CBSE SAMPLE PAPER - 02

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If the length of the perpendicular from the point (β, 0, β) ($\beta \neq 0$) to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to: **[1]**

    a) -2 b) 2

    c) 1 d) -1

2. $\int$ cosec xdx=? **[1]**

    a) log |cosec x + cot x| + C b) none of these

    c) - log |cosec x - cot x| + C d) log |cosec x - cot x| + C

3. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is **[1]**

    a) -1 b) 1

    c) 0 d) 3

4. The area of the region bounded by the curve y = $\sqrt{16 - x^2}$ and x-axis is **[1]**

    a) 16π sq units b) 8πsq units

    c) 20π sq units d) 256π sq units

5. Area enclosed between by the curve y^2 (2a - x) = x^3 and the line x = 2a above x-axis is **[1]**

    a) $\frac{3}{2}\pi a^2$ b) πa^2

    c) $3\pi a^2$ d) $2\pi a^2$

6. India play two matches each with West Indies and Australia. In any match the probabilities of India getting 0,1 and 2 points are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of **[1]**

India getting at least 7 points is

a) 0.0875

b) 0.1125

c) none of these

d) $\frac{1}{16}$

7. If two events are independent, then **[1]**

a) None of these

b) they must be mutually exclusive

c) they must be mutually exclusive and the sum of their probabilities must be equal to 1 both are correct

d) the sum of their probabilities must be equal to 1

8. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).Let F = 4x + 6y be the objective function. The Minimum value of F occurs at **[1]**

a) (0, 2) only

b) (3, 0) only

c) any point on the line segment joining the points (0, 2) and (3, 0).

d) the mid – point of the line segment joining the points (0, 2) and (3, 0) only

9. Find $|\vec{a}|$ and $\left|\vec{b}\right|$, if $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=8$ and $|\vec{a}|=8\left|\vec{b}\right|$. **[1]**

a) $\frac{16\sqrt{2}}{3\sqrt{7}},\frac{2\sqrt{2}}{3\sqrt{7}}$

b) $\frac{19\sqrt{2}}{3\sqrt{7}},\frac{2\sqrt{5}}{3\sqrt{7}}$

c) $\frac{17\sqrt{2}}{3\sqrt{7}},\frac{2\sqrt{3}}{3\sqrt{7}}$

d) $\frac{21\sqrt{2}}{3\sqrt{7}},\frac{2\sqrt{6}}{3\sqrt{7}}$

10. Consider a differential equation of order m and degree n. Which one of the following pairs is not feasible? **[1]**

a) (2, $\frac{3}{2}$)

b) (2, 4)

c) (3, 2)

d) (2, 2)

11. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2-\left(\frac{dy}{dx}\right)=y^3$, is **[1]**

a) 4

b) $\frac{1}{2}$

c) 2

d) 3

12. $\int_{\pi/4}^{\pi/2}\cot x dx=?$ **[1]**

a) 2 log 2

b) $\frac{1}{2}\log 2$

c) None of these

d) log 2

13. Find the particular solution for (x + y)dy + (x –y) dx =0; y=1 when x =1 **[1]**

a) $\log\left(x^2-\ y^3\right)-\ 2\tan^{-1}\frac{y}{x}\ =\ \frac{\pi}{2}-\log 2$

b) $\log\left(x^2+\ y^2\right)-\ 2\tan^{-1}\frac{y}{x}\ =\ \frac{\pi}{2}+\log 2$

c) $\log\left(x^2-\ y^2\right)+\ 2\tan^{-1}\frac{y}{x}\ =\ \frac{\pi}{2}+\log 2$

d) $log|x^2+y^2|+2tan^{-1}\frac{y}{x}=log2+\frac{\pi}{2}$

14. If y = $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ then $\frac{dy}{dx}=?$ **[1]**

a) 1

b) $\frac{1}{2}$

c) -1

d) $\frac{-1}{2}$

15. If A is an invertible matrix of any order, then which of the following options is NOT true? **[1]**

a) $(A^2)^{-1}=(A^{-1})^2$

b) $|A^{-1}|=|A|^{-1}$

c) $(A^T)^{-1}=(A^{-1})^T$

d) $|A|\neq 0$

16. The solution of $x\frac{dy}{dx}$ + y = e^x is: **[1]**

a) $x = \frac{e^y}{y} + \frac{k}{y}$

b) $y = \frac{e^x}{x} + \frac{k}{x}$

c) y = xe^x + k

d) y = xe^x + cx

17. $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$ is equal to **[1]**

a) $\frac{2\pi}{3}$

b) $\frac{-\pi}{3}$

c) None of these

d) $\frac{\pi}{3}$

18. Find the equation of the line in cartesian form that passes through the point with position vector$2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$. **[1]**

a) $\frac{x-5}{1} = \frac{y+1}{2} = \frac{z-5}{-1}$

b) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$

c) $\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-3}{-1}$

d) $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{-1}$

19. **Assertion (A):** Consider the linear programming problem. Maximise Z = 4x + y Subject to constraints x + y $\leq$ 50, x + y $\geq$ 100, and x, y $\geq$ 0. Then, maximum value of Z is 50. **[1]**

Reason (R): If the shaded region is not bounded then maximum value cannot be determined.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The function defined by f(x) = cos(x^2) is a continuous function. **[1]**

Reason (R): The sine function is continuous in its domain i.e. x $\in$ R.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Solve $\cos\left(\frac{dy}{dx}\right) = a$, y = 1, when x = 0 **[2]**

22. Find $\frac{dy}{dx}$, when y = e^{ax} cos (bx + c) **[2]**

23. Find the shortest distance between the two lines whose vector equations are **[2]**

$\overrightarrow{\mathrm{r}} = (6\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 2\hat{\mathrm{k}}) + \lambda(\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + 2\hat{\mathrm{k}})$ and $\vec{r} = (-4\hat{i} - \hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

OR

Find the vector equation of a line passing through the point having the position vector $(\hat{i} + 2\hat{j} - 3\hat{k})$ and parallel to the line joining the points with position vectors $(\hat{i} - \hat{j} + 5\hat{k})$ and $(2\hat{i} + 3\hat{j} - 4\hat{k})$. Also, find the Cartesian equivalents of this equation.

24. If P(A) = 0.6, P(B) = 0.5 and P(B|A) = 0.4, find P(A $\cup$ B) and P(A|B). **[2]**

25. Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$ **[2]**

Section C

26. Solve the following linear programming problem graphically: **[3]**

Minimize z = 6x + 3y

Subject to the constraints:

4x + y $\geq$ 80

x + 5y $\geq$115

$3x + 2y \leq 150$

$x > 0, y \geq 0$

27. Using integration, find the area of the region bounded by the curves $y = x^2$ and $y = x$. **[3]**

OR

Find the area of the region bounded by the curve $y = x^3$ and $y = x + 6$ and $x = 0$.

