CBSE SAMPLE PAPER - 01

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\int \sqrt{1-9x^2}\,dx = ?$ **[1]**

   a) $\frac{3x}{2}\sqrt{1-9x^2} + \frac{1}{6}\sin^{-1} 3x + C$
   b) $\frac{x}{2}\sqrt{1-9x^2} + \frac{1}{6}\sin^{-1} 3x + C$
   c) $\frac{x}{2}\sqrt{1-9x^2} + \frac{1}{18}\sin^{-1} 3x + C$
   d) None of these

2. The direction ratios of the line perpendicular to the lines $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}$ are proportional to **[1]**

   a) 4, 5, 7
   b) -4, 5, 7
   c) 4, -5, -7
   d) 4, -5, 7

3. The values of x for which the angle between $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}, \vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse and the angle between $\vec{b}$ and the z-axis is a cute and less than $\frac{\pi}{6}$ are **[1]**

   a) $\frac{1}{2} < x < 15$
   b) $x > \frac{1}{2}$ or $x < 0$
   c) ϕ
   d) $0 < x < \frac{1}{2}$

4. If A and B are two events such that A ⊂ B and P(B) ≠ 0, then which of the following is correct? **[1]**

   a) None of these
   b) P(A|B) ≥ P(A)
   c) $P(A|B) = \frac{P(B)}{P(A)}$
   d) P(A|B) < P(A)

5. $\int \frac{dx}{(4+16x^2)} = ?$ **[1]**

   a) $\frac{1}{32}\tan^{-1} 4x + C$
   b) $\frac{1}{4}\tan^{-1} \frac{x}{2} + C$
   c) $\frac{1}{16}\tan^{-1} \frac{x}{2} + C$
   d) $\frac{1}{8}\tan^{-1} 2x + C$

6. If A and B are two independent events such that P(A) = 0.3, P(A ∪ B) = 0.5, then P(A / B) - P(B / A) = **[1]**

a) $\frac{2}{7}$

b) $\frac{1}{7}$

c) $\frac{1}{70}$

d) $\frac{3}{35}$

7. The area of the region bounded by the curve $x^2 = 4y$ and the straight line x = 4y - 2 is **[1]**

a) $\frac{5}{8}$ sq.units

b) $\frac{9}{8}$ sq.units

c) $\frac{3}{8}$sq.units

d) $\frac{7}{8}$ sq.units

8. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$. **[1]**

a) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$, $\lambda \in R$

b) $\vec{r} = \widehat{2i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$ $\lambda \in R$

c) $\vec{r} = 4\hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$ $\lambda \in R$

d) $\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$ $\lambda \in R$

9. If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ, then the value of $|\vec{a} - \vec{b}|$ is **[1]**

a) $2\cos\frac{\theta}{2}$

b) $2\sin\frac{\theta}{2}$

c) $2\cos$

d) $2\sin\theta$

10. What is the equation of a curve passing through (0, 1) and whose differential equation is given by dy = y tan x dx? **[1]**

a) y = sec x

b) y = sin x

c) y = cosec x

d) y = cos x

11. The area bounded by the curves y = sin x between the ordinates x = 0, x = π and the x-axis is **[1]**

a) 2 sq. units

b) 4 sq. units

c) 3 sq. units

d) 1 sq. units

12. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals **[1]**

a) $\frac{\pi}{4}$

b) $\frac{\pi}{12}$

c) $\frac{\pi}{6}$

d) $\frac{\pi}{24}$

13. The least value of k for which $f(x) = x^2 + kx + 1$ is increasing on (1, 2), is **[1]**

a) -2

b) 2

c) 1

d) -1

14. If A is any square matrix then which of the following is not symmetric? **[1]**

a) $A+A^t$

b) $A - A^t$

c) A^tA

d) AA^t

15. If A is an invertible matrix, then det (A^{-1}) is equal to **[1]**

a) $\frac{1}{\det(A)}$

b) 1

c) det(A)

d) none of these

16. The existence of the unique solution of the system of equations: **[1]**

x + y + z= λ

$5x - y + \mu z = 10$

$2x + 3y - z = 6$ depends on

a) λ and μ both

b) λ only

c) neither λ nor μ

d) μ only

17. Range of $\sec^{-1}x$ is **[1]**

a) $[0, \pi]$

b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

c) None of these

d) $\left[0, \frac{\pi}{2}\right]$

18. The equation of the curve satisfying the differential equation $y(x + y^3)\,dx = x(y^3 - x)\,dy$ and passing through the point (1, 1) is , **[1]**

a) None of these

b) $y^3 + 2x + 3x^2y = 0$

c) $y^3 + 2x - 3x^2y = 0$

d) $y^3 - 2x + 3x^2y = 0$

19. **Assertion (A):** The function f(x) = sin x decreases on the interval $(0, \frac{\pi}{2})$. **[1]**

Reason (R): The function f(x) = cos x decreases on the interval $(0, \frac{\pi}{2})$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $\triangle = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ where, A_{ij} is cofactor of a_{ij}. **[1]**

Reason (R): $\triangle$ = Sum of the products of elements of any row (or column) with their corresponding cofactors.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the domain of $f(x) = \sin^{-1}(-x^2)$. **[2]**

22. Find the general solution of the differential equation $(x+2)\frac{dy}{dx} = x^2 + 5x - 3 (x \neq -2)$ **[2]**

23. If $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, then show that $A - 3I = 2(I + 3A^{-1})$ **[2]**

OR

Determine the values of λ for which the following system of equations fail to have a unique solution:

$\lambda x + 3y - z = 1$

$x + 2y + z = 2$

$-\lambda x + y + 2z = -1$

Does it have any solution for this value of λ?

24. If D, E, F are the mid-points of the sides BC, CA and AB respectively of a triangle ABC, write the value of $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}$. **[2]**

25. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved. **[2]**

Section C

26. Evaluate $\int_{-2}^{2} xe^{|x|}dx$ **[3]**

27. Solve : $\frac{dy}{dx}$ = cos(x + y) + sin (x + y). [Hint: Substitute x + y = z] **[3]**

OR

Solve $\left[x\sin^2\left(\frac{y}{x}\right) - y\right] dx + xdy = 0;$

$y = \pi/4$, when x = 1

28. Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = 3\hat{i} + \hat{k}$ and other is perpendicular to $\vec{b}$. **[3]**

OR

Find λ when the projection of $\overrightarrow{\mathrm{a}} = \lambda\hat{\mathrm{i}} + \hat{\mathrm{j}} + 4\hat{\mathrm{k}}$ on $\overrightarrow{\mathrm{b}} = 2\hat{\mathrm{i}} + 6\hat{\mathrm{j}} + 3\hat{\mathrm{k}}$ is 4 units.

29. Evaluate the integral: $\int \frac{x^5}{\sqrt{1+x^3}} dx$ **[3]**

OR

Evaluate: $\int \frac{x}{(1+\sin x)} dx$

30. Discuss the continuity of the f(x) at the indicated point: f(x) = |x| + |x - 1| at x = 0, 1 **[3]**

31. Find the area bounded by the curves $y = \sqrt{x},\ 2y - x + 3 = 0,\ X - axis$ and lying in the first quadrant **[3]**

Section D

32. Solve the following linear programming problem graphically: **[5]**

Maximize Z = 50x + 15y

Subject to

5x + y $\leq$ 100

x + y $\leq$ 60

x , y $\geq$ 0

33. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R. **[5]**

OR

Let n be a positive integer. Prove that the relation R on the set Z of all integers numbers defined by (x, y) $\in$ R $\Leftrightarrow$ x - y is divisible by n, is an equivalence relation on Z.

