Class XII Session 2023-24
Subject - Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A and B are matrices of same order, then (AB′ – BA′) is a **[1]**

a) null matrix
b) unit matrix
c) symmetric matrix
d) skew-symmetric matrix

2. If A is a matrix of order 3 and |A| = 8, then |adj A| = **[1]**

a) 2
b) 1
c) 2^6
d) 2^3

3. If $A = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}$, then, what can you infer correctly about $(A + B)^{-1}$? **[1]**

a) does not exist
b) None of these
c) $(A + B)^{-1}$ is a skew-symmetric matrix
d) $A^{-1} + B^{-1}$

4. If $y = e^{\sin\sqrt{x}}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{e^{\sin\sqrt{x}}\cos\sqrt{x}}{2\sqrt{x}}$
b) $\frac{e^{\sin\sqrt{x}}}{2\sqrt{x}}$
c) None of these
d) $e^{\sin\sqrt{x}}\cdot\cos\sqrt{x}$

5. The point of intersection of the line $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-3}{-2}$ and the plane 2x - y + 3z -1 = 0, is **[1]**

a) (10, -10, -3)
b) (10, 10, -3)
c) (10, -10, 3)
d) (-10, 10, 3)

6. A homogeneous equation of the form $\frac{dy}{dx} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution **[1]**

a) $y = \nu x$

b) $x = \nu y$

c) $\nu = yx$

d) $x = \nu$

7. The objective function Z = 4 x + 3 y can be maximised subjected to the constraints 3x + 4y $\leq$ 24, 8x + 6y $\leq$ 48 , x $\leq$ 5 , y $\leq$ 6 ; x, y $\geq$ 0 **[1]**

a) at only one point

b) None of these

c) at two points only

d) at an infinite number of points

8. If θ is the angle between vectors $\vec{a}$ and $\vec{b}$ then the cross product $\vec{a} \times \vec{b} =$ **[1]**

a) $2|a||b|\sin\theta\hat{n}$

b) $\left|\overrightarrow{a}\right|\left|\overrightarrow{b}\right|\sin\theta\hat{n}$

c) $|a||b|\sin\theta$

d) $|a||b|\cos\theta$

9. $\int \frac{x^4}{(1+x^2)} dx =?$ **[1]**

a) None of these

b) $\frac{x^3}{3} + x + \tan^{-1} x + C$

c) $\frac{-x^3}{3} + x - \tan^{-1} x + C$

d) $\frac{x^3}{3} - x + \tan^{-1} x + C$

10. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x is equal to **[1]**

a) 0

b) 6

c) ± 6

d) -6

11. The point at which the maximum value of x + y, subject to the constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x, y $\geq$ 0 is obtained, is **[1]**

a) (20, 35)

b) (30, 25)

c) (35, 20)

d) (40,15)

12. The scalar product of two nonzero vectors $\vec{a}$ and $\vec{b}$ is denoted by **[1]**

a) $\overrightarrow{ab}$

b) $\vec{a}.\vec{b}$

c) $\overrightarrow{a} \times \overrightarrow{b}$

d) ab

13. The vertices of a a ABC are A(-2, 4), B(2, -6) and C(5, 4). The area of a ABC is **[1]**

a) 17.5 sq units

b) 32 sq units

c) 35 sq units

d) 28 sq units

14. Two independent events A and B have P(A) = $\frac{1}{3}$ and P(B) = $\frac{3}{4}$. What is the probability that exactly one of the two events A or B occurs? **[1]**

a) $\frac{5}{6}$

b) $\frac{7}{12}$

c) $\frac{1}{4}$

d) $\frac{5}{12}$

15. What is the solution of the differential equation $\sin\left(\frac{dy}{dx}\right)$ - a = 0? Where, C is an arbitrary constant. **[1]**

a) y = x $\sin^{-1}$ a + C

b) y = x + x$\sin^{-1}$ a + C

c) x = y $\sin^{-1}$ a + C

d) y = $\sin^{-1}$ a + C

16. What is the value of λ for which **[1]**

$(\lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \times (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) = (2\hat{\mathbf{i}} - 11\hat{\mathbf{j}} - 7\hat{\mathbf{k}})$

a) 2
b) 7
c) 1
d) -2

17. Let f(x) = |x| and g(x) = $|x^3|$, then **[1]**

a) f(x) and g(x) both are differentiable at x = 0
b) f(x) is differentiable but g(x) is not differentiable at x = 0
c) f(x) and g(x) both are continuous at x = 0
d) f(x) and g(x) both are not differentiable at x = 0

18. The vector equation of the x-axis is given by **[1]**

a) $\vec{r} = \hat{j} + \hat{k}$
b) none of these
c) $\vec{r} = \hat{i}$
d) $\vec{r} = \lambda\hat{i}$

19. **Assertion (A):** The absolute maximum value of the function $2x^3 - 24x$ in the interval [1, 3] is 89. **[1]**
Reason (R): The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** A relation R = {(a, b): |a - bl <3} defined on the set A = {1, 2, 3, 4} is reflexive. **[1]**
Reason (R): A relation R on the set A is said to be reflexive if for (a, b) ∈ R and (b, c) ∈ R, we have (a, c) ∈ R.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Evaluate: $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$. **[2]**

OR

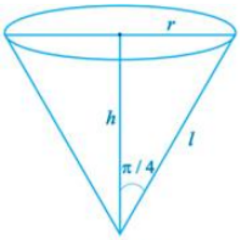
Evaluate:- $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$

22. Find the points of local maxima or local minima and corresponding values of local maximum and local minimum values of each of the function. Also, find the points of inflection, if any: f(x) = (x + 1) $(x+2)^{\frac{1}{3}}, x \geq -2$. **[2]**

23. Show that $f(x) = \frac{1}{1+x^2}$ is neither increasing nor decreasing on R. **[2]**

OR

Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of $2cm^2/\sec$ in the surface area, through a tiny hole at the vertex of the bottom. When the slant height of cone is 4 cm, find the rate of decrease of the slant height of water.



24. Evaluate: $\int \frac{\log x}{(1+\log x)^2}$ dx **[2]**

25. Find the interval in function f(x) = 6 + 12x+ 3x^2 - 2x^3 is increasing or decreasing. **[2]**

Section C

26. Evaluate: $\int \frac{1}{(2x^2+3)\sqrt{x^2-4}} dx$ **[3]**

27. The contents of three urns are as follows: Urn 1 : 7 white, 3 black balls, Urn 2: 4 white, 6 black balls, and Urn 3: 2 white, 8 black balls. One of these urns is chosen at random with probabilities 0.20, 0.60 and 0.20 respectively. From the chosen urn two balls are drawn at random without replacement. If both these balls are white, what is the probability that these came from urn 3? **[3]**

28. Evaluate: $\int_0^\pi \frac{1}{5+4\cos x} dx$ **[3]**

OR

For $x > 0$, let $f(x) = \int_1^x \frac{\log_e t}{1+t} dt$. Find the function f (x) +$f\left(\frac{1}{x}\right)$ and show that $f(e) + f\left(\frac{1}{e}\right) = \frac{1}{2}$

29. Find one-parameter families of solution curves of the differential equation: (x log x) $\frac{dy}{dx}$ + y = log x **[3]**

OR

Solve the initial value problem: tan $x\frac{dy}{dx}$ = 2x tan x + x^2 - y; tan x $\neq$ 0 given that y = 0 when x = $\frac{\pi}{2}$

30. Show the solution zone of the following inequalities on a graph paper: **[3]**

5x + y $\geq$ 10

x + y $\geq$ 6

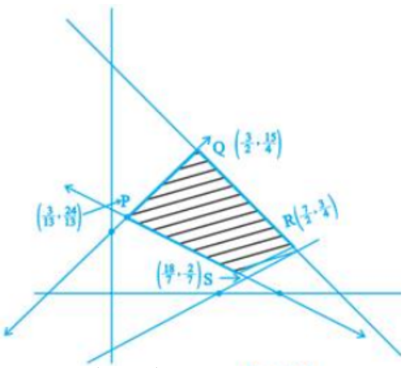
x + 4y $\geq$ 12

x $\geq$ 0, y $\geq$ 0

Find x and y for which 3x + 2y is minimum subject to these inequalities. Use a graphical method.