28. Evaluate: $\int \frac{(x^2+1)}{(x-1)^2(x+3)} dx$. **[3]**

OR

Evaluate: $\int e^{2x} \cos (3x + 4)\, dx$

29. Find the angle between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$. **[3]**

OR

Find the foot of the perpendicular drawn from the point $2\hat{i} - \hat{j} + 5\hat{k}$ to the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. Also, find the length of the perpendicular.

30. Find the area of the region bounded by the parabola $y^2 = 4x$, the x-axis, and the lines $x = 1$ and $x = 4$. **[3]**

31. If $\log \sqrt{x^2 + y^2} = \tan^{-1}\left(\frac{y}{x}\right)$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$ **[3]**

Section D

32. Write a vector of magnitude 12 units which makes 45° angle with X-axis, 60° angle with Y-axis and an obtuse angle with Z-axis. **[5]**

33. Solve the system of equation: **[5]**

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$

$\frac{6}{x} + \frac{9}{y} + \frac{-20}{z} = 2$

OR

Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of Rs.x, Rs.y, and Rs.z, respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with total prize money of Rs.37000 and the second institution decided to award respectively 5, 3 and 4 employees with total prize money of, Rs.47000. If all the three prizes per person together amount to Rs.12000, then using a matrix method, find the values of x, y, and z. What values are described in this question?

34. Let A = R – {3} and B = R – {1}. Consider the function f: A ⇒B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer. **[5]**

OR

Show that the function f : R - {3} → R - {1} given by $f(x) = \frac{x-2}{x-3}$ is a bijection.

35. Evaluate: $\int \frac{x^2}{(x^2+6x-3)} dx$ **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.

(i) If l(x) denotes the combined light intensity, then find the value of x so that I(x) is minimum.

(ii) Find the darkest spot between the two lights.

(iii) If you are in between the lamp posts, at distance x feet from the stronger light, then write the combined light intensity coming from both lamp posts as function of x.

OR

Find the minimum combined light intensity?

37. **Read the text carefully and answer the questions:** **[4]**

Consider 2 families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommended daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children.

(i) Represent the requirement of calories and proteins for each person in matrix form.

(ii) Find the requirement of calories of family A and requirement of proteins of family B.

(iii) Represent the requirement of calories and proteins If each person increases the protein intake by 5% and decrease the calories by 5% in matrix form.

OR

If A and B are two matrices such that AB = B and BA = A, then find $A^2 + B^2$ in terms of A and B.

38. **Read the text carefully and answer the questions:** **[4]**

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.

(i) Teacher ask Govind, what is the probability that tickets are drawn by Abhishek, shows a prime number on one ticket and a multiple of 4 on other ticket?

(ii) Teacher ask Girish, what is the probability that tickets drawn by Ankit, shows an even number on first ticket and an odd number on second ticket?

Solution

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Class 12 - Mathematics

Section A

1. **(d)** -1

Explanation: Given, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ = p(let) and point P(β, 0, β)

Anypoint on line A = (p, 1, - p - 1)

Now, DR of AP (p - β, 1 - 0, - p - 1 - β)

Which is perpendicular to line.

$\therefore$ (p - β)1 + 0·1 - 1(- p - 1 - β) = 0

$\Rightarrow$ p - β + p + 1 + β = 0 $\Rightarrow$ p = $\frac{-1}{2}$

$\therefore$ Point A $\left(\frac{-1}{2}, 1-\frac{1}{2}\right)$

Given that distance AP = $\sqrt{\frac{3}{2}} \Rightarrow AP^2=\frac{3}{2}$

$\Rightarrow\left(\beta+\frac{1}{2}\right)^2+1+\left(\beta+\frac{1}{2}\right)^2=\frac{3}{2}$ or $2\left(\beta+\frac{1}{2}\right)^2=\frac{1}{2}$

$\Rightarrow\left(\beta+\frac{1}{2}\right)^2=\frac{1}{4} \Rightarrow \beta$ = 0, -1, ($\beta \neq$ 0)

$\therefore \beta$ = -1

2. **(d)** log |cosec x - cot x| + C

Explanation: Formula :- $\int x^n dx=\frac{x^{n+1}}{n+1}+c$

sin (α+b)=sin α cos b+cos α sin b

$\int$ cot x=log (sin x)+c

Therefore ,

$\int cosecx \frac{\csc x-\cot x}{\csc x-\cot x} dx$

$\int \frac{cosec^2 x-\csc x \cot x}{cosecx-\cot x} dx$

Put cosecx - cot x = t (cosec^2 x-cosec x cot x)dx = dt

$\Rightarrow \int \frac{dt}{t}$

= log t + c

= log |cosec x - cot x| + c

3. **(b)** 1

Explanation: Given: $(\hat{j} \times \hat{k})+\hat{j} \cdot(\hat{i} \times \hat{k})+\hat{k}(\hat{i} \times \hat{j})$

i. $(\hat{j} \times \hat{k})+\hat{j} \cdot(\hat{i} \times \hat{k})+\hat{k} \cdot(\hat{i} \times \hat{j})$ = $\hat{\mathrm{i}} . \hat{\mathrm{I}}+\hat{\mathrm{j}} \cdot(-\hat{\mathrm{j}})+\hat{\mathrm{k}} \hat{\mathrm{k}}$

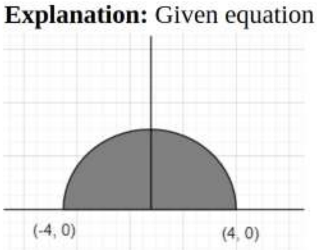
= $1-\hat{j} . \hat{j}+1$

=1 - 1 + 1

=1

4. **(b)** 8πsq units

Explanation: Given equation of curve is $y=\sqrt{16-x^2}$ and the equation of line is x - axis is



$\therefore \sqrt{16-x^2}=0$ .....(i)

$\Rightarrow$ 16 - x^2 = 0

$\Rightarrow$ x^2 = 16

$\Rightarrow$ x= $\pm$4

So, the intersections points are (4, 0) and (-4, 0).

$\therefore$ Area of the curve, A = $\int_{-4}^4\left(16-x^2\right)^{12} dx$

$= \int_{-4}^{4} \sqrt{(4^2 - x^2)} dx$

$= \left[\frac{x}{2}\sqrt{4^2 - x^2} + \frac{4^2}{2}\sin^{-1}\frac{x}{4}\right]_{-4}^{4}$

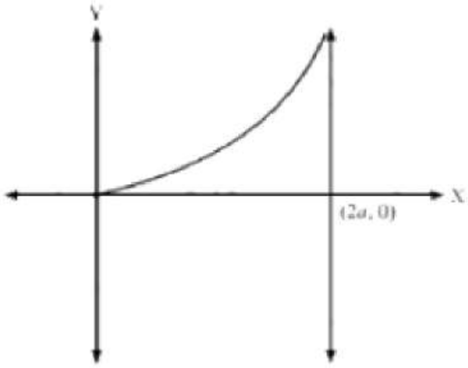
$= \left[\frac{4}{2}\sqrt{4^2 - 4^2} + 8\sin^{-1}\frac{4}{4}\right]$ - $\left[-\frac{4}{2}\sqrt{4^2 - (-4)^2} + 8\sin^{-1}\left(-\frac{4}{4}\right)\right]$

$= \left[2 \cdot 0 + 8 \cdot \frac{\pi}{2} - 0 + 8 \cdot \frac{\pi}{2}\right]$ = 8πsq. units

Which is the required solution.

