat{i} + \hat{k}$. Finds $(\vec{b_1} \times \vec{b_2}) . (\vec{a_2} - \vec{a_1})$ as $7 + 3 = 10$. Substitutes values from above steps to find distance as $\frac{10}{\sqrt{59}}$ units. **OR** **Writes that $\frac{x-4}{1} = \frac{y-2}{3} = \frac{z-1}{2} = \lambda$.** Assumes P ($x$, y, z) to be the point of intersection of the two lines. Finds $x = \lambda + 4, y = 3\lambda + 2$ and $z = 2\lambda + 1$. Takes Tara's position as T(2, -2, 1) to find the direction ratios of TP as $(\lambda + 2), (3\lambda + 4)$ and (2λ).	1 1 1 1.5 0.5 0.5 0.5 1

	Notes that the dot product of the direction ratios of the given line and TP will be 0, since they are perpendicular, and cos 90° = 0. Writes that $1(\lambda + 2) + 3(3\lambda + 4) + 2(2\lambda) = 0$. Solves the above equation to find $\lambda = (-1)$. Substitutes the value of λ to find $x = 3, y = 7$ and $z = 2$. Finds the length of TP as √83 units, using the following formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$	1.5 0.5 1
Q.35	Rewrites the given integral as: $\int x^4 x^3 \sin(2x^4)\, dx$ Substitutes $x^4 = u$ and hence $x^3\, dx = \frac{1}{4} du$ in the above integral to get: $\frac{1}{4} \int u \sin(2u)\, du$ Uses integration by parts to integrate the above expression as: $\frac{1}{4} \left[u \int \sin(2u)\, du - \int (\int \sin(2u) du)\, du\right]$ Integrates the above expression to get: $-\frac{1}{8} u \cos(2u) + \frac{1}{16} \sin(2u) + C$ Substitutes x^4 in place of u in the above expression to get: $-\frac{1}{8} x^4 \cos(2x^4) + \frac{1}{16} \sin(2x^4) + C$ **OR** Expands the denominator using the identity $(a^3 - b^3)$ as: $\int \frac{1}{(2-x)(x^2+2x+4)}\, dx$ Rewrites the integral as a sum of two integrals using partial fractions as: $\frac{1}{12} \int \frac{1}{(2-x)}\, dx + \frac{1}{12} \int \frac{x+4}{x^2+2x+4}\, dx$	0.5 1 1 2 0.5 0.5 1

	Solves the first integral as: $-\frac{1}{12}\log\lvert 2-x\rvert$ Rewrites the second integral as: $\frac{1}{24}\int\frac{2x+2}{x^2+2x+4}\,dx + \frac{1}{4}\int\frac{1}{(x+1)^2+(\sqrt{3})^2}\,dx$ Solves the above integral as: $\frac{1}{24}\log\lvert x^2+2x+4\rvert + \frac{1}{4\sqrt{3}}\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$ Concludes the final answer as: $-\frac{1}{12}\log\lvert 2-x\rvert + \frac{1}{24}\log\lvert x^2+2x+4\rvert + \frac{1}{4\sqrt{3}}\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right) + C$	0.5 1 1.5 0.5
	SECTION E - Case Studies/Passage based questions of 4 Marks each	
Q.36i)	Lists all the elements of R as: R = {(C, PB), (PB, C), (V, PB), (PB, V), (PB, SwD), (SwD, PB), (PB, ShD), (ShD, PB), (SwD, ShD), (ShD, SwD)}	1
Q.36ii)	Writes that the relation R is symmetric. Gives a reason. For example, for every $(x_1, x_2) \in R$, $(x_2, x_1) \in R$ as every direct ship/direct ferry runs in both the directions.	0.5 0.5
Q.36iii)	Writes that R is not transitive. Gives a reason. For example, (C, PB) $\in$ R as there is a direct ship from Chennai to Port Blair. (PB, SwD) $\in$ R as there is a direct ferry from Port Blair to Swaraj Dweep. But (C, SwD) $\notin$ R as there is no direct ship/ferry from Chennai to Swaraj Dweep. **OR**	0.5 1.5

