

4. Triangles

Exercise 4.1

1. Question

Fill in the blanks using the correct word given in brackets :

- (i) All circles are.....(congruent, similar).
- (ii) All squares are.....(similar, congruent).
- (iii) All.....triangles are similar (isosceles, equilaterals).
- (iv) Two triangles are similar, if their corresponding angles are.....(proportional, equal)
- (v) Two triangles are similar, if their corresponding sides are.....(proportional, equal)
- (vi) Two polygons of the same number of sides are similar, if (a) their corresponding angles are and (b) their corresponding sides are.....(equal, proportional)

Answer

- (i) similar (ii) similar
- (iii) equilateral (iv) equal
- (v) proportional (vi) equal, proportional

2. Question

Write the truth value (T/F) of each of the following statements:

- (i) Any two similar figures are congruent.
- (ii) Any two congruent figures are similar.
- (iii) Two polygons are similar, if their corresponding sides are proportional.
- (iv) Two polygons are similar if their corresponding angles are proportional.
- (v) Two triangles are similar if their corresponding sides are proportional.
- (vi) Two triangles are similar if their corresponding angles are proportional.

Answer

- (i) False (ii) True
- (iii) False (iv) False
- (v) True (vi) True

Exercise 4.2

1. Question

In a $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$

(i) If $AD = 6$ cm, $DB = 9$ cm and $AE = 8$ cm, find AC.

(ii) If $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15$ cm, find AE.

(iii) If $\frac{AD}{DB} = \frac{2}{3}$ and $AC = 18$ cm, find AE.

(iv) If $AD = 4$, $AE = 8$, $DB = x - 4$, and $EC = 3x - 19$, find x.

(v) If $AD = 8$ cm, $AB = 12$ cm and $AE = 12$ cm, find CE.

(vi) If $AD = 4$ cm, $DB = 4.5$ cm and $AE = 8$ cm, find AC.

(vii) If $AD = 2$ cm, $AB = 6$ cm and $AC = 9$ cm, find AE.

(viii) If $\frac{AD}{BD} = \frac{4}{5}$ and $EC = 2.5$ cm, find AE.

(ix) If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, find the value of x.

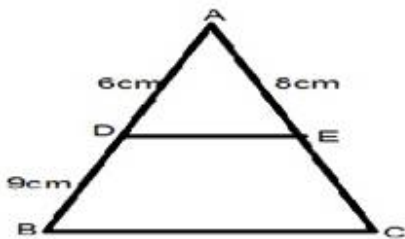
(x) If $AD = 8x - 7$, $DB = 5x - 3$, $AE = 4x - 3$ and $EC = (3x - 1)$, find the value of x.

(xi) If $AD = 4x - 3$, $AE = 8x - 7$, $BD = 3x - 1$ and $CE = 5x - 3$, find the value x.

(xii) If $AD = 2.5$ cm, $BD = 3.0$ cm and $AE = 3.75$ cm, find the length of AC.

Answer

(i)



we have

$DE \parallel BC$

Therefore by basic proportionality theorem

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{6}{9} = \frac{8}{EC}$$

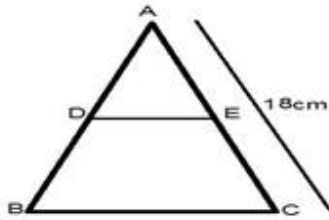
$$\frac{2}{3} = \frac{8}{EC}$$

$$EC = 3 \times \frac{8}{2}$$

$$EC = 3 \times 4$$

$$EC = 12 \text{ cm}$$

(ii)



we have

$$DE \parallel BC$$

Therefore by basic proportionality theorem

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Adding 1 both side

$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\frac{3}{4} + 1 = \frac{AE + EC}{BC}$$

$$\frac{3+4}{4} = \frac{AC}{EC} \quad [AE + EC = AC]$$

$$\frac{7}{4} = \frac{15}{EC}$$

$$EC = 15 \times \frac{4}{7}$$

$$EC = \frac{60}{7}$$

$$\text{Now } AE + EC = AC$$

$$AE + \frac{60}{7} = 15$$

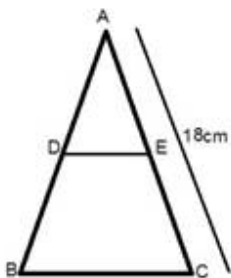
$$AE = 15 - \frac{60}{7}$$

$$AE = \frac{105 - 60}{7}$$

$$AE = \frac{45}{7}$$

$$AE = 6.43 \text{ cm}$$

(iii)



we have

$$DE \parallel BC$$

Therefore by basic proportionality theorem

$$AD/DB = AE/EC$$

Adding 1 both side

$$AD/DB + 1 = AE/EC + 1$$

$$\frac{3}{2} + 1 = \frac{EC}{AE} + 1$$

$$\frac{3+2}{2} = \frac{EC+AE}{AE}$$

$$\frac{5}{2} = AC/AE \quad [AE+EC=AC]$$

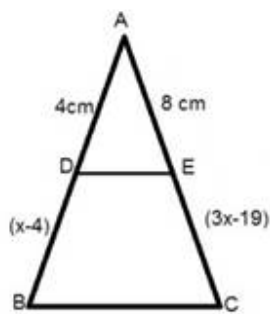
$$5/2 = 18/AE$$

$$AE = \frac{18 \times 2}{5}$$

$$AE = 36/5$$

$$AE = 7.2 \text{ cm}$$

(iv)



we have

$$DE \parallel BC$$

Therefore by basic proportionality theorem

$$AD/DB = AE/EC$$

$$\frac{4}{x-4} = \frac{8}{3x-19}$$

$$4(3x-19) = 8(x-4)$$

$$12x-76 = 8x-32$$

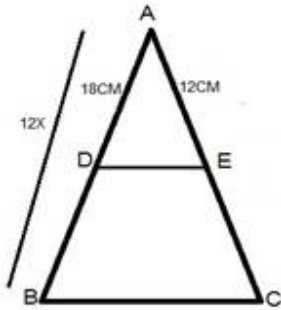
$$12x-8x = 76-32$$

$$4x = 44$$

$$x = 44/4$$

$$x = 11 \text{ cm}$$

(v)



$$AD = 8 \text{ cm}, AB = 12 \text{ cm}$$

$$\text{since } BD = AB - AD$$

$$BD = 12 - 8$$

$$BD = 4 \text{ cm}$$

$$DE \parallel BC$$

Therefore by basic proportionality theorem

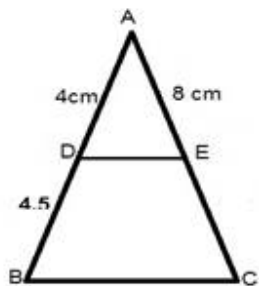
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{8}{4} = \frac{12}{EC}$$

$$EC = \frac{12 \times 4}{8}$$

$$EC = 6 \text{ cm}$$

(vi)



we have

$$DE \parallel BC$$

Therefore by basic proportionality theorem

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{4}{4.5} = \frac{8}{EC}$$

$$EC = \frac{8 \times 4.5}{4}$$

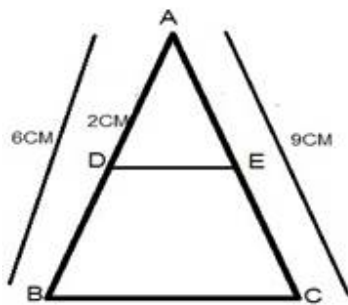
$$EC = 9\text{cm}$$

$$\text{Now } AE + EC = AC$$

$$AC = 8 + 9$$

$$AC = 17\text{ cm}$$

(vii)



$$AD = 2\text{cm}, AB = 6\text{cm}$$

$$\text{Since } BD = AB - AD$$

$$BD = 6 - 2$$

$$BD = 4\text{ cm}$$

$$DE \parallel BC$$

Therefore by basic proportionality theorem

$$AD/DB = AE/EC$$

Taking reciprocal on both side

$$DB/AD = EC/AE$$

$$4/2 = EC/AE$$

Adding 1 both side

$$AD/DB + 1 = AE/EC + 1$$

$$\frac{4}{2} + 1 = \frac{EC}{AE} + 1$$

$$\frac{4+2}{2} = \frac{EC+AE}{AE}$$

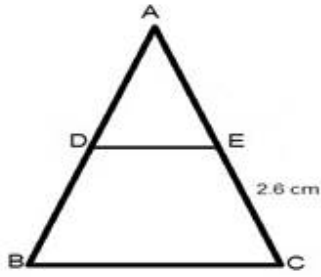
$$\frac{6}{2} = AC/AE \quad [AE + EC = AC]$$

$$3 = 9/AE$$

$$AE = \frac{9}{3}$$

$$AE = 3 \text{ cm}$$

(viii) we have



$$DE \parallel BC$$

Therefore by basic proportionality theorem

$$AD/DB = AE/EC$$

$$4/5 = AE/2.5$$

$$AE = 4 \times 2.5 / 5$$

$$AE = 10/5$$

$$AE = 2 \text{ cm}$$

(ix) we have

$$DE \parallel BC$$

Therefore by basic proportionality theorem

$$AD/DB = AE/EC$$

$$\frac{x}{x-2} = \frac{x+2}{x-1}$$

$$x(x-1) = (x+2)(x-2)$$

$$x^2 - x = x^2 - 2^2$$

$$-x = -4$$

$$x = 4 \text{ cm}$$

(x) we have

$$DE \parallel BC$$

Therefore by basic proportionality theorem

$$AD/DB = AE/EC$$

$$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$$

$$(8x-7)(3x-1) = (4x-3)(5x-3)$$

$$8x(3x-1)-7(3x-1)=4x(5x-3)-3(5x-3)$$

$$24x^2-8x-21x+7=20x^2-12x-15x+9$$

$$24x^2-20x^2-29x+27x+7-9=0$$

$$4x^2-2x-2=0$$

$$2[2x^2-x-1]=0$$

$$2x^2-x-1=0$$

$$2x^2-2x-x-1=0$$

$$2x(x-1)+1(x-1)=0$$

$$(x-1)(2x+1)=0$$

$$x-1=0$$

$$x=1$$

$$\text{or } 2x+1=0$$

$$\text{or } x=-1/2$$

-1/2 is not possible.

$$\text{So } x=1$$

(xi) we have

$$DE \parallel BC$$

Therefore by basic proportionality theorem

$$AD/DB=AE/EC$$

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$(8x-7)(3x-1)=(4x-3)(5x-3)$$

$$24x^2-8x-21x+7=20x^2-12x-15x+9$$

$$24x^2-20x^2-29x+27x+7-9=0$$

$$4x^2-2x-2=0$$

$$2[2x^2-x-1]=0$$

$$2x^2-x-1=0$$

$$2x^2-2x-x-1=0$$

$$2x(x-1)+1(x-1)=0$$

$$(x-1)(2x+1)=0$$

$$x-1=0$$

$$x=1$$

$$\text{or } 2x+1=0$$

$$\text{or } x=-1/2$$

$-1/2$ is not possible.

$$\text{So } x=1$$

(xii) we have

$$DE \parallel BC$$

Therefore by basic proportionality theorem

$$AD/DB=AE/EC$$

$$2.5/3=3.75/EC$$

$$EC=3.75 \times 3 / 2.5$$

$$EC=375 \times 3 / 250$$

$$EC=15 \times 3 / 10$$

$$EC=9/2$$

$$EC=4.5 \text{ cm}$$

$$\text{Now } AC=AE+EC$$

$$AC=3.75+4.5$$

$$AC=8.25 \text{ cm}$$

2. Question

In a $\triangle ABC$, D and E are points on the sides AB and AC respectively. For each of the following cases show that $DE \parallel BC$:

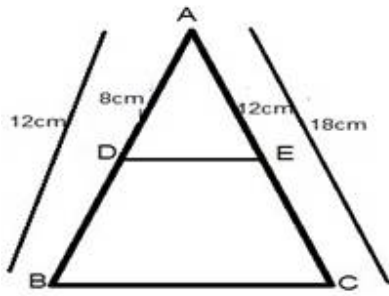
(i) $AB = 12 \text{ cm}$, $AD = 8 \text{ cm}$, $AE = 12 \text{ cm}$ and $AC = 18 \text{ cm}$.

(ii) $AB = 5.6 \text{ cm}$, $AD = 1.4 \text{ cm}$, $AE = 7.2 \text{ cm}$ and $AC = 1.8 \text{ cm}$.

(iii) $AB = 10.8 \text{ cm}$, $BD = 4.5 \text{ cm}$, $AC = 4.8 \text{ cm}$ and $AE = 2.8 \text{ cm}$.

(iv) $AD = 5.7 \text{ cm}$, $BD = 9.5 \text{ cm}$, $AE = 3.3 \text{ cm}$ and $EC = 5.5 \text{ cm}$.

Answer



(i) $AB = 12 \text{ cm}$, $AD = 8 \text{ cm}$, and $AC = 18 \text{ cm}$.

$$\therefore DB = AB - AD$$

$$= 12 - 8$$

$$= 4 \text{ cm}$$

$$EC = AC - AE$$

$$= 18 - 12$$

$$= 6 \text{ cm}$$

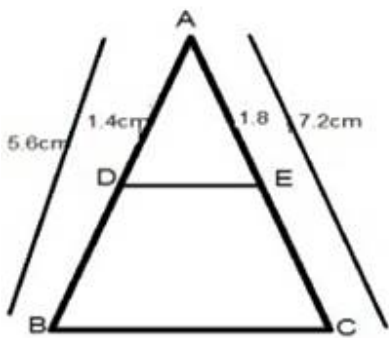
$$\text{Now } AD/DB = 8/4 = 2$$

$$AE/EC = 12/6 = 2$$

Thus DE divides side AB and AC of $\triangle ABC$ in same ratio

Then by the converse of basic proportionality theorem.

(ii)



$AB = 5.6 \text{ cm}$, $AD = 1.4 \text{ cm}$, $AE = 1.8 \text{ cm}$ and $AC = 7.2 \text{ cm}$

$$\therefore DB = AB - AD$$

$$DB = 5.6 - 1.4$$

$$DB = 4.2 \text{ cm}$$

$$\text{And } EC = AC - AE$$

$$EC = 7.2 - 1.8$$

$$EC = 5.4$$

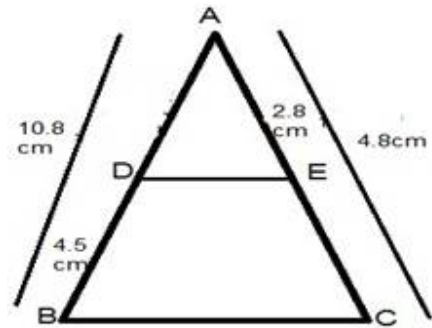
$$\text{Now } AD/DB = 1.4/4.2 = 1/3$$

$$AE/EC = 1.8/5.4 = 1/3$$

Thus DE divides side AB and AC of $\triangle ABC$ in same ratio

Then by the converse of basic proportionality theorem.

(iii)



we have

$$AB = 10.8 \text{ cm}, BD = 4.5 \text{ cm}, AC = 4.8 \text{ cm and } AE = 2.8 \text{ cm}$$

$$\therefore AD = AB - DB$$

$$AD = 10.8 - 4.5$$

$$AD = 6.3 \text{ cm}$$

$$\text{And } EC = AC - AE$$

$$EC = 4.8 - 2.8$$

$$EC = 2 \text{ cm}$$

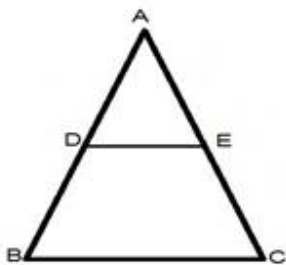
$$\text{Now } AD/DB = 6.3/4.5 = 7/5$$

$$AE/EC = 2.8/2 = 28/20 = 7/5$$

Thus DE divides side AB and AC of $\triangle ABC$ in same ratio

Then by the converse of basic proportionality theorem.

(iv)



$$DE \parallel BC$$

We have,

$$AD = 5.7 \text{ cm}, BD = 9.5 \text{ cm}, AE = 3.3 \text{ cm and } EC = 5.5 \text{ cm}$$

$$\text{Now } AD/DB = 5.7/9.5 = 57/95 = 3/5$$

$$AE/EC = 3.3/5.5 = 33/55 = 3/5$$

Thus DE divides side AB and AC of $\triangle ABC$ in same ratio

Then by the converse of basic proportionality theorem.

3. Question

In a $\triangle ABC$, P and Q are points on sides AB and AC respectively, such that $PQ \parallel BC$. If $AP = 2.4 \text{ cm}$, $AQ = 2 \text{ cm}$, $QC = 3 \text{ cm}$ and $BC = 6 \text{ cm}$, find AB and PQ.

Answer

WE have,

$$PQ \parallel BC$$

$$\text{We have } AP/PB = AQ/QC$$

$$2.4/PB = 2/3$$

$$PB = 3 \times 2.4 / 2$$

$$PB = 3 \times 1.2$$

$$PB = 3.6 \text{ cm}$$

$$\text{Now } AB = AP + PB$$

$$AB = 2.4 + 3.6$$

$$AB = 6 \text{ cm}$$

Now IN $\triangle APQ$ and $\triangle ABC$

$$\angle A = \angle A \text{ [Common]}$$

$$\angle APQ = \angle ABC \text{ [} PQ \parallel BC \text{]}$$

$$\triangle APQ \sim \triangle ABC \text{ [By AA criteria]}$$

$$AB/AP = BC/PQ$$

$$PQ = 6 \times 2.4 / 6$$

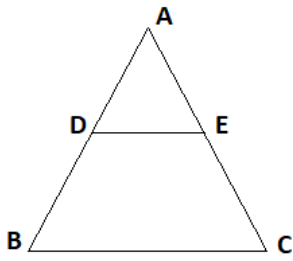
$$PQ = 2.4 \text{ cm}$$

4. Question

In a $\triangle ABC$, D and E are points on AB and AC respectively such that $DE \parallel BC$. If $AD = 2.4 \text{ cm}$, $AE = 3.2 \text{ cm}$, $DE = 2 \text{ cm}$ and $BC = 5 \text{ cm}$, find BD and CE.

Answer

In the figure given below,



Given: AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm

Let BD be x cm and CE be y cm,

Then, from, $\triangle ADE$ and $\triangle ABC$, $DE \parallel BC$, so by basic proportionality theorem we can write,

$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\text{Or } \frac{AD}{AD + BD} = \frac{DE}{BC}$$

$$\text{or } \frac{2.4}{2.4 + x} = \frac{2}{5}$$

$$\text{or } 12 = 4.8 + 2x$$

$$\text{or } x = 7.2/2$$

$$\text{or } x = DB = 3.6\text{cm}$$

Similarly, from $\triangle ADE$ and $\triangle ABC$, we can write,

$$\frac{AE}{AC} = \frac{DE}{BC}$$

$$\text{Or } \frac{AE}{AE + EC} = \frac{DE}{BC}$$

$$\text{or } \frac{3.2}{3.2 + y} = \frac{2}{5}$$

$$\text{or } 16 = 6.4 + 2y$$

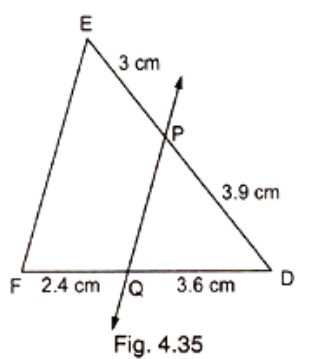
$$\text{or } y = 9.6/2$$

$$\text{or } y = CE = 4.8\text{ cm}$$

Thus, the lengths of BD and CE are 3.6 cm and 4.8 cm respectively.

5. Question

In Fig. 4.35, state if $PQ \parallel EF$.



Answer

$$DP/PE = 3.9/3 = 1.3/1 = 13/10$$

$$DQ/QF = 3.6/2.4 = 36/24 = 3/2$$

$$DP/PE \neq DQ/QF$$

So PQ is not parallel to EF

6. Question

M and N are points on the sides PQ and PR respectively of a $\triangle PQR$. For each of the following cases, state whether $MN \parallel QR$:

(i) $PM = 4$ cm, $QM = 4.5$ cm, $PN = 4$ cm, $NR = 4.5$ cm

(ii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PM = 0.16$ cm, $PN = 0.32$ cm

Answer

(i) we have $PM = 4$ cm, $QM = 4.5$ cm, $PN = 4$ cm and $NR = 4.5$ cm

$$\text{Hence } PM/QM = 4/4.5 = 40/45 = 8/9$$

$$PN/NR = 4/4.5 = 40/45 = 8/9$$

$$PM/QM = PN/NR$$

by the converse of proportionality theorem

$$MN \parallel QR$$

(ii) we have $PQ = 1.28$ cm, $PR = 2.56$ cm, $PM = 0.16$ cm and $PN = 0.32$ cm

$$\text{Hence } PQ/PR = 1.28/2.56 = 128/256 = 1/2$$

$$PM/PN = 0.16/0.32 = 16/32 = 1/2$$

$$PQ/PR = PM/PN$$

by the converse of proportionality theorem

$$MN \parallel QR$$

7. Question

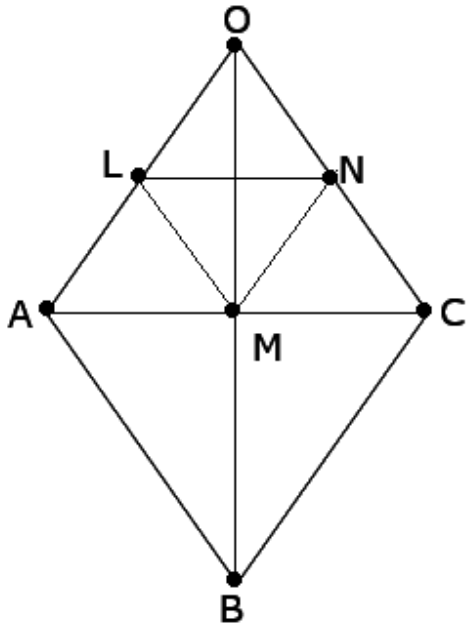
In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that $LM \parallel AB$ and $MN \parallel BC$ but neither of L, M, N nor of A, B, C are collinear. Show that $LN \parallel AC$.

Answer

Given: In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that $LM \parallel AB$ and $MN \parallel BC$ but neither of L, M, N nor of A, B, C are collinear.

To show : $LN \parallel AC$

Solution:



We have $LM \parallel AB$ and $MN \parallel BC$

by the basic proportionality theorem

$$OL/AL = OM/MB \dots\dots\dots(i)$$

$$ON/NC = OM/MB \dots\dots\dots(ii)$$

Comparing equ.(i) and (ii)

$$OL/AL = ON/NC$$

Thus LN divides side OA and OC of $\angle OAC$ in same ratio

Then by the converse of basic proportionality theorem

$$LN \parallel AC$$

8. Question

If D and E are points on sides AB and AC respectively of a $\triangle ABC$ such that $DE \parallel BC$ and $BD = CE$. Prove that $\triangle ABC$ is isosceles.

Answer

We have $DE \parallel BC$

by the converse of proportionality theorem

$$AD/DB=AE/EC$$

$$AD/DB=AE/DB \text{ [BD=CE]}$$

$$AD=AE$$

Adding D both sides

$$AD+BD=AE+DB$$

$$AD+BD=AE+EC \text{ [BD=CE]}$$

$$AB=AC$$

$\triangle ABC$ is isosceles

Exercise 4.3

1. Question

In a $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D.

- (i) If $BD = 2.5$ cm, $AB = 5$ cm and $AV = 4.2$ cm, find DC.
- (ii) If $BD = 2$ cm, $AB = 5$ cm and $DC = 3$ cm, find AC.
- (iii) If $AB = 3.5$ cm, $AC = 4.2$ cm and $DC = 2.8$ cm, find BD.
- (iv) If $AB = 10$ cm, $AC = 14$ cm and $BC = 6$ cm, find BD and DC.
- (v) If $AC = 4.2$ cm, $DC = 6$ cm and $BC = 10$ cm, find AB.
- (vi) If $AB = 5.6$ cm, $AC = 6$ cm and $DC = 6$ cm, find BC.
- (vii) If $AD = 5.6$ cm, $BC = 6$ cm and $BD = 3.2$ cm, find AC.
- (viii) If $AB = 10$ cm, $AC = 6$ cm and $BC = 12$ cm, find BD and DC.

Answer

(i) we have

$$\text{Angle } BAD = CAD$$

Here AD bisects $\angle A$

$$BD/DC=AB/AC$$

$$2.5/DC=5/4.2$$

$$DC=2.5*4.2/5$$

$$DC=2.1 \text{ cm}$$

(ii) Here AD bisects $\angle A$

$$AB/DC=AB/AC$$

$$2/3 = 5/AC$$

$$AC = 15/2$$

$$AC = 7.5 \text{ cm}$$

(iii) in $\triangle ABC$ A bisects $\angle A$

$$BD/DC = AB/BC$$

$$BD/2.8 = 3.5/4.2$$

$$BD = 3.5 * 2.8 / 4.2$$

$$BD = 7/3$$

$$BD = 2.33 \text{ cm}$$

(iv) In $\triangle ABC$, AD bisects $\angle A$

$$BD/DC = AB/AC$$

$$x/6-x = 10/14$$

$$14x = 60 - 10x$$

$$14x + 10x = 60$$

$$24x = 60$$

$$x = 60/24$$

$$x = 5/2$$

$$x = 2.5$$

$$BD = 2.5$$

$$DC = 6 - 2.5$$

$$DC = 3.5$$

$$(v) AB/AC = BD/DC$$

$$AB/4.2 = BC - DC/DC$$

$$AB/4.2 = 10 - 6/6$$

$$AB/4.2 = 4/6$$

$$AB = 4 * 4.2 / 6$$

$$AB = 2.8 \text{ cm}$$

$$(vi) BD/DC = AB/AC$$

$$BD/6 = 5.6/6$$

$$BD = 5.6$$

$$BC = BD + DC$$

$$BC=5.6+6$$

$$BC=11.6 \text{ cm}$$

(viii) In $\triangle ABC$, AD bisects $\angle A$

$$AB/AC=BD/DC$$

$$5.6/AC=3.2/BC-BD$$

$$5.6/AC=3.2/6-3.2$$

$$5.6/AC=3.2/2.8$$

$$AC \cdot 3.2 = 2.8 \cdot 5.6$$

$$AC = 2.8 \cdot 5.6 / 3.2$$

$$AC = 7 \cdot 0.7$$

$$AC = 4.9 \text{ cm}$$

(ix) let $BD=x$, then $DC=12-X$

$$BD/DC=AB/BC$$

$$x/12-x = 10/6$$

$$6x=120-10x$$

$$6x+10x=120$$

$$16x=120$$

$$x=120/16$$

$$x= 7.5$$

$$BD=7.5 \text{ cm}$$

$$DC = 12-x$$

$$DC=12-7.5$$

$$DC=4.5 \text{ cm}$$

2. Question

In Fig. 4.57, AE is the bisector of the exterior $\angle CAD$ meeting BC produced in E. If $AB = 10 \text{ cm}$, $AC = 6 \text{ cm}$ and $BC = 12 \text{ cm}$, find CE.

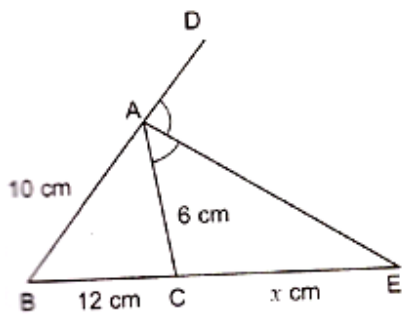


Fig. 4.57

Answer

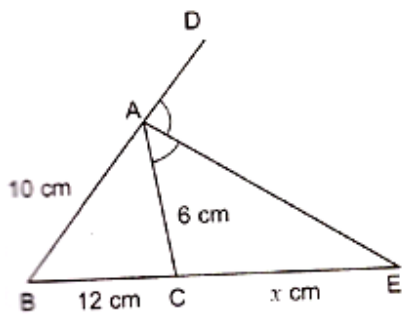


Fig. 4.57

AE is the bisector of $\angle A$

We know that external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angles.

$$\frac{BE}{CE} = \frac{AB}{AC}$$

$$\Rightarrow \frac{12+x}{x} = \frac{10}{6}$$

$$\Rightarrow 10x = 6(12+x)$$

$$\Rightarrow 10x = 72 + 6x$$

$$\Rightarrow 10x - 6x = 72$$

$$\Rightarrow 4x = 72$$

$$\Rightarrow x = 72/4$$

$$\Rightarrow x = 18$$

3. Question

In Fig. 4.58, $\triangle ABC$ is a triangle such that $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 70^\circ$, $\angle C = 50^\circ$. Find $\angle BAD$.

