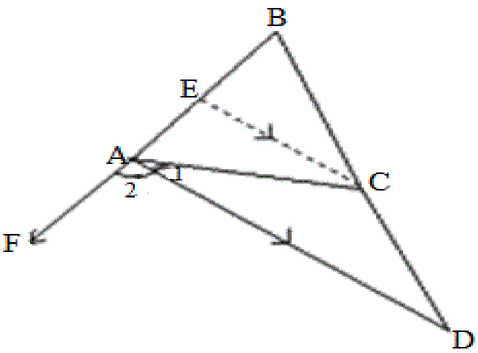
Chapter 6. Triangle

Question-1

The bisector of the exterior angle ∠A of Δ ABC intersects side BC produced at D. Prove that $\frac{AB}{AC} = \frac{BD}{DC}$.



Solution:

Given: ABC is a triangle; AD is the exterior bisector of ∠A and meets BC produced at D; BA is produced to F.

To prove: $\frac{AB}{AC} = \frac{BD}{DC}$

Construction: Draw CE||DA to meet AB at E.

Proof: In Δ ABC, CE || AD cut by AC.
∠CAD = ∠ACE (Alternate angles)

Similarly CE || AD cut by AB
∠FAD = ∠AEC (corresponding angles)

Since ∠FAD = ∠CAD (given)
∴ ∠ACE = ∠AEC

\ AC = AE (by isosceles Δ theorem)

Now in Δ BAD, CE || DA
$\frac{AE}{AB} = \frac{DC}{BD}$ (BPT)

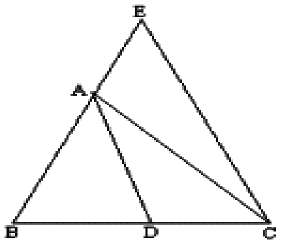
But AC = AE (proved above)

\ $\frac{AC}{AB} = \frac{DC}{BD}$ or

$\frac{AB}{AC} = \frac{BD}{DC}$ (proved).

Question-2

State and prove the converse of angle bisector theorem.



Solution:

Given: ABC is a Δ ; AD divides BC in the ratio of the sides containing the angles ∠A to meet BC at D.

i.e. $\frac{AB}{AC} = \frac{BD}{DC}$

To prove: AD bisects ∠A.

Construction: Draw CE || DA to meet BA produced at E.

Proof: In Δ ABC, CE || DA cut by AE.

\ ∠BAD = ∠AEC (corresponding angle) ----(i)

Similarly CE || DA cut by AC

\ ∠ DAC = ∠ACE (alternate angles) ----(ii)

In DBEC; CE || AD

\ $\frac{AB}{AE} = \frac{BD}{DC}$ (BPT)

But $\frac{AB}{AC} = \frac{BD}{DC}$ (given)

\ $\frac{AB}{AE} = \frac{AB}{AC}$

\ AE = AC

⇒ ∠AEC = ∠ACE (isosceles property) ----(iii)

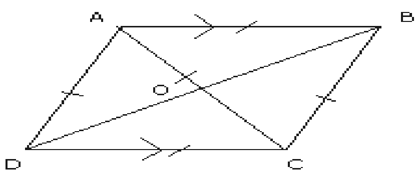
According to equation (i), (ii) and (iii) ∠BAD = ∠DAC

⇒ AD bisects ∠A.

Question-3

If a parallelogram has all its sides equal and one of its diagonal is equal to a side, show that its diagonals are in the ratio $\sqrt{3}$: 1.

Solution:



Given: ABCD is a parallelogram, where AC and BD are the diagonals meeting at O. AB = BC = AC.

To Prove: BD : AC :: $\sqrt{3}$: 1

Proof: In Δ ABC, AB = BC = CA (given).

= a (say)

Hence ABC is an equilateral triangle. (Definition of equilateral triangle)

AC and BD are the diagonals of parallelogram ABCD,

⇒ AC = BD (Diagonals of a parallelogram bisect each other)

or AO = OC.

i.e BO is the median of the equilateral ABC.

Hence BO = $\frac{\sqrt{3}}{2}$ a

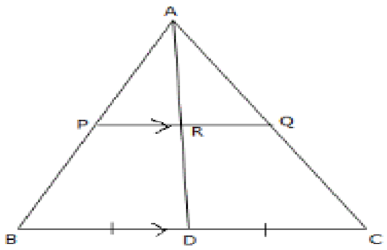
∴ BD = $\sqrt{3}$ a

⇒ BD : AC :: $\sqrt{3}$ a : a

⇒ BD : AC :: $\sqrt{3}$: 1.

Question-4

In Δ ABC, P, Q are points on AB and AC respectively and PQ || BC. Prove that the median AD bisects PQ.



Solution:

Given: ABC is a triangle, PQ || BC; AD is the median which cuts PQ at R.

To prove: AD bisects PQ at R.

Proof: In Δ ABD; PR || BD

$\frac{AP}{PB} = \frac{AR}{RD}$ (BPT)

In Δ ACD, RQ || DC

∴ $\frac{AR}{RD} = \frac{AQ}{QC}$ (BPT)

In ΔAPR and ΔABD,

∠APR = ∠ABD (corresponding angles.)