34. Show that the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$ intersect. Also, find their point intersection. **[5]**

OR

Find the vector equation of the line passing through (1,2,3) and || to the plane $\vec{r}.\left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5$ and $\vec{r}.\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6$

35. Find all the points of discontinuity of f defined by f(x) = |x| – |x + 1|. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A tin can manufacturer designs a cylindrical tin can for a company making sanitizer and disinfectors. The tin can is made to hold 3 litres of sanitizer or disinfector. The cost of material used to manufacture the tin can is

₹100/m^2.

(i) If r cm be the radius and h cm be the height of the cylindrical tin can, then express the surface area as a function of radius (r)

(ii) Find the radius of the can that will minimize the cost of tin used for making can?

(iii) Find the height that will minimize the cost of tin used for making can ?

OR

Find the minimum cost of material used to manufacture the tin can.

37. **Read the text carefully and answer the questions:** **[4]**

To promote the making of toilets for women, an organization tried to generate awareness through

i. house calls
ii. emails and
iii. announcements.

The cost for each mode per attempt is given below:

1. ₹ 50
2. ₹ 20
3. ₹ 40

The number of attempts made in the villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also, the chance of making of toilets corresponding to one attempt of given modes is

1. 2%
2. 4%
3. 20%

(i) Find total number of toilets that can be expected after the promotion in village X.

(ii) Find the percentage of toilets that can be expected after the promotion in all the three-villages?

(iii) Find the cost incurred by the organization on village X.

OR

Find the total cost incurred by the organization on for all the three villages?

38. **Read the text carefully and answer the questions:** **[4]**

In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.

(i) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is NOT processed by Govind.

(ii) Find the probability that Priyanka processed the form and committed an error.

Solution

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Class 12 - Mathematics

Section A

1. **(b)** $\frac{x}{2}\sqrt{1-9x^2}+\frac{1}{6}\sin^{-1}3x+C$

Explanation: $\int\sqrt{1-9x^2}dx=3\int\sqrt{\frac{1}{9}-x^2}dx$

$=3\left[\frac{x}{2}\sqrt{\frac{1}{9}-x^2}+\frac{1}{18}\sin^{-1}\frac{x}{(1/3)}\right]+C$

$=\frac{3x}{2}\sqrt{\frac{1}{9}-x^2}+\frac{1}{6}\sin^{-1}3x+C$

$=\frac{x}{2}\sqrt{1-9x^2}+\frac{1}{6}\sin^{-1}3x+C$

2. **(a)** 4, 5, 7

Explanation: We have,

$\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1}$

$\frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$

The direction ratios of the given lines are proportional to 2, -3, 1 and 1, 2, -2.

The vectors parallel to the given lines are $\overrightarrow{b_1}=2\hat{i}-3\hat{j}+\hat{k}$ and $\overrightarrow{b_2}=\hat{i}+2\hat{j}-2\hat{k}$

Vector perpendicular to the vectors b_1 & b_2 is ,

$\vec{b}=\overrightarrow{b_1}\times\overrightarrow{b_2}$

= $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 1\\ 1 & 2 & -2\end{vmatrix}$

= $4\hat{i}+5\hat{j}+7\hat{k}$

Hence, the direction ratios of the line perpendicular to the given two lines are proportional to 4, 5, 7.

3. **(d)** $0<x<\frac{1}{2}$

Explanation: $0<x<\frac{1}{2}$

4. **(a)** None of these

Explanation: Since, $A\subset B,\quad A\cap B=A$

$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)}{P(B)}$

5. **(d)** $\frac{1}{8}\tan^{-1}2x+C$

Explanation: $Let\ I=\int\frac{dx}{(4x)^2+2^2}$

4x = t

4dx = dt

$dx=\frac{dt}{4}$

$=\frac{1}{4}\int\frac{dt}{t^2+2^2}$

We know,

$=\frac{1}{8}\tan^{-1}\frac{t}{2}+c$

put t = 4x

$=\frac{1}{8}\tan^{-1}\frac{4x}{2}+c$

$=\frac{1}{8}\tan^{-1}2x+c$

6. **(c)** $\frac{1}{70}$

Explanation: P(A) = 0.3 , P(A∪B) = 0.5 (Given)

Since, A and B are two independent events,

P(A∩B) = P(A) P(B)

P(A∩B) = 0.3 × P(B) ...(i)

Also, according to the addition theorem of probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.5 = 0.3 + P(B) - 0.3\, P(B)$ From (Given) & (i)

$0.7\, P(B) = 0.2$

$P(B) = \frac{0.2}{0.7} = \frac{2}{7}$ ...(ii)

Putting value of P(B) in equation (i) we get,

$P(A \cap B) = 0.3 \times \frac{2}{7} = \frac{3}{10} \times \frac{2}{7}$

$P(A \cap B) = \frac{6}{70}$ ...(iii)

Now,

$P\left(\frac{A}{B}\right) - P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(B)} - \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{6}{70}}{\frac{2}{7}} - \frac{\frac{6}{70}}{\frac{3}{10}}$ From (iii) & (ii) and (Given)

$= \frac{6}{70} \times \frac{7}{2} - \frac{6}{70} \times \frac{10}{3}$

$= \frac{3}{10} - \frac{2}{7}$

$= \frac{1}{70}$

7. **(b)** $\frac{9}{8}$ sq.units

Explanation: Eliminating y , we get :

$x^2 - x - 2 = 0 \Rightarrow x = -1, 2$

Required area :

$= \int_{-1}^{2} \left(\frac{x}{4} + \frac{1}{2} - \frac{x^2}{4}\right) dx = \frac{1}{8}(4-1) + \frac{3}{2} - \frac{1}{12}(8+1) = \frac{3}{8} + \frac{3}{2} - \frac{3}{4} = \frac{9}{8}$ sq.units

8. **(a)** $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right), \lambda \in R$

Explanation: The equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$, let vector $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$ and vector $\overrightarrow{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$,

the equation of line is :

$\overrightarrow{a} + \lambda \overrightarrow{b} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$

9. **(b)** $2 \sin \frac{\theta}{2}$

Explanation: Given $\vec{a}$ and $\vec{b}$are unit vectors with inclination is θ

now, $|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}).(\vec{a} - \vec{b}) = |\vec{a}|^2 - \vec{a}.\vec{b} - \vec{b}.\vec{a} + |\vec{b}|^2$

$=1-2(\vec{a}.\vec{b}) + 1$

$=2-2|\vec{a}||\vec{b}|cos\theta$

$=2-2cos\theta$ (where vectors are unit vectors)

