3PC SOLVER 003

Buckling Analysis of Simply-Supported Anisotropic Laminated Composite Plates

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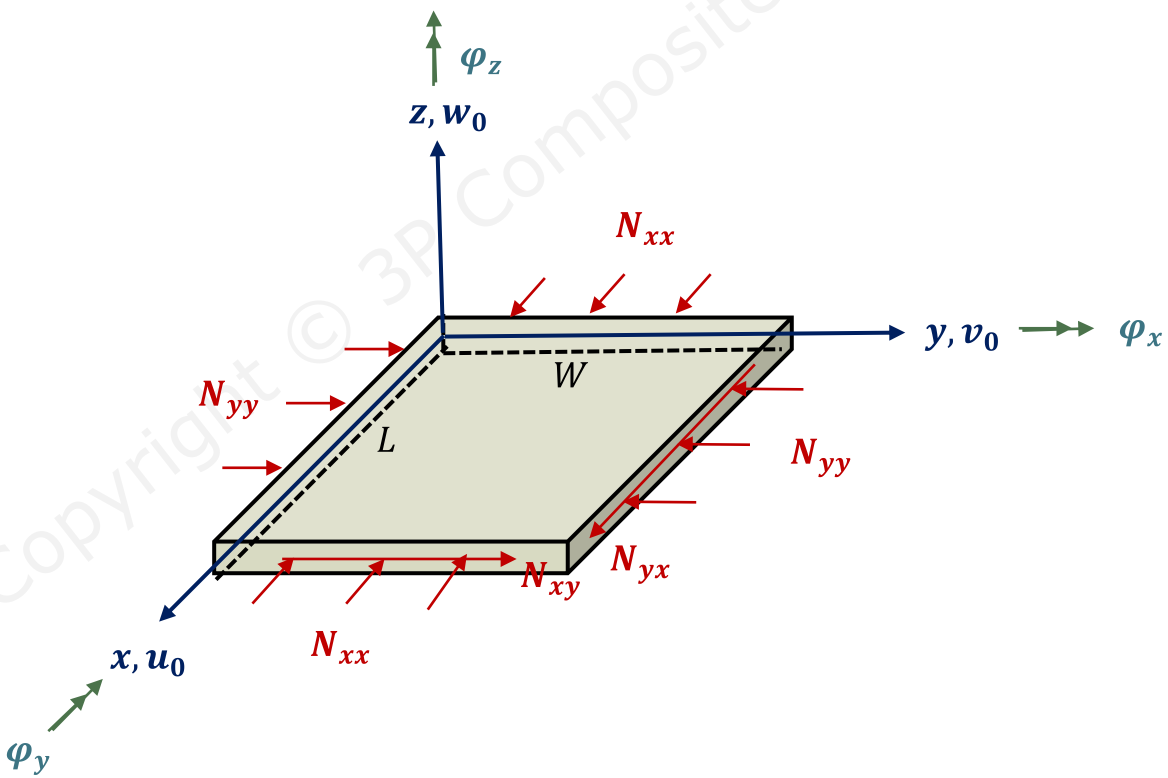
Contents: 3pcsolver003

Overview

- 3pcsolver003 performs **linear** elastic buckling analysis of simply-supported fully anisotropic composite laminates subjected combined in-plane edge compression/tension and/or edge shear loads. The edge loads can also include the effects of uniform temperature and/or moisture expansion or contraction. Simply-supported boundary condition is most widely used in the analysis of plates and shells. Applied compressive bi-axial loads and positive shear load is shown below:



Overview

❖ Following combinations of in-plane buckling loads can be analyzed using the solver 3pcsolver003:

- ✓ Longitudinal Compression N_{xx}
- ✓ Lateral Compression N_{yy}
- ✓ In-plane Shear N_{xy}
- ✓ Bi-axial Compression N_{xx} and N_{yy}
- ✓ Longitudinal Compression N_{xx} and Lateral Tension N_{yy}
- ✓ Longitudinal Tension N_{xx} and Lateral Compression N_{yy}
- ✓ Longitudinal Compression N_{xx} and In-plane Shear N_{xy}
- ✓ Longitudinal Tension N_{xx} and In-plane Shear N_{xy}
- ✓ Lateral Compression N_{yy} and In-plane Shear N_{xy}
- ✓ Lateral Tension N_{yy} and In-plane Shear N_{xy}
- ✓ Longitudinal Compression N_{xx}, Lateral Tension N_{yy} and In-plane Shear N_{xy}
- ✓ Longitudinal Tension N_{xx}, Lateral Compression N_{yy} and In-plane Shear N_{xy}
- ✓ Longitudinal and Lateral Tensions N_{xx} & N_{yy} and In-plane Shear N_{xy}
- ✓ Bi-axial Compression N_{xx} & N_{yy} and In-plane Shear N_{xy}

Applications

- Linear elastic buckling (or static stability) analysis performed by 3pcsolver003 solver is applicable to laminates built-up (or fabricated) from a LAMINA that
  - has any kind of FIBER such as boron, carbon, graphite, glass, Kevlar, Aramid, polyester, natural fibers, etc.,
  - is in any type of broad form such as short and long continuous, unidirectional, bi-directional 2D textile weaves like plain weave, twill and harness, biaxial and triaxial braids, chopped random fibers, non-crimp, nonwoven fabrics, etc.
  - Is impregnated with any RESIN/MATRIX, thermoset or thermoplastic systems such as epoxy, polyester, vinyl ester, polyurethane, phenolic, cyanate ester, bis-maleimide, polyimides, benzoxazine, Acrylic, ABS, Polylactic acid PLA, Polybenzimidazole PBI, Polyether sulfone PES, Polyoxymethylene POM, Polyether ether ketone PEEK, Polyetherimide PEI, Polyphenylene oxide PPO, Polyphenylene sulfide PPS, Polystyrene PS, Polypropylene PP, Polyvinyl chloride PVC, Teflon PTFE, etc.
  - is cured using any MANUFACTURING PROCESS such as Autoclave, Resin Transfer Molding like VARTM, SQRTM, RIM, SRIM, Filament Winding, Pultrusion, Compression Molding, Wet-lay up, etc.

- The analysis is equally applicable to Hybrid Laminates manufactured from a single or multiple types of lamina materials and/or ply broad forms or fiber types or single or multiple materials systems or their combinations

Theoretical Background

- **3pcsolver003** solver is based on First-Order Shear Deformation Laminated Plate Theory (Mindlin Type). Spatial distributions of displacements $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$, and rotations $\boldsymbol{\varphi_x}$ and $\boldsymbol{\varphi_y}$ of the plate's reference surface are assumed using double Fourier series satisfying the kinematic boundary conditions at all four simply-supported edges of the laminated plate exactly. Principle of virtual work and Ritz analysis procedure are used to obtain a highly coupled system of algebraic equations for linear elastic buckling analyses of fully anisotropic laminated plate (see below):

$$\begin{bmatrix} K_{11} & K_{12} & 0 & K_{14} & K_{15} \\ K_{21} & K_{22} & 0 & K_{24} & K_{25} \\ 0 & 0 & K_{33}+\lambda_{mn}F_{33} & K_{34} & K_{35} \\ K_{14} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{25} & K_{53} & K_{54} & K_{55} \end{bmatrix} \begin{Bmatrix} u_{mn} \\ v_{mn} \\ w_{mn} \\ \varphi_{xmn} \\ \varphi_{ymn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

- In the system of equations given above, K_{ij} are the stiffness terms containing the laminate A_{ij}, B_{ij} and D_{ij}. u_{mn}, v_{mn} and w_{mn} are the unknown coefficients of displacements, and φ_{xmn} and φ_{ymn} are the unknown coefficients of rotations of the laminated plate. It is assumed that the edge loads can be expressed as $N_{xx}^0 = \lambda N_{xx}^0$, $N_{yy}^0 = \lambda N_{yy}^0$ and $N_{xy}^0 = \pm\lambda N_{xy}^0$ (where λ is the critical buckling factor). F_{33} term contain information about the total applied edge loads from mechanical and hydrothermal loadings

Theoretical Background

- Hygrothermal effects can be accounted for the given difference in temperature ΔT and difference in moisture content ΔC. Laminated plate theory can be used to compute thermal force resultants N_{xx}^{T}, N_{yy}^{T} & N_{xy}^{T} and/or moisture force resultants N_{xx}^{M}, N_{yy}^{M} & N_{xy}^{M} as shown below

$$\begin{Bmatrix} N_{xx}^{T\,or M} \\ N_{yy}^{T\,or\,M} \\ N_{xy}^{T\,or\,M} \end{Bmatrix} = [A'] \begin{Bmatrix} \varepsilon_{xx}^{T\,or\,M} \\ \varepsilon_{yy}^{T\,or\,M} \\ \gamma_{xy}^{T\,or\,M} \end{Bmatrix} \qquad A' = -BD^{-1}B^{T} + A$$

- The total applied in-plane edge (or buckling) loads are then obtained as summation of mechanical and hydrothermal force resultants as

$$\begin{Bmatrix} N_{xx}^{Total} \\ N_{yy}^{Total} \\ N_{xy}^{Total} \end{Bmatrix} = \begin{Bmatrix} N_{xx}^{T} \\ N_{yy}^{T} \\ N_{xy}^{T} \end{Bmatrix} + \begin{Bmatrix} N_{xx}^{M} \\ N_{yy}^{M} \\ N_{xy}^{M} \end{Bmatrix} + \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}$$