∠ARP = ∠ADB (corresponding angles.)

∴ Δ APR is similar to Δ ABD (AA similarity)

∴ $\frac{AP}{AB} = \frac{AR}{AD} = \frac{PR}{BD}$ (corresponding sides of similar triangles are proportional)----(i)

Similarly Δ ARQ is similar to Δ ADC

\ $\frac{AQ}{AC} = \frac{AR}{AD} = \frac{RQ}{DC}$ -----(ii)

According to equation (i) and (ii),

$\frac{AR}{AD} = \frac{PR}{BD} = \frac{RQ}{DC}$

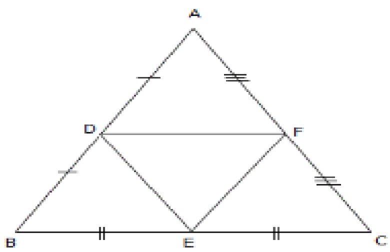
but BD = DC (given)

∴ PR = RQ

or AD bisects PQ at R (proved).

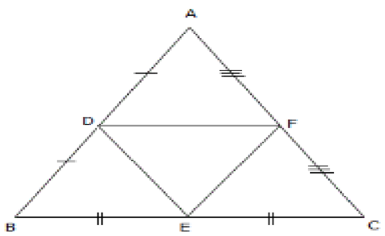
Question-5

Prove that the line joining the midpoints of the sides of the triangle form four triangles, each of which is similar to the original triangle.



Solution:

Given: In ΔABC, D, E, F are the midpoints of AB, BC and AC respectively.



To prove: ΔABC ~ ΔDEF

ΔABC ~ ΔADF

ΔABC ~ ΔBDE

ΔABC ~ ΔEFC

Proof: In ΔABC, D and F are mid points of AB and AC respectively.

∴ DF || BC (midpoint theorem)

In Δ ABC and Δ ADF

∠A is common; ∠ADF = ∠ABC (corresponding angles)

ΔABC ~ Δ DF (AA similarity) -----(1)

Similarly we can prove ΔABC ~ ΔBDE (AA similarity)-----(2)

ΔABC ~ ΔEFC (AA similarity)-----(3)

In ΔABC and ΔDEF;

since D,E, F are the midpoints of AB, BC and AC respectively,

DF= (1/2) × BC; DE = (1/2) × AC; EF = (1/2) × AB; (midpoint theorem)

$\therefore \frac{AB}{EF} = \frac{BC}{DF} = \frac{CA}{DE} = 2$

∴ Δ ABC ~ Δ EFD (SSS similarity)-----------------(4)

From (1), (2), (3) and (4)

Δ ABC ~ Δ DEF

Δ ABC ~ Δ ADF

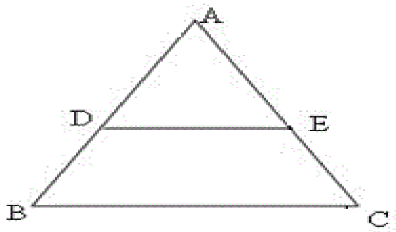
Δ ABC ~ Δ BDE

Δ ABC ~ Δ EFC.

Question-6

In triangle ABC, DE||BC and AD : DB = 2 : 3. Determine the ratio of the area triangle ADE to the area triangle ABC.

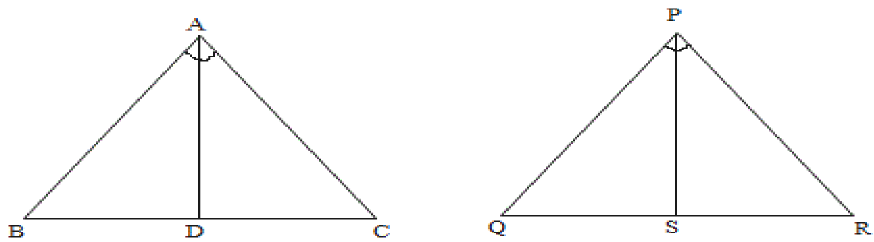
Solution:



$\therefore \frac{\text{Area } \Delta ADE}{\text{Area } \Delta ABC} = \frac{2^2}{5^2} = \frac{4}{25}$

Question-7

One angle of a triangle is equal to one angle of another triangle and the bisectors of these equal angles divide the opposite sides in the same ratio. Prove that the triangles are similar.



Solution:

Given: ΔABC and ΔPQR

∠ A = ∠ P

AD and PS bisects ∠A and ∠P respectively.

$\frac{BD}{DC} = \frac{QS}{SR}$

To prove: ΔABC ~ ΔPQR

Proof: In ΔABC and ΔPQR

AD bisects ∠A

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$ (Angle bisector theorem) -----(1)

Similarly in ΔPQR,

$\frac{PQ}{PR} = \frac{QS}{SR}$ (Angle bisector theorem) ----(2)

But $\frac{BD}{DC} = \frac{QS}{SR}$ (given)

∴ According to equation (1) and (2)

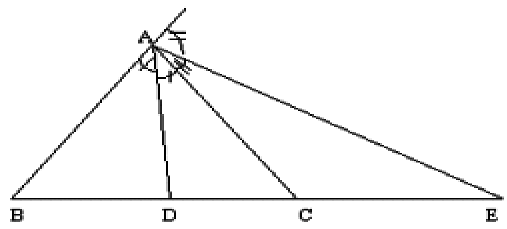
$\frac{AB}{AC} = \frac{PQ}{PR} \Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$

∠A = ∠P (given)

∴ ΔABC ~ ΔPQR (SAS similarity).

