Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

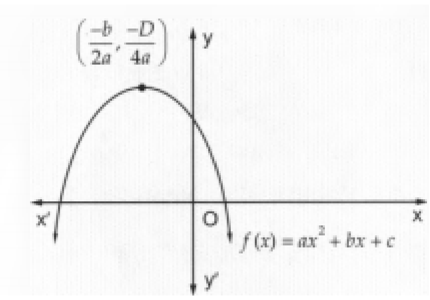
Section A

1. The least positive integer divisible by 20 and 24 is **[1]**

a) 480 b) 240

c) 360 d) 120

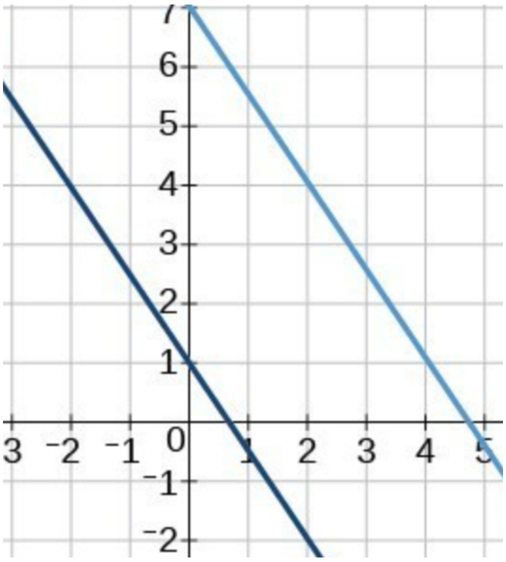
2. If the diagram in Fig. shows the graph of the polynomial f(x) = ax^2 + bx + c, then **[1]**



a) a < 0, b < 0 and c < 0 b) a < 0, b > 0 and c > 0

c) a < 0, b < 0 and c > 0 d) a < 0, b > 0 and c < 0

3. The number of solutions for two linear equations representing parallel lines is/are **[1]**



a) 2

b) ∞

c) 1

d) 0

4. If **p** is a root of the quadratic equation x^2 - (p + q) x + k = 0, then the value of **k** is **[1]**

a) p + q

b) p

c) pq

d) q

5. In an AP, if d = -4, n = 7 and a_n = 4, then the value of a is **[1]**

a) 20

b) 6

c) 7

d) 28

6. The distance between the points (6, 2) and (-6, 2) is: **[1]**

a) 12 units

b) $6\sqrt{2}$ units

c) $2\sqrt{6}$ units

d) 6 units

7. The coordinates of the point A, where AB is the diameter of the circle whose centre is (3, -2) and B(7, 4) is: **[1]**

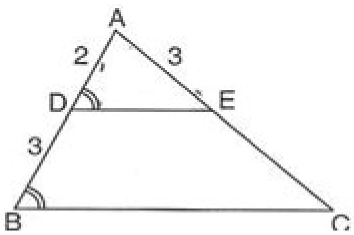
a) (1, 8)

b) (-1, -8)

c) (-1, 8)

d) (1, -8)

8. In the given figure if $\angle$ADE = $\angle$ABC, then CE is equal to **[1]**



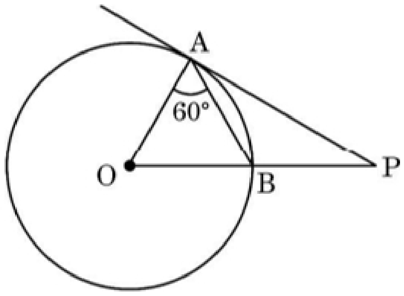
a) 4.5

b) 3

c) 2

d) 5

9. In the given figure, O is the centre of the circle and PA is a tangent to the circle. If $\angle$OAB = 60^o, then $\angle$OPA is equal to: **[1]**



a) 30^o

b) 20^o

c) 15^o

d) 60^o

10. A tangent to a circle is a line that touches the circle at: **[1]**

a) three points

b) one point only

c) infinite number of points

d) two points

11. If 2x = sec A and $\frac{2}{x}$ = tan A then $2\left(x^2 - \frac{1}{x^2}\right)$ = ? **[1]**

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{1}{16}$

d) $\frac{1}{8}$

12. If cos A = $\frac{\sqrt{3}}{2}$, $0^o < A < 90^o$, then A is equal to **[1]**

a) 30^o

b) 1

c) $\frac{\sqrt{3}}{2}$

d) 60^o

13. At some time of the day, the height and length of the shadow of a man are equal. The sun's altitude is: **[1]**

a) 45^o

b) 30^o

c) 90^o

d) 60^o

14. A horse is grazing in a field. It is tied to a pole with a rope of length 6 m. The horse moves from point A to point B making an arch with an angle of 70°. Find the area of the sector grazed by the horse. **[1]**

a) 22.99 m

b) 20.99 m

c) 21.99 m

d) 21 m

15. A car has two wipers which do not overlap. Each wiper has a blade of length 42 cm sweeping through an angle of 120^o. Find the total area cleaned at each sweep of the blades. **[1]**