$=2(1-\cos\theta)$

$=4sin^2\frac{\theta}{2}$

thus $|\vec{a} - \vec{b}|^2 = 4sin^2\frac{\theta}{2}$

$\therefore |\vec{a} - \vec{b}| = 2\sin\frac{\theta}{2}$

10. **(a)** y = sec x

Explanation: The given differential equation of the curve is,

dy = y tan x dx $\Rightarrow \int \frac{dy}{y} = \int \tan x \cdot dx$ [on integrating]

$\Rightarrow$ log y = log sec x + log C $\Rightarrow$ log y = log C sec x

$\Rightarrow$ y = C sec x ...(i)

Since, the curve passes through the origin (0, 1), then

1 = C sec 0 $\Rightarrow$ C = 1

$\therefore$ Required equation of curve is, y = sec x

11. **(a)** 2 sq. units

Explanation: $\int_0^{\pi} y dx = \int_0^{\pi} \sin x dx$

$\int_0^{\pi} y dx = -[\cos x]_0^{\pi}$

$\int_0^{\pi} y dx = -[-1-1]$

$\int_0^{\pi} y dx = 2$

12. **(d)** $\frac{\pi}{24}$

Explanation: Let $I=\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$

Taking 9 common from Denominator in I

$\Rightarrow I=\frac{1}{9}\int_0^{\frac{2}{3}} \frac{dx}{\frac{4}{9}+x^2}=\frac{1}{9}\int_0^{\frac{2}{3}} \frac{dx}{\left(\frac{2}{3}\right)^2+x^2}$ $\left[\int \frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c\right]$

$\Rightarrow \mathrm{I}=\frac{1}{9}\times\frac{3}{2}\left[\tan^{-1}\frac{x}{\frac{2}{3}}\right]_0^{\frac{2}{3}}=\frac{1}{9}\times\frac{3}{2}\left[\tan^{-1}\frac{3x}{2}\right]_0^{\frac{2}{3}}$

$\Rightarrow I=\frac{1}{6}\left[\tan^{-1}\frac{3}{2}\times\frac{2}{3}-\tan^{-1}0\right]=\frac{1}{6}\left[\tan^{-1}1-\tan^{-1}0\right]$

$\Rightarrow I=\frac{1}{6}\times\left(\frac{\pi}{4}-0\right)=\frac{\pi}{24}$

$\therefore \int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}=\frac{\pi}{24}$

13. **(a)** -2

Explanation: Given, $f(x) = x^2 + kx + 1$

For increasing

$f'(x) = 2x + k$

$k \geq -2x$

thus,

$k \geq -2x$

Least value of -2

14. **(b)** $A - A^t$

Explanation: For every square matrix (A – A') is always skew – symmetric.

15. **(a)** $\frac{1}{\det(A)}$

Explanation: Solution.

Since we know that $\left|A^{-1}\right|=\frac{1}{|A|}$

16. **(d)** μ only

Explanation: The given system of linear equation :-

$x + y + z = \lambda$

$5x - y + \mu z = 10$

$2x + 3y - z = 6$

The matrix equation corresponding to the above system is :

$$\begin{bmatrix}1 & 1 & 1\\ 5 & -1 & \mu\\ 2 & 3 & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}\lambda\\ 10\\ 6\end{bmatrix}$$

Suppose A = $\begin{bmatrix}1 & 1 & 1\\ 5 & -1 & \mu\\ 2 & 3 & -1\end{bmatrix}$

$\therefore |A| = \begin{vmatrix}1 & 1 & 1\\ 5 & -1 & \mu\\ 2 & 3 & -1\end{vmatrix} = 1(1-3\mu) -1(-5-2\mu) +1(15 + 2)$

$= 1 - 3\mu +5 +2\mu +17 = 23-\mu$

For the existence of the unique solution, the value of |A| must not be equal to 0.

Therefore, the existence of the unique solution merely depends on the value of μ.Which is the required solution.

17. **(b)** $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

Explanation:

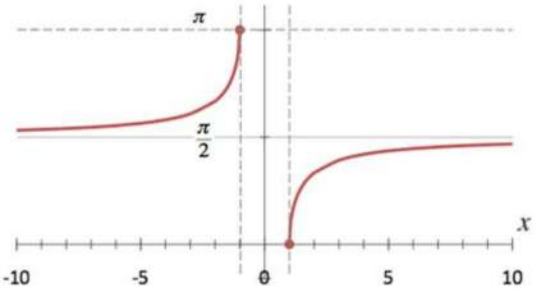
To Find: The range of $\sec^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sec^{-1}(x)$ can be obtained from the graph of

Y = sec x by interchanging x and y axes.i.e, if (a, b) is a point on Y = sec x then (b, a) is the point on the function $y = \sec^{-1}(x)$

Below is the Graph of the range of $\sec^{-1}(x)$



From the graph, it is clear that the range of $\sec^{-1}(x)$ is restricted to interval
$[0, \pi] - \left\{\frac{\pi}{2}\right\}$

18. **(c)** $y^3 + 2x - 3x^2 y = 0$

Explanation: We have,

$y\left(x + y^3\right) dx = x\left(y^3 - x\right) dy$

$xydx + y^4 dx = xy^3 dy - x^2 dy$

$xydx - xy^3 dy + y^4 dx + x^2 dy = 0$

$xydx + x^2 dy + y^4 dx - xy^3 dy = 0$

$x\left(ydx + x^2 dy\right) + y^3(ydx - xdy) = 0$

$x\left(ydx + x^2 dy\right) + x^2 y^3 \frac{(ydx - xdy)}{x^2} = 0$

$x\left(ydx + x^2 dy\right) - x^2 y^3 \frac{(xdy - ydx)}{x^2} = 0$

$x\left(ydx + x^2 dy\right) - x^2 y^3 d\left(\frac{y}{x}\right) = 0$

$x\left(ydx + x^2 dy\right) = x^2 y^3 d\left(\frac{y}{x}\right)$

$\frac{x\left(ydx + x^2 dy\right)}{x^3 y^2} = \frac{y}{x} d\left(\frac{y}{x}\right)$

$\int \frac{x\left(ydx + x^2 dy\right)}{x^3 y^2} = \int \frac{y}{x} d\left(\frac{y}{x}\right)$

$\int \frac{d(xy)}{x^2 y^2} = \int \frac{y}{x} d\left(\frac{y}{x}\right)$

$\frac{-1}{xy} = \frac{\left(\frac{y}{x}\right)^2}{2} + c$

$\frac{1}{xy} + \frac{\left(\frac{y}{x}\right)^2}{2} + c = 0$

$\Rightarrow y^3 + 2x + 2cx^2 y = 0$

Curve passes through (1, 1)

1 + 2 +2 c =0

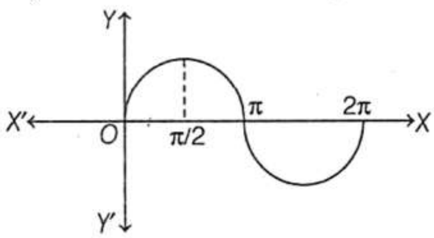
1 + 2 + 2 c = 0

c = $\frac{3}{-2}$

$\Rightarrow y^3 + 2x - 3x^2 y = 0$

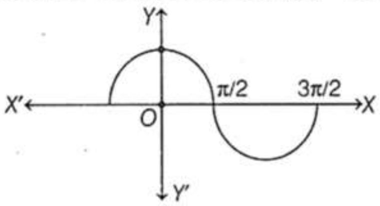
19. **(d)** A is false but R is true.