Question-8

The bisector of interior angle A of a triangle AB meets BC in D and the bisector of exterior angle A meets BC produced in E. Prove that $\frac{BD}{BE} = \frac{CD}{CE}$



Solution:

Given: ΔABC, AD bisects interior ∠A and AE bisects exterior ∠A meeting BC at D and BC produced at E.

To prove: $\frac{BD}{BE} = \frac{CD}{CE}$

Proof: In ΔABC, AD bisects interior ∠A

∴ $\frac{AB}{AC} = \frac{BD}{DC}$ (Angle Bisector theorem).....................(1)

Similarly in DABC, AE bisects exterior ∠A

∴ $\frac{AB}{AC} = \frac{BE}{CE}$ ....(2)

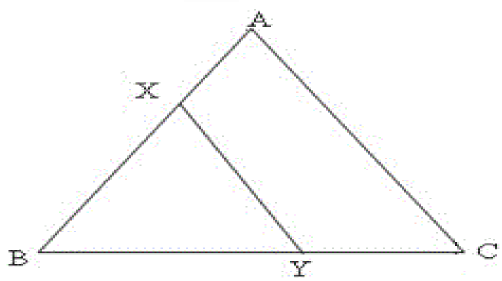
From equation (1) and (2),

$\frac{AB}{AC} = \frac{BD}{CD} = \frac{BE}{CE} \Rightarrow \frac{BD}{BE} = \frac{CD}{CE}$

Hence Proved.

Question-9

In a triangle ABC, XY || AC divides the triangle into two parts equal in areas. Determine $\frac{AX}{AB}$.



Solution:

Given: ABC is a triangle with XY || AC divides the triangle into two parts equal in areas.

To find: $\frac{AX}{AB}$

Proof:

ar ΔBXY = ar trap. XYCA (Given) ∴ ar ΔBXY = $\frac{1}{2}$ ar ΔABC

In ΔBXY andBAC,

∠BXY = ∠BAC (Corresponding angles)

∠BYX = ∠BCA (Corresponding angles)

ΔBXY ~ ΔBAC (AA similarity)

∴ $\frac{\text{Area } \Delta BXY}{\text{Area } \Delta BAC} = \frac{BX^2}{AB^2}$ (Areas of similar triangle)

∴ $\frac{1}{2} = \frac{BX^2}{AB^2}$

∴ $\frac{1}{\sqrt{2}} = \frac{BX}{AB}$

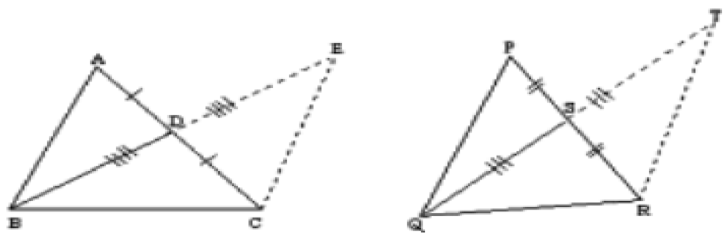
∴ AB − BX = √ 2 BX − BX

∴ AX = (√ 2 − 1)BX

$\frac{AX}{AB} = \frac{(\sqrt{2} - 1)BX}{\sqrt{2}BX} = \frac{\sqrt{2} - 1}{\sqrt{2}}$.

Question-10

If two sides and a median bisecting the third side of a Δ are respectively proportional to the corresponding sides and the median of another triangle, then prove that the two triangles are similar.



Solution:

Given: ΔABC and ΔPQR where BD and QS are the medians and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{BD}{QS}$

To prove: ΔABC ~ ΔPQR

Construction: Produce BD and QS to E and T respectively such that BD = DE and QS = ST. CE and TR are joined.

Proof: In ΔADB and DCDE,

AD = DC (given)

∠ADB = ∠CDE (Vertically opposite angles)

BD = DE.

∴ ΔADB ≅ ΔCDE (SAS ≅ axiom)

Hence AB = CE and ∠ABD = ∠DEC.

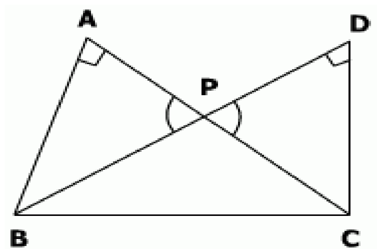
Similarly ΔPQS ≅ Δ RST,
hence PQ = TR and ∠PQS = ∠STR.
Consider Δ EBC and Δ TQR,
$\frac{BD}{QS} = \frac{2BD}{2QS} = \frac{BE}{QT}$ (from given and construction)-------(1)
AB = CE and PQ = RT (proved),
$\frac{AB}{PQ} = \frac{CE}{RT}$ ----(2)
$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{BD}{QS}$ (Given) -----(3)

From (1),(2) and (3),
$\frac{BE}{QT} = \frac{CE}{RT} = \frac{BC}{QR}$
∴ ΔEBC ~ ΔTQR(SSS similarity axiom).
⇒ ∠DBC = ∠SQR and ∠DEC = ∠STR ---(4) (corresponding angles of similar triangles are proportional)

But ∠ABD = ∠DEC and ∠PQS = ∠STR (proved)-----(5)
∴ ∠ABD = ∠PQS (from (4) and (5)) ----(6)
From (5) and (6),
∠ABC = ∠PQR -----I
In Δ ABC and Δ PQR,
$\frac{AB}{PQ} = \frac{BC}{QR}$ (given)
And ∠ABC = ∠PQR (from I)
∴ ΔABC ~ ΔPQR (SAS Similarity).