5. **(a)** $\frac{3}{2}\pi a^2$

Explanation:



y^2 (2a - x) = x^3

$y = \sqrt{\frac{x^3}{2a-x}}$

Let x = $2a\sin^2\theta$

$\mathrm{dx} = 4\mathrm{a}\sin\theta\cos\theta\mathrm{d}\theta$

Area $= \int_0^{2a} \sqrt{\frac{x^3}{2a-x}} dx$

$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{(8a^3)\sin^6\theta}{(2a)\cos^2\theta}} \cdot (4a)\sin\theta\cos\theta d\theta$

$= 8a^2 \int_0^{\frac{x}{2}} \sqrt{\sin^6\theta}\sin\theta d\theta$

$= 8a^2 \left[\int_0^{\frac{\pi}{2}} \sin^4\theta d\theta\right]$

$= 8a^2 \left[\int_0^{\frac{\pi}{2}} \sin^2\theta\left(1 - \cos^2\theta\right) d\theta\right]$

$= 8a^2 \left[\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2\theta}{2}\right) d\theta - \frac{1}{4}\int_0^{\frac{\pi}{2}} \sin^2 2\theta d\theta\right]$

$= 8a^2 \left[\frac{1}{2}[\theta]_0^{\frac{x}{2}} - \left[\frac{\sin 2\theta}{4}\right]_0^{\frac{x}{2}}\right] - \frac{1}{4}\left[\int_0^{\frac{\pi}{2}} \frac{1-\cos 4\theta}{2} d\theta\right]$

$= 8a^2 \left[\left(\frac{\pi}{4}\right) - 0\right] - \frac{1}{4}\left[\frac{\pi}{4} - 0\right]$

$= 8a^2 \left[\frac{\pi}{4} - \frac{\pi}{16}\right] = \frac{3}{2}\pi a^2$

6. **(a)** 0.0875

Explanation: Here, there are total 5 ways by which India can get at least 7 points.

2 points + 2 points + 2 points + 2 points =$(0.5 \times 0.5 \times 0.5 \times 0.5)$

1 point + 2 points + 2 points + 2 points = $(0.05 \times 0.5 \times 0.5 \times 0.5)$

2 points + 1 point + 2 points + 2 points $=$ $(0.5 \times 0.05 \times 0.5 \times 0.5)$

2 points + 2 points + 1 point + 2 points =$(0.5 \times 0.5 \times 0.05 \times 0.5)$

P(atleast 7 points) = $0.5 \times 0.5 \times 0.5 \times 0.5 + 4[0.05 \times 0.5 \times 0.5 \times 0.5]$

= 0.0625 + 4(0.00625)

= 0.0625 + 0.025

= 0.0875

7. **(a)** None of these

Explanation: If two events A and B are independent, then we know that

$P(A \cap B) = P(A) \cdot P(B), P(A) \neq 0, P(B) \neq 0$

Since, A and B have a common outcome.

Further, mutually exclusive events never have a common outcome.

In other words, two independents events having non-zero probabilities of occurrence cannot be mutually exclusive and conversely, i.e., two mutually exclusive events having non-zero probabilities of outcome cannot be independent.

8. **(c)** any point on the line segment joining the points (0, 2) and (3, 0).

Explanation: Here the objective function is given by : F = 4x +6y .

Corner points	Z = 4x +6 y
(0, 2)	12........(Min.)
(3,0)	12........(Min.)
(6,0)	24
(6 , 8)	72........(Max.)
(0 , 5)	30

Hence it is clear that the minimum value occurs at any point on the line joining the points (0,2) and (3,0)

9. **(a)** $\frac{16\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{2}}{3\sqrt{7}}$

Explanation: $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 8$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8 \Rightarrow 63|\vec{b}|^2 = 8 \Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}} = \frac{2\sqrt{2}}{3\sqrt{7}}$

$\Rightarrow |\vec{a}| = 8|\vec{b}| \Rightarrow |\vec{a}| = \frac{16\sqrt{2}}{3\sqrt{7}}$

10. **(a)** $(2, \frac{3}{2})$

Explanation: The pairs $\left(2, \frac{3}{2}\right)$ is not feasible. Because the degree of any differential equation cannot be rational type. If so, then we use rationalization and convert it into an integer.

11. **(c)** 2

Explanation: We know that,

The degree is the power of the highest order derivative.

The highest order is 2 and its power is 2.

Hence, the degree of a differential equation is 2.

12. **(b)** $\frac{1}{2}\log 2$

Explanation: I = $(\text{In}(\sin x))_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \ln\left(\sin \frac{\pi}{2}\right) - \ln\left(\sin \frac{\pi}{4}\right)$

$= \ln 1 - \ln \frac{1}{\sqrt{2}}$

$= \frac{1}{2}\ln 2$

13. **(d)** $log|x^2 + y^2| + 2tan^{-1}\frac{y}{x} = log2 + \frac{\pi}{2}$

Explanation: $\frac{dy}{dx} = \frac{y-x}{x+y}$ it is a homogenous equation Hence we put y=vx

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} let\ y = vx$

$\frac{dy}{dx} = v + x\frac{\text{dv}}{\text{dx}}$

$v + x\frac{\text{dv}}{\text{dx}} = \frac{vx-x}{vx+x}$

$x\frac{dv}{dx} = \frac{v-1}{v+1} - v$

$x\frac{dv}{dx} = \frac{v-1-v^2-v}{v+1}$

$\int \frac{(v+1)dv}{1+v^2} = -\int \frac{dx}{x}$

$\int \frac{vdv}{1+v^2} + \int \frac{dv}{1+v^2} = -\int \frac{dx}{x}$

$\frac{1}{2}log|1+v^2|+tan^{-1}v=-logx+c$
$log|1+v^2|+2tan^{-1}v+2logx=c$
Resubstituting v=y/x we get
$log|\frac{x^2+y^2}{x^2}|+2tan^{-1}\frac{y}{x}+2logx=c$
When x=y=1 we get,
$log|\frac{1^2+1^2}{1^2}|+2tan^{-1}\frac{1}{1}+log1=c$
$c=log2+\frac{\pi}{2}$
$log|\frac{x^2+y^2}{x^2}|+2tan^{-1}\frac{y}{x}+2logx=log2+\frac{\pi}{2}$
$log|x^2+y^2|+2tan^{-1}\frac{y}{x}=log2+\frac{\pi}{2}$