Explanation: Assertion: Given, function f(x) = sin x



From the graph of sin x, we observe that f(x) increases on the interval (0, $\frac{\pi}{2}$).

Reason: Given function is f(x) = cos x.



From the graph of cos x, we observe that, f(x) decreases on the interval $(0, \frac{\pi}{2})$.

Hence, Assertion is false and Reason is true.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: By expanding the determinant

$\triangle = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and R_1, we have

$\triangle = (-1)^{1+1}a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2}a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3}a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$= a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$, where A_{ij} is cofactor of a_{ij}

= Sum of products of elements of R_1 with their corresponding cofactors.

Section B

21. The domain of $\sin^{-1} x$ is [-1,1]. Therefore, $f(x) = \sin^{-1}(-x^2)$ is defined for all x satisfying $-1 \leq -x^2 \leq 1$

$\Rightarrow \quad 1 \geq x^2 \geq -1$

$\Rightarrow 0 \leq x^2 \leq 1$

$\Rightarrow x^2 \leq 1$

$\Rightarrow x^2 - 1 \leq 0$

$\Rightarrow (x - 1)(x + 1) \leq 0$

$\Rightarrow -1 \leq x \leq 1$

Hence, the domain of $f(x) = \sin^{-1}(-x^2)$ is [-1, 1].

22. The given differential equation may be written as

$\frac{dy}{dx} = \frac{x^2+5x-3}{x+2}$

$\Rightarrow \quad dy = \left(\frac{x^2+5x-3}{x+2}\right) dx$ [separating the variables]

$\Rightarrow \int dy = \int \left(\frac{x^2+5x-3}{x+2}\right) dx$

$\Rightarrow y = \int \left\{x + 3 - \frac{9}{(x+2)}\right\} dx$ +C, where C is an arbitrary constant

$\left[\text{on dividing } \left(x^2 + 5x - 3\right) \text{ by } (x + 2)\right]$

$\Rightarrow y = \frac{x^2}{2} + 3x - 9\log|x + 2| + C$

Therefore, $y = \frac{x^2}{2} + 3x - 9\log|x + 2| + C$ is the required general solution of the given differential equation.

23. We have

$A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$

$|A| = 4 - 10 = -6$ adj A = $\begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{-6}\begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$

To Show: $A - 3I = 2(I + 3A^{-1})$

LHS $A - 3I = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$

R.H.S $2(I + 3A^{-1}) = 2I + 6A^{-1} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 6\frac{1}{6}\begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$

Hence, $A - 3I = 2(I + 3A^{-1})$

OR

Given system of equations is

$\lambda x + 3y - z = 1$

$x + 2y + z = 2$

$-\lambda x + y + 2z = -1$

The given system of equations will fail to have unique solution, if D = 0 i.e. $\begin{vmatrix} \lambda & 3 & -1 \\ 1 & 2 & 1 \\ -\lambda & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow \lambda(4 - 1) - 3(2 + \lambda) - (1 + 2\lambda) = 0$

$\Rightarrow 3\lambda - 6 - 3\lambda - 1 - 2\lambda = 0$

$\Rightarrow -2\lambda - 7 = 0$

$\Rightarrow \lambda = -\frac{7}{2}$

For $\lambda = -\frac{7}{2}$, we obtain

$D_1 = \begin{vmatrix} 1 & 3 & -1 \\ 2 & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix} = -16 \neq 0$

Thus, for $\lambda = -\frac{7}{2}$, we have D = 0 and $D_1 \neq 0$

Hence, the given system of equations has no solution for $\lambda = -\frac{7}{2}$

24. Here,it is given that DD, E, F are the midpoints of the sides BC, CA, AB respectively.

Then, the position vectors of the midpoints D, E, F are given by $\frac{\vec{b}+\vec{c}}{2}, \frac{\vec{c}+\vec{a}}{2}, \frac{\vec{a}+\vec{b}}{2}$

Now we have, $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = = \left(\frac{\vec{b}+\vec{c}}{2}\right) - \vec{a} + \left(\frac{\vec{c}+\vec{a}}{2}\right) - \vec{b} + \left(\frac{\vec{a}+\vec{b}}{2}\right) - \vec{c}$

$= 2\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right) - (\vec{a} + \vec{b} + \vec{c})$

$= (\vec{a} + \vec{b} + \vec{c}) - (\vec{a} + \vec{b} + \vec{c})$

$= \overrightarrow{0}$

25. Given:

P(A) = Probability of solving the problem by A = $\frac{1}{2}$

P(B) = Probability of solving the problem by B = $\frac{1}{3}$

Since, A and B both are independent.

$\Rightarrow P(A \cap B) = P(A).P(B)$

$\Rightarrow P(A \cap B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

The problem is solved, i.e. it is either solved by A or it is solved by B.

$= P(A \cup B)$

As we know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6}$

$\Rightarrow P(A \cup B) = \frac{2}{3}$

Section C

26. The given integral can be written as

$I = \int_{-2}^{0} xe^{-x}dx + \int_{0}^{2} xe^{x}dx$

For

$\int_{-2}^{0} xe^{-x}dx$

Using Integration By parts

$\int fg = fg - \int fg'$

$f' = e^{-x}, g = x$

$f = -e^{-x}, g' = 1$

$\int_{-2}^{0} xe^{-x}dx = \{-xe^{-x}\}_{-2}^{0} + \int_{-2}^{0} e^{-x}dx$

$\int_{-2}^{0} xe^{-x}dx = \{-xe^{-x} - e^{-x}\}_{-2}^{0}$

$\int_{-2}^{0} xe^{-x}dx = \{(-1) - (2e^2 - e^2)\}$

$\int_{-2}^{0} xe^{-x}dx = \{-1 - e^2\}$

For

$\int_{0}^{2} xe^{x}dx$

Using Integration By parts

$\int fg = fg - \int fg'$

$f' = e^{x}, g = x$

$f = e^{x}, g' = 1$

$\int_{0}^{2} xe^{x}dx = \{xe^{x}\}_{0}^{2} - \int_{0}^{2} e^{x}dx$

$\int_0^2 xe^x dx = \{xe^x - e^x)_0^2$

$\int_0^2 xe^x dx = 2e^2 - e^2 + 1$

$\int_0^2 xe^x dx = e^2 + 1$

Hence answer is,

$\int_{-2}^2 xe^H dx = -1 - e^2 + e^2 + 1 = 0$

27. We have, $\frac{dy}{dx}$ = cos (x + y) + sin (x + y)

Using the hint given and substituting x + y = z

$\Rightarrow \frac{d(z-x)}{dx}$ = cos z + sin z

Differentiating z – x with respect to x

$\Rightarrow \frac{dz}{dx}$ - 1 = cos z + sin z

$\Rightarrow \frac{dz}{dx}$ = 1 + cos z + sin z

$\Rightarrow \frac{dz}{1+\cos z+\sin z}$ = dx

Integrating both sides, we have

$\Rightarrow \int \frac{dz}{1+\cos z+\sin z} = \int dx$

We know that cos 2z = 2 $\cos^2$z – 1 and sin 2z = 2 sin z cos z

$\Rightarrow \int \frac{dz}{1+2\cos^2\frac{z}{2}-1+2\sin\frac{z}{2}\cos\frac{z}{2}} = x$