Question-11

Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same sides of BC. If AC and DB intersect at P, prove that AP × PC = BP × PD.



Solution:

Given: Two right triangles ABC and BDC on the same hypotenuse BC. AC and BD intersect at P.

To prove: AP × PC = BP × PD

Proof: In ΔABP and ΔDCP

∠A = ∠D (= 90°) (given)

∠APB = ∠DPC (vertically opposite angles)

∴ ΔABP ~ ΔDCP (AA similarity axiom)

∴ $\frac{AB}{DC} = \frac{BP}{CP} = \frac{AP}{DP}$ (corresponding sides of similar Δ s are proportional) ....(1)

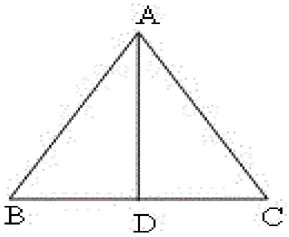
From (1) $\frac{BP}{CP} = \frac{AP}{DP}$

By cross multiplication,

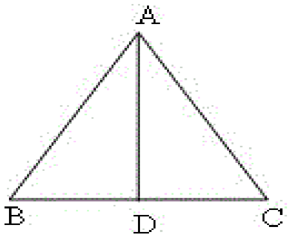
BP × DP = AP × PC (proved).

Question-12

If ABC is an equilateral triangle of side 2a prove that the altitude AD = 3a and $3AB^2 = 4AD^2$.



Solution:



Given: Δ ABC is an equilateral triangle of side 2a. AD is the altitude of triangle.

To Prove: AD = a√ 3 and $3AB^2 = 4AD^2$

Proof:

In rt. Δ ADC,

$AD^2 = AC^2 - DC^2$

$= (2a)^2 - a^2$

$= 4a^2 - a^2$

$= 3a^2$

$\therefore AD = a\sqrt{3}$

$3AB^2 = 3(2a)^2$

$= 3(2a)^2$

$= 3 ´ 4a^2$

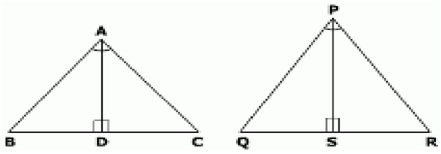
$= 4(a\sqrt{3})2$

$= 4AD^2$

$\therefore 3AB^2 = 4AD^2$.

Question-13

Two isosceles Δs have equal vertical angles and their areas are in the ratio 9 : 16. Find the ratio of their corresponding heights (altitudes).



Solution:

Given: ΔABC and ΔPQR are isosceles and ∠A = ∠P. AD, PS are the altitudes and $\frac{\Delta ABC}{\Delta PQR} = \frac{9}{16}$.

To find: $\frac{AD}{PS}$

Proof: In Δ ABC, ∠B = ∠C (isosceles Δ property)

Similarly in Δ PQR, ∠Q = ∠R.

∠A = ∠P (given)

$\therefore \angle B = \angle C = \frac{180^0 - \angle A}{2}$

Since ∠ A = ∠ P

∠ B = ∠ C = ∠ Q = ∠R

∴ Δ ABC ~ Δ PQR (AA)

If 2 triangles are similar then the ratio of areas will be equal to the square of the corresponding sides,

$\frac{ar.(\Delta ABC)}{ar.(\Delta PQR)} = \frac{AD^2}{PQ^2}$ ....(1)

In Δ ABD, Δ PQS

∠D = ∠S (= 90°)

∠B = ∠Q (given)

∴ Δ ABD ~ Δ PQS (AA)

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ ....(2)

According equation (1) $\frac{ar.(\Delta ABC)}{ar.(\Delta PQR)} = \frac{AB^2}{PS^2} = \frac{AB^2}{PQ^2}$

$\therefore \frac{9}{16} = \frac{AD^2}{PS^2}$

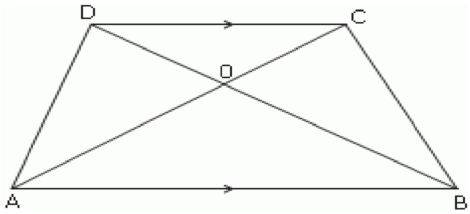
$\Rightarrow \frac{AD}{PS} = \frac{3}{4}$

∴ The ratio of their corresponding heights is 3 : 4.

Question-14

The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Solution:



Given: ABCD is a trapezium with AB || CD and the diagonals AC and BD intersect at 'O'.

To prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Proof:

In the figure consider the triangle OAB and OCD

$\angle DOC = \angle AOB$ (Vertically opposite angles are equal)

since AB || DC,

$\angle DCO = \angle OAB$ (Alternate angles are equal)

∴ By AA corollary of similar triangles.

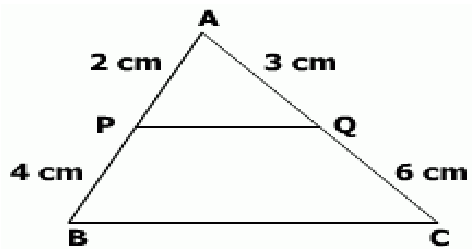
∴ Δ OAB ~ Δ OCB When the two triangle are similar, the side are proportionally.

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

Hence proved.

Question-15

P and Q are the points on the sides AB and AC respectively of a ΔABC. If AP = 2 cm, PB = 4 cm, AQ = 3 cms, QC = 6 cm, prove that BC = 3PQ.



Solution:

Given: Δ ABC, PQ are points on AB and AC such that AP = 2 cm, BP = 4 cm, AQ = 3 cm, QC = 6 cm

To prove: BC = 3PQ

Proof: In Δ ABC, $\frac{AP}{PB} = \frac{2}{4} = \frac{1}{2}; \frac{AQ}{QC} = \frac{3}{6} = \frac{1}{2}$

As $\frac{AP}{PB} = \frac{AQ}{QC}$

According to converse of BPT, PQ || BC

In Δ APQ and Δ ABC

∴ ∠ APQ = ∠ ABC (Corresponding angles)

∠ A is Common

∴ Δ APQ ~ Δ ABC (AAS similarity)

∴ $\frac{AP}{AB} = \frac{AQ}{QC}$ (corresponding sides of similar Δ s are proportional)

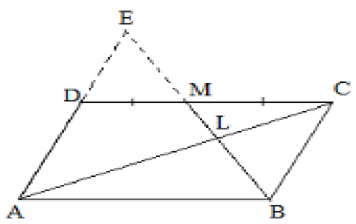
But $\frac{AP}{AB} = \frac{PQ}{BC}$

∴ $\frac{PQ}{BC} = \frac{2}{6} = \frac{1}{3}$

∴ 3PQ = BC (Proved).

Question-16

Through the midpoint of M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD produced in E. Prove that EL = 2BL.



Solution:

Given: ABCD is a parallelogram, M is the midpoint of CD. BM intersects AC at L and AD produced at E.

To prove: EL = 2BL

Proof: In Δ BMC and Δ EDM

∠DME = ∠ BMC (Vetically opposite angles)

DM = MC (given)

∠DEM = ∠ MBC (alternate angles)

∴ Δ BMC ≅ Δ EDM (ASA congruence)

∴ DE = BC (c.p.c.t)

But BC = AD (opposite sides of parallelogram ABCD)

∴ AD = DE ⇒ AE = 2AD = 2BC

In Δ AEL and Δ CBL

∠ ALE = ∠ BLC (Vertically opposite angles)

∠ AEL = ∠ LBC (alternate angles)

∴ Δ AEL ~ Δ CBL (AA similarity axiom)

$\Rightarrow \frac{AE}{BC} = \frac{AL}{LC} = \frac{EL}{BL}$

$\Rightarrow \frac{EL}{BL} = \frac{AE}{BC}$

$\Rightarrow \frac{EL}{BL} = \frac{AD+DE}{BC}$

$\Rightarrow \frac{EL}{BL} = \frac{BC+BC}{BC}$

$\Rightarrow \frac{EL}{BL} = \frac{2BC}{BC}$

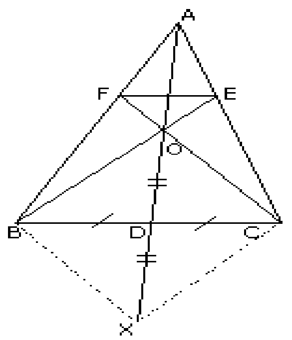
$\Rightarrow \frac{EL}{BL} = 2$

∴ EL = 2 BL

Question-17

The side BC of a triangle ABC is bisected at D; O is any point in AD. BO, CO produced meet AC, AB in E,F respectively, and AD is produced to X so that D is the mid point of OX. Prove that AO : AX = AF : AB and show that EF is parallel to BC.

Solution:



Given : The side BC of a triangle ABC is bisected at D; O is any point in AD. BO , CO produced meet AC, AB in E , F respectively, and AD is produced to X so that D is the mid point of OX.

To Prove : AO : AX = AF : AB and show that EF is parallel to BC.

Construction: Join BX and CX.

Proof: In quadrilateral BOCX, BD = DC and DO = DX (given)

∴ BOCX is a parallelogram (When the diagonals of a quadrilateral bisect each other, then the quad. is a parallelogram)

∴ BX || CO (Definition of a parallelogram)

or BX || FO.

In Δ ABX, BX || FO(proved).