Theoretical Background

- The determinant of the system of $5M \times 5N$ equations for the Eigen-value problem derived above is set to zero to obtain buckling load factors λ_{mn} for a simply-supported fully anisotropic plate subjected to various combinations of applied compression/tension and/or edge shear loads

$$\begin{vmatrix} K_{11} & K_{12} & 0 & K_{14} & K_{15} \\ K_{21} & K_{22} & 0 & K_{24} & K_{25} \\ 0 & 0 & K_{33} + \lambda_{mn} F_{33} & K_{34} & K_{35} \\ K_{14} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{25} & K_{53} & K_{54} & K_{55} \end{vmatrix} = \{0\}$$

- Solution to the Eigen-value problem is obtained for truncated Fourier series using $m = 1,2,..,M$ terms in the x −direction and $n = 1,2,..,N$ terms in the y −direction. Without loss of generality, $M = N$ is assumed for the solution. The buckling mode shapes for each buckling factor λ_{mn} can be obtained by substituting the λ_{mn} in the system of equations given on the previous slide

- Given the lamina/ply material properties, laminate stack-up and its length and width dimensions, 3pcsolver003 solver calculates buckling load factors for an anisotropic laminated plate subjected to any combination of the load described on Slide 5

Theoretical Background

- The 3pcsolver003 is a unique solver which is based on FSDT of laminated plates, employs a closed-form Ritz solution procedure, considers the fully anisotropic laminate effects, and obtain buckling load factors for combined edge compression/tension and shear loads. All types of laminate coupling terms represented by the non-zero A_{i6}, B_{ij} and D_{i6} ($i = 1{,}2$, and $j = 1{,}2{,}6$) are included in the buckling analysis of laminated composite plates. Most closed-form analyses neglect these coupling effects due to the complexity in deriving the system of equations, and hence, assume the laminated plates as being specially orthotropic (i.e. $A_{i6} = B_{ij} = D_{i6} = 0, i = 1{,}2\ , j = 1{,}2{,}6$)

- Numerous examples are solved using **3pcsolver003**, and results are compared with those (i) obtained from standard commercially available finite element analysis software, and (ii) available in open literature. Many types of material systems, ply orientations, laminate stack ups, laminate dimensions, and types of buckling loads are considered to check the accuracy of the solver. Excellent correlations are obtained in all cases. Numerical examples highlight the adverse effects of various types of laminate stiffness couplings on buckling of anisotropic laminated composite plates

- Details of the theoretical approach along with numerous verification and application examples are available in the training module **3pcmodule003**

Inputs

- All inputs should be in consistent units. Use either (N, m, Pa) OR (N, mm, MPa) or (lbs, in, Psi) consistently. Inputs in scientific notation (0.0+e) are acceptable

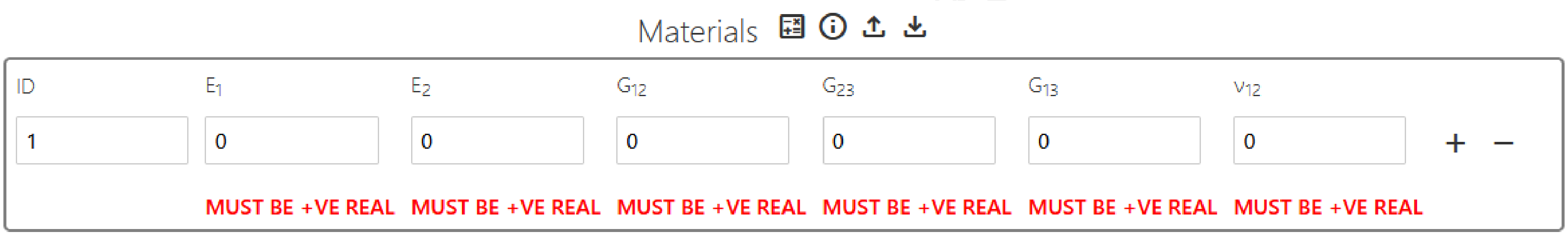
- Input process is intuitive and uses the following logical order:

  - Materials
  - Plies / Laminae
  - Laminates
  - Panels
  - Loads
  - Analysis Options

Inputs: Materials

❖ Material Properties:

In the SI system, MPa and mm or Pa and m, and in the US system Msi and in are used to input the orthotropic lamina Moduli E_1, E_2, G_{12}, G_{13} and G_{23}. ν_{12} is major Poison's ratio. Multiple lamina types and lamina materials can be input by simply clicking the '+' sign on the extreme right. Based on the type of analyses selected, the required material inputs for an orthotopic Lamina can vary as shown below:



Inputs: Plies

❖ Plies/Laminae:

Types of plies in a laminate are required as input. Each ply type is defined by its angle (or orientation) in degrees, material type and the thickness. Material of a ply/lamina can be selected from a predefined list of materials that are input in the Material Properties Section above. The thickness of the ply or lamina is in mm or m in the SI system or inch in the US system. Multiple ply or lamina types can be input by simply clicking the '+' sign on the extreme right. The required ply/lamina type inputs with few examples are shown below:

Plies

ID	Angle (deg)	Material	Thickness		
1	0	Uni	0.005	+	−
2	45	PW	0.010	+	−
3	90	Uni	0.005	+	−

Plies

ID	Angle (deg)	Material	Thickness		
1	0	CEP	0.005	+	−
2	30	Flax	0.010	+	−
3	60	CEP	0.005	+	−

Plies

ID	Angle (deg)	Material	Thickness		
1	0	CEP	0.005	+	−
2	45	CEP	0.005	+	−
3	-45	CEP	0.005	+	−
4	90	CEP	0.005	+	−
5	0	Flax	0.01	+	−

Inputs: Laminates

❖ Laminates:

Multiple laminates can be quickly created by defining their stacking sequences using the plies defined in the previous step. Laminate Offset is fixed at middle (default). Hybrid laminates can be defined using different ply and material combinations established in the previous steps. Additional laminates can be added by simply clicking the '+' sign on the extreme right. A few examples of laminates and their inputs are shown below:

Laminates

ID	Stacking Sequence	Stacking Sequence (Angle)	Offset		
CEP-QI	2,3,1,4,4,1,3,2	45,-45,0,90,90,0,-45	Middle	+	−
CEP-Cross Ply	1,4,1,4,1,4,1,4	0,90,0,90,0,90,0,90	Bottom	+	−
CEP-Angle Ply	2,3,2,3,2,3,2,3	45,-45,45,-45,45,-45	Top	+	−
CEP-Flax Hybrid	1,2,3,4,5	0,45,-45,90,0	Middle	+	−

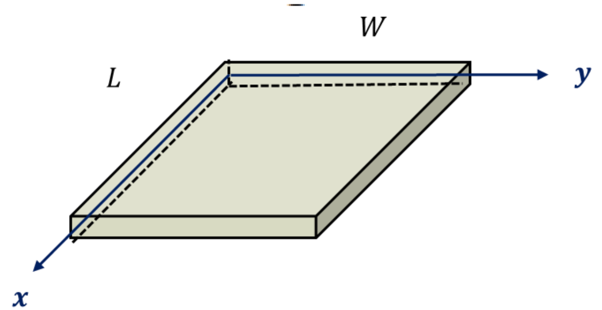
Inputs: Panels

❖ Panels:

Panels are easily created by using the predefined laminates, and by providing the length L and width W of the plate as shown below. Additional panels can be added by simply clicking the '+' sign on the extreme right (see below):

Panels

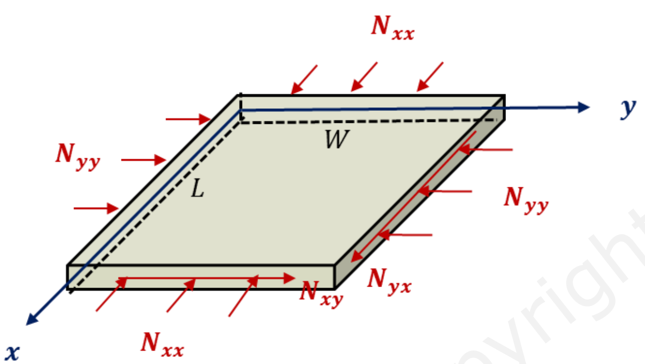
ID	Length	Width	Laminate		
1	10	10	1	+	–
2	10	10	2	+	–
3	10	10	3	+	–
4	10	10	4	+	–



Inputs: Loads

❖ Loads:

As mentioned earlier (see overview Section), various combinations of compressive/tensile and/or shear edge loads N_{xx}, N_{yy} and N_{xy} (force per unit length) can be applied to the panels (see figures below).



