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Analysis of Composite Laminates Subjected to Hygrothermal and Mechanical Loads



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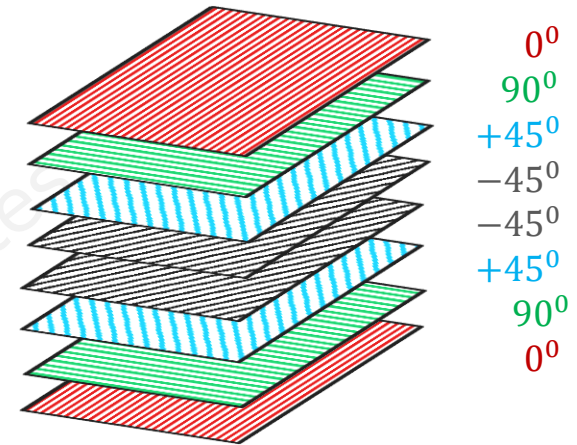
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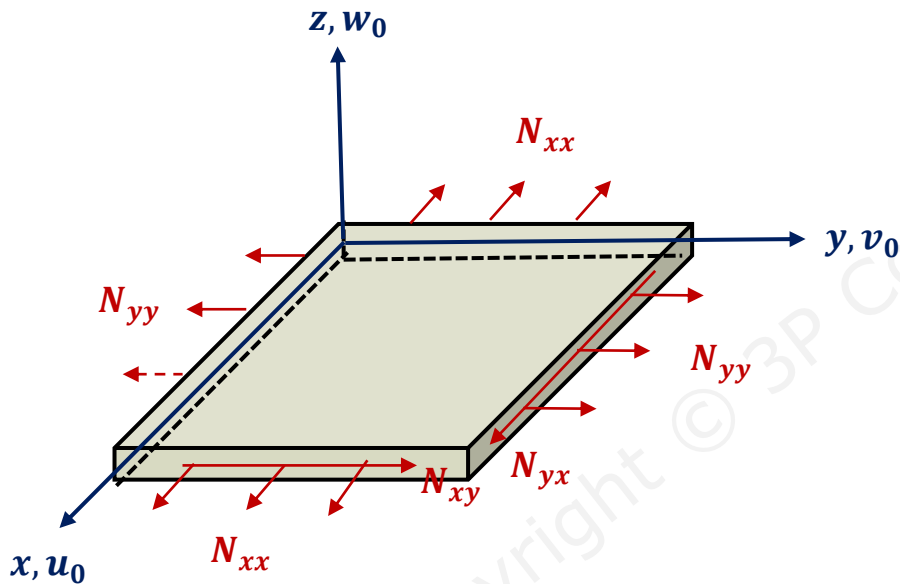
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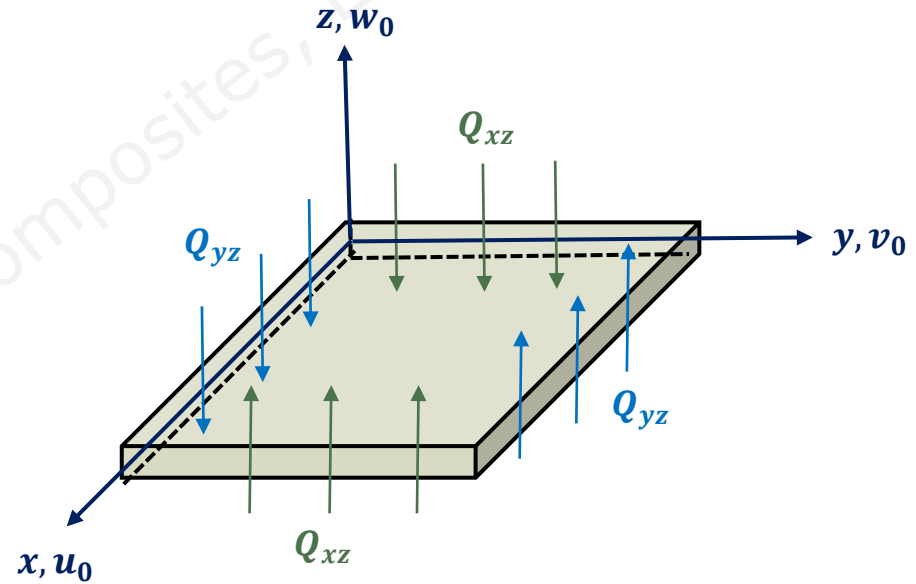


Overview

- ❖ 3pcsolver001 performs fundamental analysis of composite laminates subjected to static forces and moments per unit length, uniform temperature variations and moisture effects. Positive sign conventions for applied force and moment resultants are shown below:

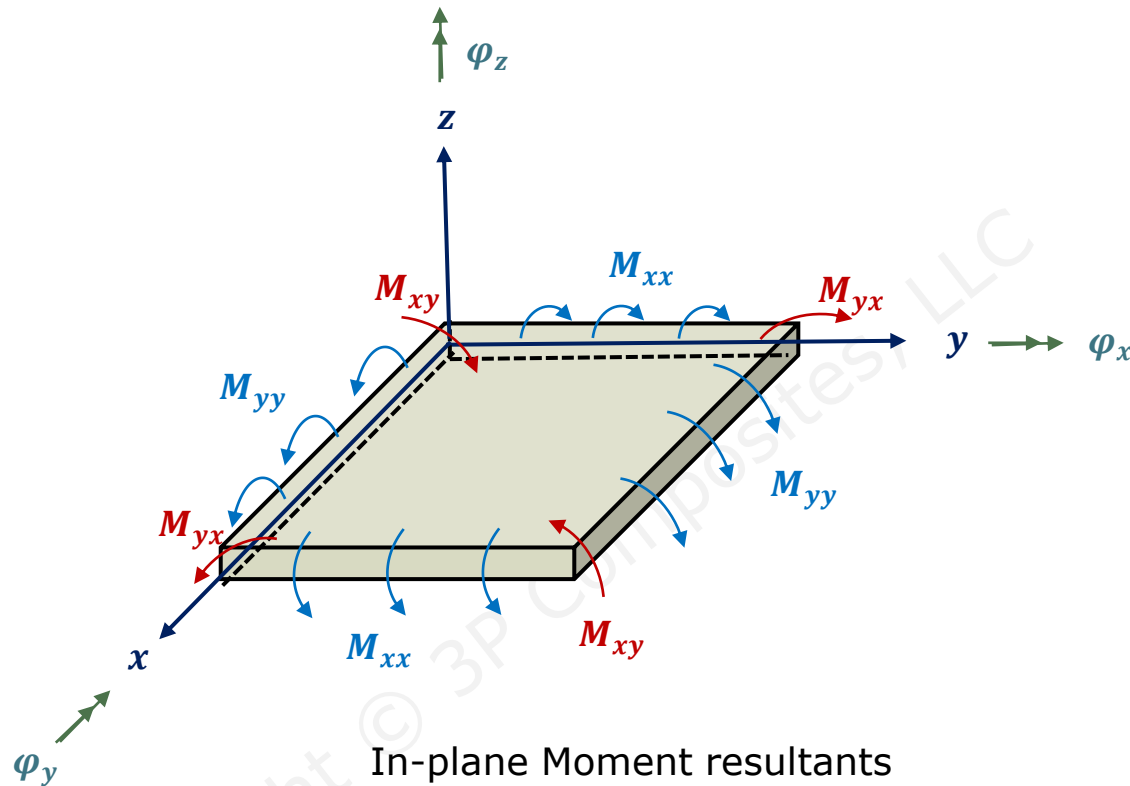


In-plane Force resultants
(Force per unit Length)



Transverse Force resultants
(Force per unit Length)

Overview



In-plane Moment resultants
(Force-length per unit Length)

- ❖ Positive bending moments M_{xx} and M_{yy} result in tension at the top surface. Conventionally the plies or laminae in the laminate are laid-up or stacked from bottom-to-top. Hence, the positive moments M_{xx} and M_{yy} would result in tension in the topmost ply and compression in the bottommost ply. Similarly, positive twisting moment M_{xy} results in a positive shear strain in the topmost ply and a negative shear strain in the bottommost ply

Applications

- ❖ The analysis is applicable to laminates built-up (or fabricated) from a LAMINA that
 - has any kind of continuous FIBER such as boron, carbon, graphite, glass, Kevlar, Aramid, polyester, natural fibers, etc.,
 - is in any type of broad form such as unidirectional, bi-directional 2D textile weaves like plain weave, twill and harness, biaxial and triaxial braids, chopped random continuous fibers, non-crimp, nonwoven fabrics, etc.
 - Is impregnated with any RESIN/MATRIX, thermoset or thermoplastic systems such as epoxy, polyester, vinyl ester, polyurethane, phenolic, cyanate ester, bis-maleimide, polyimides, benzoxazine, Acrylic, ABS, Polylactic acid PLA, Polybenzimidazole PBI, Polyether sulfone PES, Polyoxymethylene POM, Polyether ether ketone PEEK, Polyetherimide PEI, Polyphenylene oxide PPO, Polyphenylene sulfide PPS, Polystyrene PS, Polypropylene PP, Polyvinyl chloride PVC, Teflon PTFE, etc.
 - is cured using any MANUFACTURING PROCESS such as Autoclave, Resin Transfer Molding like VARTM, SQRTM, RIM, SRIM, Filament Winding, Pultrusion, Compression Molding, Wet-lay up, etc.
- ❖ The analysis is equally applicable to Hybrid Laminates manufactured from a single or multiple types of lamina materials and/or ply broad forms or fiber types or single or multiple materials systems or their combinations