∴ AO : AX = AF : AB (using B.P.T) -----------(i)

Similarly, AO : AX = AE : AC -------------(ii)

From (i) and (ii),

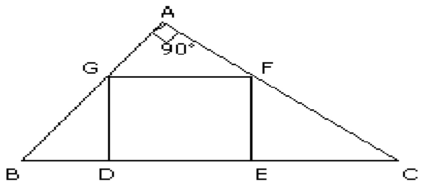
AF : AB = AE : AC

By corollary to B.P.T, EF is parallel to BC.

Question-18

ABC is a triangle in which ∠BAC = 90° and DEFG is a square, prove that DE^2 = BD × EC.

Solution:



Given: ABC is a triangle in which ∠BAC = 90° and DEFG is a square.
To prove: DE^2 = BD × EC.
Proof: In Δ AGF and Δ DBG,
∠AGF = ∠GBD (corresponding angles)
∠GAF = ∠BDG (each = 90°)
∴ ΔAGF ~ ΔDBG. ---------------------(i)
Similarly, ΔAFG ~ ΔECF (AA Similarity)----------------(ii)
From (i) and (ii), ΔDBG ~ ΔECF.

$$\frac{BD}{EF} = \frac{BG}{FC} = \frac{DG}{EC}$$

$$\frac{BD}{EF} = \frac{DG}{EC}$$

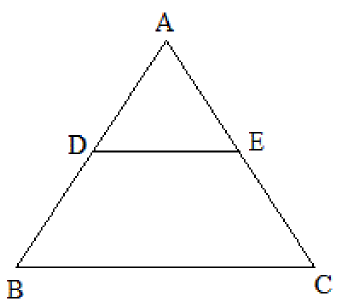
EF × DG = BD × EC. -------------------(iii)

Also DEFG is a square ⇒ DE = EF = FG = DG -------------(iv)

From (iii) and (iv), DE^2 = BD × EC.

Question-19

In fig. DE || BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value x.



Solution:

Given: ABC is a triangle, DE || BC, AD = x, DB = x − 2, AE = x + 2 and EC = x − 1.

To find: x

In Δ ABC, we have

DE || BC

Therefore [By Thale's theorem]

$\frac{AD}{DB} = \frac{AE}{EC}$

AD × EC = AE × DB

$x(x - 1) = (x - 2)(x + 2)$

$x^2 - x = x^2 - 4$

$x = 4$

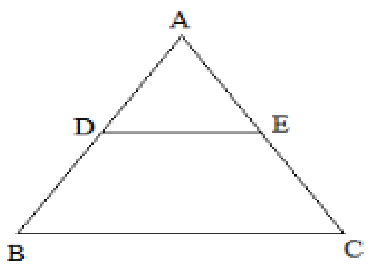
Question-20

In a Δ ABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 4 cm, AE = 8 cm, DB = x − 4 and EC = 3x − 19, find x.

Solution:

Given: In Δ ABC, D and E are points on the sides AB and AC respectively such that DE || BC. AD = 4 cm, AE = 8 cm, DB = x - 4 and EC = 3x - 19.

To find: x.



In Δ ABC, we have DE || BC

Therefore $\frac{AD}{DB} = \frac{AE}{EC}$ [By Thale's theorem]

$\frac{4}{x-4} = \frac{8}{3x-19}$

$4(3x - 19) = 8(x - 4)$

$12x - 76 = 8x - 32$

$4x = 44$

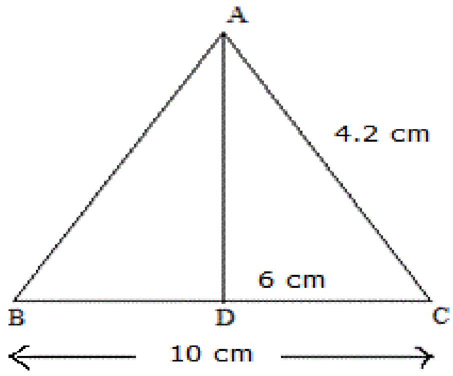
$x = 11$

Question-21

In a Δ ABC, AD is the bisector of ∠A, meeting side BC at D. If AC = 4.2 cm, DC = 6 cm, BC = 10 cm, find AB.

Solution:

Given: In a Δ ABC, AD is the bisector of ∠ A, meeting side BC at D. AC = 4.2 cm, DC = 6 cm and BC = 10 cm.



To find: AB.

In Δ ABC,

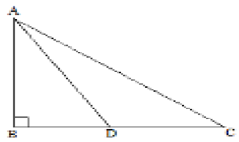
$\frac{AB}{AC} = \frac{BD}{DC}$ [By internal bisector theorem]

AB = 4 × 4.2/6 = 2.8 cm

∴ AB = 2.8 cm

Question-22

In Δ ABC, ∠ B = 90° and D is the mid-point of BC. Prove that $AC^2 = AD^2 + 3CD^2$.



Solution:

Given: In Δ ABC, ∠B = 90° and D is the mid-point of BC.