Loads

ID	Panel	N_{xx}	N_{yy}	N_{xy}
1	1	-1	0	0
2	2	-1	0	0
3	1	-1	-0.473	0
4	2	-1	-0.473	0
5	1	-1	15.6563	0
6	2	-1	15.6563	0
7	1	0	-1	0
8	2	0	-1	0

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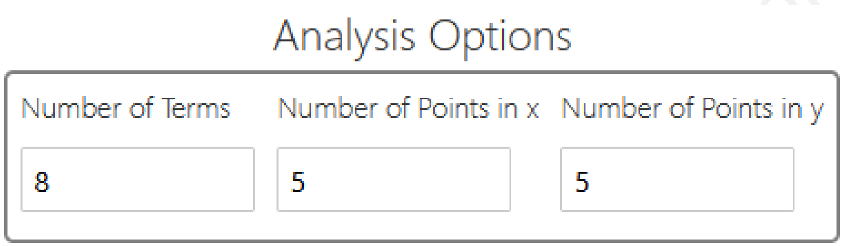
Inputs: Analysis Options

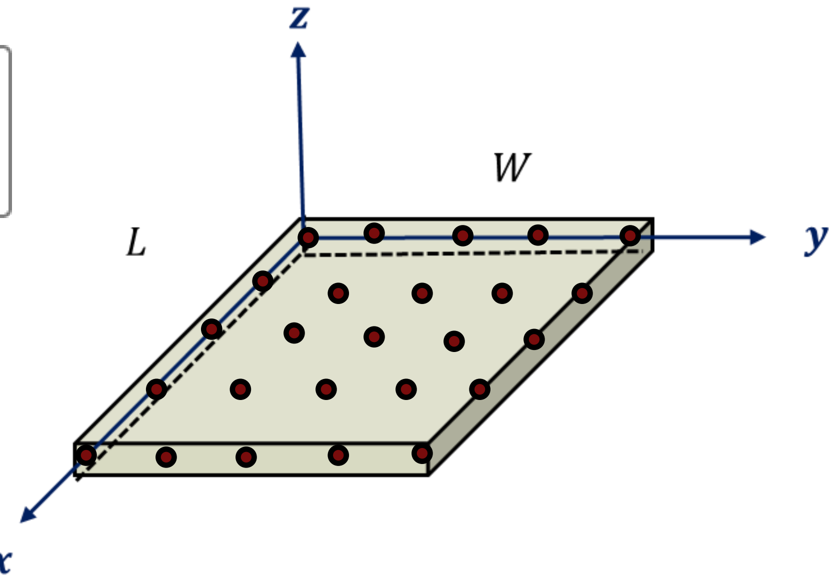
❖ Analysis Options:

User has the option to define the number of terms in Fourier series solution of the solver. By default, $M = N = 8$ is assumed. $M = N$ can be varied from 2 to 21.

Output quantities from the analysis can be requested at select number of points (a.k.a grid points) in the plate domain. By default, a 5×5 grid is assumed within the domain of the plate bounded by $0 \leq x \leq L$ and $0 \leq y \leq W$ to output the analysis solution at 25 equally divided grid points (see below)

Default analysis options are also shown below:





Outputs

❖ Analysis Outputs:

Once all the Input steps viz., Materials, Plies / Laminae, Laminates, Panels, Loads and Analysis Options are completed, analyses can be run by clicking the "submit" button

Submit

Upon completion of analyses, an output is displayed for each Load ID in the window underneath

Output

```
3pc-solver003, v1.1b1

LOADS ID  PANEL ID
1         1

PANEL GEOMETRY
LENGTH: 25.00
WIDTH : 10.00

ANALYSIS OPTIONS
m = 8
n = 8

                                 MATERIAL PROPERTIES
ID            E1            E2            G12           G23           G13           v12
aiaa-2009     1.80e+07      1.60e+06      8.70e+05      6.40e+05      8.70e+05      0.3000

     LAMINATE GEOMETRY
STACKING SEQUENCE (PLY ANG): [+45.0         , -45.0         , +0.0          , +90.0         , +90.0         , +0.0
STACKING SEQUENCE (PLY MAT): [aiaa-2009     , aiaa-2009     , aiaa-2009     , aiaa-2009     , aiaa-2009     , aiaa-2
```

Outputs

❖ Analysis Outputs:

Following information is output for each Load Case:

- Panel Geometry
- Terms in Fourier Series solution
- Number of Grid Points selected to get output information
- Material Properties and Laminate Information
- Laminate [A], [B], [D] stiffness matrices
- First Five (or lowest five) critical buckling loads
- Grid Points coordinates x and y, and transverse displacements w for first five critical buckling loads

Note that all output is consistent with the unit system used during the material, lamina, laminate, and loads Inputs.

- Laminate [A] stiffness matrices N/m or N/mm or lb/in
- Laminate [B] stiffness matrices N-m/m or N-mm/mm or lb-in/in
- Laminate [D] stiffness matrices N-m or N-mm or lb-in
- Displacements in mm, m or in and Rotations in 1/mm, 1/m or 1/in
- Critical Plate Force resultants in N/m, N/mm or lb/in

A typical output is shown below:

Output Text

3pc-solver003, v1.1b1

LOADS ID PANEL ID
1 1

PANEL GEOMETRY
LENGTH: 10.00
WIDTH : 10.00

ANALYSIS OPTIONS
m = 8
n = 8

OUTPUT OPTIONS
NUMBER OF POINTS IN X DIR: 5
NUMBER OF POINTS IN Y DIR: 5

MATERIAL PROPERTIES

ID	E1	E2	G12	G23	G13	v12
aiaa-2009	1.80e+07	1.60e+06	8.70e+05	6.40e+05	8.70e+05	0.3000

LAMINATE GEOMETRY
STACKING SEQUENCE (PLY ANG): [+45.0 , -45.0 , +45.0 , -45.0]
STACKING SEQUENCE (PLY MAT): [aiaa-2009 , aiaa-2009 , aiaa-2009 , aiaa-2009]
TOTAL THICKNESS: 0.0210
OFFSET: 0.0000

LAMINATE PROPERTIES
A MATRIX

+127080.48	+90540.48	+0.00
+90540.48	+127080.48	+0.00
+0.00	+0.00	+98649.19

A MATRIX - TRANSVERSE SHEAR

+15855.00	+0.00
+0.00	+15855.00

Output Text

B MATRIX

+0.00	+0.00	-227.84
+0.00	+0.00	-227.84
-227.84	-227.84	+0.00

D MATRIX

+4.67	+3.33	+0.00
+3.33	+4.67	+0.00
+0.00	+0.00	+3.63

LAMINATE INPLANE AND FLEXURAL ENGINEERING CONSTANTS

Ex	Ey	Gxy	vxy	vyx	Efx	Efy	Gfxy	vfxy	vfyx
+2.98e+06	+2.98e+06	+4.70e+06	+0.7125	+0.7125	+2.98e+06	+2.98e+06	+4.70e+06	+0.7125	+0.7125

	APPLIED LOADS			CRITICAL LOADS		
NUMBER	NXX	NYY	NXY	NXXCR	NYYCR	NXYCR
1	-1.0000	0.0000	0.0000	-22.1208	0.0000	0.0000
2	-1.0000	0.0000	0.0000	-22.5359	0.0000	0.0000
3	-1.0000	0.0000	0.0000	-26.2087	0.0000	0.0000
4	-1.0000	0.0000	0.0000	-31.7513	0.0000	0.0000
5	-1.0000	0.0000	0.0000	-36.0108	0.0000	0.0000

Output Text

MODE SHAPES

X	Y	W1	W2	W3	W4	W5
0.0000	0.0000	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
6.2500	0.0000	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
12.5000	0.0000	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
18.7500	0.0000	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
25.0000	0.0000	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
0.0000	2.5000	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
6.2500	2.5000	-7.0130e-01	+4.1660e-01	-7.9600e-02	+6.7060e-01	+5.2500e-01
12.5000	2.5000	+7.5400e-02	-7.2090e-01	+1.0540e-01	-5.3760e-01	+7.1160e-01
18.7500	2.5000	+7.1060e-01	+5.6450e-01	-1.7990e-01	+4.4070e-01	+4.7040e-01
25.0000	2.5000	+0.0000e+00	+0.0000e+00	-0.0000e+00	-0.0000e+00	+0.0000e+00
0.0000	5.0000	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
6.2500	5.0000	-1.0039e+00	+7.0020e-01	+7.0300e-02	+8.0130e-01	+7.0560e-01
12.5000	5.0000	-0.0000e+00	-1.0299e+00	-0.0000e+00	-7.7880e-01	+1.0090e+00
18.7500	5.0000	+1.0039e+00	+7.0020e-01	-7.0300e-02	+8.0130e-01	+7.0560e-01
25.0000	5.0000	+0.0000e+00	+0.0000e+00	-0.0000e+00	-0.0000e+00	+0.0000e+00
0.0000	7.5000	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
6.2500	7.5000	-7.1060e-01	+5.6450e-01	+1.7990e-01	+4.4070e-01	+4.7040e-01
12.5000	7.5000	-7.5400e-02	-7.2090e-01	-1.0540e-01	-5.3760e-01	+7.1160e-01
18.7500	7.5000	+7.0130e-01	+4.1660e-01	+7.9600e-02	+6.7060e-01	+5.2500e-01
25.0000	7.5000	+0.0000e+00	+0.0000e+00	-0.0000e+00	-0.0000e+00	+0.0000e+00
0.0000	10.0000	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
6.2500	10.0000	-0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
12.5000	10.0000	-0.0000e+00	-0.0000e+00	-0.0000e+00	-0.0000e+00	+0.0000e+00
18.7500	10.0000	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00	+0.0000e+00
25.0000	10.0000	+0.0000e+00	+0.0000e+00	-0.0000e+00	-0.0000e+00	+0.0000e+00