OR

In Fig, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of Z = x + 2y.



31. If x = a$\left(\frac{1+t^2}{1-t^2}\right)$ and y = $\frac{2t}{1-t^2}$, find $\frac{dy}{dx}$ **[3]**

Section D

32. Using integration, find the area of the region: {(x, y) : 9x^2 + 4y^2 $\leq$ 36, 3x + y $\geq$ 6}. **[5]**

33. Show that the function f : R - {3} $\rightarrow$ R - {1} given by $f(x) = \frac{x-2}{x-3}$ is a bijection. **[5]**

OR

Let R be a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$. Show that R is an equivalence relation.

34. Let $A=\begin{bmatrix}3 & 7\\ 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}6 & 8\\ 7 & 9\end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$ **[5]**

35. Show that the lines $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and $\vec{r}=(4\hat{i}+\hat{j})+\mu(5\hat{i}+2\hat{j}+\hat{k})$ intersect. **[5]**
Also, find their point intersection.

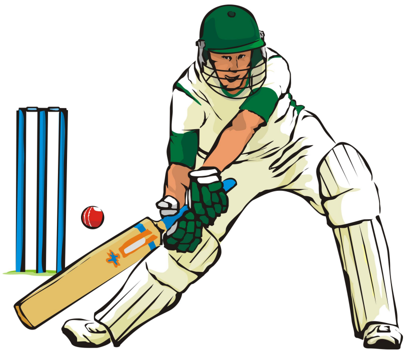
OR

Show that the lines $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$ and $\frac{x-2}{2}=\frac{y-1}{3}=\frac{z+1}{-2}$ intersect and find their point of intersection.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

In a bilateral cricket series between India and South Africa, the probability that India wins the first match is 0.6. If India wins any match, then the probability that it wins the next match is 0.4, otherwise, the probability is 0.3. Also, it is given that there is no tie in any match.



(i) Find the probability that India won the second match, if India has already loose the first match.

(ii) Find the probability that India losing the third match, if India has already lost the first two matches.

(iii) Find the probability that India losing the first two matches.

OR

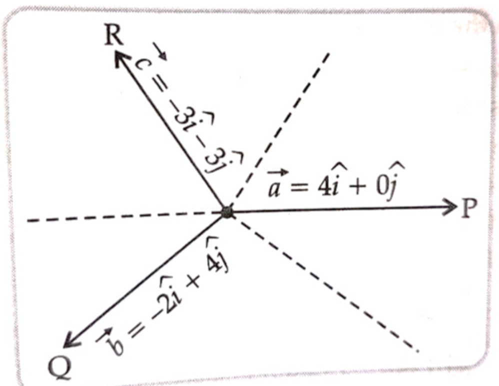
Find the probability that India winning the first three matches.

37. **Read the text carefully and answer the questions:** **[4]**

Team P, Q, R went for playing a tug of war game. Teams P, Q, R have attached a rope to a metal ring and is trying to pull the ring into their own areas (team areas when in the given figure below). Team P pulls with force $F_1 = 4\hat{i} + 0\hat{j}$ KN

Team Q pull with force $F_2 = -2\hat{i}+4\hat{j}$ KN

Team R pulls with force $F_3 = -3\hat{i}-3\hat{j}$ KN



(i) What is the magnitude of the teams combined force?

(ii) Find the magnitude of Team B.

(iii) Which team will win the game?

OR

In what direction is the ring getting pulled?

38. **Read the text carefully and answer the questions:** **[4]**

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where f(x) is the temperature in ^oF at x days.

(i) Is the function differentiable in the interval (0, 12)? Justify your answer.

(ii) If 6 is the critical point of the function, then find the value of the constant m.

Solution

Section A

1.

(d) skew-symmetric matrix

Explanation: We have matrices A and B of same order.

Let P = (AB' − BA')

Then, P′ = (AB′ − BA′)'

= (AB′)′ − (BA′)′

= (B′)′ (A)′ − (A′)′B′ = BA′ − AB′ = −(AB′ − BA′) = −P

Therefore, the given matrix (AB - BA') is a skew-symmetric matrix.

2.

(c) 2^6

Explanation: |A| = d

$|\text{adj } A| = |A|^{n-1}$

Here, n = 3, |A| = 8

$|\text{adj } A| = 8^2$

$|\text{adj } A| = (2^3)^2 = 2^6$

3.

(b) None of these

Explanation: $A=\begin{bmatrix}3 & 4\\ 2 & 4\end{bmatrix}, B=\begin{bmatrix}-2 & -2\\ 0 & -1\end{bmatrix}$

A + B = $\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$

Here, we know that $(A + B)^{-1} \neq A^{-1} + B^{-1}$

$(A+B)^T=\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}=A+B$

Hence, answer is none of these.

4. **(a)** $\frac{e^{\sin \sqrt{x}} \cos \sqrt{x}}{2 \sqrt{x}}$

Explanation: Given that $y=e^{\sin \sqrt{x}}$

Taking log both sides, we obtain

$\log_e y=\sin \sqrt{x}$

(Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we obtain

$\frac{1}{y} \frac{dy}{dx}=\cos \sqrt{x} \times \frac{1}{2 \sqrt{x}}$

Or

$\frac{dy}{dx}=\cos \sqrt{x} \times \frac{1}{2 \sqrt{x}} \times y$

Therefore $\frac{dy}{dx}=\frac{e^{\sin \sqrt{x}} \cos \sqrt{x}}{2 \sqrt{x}}$

5.

(b) (10, 10, -3)

Explanation: The given line is $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}=\lambda(\text{ say })$

A general point on this line is $(3\lambda+1, 4\lambda-2,-2\lambda+3)$

For some value of λ, let the point $P(3\lambda+1,4\lambda-2,-2\lambda+3)$ lie on the plane 2x - y + 3z - 1 = 0. Then,

$2(3\lambda+1)-(4\lambda-2)+3(-2\lambda+3)-1=0$

$\Rightarrow \quad 4\lambda=12 \Rightarrow \lambda=3$

So, the required point is P(9 + 1, 12 - 2, - 6 + 3), i.e., P (1 0 ,10, - 3).

6.

(b) $x = \nu y$

Explanation: A homogeneous equation of the form $\frac{dy}{dx} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution x= vy.so that it becomes variable separable form and integration is then possible

7.