$\Rightarrow \int \frac{dz}{2\cos^2\frac{z}{2}+2\sin\frac{z}{2}\cos\frac{z}{2}} = x$

$\Rightarrow \int \frac{dz}{2\cos\frac{z}{2}\left(\cos\frac{z}{2}+\sin\frac{z}{2}\right)} = x$

$\Rightarrow \int \frac{dz}{2\cos^2\frac{z}{2}\left(1+\frac{\sin\frac{z}{2}}{\cos\frac{z}{2}}\right)} = x$

$\Rightarrow \int \frac{\sec^2\frac{2}{2}dz}{2\left(1+\tan\frac{z}{2}\right)} = x$

Let 1 + tan $\frac{z}{2}$ = t

Differentiating with respect to z, we get

$\frac{dt}{dz} = \frac{\sec^2\frac{z}{2}}{2}$

Hence $\frac{\sec^2\frac{z}{2}dz}{2} = dt$

$\Rightarrow \int \frac{\mathrm{dt}}{\mathrm{t}} = \mathrm{x}$

⇒ log t + c = x

Resubstituting t

$\Rightarrow \log\left(1+\tan\frac{z}{2}\right) + c = x$

Resubstitute z

$\Rightarrow \log\left(1+\tan\frac{\mathrm{x+y}}{2}\right) + \mathrm{c} = \mathrm{x}$

OR

$\left\{x\sin^2\left(\frac{y}{x}\right) - y\right\}dx + xdy = 0$

$\sin^2\left(\frac{y}{x}\right) - \frac{y}{x} + \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right)$ .....(i)

let y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

put $\frac{dy}{dx}$ in eq (i)

$v + x\frac{dv}{dx} = v - \sin^2 v$

$\int \cos ec^2 v dv = \int -\frac{dx}{x}$

-cot v = -log x + c

log x - cot v = c

$\log x - \cot\left(\frac{y}{x}\right) = c$

When x = 1, $y = \frac{\pi}{4}$

c = -1

$\log x - \cot\left(\frac{y}{x}\right) = -1$

$\log x - \cot\left(\frac{y}{x}\right) = -log\, e$

$\log ex = \cot\left(\frac{y}{x}\right)$

28. Let, $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$

Also, given $\vec{b} = 3\hat{i} + \hat{k}$

Also, let

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$ ... (i)

Where **u** is parallel to **b** and **v** is perpendicular to **b.**

now **u** is parallel to **b.**

$\vec{u} = \lambda\vec{b}$

$= \lambda(3\hat{i} + \hat{k})$

$\vec{u} = 3\lambda\hat{i} + \lambda\vec{k}$.... (ii)

put value of u in equation (i)

$5\hat{i} - 2\hat{j} + 5\hat{k} = (3\lambda\hat{i} + \lambda\hat{k}) + \vec{v}$

$\vec{v} = 5\hat{j} - 2\hat{j} + 5\hat{k} - 3\lambda\hat{i} - \lambda\hat{k}$

$\vec{v} = (5 - 3\lambda)\hat{i} - 2\hat{j} + (5 - \lambda)\hat{k}$

Since, **v** is perpendicular to **b**

Then v.b =0

$[(5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}] \cdot (3\hat{i} + 0 \times \hat{j} + \tilde{k}) = 0$

(5 - 3λ)(3) + (-2)(0) + (5 - λ)(1) = 0

$15 - 9\lambda + 0 + 5 - \lambda = 0$

$20 - 10\lambda = 0$

= > -10λ = -20

=> λ= 2

putting value of λ in equation (ii)

$\vec{u} = 3\lambda\hat{i} + \lambda\vec{k}$

$= 3(2)\hat{i} + (2)\hat{k}$

$\vec{u} = 6\hat{i} + 2\vec{k}$

put the value of u in equation (i)

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$

$5\hat{i} - 2\hat{j} + 5\hat{k} = (6\hat{i} + 2\hat{k}) + \vec{v}$

$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 6\hat{i} - 2\hat{k}$

$\vec{v} = -\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{a} = (6\hat{i} + 2\vec{k}) + (-\hat{i} - 2\hat{j} + 3\hat{k})$

OR

Given vectors are, $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}, \vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

The projection of $\vec{a}$ along $\vec{b}$

$= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$\vec{a}.\vec{b} = (\lambda\hat{i} + \hat{j} + 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})$

$= 2\lambda + 6 + 12$

$= 2\lambda + 18$

$|\vec{b}| = \sqrt{2^2 + 6^2 + 3^2}$

$= \sqrt{49} = 7$

Given, the projection of vector a along vector b is 4.

$\therefore \quad \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = 4$

$\Rightarrow \quad \frac{2\lambda + 18}{7} = 4$

$\Rightarrow \quad 2\lambda + 18 = 28$

$\Rightarrow \quad 2\lambda = 10$

$\Rightarrow \quad \lambda = 5$

29. Let the given integral be,

$I = \int \frac{x^5 dx}{\sqrt{1 + x^3}}$

$= \int \frac{x^3 x^2 dx}{\sqrt{1+x^3}}$

Put $x^3 = t$

$\Rightarrow 3x^2\, dx = dt$

$\Rightarrow x^2 \mathrm{dx} = \frac{dt}{3}$

$\therefore I = \frac{1}{3} \int \frac{t \mathrm{dt}}{\sqrt{1+t}}$

$= \frac{1}{3} \int \left(\frac{1+t-1}{\sqrt{1+t}} \right) dt$

$= \frac{1}{3} \int \left(\sqrt{1+t} - \frac{1}{\sqrt{1+t}} \right) dt$

$= \frac{1}{3} \left[\frac{(1+t)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(1+t)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right] + C$

$= \frac{2}{9}(1+t)^{\frac{3}{2}} - \frac{2}{3}(1+t)^{\frac{1}{2}} + C$

$= \frac{2}{9}\left(1+x^3\right)^{\frac{3}{2}} - \frac{2}{3}\left(1+x^3\right)^{\frac{1}{2}} + C$

$= \frac{2}{9}\left(1+x^3\right)^{\frac{1}{2}} \left(1+x^3-3\right) + C$

$= \frac{2}{9}\sqrt{1+x^3}\left(x^3-2\right) + C$

OR

We can write it as $\int \frac{x}{1+\sin x} dx = \int \frac{x(1-\sin x)}{(1+\sin x)\cdot(1-\sin x)} dx$

$= \int \frac{x(1-\sin x)}{1-\sin^2 x} dx$

$= \int \frac{x(1-\sin x)}{\cos^2 x} dx$

$= \int x \sec^2 x dx - \int x \tan x \sec x\, dx$

Using by parts we have,

$\int x \sec^2 x dx - \int x \sec x \tan x\, dx = \left(x \int \sec^2 x dx - \int \left(\frac{dx}{dx} \cdot \int \sec^2 x dx \right) dx \right)$