	Writes that the function f is one-one. Gives a reason. For example, no two elements of set Y are mapped to a common element in set X. Writes that the function f is not onto. Gives a reason. For example, $C \in X$ (co-domain of f) but it has no pre-image in Y.	0.5 0.5 0.5 0.5
Q.37i)	Finds the rate at which the amount of drug is changing in the blood stream 5 hours after the drug has been administered as: $C'(\mathrm{t}) = -3t^2 + 9\mathrm{t} + 54$ $\Rightarrow \mathrm{C}'(5) = 24$ mg/hr	1
Q.37ii)	Equates the derivative $C'(\mathrm{t})$ to 0 and factorises $C'(\mathrm{t})$as $3(3 + \mathrm{t})(6 - \mathrm{t})$. Writes that for $\mathrm{t} \in (3, 4)$, $3 > 0$, $(3 + \mathrm{t}) > 0$ and $(6 - \mathrm{t}) > 0$ Therefore, $\mathrm{C}'(\mathrm{t}) > 0$. Concludes that $C(t)$ is strictly increasing in the interval (3, 4). **OR** Equates the derivative $C'(\mathrm{t}) = -3t^2 + 9\mathrm{t} + 54$ to 0 and finds the critical points as $t = 6$ hours and $t = (-3)$ hours. Differentiate $C'(\mathrm{t})$ to get $\mathrm{C}''(\mathrm{t})$ as: $\mathrm{C}''(\mathrm{t}) = -6\mathrm{t} + 9$ Finds $\mathrm{C}''(6)$ as (-27) and writes that $\mathrm{C}(\mathrm{t})$ attains its maximum at $\mathrm{t} = 6$ hours, as $\mathrm{C}''(6) = (-27) < 0$. Concludes that 6 hours after the drug is administered, C_{max} is attained.	0.5 1.5 0.5 0.5 0.5 0.5
Q.37iii)	Finds the value of $\mathrm{C}(\mathrm{t})$ at $\mathrm{t} = 6$ hours as: $\mathrm{C}(6) = -(6)^3 + 4.5(6)^2 + 54(6)$ $\Rightarrow \mathrm{C}(6) = 270$	1

	Writes the amount of drug in the bloodstream when the effect of the drug is maximum as 270 mg.	
Q.38i)	Takes P(S), P(C) and P(T) as the probabilities that a person selected randomly from the staff prefers sugar, coffee and tea respectively. Finds $\mathrm{P(T)} = \mathrm{P(C')} = 1 - 0.6 = 0.4$. Finds $\mathrm{P(S \vert T)} = 1 - 0.2 = 0.8$. Uses theorem on total probability and finds the probability that a randomly selected staff prefers a beverage with sugar as: $\mathrm{P(S)} = \mathrm{P(C)} \times \mathrm{P(S \vert C)} + \mathrm{P(T)} \times \mathrm{P(S \vert T)}$ $= 0.6 \times 0.9 + 0.4 \times 0.8 = 0.86$ or $\frac{86}{100}$ *or* $\frac{43}{50}$	1 1
Q.38ii)	Uses the sum of probabilities = 1 and finds the following probabilities: ♦ $\mathrm{P(without\ sugar \vert coffee)} = 1 - 0.9 = 0.1$ ♦ $\mathrm{P(tea)} = 1 - 0.6 = 0.4$ Uses Bayes' theorem to find the probability that a staff selected at random prefers coffee given that it is without sugar, $\mathrm{P(coffee \vert without\ sugar)}$ as: $\frac{\mathrm{P(coffee)} \times \mathrm{P(without\ sugar \vert coffee)}}{\mathrm{P(coffee)} \times \mathrm{P(without\ sugar \vert coffee)} + \mathrm{P(tea)} \times \mathrm{P(without\ sugar \vert tea)}}$ $= \frac{0.6 \times 0.1}{0.6 \times 0.1 + 0.4 \times 0.2}$ (Award 0.5 marks if only the formula for Bayes′ theorem is written correctly.) Simplifies the above expression and finds the required probability as $\frac{6}{14}$ *or* $\frac{3}{7}$.	0.5 1 0.5