(d) at an infinite number of points

Explanation: First, we will convert the given inequations into equations, we obtain the following equations:

3x + 4y = 24, 8x + 6y = 48, x = 5, y = 6, x = 0 and y = 0

The line 3x + 4y = 24 meets the coordinate axis at A(8, 0) and B(0, 6). Join these points to obtain the line 3x + 4y = 24 Clearly, (0, 0) satisfies the inequation 3x + 4y $\leq$ 24 . So, the region in x y -plane that contains the origin represents the solution set of the given equation. The line 8x + 6y = 48 meets the coordinate axis at C(6, 0) and D(0, 8) . Join these points to obtain the line 8x + 6y = 48 Clearly, (0, 0) satisfies the inequation 8x + 6y $\leq$ 48 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.

x = 5 is the line passing through x = 5 parallel to the Y axis.

y = 6 is the line passing through y = 6 parallel to the X axis.

The region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations.

These lines are drawn using a suitable scale.

The corner points of the feasible region are O(0, 0), G(5, 0), $F\left(5, \frac{4}{3}\right)$, $E\left(\frac{24}{7}, \frac{24}{7}\right)$ and B(0, 6)

The values of Z at these corner points are as follows:

Corner point : Z = 4x + 3y

O(0, 0) : $4 \times 0 + 3 \times 0 = 0$

G(5, 0) : $4 \times 5 + 3 \times 0 = 20$

$F\left(5, \frac{4}{3}\right) : 4 \times 5 + 3 \times \frac{4}{3} = 24$

$E\left(\frac{24}{7}, \frac{24}{7}\right) : 4 \times \frac{24}{7} + 3 \times \frac{24}{7} = \frac{196}{7} = 28$

B(0, 6) : $4 \times 0 + 3 \times 6 = 18$

We see that the maximum value of the objective function z is 24 which is at $F\left(5, \frac{4}{3}\right)$ and $E\left(\frac{24}{7}, \frac{24}{7}\right)$ Thus, the optimal value of Z is 24

As, we know that if an LPP has two optimal solutions, then there are an infinite number of optimal solutions. Therefore, the given objective function can be subjected at an infinite number of points.

8.

(b) $\left|\overrightarrow{a}\right|\left|\overrightarrow{b}\right| \sin \theta \hat{n}$

Explanation: If θ is the angle between vectors $\vec{a}$ and $\vec{b}$ then, the cross product: $\vec{a} \times \vec{b} = \left|\overrightarrow{a}\right|\left|\overrightarrow{b}\right| \sin \theta \hat{n}$.

9.

(d) $\frac{x^3}{3} - x + \tan^{-1} x + C$

Explanation: Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \sec^2 x dx = \tan x; \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \frac{x^4+1-1}{1+x^2} dx$

$\Rightarrow \int \frac{x^4-1}{1+x^2} dx + \int \frac{1}{1+x^2} dx = \int \frac{(x^2+1)(x^2-1)}{1+x^2} dx + \int \frac{1}{1+x^2} dx$

$\int x^2 - 1 dx + \int \frac{1}{1+x^2} dx$

$= \frac{x^2}{3} - x + \tan^{-1} x + c$

10.

(c) ± 6

Explanation: We have $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

We know that determinant of A is calculated as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

⇒ x(x) - 2(18) = 6(6) - 2(18)

⇒ x^2 - 36 = 36 - 36

⇒ x^2 = 36 - 36 + 36

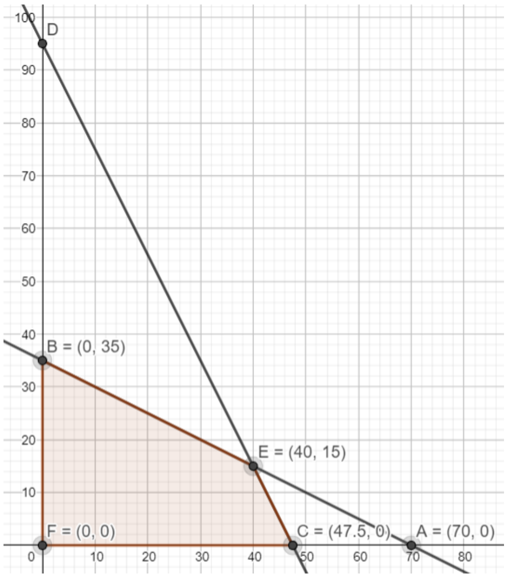
⇒ x^2 = 36

⇒ x = ± 6

11.

(d) (40,15)

Explanation: We need to maximize the function z = x + y Converting the given inequations into equations, we obtain
x + 2y = 70, 2x + y = 95, x = 0 and y = 0
Region represented by x + 2y ≤ 70 :
The line x + 2y = 70 meets the coordinate axes at A(70, 0) and B(0, 35) respectively. By joining these points we obtain the line x + 2y = 70. Clearly (0, 0) satisfies the inequation x + 2y ≤ 70 . So, the region containing the origin represents the solution set of the inequation x + 2y ≤ 70.
Region represented by 2x + y ≤ 95 :
The line 2x + y = 95 meets the coordinate axes at $C\left(\frac{95}{2}, 0\right)$ respectively. By joining these points we obtain the line 2x + y = 95
Clearly (0, 0) satisfies the inequation 2x + y ≤ 95 . So, the region containing the origin represents the solution set of the inequation 2x + y ≤ 95
Region represented by x ≥ 0 and y ≥ 0 :
since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x ≥ 0, and y ≥ 0
The feasible region determined by the system of constraints x + 2y ≤ 70, 2x + y ≤ 95, x ≥ 0, and y ≥ 0 are as follows.



The corner points of the feasible region are O(0, 0), C($\frac{95}{2}$, 0) E(40, 15) and B(0, 35).
The value fo Z at these corner points are as follows.
Corner point : z = x + y
O(0, 0) : 0 + 0 = 0
$C\left(\frac{95}{2}, 0\right) : \frac{95}{2} + 0 = \frac{95}{2}$
E(40, 15) : 40 + 15 = 55
B(0, 35) : 0 + 35 = 35
We see that maximum value of the objective function Z is 55 which is at (40, 15).

12.

(b) $\vec{a}.\vec{b}$

Explanation: The scalar product of two nonzero vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}.\vec{b}$.and is defined by $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

13.

(c) 35 sq units

Explanation: $\Delta = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 4 & -10 & 0 \\ 7 & 0 & 0 \end{vmatrix}$ = 35 sq units

14.

(b) $\frac{7}{12}$

Explanation: Given, P(A) = $\frac{1}{3}$ and P(B) = $\frac{3}{4}$

Now, P(exactly one)

= P(A) + P(B) - 2(P(A ∩ B))

= P(A) + P(B) - 2P(A) · P(B)

[∵ event A and B are independent]

$= \frac{1}{3} + \frac{3}{4} - 2 \times \frac{1}{3} \times \frac{3}{4}$

$= \frac{1}{3} + \frac{3}{4} - \frac{1}{2} = \frac{4+9-6}{12} = \frac{7}{12}$

15. **(a)** y = x $\sin^{-1}$ a + C

Explanation: Consider the given differential equation,

$\sin\left(\frac{dy}{dx}\right)$ - a = 0 ⇒ dy = ($\sin^{-1}$ a) dx

On integrating both sides w.r.t. x, we get

$\int dy = \int \left(\sin^{-1} a\right)$ dx ⇒ y = ($\sin^{-1}$ a)$\int dx \Rightarrow y$ = ($\sin^{-1}$ a) x + C

16. **(a)** 2

Explanation: Given,

$(\lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \times (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$= (2\hat{\mathbf{i}} - 11\hat{\mathbf{j}} - 7\hat{\mathbf{k}})$

$\Rightarrow \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \lambda & 1 & -1 \\ 3 & -2 & 4 \end{vmatrix} = (2\hat{\mathbf{i}} - 11\hat{\mathbf{j}} - 7\hat{\mathbf{k}})$

$\Rightarrow 2\hat{\mathbf{i}} - (4\lambda + 3)\hat{\mathbf{j}} + (-2\lambda - 3)\hat{\mathbf{k}} = 2\hat{\mathbf{i}} - 11\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$

On comparing the coefficient of, we get

$(4\lambda + 3)$ = 11 ⇒ λ = 2

17.