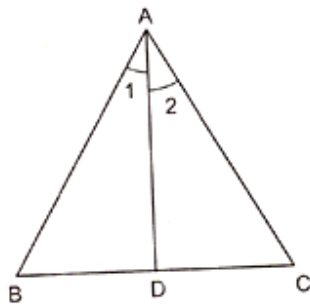


Fig. 4.58

Answer

We have

$$AB/AC = BD/DC$$

$$\therefore \angle 1 = \angle 2$$

IN $\triangle ABC$

$$\angle A + \angle B + \angle C = 180$$

$$\angle A + 70 + 50 = 180$$

$$\angle A + 120 = 180$$

$$\angle A = 180 - 120$$

$$\angle A = 60$$

$$\angle 1 + \angle 2 = 60 \quad (\angle 1 + \angle 2 = \angle A)$$

$$\angle 1 + \angle 1 = 60 \quad (\angle 1 = \angle 2)$$

$$2\angle 1 = 60$$

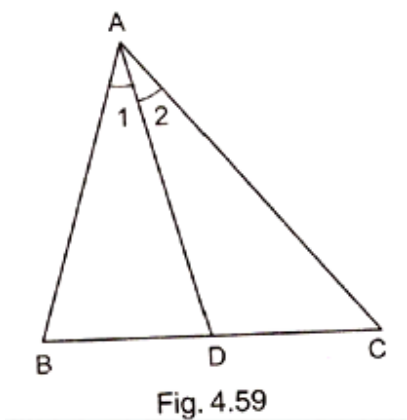
$$\angle 1 = 60/2$$

$$\angle 1 = 30$$

$$\angle BAD = 30$$

4. Question

In $\triangle ABC$ (fig. 4.59), if $\angle 1 = \angle 2$, prove that $\frac{AB}{AC} = \frac{BD}{DC}$.



Answer

$$\angle 1 = \angle 2 \text{ (Given)}$$

Draw a line $EC \parallel AD$

AC bisects them

$$\therefore \angle 2 = \angle 3 \text{ (by alternate angle) (i)}$$

$$\angle 1 = \angle 4 \text{ (corresponding angle) (ii)}$$

$$\angle 1 = \angle 2 \text{ (given)}$$

From equ (i) and equ (ii)

$$\angle 3 = \angle 4$$

$$\text{or } AE = AC \text{ (III)}$$

Now, $\triangle BCE$

$$BD/DC = BA/AE \text{ (BY PROPORTIONALITY THEORAM)}$$

$$BD/DC = AB/AC \text{ (} \because BA = AB \text{ AND } AE = AC \text{ from equ (iii))}$$

Hence $AB/AC = BA/DC$ Proved

5. Question

D, E and F are the points on sides BC, CA and AB respectively of $\triangle ABC$ such that AD bisects $\angle A$, BE bisects $\angle B$ and CF bisects $\angle C$. If $AB = 5 \text{ cm}$, $BC = 8 \text{ cm}$ and $CA = 4 \text{ cm}$, determine AF, CE and BD.

Answer

in $\triangle ABC$

CF bisects $\angle A$

$$\therefore AF/FB = AE/AC$$

$$AF/5 - AF = 4/8$$

$$2AF = 5 - AF$$

$$2AF + AF = 5$$

$$3AF=5$$

$$AF=5/3 \text{ cm}$$

$\triangle ABC$, BE bisects $\angle B$

$$\therefore AE/AC=AB/BC$$

$$4-CE/CE=5/8$$

$$5CE=32-8CE$$

$$5CE+8CE=32$$

$$13CE=32$$

$$CE=32/13 \text{ cm}$$

Similarly

$$BD/DC=AB/AC$$

$$BD/8-BD=5/4$$

$$4BD=40-5BD$$

$$4BD+5BD=40$$

$$9BD=40$$

$$BD=40/9 \text{ cm}$$

6. Question

In Fig. 4.60, check whether AD is the bisector of $\angle A$ of $\triangle ABC$ in each of the following:

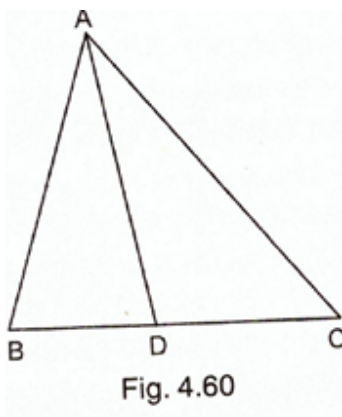
(i) $AB = 5 \text{ cm}$, $AC = 10 \text{ cm}$, $BD = 1.5 \text{ cm}$ and $CD = 3.5 \text{ cm}$

(ii) $AB = 4 \text{ cm}$, $AC = 6 \text{ cm}$, $BD = 1.6 \text{ cm}$ and $CD = 2.4 \text{ cm}$

(iii) $AB = 8 \text{ cm}$, $AC = 24 \text{ cm}$, $BD = 6 \text{ cm}$ and $BC = 24 \text{ cm}$

(iv) $AB = 6 \text{ cm}$, $AC = 8 \text{ cm}$, $BD = 1.5 \text{ cm}$ and $CD = 2 \text{ cm}$

(v) $AB = 5 \text{ cm}$, $AC = 12 \text{ cm}$, $BD = 2.5 \text{ cm}$ and $BC = 9 \text{ cm}$



Answer

(i) $BD/DC=AB/AC$

$$1.5/3.5=5/10$$

$$15/35 \times 10/10 = 1/2$$

$$3/7 = 1/2$$

Not bisects

(ii) $1.6/2.4=4/6$

$$16/24=2/3$$

$$2/3=2/3$$

bisects

(iii) $BD/CD=AB/AC$

$$BD/BC-BD=AB/AC$$

$$BD/24-6=8/24$$

$$6/18=1/3$$

$$1/3=1/3$$

bisects

(iv) $1.5/2=6/8$

$$3/4=3/4$$

bisects

(v) $BD/CD=AB/AC$

$$BD/BC-BD=AB/AC$$

$$BD/9-2.5=5/12$$

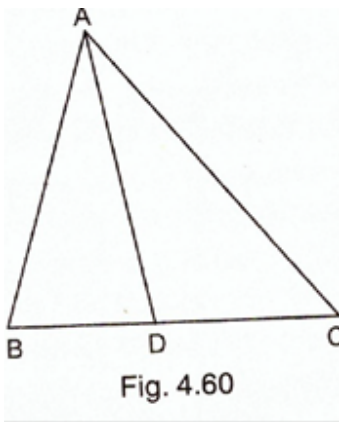
$$2.5/6.5=5/12$$

$$5/13=5/12$$

Not bisects

7. Question

In Fig. 4.60, AD bisects $\angle A$, AB = 12 cm, AC = 20 cm and BD = 5 cm, determine CD.



Answer

AD bisects $\angle A$

$$\therefore AB/AC=BD/CD$$

$$12/20=5/CD$$

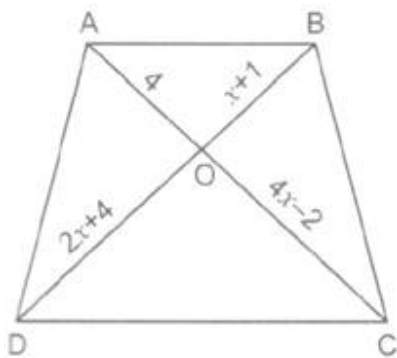
$$CD = 100/12$$

$$CD=8.33 \text{ cm}$$

Exercise 4.4

1 A. Question

(i) In fig. 4.70, if $AB \parallel CD$, find the value of x.



Answer

Diagonal of trapezium divide each other proportiona

$$AO/OC=BO/OD$$

$$4/4X-2=x+1/2x+4$$

$$4x^2-2x+4x-2=8x+16$$

$$4x^2+2x-2-8x-16=0$$

$$4x^2-6x-18=0$$

$$2(2x^2-3x-9)=0$$

$$2x^2-3x-9=0$$

$$2x^2-6x+3x-9=0$$

$$2x(x-3)+3(x-3)=0$$

$$(x-3)(2x+3)=0$$

$$x-3=0$$

$$x=3$$

$$\text{or, } 2x+3=0$$

$$2x=-3$$

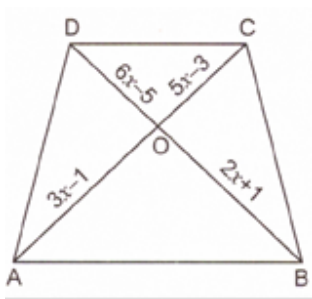
$$x=-3/2$$

$x=-3/2$ is not possible

So $x=3$

1 B. Question

In Fig. 4.71, if $AB \parallel CD$, find the value of x .



Answer

$$AO/OC=BO/OD$$

$$3x-1/5x-3=2x+1/6x-5$$

$$(3x-1)(6x-5)= (2x+1) (5x-3)$$

$$18x^2-15x-6x+5=10x^2-6x+5x-3$$

$$18x^2-21x+5=10x^2-x-3$$

$$18x^2-21x+5-10x^2+x+3=0$$

$$8x^2-20x+8=0$$

$$4(2x^2-5x+2)=0$$

$$2x^2-5x+2=0$$

$$2x^2-4x-x+2=0$$

$$2x(x-2)-1(x-2)=0$$

$$(x-2)(2x-1)=0$$

$$x-2=0$$

$$x=2$$

$$\text{Or, } 2x-1=0$$

$$2x=1$$

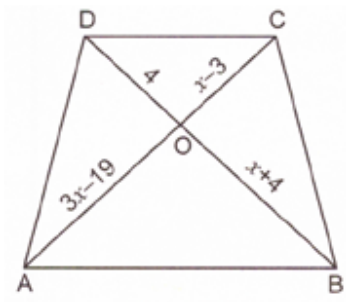
$$x=1/2$$

But $x=1/2$ is not possible

$$\text{So } x=2$$

1 C. Question

In Fig. 4.72, $AB \parallel CD$. If $OA = 3x - 19$, $OB = x - 4$, $OC = x - 3$ and $OD = 4$, find x .



Answer

$$AO/OC=BO/OD$$

$$3X-19/X-3=X-4/4$$

$$(x-3)(x-4)=4(3x-19)$$

$$X^2 -4x-3x+12=12x-76$$

$$X^2 -7x+12-12x+76=0$$

$$X^2 -19x+88=0$$

$$X^2 -11x-8x+88=0$$

$$X(x-11)-8(x-11)=0$$

$$(x-11)(x-8)=0$$

$$x-11=0$$

$$x=11$$

$$\text{or } x-8=0$$

$$x=8$$

$$x=11 \text{ or } 8$$

Exercise 4.5

1. Question

In Fig. 4.136, $\triangle ABC \sim \triangle APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm and $AP = 2.8$ cm, find CA and AQ

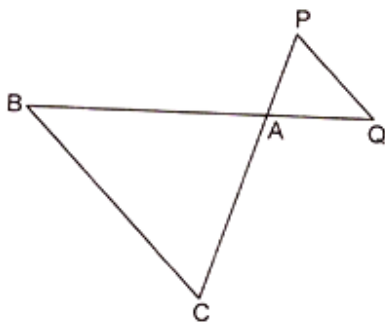


Fig. 4.136

Answer

Given $\triangle ACB \sim \triangle APQ$

Then, $AC/AP = BC/PQ = AB/AQ$

$$\text{Or } AC/2.8 = 8/4 = 6.5/AQ$$

$$\text{Or } AC/2.8 = 8/4 \text{ and } 8/4 = 6.5/AQ$$

$$\text{Or } AC = 8/4 \times 2.8 \text{ and } AQ = 6.5 \times 4/8$$

$$\text{Or } AC = 5.6 \text{ cm and } AQ = 3.25 \text{ cm}$$

2. Question

A vertical stick 10 cm long casts a shadow 8 cm long. At the same time a tower casts a shadow 30 m long. Determine the height of the tower.

Answer

Length of stick = 10 cm

Length of shadow stick = 8 cm

Length of shadow of tower = h cm

In $\triangle ABC$ and $\triangle PQR$

$\angle B = \angle C = 90^\circ$ And $\angle C = \angle R$ (Angular elevation of sun)

Then $\triangle ABC \sim \triangle PQR$ (By AA similarity)

$$\text{So, } \frac{AB}{PQ} = \frac{BC}{QR}$$

$$\text{Or } \frac{10 \text{ cm}}{8 \text{ cm}} = \frac{H}{3000}$$

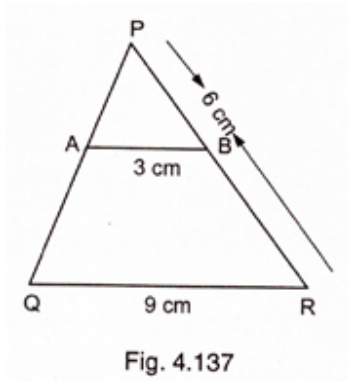
$$\text{Or } h = \frac{10}{8} \times 3000$$

$$\text{Or } 3750\text{cm}$$

$$\text{Or } 37.5\text{m}$$

3. Question

In Fig. 4.137, $AB \parallel QR$. Find the length of PB.



Answer

We have $\triangle PAB$ and $\triangle PQR$

$$\angle P = \angle P \text{ (Common)}$$

$$\angle PAB = \angle PQR \text{ (Corresponding angles)}$$

Then, $\triangle PAB \sim \triangle PQR$ (BY AA similarity)

$$\text{So, } \frac{PB}{PR} = \frac{AB}{QR} \text{ (Corresponding parts of similar triangle area proportion)}$$

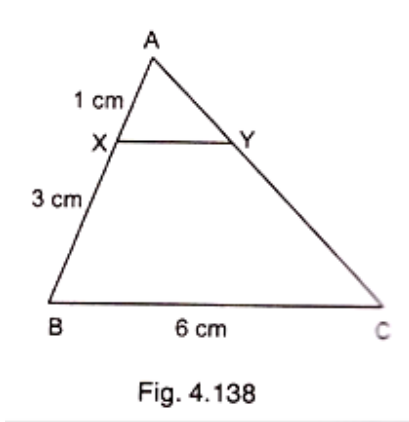
$$\text{Or, } \frac{PB}{6} = \frac{3}{9}$$

$$\text{Or } PB = \frac{3}{9} \times 6$$

$$\text{Or } PB = 2\text{cm}$$

4. Question

In Fig. 4.138, $XY \parallel BC$. Find the length of XY.



Answer

We have , $XY \parallel BC$

In ΔAXY and ΔABC

$\angle A = \angle A$ (Common)

$\angle AXY = \angle ABC$ (Corresponding angles)

Then, $\Delta AXY \sim \Delta ABC$ (By AA Similarity)

So, $\frac{AX}{AB} = \frac{XY}{BC}$ (Corresponding parts of similar triangle area proportion)

$$\text{Or } \frac{1}{4} = \frac{XY}{6}$$

$$\text{Or } XY = 6/4$$

$$\text{Or } XY = 1.5\text{cm}$$

5. Question

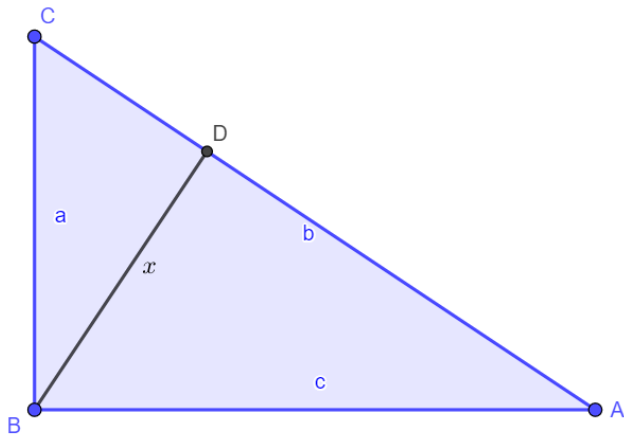
In a right angled triangle with sides a and b and hypotenuse c , the altitude drawn on the hypotenuse is x . Prove that $ab = cx$

Answer

Given: In a right angled triangle with sides a and b and hypotenuse c , the altitude drawn on the hypotenuse is x .

To prove: $ab = cx$

Proof: Let in a right-angled triangle ABC at B , a perpendicular from C to AB is drawn such that $BC = a$, $AC = b$, $AB = c$, $BD = x$



In $\triangle ABC$ and $\triangle CDB$

$\angle B = \angle B$ (Common)

$\angle ABC = \angle CDB$ (Both 90°)

Then, $\triangle ABC \sim \triangle CDB$ (By AA Similarity)

So, $\frac{AC}{CD} = \frac{AB}{CB}$ (Corresponding parts of similar triangle area proportion)

$$\text{Or } \frac{b}{x} = \frac{c}{a}$$

$$\text{Or } ab = cx$$

6. Question

In Fig. 4.139, $\angle ABC = 90^\circ$ and $BD \perp AC$. If $BD = 8$ cm and $AD = 4$ cm, find CD .

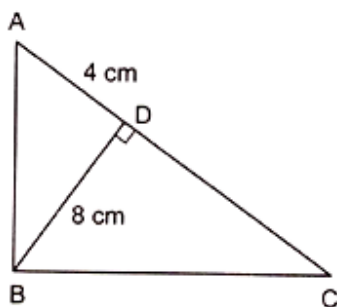


Fig. 4.139

Answer

We have, $\angle ABC = 90^\circ$ and BD perpendicular AC

Now, $\angle ABD + \angle DBC = 90^\circ$ (I) ($\angle ABC = 90^\circ$)

And $\angle C + \angle DBC = 90^\circ$ (II) (By angle sum Prop. in $\triangle BCD$) Compare equation I & II

$\angle ABD = \angle C$ (III)

In $\triangle ABD$ and $\triangle BCD$

$\angle ABD = \angle C$ (From equation I)

$\angle ADB = \angle BDC$ (Each 90°)

Then, $\triangle ABD \sim \triangle BCD$ (By AA similarity)

So, $\frac{BD}{CD} = \frac{AD}{BD}$ (Corresponding parts of similar triangle area proportion)

$$\text{Or, } \frac{8}{CD} = \frac{4}{8}$$

$$\text{Or } CD = \frac{8 \times 8}{4}$$

$$\text{Or } CD = 16\text{cm}$$

7. Question

In Fig. 4.140, $\angle ABC = 90^\circ$ and $BD \perp AC$. If $AB = 5.7$ cm, $BD = 3.8$ cm and $CD = 5.4$ cm, find BC .

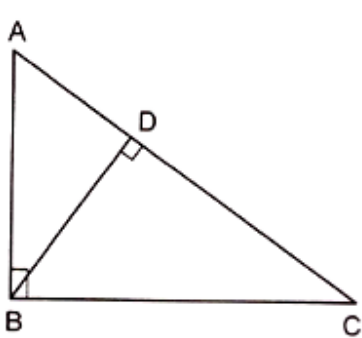


Fig. 4.140

Answer

We have, $\angle ABC = 90^\circ$ and BD Perpendicular AC

In $\triangle ABY$ and $\triangle BDC$

$\angle C = \angle C$ (Common)

$\angle ABC = \angle BDC$ (Each 90° angles)

Then, $\triangle ABC \sim \triangle BDC$ (By AA Similarity)

So, $\frac{AB}{BD} = \frac{BC}{DC}$ (Corresponding parts of similar triangle area proportion)

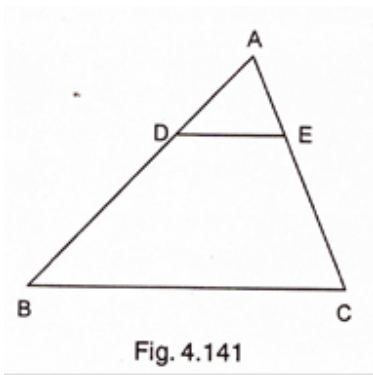
$$\text{Or } \frac{5.7}{3.8} = \frac{BC}{5.4}$$

$$\text{Or } BC = 5.7/3.8 \times 5.4$$

$$\text{Or } BC = 8.1\text{cm}$$

8. Question

In Fig. 4.141 $DE \parallel BC$ such that $AE = (1/4) AC$. If $AB = 6$ cm, find AD .



Answer

We have, $DE \parallel BC$, $AB = 6\text{cm}$ and $AE = \frac{1}{4} AC$

In $\triangle ADE$ and $\triangle ABC$

$\angle A = \angle A$ (Common)

$\angle ADE = \angle ABC$ (Corresponding angles)

Then, $\triangle ADE \sim \triangle ABC$ (By AA similarity)

So, $\frac{AD}{AB} = \frac{AE}{AC}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{AD}{6} = \frac{\frac{1}{4}AC}{AC}$ ($AE = \frac{1}{4} AC$ Given)

Or, $\frac{AD}{6} = \frac{1}{4}$

Or, $AD = \frac{6}{4}$

Or, $AD = 1.5\text{cm}$

9. Question

In Fig. 4.142, PA , QB and RC are each perpendicular to AC . Prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.

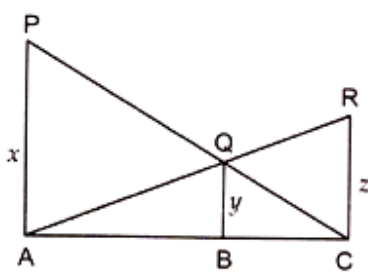


Fig. 4.142

Answer

We have, $PA \perp AC$, and $RC \perp AC$

Let $AB = a$ and $BC = b$

In $\triangle CQB$ and $\triangle CPA$

$\angle QCB = \angle PCA$ (Common)

$\angle QBC = \angle PAC$ (Each 90°)

Then, $\triangle CQB \sim \triangle CPA$ (By AA similarity)

So, $\frac{QB}{PA} = \frac{CB}{CA}$ (Corresponding parts of similar triangle area proportion)

$$\text{Or, } \frac{y}{z} = \frac{b}{a+b} \text{ -----(i)}$$

In $\triangle AQB$ and $\triangle ARC$

$\angle QAB = \angle RAC$ (Common)

$\angle ABQ = \angle ACR$ (Each 90°)

Then, $\triangle AQB \sim \triangle ARC$ (By AA similarity)

So, $\frac{QB}{RC} = \frac{AB}{CA}$ (Corresponding parts of similar triangle area proportion)

$$\text{Or, } \frac{y}{x} = \frac{a}{a+b} \text{ -----(ii)}$$

Adding equation i & ii

$$\frac{y}{x} + \frac{y}{z} = \frac{b}{a+b} + \frac{a}{a+b}$$

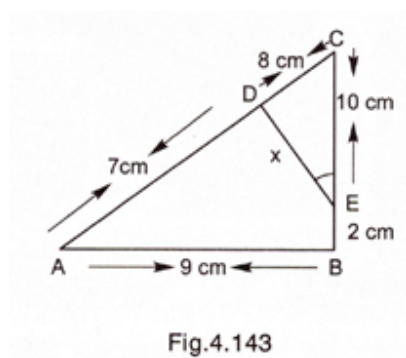
$$\text{Or, } y \left(\frac{1}{x} + \frac{1}{z} \right) = \frac{b+a}{a+b}$$

$$\text{Or, } y \left(\frac{1}{x} + \frac{1}{z} \right) = 1$$

$$\text{Or, } \frac{1}{x} + \frac{1}{z} = \frac{1}{y}$$

10. Question

In Fig. 4.143, $\angle A = \angle CED$, prove that $\triangle CAB \sim \triangle CED$. Also, find the value of x.



Answer

We have, $\angle A = \angle CED$

In $\triangle CAB$ and $\triangle CED$

$\angle C = \angle C$ (Common)

$\angle A = \angle CED$ (Given)

Then, $\triangle CAB \sim \triangle CED$ (By AA similarity)

So, $\frac{CA}{CE} = \frac{AB}{ED}$ (Corresponding parts of similar triangle area proportion)

$$\text{Or, } 15/9 = 9/x$$

$$\text{Or, } 15x = 90$$

$$\text{Or, } x = 90/6$$

$$\text{Or, } x = 6\text{cm.}$$

11. Question

The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm, what is the corresponding side of the other triangle?

Answer

Assume ABC and PQR to be 2 triangle.

We, have

$$\triangle ABC \sim \triangle PQR$$

$$\text{Perimeter of } \triangle ABC = 25\text{cm}$$

$$\text{Perimeter of } \triangle PQR = 15\text{cm}$$

$$AB = 9\text{cm}$$

$$PQ = ?$$

$$\text{Since, } \triangle ABC \sim \triangle PQR$$

Then, ratio of perimeter of triangles = ratio of corresponding sides

So, $\frac{25}{15} = \frac{AB}{PQ}$ (Corresponding parts of similar triangle area proportion)

$$\text{Or } \frac{25}{15} = \frac{9}{PQ}$$

$$\text{Or } PQ = 135/25$$

$$\text{Or } PQ = 5.4 \text{ cm}$$

12. Question

In $\triangle ABC$ and $\triangle DEF$, it is being given that: $AB = 5 \text{ cm}$, $BC = 4 \text{ cm}$ and $CA = 4.2 \text{ cm}$; $DE = 10 \text{ cm}$, $EF = 8 \text{ cm}$ and $FD = 8.4 \text{ cm}$. If $AL \perp BC$ and $DM \perp EF$, find $AL : DM$.

Answer

$$\text{Since } \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$$

Then, $\triangle ABC \sim \triangle DEF$ (By SS similarity)

Now, In $\triangle ABL \sim \triangle DEM$

$$\angle B = \angle E \text{ } (\triangle ABC \sim \triangle DEF)$$

$$\angle ALB = \angle DME \text{ (Each } 90^\circ)$$

Then, $\triangle ABL \sim \triangle DEM$ (By SS similarity)

$$\text{So, } \frac{AB}{DE} = \frac{AL}{DM} \text{ (Corresponding parts of similar triangle area proportion)}$$

$$\text{Or } \frac{5}{10} = \frac{AL}{DM}$$

$$\text{Or, } \frac{1}{2} = \frac{AL}{DM}$$

13. Question

D and E are the points on the sides AB and AC respectively of a $\triangle ABC$ such that AD = 8 cm, DB = 12 cm, AE = 6 cm and CE = 9 cm. Prove that BC = 5/2 DE.

Answer

We have ,

$$\frac{AD}{DB} = \frac{8}{12} = \frac{2}{3}$$

$$\text{And, } \frac{AE}{EC} = \frac{6}{9} = \frac{2}{3}$$

$$\text{Since, } \frac{AD}{DB} = \frac{AE}{EC}$$

Then , by converse of basic proportionality theorem.

$$DE \parallel BC$$

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \text{ (Common)}$$

$$\angle ADE = \angle B \text{ (Corresponding angles)}$$

Then, $\triangle ADE \sim \triangle ABC$ (By AA similarity)

$$\frac{AD}{AB} = \frac{DE}{BC} \text{ (Corresponding parts of similar triangle are proportion)}$$

$$\frac{8}{20} = \frac{DE}{BC}$$

$$\frac{2}{5} = \frac{DE}{BC}$$

$$BC = 5/2 DE$$

14. Question

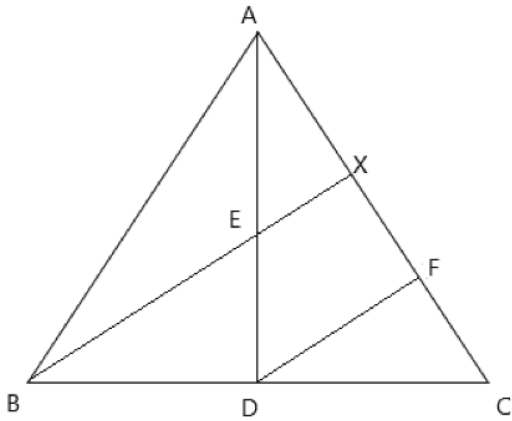
D is the mid-point of side BC of a $\triangle ABC$. AD is bisected at the point E and BE produced cuts AC at the point X. Prove that $BE : EX = 3 : 1$.