a) 5544 cm^2

b) 3696 cm^2

c) 4224 cm^2

d) 1848 cm^2

16. An event is unlikely to happen. Its probability is closest to **[1]**

a) 0.00001

b) 0.0001

c) 0.1

d) 1

17. One card is drawn at random from a well-shuffled deck of 52 cards. What is the probability of getting a 6? **[1]**

a) $\frac{1}{13}$

b) $\frac{3}{26}$

c) $\frac{1}{52}$

d) $\frac{4}{52}$

18. If every term of the statistical data consisting of n terms is decreased by 2, then the mean of the data: **[1]**

a) decreases by 1

b) remains unchanged

c) decreases by 2

d) decreases by 2n

19. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + hl)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Common difference of the AP -5, -1, 3, 7, ... is 4. **[1]**

Reason (R): Common difference of the AP a, a + d, a + 2d, ... is given by d = 2nd term - 1st term.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

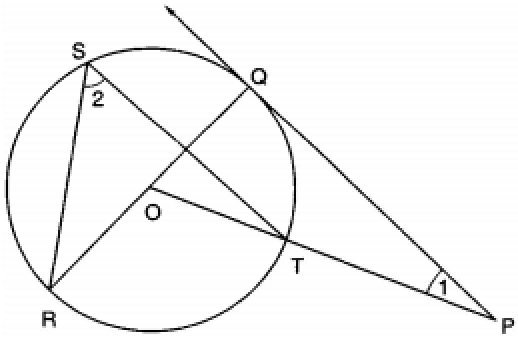
d) A is false but R is true.

Section B

21. Prove that 5 + $2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. **[2]**

22. If D and E are points on sides AB and AC respectively of a ΔABC such that $DE||BC$ and BD = CE. Prove that Δ ABC is isosceles. **[2]**

23. In figure, PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and QOR is a diameter. If $\angle$POR = 130° and S is a point on the circle, find $\angle 1 + \angle 2$. **[2]**



24. Prove that: $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$ **[2]**

OR

If m sinA + n cosA = p and m cosA - n sinA = q, prove that $m^2 + n^2 = p^2 + q^2$.

25. In a circle of radius 21 cm, an arc subtends an angle of 60^o at the centre. Find the area of the sector formed by the arc. Also, find the length of the arc. **[2]**

OR

If a chord of a circle of radius 10 cm subtends an angle of 60^o at the centre of the circle, find the area of the corresponding minor segment of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73)

Section C

26. If two positive integers p and q are written as p = a^2b^3 and q = a^3 b, a and b are a prime number then. Verify.LCM $\times$(p.q.) $\times$ HCF (p.q.) = pq **[3]**

27. If α, β are the zeroes of the $x^2 + 7x + 7$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$. **[3]**

28. Find the sum of all the natural numbers less than 100 which are divisible by 6. **[3]**

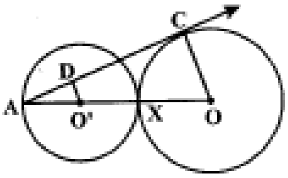
OR

The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is (2m - 1):(2n -1).

29. In two concentric circles, a chord of length 8 cm of the larger circle touches the smaller circle. If the radius of the larger circle is 5 cm then find the radius of the smaller circle. **[3]**

OR

Equal circles with centres O and O' touch each other at X. OO' produced to meet a circle with centre O', at A. AC is a tangent to the circle whose centre is O. O' D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.



30. If $\tan\theta + \frac{1}{\tan\theta} = 2$, find the value of $\tan^2\theta + \frac{1}{\tan^2\theta}$ **[3]**

31. The weights (in kg) of 50 wild animals of a National Park were recorded and the following data was obtained: **[3]**

Weight (in kg)	Number of animals
100 - 110	4
110 - 120	12
120 - 130	23
130 - 140	8
140 - 150	3

Find the mean weight (in kg) of animals, using assumed mean method.

Section D

32. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers. **[5]**

OR

A train travels at a certain average speed for a distance 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than the original speed. If it takes 3 hours to complete total journey, what is its original average speed?

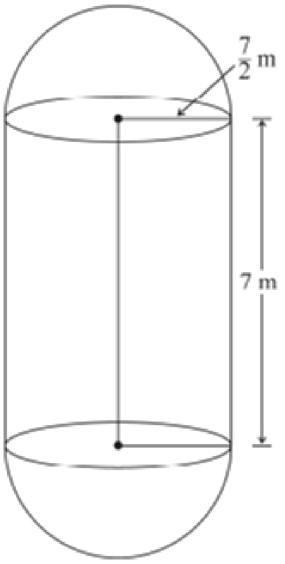
33. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval. **[5]**

34. A spherical glass vessel has a cylindrical neck 8 cm long and 1 cm in radius. The radius of the spherical part is 9 cm. Find the amount of water (in litres) it can hold, when filled completely. **[5]**

OR

The boilers are used in thermal power plants to store water and then used to produce steam. One such boiler consists of a cylindrical part in middle and two hemispherical parts at its both ends. Length of the cylindrical part is 7m and radius of cylindrical part is $\frac{7}{2}$ m.