Inputs and Outputs: Consistent Units

Quantity	SI System 1	SI system 2	US System
$E_1, E_2, G_{12}, G_{13}, G_{23}$ $E_x, E_y, G_{xy}, E_{fx}, E_{fy}, G_{fxy}$	$MPa\ (N/mm^2)$	$Pa\ (N/m^2)$	$Psi\ (lb/in^2)$
$\alpha_1\ ,\ \alpha_2\ ,\ \alpha_x\ ,\ \alpha_y, \alpha_{xy}$	$mm/mm/$℃	$m/m/$℃	$in/in/$℉
$\beta_1, \beta_2\ ,\ \beta_x\ ,\ \beta_y, \beta_{xy}$	$mm/mm/Kg/Kg$	$m/m/Kg/Kg$	$in/in/lb/lb$
$\sigma_{11}^T, \sigma_{11}^C, \sigma_{22}^T, \sigma_{22}^C\ \tau_{12}^S, \sigma_1, \sigma_2, \tau_{12}, \tau_{23}, \tau_{13},$ $\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz}$	$MPa\ (N/mm^2)$	$Pa\ (N/m^2)$	$Psi\ (lb/in^2)$
$\varepsilon_{11}^T, \varepsilon_{11}^C, \varepsilon_{22}^T, \varepsilon_{22}^C, \gamma_{12}, \varepsilon_1, \varepsilon_2, \gamma_{12}, \gamma_{13}, \gamma_{23}, \varepsilon_{x0},$ $\varepsilon_{y0}, \gamma_{xy0}, \gamma_{yz0}, \gamma_{xz0}\ \varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}$	mm/mm	m/m	in/in
$\kappa_{x0}, \kappa_{y0}, \kappa_{xy0}$	$1/mm$	$1/m$	$1/in$
$N_{xx}, N_{yy}, N_{xy}, N_{xx}^T, N_{yy}^T, N_{xy}^T,$ $N_{xx}^H, N_{yy}^H, N_{xy}^H$, [A]	N/mm	N/m	lb/in
[B]	$N-mm/mm$	$N-m/m$	$lb-in/in$
[D]	$N-mm$	$N-m$	$lb-in$
ΔT	℃	℃	℉
ΔC	Kg/Kg	Kg/Kg	lb/lb
Ply Angle, θ	Degree	Degree	Degree
Ply or Laminate thickness or Offset or w	mm	m	in

3P COMPOSITES PRODUCIBILITY PERFORMANCE PROFITABILITY

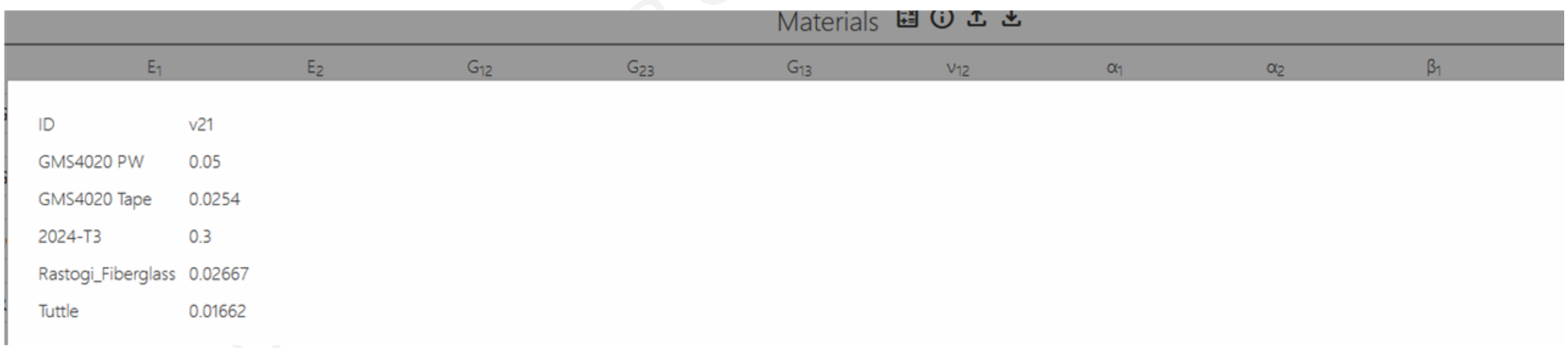
Other Features

❖ Upload/Download:

Users can upload and download Material properties, Plies, Laminates, Panels and Loads data files (*.json) using the upload and download buttons next to these inputs.

❖ Additional Output:

Users can review a few intermediate calculations such as minor Poison’s ratios ν_{21}, Q_{ij} for each ply type and laminate ABD by using the calculation button . Few such examples are shown below:



Other Features

Plies

ID	Angle (deg)	Material	Thickness
1	0	Tuttle	0.0075
2	90	Tuttle	0.0075

ID	Q	Q44	Q55	Qbar	Q44bar	Q45bar	Q55bar
1	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	0	640000
2	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[1106252.04, 376125.7, 0.0], [376125.7, 22627882.74, 0.0], [0.0, 0.0, 640000.0]]	640000	0	640000
3	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[6761596.54, 5481596.54, 5380407.67], [5481596.54, 6761596.54, 5380407.67], [5380407.67, 5380407.67, 5745470.85]]	640000	0	640000
4	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[6761596.54, 5481596.54, -5380407.67], [5481596.54, 6761596.54, -5380407.67], [-5380407.67, -5380407.67, 5745470.85]]	640000	0	640000

Laminates

ID	Stacking Sequence	Stacking Sequence (Angle)	Offset
1	1,1,1,1,1,1,1,1	0,0,0,0,0,0,0,0	Middle
2	1,2,1,2,2,1,2,1	0,90,0,90,90,0,90,0	Middle
3	3,4,3,4,4,3,4,3	45,-45,45,-45,-45,45	Middle

ID	Thickness	A	B	D	A44	A45	A55
1	0.06	[[1357672.96, 22567.54, 0.0], [22567.54, 66375.12, 0.0], [0.0, 0.0, 38400.0]]	[[0.0, 0.0, 0.0], [0.0, -0.0, 0.0], [0.0, 0.0, 0.0]]	[[407.3, 6.77, 0.0], [6.77, 19.91, 0.0], [0.0, 0.0, 11.52]]	38400	0	38400
2	0.06	[[712024.04, 22567.54, 0.0], [22567.54, 712024.04, 0.0], [0.0, 0.0, 38400.0]]	[[0.0, 0.0, 0.0], [0.0, -0.0, 0.0], [0.0, 0.0, 0.0]]	[[286.24, 6.77, 0.0], [6.77, 140.97, 0.0], [0.0, 0.0, 11.52]]	38400	0	38400
3	0.06	[[405695.79, 328895.79, 0.0], [328895.79, 405695.79, 0.0], [0.0, 0.0, 344728.25]]	[[0.0, -0.0, 0.0], [-0.0, -0.0, 0.0], [0.0, 0.0, -0.0]]	[[121.71, 98.67, 36.32], [98.67, 121.71, 36.32], [36.32, 36.32, 103.42]]	38400	0	38400

3P COMPOSITES

General Information

- Subscription fee to access 3pcsolver003 is $39/year per for a single-login license

- Training module 3pcmodule003 supports the solver 3pcsolver003. Users' can buy the training module 3pcmodule001 online at

  https://www.3pcomposites.com/

- 3P Composites, LLC can conduct online or in-class trainings for the 3pcsolver003 and 3pcmodule003. The training can be adapted to meet the requirements of individual needs and/or industrial applications

- For questions, issues, comments, suggestions, trainings, please contact us at 3pcomps@gmail.com. Your feedback is appreciated in helping us continuously improve the product

Examples: Buckling of Laminated Plates

- Lamina Properties: $E_1 = 1.8e7\ psi, E_2 = 1.6e6\ psi\ ;\ G_{12} = G_{13} = 8.7e5\ psi,$ $G_{23} = 6.4e5\ psi; \nu_{21} = 0.3, t_{ply} = 0.00525\ inch,\ \rho = 1.49 \times 10^{-4} \text{lb/in}^3$

- Plate Dimensions: L = 15 in., W = 10 in., Aspect Ratio $\frac{L}{W} = 1.5$

- Laminate I: $[\pm 45]_{2s}, D_{16} \neq D_{26} \neq 0$
  - I: Applied Axial Edge Compression N_{xx} = -10 lb/in
  - II: Applied Bi-Axial Edge Compression N_{xx} = -10 lb/in and N_{yy} = -5 lb/in
  - III: Applied Edge Shear N_{xy} = 10 lb/in
  - IV: Applied Edge Shear N_{xy} = -10 lb/in
  - V: Applied Edge Compression, Tension and Shear N_{xx} = -10 lb/in and N_{yy} = +5 lb/in, N_{xy} = -10 lb/in

Buckling Load Factor	CASE				
	I	II	III	IV	V
1	2.41	1.208	4.835	2.967	1.839
2	2.564	1.856	5.363	3.246	1.999
3	3.062	2.683	9.839	5.767	2.649
4	4.085	3.538	10.74	6.292	2.978
5	5.428	3.816	16.63	9.991	4.099