Theoretical Background

- ❖ **3pcsolver001** solver is based on Mindlin-Type First-Order Shear Deformation Theory (FSDT) of laminated plates. Based on such theory, the constitutive law for the laminated plates is given as

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} + \begin{Bmatrix} N_{xx}^T \\ N_{yy}^T \\ N_{xy}^T \\ M_{xx}^T \\ M_{yy}^T \\ M_{xy}^T \end{Bmatrix} + \begin{Bmatrix} N_{xx}^M \\ N_{yy}^M \\ N_{xy}^M \\ M_{xx}^M \\ M_{yy}^M \\ M_{xy}^M \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{66} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \\ \kappa_{xx}^0 \\ \kappa_{yy}^0 \\ \kappa_{xy}^0 \end{Bmatrix}$$

$$\begin{Bmatrix} Q_{yz} \\ Q_{xz} \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}$$

- ❖ In the equations given above, N_{ij} 's are the in-plane force resultants, M_{ij} 's are the moment resultants, and Q_{ij} 's are the transverse force resultants. Superscripts T and M denote thermal and moisture resultant terms that are dependent upon the coefficients of thermal expansions and moisture absorptions, respectively, of the laminae. ε_{xx}^0 , ε_{yy}^0 and γ_{xy}^0 are the in-plane extensional and shear strains, γ_{yz}^0 and γ_{xz}^0 are the transverse shear strains, and κ_{xx}^0 , κ_{yy}^0 and κ_{xy}^0 are bending and twisting curvatures of the reference surface of the laminate (usually the middle surface, equidistant from top and bottom plies of the laminate)

Theoretical Background

- ❖ Given the laminate stack-up, lamina/ply material properties and allowables, **3pcsolver001** solver calculates reference surface strains and curvatures, ply-by-ply strains and stresses, and failure indices / Margin of Safeties for a laminate subjected to Hygro-thermo-mechanical loads. The solver uses widely established laminated plate theories, and strain and stress transformations equations to compute $[A]$, $[B]$, $[D]$ stiffness matrices, effective in-plane and flexural engineering constants, and effective hygrothermal engineering constants for the laminate
- ❖ Details of the theoretical approach along with numerous verification and application examples are provided in the training module **3pcmodule001**

Inputs

- ❖ All inputs should be in consistent units. Use either (N, m, kg, Celsius, N/mm N-m/m) OR (N, mm, Kg, Celsius, N/mm, N-mm/mm) or (lbs, in, Fahrenheit, lb/in, lb-in/in) consistently. Inputs in scientific notation (0.0+e) are acceptable
- ❖ Input process is intuitive and uses the following logical order:
 - Analysis Options
 - Materials
 - Plies / Laminae
 - Laminates
 - Panels
 - Loads
- ❖ The type of analysis selected dictates the required inputs. In general, the loads are applied to the panels consisting of laminates, which are built from specifically oriented plies/laminae fabricated from individual materials

Inputs: Analysis Options

❖ Analysis Options:

Four types of analyses can be performed using this solver, viz. (i) Mechanical (ii) Thermo-mechanical (iii) Hygro-mechanical, and (iv) Hygro-thermo-mechanical, In addition, Failure analysis of the laminate can be performed using First-ply failure criteria using one of the four commonly used composite failure theories, viz. (i) Maximum Stress (ii) Maximum Strain (iii) Tsai-Hill, and (iv) Tsai-Wu. In total there are twenty (20) possible ways this solver can be used to perform analysis of composite laminates subjected to Hygro-thermo-mechanical loads. A few combination of analyses are shown below:

Analysis Options

MECHANICAL	NO FAILURE ANALYSIS
------------	---------------------

Analysis Options

THERMO-MECHANICAL	TSAI-HILL
-------------------	-----------

Analysis Options

HYGRO-MECHANICAL	MAX STRAIN
------------------	------------

Analysis Options

HYGRO-THERMO-MECHANICAL	MAX STRESS
-------------------------	------------

Analysis Options

MECHANICAL	MAX STRAIN
------------	------------

Analysis Options

HYGRO-THERMO-MECHANICAL	TSAI-WU
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Inputs: Materials

❖ Material Properties:

In the SI system, MPa or Pa, and in the US system Psi are used to input the orthotropic lamina Moduli E_1 , E_2 , G_{12} , G_{13} and G_{23} . Coefficient of thermal expansions α_1 and α_2 are expressed in mm/mm or m/m per degree Celsius in SI system, and in/in per Fahrenheit in the US system. Similarly, Coefficient of moisture expansions β_1 and β_2 are in mm/mm per Kg/Kg or m/m per Kg/Kg in SI system and in/in per lb/lb in the US system. Coefficients of Thermal and Moisture expansions are required to perform hygrothermal analysis due change in temperature and/or moisture content. ν_{12} is major Poison's ratio. Multiple lamina types and lamina materials can be input by simply clicking the '+' sign on the extreme right. Based on the type of analyses selected, the required material inputs for an orthotropic Lamina can vary as shown below:

Materials    

ID	E_1	E_2	G_{12}	G_{23}	G_{13}	ν_{12}	
1	0	0	0	0	0	0	+ -
MUST BE +VE REAL MUST BE +VE REAL MUST BE +VE REAL MUST BE +VE REAL MUST BE +VE REAL MUST BE +VE REAL							

Materials    

ID	E_1	E_2	G_{12}	G_{23}	G_{13}	ν_{12}	α_1	α_2	β_1	β_2	
1	0	0	0	0	0	0	0	0	0	0	+ -
MUST BE +VE REAL MUST BE +VE REAL MUST BE +VE REAL MUST BE +VE REAL MUST BE +VE REAL MUST BE +VE REAL											

Inputs: Materials

❖ Material Allowables:

Additional inputs are required to perform laminate failure analysis. Depending upon the type of failure theory selected for an orthotropic lamina, either strength ($\sigma_{11}^T, \sigma_{11}^C, \sigma_{22}^T, \sigma_{22}^C$ and τ_{12}) or strain ($\epsilon_{11}^T, \epsilon_{11}^C, \epsilon_{22}^T, \epsilon_{22}^C$ and γ_{12}) based material allowables, in tension, compression and shear should be input as shown below:

Allowables ⓘ

ID	σ_{11}^T	σ_{11}^C	σ_{22}^T	σ_{22}^C	τ_{12}
1	0	0	0	0	0

Allowables ⓘ

ID	ϵ_{11}^T	ϵ_{11}^C	ϵ_{22}^T	ϵ_{22}^C	γ_{12}
1	0	0	0	0	0

Allowables ⓘ

ID	σ_{11}^T	σ_{22}^T	τ_{12}
1	0	0	0

Strength allowables are input as MPa or Pa in SI system, and in Psi in the US systems, and should be consistent with the unit system used for input of Moduli

Inputs: Plies

❖ Plies/Laminae:

Types of plies in a laminate are required as input. Each ply type is defined by its angle (or orientation) in degrees, material type and its thickness. Material of a ply/lamina can be selected from a predefined list of materials that are input in the Material Properties Section above. The thickness of the ply or lamina is in mm or m in the SI system or inch in the US system. Multiple ply or lamina types can be input by simply clicking the '+' sign on the extreme right. The required ply/lamina type inputs with few examples are shown below:

Plies    

ID	Angle (deg)	Material	Thickness		
1	0	Uni ▾	0.005	+	-
2	45	PW ▾	0.010	+	-
3	90	Uni ▾	0.005	+	-

Plies    

ID	Angle (deg)	Material	Thickness		
1	0	CEP ▾	0.005	+	-
2	30	Flax ▾	0.010	+	-
3	60	CEP ▾	0.005	+	-

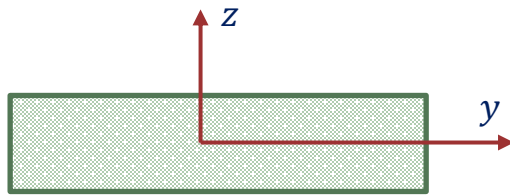
Plies    

ID	Angle (deg)	Material	Thickness		
1	0	CEP ▾	0.005	+	-
2	45	CEP ▾	0.005	+	-
3	-45	CEP ▾	0.005	+	-
4	90	CEP ▾	0.005	+	-
5	0	Flax ▾	0.01	+	-

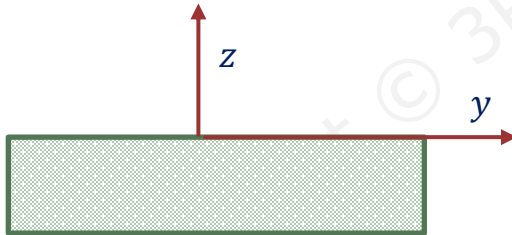
Inputs: Laminates

❖ Laminates:

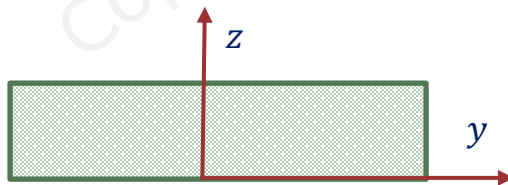
Multiple laminates can be quickly created by defining their stacking sequences using the plies defined in the previous step. Laminate Offsets can be incorporated in the analysis in three different ways, viz (i) Middle (usually default), (ii) Top and (iii) Bottom. Laminate offsets considered are depicted below:



offset = Middle(default)
Reference plane = Mid-Surface



Offset = Top
Reference plane = Upper-Surface







Offset = bottom
Reference plane = Lower-Surface

Inputs: Laminates

❖ Laminates:

Hybrid laminates can be defined using different ply and material combinations established in the previous steps. Additional laminates can be added by simply clicking the '+' sign on the extreme right. A few examples of laminates and their inputs are shown below:

Laminates    

ID	Stacking Sequence	Stacking Sequence (Angle)	Offset		
CEP-QI	2,3,1,4,4,1,3,2	45, -45, 0, 90, 90, 0, -45	Middle	▼	+ -
CEP-Cross Ply	1,4,1,4,1,4,1,4	0, 90, 0, 90, 0, 90, 0, 90	Bottom	▼	+ -
CEP-Angle Ply	2,3,2,3,2,3,2,3	45, -45, 45, -45, 45, -45	Top	▼	+ -
CEP-Flax Hybrid	1,2,3,4,5	0, 45, -45, 90, 0	Middle	▼	+ -

Inputs: Panels

❖ Panels:

Panels are defined using the laminates as shown below. While panels have same definitions as the laminates, their unique ID helps in the analyses of a specific laminate for multiple load cases more organized. Additional panels can be added by simply clicking the '+' sign on the extreme right (see below):

Panels ⓘ ⬆ ⬇

ID	Laminate		
1	CEP-QI	+	-
2	CEP-Cross Ply	+	-
3	CEP-Angle Ply	+	-
4	CEP-QI	+	-
5	CEP-Flax Hybrid	+	-
6	CEP-Flax Hybrid	+	-

Panels ⓘ ⬆ ⬇

ID	Laminate		
1	CEP-QI	+	-
2	CEP-Cross Ply	+	-
3	CEP-Angle Ply	+	-
4	CEP-Flax Hybrid	+	-

Inputs: Loads

❖ Loads:

Hygrothermomechanical loads can be applied to the panels. Single or multiple panels (or laminates) can be analyzed for single or multiple load cases (upto 100 max). For analyses of laminates subjected to mechanical loads, in-plane force N_{xx} , N_{yy} & N_{xy} and moment M_{xx} , M_{yy} & M_{xy} resultants, and transverse force resultants Q_{xz} & Q_{yz} are provided as inputs. For thermomechanical analysis, temperature change ΔT is required as input. For hygromechanical analysis, change in moisture absorption ΔC is required as input. Examples of the load inputs for typical mechanical or a complete hygrothermomechanical analyses are shown below. Additional load cases can be added by simply clicking the '+' sign on the extreme right as shown below:

Loads ⓘ ⬆ ⬇

ID	Panel	N_{xx}	N_{yy}	N_{xy}	M_{xx}	M_{yy}	M_{xy}	Q_{yz}	Q_{xz}	+	-
1	1	100	0	25	0	0	0	0	0	+	-
2	1	0	0	0	20	-10	0	0	0	+	-
3	2	10	-10	100	2	-1	-3.2	0	0	+	-
4	3	0	30	5.5	0	0	0	2.2	10	+	-
5	3	0	0	0	0	-10	0	5.2	0	+	-
6	4	20	0	0	-10	20	0	0	0	+	-
7	4	0	-10	15	0	0	2.5	0	0	+	-

Inputs: Loads

Loads ⓘ ⬆ ⬇

ID	Panel	N_{xx}	N_{yy}	N_{xy}	M_{xx}	M_{yy}	M_{xy}	Q_{yz}	Q_{xz}	ΔT	ΔC		
1	1 ▾	100	0	25	0	0	0	0	0	0	0	+	-
2	1 ▾	0	0	0	20	-10	0	0	0	0	0	+	-
3	2 ▾	10	-10	100	2	-1	-3.2	0	0	0	0	+	-
4	3 ▾	0	30	5.5	0	0	0	2.2	10	0	0	+	-
5	3 ▾	0	0	0	0	-10	0	5.2	0	0	0	+	-
6	4 ▾	20	0	0	-10	20	0	0	0	0	0	+	-
7	4 ▾	0	-10	15	0	0	2.5	0	0	0	0	+	-
8	1 ▾	50	0	0	0	10	0	0	0	-100	0	+	-
9	2 ▾	-10	0	20	0	0	0	0	0	0	1.5	+	-

In-plane force and transverse force resultants are input as N/mm or N/m in SI system, and lb/in in provided as inputs. Moment resultants are input in N-mm/mm or N-m/m in SI system, and lb-in/in in the US system. Differential Temperature is input in degree Celsius in SI system, and in Fahrenheit in the US system. Difference in moisture absorption is prescribed as Kg/Kg in SI system and lb/lb in the US system

Outputs

❖ Analysis Outputs:

Once all the Input steps viz., Analysis Options, Materials, Plies / Laminae, Laminates , Panels and Loads are completed, analyses can be run by clicking the “submit” button. Maximum 100 Load Cases can be analyzed at one time

Submit

Upon completion of analyses, an output is displayed for each Load ID in the window underneath

Output ↓

Depending upon the analysis option selected, the analyses output contains the following information at minimum.

- Material Properties and Laminate Information
- Laminate [A], [B], [D] stiffness matrices
- Effective laminate in-plane and flexural engineering constants
- Effective laminate hygrothermal engineering constants
- Applied Hygro-thermo-mechanical loads
- Laminate Reference plane strains and curvatures
- Ply-by-Ply in-plane strains and Stresses (TOTAL, MECHANICAL OR RESIDUAL) in global and ply coordinate systems
- Ply-by-Ply transverse shear strains and stresses
- Laminate/Lamina failure analysis– Failure Indices or Margins of Safety

Outputs

❖ Analysis Outputs:

Note that all output is consistent with the unit system used during the material, lamina, laminate, and loads Inputs.

- Laminate [A] stiffness matrices N/m or N/mm or lb/in
- Laminate [B] stiffness matrices N-m/m or N-mm/mm or lb-in/in
- Laminate [D] stiffness matrices N-m or N-mm or lb-in
- Effective laminate in-plane, flexural and hygrothermal engineering constants- same as material property inputs
- Laminate Reference plane strains – in/in, m/m, mm/mm
- Laminate Reference curvatures – 1/in, 1/mm, 1/m
- Ply-by-Ply strains – in/in, m/m, mm/mm
- Ply-by-Ply Stresses – Psi, MPa, Pa
- Laminate/Lamina failure analysis– Failure Indices or Margins of Safety – Dimensionless

Outputs: Laminate and Stiffnesses

3pc-solver001, v1.3b3

LOADS ID PANEL ID
1 1

MATERIAL PROPERTIES

ID	E1	E2	G12	G23	G13	v12	
Tuttle	2.25e+07	1.10e+06	6.40e+05	6.40e+05	6.40e+05	6.40e+05	0.3400

LAMINATE GEOMETRY

STACKING SEQUENCE (PLY ANG): [+0.0 , +0.0 , +0.0 , +0.0 , +0.0 , +0.0 , +0.0 , +0.0]
STACKING SEQUENCE (PLY MAT): [Tuttle , Tuttle , Tuttle , Tuttle , Tuttle , Tuttle , Tuttle , Tuttle]
TOTAL THICKNESS: 0.0600
OFFSET: 0.0000

LAMINATE PROPERTIES

A MATRIX

+1357672.96	+22567.54	+0.00
+22567.54	+66375.12	+0.00
+0.00	+0.00	+38400.00

A MATRIX - TRANSVERSE SHEAR

+38400.00	+0.00
+0.00	+38400.00

B MATRIX

+0.00	+0.00	+0.00
+0.00	-0.00	+0.00
+0.00	+0.00	+0.00

D MATRIX

+407.30	+6.77	+0.00
+6.77	+19.91	+0.00
+0.00	+0.00	+11.52

Outputs: Effective Stiffnesses and Applied Loads

LAMINATE INPLANE AND FLEXURAL ENGINEERING CONSTANTS

Ex	Ey	Gxy	vxy	vyx	Efx	Efy	Gfxy	vfx	vfy
+1.24e+04	+2.88e+04	+8.76e+03	+0.0978	+0.2267	+1.24e+04	+2.88e+04	+8.76e+03	+0.0978	+0.2267

LAMINATE HYGROTHERMAL ENGINEERING CONSTANTS

alphax	alphay	alphaxy	betax	betay	betaxy
+1.87e-05	+1.20e-05	-1.17e-05	+4.50e-01	+1.50e-01	-5.20e-01

MECHANICAL LOADS AND MOMENTS

NX	NY	NXY	MX	MY	MXY	QXZ	QYZ
+250.00	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00

THERMAL LOADS AND MOMENTS

DELTAT	NXT	NYT	NXYT	MXT	MYT	MXYT
+0.00	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00

HYGRAL LOADS AND MOMENTS

DELTAH	NXH	NYH	NXYH	MXH	MYH	MXYH
+0.00	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00