(c) f(x) and g(x) both are continuous at x = 0

Explanation: Given f(x) = |x| and g(x) = $|x^3|$,

$f(x) = \begin{cases} -x, & x \leq 0 \\ x, & x > 0 \end{cases}$

Checking differentiability and continuity,

LHL at x =0,

$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} -(0-h) = 0$

RHL at x =0,

$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} (0+h) = 0$

And f(0)=0

Hence, f(x) is continuous at x =0.

LHD at x =0,

$\lim_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h \to 0} \frac{f(0-h)-f(0)}{0-h-0}$

$= \lim_{h \to 0} \frac{(0-h)-(0)}{-h} = -1$

RHD at x =0,

$\lim_{x \to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{h \to 0} \frac{f(0+h)-f(0)}{0+h-0}$

$= \lim_{h \to 0} \frac{(0+h)-(0)}{h} = 1$

∵ LHD ≠ RHD

$\therefore$ f(x) is not differentiable at x =0.

$g(x)=\begin{cases}-x^3, & x\leq 0\\ x^3, & x>0\end{cases}$

Checking differentiability and continuity,

LHL at x =0,

$\lim_{x\to 0^-} g(x)=\lim_{h\to 0} g(0-h)=\lim_{h\to 0} -(0-h)^3=0$

RHL at x =0,

$\lim_{x\to 0^+} g(x)=\lim_{h\to 0} g(0+h)=\lim_{h\to 0} (0+h)^3=0$

And g(0)=0

Hence, g(x) is continuous at x =0.

LHD at x =0,

$\lim_{x\to 0^-}\frac{g(x)-g(0)}{x-0}=\lim_{h\to 0}\frac{g(0-h)-g(0)}{0-h-0}$

$=\lim_{h\to 0}\frac{(0-h)^3-(0)}{-h}=0$

RHD at x =0,

$\lim_{x\to 0^+}\frac{g(x)-g(0)}{x-0}=\lim_{h\to 0}\frac{g(0+h)-g(0)}{0+h-0}$

$=\lim_{h\to 0}\frac{(0+h)^3-(0)}{h}=0$

$\because$ LHD = RHD

$\therefore$ g(x) is differentiable at x =0.

18.

(d) $\vec{r}=\lambda\hat{i}$

Explanation: Vector equation needs a fixed point and a parallel vector

For x -axis we take fixed point as origin.

And parallel vector is $\hat{i}$

Equation would be $\lambda\hat{i}$

19.

(d) A is false but R is true.

Explanation: Let f(x) = $2x^3$ - 24x

$\Rightarrow$ f'(x) = $6x^2$ - 24 = 6(x^2 - 4)

= 6(x + 2) (x - 2)

For maxima or minima put f'(x) = 0.

$\Rightarrow$ 6(x + 2)(x - 2) = 0

$\Rightarrow$ x = 2, -2

We first consider the interval [1, 3].

So, we have to evaluate the value of f at the critical point x = 2 $\in$ [1, 3] and at the end points of [1, 3].

At x = 1, f(1) = 2 $\times$ 1^3 - 24 $\times$ 1 = -22

At x = 2, f(2) = 2 $\times$ 2^3 - 24 $\times$ 2 = -32

At x = 3, f(3) = 2 $\times$ 3^3 - 24 $\times$ 3 = -18

$\therefore$ The absolute maximum value of f(x) in the interval [1, 3] is -18 occurring at x = 3.

Hence, Assertion is false and Reason is true.

20.

(c) A is true but R is false.

Explanation: Assertion is true because for each element a $\in$ A , |a - a| = 0 < 3, so (1, 1) $\in R$,(2,2) $\in R$, (3, 3) $\in R$, (4,4) $\in R$ therefore R is reflexive.

Reason is false because a relation R on the set A is said to be transitive if for (a, b) $\in$R and (b, c) $\in$R, we have (a, c) $\in$ R

Section B

21. $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)=\tan^{-1}\sqrt{3}-\left[\pi-\sec^{-1}2\right]$

$=\frac{\pi}{3}-\pi+\cos^{-1}\left(\frac{1}{2}\right)$

$=-\frac{2\pi}{3}+\frac{\pi}{3}=-\frac{\pi}{3}$

OR

$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$
$=-\frac{\pi}{6}+\frac{\pi}{3}+\tan^{-1}(-1)$
$=-\frac{\pi}{6}+\frac{\pi}{3}-\frac{\pi}{4}$
$=-\frac{\pi}{12}$

22. Given: f(x) = (x + 1)($(x+2)^{\frac{1}{3}}$, x $\geq$ - 2

$\therefore$ f'(x) = $(x+2)^{\frac{1}{3}}+\frac{1}{3}$ (x + 1)$(x+2)^{\frac{-2}{3}}$

= $(x+2)^{\frac{-2}{3}}\left(x+2+\frac{1}{3}(x+1)\right)$

= $\frac{1}{3}(x+2)^{\frac{-2}{3}}$(4x + 7)

and, f "(x) = - $\frac{2}{9}(x+2)^{\frac{-5}{3}}$(4x + 7) + $\frac{1}{3}(x+2)^{\frac{-2}{3}}\times 4$

For maxima and minima, we must have

f'(x) = 0

$\Rightarrow\frac{1}{3}(x+2)^{\frac{-2}{3}}$ (4x + 7) = 0.

$\Rightarrow x=-\frac{7}{4}$

Now,

$f''\left(-\frac{7}{4}\right)=\frac{4}{3}\left(-\frac{7}{4}+2\right)^{\frac{-2}{3}}$

Which is positive.

Since the second order derivative at $-\frac{7}{4}$ is positive, therefore, $x=\frac{-7}{4}$ is point of minima

local min value = $f\left(\frac{-7}{4}\right)=\frac{-3}{4^{\frac{4}{3}}}$

23. Given:

$f(x)=\frac{1}{1+x^2}$

Let $x_1 > x_2$

$\Rightarrow\quad x_1^2>x_2^2$
$\Rightarrow\quad 1+x_1^2>1+x_2^2$
$\Rightarrow\quad \frac{1}{1+x_1^2}<\frac{1}{1+x_2^2}$
$\Rightarrow\quad f(x_1)<f(x_2)$

f(x) is decreasing on $[0,\infty)$

Case 2

$\Rightarrow\quad x_1^2<x_2^2$
$\Rightarrow\quad 1+x_1^2<1+x_2^2$
$\Rightarrow\quad \frac{1}{1+x_1^2}>\frac{1}{1+x_2^2}$
$\Rightarrow\quad f(x_1)>f(x_2)$

So, f(x) is increasing on $[0,\infty)$

Thus, f(x) is neither increasing nor decreasing on R.