14. **(b)** $\frac{1}{2}$
Explanation: Given that y = $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$
Using 1 - cos x = 2sin^2 $\frac{x}{2}$ and Using sin x = 2 sin x $\frac{x}{2}$cos $\frac{x}{2}$, we obtain
y = $\tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$ or y = $\tan^{-1}\tan\frac{x}{2}$
$y=\frac{x}{2}$
Differentiating with respect to x, we obtain
$\frac{dy}{dx}=\frac{1}{2}$

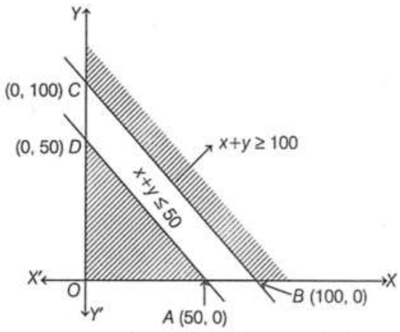
15. **(b)** $|A^{-1}| = |A|^{-1}$
Explanation: Since the determinant value of matrix and its reciprocal is same as the determinant value of an invertible matrix

16. **(b)** $y=\frac{e^x}{x}+\frac{k}{x}$
Explanation: We have, $x\frac{dy}{dx}+y=e^x$
$\Rightarrow \frac{dy}{dx}+\frac{y}{x}=\frac{e^x}{x}$
This is a linear differential equation.
On comparing it with $\frac{dy}{dx}+Py=Q$, we get
P = $\frac{1}{x}$ and Q = $\frac{e^x}{x}$
$\therefore$ IF $=e^{\int\frac{1}{x}dx}=e^{(\log x)}=x$
So, the general solution is:
$y\cdot x=\int\frac{e^x}{x}xdx$
$\Rightarrow$ y·x = $\int e^x dx$
$\Rightarrow$ y ·x = e^x + k
$\Rightarrow y=\frac{e^x}{x}+\frac{k}{x}$

17. **(d)** $\frac{\pi}{3}$
Explanation: $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$=$\cos^{-1}(\cos\frac{\pi}{3})=\frac{\pi}{3}$, because, $\cos\theta$ is positive in fourth quadrant.

18. **(b)** $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{-1}$
Explanation: We have ,
$\overrightarrow{a}=2\hat{i}-\hat{j}+4\hat{k}$ and $\overrightarrow{b}=\hat{i}+2\hat{j}-\hat{k}$, then, its Cartesian equation is given by :
$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$
Here, $x_1=2, x_2=-1, x_3=4$ And l = 1 , m = 2 and n = -1 .
Therefore , $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{-1}$

19. **(d)** A is false but R is true.
Explanation: Assertion: Given, maximise, Z = 4x + y and x + y $\leq$ 50, x + y $\geq$ 100; x, y $\geq$ 0



Hence, it is clear from the graph that it is not bounded region. So, maximum value cannot be determined. Hence Assertion is not true but Reason is true.

20. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion We have, f(x) = cos (x^2)

At x = c,

LHL = $\lim_{h\to0}$ cos $(c - h)^2$ = cos c^2

RHL = $\lim_{h\to0}$ cos $(c + h)^2$ = cos c^2

and f(c) = cos c^2

$\therefore$ LHL = RHL = f(c)

So, f(x) is continuous at x = c.

Hence, f(x) is continuous for every value of x.

Hence, both Assertion and Reason are true and Reason is not the correct explanation of Assertion.

Section B

21. $\cos\left(\frac{dy}{dx}\right) = a$

$\frac{dy}{dx} = \cos^{-1}a$

$\int dy = \int \cos^{-1}a dx$

y = $\cos^{-1}$a.x + c

$1 = 0 + c \begin{bmatrix} \because y = 1 \\ x = 0 \end{bmatrix}$

c = 1

y = $\cos^{-1}$a.x + 1

22. Given, y = e^{ax} cos (bx + c)

Using product rule, we get

$\frac{dy}{dx} = e^{ax} \times \frac{d}{dx}(\cos(bx+c)) + \cos(bx+c) \times \frac{d}{dx}(e^{ax})$

$\Rightarrow \quad \frac{dy}{dx} = e^{ax}x - \sin(bx+c) \times \frac{d}{dx}(bx+c) + \cos(bx+c) \times e^{ax} \times \frac{d}{dx}(ax)\}$ [USING CHAIN RULE]

$\Rightarrow \quad \frac{dy}{dx} = e^{ax} \times \{-\sin(bx+c)\} \times b + \cos(bx+c) \times e^{ax} \times a$

$\Rightarrow \quad \frac{dy}{dx} = e^{ax}\{-b\sin(bx+c) + a\cos(bx+c)\}$

23. Shortest distance between lines with vector equations

$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is $\left|\frac{(\overrightarrow{b_1}\times\overrightarrow{b_2})\cdot(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$

$\vec{r} = (6\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) + \lambda(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$

Comparing with $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$,

$\vec{a}_1 = 6\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b}_1 = 1\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$\vec{r} = (-4\hat{\imath} - \hat{k}) + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$

comparing with $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$,

$\vec{a}_2 = -4\hat{\imath} + 0\hat{\jmath} - 1\hat{k}$ and $\vec{b}_2 = 3\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

Now,

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (-4\hat{\imath} + 0\hat{\jmath} - 1\hat{k}) - (6\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

$= (-4 - 6)\,\hat{\imath} + (0 - 2)\hat{\jmath} + (-1 - 2)\hat{k}$

$= -10\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$

$(\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$

$= \hat{i}[(-2 \times -2) - (-2 \times 2)]$ - $\hat{j}[(1 \times -2) - (3 \times 2)]$ + $\hat{k}[(1 \times -2) - (3 \times -2)]$

= $\hat{i}$[4 + 4] - $\hat{j}$[-2 - 6] + $\hat{k}$[-2 + 6]

= $\hat{i}(8) - \hat{j}(-8) + \hat{k}(4)$

= $8\hat{i} + 8\hat{j} + 4\hat{k}$

Magnitude of $\overrightarrow{b_1} \times \overrightarrow{b_2} = \sqrt{8^2 + 8^2 + 4^2}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}|$ = $\sqrt{64 + 64 + 16}$ = $\sqrt{144}$ = 12

Also,

$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})$ = $(8\hat{i} + 8\hat{j} + 4\hat{k}) \cdot (-10\hat{i} - 2\hat{j} - 3\hat{k})$

= (8 × (-10)) + (8 × (-2)) + (4 (8 × (-3)

= -80 + (-16) + (-12)

= -108

Shortest distance = $\left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

= $\left|\frac{-108}{12}\right|$

= |-9|

= 9

Therefore, the shortest distance between the given two lines is 9.