$- \left(x \int \sec x \tan x\, dx - \int \left(\frac{dx}{dx} \cdot \int \sec x \tan x\, dx \right) dx \right)$

$=(x \tan x - \int 1.\tan x\, dx) - (x.\sec x - \int 1.\sec x\, dx)$

$= x \tan x - \ln|\sec x| - x \sec x + \ln|\sec x + \tan x| + c$

$= \mathrm{x}(\tan \mathrm{x} - \sec \mathrm{x}) + \ln\left|\frac{\sec \mathrm{x} + \tan \mathrm{x}}{\sec \mathrm{x}}\right| + \mathrm{c}$

$= x(\tan x - \sec x) + \ln|1+\sin x| + c$

30. Given function is: $f(x) = |x| + |x - 1|$

We have,

(LHL at x = 0) = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$

$= \lim_{h \to 0} [|0 - h| + |0 - h - 1|] = 1$

(RHL at x = 0) = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h)$

$= \lim_{h \to 0} [|0 + h| + |0 + h - 1|] = 1$

Also,

$f(0) = |0| + |0 - 1| = 0 + 1 = 1$

Now,

(LHL at x = 1) = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h)$

$= \lim_{h \to 0} (|1 - h| + |1 - h - 1|) = 1 + 0 = 1$

(RHL at x = 1) = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h)$

$= \lim_{h \to 0} (|1 + h| + |1 + h - 1|) = 1 + 0 = 1$

Also,

$f(1) = |1| + |1 - 1| = 1 + 0 = 1$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$ and

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

Hence, f(x) is continuous at x = 0, 1

31. Given curves are $y = \sqrt{x}$ ....(1)

and 2y-x+3=0............(2)
on solving (1) and (2), we get

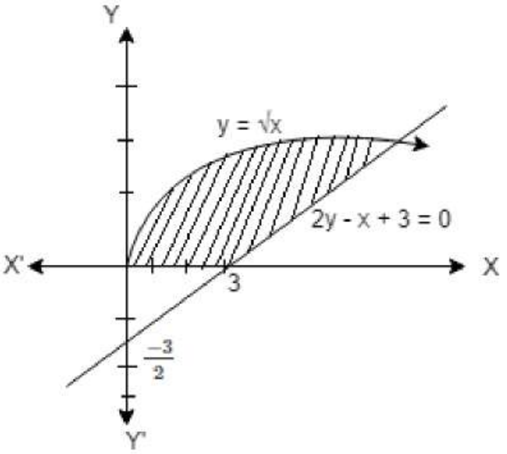
$2\sqrt{x}-(\sqrt{x})^2+3=0$

$(\sqrt{x})^2-2\sqrt{x}-3=0$

$(\sqrt{x}-3)(\sqrt{x}+1)=0$

$\sqrt{x}=3\ [as\ \sqrt{x}=-1\ is\ not\ possible]$

$\therefore\ y=3$



Hence, required area

$=\int_0^3 (x_2-x_1)dy$

$=\int_0^3 \{(2y+3)-y^2\}dy$

$=\left[y^2+3y-\frac{y^3}{3}\right]_0^3$

=9+9-9=9 sq.units

Section D

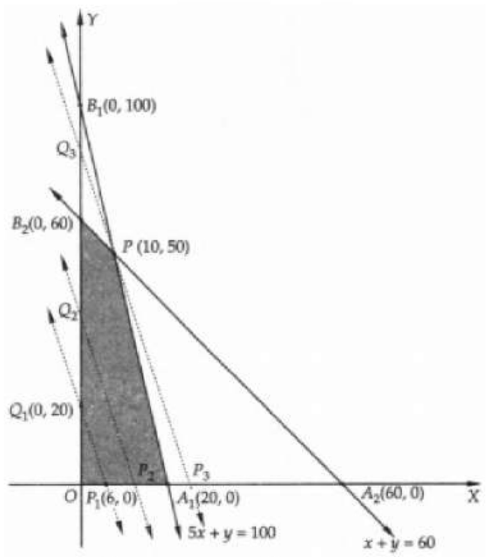
32. We first convert the inequations into equations to obtain the lines 5x + y = 100, x + y = 60, x = 0 and y = 0.

The line 5x + y = 100 meets the coordinate axes at A_1 (20, 0) and B_1 (0,100). Join these points to obtain the line 5x + y = 100.

The line x + y = 60 meets the coordinate axes at A_2 (60, 0) and B_2 (0, 60). Join these points to obtain the line x + y = 60.

Also, x = 0 is the y-axis and y = 0 is the x-axis.

The feasible region of the LPP is shaded in a figure. The coordinates of the comer-points of the feasible region OA_1PB_2 are O (0, 0), A_1 (20, 0), P (10,50) and B_2 (0, 60).



Now, we take a constant value, say 300 (i.e. 2 times the l.c.m. of 50 and 15) for Z. Then,

300 = 50x + 15y

This line meets the coordinate axes at P_1 (6, 0) and Q_1 (0, 20). Join these points by a dotted line. Now, move this line parallel to itself in the increasing direction i.e. away from the origin. P_2Q_2 and P_3Q_3 are such lines. Out of these lines locate a line that is farthest from the origin and has at least one point common to the feasible region.

Clearly, P_3Q_3 is such line and it passes through the vertex P (10, 50) the convex polygon OA_1PB_2. Hence, x = 10 and y = 50 will give the maximum value of Z.

The maximum value of Z is given by

$Z = 50 \times 10 + 15 \times 50 = 1250.$

33. R = {(a,b) = |a.b| is divisible by 2.

where $a, b \in A = \{1, 2, 3, 4, 5\}$

reflexivty

For any a∈A,|a−a|=0 Which is divisible by 2.

∴ (a, a) ∈ r for all a ∈ A

So ,R is Reflexive

Symmetric :

Let $(a, b) \in R$ for all a, b ∈ R

|a−b| is divisible by 2

|b−a| is divisible by 2

$(a, b) \in r \Rightarrow (b, a) \in R$

So, R is symmetirc .

Transitive :

Let $(a, b) \in R$ and $(b, c) \in R$ then

$(a, b) \in R$ and $(b, c) \in R$

|a−b| is divisible by 2

|b−c| is divisible by 2

Two cases :

Case 1:

When b is even

$(a, b) \in R$ and $(b, c) \in R$

|a−c| is divisible by 2

|b−c| is divisible by 2

|a−c| is divisible by 2

∴ (a, c) ∈ R

Case 2:

When b is odd

$(a, b) \in R$ and $(b, c) \in R$

|a−c| is divisible by 2

|b−c| is divisible by 2

|a−c| is divisible by 2

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

So R is transitive.

Hence , R is an equivalence relation

OR

We observe the following properties of relation R.

Reflexivity: For any a ∈N

a - a = 0 = 0 × n

⇒ a - a is divisible by n

⇒ (a, a) ∈ R

Thus, (a, a) ∈ for all a ∈ Z. So, R is reflexive on Z

Symmetry: Let (a, b) ∈R. Then,

(a, b) ∈ R

⇒ (a - b) is divisible by n

⇒ (a - b) = np for some p ∈ Z

⇒ b - a = n (-p)

⇒ b - a is divisible by n $[\because p \in Z \Rightarrow -p \in Z]$

⇒ (b, a) ∈ R

Thus, (a, b) ∈ R ⇒ (b, a) ∈R for all a, b ∈ Z.