OR

If s represents the surface area, then

$\frac{ds}{dt}=2cm^2/$ sec

Also, on using trigonometric ratios, radius of cone can be taken as

$r=l\sin\frac{\pi}{4}$

$s=\pi rl=\pi l.\sin\frac{\pi}{4}l=\frac{\pi}{\sqrt{2}}l^2$

Therefore, $\frac{ds}{dt}=\frac{2\pi}{\sqrt{2}}l.\frac{dl}{dt}=\sqrt{2}\pi l.\frac{dl}{dt}$

$\frac{dl}{dt}=\frac{1}{\sqrt{2}\pi.l}.\frac{ds}{dt}$

when $l=4cm$, $\frac{dl}{dt}=\frac{1}{\sqrt{2}\pi.4}.2=\frac{1}{2\sqrt{2}\pi}=\frac{\sqrt{2}}{4\pi}cm/s$

Thus, rate of decrease of slant height of water is $\frac{\sqrt{2}}{4\pi}cm/$ sec.

24. Let I = $\int \frac{\log x}{(1+\log x)^2}$ dx

Also let log x = t

Then, x = $e^t \Rightarrow$ dx = d(e^t) = e^t dt

$\therefore \quad I=\int \frac{te^t}{(t+1)^2}dt=\int \frac{(t+1)-1}{(t+1)^2}e^t dt$

$\Rightarrow I=\int \left\{\underset{f}{\frac{1}{t+1}}+\underset{f'}{\frac{-1}{(t+1)^2}}\right\}e^t dt$

$\Rightarrow I=\int \underset{I}{\frac{1}{t+1}}\underset{II}{e^t}dt+\int \frac{-1}{(t+1)^2}e^t dt$

$\Rightarrow I=\frac{1}{t+1}e^t-\int \frac{-1}{(t+1)^2}e^t dt+\int \frac{-1}{(t+1)^2}$ e^t dt + C

$\Rightarrow I=\frac{e^t}{t+1}+C=\frac{x}{(\log x+1)}$ + C

25. Given: f(x) = 6 + 12x + $3x^2$ - $2x^3$

f'(x) = 12 + 6x - $6x^2$

For critical points

f(x) =0

$\Rightarrow$6(2 + x - x^2)=0

$\Rightarrow$ (2 - x)(1 + x) = 0

$\Rightarrow$x = 2, -1

Clearly, f(x) > 0 if -1 < x < 2

f(x) < 0 if x < -1 and x > 2

thus, f(x) increases in (-1, 2) decreases in $(-\infty,-1)\cup(2,\infty)$

Section C

26. Let I = $\int \frac{1}{(2x^2+3)\sqrt{x^2-4}}dx$

Assume x = $\frac{1}{t} \Rightarrow$ dx = $-\frac{1}{t^2}$ dt

$\therefore I=-\int \frac{tdt}{(3t^2+2)(\sqrt{1-4t^2}}$

Assume 1 - $4t^2$ = $u^2$4tdt = udu

$\therefore I=-\frac{1}{4}\int \frac{udu}{\left(\frac{11-3u^2}{4}\right)u}$

$\Rightarrow I=-\frac{1}{3}\int \frac{du}{\left(\frac{11}{3}-u^2\right)}$

Using identity $\int \frac{dz}{(z)^2-1}=\frac{1}{2}\log\left|\frac{z-1}{z+1}\right|+c$, we get

$I=\frac{1}{2\sqrt{33}}\log\left|\frac{u-\sqrt{\frac{11}{3}}}{u+\sqrt{\frac{11}{3}}}\right|+c$

Substituting u = $\sqrt{1-4t^2}$ we get

$I=\frac{1}{2\sqrt{33}}\log\left|\frac{\sqrt{1-4t^2}-\sqrt{\frac{11}{3}}}{\sqrt{1-4t^2}+\sqrt{\frac{11}{3}}}\right|+c$

Substituting t = $\frac{1}{x}$, we get

$I=\frac{1}{2\sqrt{33}}\log\left|\frac{\sqrt{1-\frac{4}{x^2}}-\sqrt{\frac{11}{3}}}{\sqrt{1-\frac{4}{x^2}}+\sqrt{\frac{11}{3}}}\right|+c$

Hence the result

27. Let E_1, E_2 and E_3 denote the events of selecting Urn I, Urn II and Urn III, respectively.

Let A be the event that the two balls drawn are white. Therefore, we have,

P(E_1) = $\frac{20}{100}$

P(E_2) = $\frac{60}{100}$

P(E_3) = $\frac{20}{100}$

Now, we have,

$P(\frac{A}{E_1}) = \frac{^7C_2}{^{10}C_2} = \frac{21}{45}$

$P(\frac{A}{E_2}) = \frac{^4C_2}{^{10}C_2} = \frac{6}{45}$

$P(\frac{A}{E_3}) = \frac{^2C_2}{^{10}C_2} = \frac{1}{45}$

Using Bayes' theorem, we have,

Required probability = $P(\frac{E_3}{A})$ = $\frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)++P(E_3)P(A/E_3)}$

$= \frac{\frac{20}{100}\times\frac{1}{45}}{\frac{20}{100}\times\frac{21}{45}+\frac{60}{100}\times\frac{6}{45}+\frac{20}{100}\times\frac{1}{45}}$

$= \frac{1}{21+18+1} = \frac{1}{40}$

28. Let $I = \int_0^\pi \frac{1}{5+4\cos x}dx$. Then

$I = \int_0^\pi \frac{1}{5+4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx = \int_0^\pi \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+4\left(1-\tan^2\frac{x}{2}\right)}dx$

$\Rightarrow I = \int_0^\pi \frac{1+\tan^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx = \int_0^\pi \frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx$

By using substitution

Let $\tan\frac{x}{2} = t.$ Then, $d\left(\tan\frac{x}{2}\right) = dt \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt \Rightarrow dx = \frac{2dt}{\sec^2\frac{x}{2}}$

Also, $x = 0 \Rightarrow t = \tan 0 = 0$ and $x = \pi \Rightarrow t = \tan\frac{\pi}{2} = \infty$

$\therefore I = \int_0^\infty \frac{\sec^2\frac{x}{2}}{9+t^2} \times \frac{2dt}{\sec^2\frac{x}{2}}$

$\Rightarrow I = 2\int_0^\infty \frac{dt}{3^2+t^2} = \frac{2}{3}\left[\tan^{-1}\frac{t}{3}\right]_0^\infty = \frac{2}{3}\left(\tan^{-1}\infty - \tan^{-1}0\right) = \frac{2}{3}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{3}$

OR

The given function is,

$f(x) = \int_1^x \frac{\log_e t}{1+t}dt$ ...(i)

$\Rightarrow \quad f\left(\frac{1}{x}\right) = \int_1^{1/x} \frac{\log_e t}{1+t}dt$

Let $t = \frac{1}{u}$ Then, $dt = -\frac{1}{u^2}du$

Also, $t = 1 \Rightarrow u = 1$ and, $t = \frac{1}{x} \Rightarrow \frac{1}{u} = \frac{1}{x} \Rightarrow u = x$

$\therefore \quad f\left(\frac{1}{x}\right) = \int_1^x \frac{\log_e(1/u)}{1+\frac{1}{u}} \times \frac{-1}{u^2}du$

$\Rightarrow \quad f\left(\frac{1}{x}\right) = \int_1^x \frac{\log_e u}{(1+u)u}du$

$\Rightarrow \quad f\left(\frac{1}{x}\right) = \int_1^x \frac{\log_e t}{(1+t)t}dt$ ...(ii)