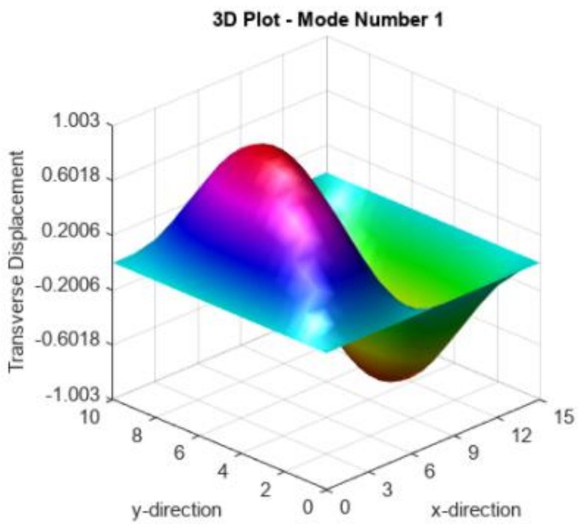
3P COMPOSITES

Buckling of Laminated Plates

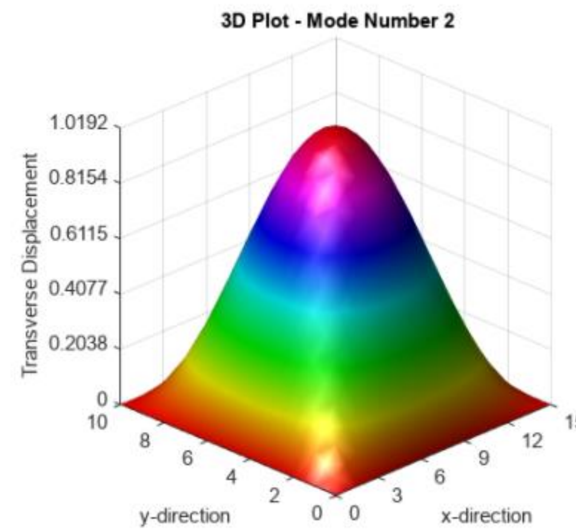
- Laminate II: $[\pm 45]_T, B_{16} \neq B_{26} \neq 0$
  - I: Applied Axial Edge Compression N_{xx} = -1 lb/in
  - II: Applied Edge Tension, Compression and Shear N_{xx} = +1 lb/in and N_{yy} = -1 lb/in, N_{xy} = 1 lb/in
- Laminate III: $[0/90]_T, B_{11} \neq B_{22} \neq 0$
  - I: Applied Bi-Axial Edge Compression N_{xx} = -1 lb/in and N_{yy} = -1 lb/in
  - II: Applied Bi-Axial Edge Compression and Tension N_{xx} = -1 lb/in and N_{yy} = +1 lb/in
- MATLAB scripts are used to create both 3D and 2D contour plots of the transverse displacement w depicting the mode shapes of laminated composite plates subjected to edge buckling loads

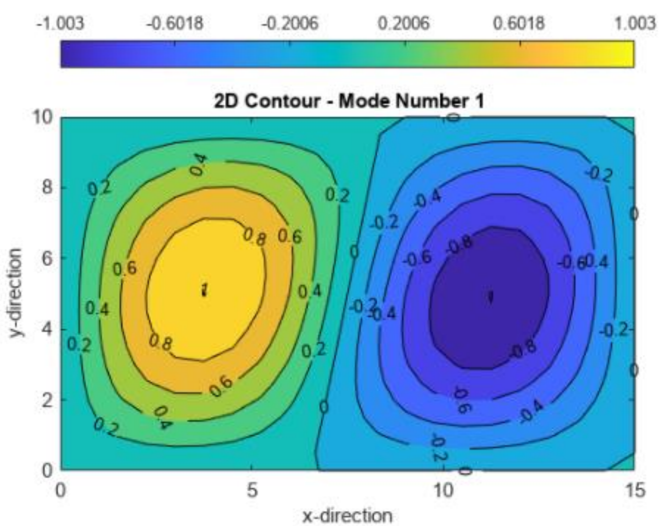
Case I: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

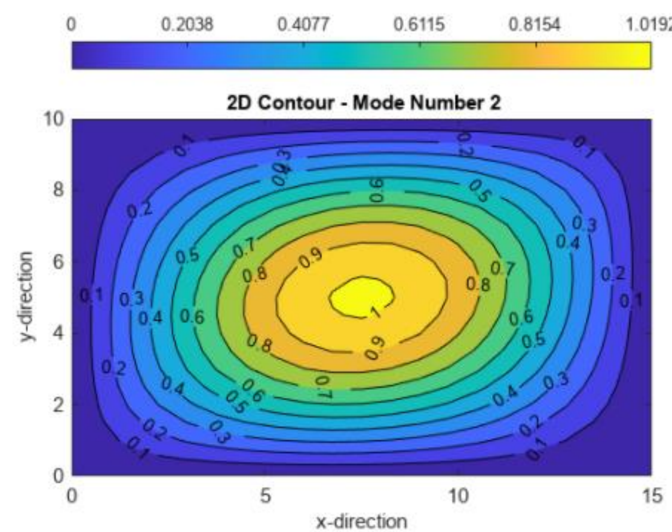
First Buckling Factor: 2.41



Second Buckling Factor: 2.564



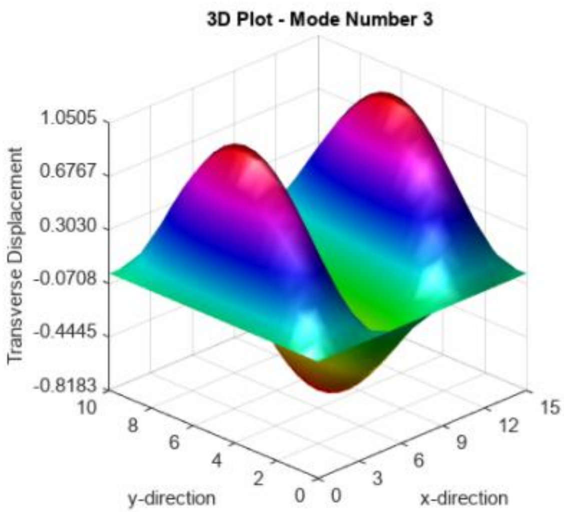


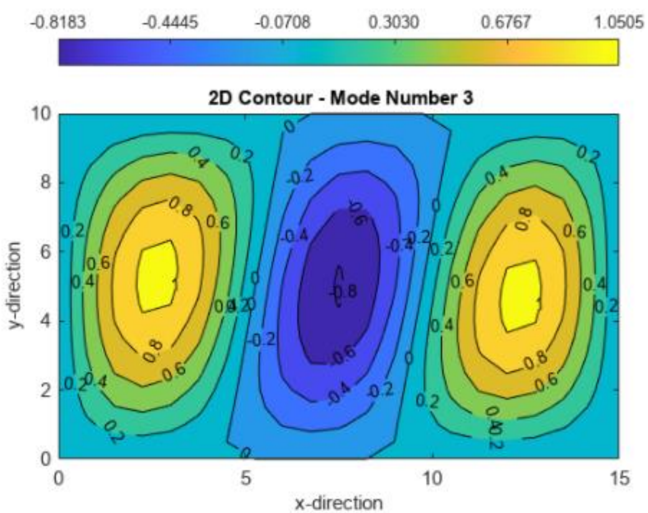


3P COMPOSITES
PRODUCIBILITY PERFORMANCE PROFITABILITY

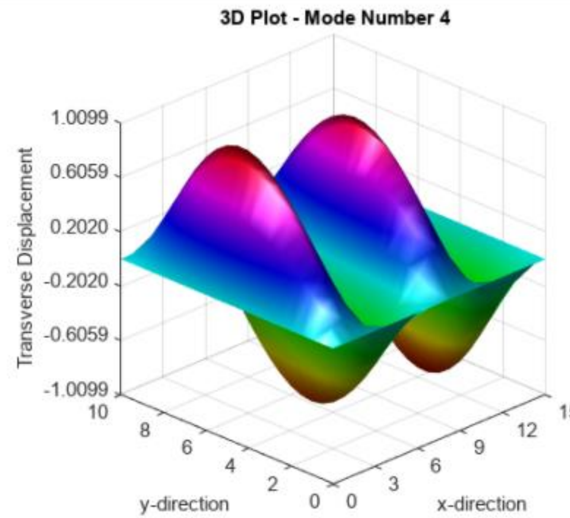
Case I: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

