Chapter 13. Surface Areas and Volume

Question-1

The diameter of the base of a right circular cylinder is 28 cm and its height is 21 cm. Find its
(i) curved surface area
(ii) total surface area
(iii) volume

Solution:
Diameter of the base of a right circular cylinder is 28 cm
Radius of the base of a right circular cylinder (r) is 14 cm
Height of a right circular cylinder is 21 cm.

(i) Curved surface area of a right circular cylinder = $2\pi rh = 2 \times \frac{22}{7} \times 14 \times 21$
$= 1848\ cm^2$

(ii) Total surface area of a right circular cylinder = $2\pi r(r + h)$
$= 2 \times \frac{22}{7} \times 14(14 + 21)$
$= 2 \times \frac{22}{7} \times 14 \times 35$
$= 3080\ cm^2$

(iii) Volume of a right circular cylinder = $\pi r^2h = \frac{22}{7} \times 14 \times 14 \times 21 = 12936$ cm^3.

Question-2

The volume of a vessel in the form of a right circular cylinder is 448π cm^3 and its height is 7 cm. Find the radius of its base.

Solution:
Height of a right circular cylinder is 7 cm
Volume of a right circular cylinder is 448π cm^3

$\pi r^2h = 448\pi\ r^2 h = 448$
$7 r^2 = 448$
$r^2 = 64$
r = 8 cm
Therefore the radius of its base is 8 cm.

Question-3

The radius of the base and the height of a right circular cone are 7 cm and 24 cm respectively. Find the volume and total surface area of the cone.

Solution:
Radius of the base of a right circular cone is 7 cm.

Height of a right circular cone is 24 cm.
Volume of a right circular cone = $\frac{1}{3}\pi r^2h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 = 1232$ cm^3

Slant height of the cone = $\sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25$ cm^3

Total surface area of a right circular cone = $\pi r(r + l)$

$= \frac{22}{7} \times 7(7 + 25)$

$= \frac{22}{7} \times 7 \times 32$

$= 704$ cm^2.

Question-4

The curved surface area of a right circular cone is 12320 cm^2. If the radius of its base is 56 cm. Find its height.

Solution:

Given, Radius of a right circular cone(r) is 56 cm.
Curved surface area of a right circular cone is 12320 cm^2
$\pi r l = 12320$
$\frac{22}{7} \times 56 \times l = 12320$
$l = \frac{12320 \times 7}{22 \times 56} = 70$
Height of the cone = $\sqrt{70^2 - 56^2} = \sqrt{4900 - 3136} = \sqrt{1736} = 42$
Therefore the height of a right circular cone is 42 m.

Question-5

Find the volume and the surface area of a metallic sphere having diameter 8.4 cm.

Solution:

Diameter of a sphere is 8.4 cm

Radius of a sphere is 4.2 cm

Volume of a metallic sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2 = 310.464$ cm^3
Surface area of a metallic sphere = $4\pi r^2 = 4 \times \frac{22}{7} \times 4.2 \times 4.2 = 221.76$ cm^2.

Question-6

The internal and external diameters of a hollow hemispherical vessel are 42 cm and 45.5 cm, respectively. Find its capacity and also its outer curved surface area.

Solution:

Internal radius of a hollow hemispherical vessel (r_1) = 21 cm

External radius of a hollow hemispherical vessel (r_2) = 22.75 cm
Capacity of a hollow hemispherical vessel = $\frac{2}{3}\pi r_1^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

= 19404 cm^3

= 19.404 litres

Curved surface area of a hollow hemispherical vessel = $2\pi r_2^2$

$= 2 \times \frac{22}{7} (22.75)^2$

= 3253.25 cm^3.

Question-7

The diameter of a metallic sphere is 6 cm. It is melted and drawn into a wire having diameter of the cross-section as 0.2 cm. Find the length of the wire.

Solution:

Radius of a metallic sphere (r) = 6 cm/2 = 3 cm
Radius of the cross section (R) = 0.2 cm/2 = 0.1 cm

Volume of a cylinder = Volume of a metallic sphere

$\pi R^2 h = \frac{4}{3}\pi r^3$

$(0.1)^2 h = \frac{4}{3} \times 3^3$

$h = \frac{4 \times 3 \times 3 \times 3}{3 \times 0.1 \times 0.1}$ = 3600 cm = 36 m

Therefore the length of the wire is 36 m.