Adding (i) and (ii), we get

$f(x) + f\left(\frac{1}{x}\right) = \int_1^x \left\{\frac{\log_e t}{1+t} + \frac{\log_e t}{(1+t)t}\right\}dt$

$\Rightarrow \quad f(x) + f\left(\frac{1}{x}\right) = \int_1^x \frac{\log_e t}{1+t}\left(\frac{1+t}{t}\right)dt$

$\Rightarrow \quad f(x) + f\left(\frac{1}{x}\right) = \int_1^x \frac{\log_e t}{t}dt$

$\Rightarrow \quad f(x) + f\left(\frac{1}{x}\right) = \int_0^{\log_e x} v\,dv,$ where $v = \log_e t$ and $\frac{1}{t}dt = dv$

$\Rightarrow \quad f(x) + f\left(\frac{1}{x}\right) = \left[\frac{v^2}{2}\right]_0^{\log_e x} = \frac{1}{2}(\log_e x)^2 - 0 = \frac{1}{2}(\log_e x)^2$

Putting x = e, we get

$f(e) + f\left(\frac{1}{e}\right) = \frac{(\log_e e)^2}{2} = \frac{1}{2}$

29. The given differential equation is,

(x log x) $\frac{dy}{dx}$ + y = log x

$\frac{dy}{dx} + \frac{y}{x\log x} = \frac{1}{x}$

It is a linear differential equation. Comparing it with,

$\frac{dy}{dx}$ + Py = Q

P = $\frac{1}{x\log x}$, Q = $\frac{1}{x}$

I.F. $= e^{\int pdx}$

$= e^{\int \frac{1}{x\log x}dx}$

= $e^{\log|\log x|}$

= log x

Solution of the equation is given by,

y $\times$ (I.F.) = $\int$ Q $\times$ (I.F.) dx + c

y (log x) = $\int \frac{1}{x}$ (log x) dx + c

y (log x) = $\frac{(\log x)^2}{2}$ + c

y = $\frac{1}{2}$ log x + $\frac{c}{\log x}$, x > 0, x $\neq$ 1

OR

The given differential equation is,

$\tan x \frac{dy}{dx}$ = 2x tan x + x^2 - y

$\Rightarrow \frac{dy}{dx} + \frac{1}{\tan x} y = \frac{2x \tan x + x^2}{\tan x}$

$\Rightarrow \frac{dy}{dx}$ + (cot x) y = 2x + x^2 cot x

This is a linear differential equation of the form $\frac{dy}{dx}$ + Py = Q

I.F. $= e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

The solution of the given differential equation is given by

y $\times$ (I.F.) = $\int$ Q $\times$ (I.F.) dx + C

y $\times$ sin x = $\int$ (2x + x^2 + cot x) sin x dx + C

y sin x = $\int$ 2x sin x dx + $\int$ x^2 cos x dx + C

y sin x = $\int$ 2x sin x dx + [x^2 $\int$ cos x dx - $\int$ ($\frac{d}{dx}x^2$ $\times$ $\int$ cos x dx] + C

y sin x = $\int$ 2x sin x dx + x^2 sin x - $\int$ 2x sin x dx + C

y sin x = x^2 sin x + C

y = x^2 + cosec x $\times$ C...(i)

It is given that, y = 0 when x = $\frac{\pi}{2}$

$\therefore$ 0 = $\left(\frac{\pi}{2}\right)^2$ + cosec $\frac{\pi}{2}$ $\times$ C

C = $-\frac{\pi^2}{4}$

Putting C = $-\frac{\pi^2}{4}$ in (i), we get

y = $x^2 - \frac{\pi^2}{4}$ cosec x

Hence, y = $x^2 - \frac{\pi^2}{4}$ cosec x is the required solution.

30. First, we will convert the given inequations into equations, we obtain the following equations:

5x + y = 10, x + y = 6, x + 4y = 12, x = 0 and y = 0

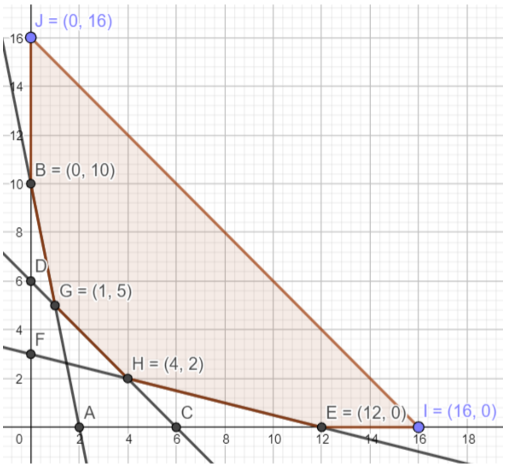
Region represented by 5x + y $\geq$ 10 :

The line 5x + y = 10 meets the coordinate axes at A(2,0) and B(0,10) respectively. By joining these points we obtain the line 5x + y = 10. Clearly (0,0) does not satisfies the inequation 5x + y $\geq$ 10 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation 5x + y $\geq$ 10 Region represented by x + y $\geq$ 6 :

The line x + y = 6 meets the coordinate axes at C(6,0) and D(0,6) respectively. By joining these points we obtain the line 2 x + 3 y = 30. Clearly (0,0) does not satisfies the inequation x + y $\geq$ 6 . So, the region which does not contain the origin represents the solution set of the inequation 2 x + 3 y $\geq$ 30 Region represented by x + 4y $\geq$ 12 The line x + 4y = 12 meets the coordinate axes at E(12,0) and F(0,3) respectively. By joining these points we obtain the line

x + 4y = 12 . Clearly (0,0) does not satisfies the inequation x + 4y $\geq$ 12 . So, the region which does not contain the origin represents the solution set of the inequation x + 4y $\geq$ 12 Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0. The feasible region determined by subject to the constraints are 5x + y $\geq$ 10, x + y $\geq$ 6, x + 4y $\geq$ 12, and the non-negative restrictions xx $\geq$ 0, and y $\geq$ 0, are as follows.



The corner points of the feasible region are B(0,10), G(1,5), H(4,2) and E(12,0)

The values of objective function Z at these corner points are as follows.

Corner point Z = 3x + 2y

B(0, 10) : $3 \times 0 + 3 \times 10 = 30$

G(1, 5) : $3 \times 1 + 2 \times 5 = 13$

H(4, 2) : $3 \times 4 + 2 \times 2 = 16$

B(12, 0) : $3 \times 12 + 2 \times 0 = 36$

Therefore, the minimum value of Z is 13 at the point G(1,5) . Hence, x = 1 and y = 5 is the optimal solution of the given LPP.

The optimal value of objective function Z is 13.

OR

From the shaded bounded region, it is clear that the coordinates of corner points are $\left(\frac{3}{13}, \frac{24}{13}\right)$, $\left(\frac{18}{7}, \frac{2}{7}\right)$, $\left(\frac{7}{2}, \frac{3}{4}\right)$ and $\left(\frac{3}{2}, \frac{15}{4}\right)$

Also, we have to determine maximum and minimum value of Z = x + 2y.