OR

Here $\vec{a} = 1 + 2j - 3\hat{k}$

The direction ratios of the line are (1 - 2) : (1 - 3) : (5 + 4)

⇒ 1 : -4 : 9

⇒ 1 : 4 : -9

Thus, $\overrightarrow{\mathrm{b}} = \hat{\mathrm{i}} + 4\hat{\mathrm{j}} - 9\hat{\mathrm{k}}$

Now,we have

Vector form:

$\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k} + \lambda(\hat{i} + 4\hat{j} - 9\hat{k})$

Cartesian form:

$\frac{x-1}{1} = \frac{y-2}{4} = \frac{z+3}{-9}$

24. P(B|A) = 0.4 ⇒ $\frac{P(B \cap A)}{P(A)}$ = 0.4 ⇒ $P(B \cap A)$ = 0.24

$P(A \cup B)$ = P(A) + P(B) - $P(A \cap B)$ = 0.6 + 0.5 - 0.24 = 0.86

P(A I B) = $\frac{P(A \cap B)}{P(B)}$ = $\frac{0.24}{0.5}$ = 0.48

25. $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$

$= \tan^{-1}\tan\left(\pi + \frac{\pi}{8}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{8}\right)\right)$

$= \frac{\pi}{8}$

Section C

26. Subject to the constraints are

4x + y $\geq$ 80[first constraint]

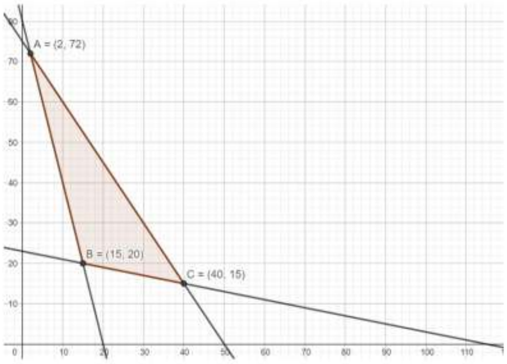
x + 5y $\geq$ 115[second constraint]

2x + 2y $\geq$ 150[third constraint]

and the non negative redtrestrict x,y ≥ 0

Converting the given inequations into equations, we get 4x + y = 80, x + 5y = 115, 2x + 2y = 150, x = 0 and y = 0

These lines are drawn on the graph and the shaded region ABC represents the feasible region of the given LPP.



It can be observed that the feasible region is bounded. The coordinates of the corner

points of the feasible region are A(2, 72), B(15, 20) and C(40, 15) The values of the objective function, Z at these corner points are given in the following table:

Corner Point Value of the Objective Function Z = 6x + 3y

A(2, 72) : $Z = 6 \times 2 + 3 \times 72 = 228$

B(15, 20) : $Z = 6 \times 15 + 3 \times 20 = 150$

C(40, 15) : $Z = 6 \times 40 + 3 \times 15 = 285$

From the table, Z is minimum at x = 15 and y = 20 and the minimum value of Z is 150. Thus, the minimum value of Z is 150.

27. Here, the given curves are:

$y = x^2$ .... (i)

y = x ..... (ii)

Using eqn. (ii) in (i), gives

or $x^2 - x = 0$

or x.(x - 1) = 0

⇒ x = 0, 1.

But y = x ⇒ y = 0, 1

Thus, (0, 0) and (1, 1) are the points of intersection.

Area between (y = x and $y = x^2$) is:

A = $\int \left|\left(x-x^2\right) \cdot dx\right|$

= $\left|\frac{(x^2)}{2}-\frac{(x^3)}{3}\right|$

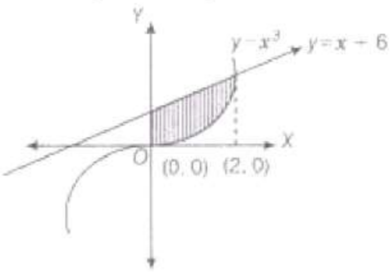
Using limits from 0, to 1, we get

A $= \left(\frac{1}{2}-\frac{1}{3}\right)-(0-0)$

$= \frac{1}{6}$ sq.unit

OR

We have, $y = x^3$ and y = x + 6 and x = 0



$\therefore x^3 = x + 6$

$\Rightarrow x^3 - x = 6$

$\Rightarrow x^3 - x - 6 = 0$

$\Rightarrow x^2(x-2) + 2x(x-2) + 3(x-2) = 0$

$\Rightarrow (x-2)\left(x^2+2x+3\right) = 0$

$\Rightarrow x = 2$ with two imaginary points

$\therefore$ Required area of shaded region $= \int_0^2 \left(x+6-x^3\right) dx$

$= \left[\frac{x^2}{2}+6x-\frac{x^4}{4}\right]_0^2$

$= \left[\frac{4}{2} + 12 - \frac{16}{4} - 0\right]$

= [2 + 12 - 4] =10 sq.units

28. Let the given integral be, $I = \int \frac{x^2+1}{(x+3)(x-1)^2}dx$

Now using partial fractions by putting, $\frac{x^2+1}{(x+3)(x-1)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$ .... (1)

$A(x - 1)^2 + B(x + 3)(x - 1) + C(x + 3) = x^2 + 1$

Now put x - 1 = 0

Therefore, x = 1

A(0) + B(0) + C(4) = 2

$c = \frac{1}{2}$

Now put x + 3 = 0

Therefore, x = -3

$A(-3 - 1)^2 + B(0) + C(0) = 9 + 1 = 10$

$A = \frac{5}{8}$

By equating the coefficient of x^2, we get, A + B = 1

$\frac{5}{8} + B = 1$

$B = 1 - \frac{5}{8} = \frac{3}{8}$

From equation (1), we get,

$\frac{x^2+1}{(x+3)(x-2)^2} = \frac{5}{8} \times \frac{1}{(x+3)} + \frac{3}{8} \times \frac{1}{(x-2)} + \frac{1}{(x-2)^2}$

$\int \frac{x^2+1}{(x+3)(x-2)^2}dx = \frac{5}{8}\int \frac{1}{(x+3)}dx + \frac{3}{8}\int \frac{1}{(x-2)}dx + \int \frac{1}{(x-2)^2}dx$

$= \frac{5}{8}\log|x+3| + \frac{3}{8}\log|x-1| - \frac{1}{2(x-1)} + c$

OR

Let the given integral be,

$l = \int e^{2x} \cos(3x + 4)\, dx$

Considering cos (3x + 4) as one function and e^{2x} as second function

$I = \cos(3x + 4) \frac{e^{2x}}{2} - \int - \sin(3x + 4) \times 3 \frac{e^{2x}}{2} dx$

$\Rightarrow I = \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}\int e^{2x} \sin(3x + 4)\, dx$