MID PLANE STRAIN AND CURVATURE

EPSX0	EPSY0	GAMMAXY0	KAPPAX0	KAPPAY0	KAPPAXY0	GAMMAYZ0	GAMMAXZ0
(x 1E-6)	(x 1E-6)	(x 1E-6)	(x 1E-6)	(x 1E-6)	(x 1E-6)	(x 1E-6)	(x 1E-6)
+4026.37	-393.88	-1616.88	+0.00	+0.00	+0.00	+0.00	+0.00

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Outputs: Ply Strains and MS

PLY BY PLY INPLANE STRAINS (TOTAL)

ANGLE	Z	EPS1 (x 1E-6)	EPS2 (x 1E-6)	GAMMA12 (x 1E-6)	EPSX (x 1E-6)	EPSY (x 1E-6)	GAMMAXY (x 1E-6)
+0.0	-0.03000	-193.63	-28674.48	-25781.25	-193.63	-28674.48	-25781.25
	-0.02625	-164.00	-24918.08	-22526.04	-164.00	-24918.08	-22526.04
	-0.02250	-134.37	-21161.68	-19270.83	-134.37	-21161.68	-19270.83
+0.0	-0.02250	-134.37	-21161.68	-19270.83	-134.37	-21161.68	-19270.83
	-0.01875	-104.74	-17405.29	-16015.63	-104.74	-17405.29	-16015.63
	-0.01500	-75.11	-13648.89	-12760.42	-75.11	-13648.89	-12760.42
+0.0	-0.01500	-75.11	-13648.89	-12760.42	-75.11	-13648.89	-12760.42
	-0.01125	-45.48	-9892.49	-9505.21	-45.48	-9892.49	-9505.21
	-0.00750	-15.85	-6136.09	-6250.00	-15.85	-6136.09	-6250.00
+0.0	-0.00750	-15.85	-6136.09	-6250.00	-15.85	-6136.09	-6250.00
	-0.00375	+13.78	-2379.70	-2994.79	+13.78	-2379.70	-2994.79
	+0.00000	+43.41	+1376.70	+260.42	+43.41	+1376.70	+260.42
+0.0	+0.00000	+43.41	+1376.70	+260.42	+43.41	+1376.70	+260.42
	+0.00375	+73.04	+5133.10	+3515.63	+73.04	+5133.10	+3515.63
	+0.00750	+102.67	+8889.49	+6770.83	+102.67	+8889.49	+6770.83
+0.0	+0.00750	+102.67	+8889.49	+6770.83	+102.67	+8889.49	+6770.83
	+0.01125	+132.30	+12645.89	+10026.04	+132.30	+12645.89	+10026.04
	+0.01500	+161.93	+16402.29	+13281.25	+161.93	+16402.29	+13281.25
+0.0	+0.01500	+161.93	+16402.29	+13281.25	+161.93	+16402.29	+13281.25
	+0.01875	+191.56	+20158.69	+16536.46	+191.56	+20158.69	+16536.46
	+0.02250	+221.19	+23915.08	+19791.67	+221.19	+23915.08	+19791.67
+0.0	+0.02250	+221.19	+23915.08	+19791.67	+221.19	+23915.08	+19791.67
	+0.02625	+250.81	+27671.48	+23046.88	+250.81	+27671.48	+23046.88
	+0.03000	+280.44	+31427.88	+26302.08	+280.44	+31427.88	+26302.08

PLY BY PLY INPLANE STRAINS (MECHANICAL OR RESIDUAL)

ANGLE	Z	EPS1 (x 1E-6)	EPS2 (x 1E-6)	GAMMA12 (x 1E-6)	EPSX (x 1E-6)	EPSY (x 1E-6)	GAMMAXY (x 1E-6)
+60.0	-2.50000	+11.05	+3621.44	-3019.61	+4026.37	-393.88	-1616.88
	+0.00000	+11.05	+3621.44	-3019.61	+4026.37	-393.88	-1616.88
	+2.50000	+11.05	+3621.44	-3019.61	+4026.37	-393.88	-1616.88

PLY BY PLY TRANSVERSE SHEAR STRAINS

ANGLE	Z	GAMMA23 (x 1E-6)	GAMMA13 (x 1E-6)	GAMMAXZ (x 1E-6)	GAMMAXZ (x 1E-6)
+60.0	-2.50000	+0.00	+0.00	+0.00	+0.00
	+0.00000	+0.00	+0.00	+0.00	+0.00
	+2.50000	+0.00	+0.00	+0.00	+0.00

Outputs: Ply Stresses and FIs

PLY BY PLY INPLANE STRESSES (TOTAL)							FAILURE ANALYSIS - TSAI-WU			
ANGLE	Z	SIG1	SIG2	TAU12	SIGX	SIGY	TAUXY	FI	MS	
+0.0	-0.03000	-15166.64	-31794.03	-16500.00	-15166.64	-31794.03	-16500.00	-16500.00	+2.58	-0.38
	-0.02625	-13083.30	-27627.36	-14416.67	-13083.30	-27627.36	-14416.67	-14416.67	+1.95	-0.28
	-0.02250	-10999.97	-23460.70	-12333.33	-10999.97	-23460.70	-12333.33	-12333.33	+1.41	-0.16
+0.0	-0.02250	-10999.97	-23460.70	-12333.33	-10999.97	-23460.70	-12333.33	-12333.33	+1.41	-0.16
	-0.01875	-8916.64	-19294.03	-10250.00	-8916.64	-19294.03	-10250.00	-10250.00	+0.96	+0.02
	-0.01500	-6833.30	-15127.36	-8166.67	-6833.30	-15127.36	-8166.67	-8166.67	+0.60	+0.29
+0.0	-0.01500	-6833.30	-15127.36	-8166.67	-6833.30	-15127.36	-8166.67	-8166.67	+0.60	+0.29
	-0.01125	-4749.97	-10960.70	-6083.33	-4749.97	-10960.70	-6083.33	-6083.33	+0.32	+0.77
	-0.00750	-2666.64	-6794.03	-4000.00	-2666.64	-6794.03	-4000.00	-4000.00	+0.13	+1.80
+0.0	-0.00750	-2666.64	-6794.03	-4000.00	-2666.64	-6794.03	-4000.00	-4000.00	+0.13	+1.80
	-0.00375	-583.30	-2627.36	-1916.67	-583.30	-2627.36	-1916.67	-1916.67	+0.02	+5.53
	+0.00000	+1500.03	+1539.30	+166.67	+1500.03	+1539.30	+166.67	+166.67	+0.01	+12.08
+0.0	+0.00000	+1500.03	+1539.30	+166.67	+1500.03	+1539.30	+166.67	+166.67	+0.01	+12.08
	+0.00375	+3583.36	+5705.97	+2250.00	+3583.36	+5705.97	+2250.00	+2250.00	+0.08	+2.65
	+0.00750	+5666.70	+9872.64	+4333.33	+5666.70	+9872.64	+4333.33	+4333.33	+0.23	+1.08
+0.0	+0.00750	+5666.70	+9872.64	+4333.33	+5666.70	+9872.64	+4333.33	+4333.33	+0.23	+1.08
	+0.01125	+7750.03	+14039.30	+6416.67	+7750.03	+14039.30	+6416.67	+6416.67	+0.47	+0.45
	+0.01500	+9833.36	+18205.97	+8500.00	+9833.36	+18205.97	+8500.00	+8500.00	+0.80	+0.12
+0.0	+0.01500	+9833.36	+18205.97	+8500.00	+9833.36	+18205.97	+8500.00	+8500.00	+0.80	+0.12
	+0.01875	+11916.70	+22372.64	+10583.33	+11916.70	+22372.64	+10583.33	+10583.33	+1.22	-0.09
	+0.02250	+14000.03	+26539.30	+12666.67	+14000.03	+26539.30	+12666.67	+12666.67	+1.72	-0.24
+0.0	+0.02250	+14000.03	+26539.30	+12666.67	+14000.03	+26539.30	+12666.67	+12666.67	+1.72	-0.24
	+0.02625	+16083.36	+30705.97	+14750.00	+16083.36	+30705.97	+14750.00	+14750.00	+2.31	-0.34
	+0.03000	+18166.70	+34872.64	+16833.33	+18166.70	+34872.64	+16833.33	+16833.33	+2.99	-0.42

PLY BY PLY INPLANE STRESSES (MECHANICAL OR RESIDUAL)							FAILURE ANALYSIS - TSAI-WU			
ANGLE	Z	SIG1	SIG2	TAU12	SIGX	SIGY	TAUXY	FI	MS	
+60.0	-2.50000	+12.50	+37.50	-21.65	+50.00	+0.00	-0.00	+0.00	+49.60	
	+0.00000	+12.50	+37.50	-21.65	+50.00	+0.00	-0.00	+0.00	+49.60	
	+2.50000	+12.50	+37.50	-21.65	+50.00	+0.00	-0.00	+0.00	+49.60	

PLY BY PLY TRANSVERSE SHEAR STRESSES					
ANGLE	Z	TAU23	TAU13	TAUYZ	TAUXZ
+60.0	-2.50000	+0.00	+0.00	+0.00	+0.00
	+0.00000	+0.00	+0.00	-0.00	-0.00
	+2.50000	+0.00	+0.00	+0.00	+0.00