Answer

Given:- In $\triangle ABC$, D is the midpoint of BC and E is the midpoint of AD.

To prove:- $BE : EX = 3 : 1$

Proof:Const:- Through D, Draw $DF \parallel BX$



In $\triangle EAX$ and $\triangle ADF$

$$\angle EAX = \angle DAF \text{ (Common)}$$

$$\angle AXE = \angle DFA \text{ (Corresponding angles)}$$

By AA similarity,

$$\triangle EAX \sim \triangle ADF$$

$$\text{So, } \frac{EX}{DF} = \frac{AE}{AD} \text{ (Corresponding parts of similar triangle are proportion)}$$

As E is mid point of AD

$$\Rightarrow \frac{EX}{DF} = \frac{AE}{2AE}$$

$$\text{Or, } DF = 2EX. \dots\dots\dots(i)$$

In $\triangle DCF$ and $\triangle BCX$ $\angle DCY = \angle BCX$ (common) $\angle CFD = \angle CXB$ (Corresponding angles) By AA similarity, $\triangle DCF \sim \triangle BCX$

$$\text{SO, } \frac{CD}{CB} = \frac{DF}{BX} \text{ (Corresponding parts of similar triangle area proportion)}$$

As D is mid point of BC and E is mid point of AD.

$$\Rightarrow \frac{CD}{2CD} = \frac{DF}{BE + EX}$$

$$\text{Or } \frac{1}{2} = \frac{DF}{BE + EX}$$

Or $BE + EX = 2DF$ From (i)

$$BE + EX = 4EX$$

$$\Rightarrow BE = 4EX - EX$$

$$\Rightarrow BE = 4EX - EX$$

$$\Rightarrow BE = 3EX$$

$$\Rightarrow BE/EX = 3/1$$

$$\Rightarrow BE:EX = 3:1$$

15. Question

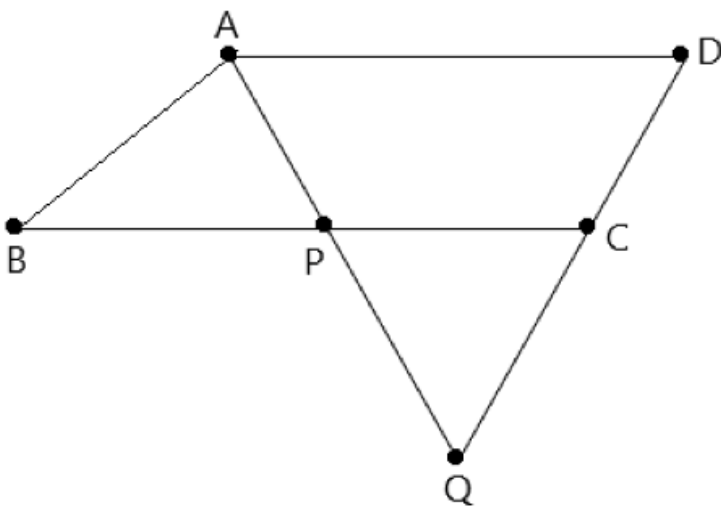
ABCD is a parallelogram and APQ is a straight line meeting BC at P and DC produced at Q. Prove that the rectangle obtained by BP and DQ is equal to the rectangle contained by AB and BC.

Answer

Given :- ABCD is a parallelogram

To prove :- $BP \times DQ = AB \times BC$

Proof:-



In $\triangle ABP$ and $\triangle QDA$

$\angle B = \angle D$ (Opposite angles of parallelogram)

$\angle BAP = \angle AQD$ (Alternative interior angle)

Then, $\triangle ABP \sim \triangle QDA$

So, $\frac{AB}{QD} = \frac{BP}{DA}$ (Corresponding parts of similar triangle area proportion) But, $DA = BC$ (Opposite side of parallelogram) But $DA = BC$ (opposite sides of parallelogram)

Then, $\frac{AB}{QD} = \frac{BP}{BC}$

Or, $AB \times BC = QD \times BP$ Hence proved

16. Question

In $\triangle ABC$, AL and CM are the perpendiculars from the vertices A and C to BC and AB respectively. If AL and CM intersect at O , prove that :

(i) $\triangle OMA \sim \triangle OLC$

(ii) $\frac{OA}{OC} = \frac{OM}{OL}$

Answer

We have

$AL \perp BC$ and $CM \perp AB$

In $\triangle OMA$ and $\triangle OLC$

$\angle MOA = \angle LOC$ (Vertically opposite angles)

$\angle AMO = \angle OLC$ (Each 90°)

Then, $\triangle OMA \sim \triangle OLC$ (By AA Similarity)

So, $\frac{OA}{OC} = \frac{OM}{OL}$ (Corresponding parts of similar triangle area proportion)

17. Question

In fig. 4.144, we have $AB \parallel CD \parallel EF$. If $AB = 6$ cm, $CD = x$ cm, $EF = 10$ cm, $BD = 4$ cm and $DE = y$ cm, calculate the values of x and y .

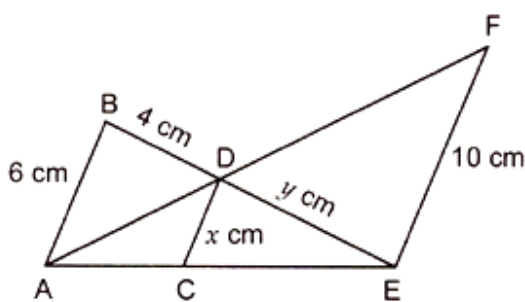


Fig. 4.144

Answer

We have $AB \parallel CD$. If $AB = 6$ cm, $CD = x$ cm, $EF = 10$ cm, $BD = 4$ cm and $DE = y$ cm

In $\triangle ECD$ and $\triangle EAB$

$\angle ECD = \angle EAB$ (Corresponding angles)

Then, $\triangle ECD \sim \triangle EAB$ (i) (By AA similarity)

So, $\frac{EC}{EA} = \frac{CD}{AB}$ (Corresponding parts of similar triangle are proportion)

Or $\frac{EC}{EA} = \frac{x}{6}$ (ii)

In $\triangle ACD$ and $\triangle AEF$

$\angle CAD = \angle EAF$ (Common)

$\angle ACD = \angle AEF$ (Corresponding angles)

Then, $\triangle ACD \sim \triangle AEF$ (By AA similarity)

So, $\frac{AC}{AE} = \frac{CD}{EF}$

Or, $\frac{AC}{AE} = \frac{x}{10}$ (iii)

Adding equation iii & ii

So, $\frac{AC}{AE} + \frac{EC}{EA} = \frac{x}{6} + \frac{x}{10}$

Or, $\frac{AE}{AE} = \frac{5x+3x}{30}$

Or, $1 = \frac{8x}{30}$

Or, $x = \frac{30}{8}$

Or, $x = 3.75\text{cm}$

From (i) $\frac{DC}{AB} = \frac{CD}{BE}$

Or, $\frac{3.75}{6} = \frac{y}{y+4}$

Or, $6y = 3.75y + 15$

Or, $2.25y = 15$

Or, $y = \frac{15}{2.25}$

Or, $y = 6.67\text{cm}$

18. Question

ABCD is a quadrilateral in which $AD = BC$. If P, Q, R, S be the mid-points of AB, AC, CD and BD respectively, show that PQRS is a rhombus.

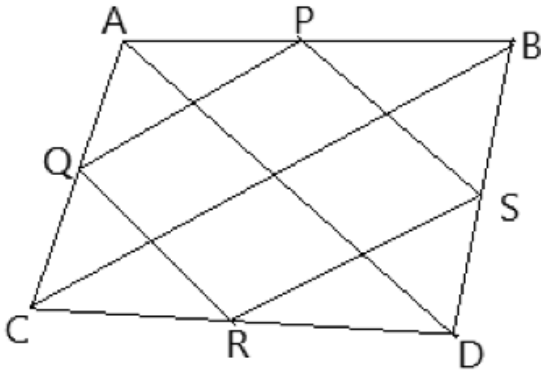
Answer

Given: ABCD is a quadrilateral in which $AD = BC$. P, Q, R, S be the mid-points of AB, AC, CD and BD respectively.

To show: PQRS is a rhombus.

Solution: So, we have, a quadrilateral ABCD where $AD = BC$

And P, Q, R and S are the mid-point of the sides AB, AC, and BD.



We need to prove that PQRS is a rhombus.

In $\triangle BAD$, P and S are the mid points of the sides AB and BD respectively, By midpoint theorem which states that the line joining mid-points of a triangle is parallel to third side we get,

$PS \parallel AD$ and $PS = \frac{1}{2} AD$(i)

In $\triangle CAD$, Q and R are the mid points of the sides CA and CD respectively, by midpoint theorem we get,

$QR \parallel AD$ and $QR = \frac{1}{2} AD$ (ii)

Compare (i) and (ii)

$PS \parallel QR$ and $PS = QR$

Since one pair of opposite sides is equal and parallel,

Then, we can say that PQRS is a parallelogram.....(iii)

Now, In $\triangle ABC$, P and Q are the mid points of the sides AB and AC respectively, by midpoint theorem,

$PQ \parallel BC$ and $PQ = \frac{1}{2} BC$(iv)

And $AD = BC$ (v) (given)

Compare equations (i) (iv) and (v), we get,

$PS = PQ$ (vi)

From (iii) and (vi), we get,

$PS = QR = PQ$ Therefore, PQRS is a rhombus.

19. Question

In Fig. 4.145, If $AB \perp BC$, $DC \perp BC$ and $DE \perp AC$, prove that $\triangle CED \sim \triangle ABC$.

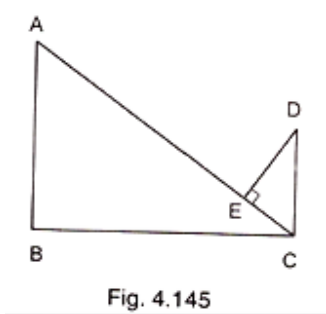


Fig. 4.145

Answer

Given $AB \perp BC$, $DC \perp BC$ and $DE \perp AC$

To prove:- $\triangle CED \sim \triangle ABC$

Proof:-

$\angle BAC + \angle BCA = 90^\circ$ (i) (By angle sum property)

And, $\angle BCA + \angle ECD = 90^\circ$ (ii) ($DC \perp BC$ given)

Compare equation (i) and (ii)

$\angle BAC = \angle ECD$ (iii)

In $\triangle CED$ and $\triangle ABC$

$\angle CED = \angle ABC$ (Each 90°)

$\angle ECD = \angle BAC$ (From equation iii)

Then, $\triangle CED \sim \triangle ABC$.

20. Question

In an isosceles $\triangle ABC$, the base AB is produced both the ways to P and Q such that $AP \times BQ = AC^2$. Prove that $\triangle APC \sim \triangle BCQ$.

Answer

Given : In $\triangle ABC$, $CA = CB$ and $AP \times BQ = AC^2$

To prove :- $\triangle APC \sim \triangle BCQ$

Proof:-

$AP \times BQ = AC^2$ (Given)

Or, $AP \times BC = AC \times AC$

Or, $AP \times BC = AC \times BC$ ($AC = BC$ given)

Or, $AP/BC = AC/PQ$ (i)

Since, $CA = CB$ (Given)

Then, $\angle CAB = \angle CBA$ (ii) (Opposite angle to equal sides)

NOW, $\angle CAB + \angle CAP = 180^\circ$ (iii) (Linear pair of angle)

And $\angle CBA + \angle CBQ = 180^\circ$ (iv) (Linear pair of angle)

Compare equation (ii) (iii) & (iv)

$\angle CAP = \angle CBQ$(v)

In $\triangle APC$ and $\triangle BCQ$

$\angle CAP = \angle CBQ$ (From equation v)

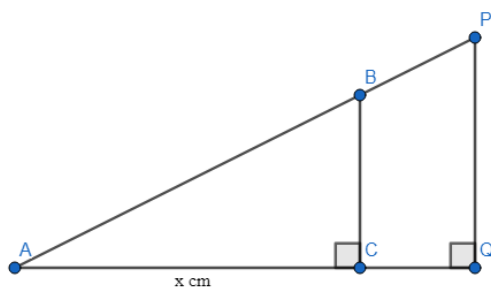
$AP/BC = AC/PQ$ (From equation i)

Then , $\triangle APC \sim \triangle BCQ$ (By SAS similarity)

21. Question

A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/sec. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Answer



We have,

Let P be a lamp at a height of 3.6 m above that ground i.e. $PQ = 3.6$ m

Let BC be a girl, such that CQ is distance she covered and Let AC be her shadow, Height of girl = $AB = 90\text{cm} = 0.9\text{m}$

Height of lamp post = $PQ = 3.6\text{m}$

Speed of girl = 1.2 m/sec

So, Distance moved by the girl(CQ) = Speed x time

$= 1.2 \times 4 = 4.8\text{m}$

Let length of shadow (AC) = 'x' cm

Then, $AQ = AC + CQ = x + 4.8$

In $\triangle ABC$ and $\triangle APQ$

$\angle ACB = \angle AQP$ (Each 90°)

$\angle BAC = \angle PAQ$ (Common)

Then , $\triangle ABC \sim \triangle APQ$ (By AA similarity)

So, $AC/AQ = BC/ PQ$ (Corresponding parts of similar triangle are proportional)

$$\text{Or, } x/x + 4.8 = 0.9/3.6$$

$$\text{Or, } x/x + 4.8 = 1/4$$

$$\text{Or, } 4x = x + 4.8$$

$$\text{Or, } 4x - x = 4.8$$

$$\text{Or, } 3x = 4.8$$

$$\text{Or } x = 4.8/3$$

Or $x = 1.6$ m. i.e. length of shadow is 1.6 m.

22. Question

Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Answer

We have,

ABCD is a trapezium with $AB \parallel DC$

In $\triangle AOB$ and $\triangle COD$ $\angle AOB = \angle COD$ (Vertically opposite angle)

$\angle OAB = \angle OCD$ (Alternate interior angle)

Then, $\triangle AOB \sim \triangle COD$ (By AA similarity)

So, $OA/OC = OB/OD$ (Corresponding parts of similar triangle are proportional)

23. Question

If $\triangle ABC$ and $\triangle AMP$ are two right triangles, right angled at B and M respectively such that $\angle MAP = \angle BAC$. Prove that

$$(i) \triangle ABC \sim \triangle AMP$$

$$(ii) \frac{CA}{PA} = \frac{BC}{MP}$$

Answer

We have,

$$\angle B = \angle M = 90^\circ$$

$$\text{And, } \angle BAC = \angle MAP$$

In $\triangle ABC$ and $\triangle AMP$

$$\angle B = \angle M \text{ (each } 90^\circ)$$

$$\angle BAC = \angle MAP \text{ (Given)}$$

Then, $\triangle ABC \sim \triangle AMP$ (By AA similarity)

So, $CA/PM = BC/MP$ (Corresponding parts of similar triangle are proportional)

24. Question

A vertical stick of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer

Let AB be a tower

CD be a stick, $CD = 6\text{ m}$

Shadow of AB is $BE = 28\text{ m}$

Shadow of CD is $DF = 4\text{ m}$

At same time light rays from sun will fall on tower and stick at the same angle

So, $\angle DCF = \angle BAE$

And $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$ (Tower and stick are vertically to ground)

Therefore $\triangle ABE \sim \triangle CDF$ (By AAA similarity)

So, $AB/CD = BE/DF$

$AB/6 = 28/4$

$AB/6 = 7$

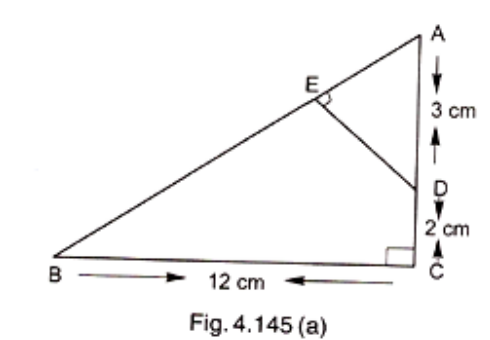
$AB = 7 \times 6$

$AB = 42\text{ m}$

So, height of tower will be 42 meter.

25. Question

In Fig. 4.145 (a) $\triangle ABC$ is right angled at C and $DE \perp AB$. Prove that $\triangle ABC \sim \triangle ADE$ and hence find the lengths of AE and DE.



Answer

In $\triangle ABC$, by Pythagoras theorem

$$AB^2 = AC^2 + BC^2$$

$$\text{Or, } AB^2 = 5^2 + 12^2$$

$$\text{Or, } AB^2 = 25 + 144$$

$$\text{Or, } AB^2 = 169$$

$$\text{Or } AB = 13 \text{ (Square root both side)}$$

In $\triangle AED$ and $\triangle ACB$

$$\angle A = \angle A \text{ (Common)}$$

$$\angle AED = \angle ACB \text{ (Each } 90^\circ)$$

Then, $\triangle AED \sim \triangle ACB$ (Corresponding parts of similar triangle are proportional)

$$\text{So, } AE/AC = DE/CB = AD/AB$$

$$\text{Or, } AE/5 = DE/12 = 3/13$$

$$\text{Or, } AE/5 = 3/13 \text{ and } DE/12 = 3/13$$

$$\text{Or, } AE = 15/13 \text{ cm and } DE = 36/13 \text{ cm}$$

Exercise 4.6

1. Question

Triangles ABC and DEF are similar.

(i) If area ($\triangle ABC$) = 16 cm^2 , area ($\triangle DEF$) = 25 cm^2 and $BC = 2.3 \text{ cm}$, find EF.

(ii) If area ($\triangle ABC$) = 9 cm^2 , area ($\triangle DEF$) = 64 cm^2 and $DE = 5.1 \text{ cm}$, find AB.

(iii) If $AC = 19 \text{ cm}$ and $DF = 8 \text{ cm}$, find the ratio of the area of two triangles.

(iv) If area ($\triangle ABC$) = 36 cm^2 , area ($\triangle DEF$) = 64 cm^2 and $DE = 6.2 \text{ cm}$, find AB.

(v) If $AB = 1.2 \text{ cm}$ and $DE = 1.4 \text{ cm}$, find the ratio of the areas of $\triangle ABC$ and $\triangle DEF$.

Answer

(i) We have

$$\triangle ABC \sim \triangle DEF$$

$$\text{Area } (\triangle ABC) = 16 \text{ cm}^2$$

$$\text{Area } (\triangle DEF) = 25 \text{ cm}^2$$

$$\text{And } BC = 2.3 \text{ cm}$$

$$\text{Since, } \triangle ABC \sim \triangle DEF$$

$$\text{Then, } \text{Area } (\triangle ABC) / \text{Area } (\triangle DEF)$$

$$= BC^2/EF^2 \text{ (By area of similar triangle theorem)}$$

$$\text{Or, } 16/25 = (23)^2/EF^2$$

$$\text{Or, } 4/5 = 2.3/EF \text{ (By taking square root)}$$

$$\text{Or, } EF = 11.5/4$$

$$\text{Or, } EF = 2.875\text{cm}$$

(ii) We have

$$\Delta ABC \sim \Delta DEF$$

$$\text{Area } (\Delta ABC) = 9\text{cm}^2$$

$$\text{Area } (\Delta DEF) = 64\text{cm}^2$$

$$\text{And } BC = 5.1\text{cm}$$

$$\text{Since, } \Delta ABC \sim \Delta DEF$$

$$\text{Then, Area } (\Delta ABC)/\text{Area } (\Delta DEF)$$

$$= AB^2/DE^2 \text{ (By area of similar triangle theorem)}$$

$$\text{Or, } 9/64 = AB^2/(5.1)^2$$

$$\text{Or, } AB = 3 \times 5.1/8 \text{ (By taking square root)}$$

$$\text{Or, } AB = 1.9125\text{cm}$$

(iii) We have,

$$\Delta ABC \sim \Delta DEF$$

$$AC = 19\text{cm and } DF = 8\text{cm}$$

By area of similar triangle theorem

$$\text{Then, Area of } \Delta ABC/\text{Area of } \Delta DEF = AC^2/DE^2 \text{ (By area of similar triangle theorem)}$$

$$(19)^2/(8)^2 = 364/64$$

(iv) We have

$$\text{Area } \Delta ABC = 36\text{cm}^2$$

$$\text{Area } \Delta DEF = 64 \text{ cm}^2$$

$$DE = 6.2 \text{ cm}$$

$$\text{And, } \Delta ABC \sim \Delta DEF$$

By area of similar triangle theorem

$$\text{Area of } \Delta ABC/\text{Area of } \Delta DEF = AB^2/DE^2$$

$$\text{Or, } 36/64 = 6 \times 6.2/8 \text{ (By taking square root)}$$

Or, $AB = 4.65\text{cm}$

(V) We have

$$\Delta ABC \sim \Delta DEF$$

$$AB = 12\text{cm and } DF = 1.4\text{ cm}$$

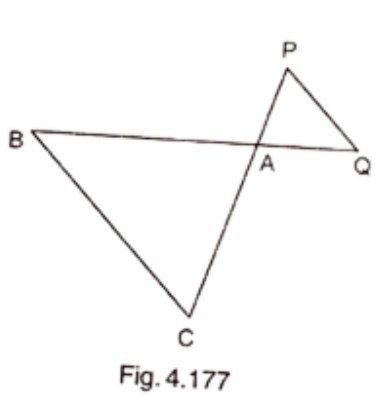
By area of similar triangle theorem

$$\text{Area of } \Delta ABC / \text{Area of } \Delta DEF = AB^2 / DE^2$$

$$\text{Or, } (1.2)^2 / (1.4)^2 = 1.44x / 1.96$$

2. Question

In Fig. 4.177, $\Delta ACB \sim \Delta APQ$. If $BC = 10\text{ cm}$, $PQ = 5\text{ cm}$, $BA = 6.5\text{ cm}$ and $AP = 2.8\text{ cm}$, find CA and AQ . Also, find the area (ΔACB) : area (ΔAPQ).



Answer

We have,

$$\Delta ACB \sim \Delta APQ$$

Then, $AC/AP = CB/PQ = AB/AQ$ [Corresponding parts of similar Δ are proportional]

$$\text{Or, } AC/2.8 = 10/5 = 6.5/AQ$$

$$\text{Or, } AC/2.8 = 10/5 \text{ and } 10/5 = 6.5/AQ$$

$$\text{Or, } AC = 5.6\text{cm and } AQ = 3.25\text{cm}$$

By area of similar triangle theorem

$$\text{Area of } \Delta ACB / \text{Area of } \Delta APQ = BC^2 / PQ^2$$

$$= (10)^2 / (5)^2$$

$$= 100/25$$

$$= 4\text{ cm}$$

3. Question

The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. Find the ratio of their corresponding heights. What is the ratio of their corresponding medians?

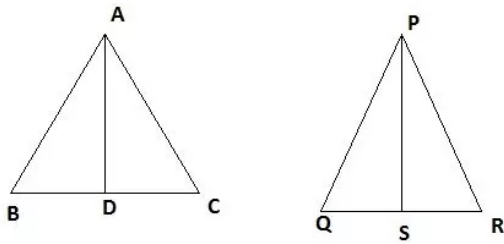
Answer

Given : $\Delta ABC \sim \Delta PQR$

$$\text{Area } (\Delta ABC) = 81 \text{ cm}^2$$

$$\text{Area } (\Delta PQR) = 49 \text{ cm}^2$$

Figure:



And AD and PS are the altitudes

By area of similar triangle theorem: The ratio of the areas of two similar triangles equal to the ratio of squares of the corresponding sides of triangles.

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AB^2}{PQ^2}$$

$$\frac{AB^2}{PQ^2} = \frac{81}{49}$$

$$\frac{AB}{PQ} = \sqrt{\frac{81}{49}}$$

$$\frac{AB}{PQ} = \frac{9}{7}$$

We also know that:

$$\frac{AD}{PS} = \frac{AB}{PQ}$$

$$\text{Therefore, } \frac{AD}{PS} = \frac{9}{7}$$

So, Ratio of altitudes = $9/7$

Hence, ratio of altitudes = Ratio of medians = $9:7$

4. Question

The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm , find the longest side of the smaller triangle.

Answer

We have,

$$\Delta ABC \sim \Delta PQR$$

$$\text{Area}(\Delta ABC) = 169\text{cm}^2$$

$$\text{Area}(\Delta PQR) = 121\text{ cm}^2$$

$$\text{And } AB = 26\text{ cm}$$

By area of similar triangle theorem

$$\text{Area of } \Delta ABC / \text{Area of } \Delta PQR = AB^2 / PQ^2$$

$$\text{Or, } 169/121 = 26^2 / PQ^2$$

$$\text{Or, } 13/11 = 26/PQ \text{ (Taking square root)}$$

$$\text{Or, } PQ = 11/13 \times 26$$

$$\text{Or, } PQ = 22\text{cm}$$

5. Question

Two isosceles triangles have equal vertical angles and their areas are in the ratio 36 : 25.. Find the ratio of their corresponding heights.

Answer

Given : - $AB = AC$, $PQ = PR$ and $\angle A = \angle P$

And AD and PS are altitudes

$$\text{And, Area}(\Delta ABC) / \text{Area of}(\Delta PQR) = 36/25 \dots\dots\dots(i)$$

To find: AD/PS

Proof:- Since, $AB = AC$ and $PQ = PR$

$$\text{Then, } AB/AC = 1 \text{ and } PQ/PR = 1$$

$$\text{So, } AB/AC = PQ/PR$$

$$\text{Or, } AB/PQ = AC/PR \dots\dots\dots(ii)$$

In ΔABC and ΔPQR

$$\angle A = \angle P \text{ (Given)}$$

$$AB/PQ = AC/PR \text{ (From equation ii)}$$

Then, $\Delta ABC \sim \Delta PQR$ (BY AA similarity)

$$\text{So, Area of } \Delta ABC / \text{Area of } \Delta PQR = AB^2 / PQ^2 \dots\dots(iii) \text{ (By area of similar triangle)}$$

Compare equation I and II

$$AB^2/PQ^2 = 36/25$$

$$\text{Or, } AB/PQ = 6/5$$

In $\triangle ABD$ and $\triangle PQS$

$$\angle B = \angle Q \text{ (}\triangle ABC \sim \triangle PQR\text{)}$$

$$\angle ADB = \angle PSQ \text{ (Each } 90^\circ\text{)}$$

Then, $\triangle ABD \sim \triangle PQS$ (By AA similarity)

$$\text{So, } AB/PQ = AD/PS$$

$$6/5 = AD/PS \text{ (From iv)}$$

6. Question

The areas of two similar triangles are 25 cm^2 and 36 cm^2 respectively. If the altitude of the first triangle is 2.4 cm , find the corresponding altitude of the other.