Find the total surface area and the volume of the boiler. Also, find the ratio of the volume of cylindrical part to the volume of one hemispherical part.



35. Find the mode, median and mean for the following data: **[5]**

Marks Obtained	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of students	7	31	33	17	11	1

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A coaching institute of Mathematics conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, there are 20 poor and 5 rich children, whereas in batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch I is ₹ 9000 and from batch II is ₹ 26,000. Assume that each poor child pays ₹ x per month and each rich child pays ₹ y per month.

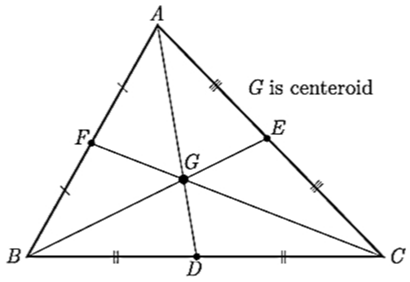
i. Represent the information given above in terms x and y. (1)
ii. Find the monthly fee paid by a poor child. (1)
iii. Find the difference in the monthly fee paid by a poor child and a rich child. (2)

OR

If there are 10 poor and 20 rich children in batch II, what is the total monthly collection of fees from batch II? (2)

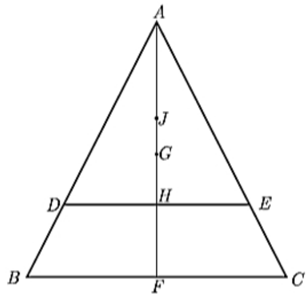
37. **Read the following text carefully and answer the questions that follow:** **[4]**

The centroid is the centre point of the object. It is also defined as the point of intersection of all the three medians. The median is a line that joins the midpoint of a side and the opposite vertex of the triangle. The centroid of the triangle separates the median in the ratio of 2 : 1. It can be found by taking the average of x-coordinate points and y-coordinate points of all the vertices of the triangle. See the figure given below



Here D, E and F are mid points of sides BC, AC and AB in same order. G is centroid, the centroid divides the median in the ratio 2 : 1 with the larger part towards the vertex. Thus AG : GD = 2 : 1

On the basis of above information read the question below. If G is Centroid of $\triangle$ABC with height h and J is Centroid of $\triangle$ADE. Line DE parallel to BC, cuts the $\triangle$ABC at a height $\frac{h}{4}$ from BC. HF = $\frac{h}{4}$



i. What is the length of AH? (1)

ii. What is the distance of point A from point G? (1)

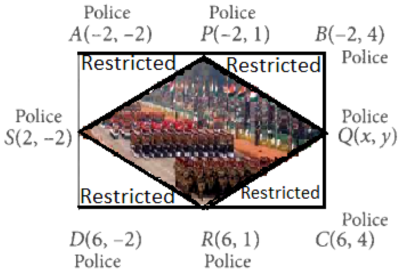
iii. What is the distance of point A from point J? (2)

OR

What is the distance GJ? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

In order to facilitate smooth passage of the parade, movement of traffic on certain roads leading to the route of the Parade and Tableaux ah rays restricted. To avoid traffic on the road Delhi Police decided to construct a rectangular route plan, as shown in the figure.



i. If Q is the mid point of BC, then what are the coordinates of Q? (1)

ii. What is the length of the sides of quadrilateral PQRS? (2)

iii. What is the length of route PQRS? (2)

OR

What is the length of route ABCD? (2)

Solution

Section A

1.

(d) 120

Explanation: Least positive integer divisible by 20 and 24 is
LCM of (20, 24).

20 = $2^2 \times 5$

24 = $2^3 \times 3$

$\therefore$ LCM (20, 24) = $2^3 \times 3 \times 5$ = 120

Thus 120 is divisible by 20 and 24.

2.

(c) a < 0, b < 0 and c > 0

Explanation: Clearly, $f(x) = ax^2 + bx + c$ represent a parabola opening downwards.

Clearly a < 0

Let, y = ax^2 + bx + c cuts y-axis at P which lies on OY.

Putting x = 0 in $y = ax^2 + bx + c$, we get y = c. So the coordinates of P are $(0, c)$.

Clearly, P lies on OY. Therefore c > 0

The vertex $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$ of the parabola is in the second quadrant.

Therefore, $\frac{-b}{2a} < 0$, b < 0

Therefore a < 0, b < 0 and c > 0.

3.

(d) 0

Explanation: The number of solutions of two linear equations representing parallel lines is 0 because two linear equations representing parallel lines has no solution and they are inconsistent.

4.

(c) pq

Explanation: Let the roots of given quadratic equation be α and β.