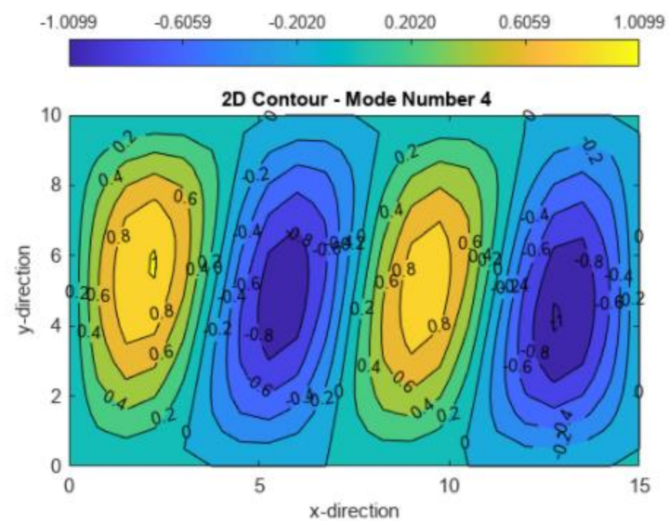
Third Buckling Factor: 3.062





Fourth Buckling Factor: 4.085

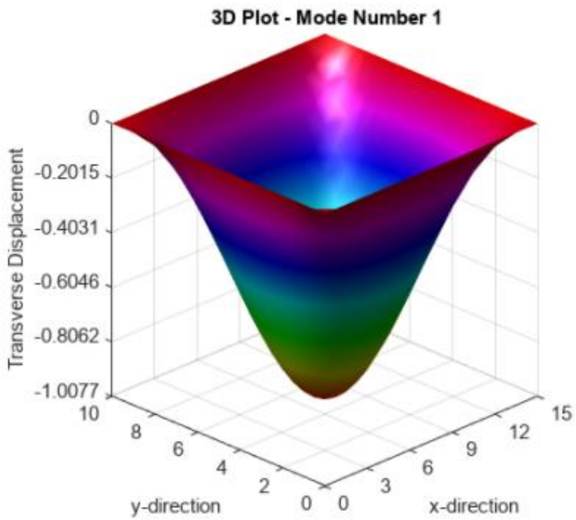


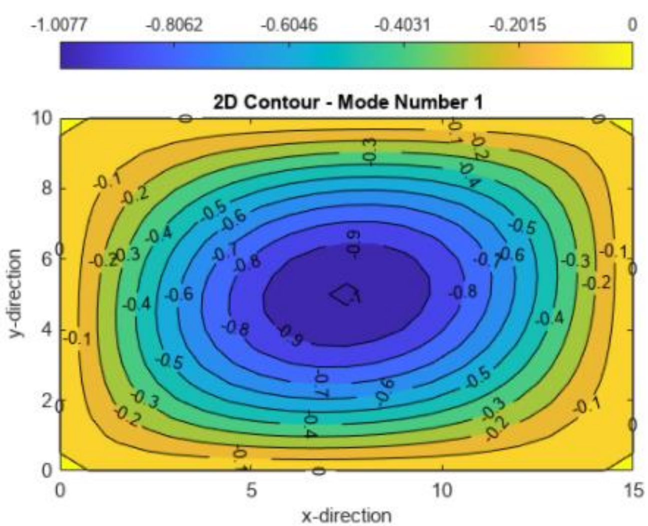


3P COMPOSITES

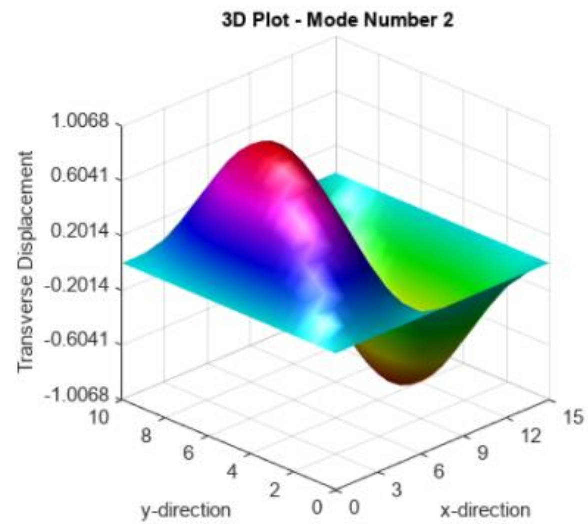
Case II: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

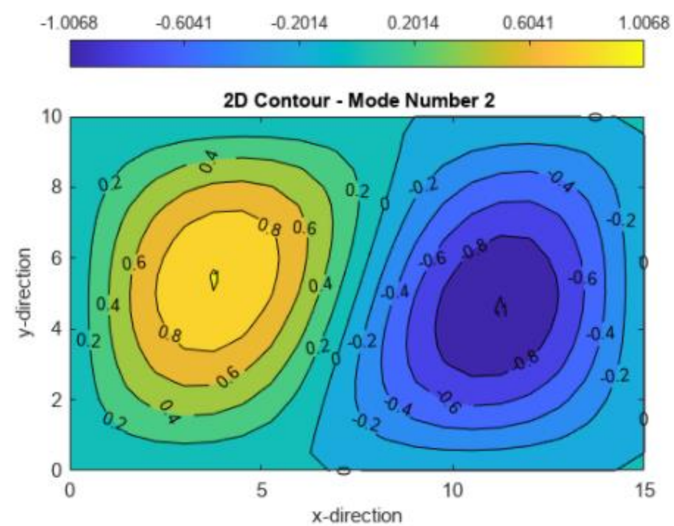
First Buckling Factor: 1.21





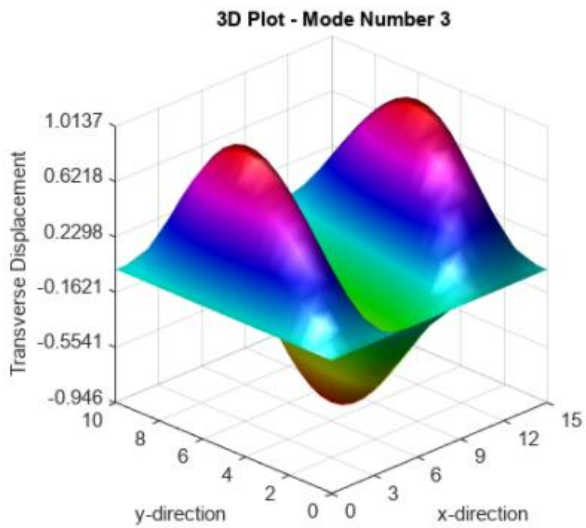
Second Buckling Factor: 1.856



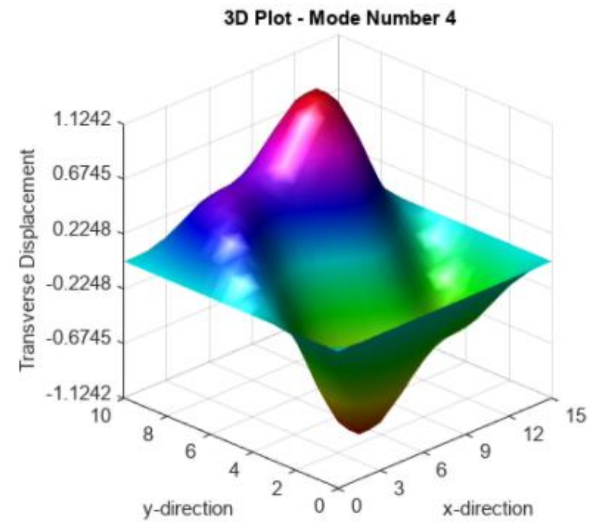


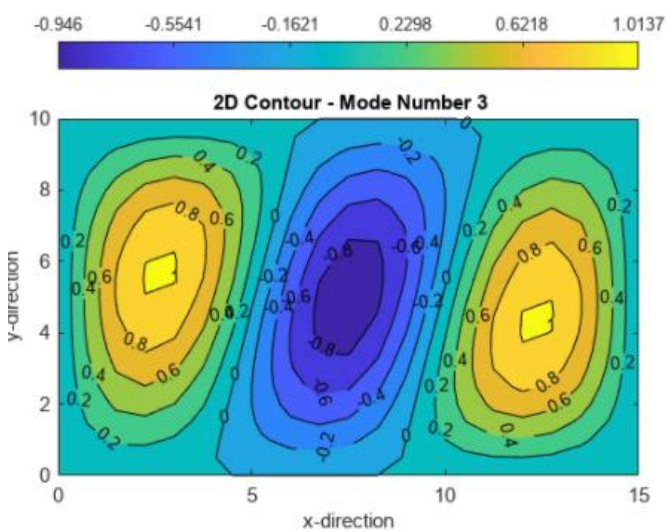
Case II: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