$\Rightarrow I = \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}I_1$ ...(i)

where $I_1 = \int e^{2x} \sin(3x + 4)\, dx$

Considering cos (3x + 4) as first function and e^{2x} as second function

$I_1 = \sin(3x + 4) \frac{e^{2x}}{2} - \int 3\cos(3x + 4) \frac{e^{2x}}{2} dx$

$\Rightarrow I_1 = \frac{e^{2x}\sin(3x+4)}{2} - \frac{3}{2}\int e^{2x} \cos(3x + 4)\, dx$

$\Rightarrow I_1 = \frac{e^{2x}\sin(3x+4)}{2} - \frac{3}{2}I$ ...(ii)

from (i) and (ii)

$I = \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{4} e^{2x} \sin(3x + 4) - \frac{9}{4} I$

$\Rightarrow I + \frac{9}{4}I = \frac{2e^{2x}\cos(3x+4)+3e^{2x}\sin(3x+4)}{4}$

$\Rightarrow I = \frac{e^{2x}}{13}$ [2 cos (3x + 4) + 3 sin (3x + 4)] + C

29. We know that for the lines $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$ and $\frac{x-c}{l} = \frac{y-d}{m} = \frac{z-\epsilon}{n}$

the angle between them 'θ' is given by

$\theta = \cos^{-1}\frac{pl+qm+rn}{\sqrt{p^2+q^2+r^2}\sqrt{l^2+m^2+n^2}}$

The lines are $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

Here p = 2, q = 3, r = 4 and l = 3, m = 4, n = 5

$\theta =^{-1} \frac{2(3)+3(4)+4(5)}{\sqrt{2^2+3^2+4^2}\sqrt{3^2+4^2+5^2}} = \cos^{-1}\frac{6+12+20}{\sqrt{4+9+16}\sqrt{9+16+25}}$

$\theta = \cos^{-1}\frac{6+12+20}{\sqrt{29}\sqrt{50}} = \cos^{-1}\frac{38}{5\sqrt{58}}$

$\theta = \cos^{-1}\frac{38}{5\sqrt{58}}$

The angle between the $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-5}{5}$ is $\cos^{-1}\frac{38}{5\sqrt{58}}$

OR

Let L be the foot of the perpendicular drawn from P $(2\hat{i}-\hat{j}+5\hat{k})$ on the line

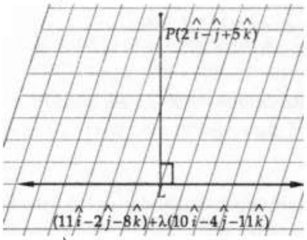
$\vec{r} = 11\hat{i}-2\hat{j}-8\hat{k}+\lambda(10\hat{i}-4\hat{j}-11\hat{k})$

Let the position vector of L be

$\vec{r} = 11\hat{i}-2\hat{j}-8\hat{k}+\lambda(10\hat{i}-4\hat{j}-11\hat{k})$

= $(11+10\lambda)\hat{i}+(-2-4\lambda)\hat{j}+(-8-11\lambda)\hat{k}$ ....(i)

Then, $\overrightarrow{PL}$ = Position vector of L - Position vector of P



$\Rightarrow \overrightarrow{PL}$ = $\{(11+10\lambda)\hat{i}+(-2-4\lambda)\hat{j}+(-8-11\lambda)\ddot{k}|-(2\hat{i}-\hat{j}+5\hat{k})$

$\Rightarrow \overrightarrow{PL}$ = $(9+10\lambda)\hat{i}+(-1-4\lambda)\hat{j}+(-13-11\lambda)\hat{k}$ .....(ii)

Since $\overrightarrow{PL}$ is perpendicular to the given line which is parallel to $\vec{b}=10\hat{i}-4\hat{j}-11\hat{k}$

$\therefore \overrightarrow{PL}\cdot\vec{b}=0$

$\Rightarrow \{(9+10\lambda)\hat{i}+(-1-4\lambda)\hat{j}+(-13-11\lambda)\hat{k}\cdot(10\hat{i}-4\hat{j}-11\hat{k})$ = 0

$\Rightarrow$ 10 (9 + 10λ) - 4 (- 1 - 4λ) - 11(-13 -11λ)) = 0

$\Rightarrow$ 90 + 100λ + 4 + 16λ + 143 + 121λ = 0

$\Rightarrow$ 237λ = -237 $\Rightarrow$ λ = - 1

Putting the value of λ, in (i), we obtain the position vector of L as $\hat{i}+2\hat{j}+3\hat{k}$

Putting $\lambda=-1$ in (ii), we obtain

$\overrightarrow{PL}$ = $(\hat{i}+2\hat{j}+3\hat{k})$ - $(2\hat{i}-\hat{j}+5\hat{k})=-\hat{i}+3\hat{j}-2\hat{k}$

$\Rightarrow |\overrightarrow{PL}|$ = $\sqrt{1+9+4}=\sqrt{14}$

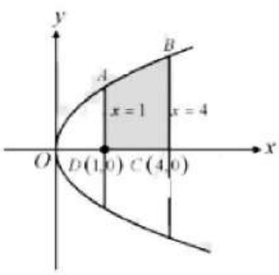
Hence, length of the perpendicular from P on the give line is 14 units.

30. The given equation, y^2 = 4x is a right handed Parabola with its vertex at the origin and x = 1, x = 4 is the line parallel to y-axis at distance of x = 1 to x = 4 units.

Also, y^2 = 4x contains only even power of y.

So, it is symmetrical about the x-axis

$\therefore$ Required area = area of ABCD



$\Rightarrow \int_1^4 y dx = \int_1^4 \sqrt{4x}dx = 2\int_1^4 \sqrt{x}dx$

$\Rightarrow 2\cdot\left[\frac{x^{3/2}}{3/2}\right]_1^4 = \frac{4}{3}\left[(4)^{1/2}-(1)^{2/2}\right] = \frac{4}{3}[8-1]$

$\Rightarrow \frac{4}{3}\times 7 = \frac{28}{3}$ sq. units.