On comparing equation x^2 - (p - q) x + k = 0

with ax^2 + bx + c = 0, we have

a = 1, b = -(p + q), c = k

We know that

$\Rightarrow \alpha + \beta = \frac{-b}{a}$

Put the value a and b

$\Rightarrow \alpha + \beta = \frac{p+q}{1}$

$\Rightarrow \alpha + \beta$ = p + q ...(i)

Given α = p

Put the value of α in equation (i),

$\Rightarrow$ p + β = p + q

$\Rightarrow$ β = q

But we know that

$\alpha \cdot \beta = \frac{c}{a}$

Put the values

$\text{p.q.} = \frac{k}{1}$

Then, k = pq.

5.

(d) 28

Explanation: Given: d = -4, n = 7 and a_n = 4

$\therefore a_n = a + (n - 1)d$

$\Rightarrow 4 = a + (7 - 1) \times (-4)$

$\Rightarrow 4 = a + 6 \times -4$

$\Rightarrow 4 = a - 24$

$\Rightarrow a = 28$

6. **(a)** 12 units

Explanation: 12 units

7.

(b) (-1, -8)

Explanation: (-1, -8)

8. **(a)** 4.5

Explanation: $\angle$ADE = $\angle$ABC and $\angle$DAE = $\angle$BAC. Hence ΔADE $\sim$ ΔABC(AA similarity)

hence the corresponding sides are in proportion

$\frac{AD}{AB} = \frac{AE}{Ac}$

$\Rightarrow \frac{2}{5} = \frac{3}{CE+3}$

$\Rightarrow$ CE = 4.5

9. **(a)** 30^o

Explanation: $\angle OAB = 60^o$ (given)

$\angle OAB = \angle OBA$ ($\because$ OA = OB = r)

$\therefore \angle OBA = 60^o$

Now, in $\triangle$OAB

$\angle AOB = 180^o - 60^o - 60^o$

$\angle AOB = 60^o$

Now, In $\triangle$AOP

$\angle OPA + \angle OAP + \angle AOP = 180^o$ (angle sum property of $\triangle$)

$\angle OPA + 90^o + 60^o = 180^o$

$\angle OPA = 180^o - 150^o$

$\angle OPA = 30^o$

10.

(b) one point only

Explanation: one point only

11. **(a)** $\frac{1}{2}$

Explanation: We know that $\sec^2A - \tan^2A = 1$.

$\therefore (2x)^2 - \left(\frac{2}{x}\right)^2 = 1 \Rightarrow 4x^2 - \frac{4}{x^2} = 1 \Rightarrow 4\left(x^2 - \frac{1}{x^2}\right) = 1$

$\Rightarrow \left(x^2 - \frac{1}{x^2}\right) = \frac{1}{4} \Rightarrow 2\left(x^2 - \frac{1}{x^2}\right) = 2 \times \frac{1}{4} = \frac{1}{2}$

12. **(a)** 30^o

Explanation: Cos A = $\frac{\sqrt{3}}{2}$

Cos A = Cos 30^o

A = 30^o

13. **(a)** 45^o

Explanation: 45^o

14.

(c) 21.99 m

Explanation: The area of the sector $= \frac{x^\circ}{360^\circ} \times \pi r^2$

$= \frac{70^\circ}{360^\circ} \times \frac{22}{7} \times 6^2$
21.99 m

15.

(b) 3696 cm^2

Explanation: Clearly, each wiper sweeps a sector of a circle of radius 42 cm and sector angle 120^o.

$\therefore$ Total area cleaned at each sweep = 2 $\times \frac{\theta}{360^\circ} \times \pi r^2$

= 2 $\times \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 42 \times$ 42 cm^2 = 3696 cm^2

16. **(a)** 0.00001

Explanation: An event is unlikely to happen. Its probability is very very close to zero but not zero, So it is equal to 0.00001

17. **(a)** $\frac{1}{13}$

Explanation: Total number of cards = 52.
Number of 6 s = 4.
$\therefore$ P (getting a 6) = $\frac{4}{52} = \frac{1}{13}$

18.

(c) decreases by 2

Explanation: decreased by 2.

19.

(d) A is false but R is true.

Explanation: A is false but R is true.

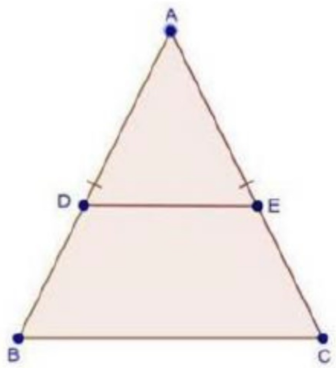
20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Common difference, d = -1 - 1(-5) = 4
So, both A and R are true and R is the correct explanation of A.

Section B

21. Given
$\sqrt{3}$ is an irrational number
Let 5 + 2$\sqrt{3}$ is a rational number
$\therefore$ we can write 5 + 2$\sqrt{3}$ = $\frac{p}{q}$, where p and q are integers
$\Rightarrow$ 2$\sqrt{3}$ = $\frac{p}{q} - 5 = \frac{p-5q}{q}$
$\sqrt{3}$ = $\frac{p-5q}{2q}$
Here, $\frac{p-5q}{2q}$ is a rational number
So, $\sqrt{3}$ is also a rational number.
But it is given that $\sqrt{3}$ is irrational number.
$\Rightarrow$ our assumption was wrong
$\Rightarrow$ 5 + 2$\sqrt{3}$ is an irrational number.