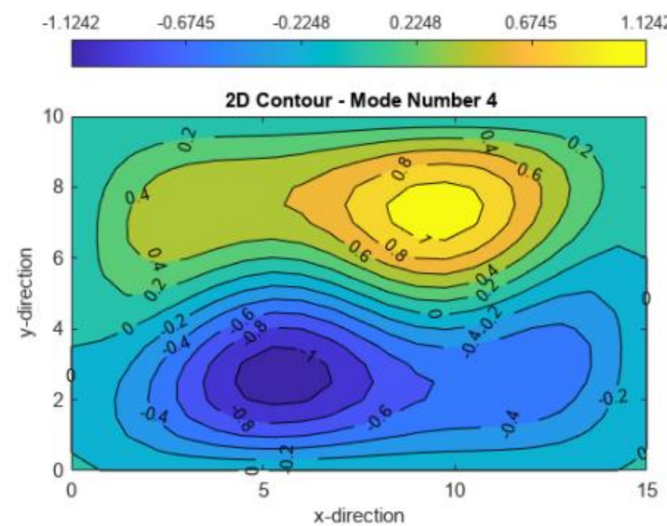
Third Buckling Factor: 2.683



Fourth Buckling Factor: 3.538

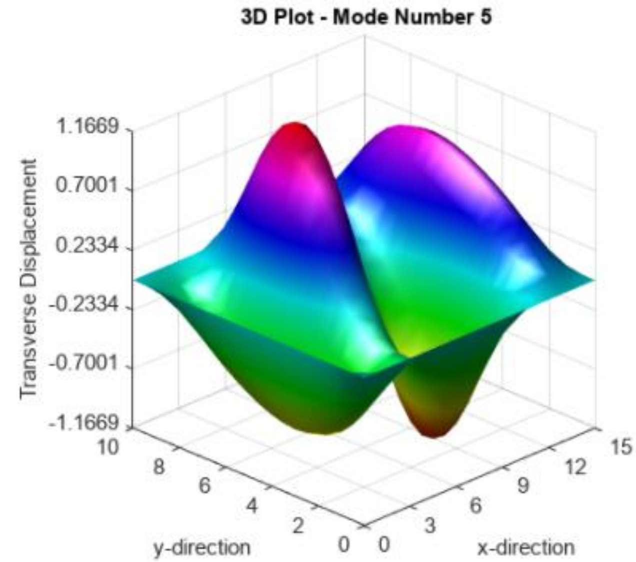


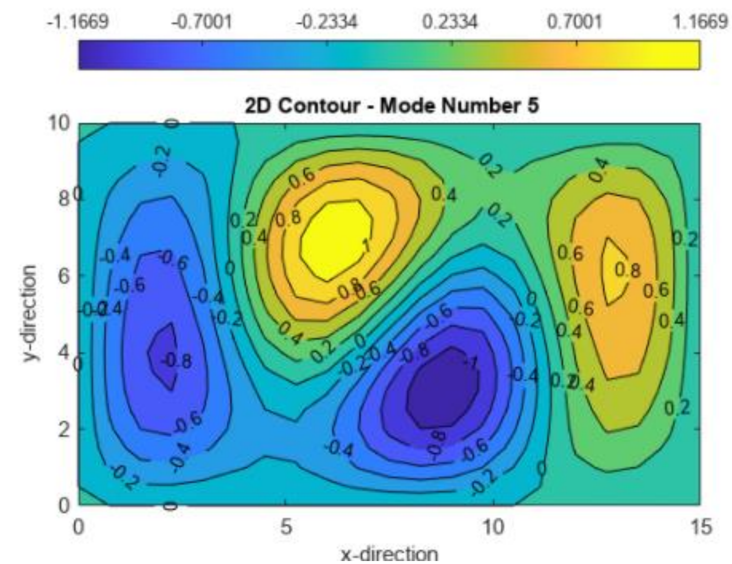




Case II: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

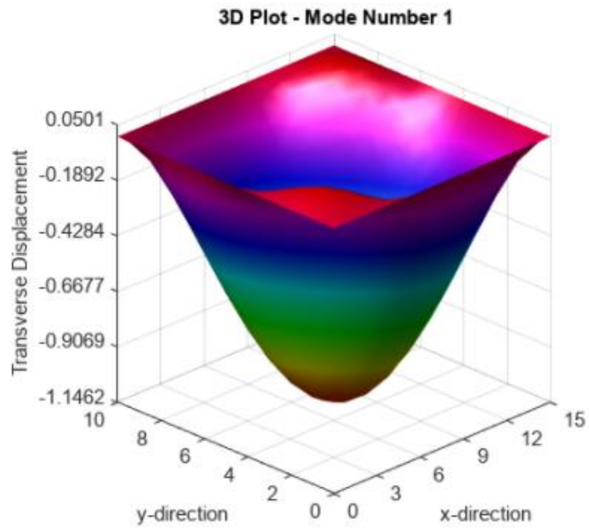
Fifth Buckling Factor: 3.816



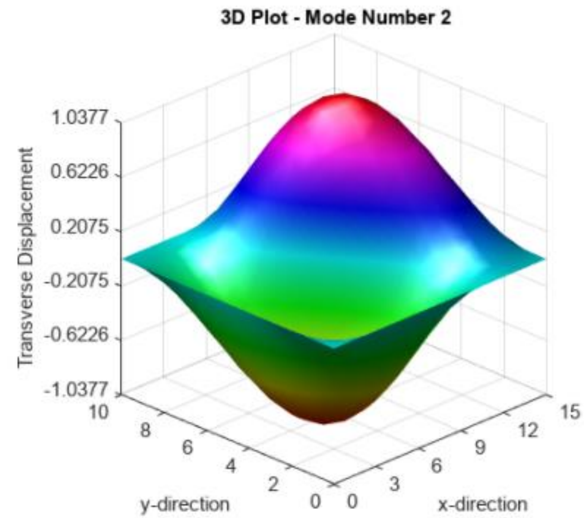


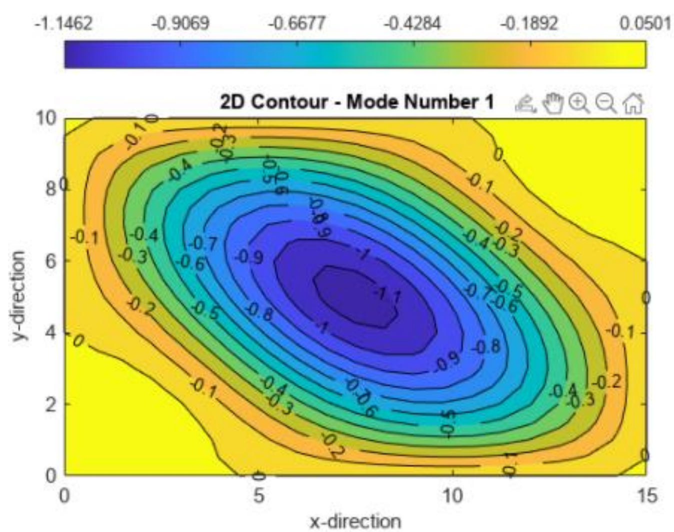
Case III: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

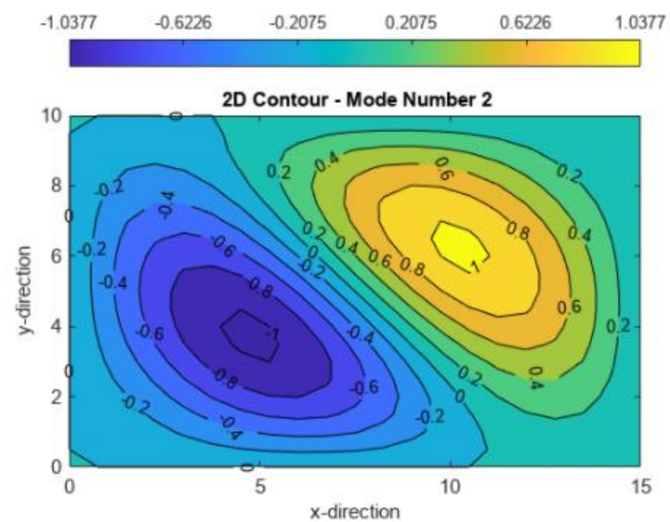
First Buckling Factor: 4.835



Second Buckling Factor: 5.363

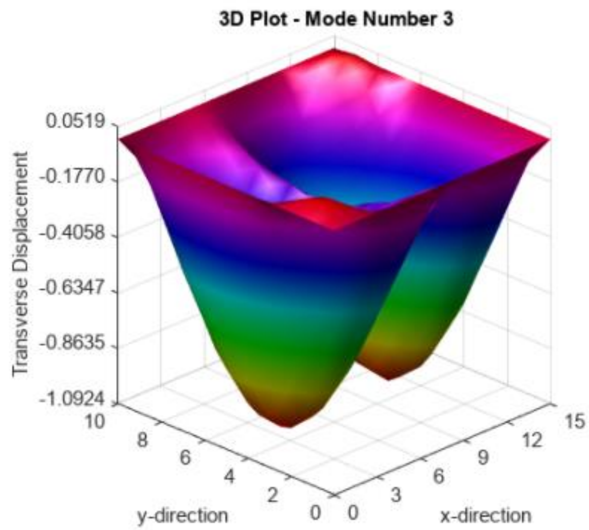




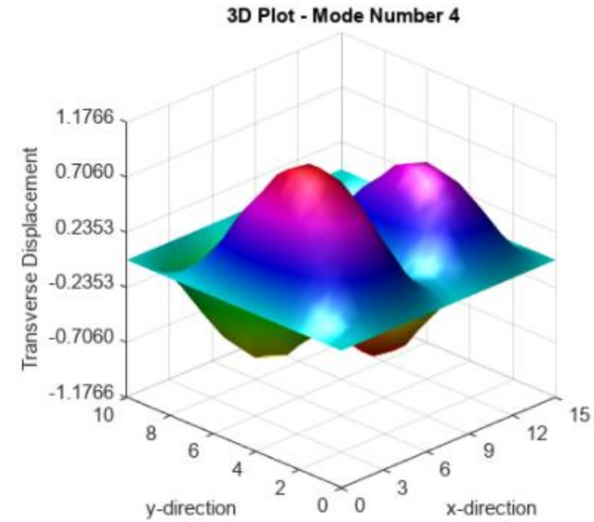


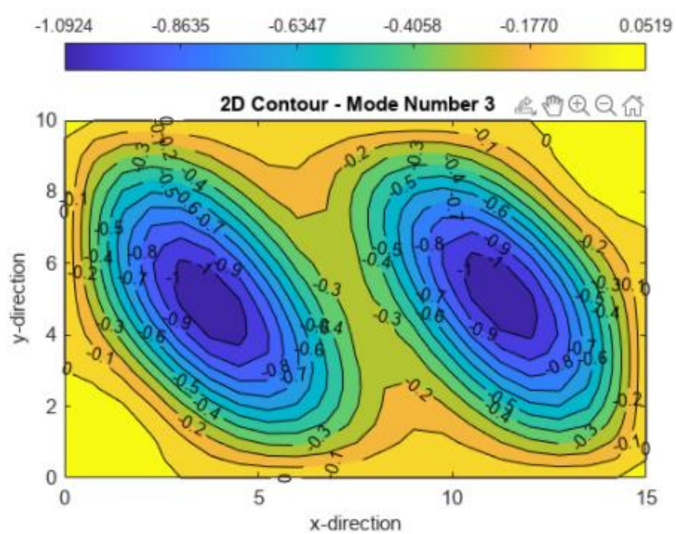
Case III: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