Inputs and Outputs: Consistent Units


Quantity	SI System 1	SI system 2	US System
$E_1, E_2, G_{12}, G_{13}, G_{23}$ $E_x, E_y, G_{xy}, E_{fx}, E_{fy}, G_{fxy}$	MPa (N/mm ²)	Pa (N/m ²)	Psi (lb/in ²)
$\alpha_1, \alpha_2, \alpha_x, \alpha_y, \alpha_{xy}$	mm/mm/°C	m/m/°C	in/in/°F
$\beta_1, \beta_2, \beta_x, \beta_y, \beta_{xy}$	mm/mm/Kg/Kg	m/m/Kg/Kg	in/in/lb/lb
$\sigma_{11}^T, \sigma_{11}^C, \sigma_{22}^T, \sigma_{22}^C, \tau_{12}^S, \sigma_1, \sigma_2, \tau_{12}, \tau_{23}, \tau_{13},$ $\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz}$	MPa (N/mm ²)	Pa (N/m ²)	Psi (lb/in ²)
$\varepsilon_{11}^T, \varepsilon_{11}^C, \varepsilon_{22}^T, \varepsilon_{22}^C, \gamma_{12}, \varepsilon_1, \varepsilon_2, \gamma_{12}, \gamma_{13}, \gamma_{23}, \varepsilon_{x0},$ $\varepsilon_{y0}, \gamma_{xy0}, \gamma_{yz0}, \gamma_{xz0}, \varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}$	mm/mm	m/m	in/in
Ply Angle, θ	Degree	Degree	Degree
Ply or Laminate thickness or Offset	mm	m	in
$N_{xx}, N_{yy}, N_{xy}, Q_{yz}, Q_{xz},$ $N_{xx}^T, N_{yy}^T, N_{xy}^T, N_{xx}^H, N_{yy}^H, N_{xy}^H$ [A]	N/mm	N/m	lb/in
$M_{xx}, M_{yy}, M_{xy}, M_{xx}^T, M_{yy}^T, M_{xy}^T,$ $M_{xx}^H, M_{yy}^H, M_{xy}^H$ [B]	N – mm/mm	N – m/m	lb – in/in
ΔT	°C	°C	°F
ΔC	Kg/Kg	Kg/Kg	lb/lb
[D]	N – mm	N – m	lb – in
K_{x0}, K_{y0}, K_{xy0}	1/mm	1/m	1/in





Other Features

❖ Upload/Download:

Users can upload and download material properties, Plies, Laminates, Panels and Loads data files (*.json) using the upload  and download  buttons next to these inputs. Sample input and output files can be downloaded from the 3p Composites website at www.3pcomposites.com

❖ Additional Output:

Users can review a few intermediate calculations such as minor Poisson's ratios ν_{21} , Q_{ij} for each ply type and laminate ABD by using the calculation button . Examples are shown below:

		Materials    							
	E ₁	E ₂	G ₁₂	G ₂₃	G ₁₃	ν_{12}	α_1	α_2	β_1
ID	v21								
GMS4020 PW	0.05								
GMS4020 Tape	0.0254								
2024-T3	0.3								
Rastogi_Fiberglass	0.02667								
Tuttle	0.01662								

Other Features

Plies

ID	Angle (deg)	Material	Thickness		
1	0	Tuttle	0.0075	+	-
2	90	Tuttle	0.0075	+	-

ID	Q	Q44	Q55	Qbar	Q44bar	Q45bar	Q55bar
1	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	0	640000
2	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[1106252.04, 376125.7, 0.0], [376125.7, 22627882.74, 0.0], [0.0, 0.0, 640000.0]]	640000	0	640000
3	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[6761596.54, 5481596.54, 5380407.67], [5481596.54, 6761596.54, 5380407.67], [5380407.67, 5380407.67, 5745470.85]]	640000	0	640000
4	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[6761596.54, 5481596.54, -5380407.67], [5481596.54, 6761596.54, -5380407.67], [-5380407.67, -5380407.67, 5745470.85]]	640000	0	640000

Laminates

ID	Stacking Sequence	Stacking Sequence (Angle)	Offset		
1	1,1,1,1,1,1,1	0,0,0,0,0,0,0	Middle	+	-
2	1,2,1,2,1,2,1	0,90,0,90,90,0,90,0	Middle	+	-
3	3,4,3,4,4,3,4,3	45,-45,45,-45,-45,45	Middle	+	-

ID	Thickness	A	B	D	A44	A45	A55
1	0.06	[[1357672.96, 22567.54, 0.0], [22567.54, 66375.12, 0.0], [0.0, 0.0, 38400.0]]	[[0.0, 0.0, 0.0], [0.0, -0.0, 0.0], [0.0, 0.0, 0.0]]	[[407.3, 6.77, 0.0], [6.77, 19.91, 0.0], [0.0, 0.0, 11.52]]	38400	0	38400
2	0.06	[[712024.04, 22567.54, 0.0], [22567.54, 712024.04, 0.0], [0.0, 0.0, 38400.0]]	[[0.0, 0.0, 0.0], [0.0, -0.0, 0.0], [0.0, 0.0, 0.0]]	[[286.24, 6.77, 0.0], [6.77, 140.97, 0.0], [0.0, 0.0, 11.52]]	38400	0	38400
3	0.06	[[405695.79, 328895.79, 0.0], [328895.79, 405695.79, 0.0], [0.0, 0.0, 344728.25]]	[[0.0, -0.0, 0.0], [-0.0, -0.0, 0.0], [0.0, 0.0, -0.0]]	[[121.71, 98.67, 36.32], [98.67, 121.71, 36.32], [36.32, 36.32, 103.42]]	38400	0	38400



General Information

- ❖ Subscription fee to access **3pcsolver001** is \$39/year per for a single-login license
- ❖ Training module **3pcmodule001** supports the solver **3pcsolver001**. Users' can buy the training module **3pcmodule001** online at

<https://www.3pcomposites.com/>
- ❖ 3P Composites, LLC can conduct online or in-class trainings for the **3pcsolver001** and **3pcmodule001**. The training can be adapted to meet the requirements of individual needs and/or industrial applications
- ❖ For questions, issues, comments, suggestions, trainings, please contact us at 3pcomps@gmail.com. Your feedback is appreciated in helping us continuously improve the product



Example: Strength Analysis of a Laminate

❖ Unidirectional Lamina Properties:

$E_1 = 1.8e7 \text{ psi}, E_2 = 1.6e6 \text{ psi}; G_{12} = G_{13} = 8.7e5 \text{ psi}, G_{23} = 6.4e5 \text{ psi}; \nu_{21} = 0.3, t_{ply} = 0.00525 \text{ inch}$

❖ Unidirectional Lamina Allowables:

$\varepsilon_{11}^T = 5000 \mu\varepsilon, \varepsilon_{11}^C = -5000 \mu\varepsilon, \varepsilon_{22}^T = 10000 \mu\varepsilon, \varepsilon_{22}^C = -10000 \mu\varepsilon, \gamma_{12} = 10000 \mu\varepsilon;$
 $\sigma_{11}^T = 20 \text{ ksi}, \sigma_{11}^C = -20 \text{ ksi}, \sigma_{22}^T = 50 \text{ ksi}, \sigma_{22}^C = -50 \text{ ksi}, \tau_{12} = 100 \text{ ksi};$

❖ Laminate Stack-up [15/30/45/60]_{3T}

❖ Applied Loads:

$N_{xx} = -100 \text{ lb/in}, N_{yy} = 10 \text{ lb/in}, N_{xy} = 10 \text{ lb/in}, M_{xx} = 10 \text{ lb-in/in}, M_{yy} = 20 \text{ lb-in/in}, M_{xy} = 10 \text{ lb/in}, Q_{xz} = 110 \text{ lb/in}, Q_{yz} = 10 \text{ lb/in}$

Strength Analysis of a Laminate

❖ Laminate $[15/30/45/60]_{3T}$ Stiffnesses

$$[A] = \begin{bmatrix} 569346.12 & 196265.96 & 236496.73 \\ 196265.96 & 343847.77 & 184288.91 \\ 236496.73 & 184288.91 & 220592.09 \end{bmatrix} \text{ lb/in}$$

$$[B] = \begin{bmatrix} -1848.34 & 276.93 & -269.13 \\ 276.93 & 1294.47 & 690.19 \\ -269.13 & 690.19 & 276.93 \end{bmatrix} \text{ lb}$$

$$[D] = \begin{bmatrix} -1848.34 & 276.93 & -269.13 \\ 276.93 & 1294.47 & 690.19 \\ -269.13 & 690.19 & 276.93 \end{bmatrix} \text{ lb-in}$$

$$[A]_{\text{Transverse shear}} = \begin{bmatrix} 45996.41 & 0.00 \\ 0.00 & 49133.5 \end{bmatrix} \text{ lb/in}$$

❖ Equivalent or smeared properties of laminate $[15/30/45/60]_{3T}$

$$E_x = 5.01e6 \text{ psi}, E_y = 3.01e6 \text{ psi}, G_{xy} = 1.34e6 \text{ psi}, \nu_{yx} = -0.0069, t_{lam} = 0.063 \text{ inch}$$

$$E_{xf} = 5.15e6 \text{ psi}, E_{yf} = 3.07e6 \text{ psi}, G_{xyf} = 1.33e6 \text{ psi}, \nu_{yxf} = -0.0196$$