22. We have, DE || BC

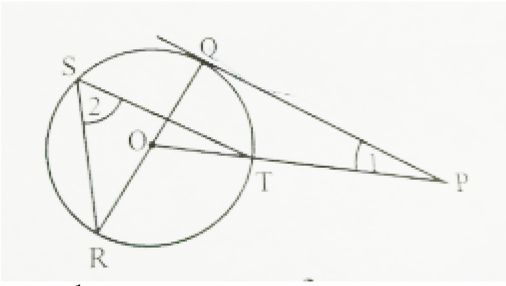


Therefore, by BPT, $\frac{AD}{BD} = \frac{AE}{EC} \Rightarrow AD = AE$
Adding DB on both sides
⇒ AD + DB = AE + DB
⇒ AD + DB = AE + EC   [$\therefore$ BD = CE]

⇒ AB = AC

∴ Δ ABC is isosceles triangle.

23. In the given figure



$\angle 2 = \frac{1}{2}\angle ROT$ (Angle subtended at the center by same arc)

$\angle 2 = \frac{1}{2} \times 130^\circ = 65^\circ$

$\angle ROT = \angle 1 + \angle PQO$

$\angle 1 = 130^\circ - 90^\circ = 40^\circ$

$\therefore \quad \angle 1 + \angle 2 = 65^\circ + 40^\circ = 105^\circ$

24. $= \sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$

$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$

$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

= sec A + tan A

= RHS

OR

Given

m sinA + n cosA = p......(1)

m cosA - n sinA = q........(2)

Squaring (1) and (2) we get,

$m^2 \sin^2 A + n^2 \cos^2 A + 2mn \sin A \cos A = p^2$ ......(3)

$m^2 \cos^2 A + n^2 \sin^2 A - 2mn \sin A \cos A = q^2$ .... (4)

Adding (3) and (4) we get,

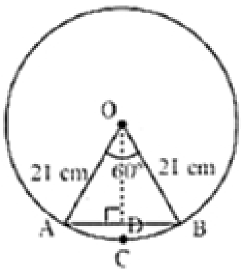
$m^2(\sin^2 A + \cos^2 A) + n^2(\sin^2 A + \cos^2 A) = p^2 + q^2$

$\Rightarrow m^2 + n^2 = p^2 + q^2$ [$\because \sin^2 A + \cos^2 A = 1$]

25. Radius (r) of circle = 21 cm

Angle subtended by the given arc = 60^o

Length of an arc of a sector of angle $\theta = \frac{\theta}{360^\circ} \times 2\pi r$



Area of sector OACB = $\frac{60^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$

= 231 cm^2

Length of arc ACB = $\frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$

= $\frac{1}{6} \times 2 \times 22 \times 3$

= 22 cm

OR

Area of minor segment = $\frac{3.14\times(10)^2\times 60^\circ}{360^\circ} - \frac{1}{2}\times(10)^2\times\frac{\sqrt{3}}{2}$

$= \frac{314}{6} - \frac{173}{4}$

$= 9\frac{1}{12}$ or 9.08

Hence, area of minor segment is 9.08 cm^2.

Section C

26. Given, $p = a^2b^3$

and $q = a^3b$

HCF(p,q) = a^2b

LCM(p, q) = a^3b^3

$pq = a^2b^3\times a^3b = a^5b^4 ------------(1)$

$LCM(p,q)\times HCF(p,q) = a^3b^3\times a^2b = a^5b^4 ----(2)$

$From\ equation\ (1)\ and\ (2)\ We\ get$

$LCM(p,q)\times HCF(p,q) = pq$

27. Let the given polynomial is $p(x) = x^2 + 7x + 7$

Here, a = 1, b = 7, c = 7

$\therefore \alpha, \beta$ are both zeroes of p(x)

$\therefore \quad \alpha+\beta = \frac{-b}{a}$ = -7.........(i)

$\alpha\beta = \frac{c}{a} = 7$ ....(ii)

Now,

$\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta}-2\alpha\beta$

$= \frac{-7}{7} - 2\times 7$

= -1 - 14

= -15

Hence the value of $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$ is - 15.

28. All the natural numbers less than 100 which are divisible by 6 are

6, 12, 18, 24,................., 96

Here, $a_1 = 6$

$a_2 = 12$

$a_3 = 18$

$a_4 = 24$

: :

$\therefore a_2 - a_1 = 12 - 6 = 6$

$a_3 - a_2 = 18 - 12 = 6$

$a_4 - a_3 = 24 - 18 = 6$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 6$ (=6 in each case)

$\therefore$ This sequence is an arithmetic progression whose difference is 6.