To Prove: $AC^2 = AD^2 + 3CD^2$

Proof:

In Δ ABD,

$AD^2 = AB^2 + BD^2$

$AB^2 = AD^2 - BD^2$ ....(i)

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 - BC^2$ ....(ii)

Equating (i) and (ii)

$AD^2 - BD^2 = AC^2 - BC^2$

$AD^2 - BD^2 = AC^2 - (BD + DC)^2$

$AD^2 - BD^2 = AC^2 - BD^2 - DC^2 - 2BD \times DC$

$AD^2 = AC^2 - DC^2 - 2DC^2$ (DC = BD)

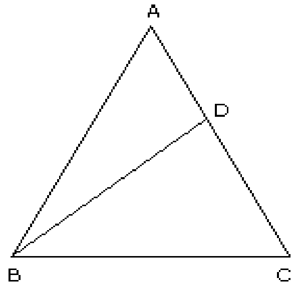
$AD^2 = AC^2 - 3DC^2$

Question-23

ABC is a triangle in which AB = AC and D is a point on the side AC such that BC^2 = AC × CD. Prove that BD = BC.

Solution:

Given: A Δ ABC in which AB = AC. D is a point on AC such that BC^2 = AC × CD.



To prove: BD = BC
Proof: Since BC^2 = AC × CD
Therefore BC × BC = AC × CD
AC/BC = BC/CD ....(i)

Also ∠ACB = ∠BCD
Since ΔABC ~ ΔBDC [By SAS Axiom of similar triangles]
AB/AC = BD/BC ....(ii)

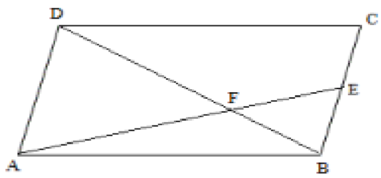
But AB = AC (Given) ....(iii)

From (i), (ii) and (iii) we get
BD = BC.

Question-24

The diagonal BD of a parallelogram ABCD intersects the segment AE at the point F, where E is any point on the side BC. Prove that DF × EF = FB × FA.

Solution:



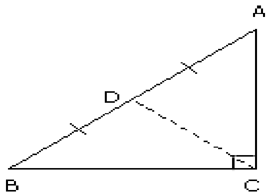
Given: The diagonal BD of parallelogram ABCD intersects the segment AE at F, where E is any point on BC.
To prove: DF × EF = FB × FA

Proof: In triangles AFD and BFE,
$\angle FAD = \angle FEB$ (Alternate angles)
$\angle AFD = \angle BFE$ (Vertically opposite angles)
Therefore $\Delta ADF \sim \Delta BFE$ (AA similarity)
$\frac{DF}{FA} = \frac{FB}{EF}$
Hence $DF \times EF = FB \times FA$

Question-25

ABC is a triangle, right-angled at C and AC = $\sqrt{3}$ BC. Prove that $\angle ABC = 60°$.

Solution:
Given: ΔABC is right angled at C and $AC = \sqrt{3}BC$.

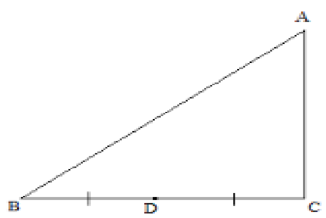


To prove: $\angle ABC = 60°$.
Proof:

Let D be the midpoint of AB. Join CD.
Now, $AB^2 = BC^2 + AC^2 = BC^2 + (\sqrt{3}BC)^2 = 4BC^2$
Therefore $AB = 2BC$.
Now, $BD = \frac{1}{2}AB = \frac{1}{2}(2BC) = BC$.
But, D being the midpoint of hypotenuse AB, it is equidistant from all the three vertices.
Therefore $CD = BD = DA$ or $CD = \frac{1}{2}AB = BC$.
Thus, $BC = BD = CD$,
i.e., ΔBCD is a equilateral triangle.
Hence, $\angle ABC = 60°$.

Question-27

Let ABC be a triangle, right-angled at C. If D is the mid-point of BC, prove that $AB^2 = 4AD^2 - 3AC^2$.



Solution:

Given: ABC be a triangle, right-angled at C and D is the mid-point of BC.

To Prove: $AB^2 = 4AD^2 - 3AC^2$.

Proof:

From right triangle ACB, we have,

$AB^2 = AC^2 + BC^2$

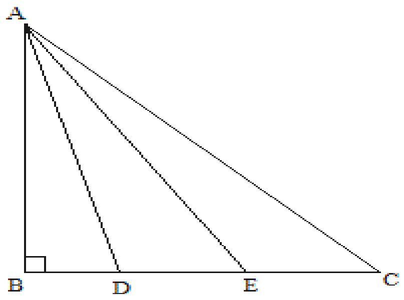
$= AC^2 + (2CD)^2 = AC^2 + 4CD^2$ [Since BC = 2CD]

$= AC^2 + 4(AD^2 - AC^2)$ [From right Δ ACD]

$= 4AD^2 - 3AC^2$.

Question-28

In the given figure, points D and E trisect BC and $\angle B = 90°$. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



Solution:

Given: In Δ ABC, points D and E trisect on BC and $\angle B = 90°$.

To Prove: $8AE^2 = 3AC^2 + 5AD^2$.

Proof: Let ABC be the triangle in which h $\angle$ B = 90°. Let the points D and E trisect BC.