Question-8

50 circular plates, each of radius 7 cm and thickness $\frac{1}{2}$ cm, are placed one above another to form a solid right circular cylinder. Find the total surface area and the volume of the cylinder so formed.

Solution:

Volume of the circular plate = $\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times \frac{1}{2}$ = 77 cm^3
Volume of the cylinder so formed by 50 circular plate = 50 × 77 = 3850 cm^3
Total surface area of the cylinder = $2\pi r(r + h) = 2 \times \frac{22}{7} \times 7(7 + 25)$ = 1408 cm^2.

Question-9

A sphere of diameter 5 cm is dropped into a cylinder vessel partly filled with water. The diameter of the base of the vessel is 10 cm. If the sphere is completely submerged, by how much will the level of water rise?

Solution:

Diameter of the sphere is 5 cm.

Radius of the sphere is 2.5 cm.

Diameter of the base of the vessel is 10 cm.

Radius of the base of the vessel is 5 cm.

Volume of the sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 2.5$

Increase in the volume of water = Volume of the sphere

$\frac{22}{7} \times 5 \times 5 \times h = \frac{4}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 2.5$

$h = \frac{4 \times 2.5 \times 2.5 \times 2.5}{3 \times 5 \times 5} = \frac{5}{6}$

Therefore the rise in the level of water is $\frac{5}{6}$ cm.

Question-10

A conical flask is full of water. The flask has base-radius r and height h. The water is poured into a cylindrical flask of base-radius *m*r. Find the height of water in the cylindrical flask.

Solution:

Radius of the conical flask = r

Height of the conical flask = h

Radius of the cylindrical flask (R) = m r

Height of water in the cylindrical flask = $\frac{\text{Volume of conical flask}}{\pi R^2}$

$$= \frac{\frac{1}{3} \times \pi \times r^2 \times h}{\pi R^2}$$

$$= \frac{\frac{1}{3} \times r^2 \times h}{(mr)^2}$$

$$= \frac{h}{3m^2}.$$

Question-11

A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with conical ends each of axis length 9 cm. Determine the capacity of the tank.

Solution:

Radius of the cylinder = 10.5 cm

Radius of the cone = 10.5 cm

Height of the cylinder = 18

Height of the cone = 9 cm

Capacity of the tank =

Volume of the cylinder + 2 × Volume of the cone

$= \frac{22}{7} \times 10.5 \times 10.5 \times 18 + 2 \times \frac{1}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 9$

= 6237 + 2079

= 8316 cm^3.

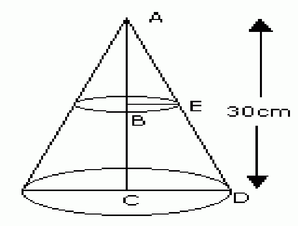
Question-12

The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made?

Solution:

Height of the cone is 30 cm.

Let the height of the cone cut off be h cm



$\Delta ABE \sim \Delta ACD$

Therefore $\frac{r}{R} = \frac{h}{H}$ ....(1)

Volume of the small cone = $\frac{1}{27}$th volume of the given cone

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times h = \frac{1}{27} \times \frac{1}{3} \times \frac{22}{7} \times R^2 \times H$

$r^2 \times h = \frac{1}{27} \times R^2 \times 30$

$(r/R)^2 h = 30$ ´ $\frac{1}{27}$

$(h/H)^2 h = 30$ ´ $\frac{1}{27}$ ....[using (1)]

$h^3 = 30 \times 30 \times 30$ ´ $\frac{1}{27}$

$h = 30/3 = 10$ cm

Therefore the height of the above section is (30 - 10) cm = 20 cm.

Question-13

A hemispherical tank of radius $1\frac{3}{4}$ m is full of water. A pipe connected to it empties it at the rate of 7 litres per second. How much time will it take to empty the tank?

Solution:

Radius of the hemispherical tank = $1\frac{3}{4}$ m

= $\frac{7}{4}$ m $\therefore$ Volume of water in the tank = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4}$

$= \frac{539}{48}$ cu. m

$= \frac{539000000}{48}$ cu. cm

$= \frac{539000000}{48} \times \frac{1}{1000}$ litres

Water emptied per second = 7 litres ∴ Time required to empty the tank = $\frac{539000000}{48} \times \frac{1}{1000} \times \frac{1}{7}$