Here, a = 6

d = 6

l = 96

Let the number of terms be n. Then,

l = a + (n - 1)d

$\Rightarrow$ 96 = 6 + (n - 1)6

$\Rightarrow$ 96 - 6 = (n - 1)6

$\Rightarrow$ 90 = (n - 1)6

$\Rightarrow$ (n - 1)6 = 90

$\Rightarrow n - 1 = \frac{90}{6}$

$\Rightarrow$ n - 1 = 15

$\Rightarrow$ n = 15 + 1

$\Rightarrow$ n = 16

$\therefore S_n = \frac{n}{2}(a+l)$

$= \left(\frac{16}{2}\right)(6+96)$

= (8) (102)

= 816

OR

Let first term of given A.P. be a and common difference be d also sum of first m and first n terms be S_m and S_n respectively

$\therefore \quad \frac{S_m}{S_n} = \frac{m^2}{n^2}$

or, $\frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$

or, $\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m^2}{n^2} \times \frac{n}{m}$

or, $\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$

or, $m(2a+(n-1)d) = n[2a+(m-1)d]$

Now, $\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d}$

$= \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$

or, $= \frac{a+2ma-2a}{a+2na-2a}$

or, $= \frac{2ma-a}{2na-a}$

or, $= \frac{a(2m-1)}{a(2n-1)}$

or, $= \frac{(2m-1)}{(2n-1)}$

= $2m-1 : 2n-1$

The ratio of its m^{th} and n^{th} terms is $2m-1 : 2n-1$.

Hence proved

29. In two concentric circles with center O, a chord AB of the larger circle touches the smaller circle at C.

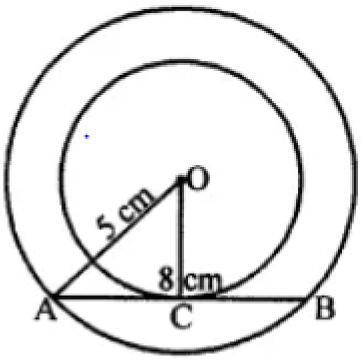
AB = 8 cm and radius of larger circle = 5 cm

Join OA, OC

To find, the radius of the smaller circle,

AB is the tangent and OC is the radius

OC $\perp$ AB



AC = CB = $\frac{8}{2}$ = 4 cm

OA = 5 cm

In right $\triangle$OCA,

$OA^2 = OC^2 + AC^2$ (Pythagoras Theorem)

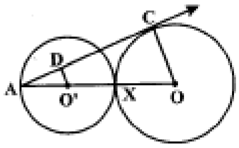
$(5)^2 = OC^2 + (4^2$

$OC^2 = (5)^2 - (4)^2 = 25 - 16 = 9 = (3^2)$

OC = 3

Radius of smaller circle = 3 cm

OR



Let the radius of both the circles is r.
In the fig, O'D⊥AC and AC is tangent of circle (O,r)
So, OC ⊥ AC (as line joining center to tangent is ⊥ to the tangent)
Now in ΔAO'D and ΔAOC,
∠O'DA = ∠OCA = 90°
∠A = ∠A (common)
Therefore, ΔAO'D∼ΔAOC [by AA rule]
So, $\frac{DO'}{CO} = \frac{AO'}{AO}$-----------(1)
Now, AO= r + r + r = 3r
and O'A=r
Putting the value of AO and AO' in equation (1), we get
$\frac{DO'}{CO} = \frac{r}{3r} = \frac{1}{3}$
Therefore, DO':CO = 1:3

30. We have,
$\tan\theta + \frac{1}{\tan\theta} = 2$
Squaring both sides, we get
$\Rightarrow \left(\tan\theta + \frac{1}{\tan\theta}\right)^2 = 2^2$
$\Rightarrow \quad \tan^2\theta + \frac{1}{\tan^2\theta} + 2 \times \tan\theta \times \frac{1}{\tan\theta} = 4$
$\Rightarrow \quad \tan^2\theta + \frac{1}{\tan^2\theta} + 2 = 4$
$\Rightarrow \quad \tan^2\theta + \frac{1}{\tan^2\theta} = 2$
Alternate method, We have
$\tan\theta + \frac{1}{\tan\theta} = 2$
$\Rightarrow \quad \tan^2\theta + 1 = 2\tan\theta$
$\Rightarrow \quad \tan^2\theta - 2\tan\theta + 1 = 0$
$\Rightarrow \quad (\tan\theta - 1)^2 = 0$
$\Rightarrow \quad \tan\theta = 1$
$\therefore \quad \tan^2\theta + \frac{1}{\tan^2\theta} = 1 + 1 = 2$

31.

Weight (in kg) (Class Interval)	Number of animals (f_i)	Mid point (x_i)	$d_i = x_i - a$	$f_i d_i$
100-110	4	105	-20	-80
110-120	12	115	-10	-120
120-130	23	125	0	0
130-140	8	135	10	80
140-150	3	145	20	60
Total	50			-60

let a= 125, $\sum f_i$ = 50, $\sum f_i d_i$ = 60
Now, Mean = $a + \frac{\sum f_i d_i}{\sum f_i}$
Mean = $125 + \frac{(-60)}{50}$
Mean = 125 - 1.2
Mean = 123.8

Section D

32. Let the first number be x
$\therefore$ Second number = x + 5

Now according to the question

$\frac{1}{x}-\frac{1}{x+5}=\frac{1}{10}$

$\Rightarrow \quad \frac{x+5-x}{x(x+5)}=\frac{1}{10}$

$\Rightarrow \quad 50=x^2+5x$

$\Rightarrow \quad x^2+5x-50=0$

$\Rightarrow \quad x^2+10x-5x-50=0$

$\Rightarrow x(x+10)-5(x+10)=0$

$\Rightarrow \quad (x+10)(x-5)=0$

x = 5 ,- 10 rejected

The numbers = 5 and 10.