So, R is symmetric on Z.

Transitivity: Let a, b, c $\in$ Z such that (a, b) $\in$R and (b, c) $\in$ R. Then,

(a, b) $\in$ R

$\Rightarrow$ (a - b) is divisible by n

$\Rightarrow$ a - b = np for some p $\in$ Z

and, (b, c) $\in$ R

$\Rightarrow$ (b - c) is divisible by n

$\Rightarrow$ b - c = nq for some q $\in$ Z

$\therefore$ (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow$ a - b = np and b - c = nq

$\Rightarrow$ (a - b) + (b - c) = np + nq

$\Rightarrow$ a - c = n (p + q)

$\Rightarrow$ a - c is divisible by n $[\because p, q \in Z \Rightarrow p + q \in Z]$

$\Rightarrow$ (a, c) $\in$ R

Thus, (a, b) $\in$ R and (b, c) $\in$ R $\Rightarrow$(a, c) $\in$ R for all a, b, c $\in$ Z.

34. Here,it is given that

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k})\ + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\vec{r} = (4\hat{i} + \hat{j})\ + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

Here,

$\overrightarrow{a_1} = i + 2\hat{j} + 3\hat{k}$

$\overrightarrow{\mathrm{b}_1} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{\mathrm{k}}$

$\overrightarrow{a_2} = 4\hat{\imath} + \hat{\jmath}$

$\overrightarrow{\mathrm{b}_2} = 5\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \hat{\mathrm{k}}$

Thus,

$$\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(3 - 8) - \hat{j}(2 - 20) + \hat{k}(4 - 15)$

$\therefore \overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2} = -5\hat{\mathrm{i}} + 18\hat{\mathrm{j}} - 11\hat{\mathrm{k}}$

$\therefore |\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$

$= \sqrt{25 + 324 + 121}$

$= \sqrt{470}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = (4 - 1)\hat{i} + (1 - 2)\hat{\jmath} + (0 - 3)\hat{\mathrm{k}}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 3\hat{\imath} - \hat{\jmath} - 3\hat{k}$

Now,we have

$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (-5\hat{i} + 18\hat{j} - 11\hat{k}) \cdot (3\hat{i} - \hat{j} - 3\hat{k})$

$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$

= -15 - 18 + 33

= 0

Thus, the distance between the given lines is

$$\mathrm{d} = \left| \frac{(\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}) \cdot (\overrightarrow{\mathrm{a}_2} - \overrightarrow{\mathrm{a}_1})}{|\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}|} \right|$$

$\therefore \mathrm{d} = \left| \frac{0}{\sqrt{470}} \right|$

$\therefore$ d = 0 units

As d = 0

Thus, the given lines intersect each other.

Now, to find a point of intersection, let us convert given vector equations into Cartesian equations.

For that putting $\vec{\mathrm{r}} = \mathrm{x}\hat{\mathrm{i}} + \mathrm{y}\hat{\mathrm{j}} + \mathrm{z}\hat{\mathrm{k}}$ in given equations,

$\Rightarrow \vec{L}_1 : \mathrm{x}\hat{\mathrm{i}} + \mathrm{y}\hat{\mathrm{j}} + \mathrm{z}\hat{\mathrm{k}} = (i + 2j + 3\hat{k}) + \lambda(2i + 3\hat{j} + 4\hat{k})$

$\Rightarrow \vec{L}_2 : \mathrm{x}\hat{\mathrm{i}} + \mathrm{y}\hat{\mathrm{j}} + \mathrm{z}\hat{\mathrm{k}} = (4\hat{i} + \hat{\jmath}) + \mu(5\hat{\imath} + 2\hat{\jmath} + \hat{k})$

$\Rightarrow \vec{L}_1 : (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{\imath} + 3\lambda\hat{\jmath} + 4\lambda\hat{k}$

$\Rightarrow \vec{L}_2 : (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{\imath} + 2\mu\hat{\jmath} + \mu\hat{k}$

$\Rightarrow \vec{L}_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$\therefore \vec{L}_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$

General point on L1 is

$x_1 = 2\lambda + 1$, $y_1 = 3\lambda + 2$, $z_1 = 4\lambda + 3$

Suppose, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Thus, point P satisfies the equation of line $\vec{L}_2$.

$\Rightarrow \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = \frac{4\lambda+3-0}{1}$

$\therefore \frac{2\lambda-3}{5} = \frac{3\lambda+1}{2}$

$\Rightarrow 4\lambda - 6 = 15\lambda + 5$

$\Rightarrow 11\lambda = -11$

$\Rightarrow \lambda = -1$

Thus, $x_1 = 2(-1) + 1$, $y_1 = 3(-1) + 2$, $z_1 = 4(-1) + 3$

$\Rightarrow x_1 = -1$, $y_1 = -1$, $z_1 = -1$

Therefore, point of intersection of given lines is (-1, -1, -1).

OR

Line passing through (1, 2, 3)

ie $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and parallel to the given planes is perpendicular to the vectors

$\vec{b}_1 = \hat{i} - \hat{j} + 2\hat{k}$ and

$\vec{b}_2 = 3\hat{i} + \hat{j} + \hat{k}$

Required line is parallel to $\vec{b}_1 \times \vec{b}_2$

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = \vec{i}(-1-2) - \vec{j}(1-6) + \vec{k}(1+3) = -3\vec{i} + 5\vec{j} + 4\vec{k}$$

Required education of line is : -

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$

35. It is given that f(x) = |x| - |x + 1|

The given function f is defined for real number and f can be written as the composition of two functions, as

f = goh, where g(x) = |x| and h(x) = |x + 1|

Then, f = g - h

First we have to prove that g(x) = |x| and h(x) = |x + 1| are continuous functions.

g(x) = |x| can be written as

$$g(x) = \begin{cases} -x, \text{ if } x < 0 \\ x, \text{ if } x \geq 0 \end{cases}$$

Now, g is defined for all real number.

Let k be a real number.

Case I: If k < 0,

Then g(k) = -k

And $\lim_{x \to \mathbf{k}} g(x) = \lim_{x \to k}(-x) = -k$

Thus, $\lim_{x \to \mathbf{k}} g(x) = g(k)$

Therefore, g is continuous at all points x, i.e., x > 0|

Case II: If k > 0,

Then g(k) = k and

$\lim_{x \to \mathbf{k}} g(x) = \lim_{x \to k} x = k$

Thus $\lim_{\mathbf{x} \to \mathbf{k}} \mathrm{g}(\mathrm{x}) = \mathrm{g}(\mathrm{k})$

Therefore, g is continuous at all points x, i.e., x < 0.

Case III: If k = 0,

Then, g(k) = g(0) = 0

$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} (-x) = 0$

$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} (x) = 0$

$\therefore \lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x) = g(0)$

Therefore, g is continuous at x = 0

From the above 3 cases, we get that g is continuous at all points.

g(x) = |x + 1| can be written as

$g(x) = \begin{cases} -(x+1), \text{ if } x < -1 \\ x+1, \text{ if } x \geqslant -1 \end{cases}$

Now, h is defined for all real number.