Answer

We have,

$$\triangle ABC \sim \triangle PQR$$

$$\text{Area } (\triangle ABC) = 25 \text{ cm}^2$$

$$\text{Area } (PQR) = 36 \text{ cm}^2$$

$$\text{And } AD = 2.4 \text{ cm}$$

And AD and PS are the altitudes

To find: PS

Proof: Since, $\triangle ABC \sim \triangle PQR$

Then, by area of similar triangle theorem

$$\text{Area of } \triangle ABC / \text{Area of } \triangle PQR = AB^2 / PQ^2$$

$$25/36 = AB^2/PQ^2$$

$$5/6 = AB/PQ \dots\dots\dots(i)$$

In $\triangle ABD$ and $\triangle PQS$

$$\angle B = \angle Q \text{ (}\triangle ABC \sim \triangle PQR\text{)}$$

$$\angle ADB = \angle PSQ \text{ (Each } 90^\circ\text{)}$$

Then, $\triangle ABD \sim \triangle PQS$ (By AA similarity)

$$\text{So, } AB/PS = AD/PS \dots\dots\dots(ii) \text{ (Corresponding parts of similar } \triangle \text{ are proportional)}$$

Compare (i) and (ii)

$$AD/PS = 5/6$$

$$2.4/PS = 5/6$$

$$PS = 2.4 \times 6/5$$

$$PS = 2.88\text{cm}$$

7. Question

The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Answer

We have,

$$\Delta ABC \sim \Delta PQR$$

$$AD = 6\text{cm}$$

$$PS = 9\text{cm}$$

By area of similar triangle theorem

$$\text{Area of } \Delta ABC / \text{Area of } \Delta PQR = AB^2 / PQ^2 \dots\dots\dots(i)$$

In ΔABD and ΔPQS

$$\angle B = \angle Q \text{ } (\Delta ABC \sim \Delta PQS)$$

$$\angle ADB = \angle PSQ \text{ (Each } 90^\circ)$$

Then, $\Delta ABD \sim \Delta PQS$ (By AA Similarity)

So, $AB/PQ = AD/PS$ (Corresponding parts of similar Δ are proportional)

$$\text{Or, } AB/PQ = 6/9$$

$$\text{Or, } AB/PQ = 2/3 \dots\dots\dots(ii)$$

Compare equation (i) and (ii)

$$\text{Area of } \Delta ABC / \text{Area of } \Delta PQR = (2/3)^2 = 4/9$$

8. Question

ABC is a triangle in which $\angle A = 90^\circ$, $AN \perp BC$, $BC = 12$ cm and $AC = 5$ cm. Find the ratio of the areas of ΔANC and ΔABC .

Answer

In ΔANC and ΔABC

$$\angle C = \angle C \text{ (Common)}$$

$$\angle ANC = \angle BAC \text{ (Each } 90^\circ)$$

Then, $\Delta ANC \sim \Delta ABC$ (By AA similarity)

By area of similarity triangle theorem.

$$\text{Area of } \triangle ABC / \text{Area of } \triangle PQR = AC^2 / BC^2$$

$$\text{Or, } 5^2 / 12^2$$

$$\text{Or, } 25 / 144$$

9. Question

In Fig. 4.178, $DE \parallel BC$

(i) If $DE = 4$ cm, $BC = 6$ cm and area ($\triangle ADE$) = 16 cm^2 , find the area of $\triangle ABC$.

(ii) If $DE = 4$ cm, $BC = 8$ cm and area ($\triangle ADE$) = 25 cm^2 , find the area of $\triangle ABC$.

(iii) If $DE : BC = 3 : 5$. Calculate the ratio of the areas of $\triangle ADE$ and the trapezium BCED.

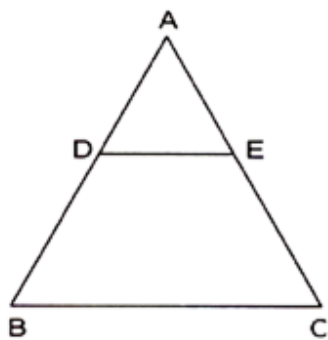


Fig. 4.178

Answer

(i) We have, $DE \parallel BC$, $DE = 4$ cm, $BC = 6$ cm and area ($\triangle ADE$) = 16 cm^2

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \text{ (Common)}$$

$$\angle ADE = \angle ABC \text{ (Corresponding angles)}$$

Then, $\triangle ADE \sim \triangle ABC$ (BY AA similarity)

So, By area of similar triangle theorem

$$\text{Area of } \triangle ADE / \text{Area of } \triangle ABC = DE^2 / BC^2$$

$$16 / \text{Area of } \triangle ABC = 4^2 / 6^2$$

$$\text{Or, Area } (\triangle ABC) = 16 \times 36 / 16$$

$$= 36 \text{ cm}^2$$

(ii) We have, $DE \parallel BC$, $DE = 4$ cm, $BC = 8$ cm and area ($\triangle ADE$) = 25 cm^2

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \text{ (Common)}$$

$\angle ADE = \angle ABC$ (Corresponding angles)

Then, $\triangle ADE \sim \triangle ABC$ (BY AA similarity)

So, By area of similar triangle theorem

$$\text{Area of } \triangle ADE / \text{Area of } \triangle ABC = DE^2 / BC^2$$

$$25 / \text{Area of } \triangle ABC = 4^2 / 8^2$$

$$\text{Or, Area } (\triangle ABC) = 25 \times 64 / 16$$

$$= 100 \text{ cm}^2$$

(iii) We have $DE \parallel BC$, And $DE/BC = 3/5$ (i)

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \text{ (Common)}$$

$$\angle ADE = \angle ABC \text{ (Corresponding angles)}$$

Then, $\triangle ADE \sim \triangle ABC$ (BY AA similarity)

So, By area of similar triangle theorem

$$\text{Area of } \triangle ADE / \text{Area of } \triangle ABC = DE^2 / BC^2$$

$$\text{Area of } \triangle ADE / \text{Area of } \triangle ADE + \text{Area of trap. DECB} = 3^2 / 5^2$$

$$\text{Or, } 25 \text{ area } \triangle ADE = 9 \text{ Area of } \triangle ADE + 9 \text{ Area of trap. DECB}$$

$$\text{Or } 25 \text{ area } \triangle ADE - 9 \text{ Area of } \triangle ADE = 9 \text{ Area of trap. DECB}$$

$$\text{Or, } 16 \text{ area } \triangle ADE = 9 \text{ Area of trap. DECB}$$

$$\text{Or, area } \triangle ADE / \text{Area of trap. DECB} = 9/16$$

10. Question

In $\triangle ABC$, D and E are the mid-points of AB and AC respectively. Find the ratio of the areas of $\triangle ADE$ and $\triangle ABC$.

Answer

We have, D and E as the midpoint of AB and AC

So, according to the midpoint theorem

$$DE \parallel BC \text{ and } DE = 1/2 BC \text{(i)}$$

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \text{ (Common)}$$

$$\angle ADE = \angle B \text{ (Corresponding angles)}$$

Then, $\triangle ADE \sim \triangle ABC$ (By AA similarity)

By area of similar triangle theorem

$$\text{Area } \triangle ADE / \text{Area } \triangle ABC = DE^2 / BC^2$$

$$\text{Or, } (1/2 BC)^2 / (BC)^2$$

$$\text{Or, } 1/4$$

11. Question

In Fig. 4.179, $\triangle ABC$ and $\triangle DBC$ are on the same base BC. If AD and BC intersect at O. Prove that

$$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DBC)} = \frac{AO}{DO}$$

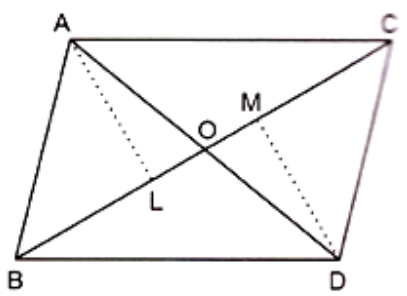


Fig. 4.179

Answer

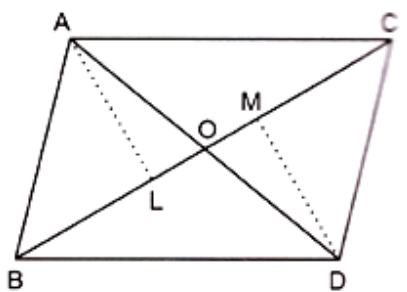


Fig. 4.179

We know that area of a triangle = $1/2 \times \text{base} \times \text{height}$

Since, $\triangle ABC$ and $\triangle DBC$ are on the same base.

Therefore ratio between their areas will be as ratio of their heights.

Let us draw two perpendiculars AP and DM on line BC

In $\triangle ALO$ and $\triangle DMO$,

$$\angle ALO = \angle DMO \text{ (Each is } 90^\circ)$$

$$\angle AOL = \angle DOM \text{ (Vertically opposite angle)}$$

$$\angle OAL = \angle ODM \text{ (remaining angle)}$$

Therefore $\triangle ALO \sim \triangle DMO$ (By AAA rule)

$$\text{Therefore } AL/DM = AO/DO$$

Therefore, $\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DBC)} = \frac{AO}{DO}$

12. Question

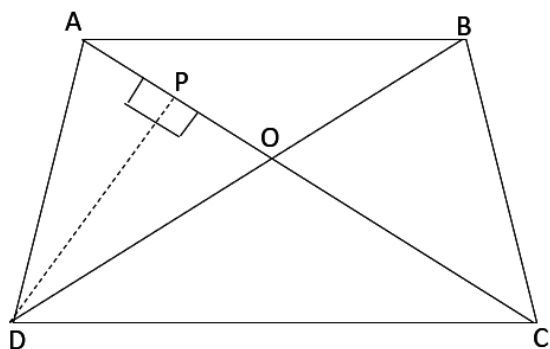
ABCD is a trapezium in which $AB \parallel CD$. The diagonals AC and BD intersect at O. Prove that : (i) $\triangle AOB \sim \triangle COD$

(ii) If $OA = 6$ cm, $OC = 8$ cm, Find:

(a) $\frac{\text{Area}(\triangle AOB)}{\text{Area}(\triangle COD)}$ (b) $\frac{\text{Area}(\triangle AOD)}{\text{Area}(\triangle COD)}$

Answer

We have,



$AB \parallel DC$

In $\triangle AOB$ and $\triangle COD$ $\angle AOB = \angle COD$ (Vertically opposite angles)

$\angle OAB = \angle OCD$ (Alternate interior angle)

Then , $\triangle AOB \sim \triangle COD$ (By AA similarity)

(a) By area of similar triangle theorem.

$$\frac{\text{Area of } \triangle AOB}{\text{Area of } \triangle COD} = \frac{OA^2}{OC^2}$$

$$\Rightarrow \frac{\text{Area of } \triangle AOB}{\text{Area of } \triangle COD} = \frac{6^2}{8^2}$$

$$\Rightarrow \frac{\text{Area of } \triangle AOB}{\text{Area of } \triangle COD} = \frac{36}{64} = \frac{9}{16}$$

b) Draw $DP \perp AC$

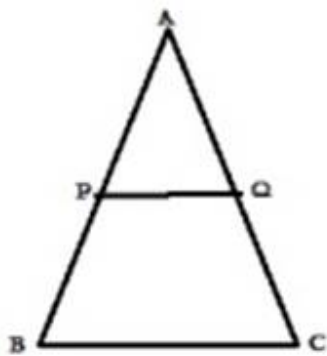
$$\Rightarrow \frac{\text{Area of } \triangle AOD}{\text{Area of } \triangle COD} = \frac{\frac{1}{2} \times OA \times DP}{\frac{1}{2} \times OC \times DP}$$

$$\Rightarrow \frac{\text{Area of } \triangle AOD}{\text{Area of } \triangle COD} = \frac{6}{8} = \frac{3}{4}$$

13. Question

In $\triangle ABC$, P divides the side AB such that $AP : PB = 1 : 2$. Q is a point in AC such that $PQ \parallel BC$. Find the ratio of the areas of $\triangle APQ$ and trapezium BPQC.

Answer



We know

$PQ \parallel BC$

1 = AP

2 = PB

In $\triangle APQ$ and $\triangle ABC$

$\angle A = \angle A$ [Common]

$\angle APQ = \angle B$ [Corresponding angle]

$\triangle ABC \sim \triangle APQ$

$\text{Area}(\triangle APQ) = AP^2$

$\text{Area}(\triangle ABC) = AB^2$

$\frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ABC)} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{1}{3}\right)^2$

$\text{ar}(\triangle APQ) + \text{ar}(\text{trap}BPQC)$

$9\text{ar}(\triangle APQ) = \text{ar}(\triangle ABC) = \text{ar}(\triangle APQ) + \text{ar}(\text{trap}BPQC)$

$$9\text{ar}(\triangle APQ) - \text{ar}(\triangle APQ) = \text{ar}(\text{trap}BPQC)$$

$$8\text{ar}(\triangle APQ) = \text{ar}(\text{trap}BPQC)$$

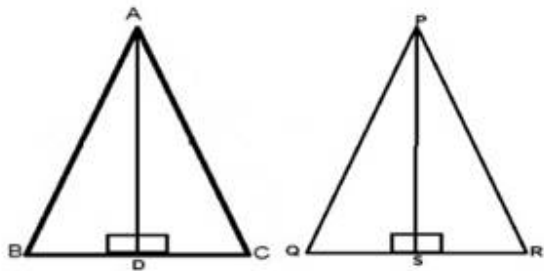
$$\text{ar}(\triangle APQ) = \frac{1}{8}$$

$$\text{ar}(\text{trap}BPQC)$$

14. Question

The areas of two similar triangles are 100 cm^2 and 49 cm^2 respectively. If the altitude of the bigger triangle is 5 cm , find the corresponding altitude of the other.

Answer



We have,

$$\triangle ABC \sim \triangle PQR$$

$$\text{Area}(\triangle ABC) = 100 \text{ cm}^2$$

$$\text{Area}(\triangle PQR) = 49 \text{ cm}^2$$

$$AD = 5 \text{ cm}$$

AD and PS are the altitudes

by area of similar triangle theorem

$$\text{Area}(\triangle ABC) = AB^2$$

$$\text{Area}(\triangle PQR) = PQ^2$$

$$AB^2 = 100/49$$

$$PQ^2$$

$$AB/PQ = 10/7 \dots\dots\dots(i)$$

In $\triangle ABD$ and $\triangle PQS$

$$\angle B = \angle Q [\triangle ABC \sim \triangle PQR]$$

$$\angle ADB = \angle PQS = 90^\circ$$

$$\triangle ABD \sim \triangle PQS [\text{By AA similarity}]$$

$$AB/PQ=AD/PS \dots\dots(ii)$$

Compare equ. (i) and (ii)

$$AD/PS=10/7$$

$$5/PS=10/7$$

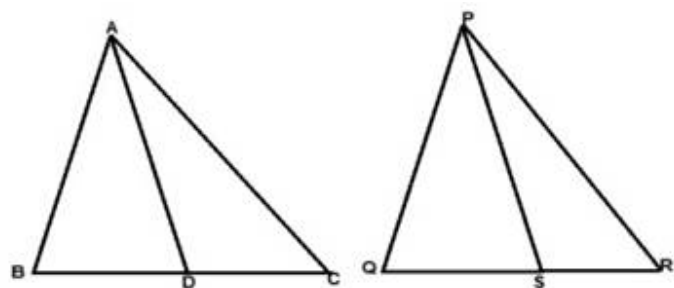
$$PS=35/10$$

$$PS=3.5 \text{ cm}$$

15. Question

The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm , find the corresponding median of the other.

Answer



We have,

$$\triangle ABC \sim \triangle PQR$$

$$\text{Area}(\triangle ABC) = 121 \text{ cm}^2$$

$$\text{Area}(\triangle PQR) = 64 \text{ cm}^2$$

$$AD = 12.1 \text{ cm}$$

AD and PS are the medians

By area of similar triangle theorem

$$\text{Area}(\triangle ABC) = AB^2$$

$$\text{Area}(\triangle PQR) = PQ^2$$

$$AB^2 = 121$$

$$PQ^2 = 64$$

$$AB = 11 \dots\dots\dots (i)$$

$$PQ = 8$$

$$\triangle ABC \sim \triangle PQR$$

$AB/PQ=BC/QR$ [Corresponding parts of similar triangles are proportional] $AB/PQ=2BD/2QS$ [AD and BD are medians]

$$AB/PQ=BD/QS \dots\dots\dots (ii)$$

In $\triangle ABD$ and $\triangle PQS$

$$\angle B = \angle Q \text{ [} \triangle ABC \sim \triangle PQS \text{]}$$

$$AB/PQ=BD/QS \text{ [from (ii)]}$$

$$\triangle ABD \sim \triangle PQS \text{ [By AA similarity]}$$

$$AB/PQ=AD/PS \text{ Compare equ. (i) and (ii)}$$

$$AD/PS=11/8$$

$$12.1/PS=11/8$$

$$PS=12.1 \times 8/11$$

$$PS= 8.8 \text{ cm}$$

16. Question

If $\triangle ABC \sim \triangle DEF$ such that $AB = 5 \text{ cm}$, $\text{area}(\triangle ABC) = 20 \text{ cm}^2$ and $\text{area}(\triangle DEF) = 45 \text{ cm}^2$, determine DE.

Answer

We have

$$\triangle ABC \sim \triangle DEF$$

$$\text{Where } AB = 5 \text{ cm}$$

$$\text{Area}(\triangle ABC) = 20 \text{ cm}^2$$

$$\text{Area}(\triangle DEF) = 45 \text{ cm}^2$$

By area of similar triangle theorem

$$\text{Area}(\triangle ABC) = AB^2$$

$$\text{Area}(\triangle DEF) = DE^2$$

$$5^2/DE^2 = 20/45$$

$$25/DE^2 = 4/9$$

$$5/DE = 2/3$$

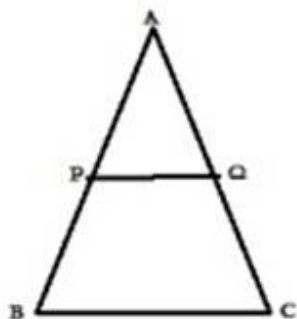
$$DE = 3 \times 5/2$$

$$DE = 7.5 \text{ cm}$$

17. Question

In $\triangle ABC$, PQ is a line segment intersecting AB at P and AC at Q such that $PQ \parallel BC$ and PQ divides $\triangle ABC$ into two parts equal in area. Find $\frac{BP}{AB}$.

Answer



We know

$PQ \parallel BC$

Area ($\triangle APQ$) = Area (*trap* $PQCB$)

Area ($\triangle APQ$) = Area ($\triangle ABC$) - Area ($\triangle APQ$)

2Area ($\triangle APQ$) = Area ($\triangle ABC$)(i)

In $\triangle APQ$ and $\triangle ABC$

$\angle A = \angle A$ [Common]

$\angle APQ = \angle B$ [Corresponding angle]

$\triangle ABC \sim \triangle APQ$

Area($\triangle APQ$) = AP^2

Area ($\triangle ABC$) = AB^2

Area($\triangle APQ$) = AP^2

Area ($\triangle APQ$) = AB^2 [By using (i)]

$1 = \frac{AP^2}{AB^2}$

$2 = \frac{AB^2}{AB^2}$

$\frac{1}{\sqrt{2}} = \frac{AP}{AB}$

$\frac{1}{\sqrt{2}} = \frac{AB - BP}{AB}$

$\frac{1}{\sqrt{2}} = \frac{AB}{AB} - \frac{BP}{AB}$

$$\frac{1}{\sqrt{2}} = 1 - BP/AB$$

$$BP/AB = 1 - \frac{1}{\sqrt{2}}$$

$$BP/AB = \frac{\sqrt{2}-1}{\sqrt{2}}$$

18. Question

The areas of two similar triangles ABC and PQR are in the ratio 9 : 16. If BC = 4.5 cm, find the length of QR.

Answer

We have,

$$\triangle ABC \sim \triangle PQR$$

$$\text{Area}(\triangle ABC) = BC^2$$

$$\text{Area}(\triangle PQR) = QR^2$$

$$(4.5)^2 / QR^2 = 9/16$$

$$4.5 / QR = 3/4$$

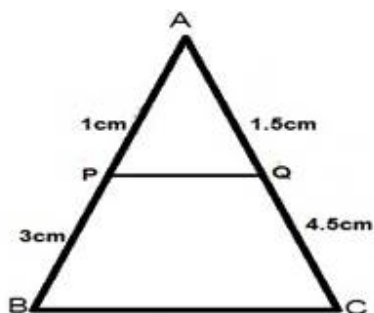
$$QR = 4 \times 4.5 / 3$$

$$QR = 6 \text{ cm}$$

19. Question

ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 cm, prove that area of $\triangle APQ$ is one-sixteenth of the area of $\triangle ABC$.

Answer



$$AP = 1 \text{ cm}, PB = 3 \text{ cm}, AQ = 1.5 \text{ cm}, \text{ and } QC = 4.5 \text{ cm}$$

In $\triangle APQ$ and $\triangle ABC$

$$\angle A = \angle A \text{ [Common]}$$

$$AP/AB = AQ/AC \text{ [Each equal to } 1/4]$$

$$\triangle APQ \sim \triangle ABC \text{ [By SAS]}$$

By area of similar triangle theorem

$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ABC} = \left(\frac{AP}{AB}\right)^2$$

$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ABC} = \left(\frac{1}{4}\right)^2$$

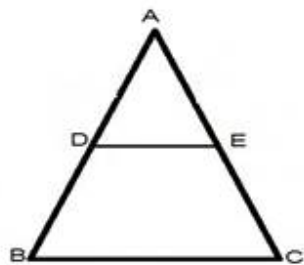
$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ABC} = \frac{1}{16}$$

$$\text{Area}(\triangle ABC) = 16 \times \text{ar}(\triangle APQ)$$

20. Question

If D is a point on the side AB of $\triangle ABC$ such that $AD : DB = 3 : 2$ and E is a point on BC such that $DE \parallel AC$. Find the ratio of areas of $\triangle ABC$ and $\triangle BDE$.

Answer



We have

$$AD/DB = 3/2$$

In $\triangle BDE$ and $\triangle BAC$

$$\angle B = \angle B \text{ [Common]}$$

$$\angle BDE = \angle A \text{ [Corresponding]}$$

$$\triangle BDE \sim \triangle BAC$$

$$\text{Area}(\triangle ABC) = AB^2$$

$$\text{Area}(\triangle BDE) = BD^2$$

$$= \left(\frac{2}{5}\right)^2 \text{ [AD/DB=3/2]}$$

$$= 4/25$$

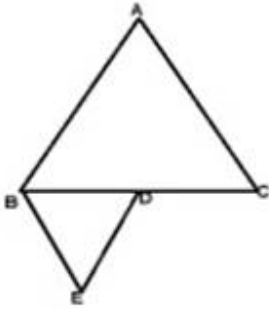
Area($\triangle ABC$)

Area ($\triangle BDE$) = 25:4

21. Question

If $\triangle ABC$ and $\triangle BDE$ are equilateral triangles, where D is the mid point of BC, find the ratio of areas of $\triangle ABC$ and $\triangle BDE$.

Answer



$\triangle ABC$ and $\triangle BDE$ is an equilateral triangles

$\triangle ABC \sim \triangle BDE$ [By SAS]

By area of similar triangle theorem

Area($\triangle ABC$) = AB^2 [D is the midpoint of BC]

Area ($\triangle BDE$) BD^2

$= 4BD^2 / BD^2$

$= 4/1$

Area($\triangle ABC$) = 4:1 Area ($\triangle BDE$)

22. Question

AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that Area ($\triangle ADE$) : Area ($\triangle ABC$) = 3 : 4.

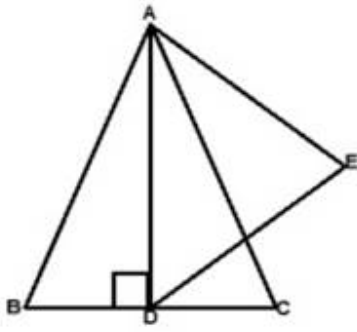
Answer

Given: AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed

To prove: Area ($\triangle ADE$) : Area ($\triangle ABC$) = 3 : 4.

Proof:

Construct the figure according to the conditions given.



We have,

$\triangle ABC$ is an equilateral triangle

Let one side AB be $2x$ Since in equilateral triangle all the sides are of equal length.

$$\Rightarrow AB=BC=AC= 2x$$

$\therefore AD \perp BC$ Since perpendicular bisects the given side into two equal parts, then $BD=DC=x$

Now, In $\triangle ADB$

By Pythagoras theorem, $AB^2 = AD^2 + BD^2$

$$AD^2 = AB^2 - BD^2 \quad AD^2 = (2x)^2 - (x)^2 \quad AD^2 = 3x^2$$

$$AD = \sqrt{3x} \text{ cm}$$

$\triangle ABC$ and $\triangle ADE$ both are equilateral triangles

Since, all the angles of the equilateral triangle are of 60° .

$\therefore \triangle ABC \sim \triangle ADE$ [By AA similarity]

By the theorem which states that the areas of two similar triangles are in the ratio of the squares of the any two corresponding sides.

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \frac{AD^2}{AB^2}$$

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \frac{(\sqrt{3x})^2}{(2x)^2}$$

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \frac{3x^2}{4x^2}$$

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \frac{3}{4}$$

Hence, Area ($\triangle ADE$) : Area ($\triangle ABC$) = 3 : 4

Exercise 4.7

1. Question

If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is a right-angled triangle.

Answer

We have,

$$AB=3\text{cm}, BC=4\text{cm}, AC=6\text{cm}$$

$$\therefore AB^2 = 3^2 = 9$$

$$BC^2 = 4^2 = 16$$

$$AC^2 = 6^2 = 36$$

$$\text{Since } AB^2 + BC^2 \neq AC^2$$

SO Triangle is not a right angle.

2. Question

The sides of certain triangles are given below. Determine which of them are right triangles.

$$(i) a = 7 \text{ cm}, b = 24 \text{ cm and } c = 25 \text{ cm}$$

$$(ii) a = 9 \text{ cm}, b = 16 \text{ cm and } c = 18 \text{ cm}$$

$$(iii) a = 1.6 \text{ cm}, b = 3.8 \text{ cm and } c = 4 \text{ cm}$$

$$(iv) a = 8 \text{ cm}, b = 10 \text{ cm and } c = 6 \text{ cm}$$

Answer

$$(i) a = 7, b = 24, c = 25$$

$$\text{Here } a^2 = 49, b^2 = 576, c^2 = 625$$

$$= a^2 + b^2$$

$$= 49 + 576$$

$$= 625 = c^2$$

\therefore So given triangle is a right angle.

$$(ii) a = 9, b = 16, c = 18$$

$$\text{Here } a^2=81, b^2= 256, c^2= 324$$

$$=a^2+b^2$$

$$=81+256$$

$$=337 \neq c^2$$

So given Triangle is not a right angle.

$$\text{(iii) } a=1.6, b=3.8, c= 4$$

$$\text{Here } a^2=2.56, b^2= 14.44, c^2= 16$$

$$=a^2+b^2$$

$$=2.56+14.44$$

$$=17 \neq c^2$$

So given Triangle is not a right angle.

$$\text{(iv) } a=8, b=10, c= 6$$

$$\text{Here } a^2=64, b^2= 100, c^2= 36$$

$$=a^2+c^2$$

$$=64+36$$

$$=100 = b^2$$

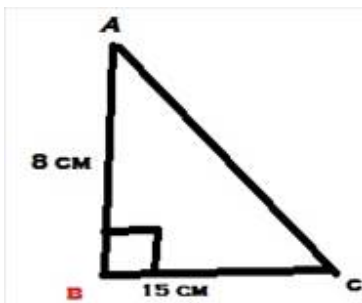
So given Triangle is a right angle.

3. Question

A man goes 15 metres due west and then 8 metres due north. How far is he from the starting point?

Answer

Let the man starts walk from point A and finished at



Point C.

\therefore In $\triangle ABC$

$$\text{SO } AC^2=AB^2+BC^2$$

$$AC^2=8^2+15^2$$

$$AC^2=64+225$$

$$AC^2=289$$

$$AC=\sqrt{289}$$

$$AC=17 \text{ m}$$

The man is 17 m far from the starting point.

4. Question

A ladder 17 m long reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

Answer

In $\triangle ABC$

$$AC^2=AB^2+BC^2$$

$$17^2=15^2+BC^2$$

$$289=225+BC^2$$

$$BC^2=289 - 225$$

$$BC^2=64$$

$$BC=\sqrt{64}$$

$$BC=8 \text{ m}$$

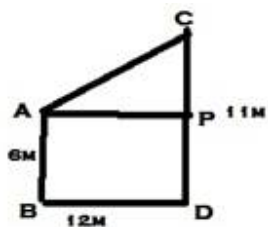
Distance of the foot of ladder is 8 m from the building.