= 1604.17 seconds

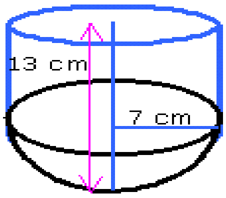
= $\frac{1604.17}{60}$

= 26.74 minutes

Question-14

A vessel is in the form of a hemi-spherical bowl mounted by a hollow cylinder. The diameter of the sphere is 14 cm and the total height of the vessel is 13 cm. Find the capacity. (π = 22/7)

Solution:



Diameter of the sphere = d = 14 cm ⇒ Radius of the sphere = r = 7 cm
Total height of the vessel = 13 cm ⇒ Height of the cylinder = h = 13 − 7 = 6 cm ∴ Volume of the hemispherical part = $\frac{2}{3}\pi r^3$

= $\frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

= 718.67 cu. cm ∴ Volume of the cylindrical part = $\pi r^2 h$

= $\frac{22}{7} \times 7 \times 7 \times 6$

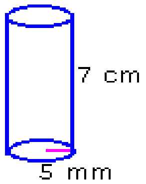
= 924 cu. cm ∴ Capacity of the vessel = (718.67 + 924) cu. cm

= 1642.67cu. cm

Question-15

The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used up on writing 330 words on an average. How many words would use up a bottle of ink containing one fifth of a litre?

Solution:



Diameter of the barrel of the fountain pen = 5 mm = 0.5 cm ⇒ Radius of the barrel of the fountain pen = 0.25 cm

Volume of the barrel of the fountain pen = πr^2h

$= \frac{22}{7} \times 0.25 \times 0.25 \times 7$

= 1.375 cu. cm ⇒ 1.375 cu. cm of ink writes 330 words on an average. ∴ One-fifth of a litre = $\frac{1}{5} \times 1000$ = 200 cu. cm ∴

Number of words 200 cu. cm of ink writes on an average = $200 \times \frac{330}{1.375}$

= 48000

The bottle of ink containing one-fifth of a litre of ink writes 48000 words on an average.

Question-16

A rectangular sheet of metal 44 cm long and 20 cm broad is rolled along its length into a right circular cylinder so that the cylinder has 20 cm as its height. Find the volume and curved surface are of the cylinder so formed.

Solution:

Length of the rectangular sheet = 44 cm

Breadth of the rectangular sheet = 20 cm

Area of the sheet = 44 × 20 = 880 sq. cm ∴ Curved surface area of the cylinder = Area of the sheet

= 880 sq. cm

Height of the cylinder = 20 cm

Circumference of the cylinder = 44 cm ⇒ $2\pi r = 44$ ∴ $r = \frac{44}{2\pi} = \frac{44}{2 \times \frac{22}{7}} = \frac{44 \times 7}{2 \times 22}$ = 7 cm ∴ Volume of the cylinder = πr^2h

$= \frac{22}{7} \times 7 \times 7 \times 20$

= 3080 cu. cm ∴ Volume of the cylinder is 3080 cu. cm and the curved surface area of the cylinder is 880 sq. cm.

Question-17

A cone of height 24 cm and radius of base 6 cm is made up of modeling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

Solution:

Radius of the base of the cone = 6 cm

Height of the cone = 24 cm ∴ Volume of clay in the cone = $\frac{1}{3}\pi r^2h = \frac{1}{3} \times \pi \times 6 \times 6 \times 24$ cm^3 ∴ Volume of the cone = $\frac{1}{3} \times \pi \times 6 \times 6 \times 24$ cm^3

Let R cm be the radius of the sphere. ∴ Volume of the sphere = $\frac{4}{3}$ π R^3 ∴ $\frac{4}{3}$

π R^3 = $\frac{1}{3}$ × π × 6 × 6 × 24 ⇒ 4 π R^3 = π × 6 × 6 × 24

Þ $R^3 = \frac{\pi \times 6 \times 6 \times 24}{4 \times \pi}$

= 6^3 ∴ R = 6 cm ∴ Radius of the sphere = 6 cm.

Question-18

A Joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Solution:

Base radius (r) = 7 cm

Height (h) = 24 cm

∴ Slant height (l) = $\sqrt{r^2 + h^2}$ = $\sqrt{(7)^2 + (24)^2}$ = $\sqrt{49 + 576}$ = $\sqrt{625}$ = 25 cm

∴ Curved surface area of a cap = πrl = $\frac{22}{7} \times 7 \times 25$ = 550 cm^2

∴ Curved surface area of 10 caps = 550 × 10 = 5500 cm^2

Hence the area of the sheet required to make 10 such caps is 5500 cm^2.