Let k be a real number.

Case I: If k < -1,

Then h(k) = -(k + 1)

And $\lim_{x\to \mathbf{k}} \mathrm{h(x)} = \lim_{\mathrm{x}\to \mathrm{k}}[-(\mathrm{x}+1)] = -(\mathrm{k}+1)$

Thus, $\lim_{\mathbf{x}\to \mathbf{k}} \mathrm{h(x)} = \mathrm{h(k)}$

Therefore, h is continuous at all points x, i.e., x < -1

Case II: If k > -1,

Then h(k) = k + 1 and

$\lim_{x\to \mathbf{k}} h(x) = \lim_{x\to k}(x+1) = k+1$

Thus, $\lim_{x\to \mathbf{k}} h(x) = h(k)$

Therefore, h is continuous at all points x, i.e., x > -1.

Case III: If k = -1,

Then, h(k) = h(-1) = -1 + 1 = 0

$\lim_{x\to 1^-} h(x) = \lim_{x\to 1^-}$ [-(x + 1)] = -(-1 + 1) = 0

$\lim_{x\to 1^+} h(x) = \lim_{x\to 1^+} (x+1)$ = -(-1 + 1) = 0

$\therefore \lim_{x\to 1^-} h(x) = \lim_{x\to 1^+} h(x) = h(-1)$

Therefore, g is continuous at x = -1

From the above 3 cases, we get that h is continuous at all points.

Hence, g and h are continuous function.

Therefore, f = g – h is also a continuous function.

Section E

36. **Read the text carefully and answer the questions:**

A tin can manufacturer designs a cylindrical tin can for a company making sanitizer and disinfectors. The tin can is made to hold 3 litres of sanitizer or disinfector. The cost of material used to manufacture the tin can is ₹100/m^2.

(i) Given, r cm is the radius and h cm is the height of required cylindrical can.

Given that, volume of cylinder = 3l = 3000 cm^3 ($\because$ 1l = 1000 cm^3)

$\Rightarrow \pi r^2 h$ = 3000 $\Rightarrow$ h = $\frac{3000}{\pi r^2}$

Now, the surface area, as a function of r is given by

S(r) = $2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r\left(\frac{3000}{\pi r^2}\right)$

= $2\pi r^2 + \frac{6000}{r}$

(ii) Now, S(r) = $2\pi r^2 + \frac{6000}{r}$

$\Rightarrow$ S'(r) = $4\pi r - \frac{6000}{r^2}$

To find critical points, put S'(r) = 0

$\Rightarrow \frac{4\pi r^3 - 6000}{r^2} = 0$

$\Rightarrow r^3 = \frac{6000}{4\pi} \Rightarrow r = \left(\frac{1500}{\pi}\right)^{1/3}$

Also, $S''(r)|_r = \sqrt[3]{\frac{1500}{\pi}} = 4\pi + \frac{12000 \times \pi}{1500}$

= $4\pi + 8\pi$ = $12\pi > 0$

Thus, the critical point is the point of minima.

(iii) The cost of material for the tin can is minimized when r = $\sqrt[3]{\frac{1500}{\pi}}$ cm and the height is $\frac{3000}{\pi\left(\sqrt[3]{\frac{1500}{\pi}}\right)^2} = 2\sqrt[3]{\frac{1500}{\pi}}$ cm.

OR

We have, minimum surface area = $\frac{2\pi r^3 + 6000}{r}$

= $\frac{2\pi \cdot \frac{1500}{\pi} + 6000}{\sqrt[3]{\frac{1500}{\pi}}} = \frac{9000}{7.8}$ = 1153.84 cm^2

Cost of 1 m^2 material = ₹100

$\therefore$ Cost of 1 cm^2 material = ₹ $\frac{1}{100}$

$\therefore$ Minimum cost = ₹$\frac{1153.84}{100}$ = ₹11.538

37. **Read the text carefully and answer the questions:**

To promote the making of toilets for women, an organization tried to generate awareness through

i. house calls
ii. emails and
iii. announcements.

The cost for each mode per attempt is given below:

1. ₹ 50
2. ₹ 20
3. ₹ 40

The number of attempts made in the villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also, the chance of making of toilets corresponding to one attempt of given modes is

1. 2%
2. 4%
3. 20%

(i) Total number of toilets that can be expected in each village is given by the following matrix.

$$\begin{matrix} X \\ Y \\ Z \end{matrix} \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix}$$

$$\begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 8 & + & 12 & + & 20 \\ 6 & + & 10 & + & 15 \\ 10 & + & 16 & + & 30 \end{bmatrix} = \begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 40 \\ 31 \\ 56 \end{bmatrix}$$

X = 40, Y = 31, Z = 56

(ii) $$\begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix}$$

$$\begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 8 & + & 12 & + & 20 \\ 6 & + & 10 & + & 15 \\ 10 & + & 16 & + & 30 \end{bmatrix} = \begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 40 \\ 31 \\ 56 \end{bmatrix}$$

Total attempt made in all the villages = 2475

Total number of toilets that can be expected after the promotion in all the three-villages = 40 + 31 + 56 = 127

The percentage of toilets that can be expected after the promotion in all the three-villages = $\frac{127}{2475} \times 100 = 5.13\%$

(iii) Let ₹A, ₹B and ₹C be the cost incurred by the organization for villages X, Y and Z respectively. Then A, B, C will be given by the following matrix equation.

$$\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 400 \times 50 + 300 \times 20 + 100 \times 40 \\ 300 \times 50 + 250 \times 20 + 75 \times 40 \\ 500 \times 50 + 400 \times 20 + 150 \times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 20000 + 6000 + 4000 \\ 15000 + 5000 + 3000 \\ 25000 + 8000 + 6000 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$$

Cost is ₹30,000.

OR

$$\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 400 \times 50 + 300 \times 20 + 100 \times 40 \\ 300 \times 50 + 250 \times 20 + 75 \times 40 \\ 500 \times 50 + 400 \times 20 + 150 \times 40 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 20000 + 6000 + 4000 \\ 15000 + 5000 + 3000 \\ 25000 + 8000 + 6000 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$$

Hence total cost is = ₹92000

38. **Read the text carefully and answer the questions:**

In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.

(i) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5$, $P(E_2) = 0.2$, $P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06$, $P(\frac{A}{E_2}) = 0.04$, $P(\frac{A}{E_3}) = 0.03$

Using Bayes' theorem, we have

$P(\frac{E_1}{A}) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)}$

$= \frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03} = \frac{30}{47}$

$\therefore$ Required probability = $P\left(\frac{\bar{E}_1}{A}\right)$

$= 1 - P\left(\frac{E_1}{A}\right) = 1 - \frac{30}{47} = \frac{17}{47}$

(ii) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5$, $P(E_2) = 0.2$, $P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06$, $P(\frac{A}{E_2}) = 0.04$, $P(\frac{A}{E_3}) = 0.03$

$P(A \cap E_2) = P\left(\frac{A}{E_2}\right) . P(E_2)$

$\Rightarrow 0.04 \times 0.2 = 0.008$