Corner Points	**Corresponding value of Z**
$\left(\frac{3}{13}, \frac{24}{13}\right)$	$\frac{3}{13} + \frac{48}{13} = \frac{51}{13} = 3\frac{12}{13}$
$\left(\frac{18}{7}, \frac{2}{7}\right)$	$\frac{18}{7} + \frac{4}{7} = \frac{22}{7} = 3\frac{1}{7}$ (Minimum)
$\left(\frac{7}{2}, \frac{3}{4}\right)$	$\frac{7}{2} + \frac{6}{4} = \frac{20}{4} = 5$
$\left(\frac{3}{2}, \frac{15}{4}\right)$	$\frac{3}{2} + \frac{30}{4} = \frac{36}{4} = 9$ (Maximum)

Hence, the maximum and minimum value of are 9 and $3\frac{1}{7}$ respectively.

31. We are given with, x = a$\left(\frac{1+t^2}{1-t^2}\right)$

$\Rightarrow \frac{dx}{dt} = a\left[\frac{(1-t^2)\frac{d}{dt}(1+t^2)-(1+t^2)\frac{d}{dt}(1-t^2)}{(1-t^2)^2}\right]$ [From quotient rule]

$\Rightarrow \frac{dx}{dt} = a\left[\frac{(1-t^2)(2t)-(1+t^2)(-2t)}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dx}{dt} = a\left[\frac{2t-2t^3+2t+2t^3}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dx}{dt} = \frac{4at}{(1-t^2)^2}$ ...(i)

and

y = $\frac{2t}{1-t^2}$

$\Rightarrow \frac{dy}{dt} = 2\left[\frac{(1-t^2)\frac{d}{dt}(t)-t\frac{d}{dt}(1-t^2)}{(1-t^2)^2}\right]$ [From quotient rule]

$\Rightarrow \frac{dy}{dt} = 2\left[\frac{(1-t^2)(1)-t(-2t)}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dy}{dt} = 2\left[\frac{1-t^2+2t^2}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dy}{dt} = \frac{2(1+t^2)}{(1-t^2)^2}$ ...(ii)

Dividing equation (ii) by (i),

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(1+t^2)}{(1-t^2)^2} \times \frac{(1-t^2)^2}{4at}$

$\Rightarrow \frac{dy}{dx} = \frac{(1+t^2)}{2at}$

Section D

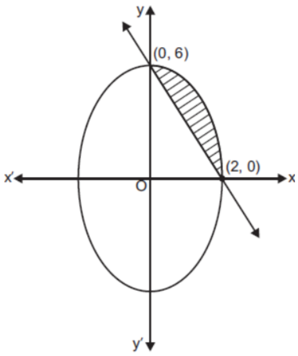
32. Given region is {(x, y): $9x^2 + 4y^2 \leq 36$ and $3x + y \geq 6$}

We draw the curves corresponding to equations

$9x^2 + 4y^2 = 36$ or $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and $3x + y = 6$

The curves intersect at (2, 0) and (0, 6)

Shaded area is the area enclosed by the two curves and is given as,



$= \int_0^2 \sqrt{9\left(1 - \frac{x^2}{4}\right) dx}$ - $\int_0^2 (6 - 3x)dx$

$= 3\left[\frac{x}{4}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} - 2x + \frac{x^2}{2}\right]_0^2$

$= 3\left[\frac{2}{4}\sqrt{4 - 4} + \frac{4}{2}\sin^{-1}\frac{2}{2} - 4 + \frac{4}{2} - 0\right]$

$= 3\left[2\frac{\pi}{2} - 2\right]$ = $3(\pi - 2)$ square units.

33. A = R - {3}, B = R - {1}

f : A $\rightarrow$ B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let x, y $\in$ A such that f(x) = f(y).

$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$

$\Rightarrow$ (x - 2) (y - 3) = (y - 2) (x - 3)

$\Rightarrow$ xy - 3x - 2y + 6 = xy - 3y - 2x + 6

$\Rightarrow$ -3x - 2y = -3y - 2x

$\Rightarrow$ 3x - 2x = 3y - 2y

$\Rightarrow$ x = y

Therefore, f is one-one.

Let y $\in$ B = R - {1}

Then, $y \neq 1$.

The function f is onto if there exists x $\in$ A such that f(x) = y.

Now, f (x) = y

$\Rightarrow \frac{x-2}{x-3} = y$

$\Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = -3y + 2

$\Rightarrow x = \frac{2-3y}{1-y} \in \mathbf{A} \qquad [y \neq 1]$

Thus, for any y $\in$ B, there exists $\frac{2-3y}{1-y} \in \mathrm{A}$ such that

$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$

$\therefore$ f is onto.

Hence, function f is one-one and onto.

OR

Here R is a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$

We shall show that R satisfies the following properties

i. Reflexivity:

We know that a + b = b + a for all a, b $\in$ N.

$\therefore$ (a, b) R (a, b) for all (a, b) $\in (N \times N)$

So, R is reflexive.

ii. Symmetry:

Let (a, b) R (c, d). Then,

(a, b) R (c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R (a, b).

$\therefore$ (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in N \times N$

This shows that R is symmetric.

iii. Transitivity:

Let (a, b) R (c, d) and (c, d) R (e, f). Then,

(a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow$ a + d = b + c and c + f = d + e

$\Rightarrow$ a + d + c + f = b + c + d + e

$\Rightarrow$ a + f = b + e

$\Rightarrow$ (a, b) R (e, f).

Thus, (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f)

This shows that R is transitive.

$\therefore$ R is reflexive, symmetric and transitive

Hence, R is an equivalence relation on $N \times N$

34. Given: Matrix $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} = 15 - 14 = 1 \neq 0$

$\therefore A^{-1} = \frac{1}{|A|} adj.\, A = \frac{1}{1} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

Matrix $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$\therefore |B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} = 54 - 56 = -2 \neq 0$

$\therefore B^{-1} = \frac{1}{|B|} adj.\, B = \frac{1}{-2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

Now $AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

$\therefore |AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix}$ = 67(61) - 87(47) = 4087 - 4089 $= -2 \neq 0$

Now L.H.S. = $(AB)^{-1} = \frac{1}{|AB|} adj.\,(AB) = \frac{1}{-2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$ ....(i)

R.H.S. = $B^{-1}A^{-1} = \frac{1}{-2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{-2} \begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix}$

$= \frac{1}{-2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$ ....(ii)

$\therefore$ From eq. (i) and (ii), we get

L.H.S. = R.H.S.

$\Rightarrow (AB)^{-1} = B^{-1}A^{-1}$

35. Here,it is given that

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

Here,

$\overrightarrow{a_1} = i + 2\hat{j} + 3\hat{k}$
$\overrightarrow{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$
$\overrightarrow{a_2} = 4\hat{i} + \hat{j}$
$\overrightarrow{b_2} = 5\hat{i} + 2\hat{j} + \hat{k}$
Thus,