OR

Let the original average speed of the train be x km/hr.

Time taken to cover 63 km $=\frac{63}{x}$ hours

Time taken to cover 72 km when the speed is increased by 6 km/hr $=\frac{72}{x+6}$ hours

By the question,we have,

$\frac{63}{x}+\frac{72}{x+6}=3$

$\Rightarrow \frac{21}{x}+\frac{24}{x+6}=1$

$\Rightarrow \frac{21x+126+24x}{x^2+6x}=1$

$\Rightarrow$ 45x + 126 = x^2 + 6x

$\Rightarrow$ x^2 - 39x - 126 = 0

$\Rightarrow$ x^2 - 42x + 3x - 126 = 0

$\Rightarrow$ x(x - 42) + 3(x - 42) = 0

$\Rightarrow$ (x - 42)(x + 3) = 0

$\Rightarrow$ x - 42 = 0 or x + 3 = 0

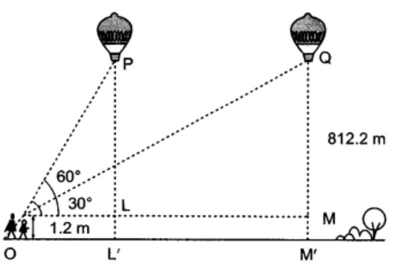
$\Rightarrow$ x = 42 or x = -3

Since the speed cannot be negative, $x \neq -3$.

Thus, the original average speed of the train is 42 km/hr.

33. Let P be the position of the balloon when its angle of elevation from the eyes of the girl is 60° and Q be the position when angle of elevation is 30°.

In $\triangle OLP$, we have



$\tan 60^\circ=\frac{PL}{OL}$

$\Rightarrow \quad \sqrt{3}=\frac{PL'-LL'}{OL}=\frac{88.2-1.2}{OL}$

$\Rightarrow \sqrt{3}=\frac{87}{OL}$

$\Rightarrow \quad OL=\frac{87}{\sqrt{3}}$

In $\triangle OMQ$,we have

$\tan 30^\circ=\frac{QM}{OM}=\frac{QM'-MM'}{OM}$

$\Rightarrow \quad \frac{1}{\sqrt{3}}=\frac{88.2-1.2}{OM}$

$\Rightarrow \quad \frac{1}{\sqrt{3}}=\frac{87}{OM}$

$\Rightarrow \quad OM=87\times\sqrt{3}$

$\therefore$ Distance travelled by the balloon = PQ = LM = OM - OL

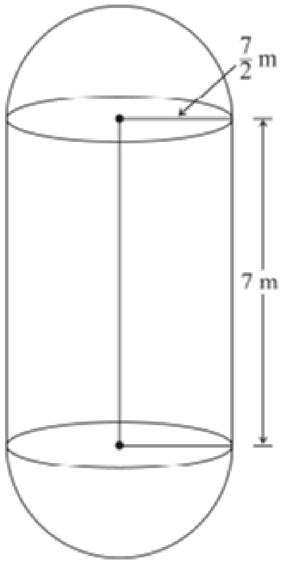
$=\left(87\times\sqrt{3}-\frac{87}{\sqrt{3}}\right)\mathrm{m}$

$= 87 \times \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)\text{m} = \frac{87\times 2}{\sqrt{3}}\text{m} = \frac{174}{\sqrt{3}}\text{m}$
$= \frac{174}{3}\sqrt{3}\text{m} = 58\sqrt{3}\text{m}.$

34. The volume of the spherical vessel is
calculated by the given formula
V = $\frac{4}{3}\pi \times r^3$
Now,
V = $\frac{4}{3} \times \frac{22}{7} \times 9 \times 9 \times 9$
V = 3,054.85 cm^3
The volume of the cylinder neck is calculated by the given formula.
V = $\pi \times R^2 \times h$
Now,
V = $\frac{22}{7} \times 1 \times 1 \times 8$
V = 25.14 cm^3
The total volume of the vessel is equal to the volume of the spherical shell and the volume of its cylindrical neck.
3054.85 + 25.14 = 3, 080 cm^3
The total volume of the vessel is 3,080 cm^3.
As we know,
1 L = 1000 cm^3
$\frac{3080}{1000}$ = 3.080 L
Thus, the amount of water (in litres) it can hold is 3.080 L.