5. Question

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Answer

Let AB and CD be the poles.



$$AB=PD= 6\text{m}, CD=11\text{m}$$

$$BD=AP=12\text{m}$$

$$CP = CD - PD$$

$$CP = 11 - 6$$

$$CP = 5$$

In $\triangle APC$

$$AC^2 = CP^2 + AP^2$$

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

$$AC = \sqrt{169}$$

$$AC = 13\text{m}$$

6. Question

In an isosceles triangle ABC, $AB = AC = 25\text{ cm}$, $BC = 14\text{ cm}$. Calculate the altitude from A on BC.

Answer

We have,

$$AB = AC = 25\text{cm}$$

$$BC = 14\text{cm}$$

In $\triangle ACD$ and $\triangle ABD$

$$\angle ADB = \angle ADC = 90^\circ$$

$$AB = AC = 25\text{cm}$$

$$AD = AD \text{ (Common)}$$

$$\triangle ABD \cong \triangle ACD$$

$$\therefore BD = CD = 7\text{cm (By c.p.c.t)}$$

In $\triangle ACD$

$$AB^2 = AD^2 + BD^2$$

$$25^2 = AD^2 + 7^2$$

$$625 = AD^2 + 49$$

$$AD^2 = 625 - 49$$

$$AD^2 = 576$$

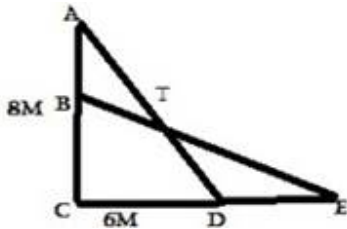
$$AD = \sqrt{576}$$

$$AD=24 \text{ cm}$$

7. Question

The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?

Answer



Let length of ladder be $AD=BE=1m$

In $\triangle ACD$

$$AD^2=AC^2+CD^2$$

$$t^2= 8^2+6^2 \dots\dots\dots(i)$$

In $\triangle BCE$

$$BE^2=BC^2+EC^2$$

$$t^2= BC^2+8^2 \dots\dots\dots (II)$$

From (i) and (ii)

$$BC^2+8^2=8^2+6^2$$

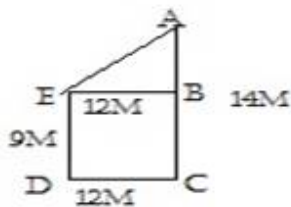
$$BC^2=6^2$$

$$BC=6m$$

8. Question

Two poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Answer



We have,

$$AC=14m, DC=12m, ED=BC=9m$$

Draw $EB \perp AC$

$$\therefore AB = AC - BC$$

$$AB = 14 - 9 = 5\text{m}$$

$$EB = DC = 12\text{m}$$

In $\triangle ABE$

$$AE^2 = AB^2 + BE^2$$

$$AE^2 = 5^2 + 12^2$$

$$AE^2 = 25 + 144$$

$$AE^2 = 169$$

$$AE = \sqrt{169}$$

$$AE = 13\text{m}$$

9. Question

Using Pythagoras theorem determine the length of AD in terms of b and c shown in Fig 4.219.

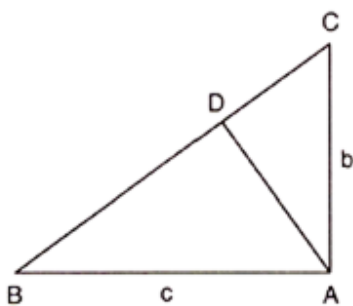


Fig. 4.219

Answer

In $\triangle ABC$

$$BC^2 = AB^2 + AC^2$$

$$BC^2 = c^2 + b^2$$

$$BC = \sqrt{c^2 + b^2} \dots\dots\dots(i)$$

In $\triangle ABC$ and In $\triangle CBA$

$$\angle B = \angle B \text{ (Common)}$$

$$\angle ADB = \angle BAC = 90^\circ$$

$$\therefore \triangle ABD \sim \triangle CBA$$

$$\therefore AB/CB = AD/CA$$

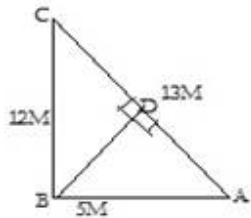
$$c/\sqrt{c^2 + b^2} = AD/b$$

$$AD = bc/\sqrt{c^2 + b^2}$$

10. Question

A triangle has sides 5 cm, 12 cm and 13 cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13 cm.

Answer



Here $AB = 5\text{cm}$, $BC = 12\text{cm}$, $AC = 13\text{cm}$.

$$AC^2 = AB^2 + BC^2$$

$\triangle ABC$ is a right angled triangle at $\angle B$.

$$\text{Area } \triangle ABC = \frac{1}{2}(BC \times BA)$$

$$= \frac{1}{2}(12 \times 5)$$

$$= \frac{1}{2} \times 60$$

$$= 30\text{cm}^2$$

$$\text{Also Area of } \triangle ABC = \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2}(13 \times BD)$$

$$30 = \frac{1}{2}(13 \times BD)$$

$$13 \times BD = 30 \times 2$$

$$BD = 60/13$$

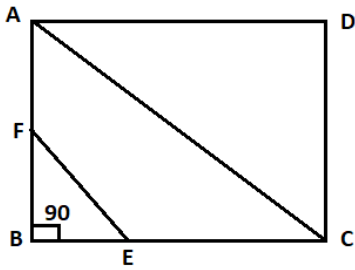
$$BD = 4.6 \text{ cm}$$

11. Question

ABCD is a square. F is the mid-point of AB. BE is one third of BC. If the area of $\triangle FBE = 108 \text{ cm}^2$, find the length of AC.

Answer

According to the question, the figure is :



\therefore ABCD is a square. Hence, $AB = BC = CD = DA$

\therefore F is the midpoint of AB.

\therefore Length of $BF = AB/2 = BC/2$ ($\because AB = BC$)

Given that, $BE = BC/3$

In $\triangle FBE$, $\angle B = 90^\circ$ and Area of $\triangle FBE = 108 \text{ cm}^2$

$$\therefore \frac{1}{2} \times BE \times BF = 108$$

$$\Rightarrow \frac{1}{2} \times \frac{BC}{3} \times \frac{BC}{2} = 108$$

$$\Rightarrow BC^2 = 108 \times 12$$

$$\Rightarrow BC^2 = 36 \times 36$$

$$\Rightarrow BC = 36 \text{ cm}$$

AC is the diagonal of the ABCD.

$$\therefore \text{Length of } AC = \sqrt{BC^2 + AB^2}$$

$$\Rightarrow AC = \sqrt{36^2 + 36^2}$$

$$\Rightarrow AC = \sqrt{36^2 + 36^2}$$

$$\Rightarrow AC = 36\sqrt{2} = 50.904 \text{ cm}$$

12. Question

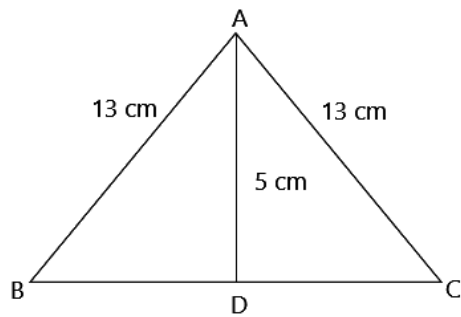
In an isosceles triangle ABC, if $AB = AC = 13 \text{ cm}$ and the altitude from A on BC is 5 cm, find BC.

Answer

Given: isosceles triangle ABC, where $AB = AC = 13 \text{ cm}$ and the altitude from A on BC is 5 cm.

To find: The value of BC.

Solution:



In $\triangle ADB$

$$AD^2 + BD^2 = AB^2$$

$$5^2 + BD^2 = 13^2$$

$$25 + BD^2 = 169$$

$$BD^2 = 169 - 25$$

$$BD^2 = 144$$

$$BD = \sqrt{144}$$

$$BD = 12\text{cm}$$

In $\triangle ADB$ and $\triangle ADC$

$$\angle ADB = \angle ADC = 90^\circ$$

$$AB = AC = 13\text{cm}$$

$$AD = AD \text{ (Common)}$$

$$\triangle ADB \cong \triangle ADC \text{ (By RHS condition)}$$

$$BD = CD = 12\text{cm (c.p.c.t)}$$

$$\text{As } BC = BD + DC$$

$$BC = 12 + 12$$

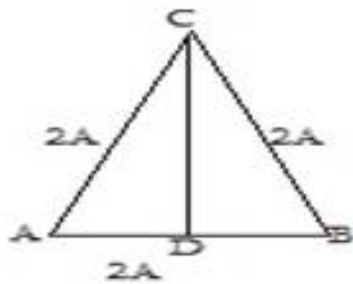
$$BC = 24\text{cm}$$

13. Question

In a $\triangle ABC$, $AB = BC = CA = 2a$ and $AD \perp BC$. Prove that

$$(i) AD = a\sqrt{3} \quad (ii) \text{Area}(\triangle ABC) = \sqrt{3}a^2$$

Answer



(i) In $\triangle ABD$ and $\triangle ACD$

$$\angle ADB = \angle ADC = 90^\circ$$

$$AB = AC \text{ (given)}$$

$$AD = AD \text{ (common)}$$

$$\triangle ADB \cong \triangle ACD$$

$$BD = CD = a \text{ (By c.p.c.t)}$$

In $\triangle ADB$

$$AD^2 + BD^2 = AB^2$$

$$AD^2 + a^2 = (2a)^2$$

$$AD^2 = 4a^2 - a^2$$

$$AD^2 = 3a^2$$

$$AD = a\sqrt{3}$$

(ii) Area of $\triangle ABC = \frac{1}{2} \times BC \times AD$

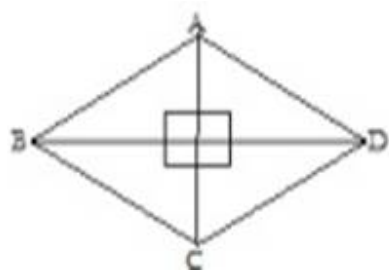
$$= \frac{1}{2} \times 2a \times a\sqrt{3}$$

$$= \sqrt{3}a^2$$

14. Question

The lengths of the diagonals of a rhombus are 24 cm and 10 cm. Find each side of the rhombus.

Answer



We have,

ABCD is a rhombus

AC and BD are the diagonals with length 10cm and 24 cm respectively.

We know that rhombus of diagonal bisects each other at 90°

$\therefore AO=OC=5\text{cm}$ and $BO=OD=12\text{cm}$

In $\triangle AOB$

$$AB^2 = AO^2 + BO^2$$

$$AB^2 = 5^2 + 12^2$$

$$AB^2 = 25 + 144$$

$$AB^2 = 169$$

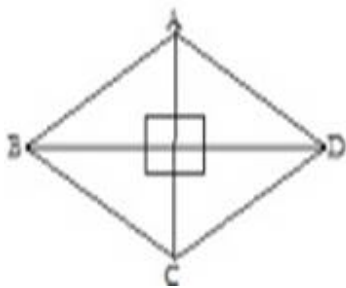
$$AB = \sqrt{169}$$

$$AB = 13 \text{ cm}$$

15. Question

Each side of a rhombus is 10 cm. If one of its diagonals is 16 cm find the length of the other diagonal.

Answer



We have,

ABCD is a rhombus with side 10 cm and diagonal BD=16 CM

We know that rhombus of diagonal bisects each other at 90°

$BO=OD=8\text{cm}$

In $\triangle AOB$

$$AB^2 = AO^2 + BO^2$$

$$10^2 = AO^2 + 8^2$$

$$100 = AO^2 + 64$$

$$AO^2 = 100 - 64$$

$$AO^2 = 36$$

$$AO = \sqrt{36}$$

$$AO = 6 \text{ cm}$$

$$\therefore AC = AO + OC$$

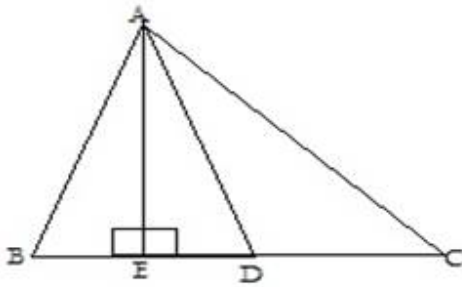
$$AC = 6 + 6$$

$$AC = 12 \text{ cm}$$

16. Question

In an acute-angled triangle, express a median in terms of its sides.

Answer



We have

In $\triangle ABC$, AD is median

$$AE \perp BC$$

In $\triangle AEB$

$$AB^2 = AE^2 + BE^2$$

$$AB^2 = AD^2 - DE^2 + (BD - DE)^2$$

$$AB^2 = AD^2 - DE^2 + BD^2 - 2 \times BD \times DE + DE^2$$

$$AB^2 = AD^2 + BD^2 - 2 \times BD \times DE$$

$$AB^2 = AD^2 + BC^2/4 - BC \times DE \dots\dots\dots (I) \text{ [GIVEN } BC = 2BD]$$

In $\triangle AEC$

$$AC^2 = AE^2 + EC^2$$

$$AC^2 = AD^2 - DE^2 + (DE + CD)^2$$

$$AC^2 = AD^2 - DE^2 + 2CD \times DE$$

$$AC^2 = AD^2 + BC^2/4 + BC \times DE \dots\dots\dots (II) \text{ [BC = 2CD]}$$

By adding equ. (i) and (ii) we get

$$AB^2 + AC^2 = 2AD^2 + BC^2/2$$

$$2AB^2 + 2AC^2 = 4AD^2 + BC^2 \text{ [MULTIPLY BY 2]}$$

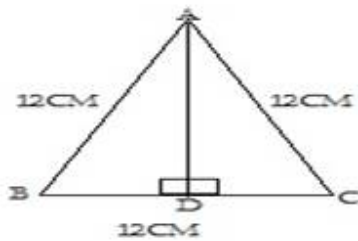
$$4AD^2 = 2AB^2 + 2AC^2 - BC^2$$

$$AD^2 = 2AB^2 + 2AC^2 - BC^2$$

17. Question

Calculate the height of an equilateral triangle each of whose sides measures 12 cm.

Answer



$\triangle ABC$ is an equilateral triangle with side 12cm

$AE \perp BC$

In $\triangle ABD$ and $\triangle ACD$

$$\angle ADB = \angle ADC = 90^\circ$$

$$AB = AC = 12\text{cm}$$

$$AD = AD \text{ (COMMON)}$$

$$\triangle ABD \cong \triangle ACD$$

$$AD^2 + BD^2 = AB^2$$

$$AD^2 + 6^2 = 12^2$$

$$AD^2 + 36 = 144$$

$$AD^2 = 144 - 36$$

$$AD^2 = 108$$

$$AD = \sqrt{108}$$

$$AD = 10.39 \text{ cm}$$

18. Question

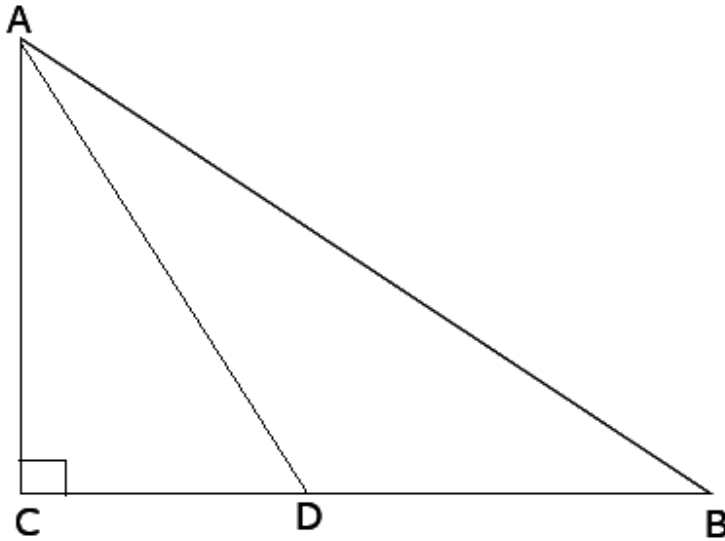
In right-angled triangle ABC in which $\angle C = 90^\circ$, if D is the mid-point of BC, prove that $AB^2 = 4AD^2 - 3AC^2$.

Answer

Given: In right-angled triangle ABC in which $\angle C = 90^\circ$, if D is the mid-point of BC.

To prove: $AB^2 = 4AD^2 - 3AC^2$

Solution:



We have

$\angle C = 90^\circ$ and D is the midpoint of BC

In $\triangle ABC$

$$AB^2 = AC^2 + BC^2$$

As $BC = CD + BD$ D is the mid point of BC $\Rightarrow CD = BD$ So, $AB^2 = AC^2 + (CD + CD)^2$

$$\Rightarrow AB^2 = AC^2 + (2CD)^2$$

$$\Rightarrow AB^2 = AC^2 + 4CD^2$$

Also In $\triangle ACD$ $AD^2 = AC^2 + CD^2 \Rightarrow CD^2 = AD^2 - AC^2$ So,

$$\Rightarrow AB^2 = AC^2 + 4(AD^2 - AC^2)$$

$$AB^2 = AC^2 + 4AD^2 - 4AC^2$$

$$AB^2 = 4AD^2 - 3AC^2$$

19. Question

In Fig. 4.220, D is the mid-point of side BC and $AE \perp BC$. If $BC = a$, $AC = b$, $AB = c$, $ED = x$, $AD = p$ and $AE = h$, prove that:

$$(i) \ b^2 = p^2 + a + \frac{a^2}{4} \quad (ii) \ c^2 = p^2 - ax + \frac{a^2}{4}$$

$$(iii) \ b^2 + c^2 = 2p^2 + \frac{a^2}{2}$$

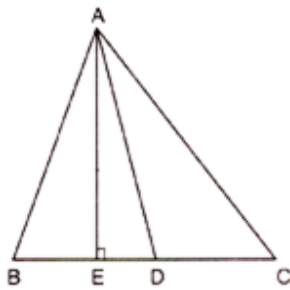


Fig. 4.220

Answer

We have

D is the midpoint of BC

(i) In $\triangle AEC$

$$AC^2 = AE^2 + EC^2$$

$$b^2 = AE^2 + (ED + DC)^2$$

$$b^2 = AD^2 + DC^2 + 2 \times ED \times DC \quad (\text{Given } BC = 2CD)$$

$$b^2 = p^2 + (a/2)^2 + 2(a/2)x$$

$$b^2 = p^2 + a^2/4 + ax$$

$$b^2 = p^2 + ax + a^2/4 \dots\dots\dots (i)$$

(ii) In $\triangle AEB$

$$AB^2 = AE^2 + BE^2$$

$$c^2 = AD^2 - ED^2 + (BD - ED)^2$$

$$c^2 = p^2 - ED^2 + BD^2 + ED^2 - 2BD \times ED$$

$$c^2 = p^2 + (a/2)^2 - 2(a/2)x$$

$$c^2 = p^2 - ax + a^2/4 \dots\dots\dots (ii)$$

(iii) Adding equ. (i) and (ii) we get

$$b^2 + c^2 = 2p^2 + a^2/2$$

20. Question

In Fig. 4.221, $\angle B < 90^\circ$ and segment $AD \perp BC$, show that

(i) $b^2 = h^2 + a^2 + x^2 - 2ax$

(ii) $b^2 = a^2 + c^2 - 2ax$

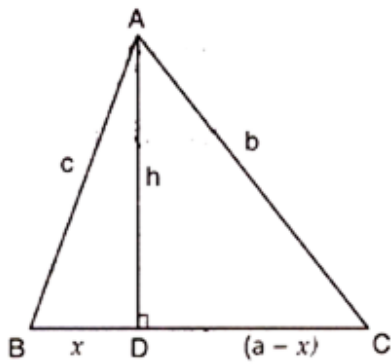


Fig. 4.221

Answer

In $\triangle ADC$

$$AC^2 = AD^2 + DC^2$$

$$b^2 = h^2 + (a-x)^2$$

$$b^2 = h^2 + a^2 - 2ax + x^2 \dots\dots\dots (i)$$

$$b^2 = h^2 + x^2 - 2ax$$

$$b^2 = a^2 + (h^2 + x^2) - 2ax \text{ (from equ.i)}$$

$$b^2 = a^2 + c^2 - 2ax \text{ [} h^2 + x^2 = c^2 \text{]}$$

21. Question

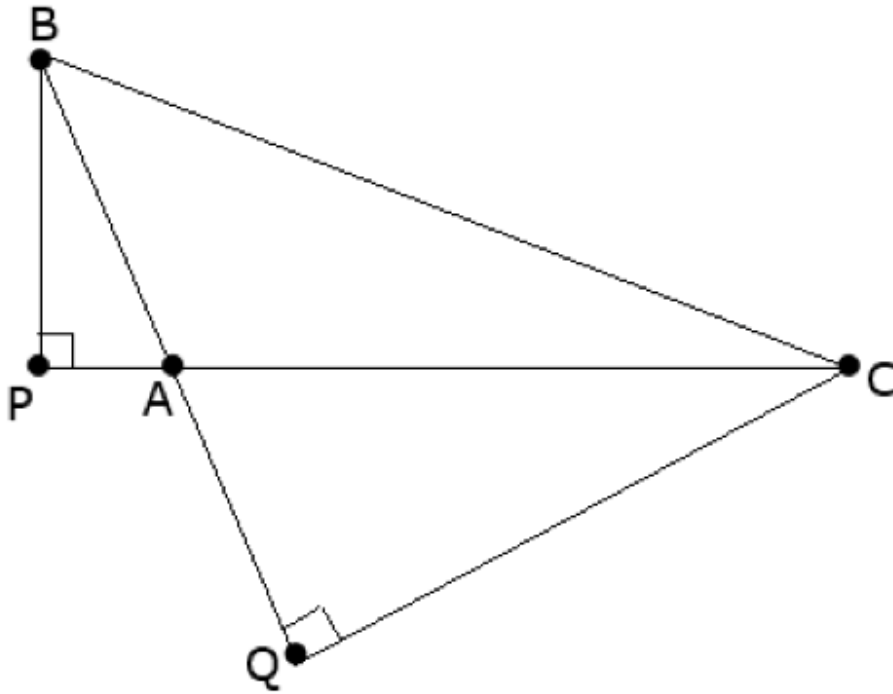
In $\triangle ABC$, $\angle A$ is obtuse, $PB \perp AC$ and $QC \perp AB$. Prove that:

(i) $AB \times AQ = AC \times AP$

(ii) $BC^2 = (AC \times CP + AB \times BQ)$

Answer

Draw the diagram according to given questions.



(I) In $\triangle APB$ and $\triangle AQC$ $\angle A = \angle A$

(common) $\angle P = \angle Q$ (both 90°) $\therefore \triangle APB \sim \triangle AQC$ [By AA similarity]

$$\Rightarrow \frac{AP}{AQ} = \frac{AB}{AC} \quad \{\text{Corresponding part of similar triangle are proportional}\}$$

$$AP \times AC = AQ \times AB \dots\dots\dots(1)$$

(II)

In $\triangle BPC$ By pythagoras theorem,

$$BC^2 = BP^2 + PC^2 \text{ Also in } \triangle BPA$$

$$BP^2 = AB^2 - AP^2 \text{ Also } PC = PA + AC$$

$$\Rightarrow BC^2 = AB^2 - AP^2 + (AP + AC)^2$$

Apply the theorem $(a + b)^2 = a^2 + b^2 + 2ab$ in $(AP + AC)^2$

$$\Rightarrow BC^2 = AB^2 - AP^2 + AP^2 + AC^2 + 2AP \times AC$$

$$BC^2 = AB^2 + AC^2 + 2AP \times AC \dots\dots\dots(ii)$$

In $\triangle BQC$

$$BC^2 = CQ^2 + BQ^2$$

$$BC^2 = AC^2 - AQ^2 + (AB + AQ)^2$$

$$BC^2 = AC^2 - AQ^2 + AB^2 + 2AB \times AQ$$

$$BC^2 = AC^2 + AB^2 + AQ^2 + 2AB \times AQ \dots\dots\dots(iii)$$

Adding equ. (ii) and (iii)

$$BC^2 + BC^2 = AB^2 + AC^2 + 2AP \times AC + AC^2 + AB^2 + AQ^2 + 2AB \times AQ$$

$$\Rightarrow 2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$$

$$\Rightarrow 2BC^2 = 2AC[AC + AP] + AB[AB + AQ]$$

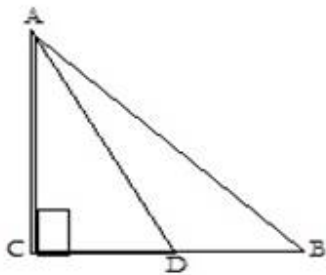
$$\Rightarrow 2BC^2 = 2AC \times PC + 2AB \times BQ$$

$$\Rightarrow BC^2 = AC \times PC + AB \times BQ \text{ Hence proved.}$$

22. Question

In a right $\triangle ABC$ right-angled at C, if D is the mid-point of BC, prove that $BC^2 = 4(AD^2 - AC^2)$.

Answer



We have

$\angle C = 90^\circ$ and D is the midpoint of BC

$$\text{LHS} = BC^2$$

$$= (2CD)^2$$

$$= 4CD^2$$

$$= 4(AD^2 - AC^2) = \text{RHS}$$

23. Question

In a quadrilateral ABCD, $\angle B < 90^\circ$, $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90^\circ$.

Answer

We have

$\angle B = 90^\circ$ and

$$AD^2 = AB^2 + BC^2 + CD^2 \text{ (Given)}$$

$$\text{But } AB^2 + BC^2 = AC^2$$

$$AD^2 = AC^2 + CD^2$$

By converse of Pythagoras

$$\angle ACD = 90^\circ$$

24. Question

In an equilateral $\triangle ABC$, $AD \perp BC$, prove that $AD^2 = 3BD^2$.