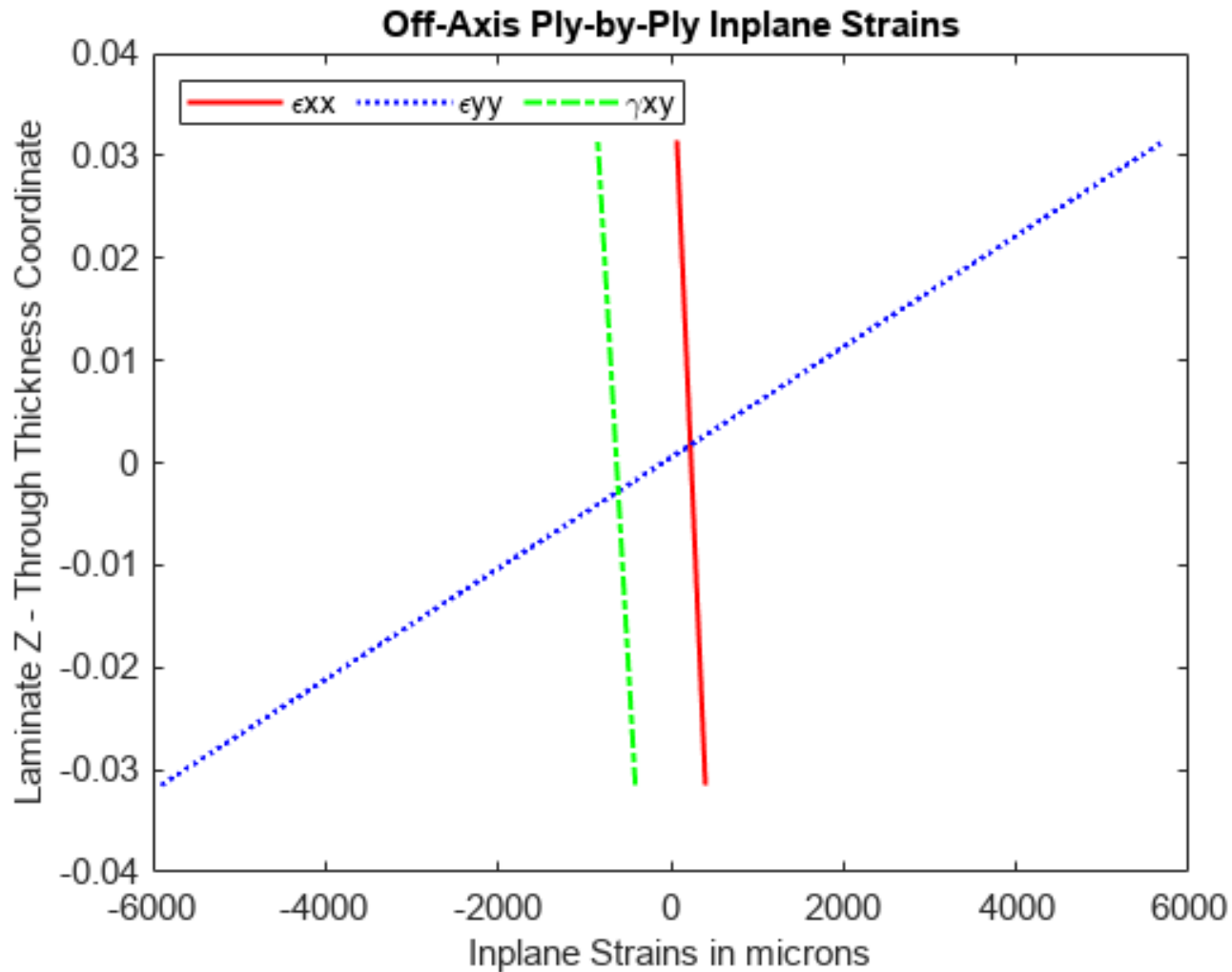
Strength Analysis of a Laminate

- ❖ Mid Plane Strains and Curvatures:

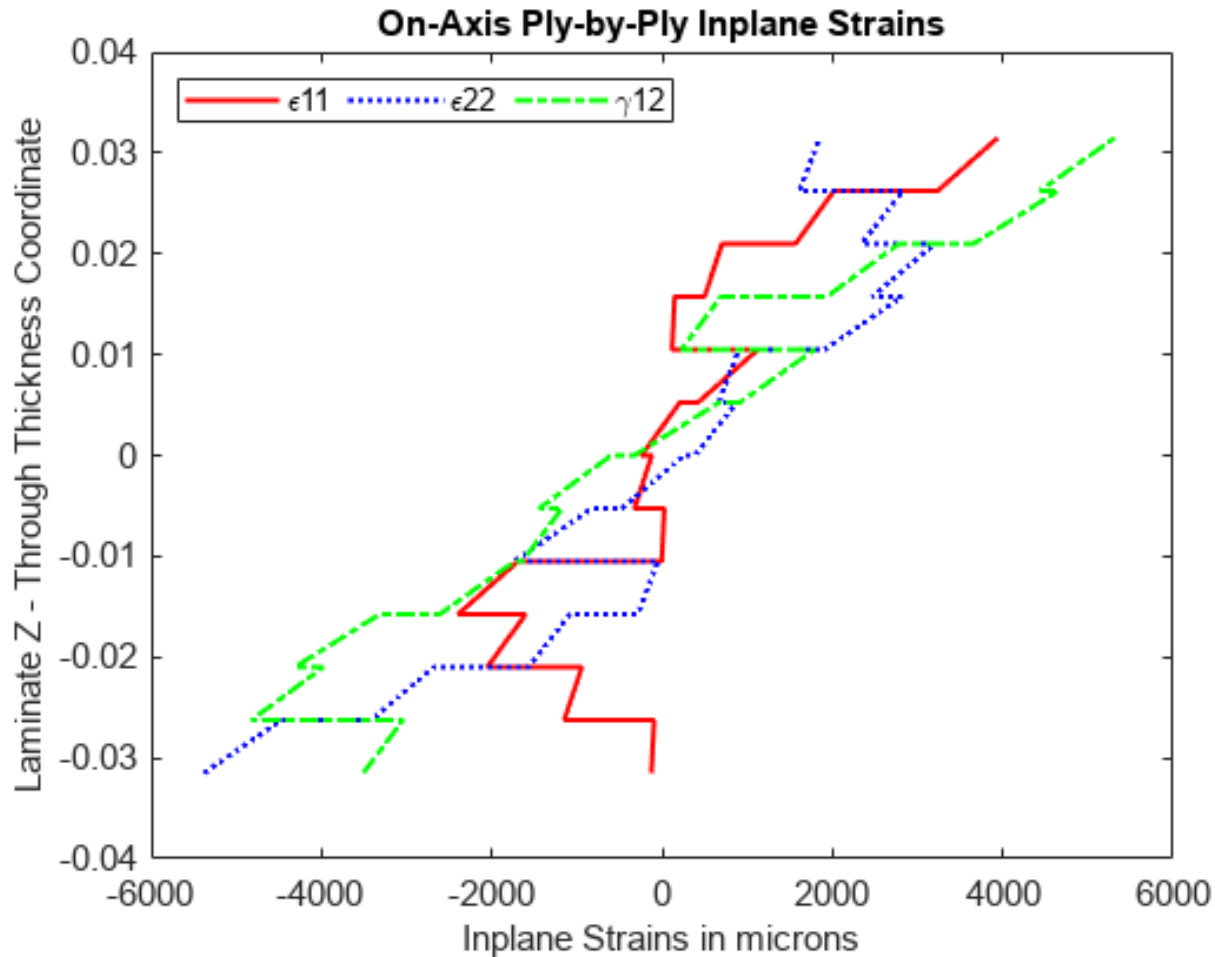
$$\varepsilon_{x0} = 235 \mu\varepsilon, \varepsilon_{y0} = -94.7 \mu\varepsilon, \gamma_{xy0} = -629.5 \mu\varepsilon, \kappa_{x0} = -0.005231 \text{ 1/in}, \kappa_{y0} = 0.18453 \text{ 1/in}, \\ \kappa_{xy0} = -0.0069083 \text{ 1/in}, \gamma_{yz0} = 217 \mu\varepsilon, \gamma_{xz0} = 2239 \mu\varepsilon$$

- ❖ MATLAB scripts are used to plot ply-by-ply strains, stresses and Failure Indices/Margin of Safeties through the thickness of the laminate