31. We have, $\log\sqrt{x^2+y^2}=\tan^{-1}\left(\frac{x}{y}\right)$

$\Rightarrow \log\left(x^2+y^2\right)^{\frac{1}{2}}=\tan^{-1}\left(\frac{y}{x}\right)$

$\Rightarrow \frac{1}{2}\log\left(x^2+y^2\right)=\tan^{-1}\left(\frac{y}{x}\right)$

Differentiate with respect to x, we get

$\Rightarrow \frac{1}{2} \frac{d}{dx} \log\left(x^2+y^2\right)=\frac{d}{dx} \tan^{-1}\left(\frac{y}{x}\right)$

$\Rightarrow \frac{1}{2}\left(\frac{1}{x^2+y^2}\right) \frac{d}{dx}\left(x^2+y^2\right)=\frac{1}{1+\left(\frac{y}{x}\right)^2} \frac{d}{dx}\left(\frac{y}{x}\right)$

$\Rightarrow \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\left[2x+2y \frac{dy}{dx}\right]=\frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x \frac{dy}{dx}-y \frac{d}{dx}(x)}{x^2}\right]$

$\Rightarrow\left(\frac{1}{x^2+y^2}\right)\left(x+y \frac{dy}{dx}\right)=\frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x \frac{dy}{dx}-y \frac{d}{dx}(x)}{x^2}\right]$

$\Rightarrow\left(\frac{1}{x^2+y^2}\right)\left(x+y \frac{dy}{dx}\right)=\frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x \frac{dy}{dx}-y(1)}{x^2}\right]$

$\Rightarrow x+y \frac{dy}{dx}=x \frac{dy}{dx}-y$

$\Rightarrow y \frac{dy}{dx}-x \frac{dy}{dx}=-y-x$

$\Rightarrow \frac{dy}{dx}$(y - x) = -(y + x)

$\Rightarrow \frac{dy}{dx}=\frac{-(y+x)}{y-x}$

$\Rightarrow \frac{dy}{dx}=\frac{x+y}{x-y}$

LHS=RHS

Hence Proved.

Section D

32. Suppose that the required vector $=\vec{r}$

If α, β, γ are angle that $\vec{r}$ makes with the coordinate axes x, y and z respectively then

$l=\cos \alpha$

$=\cos 45^{\circ}$

$l=\frac{1}{\sqrt{2}}$

$m=\cos \beta$

$=\cos 60^{\circ}$

$m=\frac{1}{2}$

$n=\cos \gamma$

We know that,

$l^2+m^2+n^2=1$

$\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{2}\right)^2+\left(\cos^2 r\right)=1$

$\Rightarrow \frac{1}{2}+\frac{1}{4}+\cos^2 \gamma=1$

$\cos^2 \gamma=1-\frac{1}{2}-\frac{1}{4}$

$=\frac{4-2-1}{4}$

$=\frac{1}{4}$

$\cos \gamma=\pm \frac{1}{2}$

$\cos \gamma=\frac{1}{2}$

$\gamma=\cos^{-1}\left(\frac{1}{2}\right)$

$\gamma=\frac{\pi}{3}$

Neglecting since γ is an obtuse angle.

$\cos \gamma=-\frac{1}{2}$

$\gamma=\cos^{-1}\left(-\frac{1}{2}\right)$

$=\pi-\frac{\pi}{3}$

$\gamma=\frac{2\pi}{3}$

Therefore, $n=\cos \gamma$

$=\cos\left(\frac{2\pi}{3}\right)$

$=\cos\left(\pi-\frac{\pi}{3}\right)$

$=-\cos \frac{\pi}{3}$

$n=-\frac{1}{2}$

Therefore,

vector $\vec{r} = \vec{r}(\{l\hat{i} + m\hat{j} + n\hat{k})$

$= 12\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} - \frac{1}{2}n\hat{k}\right)$

$= 12\left(\frac{\sqrt{2}\hat{i}+\hat{j}-\hat{k}}{2}\right)$

$= 6(\sqrt{2}i + \hat{j} - \hat{k})$

$\vec{r} = 6(\sqrt{2}i + \hat{j} - \hat{k})$

33. Let $\frac{1}{x} = u, \frac{1}{y} = v$ and $\frac{1}{z} = w$

2u + 3v + 10w = 4

4u – 6v+ 5w = 1

6u + 9v – 20w = 2

$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} u \\ v \\ w \end{bmatrix}\ B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

= 2[120 - 45] -3[-80 - 30] +10[36 + 36]

= 150 + 330 + 720 = 1200$\neq$ 0

$\Rightarrow$A is non-singular and hence A^{-1} exists.

Now, $A_{11} = 75$, $A_{12} = 110$, $A_{13} = 72$

$A_{21} = 150$, $A_{22} = -100$, $A_{23} = 0$

$A_{31} = 75$, $A_{32} = 30$, $A_{33} = -2$

$\therefore adjA = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$X = A^{-1}B$

$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

$\begin{bmatrix} y \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

$u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5}$

$\frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$

x = 2, y = 3, z = 5

OR

Let the numbers are x, y, z be the cash awards for Resourcefulness, Competence, and Determination respectively

4x + 3y + 2z = 37000 …… (i)

Also,

5x + 3y + 4z = 47000 …… (ii)

Again,

x + y + z = 12000 …… (iii)

$\begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$

A X = B

|A| = 4(3 - 4) - 3(5 - 4) + 2(5 - 3)

= 4(-1) - 3(1) + 2(2)

= -4 - 3 + 4

= -3

Hence, the unique solution given by $X = A^{-1}B$

$C_{11} = (-1)^{1+1}(3 - 4) = -1$

$C_{12} = (-1)^{1+2}(5 - 4) = -1$

$C_{13} = (-1)^{1+3}(5 - 3) = 2$

$C_{21} = (-1)^{2+1}(3 - 2) = -1$

$C_{22} = (-1)^{2+2}(4 - 2) = 2$

$C_{23} = (-1)^{2+3}(4 - 3) = -1$

$C_{31} = (-1)^{3+1}(12 - 6) = 6$

$C_{32} = (-1)^{3+2}(16 - 10) = -6$

$C_{33} = (-1)^{3+3}(12 - 15) = -3$

$$\text{Adj } A = \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 6 & -6 & -3 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

$X = A^{-1}B = \frac{1}{|A|}(\text{Adj } A)B$

$$X = \frac{1}{-3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$X = \frac{1}{-3}\begin{bmatrix} -37000 - 47000 + 72000 \\ -37000 + 94000 - 72000 \\ 74000 - 47000 - 36000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4000 \\ 5000 \\ 3000 \end{bmatrix}$$

Hence, x = 4000, y = 5000 and z = 3000

Thus, the value x, y, z describes the number of prizes per person for Resourcefulness, Competence, and Determination.

34. A = R – {3} and B = R – {1} and $f(x) = \left(\frac{x-2}{x-3}\right)$

Let $x_1, x_2 \in$ A, then $f(x_1) = \frac{x_1-2}{x_1-3}$ and $f(x_2) = \frac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-3}{x_2-3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$= x_1 = x_2$

$\therefore f$ is one-one function.

Now $y = \frac{x-2}{x-3}$

$\Rightarrow y(x - 3) = x - 2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y - 1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1}$

$$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = \frac{3y-2-2y+2}{2y-2-3y+3} = y$$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

OR

A = R - {3}, B = R - {1}

f : A $\rightarrow$ B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let x, y $\in$ A such that f(x) = f(y).

$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$

$\Rightarrow$ (x - 2) (y - 3) = (y - 2) (x - 3)

$\Rightarrow$ xy - 3x - 2y + 6 = xy - 3y - 2x + 6

$\Rightarrow$ -3x - 2y = -3y - 2x

$\Rightarrow$ 3x - 2x = 3y - 2y

$\Rightarrow$ x = y

Therefore, f is one-one.