Answer

We have $\triangle ABC$ is an equilateral triangle and $AD \perp BC$

In $\triangle ADB$ and $\triangle ADC$

$$\angle ADB = \angle ADC = 90^\circ \quad AB = AC \text{ (Given)}$$

$$AD = AD \text{ (Common)}$$

$$\triangle ADB \cong \triangle ADC \text{ (By RHS condition)}$$

$$\therefore BD = CD = BC/2 \dots\dots (i)$$

In $\triangle ABD$

$$BC^2 = AD^2 + BD^2$$

$$BC^2 = AD^2 + BD^2 \text{ [Given } AB = BC]$$

$$(2BD)^2 = AD^2 + BD^2 \text{ [From (i)]}$$

$$4BD^2 - BD^2 = AD^2$$

$$AD^2 = 3BD^2$$

25. Question

$\triangle ABC$ is a right triangle right-angled at A and $AC \perp BD$. Show that

$$(i) AB^2 = BC \cdot BD \quad (ii) AC^2 = BC \cdot DC$$

$$(iii) AD^2 = BD \cdot CD \quad (iv) \frac{AB^2}{AC^2} = \frac{BD}{DC}$$

Answer

(i) In $\triangle ABD$ and In $\triangle CAB$

$$\angle DAB = \angle ACB = 90^\circ$$

$$\angle ABD = \angle CBA \text{ [Common]}$$

$$\angle ADB = \angle CAB \text{ [remaining angle]}$$

So, $\triangle ADB \cong \triangle CAB$ [By AAA Similarity]

$$\therefore AB/CB = BD/AB$$

$$AB^2 = BC \times BD$$

(ii)

Let $\angle CAB = x$

In $\triangle CBA = 180^\circ - 90^\circ - x$

$\angle CBA = 90^\circ - x$

Similarly in $\triangle CAD$

$\angle CAD = 90^\circ - \angle CAB = 90^\circ - x$

$\angle CDA = 90^\circ - \angle CAB$

$= 90^\circ - x$

$\angle CDA = 180^\circ - 90^\circ - (90^\circ - x)$

$\angle CDA = x$

Now in $\triangle CBA$ and $\triangle CAD$ we may observe that

$\angle CBA = \angle CAD$

$\angle CAB = \angle CDA$

$\angle ACB = \angle DCA = 90^\circ$

Therefore $\triangle CBA \sim \triangle CAD$ (by AAA rule)

Therefore $AC/DC = BC/AC$

$AC^2 = DC \times BC$

(iii) In $\triangle DCA$ and $\triangle DAB$

$\angle DCA = \angle DAB$ (both angles are equal to 90°)

$\angle CDA = \angle ADB$ (common)

$\angle DAC = \angle DBA$

$\triangle DCA \sim \triangle DAB$ (AAA condition)

Therefore $DC/DA = DA/DB$

$AD^2 = BD \times CD$

(iv) From part (I) $AB^2 = CB \times BD$

From part (II) $AC^2 = DC \times BC$

Hence $AB^2/AC^2 = CB \times BD/DC \times BC$

$AB^2/AC^2 = BD/DC$

Hence proved

26. Question

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer

Let OB be the pole and AB be the wire.

$$AB^2 = OB^2 + OA^2$$

$$24^2 = 18^2 + OA^2$$

$$OA^2 = 576 - 324$$

$$OA^2 = 252$$

$$AO = \sqrt{252}$$

$$AO = 6\sqrt{7} \text{ m.}$$

$$\text{Distance from base} = 6\sqrt{7} \text{ m}$$

27. Question

An aeroplane leaves an airport and flies due north at a speed of 1000 km/hr. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer

Distance traveled by the plane flying towards north in $1\frac{1}{2}$ hrs

$$= 1000 \times 1\frac{1}{2} = 1500 \text{ km}$$

Similarly Distance traveled by the plane flying towards west in $1\frac{1}{2}$ hrs

$$= 1200 \times 1\frac{1}{2} = 1800 \text{ km}$$

Let this distance is represented by OA and OB

$$\text{Distance between these place after } 1\frac{1}{2} \text{ hrs } AB = \sqrt{OA^2 + OB^2}$$

$$= \sqrt{\{1500\}^2 + \{1800\}^2} = \sqrt{2250000 + 3240000}$$

$$= \sqrt{5490000} = \sqrt{9 \times 610000} = 300\sqrt{61}$$

$$= 300 \times 7.8102$$

$$= 2343.07 \text{ km}$$

So, distance between these places will be 2343 km (Approx) km, after $1\frac{1}{2}$ hrs

28. Question

Determine whether the triangle having sides $(a - 1)$ cm, $2\sqrt{a}$ cm and $(a + 1)$ cm is a right angled triangle.

Answer

Let ABC be the triangle

Where $AB = (a-1)^2$ cm

$BC = 2\sqrt{a}$ cm

$CA = (a+1)$ cm

$AB^2 = (a-1)^2 = a^2 + 1 - 2a$

$BC^2 = (2\sqrt{a})^2 = 4a$

$CA^2 = (a+1)^2 = a^2 + 1 + 2a$

Hence $AB^2 + BC^2 = AC^2$

SO $\triangle ABC$ is a right angles triangle at B

CCE - Formative Assessment**1. Question**

State basic proportionality theorem and its converse.

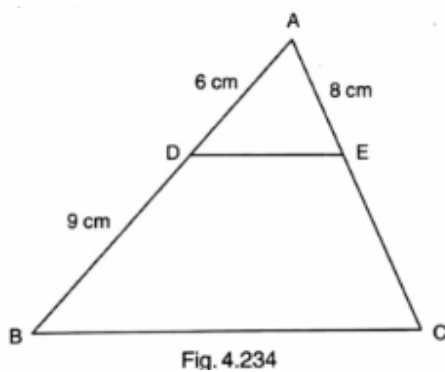
Answer

Basic Proportionality Theorem: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Converse of Basic Proportionality Theorem: If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

2. Question

In the adjoining figure, find AC.

**Answer**

From the given figure $\triangle ABC$, $DE \parallel BC$.

Let $EC = x$ cm.

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the

same ratio.

$$\text{Then } \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{6}{9} = \frac{8}{x}$$

$$\Rightarrow x = \frac{8(9)}{6}$$

$$\Rightarrow x = 12 \text{ cm} = EC$$

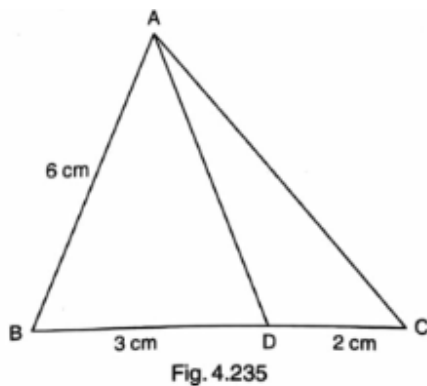
$$\text{Here, } AC = AE + EC$$

$$\Rightarrow AC = 8 + 12 = 20 \text{ cm}$$

$$\therefore AC = 20 \text{ cm}$$

3. Question

In the adjoining figure, if AD is the bisector of $\angle A$, what is AC?



Answer

Given AD is the bisector of $\angle A$ in $\triangle ABC$. Let AC be x cm.

We know that the angle bisector theorem states that the internal bisector of an angle of a triangle divides the opposite side in two segments that are proportional to the other two sides of the triangle.

$$\Rightarrow \frac{AB}{AC} = \frac{DB}{DC}$$

$$\Rightarrow \frac{6}{x} = \frac{3}{2}$$

$$\Rightarrow x = \frac{6(2)}{3}$$

$$\Rightarrow x = 4 \text{ cm}$$

$$\therefore AC = 4 \text{ cm}$$

4. Question

State AAA similarity criterion.

Answer

AAA similarity criterion: In two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

5. Question

State SSS similarity criterion.

Answer

SSS similarity criterion: If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

6. Question

State SAS similarity criterion.

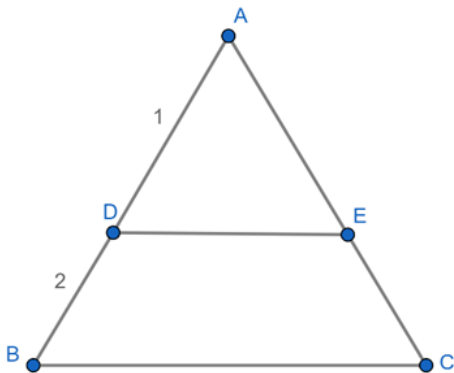
Answer

SAS similarity criterion: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

7. Question

In the adjoining figure, DE is parallel to BC and AD = 1 cm, BD = 2 cm. What is the ratio of the area of $\triangle ABC$ to the area of $\triangle ADE$?

Answer



Given $DE \parallel BC$, $AD = 1$ cm and $DB = 2$ cm.

So, $AB = 3$ cm.

In $\triangle ABC$ and $\triangle ADE$,

$\angle ABC = \angle ADE$ [corresponding angles]

$\angle ACB = \angle AED$ [corresponding angles]

$\angle A = \angle A$ [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$$\therefore \triangle ABC \sim \triangle ADE$$

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

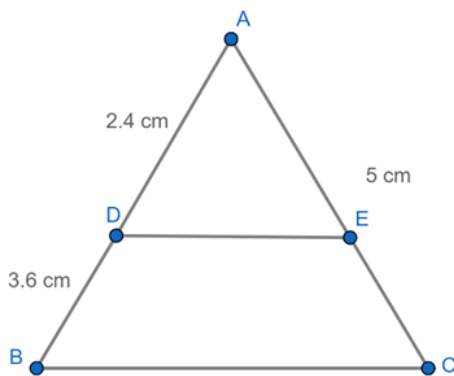
$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle ADE)} = \frac{(AB)^2}{(AD)^2} = \frac{3^2}{1^2} = \frac{9}{1}$$

$$\therefore ar(\triangle ABC) : ar(\triangle ADE) = 9 : 1$$

8. Question

In the figure given below $DE \parallel BC$. If $AD = 2.4$ cm, $DB = 3.6$ cm and $AC = 5$ cm. Find AE .

Answer



Given $DE \parallel BC$, $AD = 2.4$ cm, $DB = 3.6$ cm and $AC = 5$ cm.

We have to find AE .

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{AC - AE}$$

$$\Rightarrow \frac{2.4}{3.6} = \frac{AE}{5 - AE}$$

$$\Rightarrow 2.4 (5 - AE) = 3.6 AE$$

$$\Rightarrow 12 - 2.4 AE = 3.6 AE$$

$$\Rightarrow 12 = 3.6 AE + 2.4 AE$$

$$\Rightarrow 12 = 6 AE$$

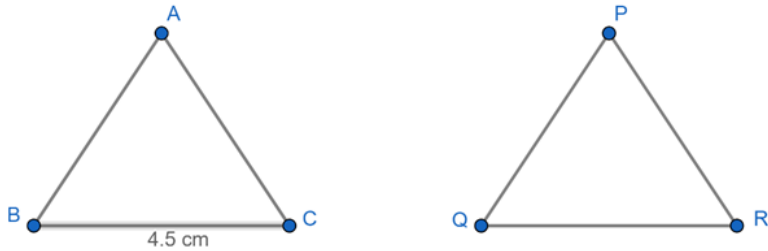
$$\Rightarrow AE = 12/6$$

$$\therefore AE = 2 \text{ cm}$$

9. Question

If the areas of two similar triangles ABC and PQR are in the ratio 9 : 16 and $BC = 4.5 \text{ cm}$, what is the length of QR?

Answer



Given $\triangle ABC \sim \triangle PQR$, $\text{ar}(\triangle ABC) : \text{ar}(\triangle PQR) = 9 : 16$ and $BC = 4.5 \text{ cm}$

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{(BC)^2}{(QR)^2}$$

$$\Rightarrow \frac{9}{16} = \frac{(4.5)^2}{(QR)^2}$$

$$\Rightarrow QR^2 = \frac{20.25(16)}{9}$$

$$\Rightarrow QR^2 = 2.25 (16)$$

$$\Rightarrow QR^2 = 36$$

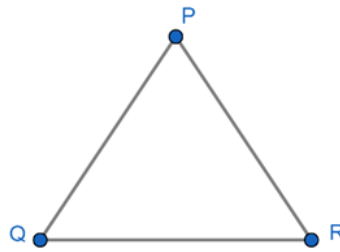
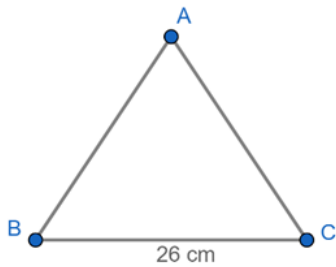
$$\Rightarrow QR = 6$$

\therefore The length of QR is 6 cm.

10. Question

The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, what is the length of the longest side of the smaller triangle?

Answer



Given $\triangle ABC \sim \triangle PQR$, ar $(\triangle ABC)$: ar $(\triangle PQR) = 169: 121$ and $BC = 26$ cm

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{(BC)^2}{(QR)^2}$$

$$\Rightarrow \frac{169}{121} = \frac{(26)^2}{(QR)^2}$$

$$\Rightarrow QR^2 = \frac{26(26)(121)}{169}$$

$$\Rightarrow QR^2 = 4 (121)$$

$$\Rightarrow QR^2 = 484$$

$$\Rightarrow QR = 22$$

\therefore The length of QR is 22 cm.

11. Question

If ABC and DEF are similar triangles such that $\angle A = 57^\circ$ and $\angle E = 73^\circ$, what is the measure of $\angle C$?

Answer

Given ABC and DEF are two similar triangles, $\angle A = 57^\circ$ and $\angle E = 73^\circ$

We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

In $\triangle ABC$ and $\triangle DEF$,

if $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$, then $\triangle ABC \sim \triangle DEF$

So, $\angle A = \angle D$

$$\Rightarrow \angle D = 57^\circ \dots (1)$$

Similarly, $\angle B = \angle E$

$$\Rightarrow \angle B = 73^\circ \dots (2)$$

We know that the sum of all angles of a triangle is equal to 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 57^\circ + 73^\circ + \angle C = 180^\circ$$

$$\Rightarrow 130^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 130^\circ = 50^\circ$$

$$\therefore \angle C = 50^\circ$$

12. Question

If the altitude of two similar triangles are in the ratio 2 : 3, what is the ratio of their areas?

Answer

Given altitudes of two similar triangles are in ratio 2: 3.

Let first triangle be $\triangle ABC$ and second triangle be $\triangle PQR$.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{(2)^2}{(3)^2}$$

$$\therefore ar(\triangle ABC): ar(\triangle PQR) = 4: 9$$

13. Question

If $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{3}{4}$, then write Area ($\triangle ABC$): Area ($\triangle DEF$).

Answer

Given that $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{3}{4}$

Here, the corresponding sides are given proportional.

We know that two triangles are similar if their corresponding sides are proportional.

And we know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{(3)^2}{(4)^2}$$

$$\therefore \text{Area } (\triangle ABC): \text{Area } (\triangle DEF) = 9: 16$$

14. Question

If $\triangle ABC$ and $\triangle DEF$ are similar triangles such that $AB = 3$ cm, $BC = 2$ cm $CA = 2.5$ cm and $EF = 4$ cm, write the perimeter of $\triangle DEF$.

Answer

Given that $\triangle ABC$ and $\triangle DEF$ are similar triangles such that $AB = 3$ cm, $BC = 2$ cm, $CA = 2.5$ cm and $EF = 4$ cm.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

First consider,

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\Rightarrow \frac{3}{2} = \frac{DE}{4}$$

$$\Rightarrow DE = 6 \text{ cm} \dots (1)$$

Now,

$$\frac{CA}{BC} = \frac{DF}{EF}$$

$$\Rightarrow \frac{2.5}{2} = \frac{DF}{4}$$

$$\Rightarrow DF = 5 \text{ cm} \dots (2)$$

Then, perimeter of $\triangle DEF = DE + EF + DF = 6 + 4 + 5$

\therefore Perimeter of $\triangle DEF = 15$ cm

15. Question

State Pythagoras theorem and its converse.

Answer

Pythagoras Theorem: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Converse of Pythagoras Theorem: In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to first side is a right angle.

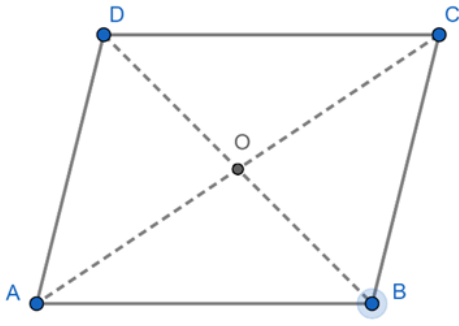
16. Question

The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus. [CBSE 2008]

Answer

Given the lengths of the diagonals of a rhombus are 30 cm and 40 cm.

Let the diagonals AC and BD of the rhombus ABCD meet at point O.



We know that the diagonals of the rhombus bisect each other perpendicularly.

Also we know that Pythagoras theorem states that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Consider right triangle AOD,

$$\Rightarrow AD^2 = AO^2 + OD^2$$

$$= 15^2 + 20^2$$

$$= 225 + 400$$

$$= 625$$

$$\Rightarrow AD = 25 \text{ cm}$$

\therefore The side of the rhombus is 25 cm.

17. Question

In Fig. 4.236, $PQ \parallel BC$ and $AP : PB = 1 : 2$. Find $\frac{\text{area}(\triangle APQ)}{\text{area}(\triangle ABC)}$ [CBSE 2008]

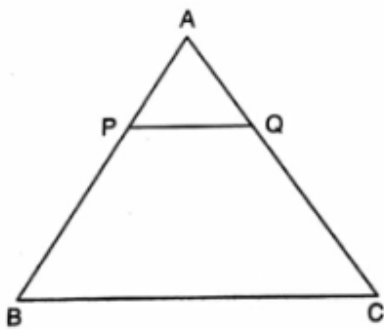


Fig. 4.236

Answer

Given in the given figure $PQ \parallel BC$ and $AP : PB = 1 : 2$

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Since ΔAPQ and ΔABC are similar, $\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$

Given $\frac{AP}{PB} = \frac{1}{2}$

$\Rightarrow PB = 2AP$

So, $\frac{AP}{AB} = \frac{AP}{AP+PB} = \frac{AP}{AP+2AP} = \frac{1}{3}$

we know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

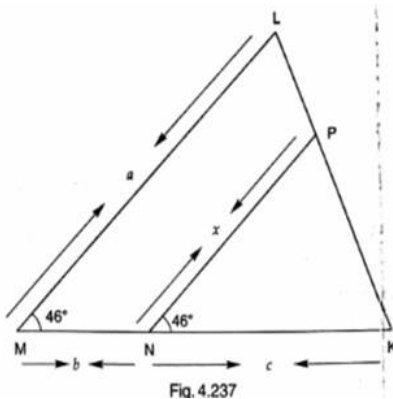
$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \left(\frac{AP}{AB}\right)^2$$

$$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$\therefore \text{Area } (\Delta APB) : \text{Area } (\Delta ABC) = 1 : 9$

18. Question

In Fig. 4.237, $\angle M = \angle N = 46^\circ$. Express x in terms of a , b and c where a , b , c are lengths of LM , MN and NK respectively.



Answer

Given $\angle M = \angle N = 46^\circ$

It forms a pair of corresponding angles, hence $LM \parallel PN$.

In ΔLMK and ΔPNK ,

$\angle LMK = \angle PNK$ [corresponding angles]

$\angle MLK = \angle NPK$ [corresponding angles]

$\angle K = \angle K$ [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$\therefore \Delta LMK \sim \Delta PNK$

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{ML}{NP} = \frac{MK}{NK}$$

$$\Rightarrow \frac{a}{x} = \frac{b+c}{c}$$

$$\therefore x = \frac{ac}{b+c}$$

19. Question

In Fig. 4.238, S and T are points on the sides PQ and PR respectively of $\triangle PQR$ such that $PT = 2$ cm, $TR = 4$ cm and ST is parallel to QR . Find the ratio of the areas of $\triangle PST$ and $\triangle PQR$. [CBSE 2010]

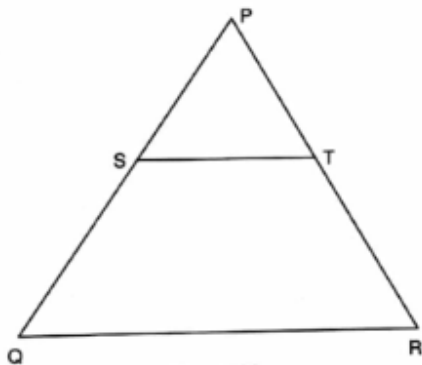


Fig. 4.238

Answer

Given $ST \parallel QR$, $TR = 4$ cm and $PT = 2$ cm.

So, $PR = 6$ cm.

In $\triangle PST$ and $\triangle PQR$,

$\angle PST = \angle PQR$ [corresponding angles]

$\angle PTS = \angle PRQ$ [corresponding angles]

$\angle P = \angle P$ [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$\therefore \triangle PST \sim \triangle PQR$

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\triangle PST)}{ar(\triangle PQR)} = \frac{(PT)^2}{(PR)^2} = \frac{2^2}{6^2} = \frac{4}{36} = \frac{1}{9}$$

$\therefore ar(\triangle PST) : ar(\triangle PQR) = 1 : 9$

20. Question

Figure 4.239 shows two triangles, $\triangle ABC$ and $\triangle HKC$, sharing a common vertex C . A line segment AC is drawn, and a line segment HK is drawn parallel to AC . The triangles are similar by the AA criterion.

A diagram of a triangle with vertices labeled B (top left), C (top right), and A (bottom). A horizontal line segment DE is drawn inside the triangle, with D on side AB and E on side AC. The line segment DE is parallel to the base BC.

$$\angle ABC = \angle ADE \text{ [corresponding angles]}$$

$$\angle ACB = \angle AED \text{ [corresponding angles]}$$

$$\angle A = \angle A \text{ [common angle]}$$

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$$\therefore \triangle ABC \sim \triangle ADE$$

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{BC}{DE} = \frac{AB}{DA}$$

$$\Rightarrow \frac{8}{DE} = \frac{6}{1.5}$$

$$\therefore DE = 2 \text{ cm}$$

22. Question

In Fig. 4.241, $DE \parallel BC$ and $AD = \frac{1}{2} BD$. If $BC = 4.5 \text{ cm}$, find DE .

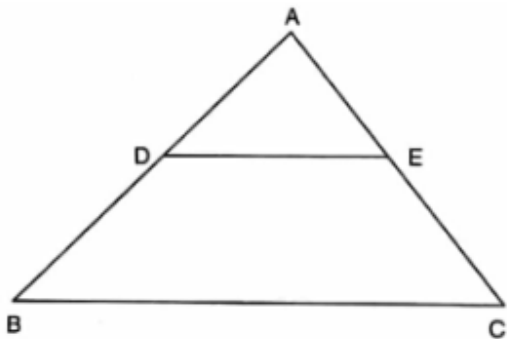


Fig. 4.241

Answer

Given $DE \parallel BC$, $AD = \frac{1}{2} BD$ and $BC = 4.5 \text{ cm}$

In $\triangle ABC$ and $\triangle ADE$,

$$\angle ABC = \angle ADE \text{ [corresponding angles]}$$

$$\angle ACB = \angle AED \text{ [corresponding angles]}$$

$$\angle A = \angle A \text{ [common angle]}$$

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$$\therefore \triangle ABC \sim \triangle ADE$$

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{AD}{AD + BD} = \frac{DE}{BC}$$

$$\Rightarrow \frac{\frac{1}{2}BD}{\frac{1}{2}BD + BD} = \frac{DE}{BC}$$

$$\Rightarrow \frac{1}{3} = \frac{DE}{BC}$$

$$\Rightarrow \frac{1}{3} = \frac{DE}{4.5}$$

$$\therefore DE = 1.5 \text{ cm}$$

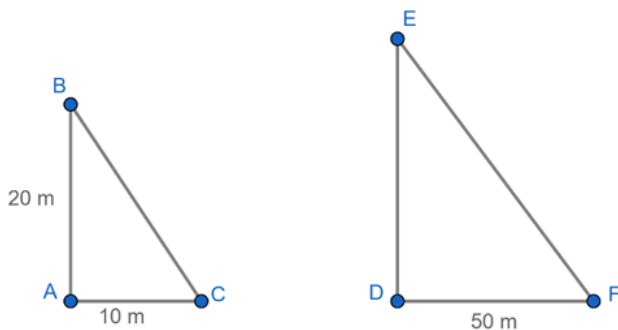
1. Question

A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time, a tower casts a shadow 50 m long on the ground. The height of the tower is

- A. 100 m
- B. 120 m
- C. 25 m
- D. 200 m.

Answer

Given A vertical stick 20 m long casts a shadow 10 m long on the ground and a tower casts a shadow 50 m long on the ground.



We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

In $\triangle ABC$ and $\triangle DEF$,

$$\angle A = \angle D = 90^\circ, \angle C = \angle F$$

$$\therefore \triangle ABC \sim \triangle DEF$$

We know that if two triangles are similar then their sides are proportional.

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$$

$$\Rightarrow \frac{20}{DE} = \frac{10}{50}$$

$$\therefore DE = 100 \text{ m}$$

2. Question

Sides of two similar triangles are in the ratio 4 : 9 . Areas of these triangles are in the ratio.

- A. 2 : 3
- B. 4 : 9
- C. 81 : 16
- D. 16 : 81

Answer

Given sides of two similar triangles are in the ratio 4: 9.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta 1)}{ar(\Delta 2)} = \frac{(side1)^2}{(side2)^2} = \frac{4^2}{9^2} = \frac{16}{81}$$

$$\therefore ar (\Delta 1): ar (\Delta 2) = 16: 81$$

3. Question

The areas of two similar triangles are in respectively 9 cm² and 16 cm². The ratio of their corresponding sides is

- A. 3:4
- B. 4 : 3
- C. 2 : 3
- D. 4 : 5

Answer

Given that area of two similar triangles are 9 cm² and 16 cm².

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta 1)}{ar(\Delta 2)} = \frac{(side1)^2}{(side2)^2}$$

$$\Rightarrow \frac{9}{16} = \frac{(side1)^2}{(side2)^2}$$

$$\Rightarrow \frac{side1}{side2} = \frac{3}{4}$$

∴ Ratio of their corresponding sides is 3: 4.

4. Question

The areas of two similar triangles $\triangle ABC$ and $\triangle DEF$ are 144 cm^2 and 81 cm^2 respectively. If the longest side of larger $\triangle ABC$ be 36 cm, then. the longest side of the smaller triangle $\triangle DEF$ is

- A. 20 cm
- B. 26 cm
- C. 27 cm
- D. 30 cm

Answer

Given that area of two similar triangles $\triangle ABC$ and $\triangle DEF$ are 144 cm^2 and 81 cm^2 . Also the longest side of larger $\triangle ABC$ is 36 cm.

We have to find the longest side of the smaller triangle $\triangle DEF$. Let it be x.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{(longest\ side\ of\ \triangle ABC)^2}{(longest\ side\ of\ \triangle DEF)^2}$$

$$\Rightarrow \frac{144}{81} = \frac{(36)^2}{(x)^2}$$

$$\Rightarrow \frac{36}{x} = \frac{12}{9}$$

$$\Rightarrow x = 27 \text{ cm}$$

∴ Longest side of $\triangle DEF$ is 27 cm.

5. Question

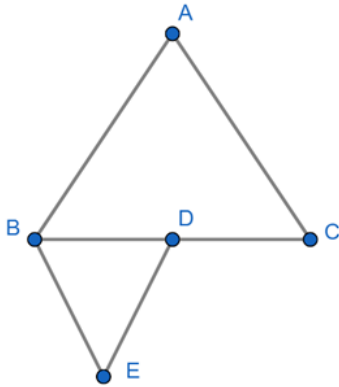
$\triangle ABC$ and $\triangle BDE$ are two equilateral triangles such that D is the mid-point of BC. The ratio of the areas of triangles ABC and BDE is

- A. 2 : 1
- B. 1 : 2
- C. 4 : 1

D. 1 : 4

Answer

Given $\triangle ABC$ and $\triangle BDE$ are two equilateral triangles such that D is the midpoint of BC.



Since the given triangles are equilateral, they are similar triangles.

And also since D is the mid-point of BC, $BD = DC$.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle BDE)} = \frac{(BC)^2}{(BD)^2}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle BDE)} = \frac{(BD + DC)^2}{(BD)^2}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle BDE)} = \frac{(BD + BD)^2}{(BD)^2}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle BDE)} = \frac{(2BD)^2}{(BD)^2}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle BDE)} = \frac{4}{1}$$

$$\therefore ar(\triangle ABC) : ar(\triangle BDE) = 4 : 1$$

6. Question

Two isosceles triangles have equal angles and their areas are in the ratio 16 : 25. The ratio of their corresponding heights is

A. 4 : 5

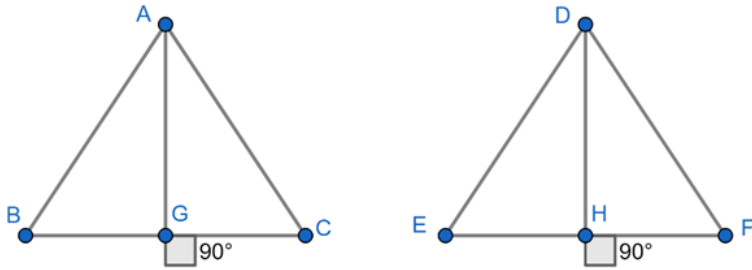
B. 5 : 4

C. 3 : 2

D. 5 : 7

Answer

Given two isosceles triangles have equal angles and their areas are in the ratio 16 : 25.



We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

In $\triangle ABC$ and $\triangle DEF$,

if $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$, then $\triangle ABC \sim \triangle DEF$

We know that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes.

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \left(\frac{AG}{DH}\right)^2$$

$$\Rightarrow \frac{16}{25} = \left(\frac{AG}{DH}\right)^2$$

$$\Rightarrow \frac{AG}{DH} = \frac{4}{5}$$

$\therefore AG : DH = 4 : 5$

7. Question

If $\triangle ABC$ and $\triangle DEF$ are similar such that $2AB = DE$ and $BC = 8$ cm, then $EF =$

- A. 16 cm
- B. 12 cm
- C. 8 cm
- D. 4 cm.