OR

Given that,



Length of cylindrical part = 7 m
Radius of cylindrical part = $\frac{7}{2}$ m
Total surface area of figure = 2πrh + 2(2πr^2)
= $2\pi\left[\frac{7}{2} \times 7 + 2 \times \left(\frac{7}{2}\right)^2\right]$
= 308 m^2
Volume of boiler = Volume of cylindrical part + Volume of two hemispherical parts
= $\pi r^2 h + \left(\frac{4}{3}\right)\pi r^3$
= $\pi\left(\frac{7}{2}\right)^2 \times (7) + \left(\frac{4}{3}\right)\pi\left(\frac{7}{2}\right)^3$
= 269.5 + 179.66
= 449.167 m^3
Required ratio = $\frac{\text{Volume of cylindrical part}}{\text{Volume of one hemispherical part}}$

$= \frac{269.5}{89.83}$

= 3

35. Table:

Class	Frequency	Mid value x_i	f_ix_i	Cumulative frequency
25 - 35	7	30	210	7
35 - 45	31	40	1240	38
45 - 55	33	50	1650	71
55 - 65	17	60	1020	88
65 - 75	11	70	770	99
75 - 85	1	80	80	100
	N = 100		$\sum f_ix_i = 4970$	

i. Mean
$\frac{\sum f_ix_i}{\sum f_i} = \frac{4970}{100} = 49.70$

ii. N = 100, $\frac{N}{2}$= 50
Median Class is 45 - 55
$l = 45, h = 10, N = 100, c = 38, f = 33$
$\therefore$Median = $l + h\left(\frac{\frac{N}{2}-c}{f}\right)$
$= 45 + \left\{10 \times \frac{50-38}{33}\right\}$
$= 45 + 3.64 = 48.64$

iii. we know that, Mode = $3 \times \text{ median } - 2 \times \text{mean}$
$= 3 \times 48.64 - 2 \times 49.70$
$= 145.92 - 99.4 = 46.52$

Section E

36. i. Since, each poor child pays ₹ x
and each rich child pays ₹ y
$\therefore$ In batch I, 20 poor and 5 rich children pays ₹ 9000 can be represented as 20x + 5y = 9000
and in batch II, 5 poor and 25 rich children pays ₹ 26,000 can be represented as 5x + 25y = 26,000

ii. As we have 20x + 5y = 9,000 ...(i)
and 5x + 25y = 26,000
or x + 5y = 5,200 ...(ii)
On subtracting (ii) from (i), we get
19x = 3,800
$\Rightarrow$ x = 200
$\therefore$ Monthly fee paid by a poor child = ₹ 200

iii. As we have,
20x + 5y = 9000 ...(i)
and 5x + 25y = 26000
x + 5y = 5200 ...(ii)
On subtracting equation (ii) from (i), we have
19x = 3800
x = $\frac{3800}{19}$
= 200
Put the value of x in equation (ii), we get
200 + 5y = 5200
5y = 5200 - 200
y = 1000
$\therefore$ y - x = 1000 - 200

= 800

Hence, difference in the monthly fee paid by a poor child and a rich child is ₹ 800.

OR

Total monthly fee = 10x + 20y

= 10(200) + 20(1,000)

= 2,000 + 20,000

= ₹ 22,000

37. i. $\therefore$ AF = h (Given)

$\therefore$ AF = AH + HF

h = AH + $\frac{h}{4}$

AH = h - $\frac{h}{4}$

AH = $\frac{3h}{4}$

ii. $\because$ AF = h (Given)

$\therefore$ AG = $\frac{2}{3}$ AF

$\because$ centroid divide the median in 2 : 1

iii. AH = $\frac{3h}{4}$

J is centroid of $\triangle$ADE

AJ : JH = 2 : 1

let AJ = 2x and JH = x

2x + x = $\frac{3h}{4}$

x = $\frac{h}{4}$

AJ = 2 $\times$ $\frac{h}{4}$ = $\frac{h}{2}$

AG = AJ + GJ

$= \frac{h}{2} + \frac{h}{6}$

$= \frac{2h}{3}$

But AJ = $\frac{h}{2} \times \frac{2}{3}$

AJ = $\frac{3}{4}$ AG

OR

GJ = AG - AJ

= AG - $\frac{3}{4}$ AG

GJ = $\frac{1}{4}$ AG

38. i. Q(x, y) is mid-point of B(-2, 4) and C(6, 4)

$\therefore$ (x, y) = $\left(\frac{-2+6}{2}, \frac{4+4}{2}\right) = \left(\frac{4}{2}, \frac{8}{2}\right)$ = (2, 4)

ii. Since PQRS is a rhombus, therefore, PQ = QR = RS = PS.

$\therefore$ PQ = $\sqrt{(-2-2)^2 + (1-4)^2}$ = $\sqrt{16+9} = \sqrt{25}$ = 5 units

Thus, length of each side of PQRS is 5 units.

iii. Length of route PQRS = 4 PQ

= 4 $\times$ 5 = 20 units

OR

Length of CD = 4 + 2 = 6 units and length of AD = 6 + 2 = 8 units

$\therefore$ Length of route ABCD - 2(6 + 8) = 28 units