Let y $\in$ B = R - {1}

Then, y $\neq$ 1.

The function f is onto if there exists x $\in$ A such that f(x) = y.

Now, f (x) = y

$\Rightarrow \frac{x-2}{x-3} = y$

$\Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = -3y + 2

$\Rightarrow x = \frac{2-3y}{1-y} \in \mathbf{A} \qquad [y \neq 1]$

Thus, for any y $\in$ B, there exists $\frac{2-3y}{1-y} \in \mathbf{A}$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$$

$\therefore$ f is onto.

Hence, function f is one-one and onto.

35. To find: $\int \frac{x^2}{(x^2+6x-3)}dx$

We will use following Formula ;

i. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

ii. $\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C$

Given equation can be rewritten as following:

$\Rightarrow \int \frac{x^2+(6x-3)-(6x-3)}{(x^2+6x-3)}dx$

$\Rightarrow \int \frac{(x^2+6x-3)-(6x-3)}{(x^2+6x-3)}dx$

$\Rightarrow x - \int \frac{6x-3}{x^2+6x-3}dx$

Let $I = \int \frac{6x-3}{x^2+6x-3}dx$ ...(ii)

Using partial fractions,

$(6x-3) = A\left(\frac{d}{dx}(x^2+6x-3)\right) + B$

6x - 3 = A (2x + 6) + B

Equating the coefficients of x,

6 = 2A

A = 3

Also, -3 = 6A + B

$\Rightarrow$ B = -21

Substituting in (I),

$\Rightarrow \int \frac{3(2x+6)-21}{(x^2+6x-3)}dx$

$\Rightarrow 3 \times \log\left|x^2+6x-3\right| + C_1 - 21\int \frac{1}{(x+3)^2-(\sqrt{12})^2}dx$

$\Rightarrow 3 \times \log\left|x^2+6x-3\right| + C_1 - 21 \times \frac{1}{2\sqrt{12}} \times \log\left|\frac{x+3-\sqrt{12}}{x+3+\sqrt{12}}\right| + C_2$

$I = 3\log\left|x^2+6x-3\right| - \frac{7\sqrt{3}}{4} \times \log\left|\frac{x+3-2\sqrt{3}}{x+3+2\sqrt{3}}\right| + C$

Therefore,

$$\int \frac{x^2}{(x^2+6x-3)}dx = x - 3\log|x^2+6x-3| + \frac{7\sqrt{3}}{4} \times \log\left|\frac{x+3-2\sqrt{3}}{x+3+2\sqrt{3}}\right| + c$$

Section E

36. **Read the text carefully and answer the questions:**

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.

(i) We have, I(x) = $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$

$\Rightarrow$ I'(x) = $\frac{-2000}{x^3} + \frac{250}{(600-x)^3}$ and

$\Rightarrow$ I''(x) = $\frac{6000}{x^4} + \frac{750}{(600-x)^4}$

For maxima/minima, I'(x) - 0

$\Rightarrow \frac{2000}{x^3} = \frac{250}{(600-x)^3} \Rightarrow 8(600 - x)^3 = x^3$

Taking cube root on both sides, we get

2(600 - x) = x $\Rightarrow$ 1200 = 3x $\Rightarrow$ x = 400

Thus, I(x) is minimum when you are at 400 feet from the strong intensity lamp post.

(ii) At a distance of 200 feet from the weaker lamp post.

Since I(x) is minimum when x = 400 feet, therefore the darkest spot between the two light is at a distance of 400 feet from a stronger lamp post, i.e., at a distance of 600 - 400 = 200 feet from the weaker lamp post.

(iii) $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$

Since, the distance is x feet from the stronger light, therefore the distance from the weaker light will be 600 - x.

So, the combined light intensity from both lamp posts is given by $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$.

OR

We know that l(x) = $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$

When x = 400

l(x) = $\frac{1000}{160000} + \frac{125}{(600-400)^2}$

$= \frac{1}{160} + \frac{125}{40000} = \frac{1}{160} + \frac{1}{320} = \frac{3}{320}$ units

37. **Read the text carefully and answer the questions:**

Consider 2 families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommended daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children.

(i) Let F be the matrix representing the number of family members and R be the matrix representing the requirement of calories and proteins for each person. Then

$$F = \begin{matrix} Family\ A \\ Family\ B \end{matrix} \overset{\begin{matrix} Men & Women & Children \end{matrix}}{\begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix}}$$

$$R = \begin{matrix} Man \\ woman \\ Children \end{matrix} \overset{\begin{matrix} Calories & Proteins \end{matrix}}{\begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}}$$

(ii) The requirement of calories and proteins for each of the two families is given by the product matrix FR.

$$FR = \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$= \begin{bmatrix} 4(2400+1900+1800) & 4(45+55+33) \\ 2(2400+1900+1800) & 2(45+55+33) \end{bmatrix}$$

$$FR = \overset{\begin{matrix} Calories & Proteins \end{matrix}}{\begin{bmatrix} 24400 & 532 \\ 12200 & 266 \end{bmatrix}} \begin{matrix} Family\ A \\ Family\ B \end{matrix}$$

(iii)

$$R' = \begin{bmatrix} 2400 - 2400 \times 5\% & 45 + 45 \times 5\% \\ 1900 - 1900 \times 5\% & 55 + 55 \times 5\% \\ 1800 - 1800 \times 5\% & 33 + 33 \times 5\% \end{bmatrix}$$

$$\Rightarrow R' = \begin{bmatrix} 2400 - 120 & 45 + 2.25 \\ 1900 - 95 & 55 + 2.75 \\ 1800 - 90 & 33 + 1.65 \end{bmatrix}$$

$$\Rightarrow R' = \begin{matrix} Man \\ Woman \\ Children \end{matrix} \overset{\begin{matrix} Calories & Proteins \end{matrix}}{\begin{bmatrix} 2280 & 45.25 \\ 1805 & 55.75 \\ 1710 & 34.65 \end{bmatrix}}$$

OR

Since, AB = B ...(i) and BA = A ..(ii)

$\therefore A^2 + B^2 = A \cdot A + B \cdot B$

= A(BA) + B(AB) [using (i) and (ii)]

- (AB)A + (BA)B [Associative law]

= BA + AB [using (i) and (ii)]

= A + B

38. **Read the text carefully and answer the questions:**

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.

(i) Required probability = P(one ticket with prime number and other ticket with a multiple of 4)

$= 2\left(\frac{15}{50} \times \frac{12}{49}\right) = \frac{36}{245}$

(ii) P(First ticket shows an even number and second ticket shows an odd number)

$= \frac{25}{50} \times \frac{25}{49} = \frac{25}{98}$