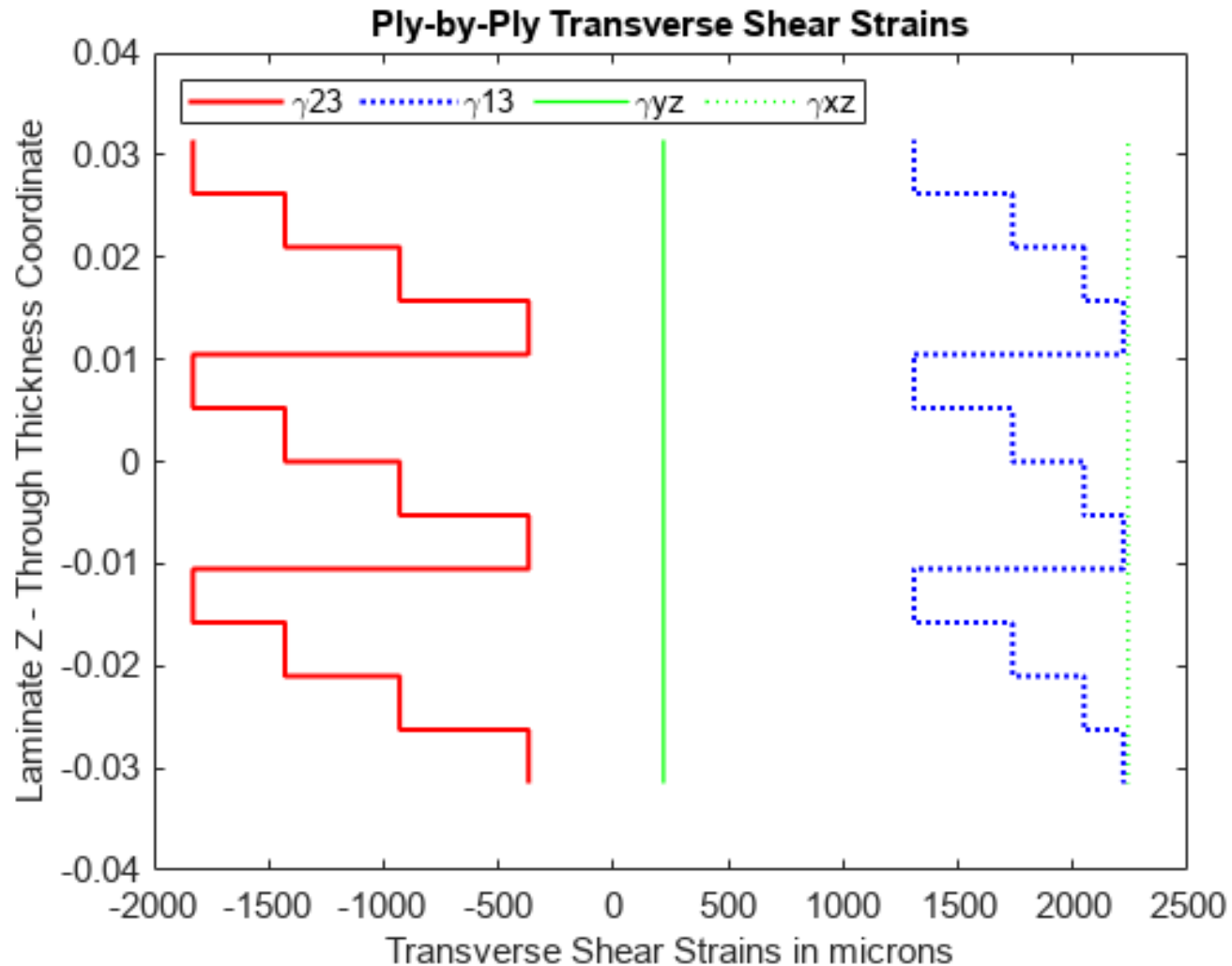
Off-Axis Ply-by-Ply In-plane Strains



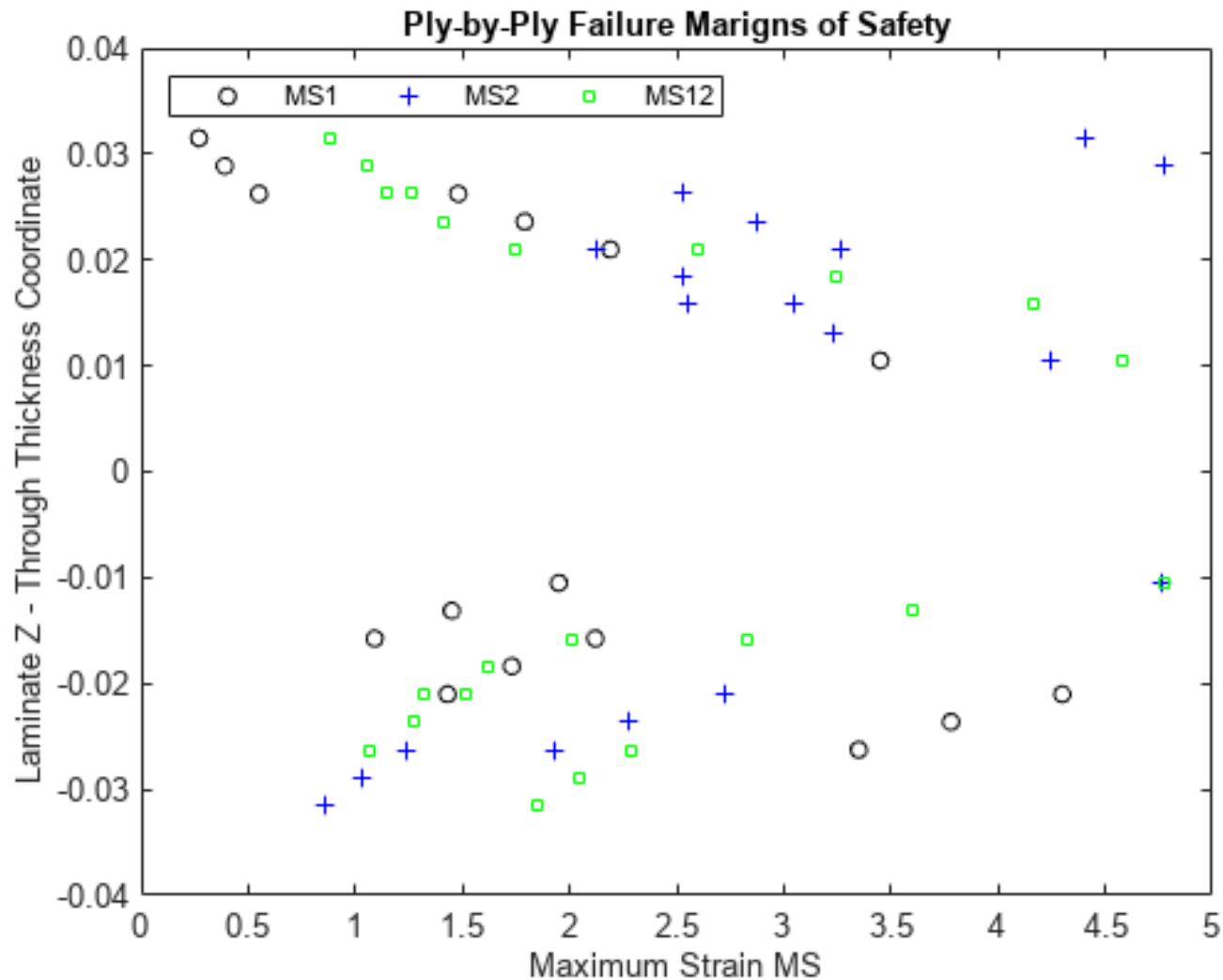
On-Axis Ply-by-Ply In-plane Strains



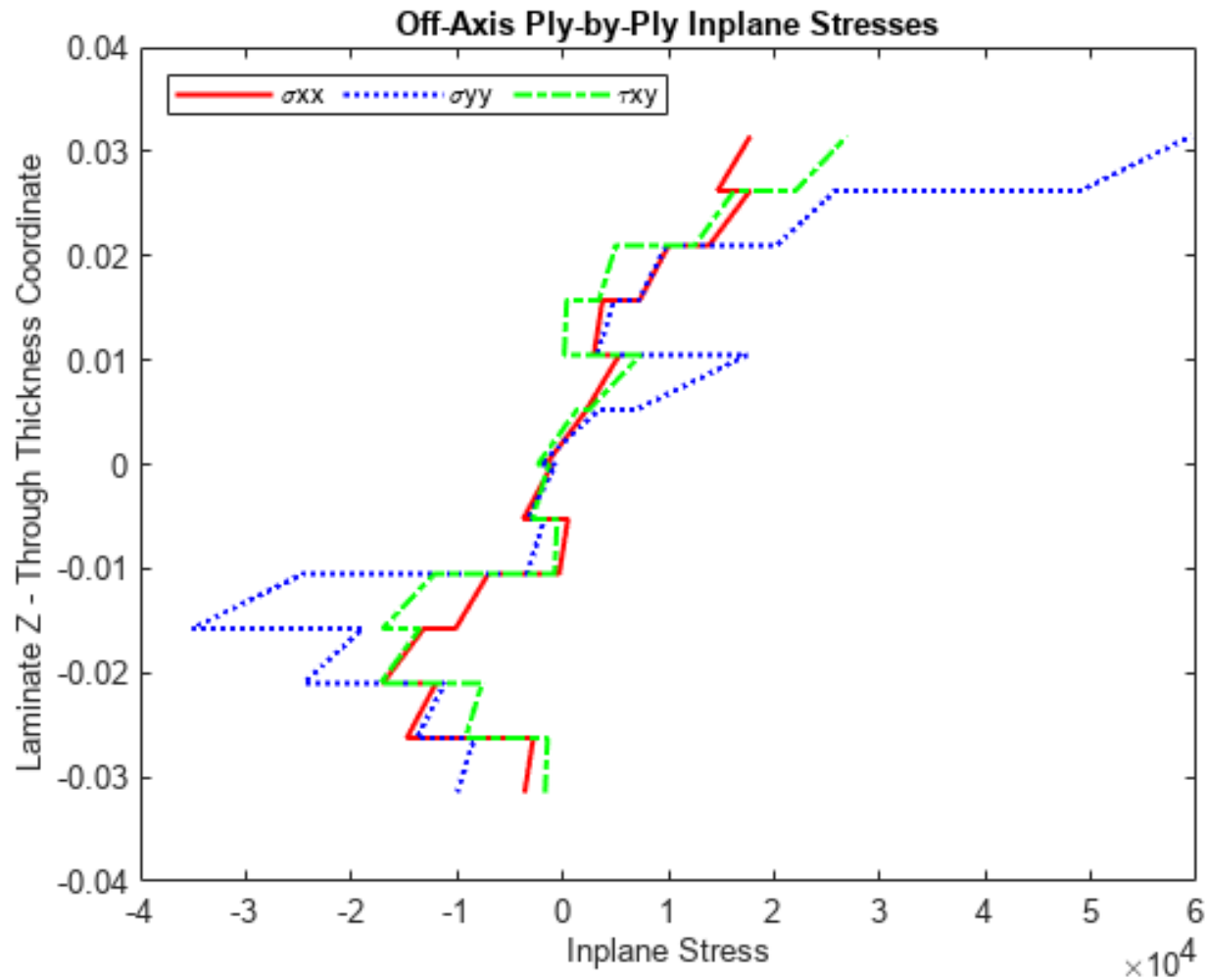
Ply-by-Ply Transverse Shear Strains



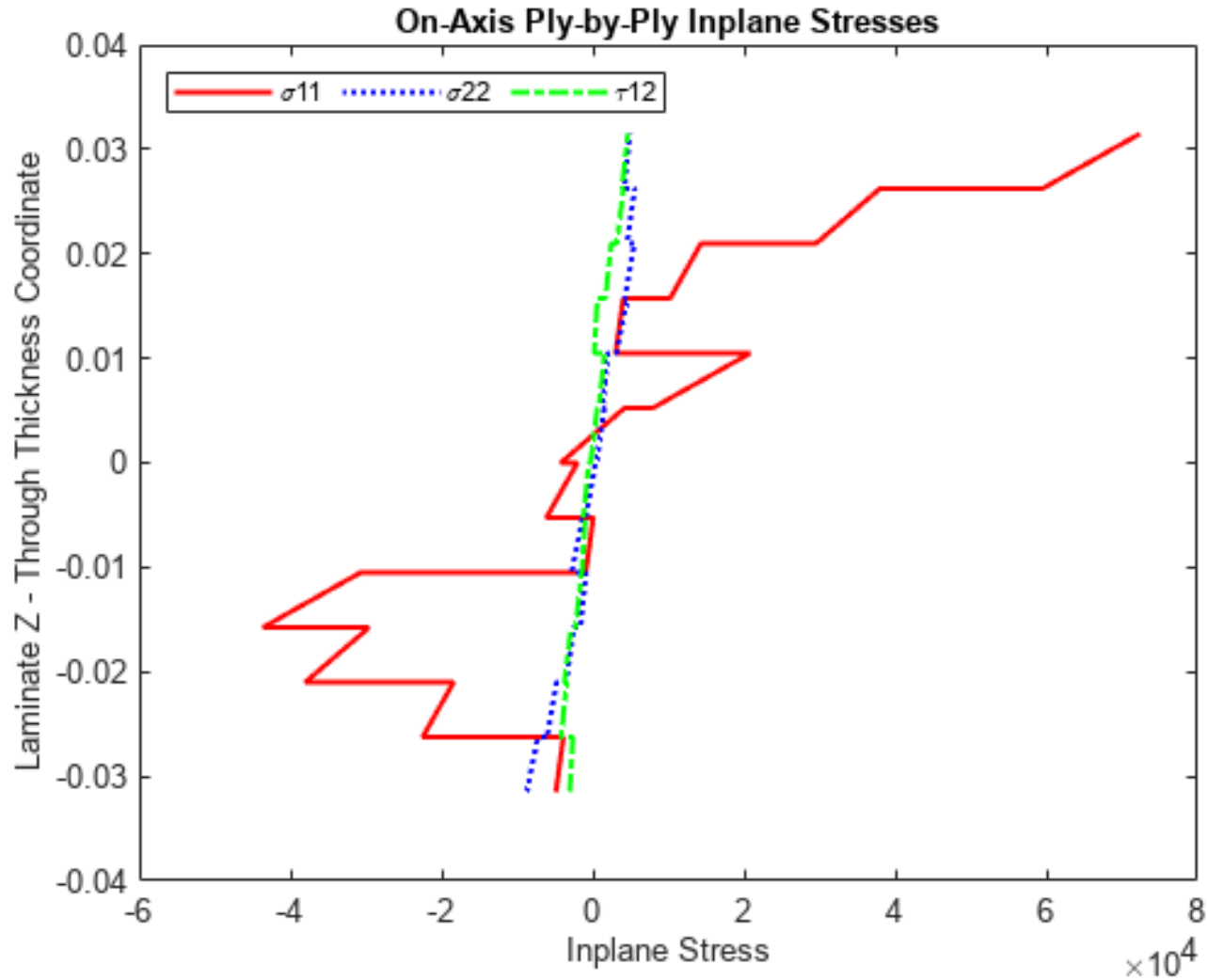