Answer

Given $\triangle ABC$ and $\triangle DEF$ are similar triangles such that $2AB = DE$ and $BC = 8$ cm

We know that if two triangles are similar then their sides are proportional.

For $\triangle ABC$ and $\triangle DEF$,

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \frac{1}{2} = \frac{8}{EF}$$

$$\therefore EF = 16 \text{ cm}$$

8. Question

If $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2}{5}$, then Area ($\triangle ABC$): Area ($\triangle DEF$) =

- A. 2 : 5
- B. 4 : 25
- C. 4 : 15
- D. 8 : 125

Answer

Given $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2}{5}$

We know that if two triangles are similar then their sides are proportional.

Since $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, $\triangle ABC$ and $\triangle DEF$ are similar.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{(AB)^2}{(DE)^2}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{(2)^2}{(5)^2}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{4}{25}$$

$$\therefore ar(\triangle ABC): ar(\triangle DEF) = 4: 25$$

9. Question

$\triangle ABC$ is such that $AB = 3 \text{ cm}$, $BC = 2 \text{ cm}$ and $CA = 2.5 \text{ cm}$. If $\triangle DEF \sim \triangle ABC$ and $EF = 4 \text{ cm}$, then perimeter of $\triangle DEF$ is

- A. 7.5 cm

- B. 15 cm
- C. 22.5 cm
- D. 30 cm.

Answer

Given that $\triangle ABC$ and $\triangle DEF$ are similar triangles such that $AB = 3$ cm, $BC = 2$ cm, $CA = 2.5$ cm and $EF = 4$ cm.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

First consider,

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\Rightarrow \frac{3}{2} = \frac{DE}{4}$$

$$\Rightarrow DE = 6 \text{ cm ... (1)}$$

Now,

$$\frac{CA}{BC} = \frac{DF}{EF}$$

$$\Rightarrow \frac{2.5}{2} = \frac{DF}{4}$$

$$\Rightarrow DF = 5 \text{ cm ... (2)}$$

Then, perimeter of $\triangle DEF = DE + EF + DF = 6 + 4 + 5$

\therefore Perimeter of $\triangle DEF = 15$ cm

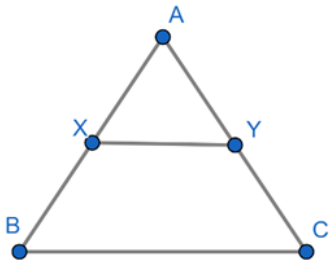
10. Question

XY is drawn parallel to the base BC of $\triangle ABC$ cutting AB at X and AC at Y. If $AB = 4$ BX and $YC = 2$ cm, then AY =

- A. 2 cm
- B. 4 cm
- C. 6 cm
- D. 8 cm.

Answer

Given XY is drawn parallel to the base BC of a $\triangle ABC$ cutting AB at X and AC at Y. $AB = 4$ BX and $YC = 2$ cm.



In $\triangle AXY$ and $\triangle ABC$,

$$\angle AXY = \angle ABC \text{ [corresponding angles]}$$

$$\angle AYX = \angle ACB \text{ [corresponding angles]}$$

$$\angle A = \angle A \text{ [common angle]}$$

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$$\therefore \triangle AXY \sim \triangle ABC$$

Let $BX = x$, so $AB = 4x$ and $AX = 3x$.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AX}{BX} = \frac{AY}{YC}$$

$$\Rightarrow \frac{3x}{x} = \frac{AY}{2}$$

$$\therefore AY = 6 \text{ cm}$$

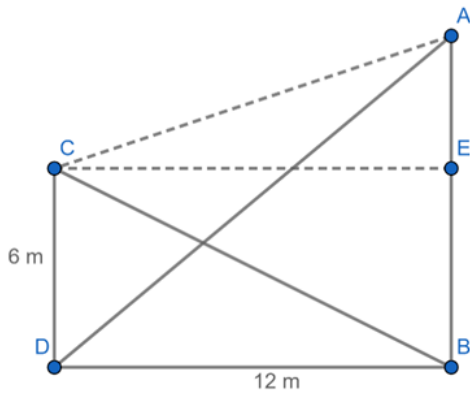
11. Question

Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, the distance between their tops is

- A. 12 m
- B. 14 m
- C. 13 m.
- D. 11 m

Answer

Given two poles of heights 6 m and 11 m stand vertically upright on a plane ground. Distance between their foot is 12 m.



Let CD be the pole with height 6 m. AB is the pole with height 11m and DB = 12 m

Let us assume a point E on the pole AB which is 6m from the base of AB.

Hence $AE = AB - 6 = 11 - 6 = 5\text{m}$

We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle AEC,

$$\Rightarrow AC^2 = AE^2 + EC^2$$

Since CDEB forms a rectangle and opposite sides of rectangle are equal,

$$\Rightarrow AC^2 = 5^2 + 12^2$$

$$= 25 + 144$$

$$= 169$$

$$\Rightarrow AC = 13$$

\therefore The distance between their tops is 13 m.

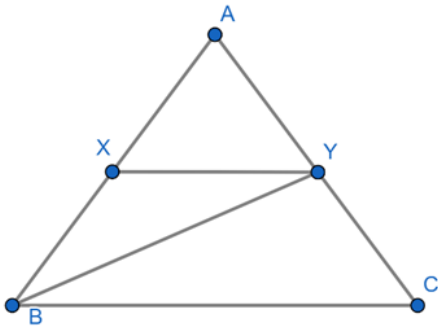
12. Question

In $\triangle ABC$, a line XY parallel to BC cuts AB at X and AC at Y. If BY bisects $\angle XYC$, then

- A. $BC = CY$
- B. $BC = BY$
- C. $BC \neq CY$
- D. $BC \neq BY$

Answer

Given in $\triangle ABC$, $XY \parallel BC$ and BY is a bisector of $\angle XYC$.



Since $XY \parallel BC$,

$\angle YBC = \angle BYC$ [alternate angles]

Now, in $\triangle BYC$, two angles are equal.

Hence, two corresponding sides will be equal.

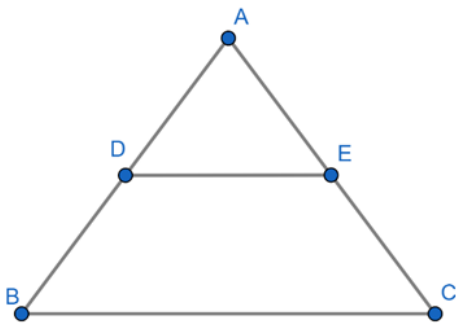
$\therefore BC = CY$

13. Question

In $\triangle ABC$, D and E are points on side AB and AC respectively such that $DE \parallel BC$ and $AD : DB = 3 : 1$.
If $EA = 3.3$ cm, then $AC =$

- A. 1.1 cm
- B. 4 cm
- C. 4.4 cm
- D. 5.5 cm

Answer



From the given figure $\triangle ABC$, $DE \parallel BC$.

Let $AC = x$ cm.

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Then $\frac{AD}{AB} = \frac{AE}{AC}$

$$\Rightarrow \frac{AD}{AD + BD} = \frac{3.3}{x}$$

$$\Rightarrow \frac{AD}{AD + \frac{1}{3}AD} = \frac{3.3}{x}$$

$$\Rightarrow x = 4.4 \text{ cm}$$

$$\therefore AC = 4.4 \text{ cm}$$

14. Question

In triangles ABC and DEF, $\angle A = \angle E = 40^\circ$, $AB : ED = AC : EF$ and $\angle F = 65^\circ$, then $\angle B =$

A. 35°

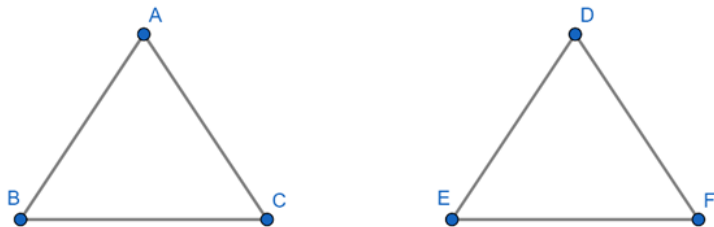
B. 65°

C. 75°

D. 85°

Answer

Given in triangles ABC and DEF, $\angle A = \angle E = 40^\circ$, $AB : ED = AC : EF$ and $\angle F = 65^\circ$.



We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

In $\triangle ABC$ and $\triangle DEF$,

$\angle A = \angle E$ and $AB : ED = AC : EF$ then $\triangle ABC \sim \triangle DEF$

So, $\angle A = \angle E = 40^\circ$

$\Rightarrow \angle C = \angle F = 65^\circ$

Similarly, $\angle B = \angle D$

We know that the sum of all angles of a triangle is equal to 180° .

$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 40^\circ + \angle B + 65^\circ = 180^\circ$

$$\Rightarrow \angle B + 115^\circ = 180^\circ$$

$$\Rightarrow \angle B = 180^\circ - 115^\circ = 75^\circ$$

$$\therefore \angle B = 75^\circ$$

15. Question

If ABC and DEF are similar triangles such that $\angle A = 47^\circ$ and $\angle E = 83^\circ$, then $\angle C =$

A. 50°

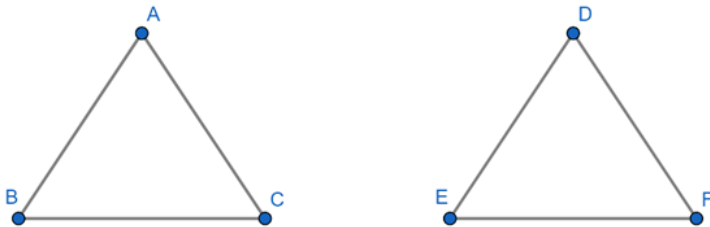
B. 60°

C. 70°

D. 80°

Answer

Given ABC and DEF are two similar triangles, $\angle A = 47^\circ$ and $\angle E = 83^\circ$



We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

In $\triangle ABC$ and $\triangle DEF$,

if $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$, then $\triangle ABC \sim \triangle DEF$

So, $\angle A = \angle D$

$$\Rightarrow \angle D = 47^\circ \dots (1)$$

Similarly, $\angle B = \angle E$

$$\Rightarrow \angle B = 83^\circ \dots (2)$$

We know that the sum of all angles of a triangle is equal to 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 47^\circ + 83^\circ + \angle C = 180^\circ$$

$$\Rightarrow 130^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 130^\circ = 50^\circ$$

$$\therefore \angle C = 50^\circ$$

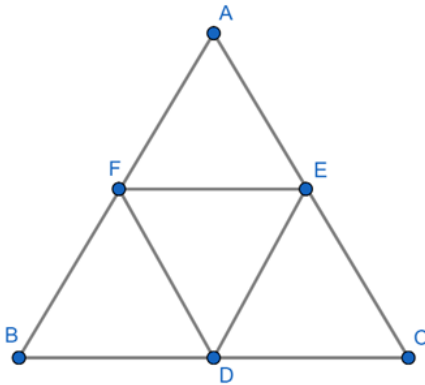
16. Question

If D, E, F are the mid-points of sides BC, CA and AB respectively of ΔABC , then the ratio of the areas of triangles DEF and ABC is

- A. 1 : 4
- B. 1 : 2
- C. 2 : 3
- D. 4 : 5

Answer

Given D, E and F are the mid-points of sides BC, CA and AB respectively of ΔABC .



Then $DE \parallel AB$, $DE \parallel FA$... (1)

And $DF \parallel CA$, $DF \parallel AE$... (2)

From (1) and (2), we get AFDE is a parallelogram.

Similarly, BDEF is a parallelogram.

In ΔADE and ΔABC ,

$\Rightarrow \angle FDE = \angle A$ [Opposite angles of \parallel gm AFDE]

$\Rightarrow \angle DEF = \angle B$ [Opposite angles of \parallel gm BDEF]

\therefore By AA similarity criterion, $\Delta ABC \sim \Delta DEF$.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{(DE)^2}{(AB)^2}$$

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{\left(\frac{1}{2}AB\right)^2}{(AB)^2}$$

$$\Rightarrow \frac{ar(\triangle DEF)}{ar(\triangle ABC)} = \frac{1}{4}$$

$$\therefore ar(\triangle DEF): ar(\triangle ABC) = 1: 4$$

17. Question

In a $\triangle ABC$, $\angle A = 90^\circ$, $AB = 5$ cm and $AC = 12$ cm. If $AD \perp BC$, then $AD =$

A. $\frac{13}{2}$ cm

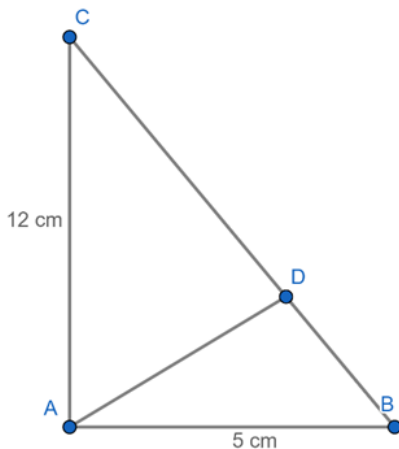
B. $\frac{60}{13}$ cm

C. $\frac{13}{60}$ cm

D. $\frac{2\sqrt{15}}{13}$ cm

Answer

Given in $\triangle ABC$, $\angle A = 90^\circ$, $AB = 5$ cm, $AC = 12$ cm and $AD \perp BC$



In $\triangle ACB$ and $\triangle ADC$,

$$\angle CAB = \angle ADC [90^\circ]$$

$$\angle ABC = \angle CAD [\text{corresponding angles}]$$

$$\angle C = \angle C [\text{common angle}]$$

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$$\therefore \triangle ACB \sim \triangle ADC$$

$$\Rightarrow \frac{AD}{AB} = \frac{AC}{BC}$$

$$\Rightarrow AD = \frac{AB(AC)}{BC}$$

$$\Rightarrow AD = \frac{12(5)}{13}$$

$$\Rightarrow AD = \frac{60}{13}$$

$$\therefore AD = 60/13 \text{ cm}$$

18. Question

In an equilateral triangle ABC, if $AD \perp BC$, then

A. $2AB^2 = 3AD^2$

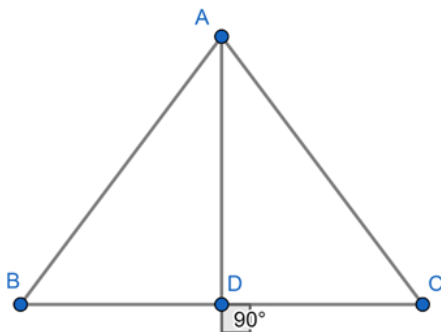
B. $4AB^2 = 3AD^2$

C. $3AB^2 = 4AD^2$

D. $3AB^2 = 2AD^2$

Answer

Given in equilateral $\triangle ABC$, $AD \perp BC$.



We know that the Pythagoras theorem states that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In $\triangle ABD$,

$$\Rightarrow AB^2 = AD^2 + BD^2$$

$$\Rightarrow AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 \quad [\because BD = \frac{1}{2}BC]$$

$$\Rightarrow AB^2 = AD^2 + \left(\frac{1}{2}AB\right)^2 \quad [\because AB = BC]$$

$$\Rightarrow AB^2 = AD^2 + \frac{1}{4}AB^2$$

$$\therefore 3AB^2 = 4AD^2$$

19. Question

If $\triangle ABC$ is an equilateral triangle such that $AD \perp BC$, then $AD^2 =$

A. $\frac{3}{2} DC^2$

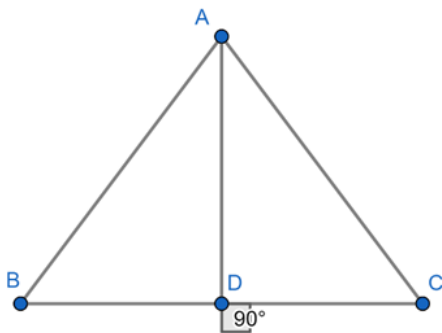
B. $2 DC^2$

C. $3 CD^2$

D. $4 DC^2$

Answer

Given in an equilateral $\triangle ABC$, $AD \perp BC$



Since $AD \perp BC$, $BD = CD = BC/2$

We know that the Pythagoras theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ADC ,

$$\Rightarrow AC^2 = AD^2 + DC^2$$

$$\Rightarrow BC^2 = AD^2 + DC^2$$

$$\Rightarrow (2DC)^2 = AD^2 + DC^2$$

$$\Rightarrow 4DC^2 = AD^2 + DC^2$$

$$\Rightarrow 3DC^2 = AD^2$$

$$\therefore 3CD^2 = AD^2$$

20. Question

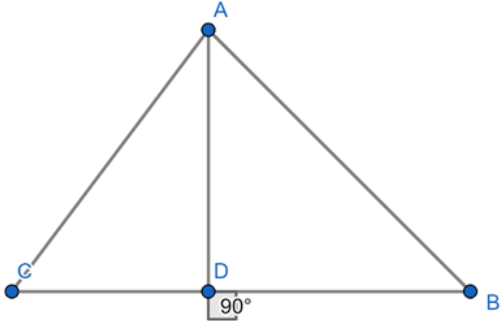
In a $\triangle ABC$, perpendicular AD from A on BC meets BC at D . If $BD = 8$ cm, $DC = 2$ cm and $AD = 4$ cm, then

A. $\triangle ABC$ is isosceles

- B. $\triangle ABC$ is equilateral
- C. $AC = 2 AB$
- D. $\triangle ABC$ is right-angled at A.

Answer

Given in $\triangle ABC$, $AD \perp BC$, $BD = 8$ cm, $DC = 2$ cm and $AD = 4$ cm.



We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ADC,

$$\Rightarrow AC^2 = AD^2 + DC^2$$

$$\Rightarrow AC^2 = (4)^2 + (2)^2$$

$$= 16 + 4$$

$$\therefore AC^2 = 20 \dots (1)$$

In $\triangle ADB$,

$$\Rightarrow AB^2 = AD^2 + BD^2 = 4^2 + 8^2 = 16 + 64$$

$$\therefore AB^2 = 80 \dots (2)$$

Now, in $\triangle ABC$,

$$\Rightarrow BC^2 = (CD + DB)^2 = (2 + 8)^2 = 10^2 = 100$$

$$\text{And } AB^2 + CA^2 = 80 + 20 = 100$$

$$\therefore AB^2 + CA^2 = BC^2$$

Hence, $\triangle ABC$ is right angled at A.

21. Question

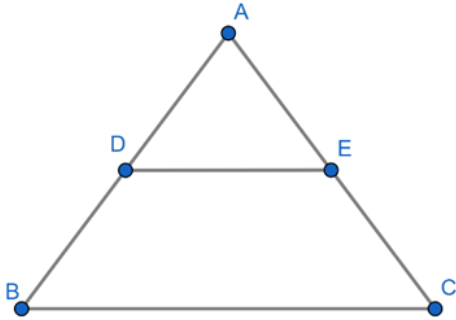
In a $\triangle ABC$, point D is on side AB and point E is on side AC, such that BCED is a trapezium. If $DE : BC = 3 : 5$, then Area ($\triangle ADE$): Area ($\triangle BCED$) =

- A. 3 : 4

- B. 9: 16
C. 3: 5
D. 9 : 25

Answer

Given in $\triangle ABC$, point D is on side AB and point E is on side AC, such that BCED is a trapezium and $DE: BC = 3: 5$.



In $\triangle ABC$ and $\triangle ADE$,

$$\angle ABC = \angle ADE \text{ [corresponding angles]}$$

$$\angle ACB = \angle AED \text{ [corresponding angles]}$$

$$\angle A = \angle A \text{ [common angle]}$$

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$$\therefore \triangle ABC \sim \triangle ADE$$

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\triangle ADE)}{ar(\triangle ABC)} = \frac{(DE)^2}{(BC)^2} = \frac{3^2}{5^2} = \frac{9}{25}$$

$$\text{Let } ar(\triangle ADE) = 9x \text{ sq. units and } ar(\triangle ABC) = 25x \text{ sq. units}$$

$$\Rightarrow ar(\text{trap } BCED) = ar(\triangle ABC) - ar(\triangle ADE)$$

$$= 25x - 9x$$

$$= 16x \text{ sq. units}$$

Now,

$$\Rightarrow \frac{ar(\triangle ADE)}{ar(\text{trap } BCED)} = \frac{9x}{16x} = \frac{9}{16}$$

$$\therefore ar(\triangle ADE): ar(\text{trap } BCED) = 9: 16$$

22. Question

In a $\triangle ABC$, AD is the bisector of $\angle BAC$. If AB = 6 cm, AC = 5 cm and BD = 3 cm,, then DC =

- A. 11.3 cm
- B. 2.5 cm
- C. 3 5 cm
- D. None of these.

Answer

Given AD is the bisector of $\angle BAC$. AB = 6 cm, AC = 5 cm and BD = 3 cm.

We know that the internal bisector of angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \frac{6}{5} = \frac{3}{DC}$$

$$\therefore DC = 2.5 \text{ cm}$$

23. Question

In a $\triangle ABC$, AD is the bisector of $\angle BAC$. If AB = 8 cm, BD = 6 cm and DC = 3 cm. Find AC

- A. 4 cm
- B. 6 cm
- C. 3 cm
- D. 8 cm

Answer

Given AD is the bisector of $\angle BAC$. AB = 8 cm, DC = 3 cm and BD = 6 cm.

We know that the internal bisector of angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \frac{8}{AC} = \frac{6}{3}$$

$$\therefore AC = 4 \text{ cm}$$

24. Question

ABCD is a trapezium such that $BC \parallel AD$ and AB = 4 cm. If the diagonals AC and BD intersect at O

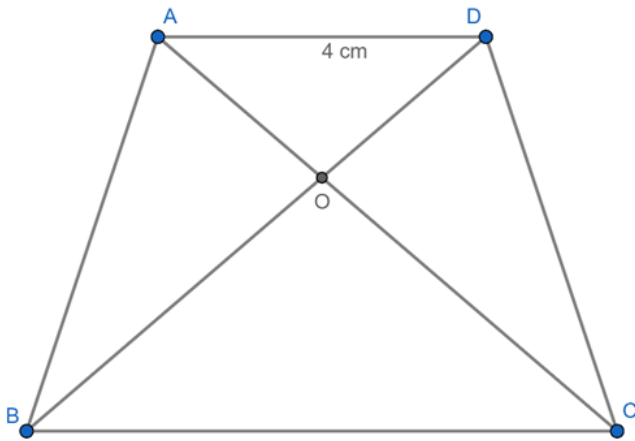
such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then BC =

- A. 7 cm
- B. 8 cm
- C. 9 cm
- D. 6 cm

Answer

Given ABCD is a trapezium in which $BC \parallel AD$ and $AD = 4$ cm.

Also, the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$



In $\triangle AOD$ and $\triangle COB$,

$\angle OAD = \angle OCB$ [alternate angles]

$\angle ODA = \angle OBC$ [alternate angles]

$\angle AOD = \angle BOC$ [vertically opposite angles]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$\therefore \triangle AOD \sim \triangle COB$

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AO}{CO} = \frac{DO}{BO} = \frac{AD}{BC}$$

$$\Rightarrow \frac{1}{2} = \frac{AD}{BC}$$

$$\Rightarrow \frac{1}{2} = \frac{4}{BC}$$

$\therefore BC = 8$ cm

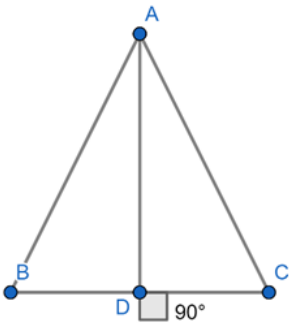
25. Question

If ABC is an isosceles triangle and D is a point on BC such that $AD \perp BC$, then

- A. $AB^2 - AD^2 = BD \cdot DC$
- B. $AB^2 - AD^2 = BD^2 - DC^2$
- C. $AB^2 + AD^2 = BD \cdot DC$
- D. $AB^2 + AD^2 = BD^2 - DC^2$

Answer

Given ABC is an isosceles triangles and $AD \perp BC$.



We know that in an isosceles triangle, the perpendicular from the vertex bisects the base.

$$\therefore BD = DC$$

We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ABD,

$$\Rightarrow AB^2 = AD^2 + BD^2$$

$$\Rightarrow AB^2 - AD^2 = BD^2$$

$$\Rightarrow AB^2 - AD^2 = BD (BD)$$

Since $BD = DC$,

$$\therefore AB^2 - AD^2 = BD (DC)$$

26. Question

$\triangle ABC$ is a right triangle right-angled at A and $AD \perp BC$. Then, $\frac{BD}{DC} =$

A. $\left(\frac{AB}{AC} \right)^2$

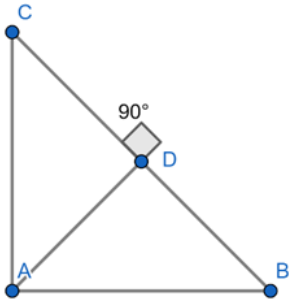
B. $\frac{AB}{AC}$

C. $\left(\frac{AB}{AD}\right)^2$

D. $\frac{AB}{AD}$

Answer

Given $\triangle ABC$ is a right triangle right-angled at A and $AD \perp BC$.



$$\Rightarrow \angle CAD + \angle BAD = 90^\circ \dots (1)$$

$$\Rightarrow \angle BAD + \angle ABD = 90^\circ \dots (2)$$

From (1) and (2),

$$\angle CAD = \angle ABD$$

By AA similarity,

In $\triangle ADB$ and $\triangle ADC$,

$$\Rightarrow \angle ADB = \angle ADC [90^\circ \text{ each}]$$

$$\Rightarrow \angle ABD = \angle CAD$$

$$\therefore \triangle ADB \sim \triangle ADC$$

We know that if two triangles are similar, their corresponding angles are equal and corresponding sides are proportional.

$$\therefore \frac{BD}{DC} = \frac{AB}{AC}$$

27. Question

If ABC is a right triangle right-angled at B and M, N are the mid-points of AB and BC respectively, then $4(AN^2 + CM^2) =$

A. $4 AC^2$

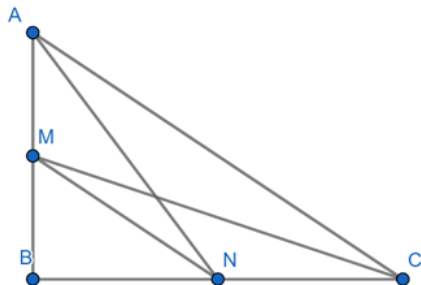
B. $5 AC^2$

C. $\frac{5}{4} AC^2$

D. $6 AC^2$

Answer

Given ABC is a right triangle right-angled at B and M, N are mid-points of AB and BC respectively.



M is the mid-point of AB.

$$\Rightarrow BM = \frac{AB}{2}$$

And N is the mid-point of BC.

$$\Rightarrow BN = \frac{BC}{2}$$

Now,

$$\Rightarrow AN^2 + CM^2 = (AB^2 + (\frac{1}{4}BC^2)) + ((\frac{1}{4}AB^2) + BC^2)$$

$$= AB^2 + \frac{1}{4}BC^2 + \frac{1}{4}AB^2 + BC^2$$

$$= \frac{5}{4} (AB^2 + BC^2)$$

$$\therefore 4 (AN^2 + CM^2) = 5AC^2$$

Hence proved.

28. Question

If E is a point on side CA of an equilateral triangle ABC such that $BE \perp CA$, then $AB^2 + BC^2 + CA^2 =$

A. $2 BE^2$

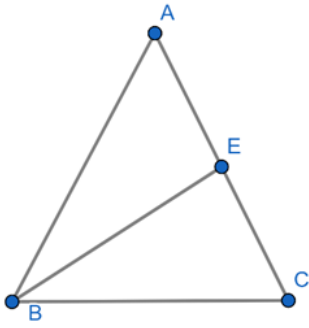
B. $3 BE^2$

C. $4 BE^2$

D. $6 BE^2$

Answer

Given in equilateral $\triangle ABC$, $BE \perp AC$.