Join AD and AE. Then,

$AC^2 = AB^2 + BC^2$

$3AC^2 = 3AB^2 + 3BC^2$ ....(i)

$AD^2 = AB^2 + BD^2$

$5AD^2 = 5AB^2 + 5BD^2$ ....(ii)

Therefore $3AC^2 + 5AD^2 = 8AB^2 + 3BC^2 + 5BD^2$

$= 8AB^2 + 3.\left(\frac{3}{2}BE\right)^2 + 5\left(\frac{1}{2}BE\right)^2$

$= 8AB^2 + \left(\frac{27}{4} + \frac{5}{4}\right)BE^2$

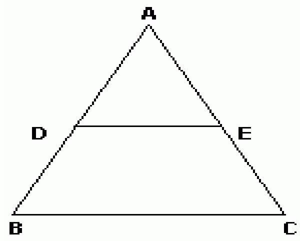
$= 8AB^2 + 8BE^2$

$= 8(AB^2 + BE^2)$

$= 8AE^2$.

Question-29

In fig., ABC is a triangle in which AB = AC. D and E are points on the sides AB and AC respectively such that AD = AE. Show that the points B, C, E and D are concyclic.



Solution:

Given: In ΔABC, AB = AC. D and E are points on the sides AB and AC respectively such that AD = AE.

To Prove: Points B, C, E and D are concyclic.

Proof: In order to prove that the points B, C, E and D are concyclic, it is sufficient to show that ∠ABC + ∠CED = 180° and ∠ACB + ∠BDE = 180°.

In ΔABC, we have

AB = AC and AD = AE

AB - AD = AC - AE

DB = EC

Thus, we have

AD = AE and DB = EC.

$\frac{AD}{DB} = \frac{AE}{EC}$

DE || BC [By the converse of Thale's Theorem]

∠ABC = ∠ADE [Corresponding angles]

∠ABC + ∠BDE = ∠ADE + ∠BDE [adding ∠BDE on both sides]

∠ABC + ∠BDE = 180°

∠ACB + ∠BDE = 180° [Since AB = AC Therefore ∠ABC = ∠ACB]

Again DE||BC

∠ACB = ∠AED

∠ACB + ∠CED = ∠AED + ∠CED [Adding ∠CED on both sides]

∠ACB + ∠CED = 180°

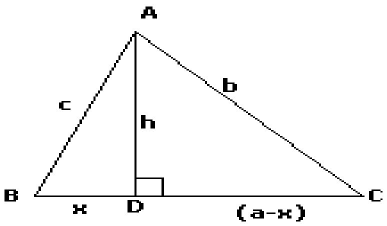
∠ABC + ∠CED = 180° [Since ∠ABC = ∠ACB]

Therefore BDEC is a cyclic quadrilateral.

Hence, B, C, E and D are concyclic points.

Question-30

In fig, ∠ B < 90° and segment AD ^ BC, show that $b^2 = h^2 + a^2 + x^2 - 2ax$



Solution:

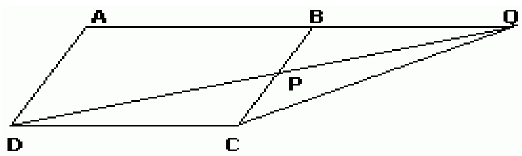
Given: In ΔABC, ∠ B < 90° and segment AD ⊥ BC.

To prove: $b^2 = h^2 + a^2 + x^2 - 2ax$

Proof:
$b^2 = h^2 + (a - x)^2$
$b^2 = h^2 + a^2 + x^2 - 2ax$.

Question-31

In the given figure, ABCD is a parallelogram P is a point on BC, such that BP : PC = 1 : 2. DP produced meets AB produced at Q. Given area of triangle CPQ = 20 m^2, calculate the area of triangle DCP.

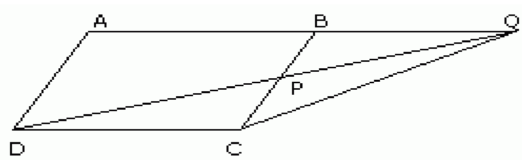


Solution:

Given: ABCD is a parallelogram P is a point on BC, such that BP : PC = 1 : 2.
DP produced meets AB produced at Q.
Area of triangle CPQ = 20 m^2.

To Find: Area of triangle DCP.

Construction: Join DB.



∠ BPQ = ∠ DPC (Vertically opposite angles)

∠ BQP = ∠ PDC (alternate angles, BQ || DC, DQ meets them)

∴ Δ BPQ ~ Δ CPD (AA similarity)

BP/CP = ½ (Given)

$\frac{\text{area}\Delta BPQ}{\text{area}\Delta CPD} = \left(\frac{BP}{CP}\right)^2 = \frac{1}{4}$

∴ Area D CPD = Area D BPQ

$\frac{\text{area}\Delta BPQ}{\text{area}\Delta CPD} = \left(\frac{BP}{CP}\right) = \frac{1}{2}$ and area ΔCPQ = 20 cm^2 (Given)

Area ΔBPQ = 10 cm^2

Area ΔCPD = 40 cm^2

$\frac{\text{area}\Delta DBC}{\text{area}\Delta DPC} = \frac{3}{2}$ (Proportional to bases BC and PC)

$\frac{\text{area}\Delta DBC}{40} = \frac{3}{2}$ ∴ Area Δ DBC = 40 × 3/2 = 60 cm^2.