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(3-8) - \hat{j}(2-20) + \hat{k}(4-15)$
$\therefore \overrightarrow{b_1} \times \overrightarrow{b_2} = -5\hat{i} + 18\hat{j} - 11\hat{k}$
$\therefore |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$
$= \sqrt{25 + 324 + 121}$
$= \sqrt{470}$
$\overrightarrow{a_2} - \overrightarrow{a_1} = (4-1)\hat{i} + (1-2)\hat{j} + (0-3)\hat{k}$
$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 3\hat{i} - \hat{j} - 3\hat{k}$
Now,we have
$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (-5\hat{i} + 18\hat{j} - 11\hat{k}) \cdot (3\hat{i} - \hat{j} - 3\hat{k})$
$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$
= -15 - 18 + 33
= 0
Thus, the distance between the given lines is

$$d = \left| \frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|} \right|$$

$$\therefore d = \left| \frac{0}{\sqrt{470}} \right|$$

$\therefore$ d = 0 units
As d = 0
Thus, the given lines intersect each other.
Now, to find a point of intersection, let us convert given vector equations into Cartesian equations.
For that putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,
$\Rightarrow \vec{L}_1 : x\hat{i} + y\hat{j} + z\hat{k} = (i + 2j + 3\hat{k}) + \lambda(2i + 3\hat{j} + 4\hat{k})$
$\Rightarrow \vec{L}_2 : x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$
$\Rightarrow \vec{L}_1 : (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$
$\Rightarrow \vec{L}_2 : (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$
$\Rightarrow \vec{L}_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$
$\therefore \vec{L}_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$
General point on L1 is
$x_1 = 2\lambda + 1$, $y_1 = 3\lambda + 2$, $z_1 = 4\lambda + 3$
Suppose, P(x_1, y_1, z_1) be point of intersection of two given lines.
Thus, point P satisfies the equation of line $\vec{L}_2$.
$\Rightarrow \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = \frac{4\lambda+3-0}{1}$
$\therefore \frac{2\lambda-3}{5} = \frac{3\lambda+1}{2}$
$\Rightarrow 4\lambda - 6 = 15\lambda + 5$
$\Rightarrow 11\lambda = -11$
$\Rightarrow \lambda = -1$
Thus, x_1 = 2(-1) + 1, y_1 = 3(-1) + 2, z_1 = 4(-1) + 3
$\Rightarrow x_1 = -1$, $y_1 = -1$, $z_1 = -1$
Therefore, point of intersection of given lines is (-1, -1, -1).

OR

Given Cartesian equations of lines

$L_1 = \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$

Line L1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)

Thus, vector equation of line L1 is

$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$

And

$L_2 : \frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$

Line L_2 is passing through point (2, -1, 1) and has direction ratios (2, 3, -2)

Thus, vector equation of line L2 is

$\vec{\mathbf{r}} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + \mu(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Now, to calculate distance between the lines,

$\vec{r} = (\hat{\imath} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$

$\vec{\mathbf{r}} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + \mu(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Here,we have

$\overrightarrow{a_1} = 1 - j + \hat{k}$

$\overrightarrow{b_1} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

$\overrightarrow{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\overrightarrow{b_2} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

Thus,

$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 2 & 5 \\ 2 & 3 & -2 \end{vmatrix}$

$= \hat{\imath}(-4 - 15) - \hat{\jmath}(-6 - 10) + \hat{k}(9 - 4)$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -19\hat{i} + 16\hat{j} + 5\hat{k}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-19)^2 + 16^2 + 5^2}$

$= \sqrt{361 + 256 + 25}$

$= \sqrt{642}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = (2 - 1)\hat{\imath} + (1 + 1)\hat{\jmath} + (-1 - 1)\hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 1 + 2\hat{\jmath} - 2\hat{k}$

Now,

$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (-19\hat{i} + 16\hat{\jmath} + 5\hat{k}) \cdot (\hat{i} + 2\hat{j} - 2\hat{k})$

$= ((-19) \times 1) + (16 \times 2) + (5 \times (-2))$

= -19 + 32 - 10

= 3

Thus, the shortest distance between the given lines is

$d = \left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

$\Rightarrow d = \left|\frac{3}{\sqrt{642}}\right|$

$\therefore d = \frac{3}{\sqrt{642}}$ units

As $d \neq 0$

Hence, given lines do not intersect each other.

Section E

36. **Read the text carefully and answer the questions:**

In a bilateral cricket series between India and South Africa, the probability that India wins the first match is 0.6. If India wins any match, then the probability that it wins the next match is 0.4, otherwise, the probability is 0.3. Also, it is given that there is no tie in any match.

(i) It is given that if India loose any match, then the probability that it wins the next match is 0.3.

$\therefore$ Required probability = 0.3

(ii) It is given that, if India loose any match, then the probability that it wins the next match is 0.3.

$\therefore$ Required probability = 1 - 0.3 = 0.7

(iii) Required probability = P(India losing first match) $\cdot$ P(India losing second match when India has already lost first match)

= 0.4 $\times$ 0.7 = 0.28

OR

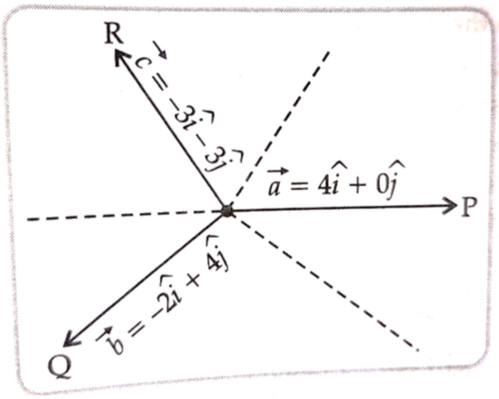
Required probability = P(India winning first match) $\cdot$ P(India winning second match if India has already won first match) $\cdot$ P(India winning third match if India has already won first two matches) = 0.6 $\times$ 0.4 $\times$ 0.4 = 0.096

37. **Read the text carefully and answer the questions:**

Team P, Q, R went for playing a tug of war game. Teams P, Q, R have attached a rope to a metal ring and is trying to pull the ring into their own areas (team areas when in the given figure below). Team P pulls with force $F_1 = 4\hat{i} + 0\hat{j}$ KN

Team Q pull with force $F_2 = -2\hat{i} + 4\hat{j}$ KN

Team R pulls with force $F_3 = -3\hat{i} - 3\hat{j}$ KN



(i) Let F be the combined force,

$\therefore \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

= $(4\hat{i} + 0\hat{j}) + (-2\hat{i} + 4\hat{j}) + (-3\hat{i} - 3\hat{j})$

= $(4 - 2 - 3)\hat{i} + (0 + 4 - 3)\hat{j}$

= $-\hat{i} + \hat{j}$

$\therefore \vec{F}| = \left|\sqrt{(-1)^2 + 1^2}\right|$

= $|\sqrt{2}|$ KN

(ii) Magnitude of force of Team B =

$\left|\vec{F}_2\right| = \left|\sqrt{(-2)^2 + 4^2}\right| = \sqrt{20}$ KN

= $2\sqrt{5}$ KN

(iii) We have,

$\left|\vec{F}_1\right| = \left|\sqrt{(4)^2 + 0^2}\right|$ = 4 KN

$\left|\vec{F}_2\right| = \left|\sqrt{(-2)^2 + 4^2}\right|$ = $\sqrt{20}$ KN

and $\left|\vec{F}_3\right| = \left|\sqrt{(-3)^2 + (-3)^2}\right|$ = $\sqrt{18}$ KN

Here, magnitude of force F_2 is greater, therefore team Q will win the game.

OR

We have,

Combined force, $\vec{F} = -\hat{i} + \hat{j}$

$\therefore \theta$ $\tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{1}{-1}\right)$

$= \tan^{-1}(1)$

$= \tan^{-1}\left(\tan \frac{3\pi}{4}\right)$

$= \frac{3\pi}{4}$ radians

38. **Read the text carefully and answer the questions:**

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where f(x) is the temperature in ^{o}F at x days.

(i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12).

(ii) $f(x) = -0.2x + m$

At Critical point

$0 = -0.2 \times 6 + m$

$m = 1.2$