We know that in an equilateral triangle, the perpendicular from the vertex bisects the base.

$$\therefore CE = AE = AC/2$$

In $\triangle ABE$,

$$\Rightarrow AB^2 = BE^2 + AE^2$$

Since $AB = BC = AC$,

$$\Rightarrow AB^2 = BC^2 = AC^2 = BE^2 + AE^2$$

$$\Rightarrow AB^2 + BC^2 + AC^2 = 3BE^2 + 3AE^2$$

Since BE is an altitude, $BE = \frac{\sqrt{3}}{2} AB$

$$\Rightarrow BE = \frac{\sqrt{3}}{2} AB$$

$$= \frac{\sqrt{3}}{2} AC = \frac{\sqrt{3}}{2} (2AE)$$

$$BE = \sqrt{3} AE$$

$$\Rightarrow AB^2 + BC^2 + AC^2 = 3BE^2 + 3 \left(\frac{BE}{\sqrt{3}} \right)^2$$

$$= 3BE^2 + BE^2$$

$$\therefore AB^2 + BC^2 + AC^2 = 4BE^2$$

29. Question

In a right triangle ABC right-angled at B, if P and Q are points on the sides AB and AC respectively, then

A. $AQ^2 + CP^2 = 2 (AC^2 + PQ^2)$

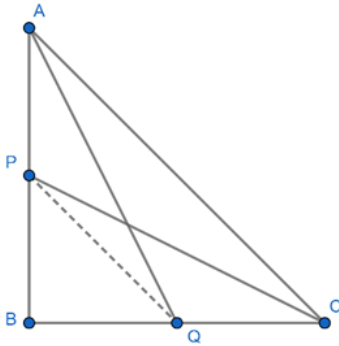
B. $2 (AQ^2 + CP^2) = AC^2 + PQ^2$

C. $AQ^2 + CP^2 = AC^2 + PQ^2$

$$D. AQ + CP = \frac{1}{2} (AC + PQ).$$

Answer

Given in right triangle ABC right-angled at B, P and Q are points on the sides AB and BC respectively.



Applying Pythagoras Theorem,

In $\triangle AQB$,

$$\Rightarrow AQ^2 = AB^2 + BQ^2 \dots (1)$$

In $\triangle PBC$,

$$\Rightarrow CP^2 = PB^2 + BC^2 \dots (2)$$

Adding (1) and (2),

$$\Rightarrow AQ^2 + CP^2 = AB^2 + BQ^2 + PB^2 + BC^2 \dots (3)$$

In $\triangle ABC$,

$$\Rightarrow AC^2 = AB^2 + BC^2 \dots (4)$$

In $\triangle PBQ$,

$$\Rightarrow PQ^2 = PB^2 + BQ^2 \dots (5)$$

From (3), (4) and (5),

$$\therefore AQ^2 + CP^2 = AC^2 + PQ^2$$

30. Question

If in $\triangle ABC$ and $\triangle DEF$, $\frac{AB}{DE} = \frac{BC}{FD}$, then $\triangle ABC \sim \triangle DEF$ when

A. $\angle A = \angle F$

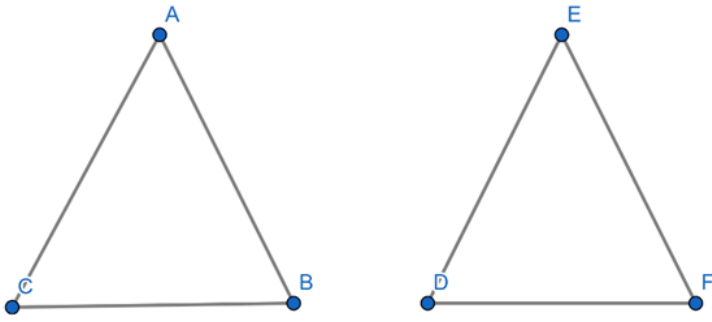
B. $\angle A = \angle D$

C. $\angle B = \angle D$

D. $\angle B = \angle E$

Answer

Given in $\triangle ABC$ and $\triangle DEF$, $\frac{AB}{DE} = \frac{BC}{FD}$



We know that if in two triangles, one pair of corresponding sides are proportional and included angles are equal, then the two triangles are similar.

Hence, $\triangle ABC$ is similar to $\triangle DEF$, we should have $\angle B = \angle D$.

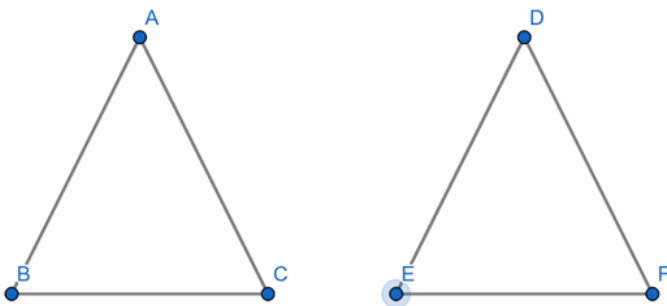
31. Question

If in two triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FE} = \frac{CA}{FD}$, then

- A. $\triangle FDE \sim \triangle CAB$
- B. $\triangle FDE \sim \triangle ABC$
- C. $\triangle CBA \sim \triangle FDE$
- D. $\triangle BCA \sim \triangle FDE$

Answer

Given that $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$



We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$$\Rightarrow \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$$

$$\therefore \triangle CAB \sim \triangle FDE$$

Hence proved.

32. Question

$\triangle ABC \sim \triangle DEF$, ar ($\triangle ABC$) = 9 cm^2 , ar ($\triangle DEF$) = 16 cm^2 . If $BC = 2.1 \text{ cm}$, then the measure of EF is

- A. 2.8 cm
- B. 4.2 cm
- C. 2.5 cm
- D. 4.1 cm

Answer

Given Ar ($\triangle ABC$) = 9 cm^2 , ar ($\triangle DEF$) = 16 cm^2 and $BC = 2.1 \text{ cm}$

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{9}{16} = \frac{2.1^2}{EF^2}$$

$$\Rightarrow \frac{3}{4} = \frac{2.1}{EF}$$

$$\therefore EF = 2.8 \text{ cm}$$

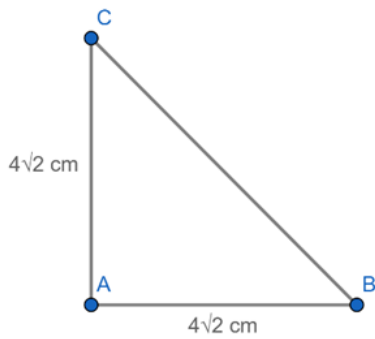
33. Question

The length of the hypotenuse of an isosceles right triangle whose one side is $4\sqrt{2} \text{ cm}$ is

- A. 12 cm
- B. 8 cm
- C. $8\sqrt{2} \text{ cm}$
- D. $12\sqrt{2} \text{ cm}$

Answer

Given that one side of isosceles right triangle is $4\sqrt{2} \text{ cm}$.



We know that in isosceles triangle two sides are equal.

In isosceles triangle ABC, let AB and AC be two equal sides of measure $4\sqrt{2}$ cm.

We know that the Pythagoras theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ABC,

$$\Rightarrow BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC^2 = (4\sqrt{2})^2 + (4\sqrt{2})^2$$

$$= 32 + 32$$

$$= 64$$

$$\therefore BC = 8 \text{ cm}$$

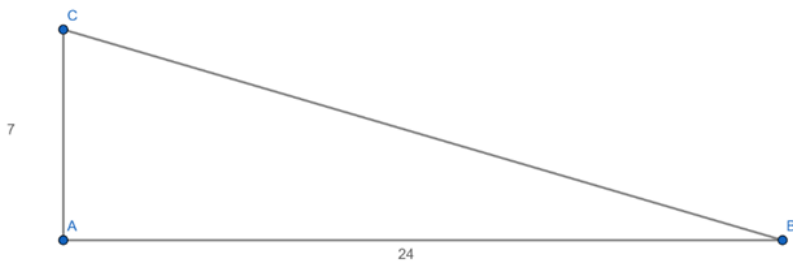
34. Question

A man goes 24 m due west and then 7 m due north. How far is he from the starting point?

- A. 31 m
- B. 17 m
- C. 25 m
- D. 26 m

Answer

Given a man goes 24 m due west and then 7 m due north.



We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ABC,

$$\Rightarrow BC^2 = AB^2 + AC^2$$

$$= 24^2 + 7^2$$

$$= 576 + 49$$

$$= 625$$

$$\therefore BC = 25 \text{ m}$$

35. Question

$\triangle ABC \sim \triangle DEF$. If $BC = 3 \text{ cm}$, $EF = 4 \text{ cm}$ and $\text{ar}(\triangle ABC) = 54 \text{ cm}^2$, then $\text{ar}(\triangle DEF) =$

A. 108 cm^2

B. 96 cm^2

C. 48 cm^2

D. 100 cm^2

Answer

Given $\triangle ABC \sim \triangle DEF$, $BC = 3 \text{ cm}$, $EF = 4 \text{ cm}$ and $\text{ar}(\triangle ABC) = 54 \text{ cm}^2$

We know that ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{54}{\text{ar}(\triangle DEF)} = \frac{3^2}{4^2}$$

$$\Rightarrow \frac{54}{\text{ar}(\triangle DEF)} = \frac{9}{16}$$

$$\Rightarrow \text{ar}(\triangle DEF) = \frac{16(54)}{9}$$

$$\therefore \text{ar}(\triangle DEF) = 96 \text{ cm}^2$$

36. Question

$\triangle ABC \sim \triangle DEF$. such that $\text{ar}(\triangle ABC) = 4 \text{ ar}(\triangle PQR)$. If $BC = 12 \text{ cm}$, then $QR =$

A. 9 cm

B. 10 cm

C. 6 cm

D. 8 cm

Answer

Given $\text{ar}(\Delta ABC) \sim \text{ar}(\Delta PQR)$ such that $\text{ar}(\Delta ABC) = 4 \text{ ar}(\Delta PQR)$ and $BC = 12 \text{ cm}$

We know that ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \frac{4\text{ar}(\Delta PQR)}{\text{ar}(\Delta PQR)} = \frac{12^2}{QR^2}$$

$$\Rightarrow \frac{4}{1} = \frac{12^2}{QR^2}$$

$$\Rightarrow \frac{2}{1} = \frac{12}{QR}$$

$$\therefore QR = 6 \text{ cm}$$

37. Question

The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm, then the corresponding median of the other triangle is

A. 11 cm

B. 8.8 cm

C. 11.1 cm

D. 8.1 cm

Answer

Given areas of two similar triangles 121 cm^2 and 64 cm^2 respectively. The median of the first triangle is 12.1 cm.

We know that ratio of areas of two similar triangles is equal to the ratio of squares of their medians.

$$\Rightarrow \frac{\text{ar}(\Delta 1)}{\text{ar}(\Delta 2)} = \frac{\text{median}_1^2}{\text{median}_2^2}$$

$$\Rightarrow \frac{121}{64} = \frac{12.1^2}{\text{median}_2^2}$$

$$\Rightarrow \frac{11}{8} = \frac{12.1}{\text{median}_2}$$

$$\therefore \text{Median}_2 = 8.8 \text{ cm}$$

38. Question

If $\triangle ABC \sim \triangle DEF$ such that $DE = 3 \text{ cm}$, $EF = 2 \text{ cm}$, $DF = 2.5 \text{ cm}$, $BC = 4 \text{ cm}$, then perimeter of $\triangle ABC$ is

- A. 18 cm
- B. 20 cm
- C. 12 cm
- D. 15 cm

Answer

Given that $\triangle ABC$ and $\triangle DEF$ are similar triangles such that $AB = 3 \text{ cm}$, $DE = 3 \text{ cm}$, $DF = 2.5 \text{ cm}$ and $EF = 2 \text{ cm}$.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

First consider,

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\Rightarrow \frac{AB}{4} = \frac{3}{2}$$

$$\Rightarrow AB = 6 \text{ cm} \dots (1)$$

Now,

$$\frac{CA}{BC} = \frac{DF}{EF}$$

$$\Rightarrow \frac{CA}{4} = \frac{2.5}{2}$$

$$\Rightarrow CA = 5 \text{ cm} \dots (2)$$

Then, perimeter of $\triangle ABC = AB + BC + CA = 6 + 4 + 5$

$$\therefore \text{Perimeter of } \triangle ABC = 15 \text{ cm}$$

39. Question

In an equilateral triangle ABC if $AD \perp BC$, then $AD^2 =$

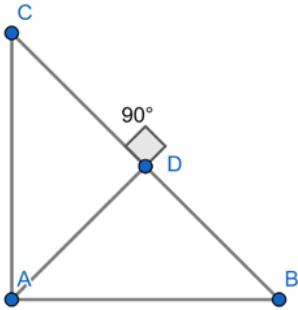
- A. CD^2
- B. $2CD^2$

C. $3CD^2$

D. $4CD^2$

Answer

Given in equilateral triangle ABC, $AD \perp BC$.



We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ADC,

$$\Rightarrow AC^2 = AD^2 + DC^2$$

$$\Rightarrow BC^2 = AD^2 + DC^2 \quad [\because AC = BC]$$

$$\Rightarrow (2DC)^2 = AD^2 + DC^2 \quad [\because BC = 2DC]$$

$$\Rightarrow 4DC^2 = AD^2 + DC^2$$

$$\Rightarrow 3DC^2 = AD^2$$

$$\therefore 3CD^2 = AD^2$$

40. Question

In an equilateral triangle ABC if $AD \perp BC$, then

A. $5AB^2 = 4AD^2$

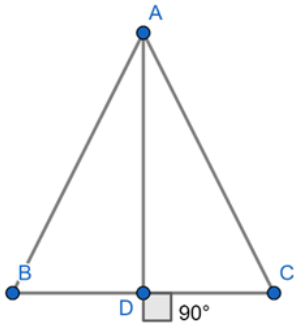
B. $3AB^2 = 4AD^2$

C. $4AB^2 = 3AD^2$

D. $2AB^2 = 3AD^2$

Answer

Given in equilateral triangle ABC if $AD \perp BC$.



We know that the Pythagoras theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ABD,

$$\Rightarrow AB^2 = AD^2 + BD^2$$

$$\Rightarrow AB^2 = AD^2 + (\frac{1}{2} BC)^2 \quad [\because BD = \frac{1}{2} BC]$$

$$\Rightarrow AB^2 = AD^2 + (\frac{1}{2} AB)^2 \quad [\because AB = BC]$$

$$\Rightarrow AB^2 = AD^2 + (\frac{1}{4} AB)^2$$

$$\therefore 3AB^2 = 4AD^2$$

41. Question

If $\triangle ABC \sim \triangle DEF$ such that $AB = 9.1$ cm and $DE = 6.5$ cm. If the perimeter of $\triangle DEF$ is 25 cm, then the perimeter of $\triangle ABC$ is

- A. 36 cm
- B. 30 cm
- C. 34 cm
- D. 35 cm

Answer

Given $\triangle ABC \sim \triangle DEF$ such that $AB = 9.1$ cm and $DE = 6.5$ cm.

Given that $\triangle ABC$ and $\triangle DEF$ are similar triangles such that $AB = 3$ cm, $BC = 2$ cm, $CA = 2.5$ cm and $EF = 4$ cm.

We know that ratio of corresponding sides of similar triangles is equal to the ratio of the perimeters.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{P_1}{P_2}$$

Consider,

$$\frac{AB}{DE} = \frac{P(\triangle ABC)}{P(\triangle DEF)}$$

$$\Rightarrow \frac{9.1}{6.5} = \frac{P(\Delta ABC)}{25}$$

$$\therefore P(\Delta ABC) = 35 \text{ cm}$$

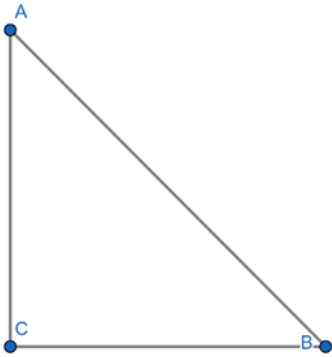
42. Question

In an isosceles triangle ABC if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C =$

- A. 30°
- B. 45°
- C. 90°
- D. 60°

Answer

Given in isosceles ΔABC , $AC = BC$ and $AB^2 = 2AC^2$



In isosceles ΔABC ,

$AC = BC$, so $\angle B = \angle A$ [Equal sides have equal angles opposite to them]

$$\Rightarrow AB^2 = 2AC^2$$

$$\Rightarrow AB^2 = AC^2 + AC^2$$

$$\Rightarrow AB^2 = AC^2 + BC^2$$

$\therefore \Delta ABC$ is right angle triangle with $\angle C = 90^\circ$

43. Question

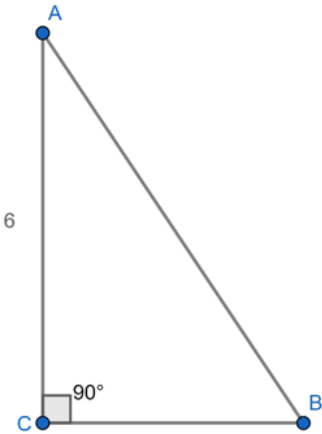
ΔABC is an isosceles triangle in which $\angle C = 90^\circ$. If $AC = 6 \text{ cm}$, then $AB =$

- A. $6\sqrt{2} \text{ cm}$
- B. 6 cm
- C. $2\sqrt{6} \text{ cm}$

D. $4\sqrt{2}$ cm

Answer

Given in an isosceles triangle ABC, $\angle C = 90^\circ$ and $AC = 6$ cm.



$$\Rightarrow BC = AC = 6 \text{ cm}$$

We know that the Pythagoras theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ABC,

$$\Rightarrow AB^2 = AC^2 + BC^2$$

$$= 6^2 + 6^2$$

$$= 36 + 36$$

$$= 72$$

$$\therefore AB = 6\sqrt{2} \text{ cm}$$

44. Question

If in two triangles ABC and DEF, $\angle A = \angle E$, $\angle B = \angle F$, then which of the following is not true?

A. $\frac{BC}{DF} = \frac{AC}{DE}$

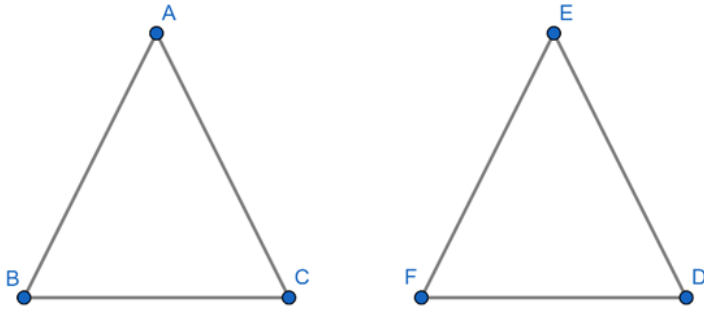
B. $\frac{AB}{DE} = \frac{BC}{DF}$

C. $\frac{AB}{EF} = \frac{AC}{DE}$

D. $\frac{BC}{DF} = \frac{AB}{EF}$

Answer

Given that $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\angle A = \angle E$ and $\angle B = \angle F$.



We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$$\Rightarrow \frac{AB}{EF} = \frac{BC}{FD} = \frac{CA}{DE}$$

$$\therefore \triangle ABC \sim \triangle DEF$$

Hence proved.

45. Question

In an isosceles triangle ABC, if $AB = AC = 25$ cm and $BC = 14$ cm, then the measure of altitude from A on BC is

- A. 20 cm
- B. 22 cm
- C. 18 cm
- D. 24 cm

Answer

Given in an isosceles $\triangle ABC$, $AB = AC = 25$ cm and $BC = 14$ cm

Here altitude from A to BC is AD.

We know in isosceles triangle altitude on non-equal sides is also median.

$$\Rightarrow BD = CD = BC/2 = 7 \text{ cm}$$

Applying Pythagoras Theorem,

$$\Rightarrow AB^2 = BD^2 + AD^2$$

$$\Rightarrow 25^2 = 7^2 + AD^2$$

$$\Rightarrow AD^2 = 625 - 49 = 576$$

$$\Rightarrow AD = 24$$

\therefore Measure of altitude from A to BC is 24 cm

46. Question

In Fig. 4.242 the measures of $\angle D$ and $\angle F$ are respectively

- A. $50^\circ, 40^\circ$
- B. $20^\circ, 30^\circ$
- C. $40^\circ, 50^\circ$
- D. $30^\circ, 20^\circ$

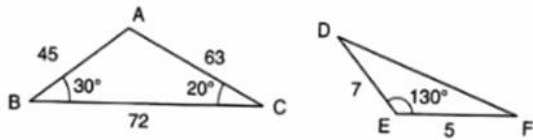


Fig. 4.242

Answer

In $\triangle ABC$ and $\triangle DEF$,

$$\Rightarrow \frac{AB}{AC} = \frac{EF}{ED}$$

$$\Rightarrow \angle A = \angle E = 130^\circ$$

We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

$$\therefore \triangle ABC \sim \triangle DEF$$

$$\text{Hence, } \angle F = \angle B = 30^\circ$$

$$\text{And } \angle D = \angle C = 20^\circ$$

47. Question

In Fig. 4.243, the value of x for which $DE \parallel AB$ is

- A. 4
- B. 1
- C. 3
- D. 2

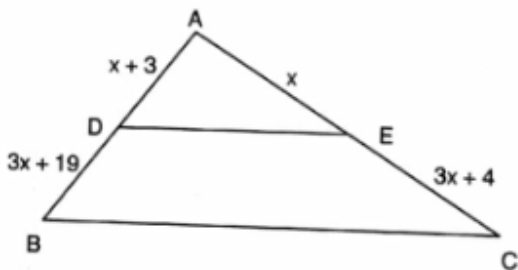


Fig. 4.243

Answer

Given in $\triangle ABC$, $DE \parallel AB$.

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$\text{Then } \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$$\Rightarrow (x+3)(3x+4) = x(3x+19)$$

$$\Rightarrow 3x^2 + 4x + 9x + 12 = 3x^2 + 19x$$

$$\Rightarrow 19x - 13x = 12$$

$$\Rightarrow 6x = 12$$

$$\therefore x = 2 \text{ cm}$$

48. Question

In Fig. 4.244, if $\angle ADE = \angle ABC$, then $CE =$

- A. 2
- B. 5
- C. $9/2$
- D. 3

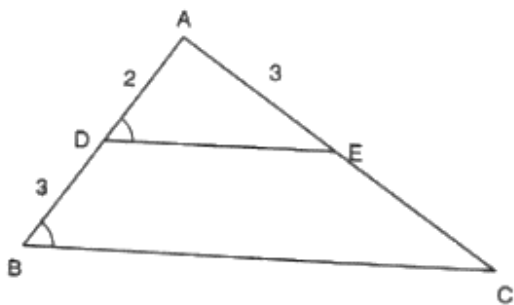


Fig. 4.244

Answer

Given $\angle ADE = \angle ABC$

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$\text{Then } \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{2}{3} = \frac{3}{EC}$$

$$\Rightarrow EC = \frac{3(3)}{2}$$

$$\therefore EC = 9/2 \text{ cm}$$

49. Question

In Fig. 4.245, $RS \parallel DB \parallel PQ$. If $CP = PD = 11 \text{ cm}$ and $DR = RA = 3 \text{ cm}$. Then the values of x and y are respectively

- A. 12, 10
- B. 14, 6
- C. 10, 7
- D. 16, 8

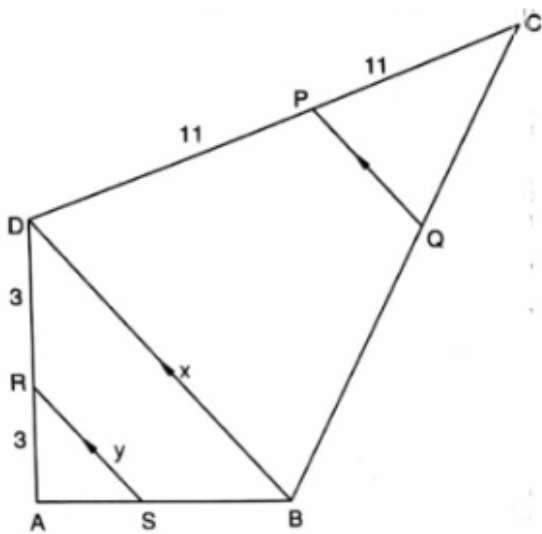


Fig. 4.245

Answer

Given in figure $RS \parallel DB \parallel PQ$, $CP = PD = 11 \text{ cm}$ and $DR = RA = 3 \text{ cm}$.

In $\triangle ASR$ and $\triangle ABD$,

$$\angle ASR = \angle ABD \text{ [corresponding angles]}$$

$$\angle ARS = \angle ADB \text{ [corresponding angles]}$$

$$\angle A = \angle A \text{ [common]}$$

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$$\therefore \triangle ASR \sim \triangle ABD$$

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AR}{AD} = \frac{AS}{AB} = \frac{RS}{DB}$$

$$\Rightarrow \frac{3}{6} = \frac{RS}{DB}$$

$$\Rightarrow \frac{1}{2} = \frac{x}{y}$$

$$\therefore x = 2y$$

$$\therefore x = 16 \text{ cm and } y = 8 \text{ cm}$$

50. Question

In Fig. 4.246, if $PB \parallel CF$ and $DP \parallel EF$, then $\frac{AD}{DE} =$

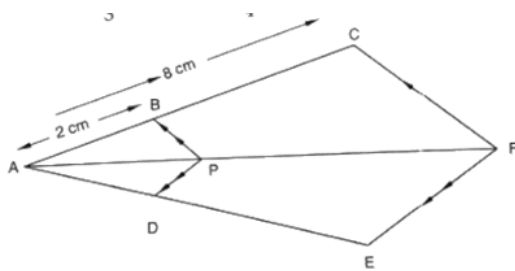


Fig. 4.246

Answer

Given $PB \parallel CF$, $DP \parallel EF$, $AB = 2 \text{ cm}$ and $AC = 8 \text{ cm}$

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

In $\triangle ACF$, $PB \parallel CF$,

$$\text{Then } \frac{AB}{BC} = \frac{AP}{PF}$$

$$\Rightarrow \frac{AP}{PF} = \frac{2}{8-2} = \frac{2}{6} = \frac{1}{3}$$

And $DP \parallel EF$

$$\Rightarrow \frac{AD}{DE} = \frac{AP}{PF}$$

$$\therefore \frac{AD}{DE} = \frac{1}{3}$$

51. Question

A chord of a circle of radius 10 cm subtends a right angle at the centre. The length of the chord (in cm) is

A. $5\sqrt{2}$

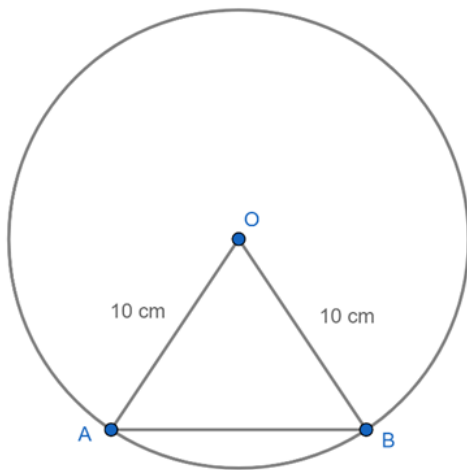
B. $10\sqrt{2}$

C. $\frac{5}{\sqrt{2}}$

D. $10\sqrt{3}$ [CBSE 2014]

Answer

Given A chord of a circle of radius 10 cm subtends a right angle at the centre.



We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle OAB,

$$\Rightarrow AB^2 = OA^2 + OB^2$$

$$= 10^2 + 10^2$$

$$= 100 + 100$$

$$= 200$$

$$\therefore AB = 10\sqrt{2} \text{ cm}$$