Ply-by-Ply Max Strain Margins of Safety



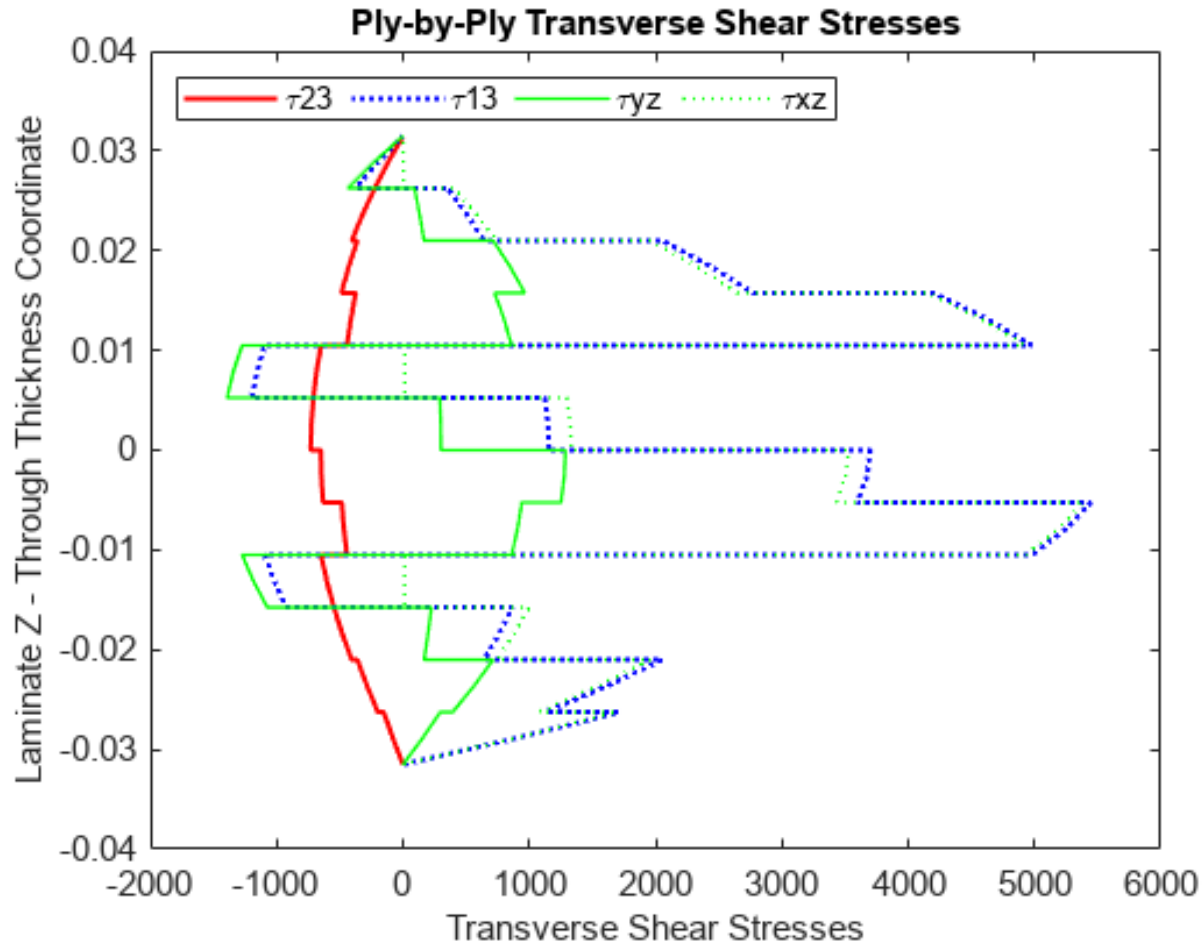
Off-Axis Ply-by-Ply In-plane Stresses



On-Axis Ply-by-Ply In-plane Stresses



Ply-by-Ply Transverse Shear Stresses



Ply-by-Ply Tsai-Wu/Hill Failure Indices & Max Stress Margins of Safety