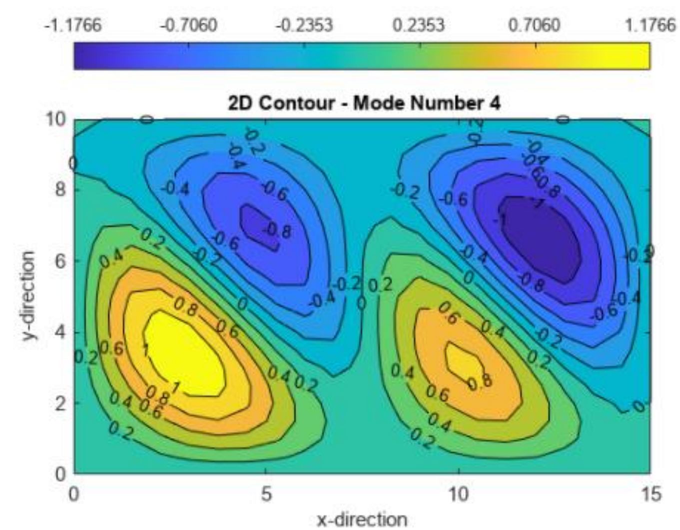
Third Buckling Factor: 9.84



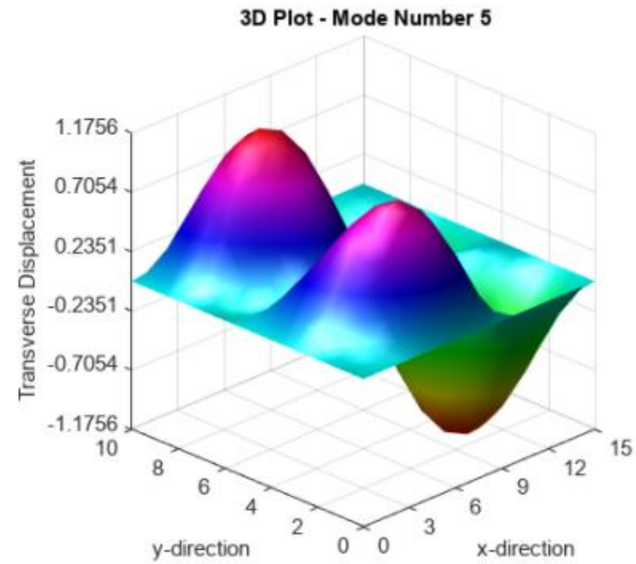
Fourth Buckling Factor: 10.74



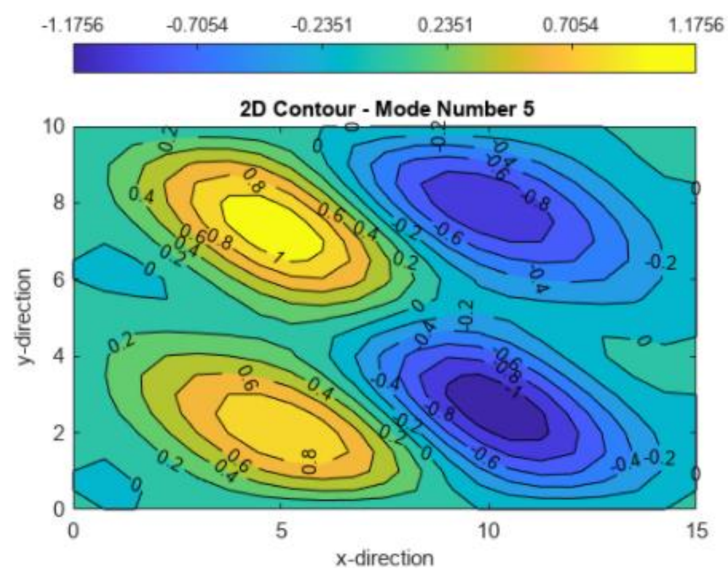




Case III: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

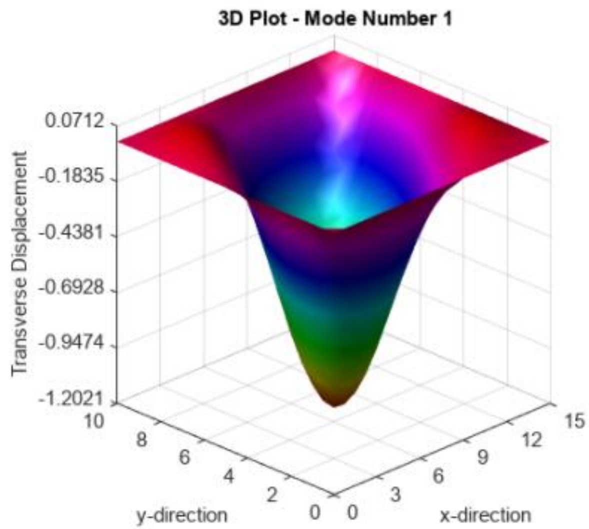


Fifth Buckling Factor: 16.63

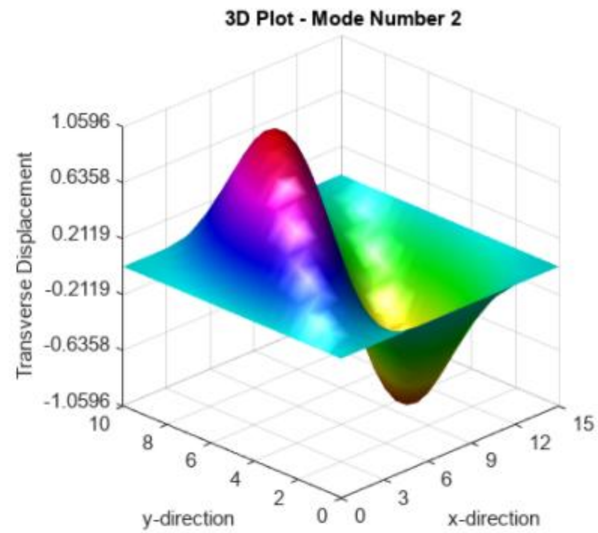


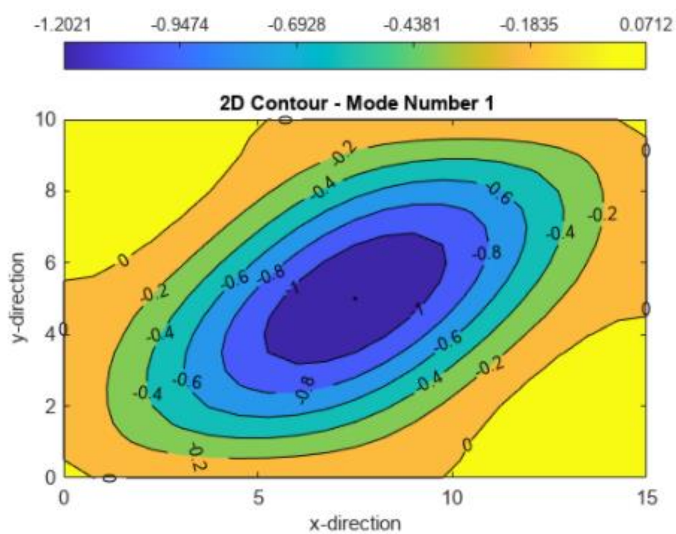
Case IV: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

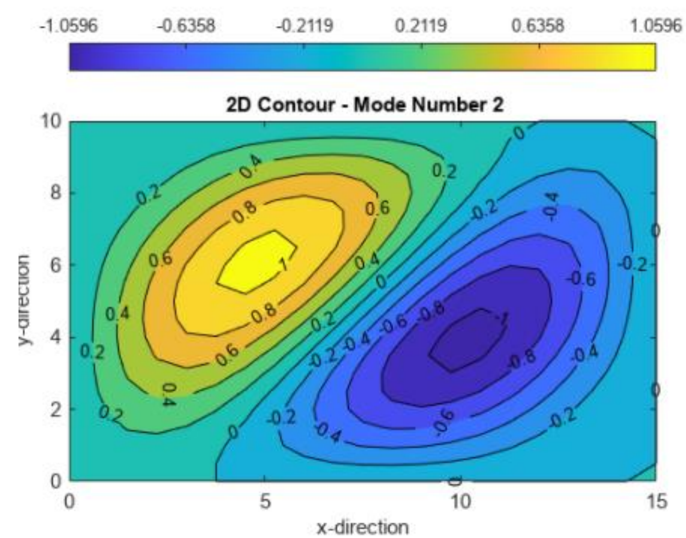
First Buckling Factor: 2.967



Second Buckling Factor: 3.246

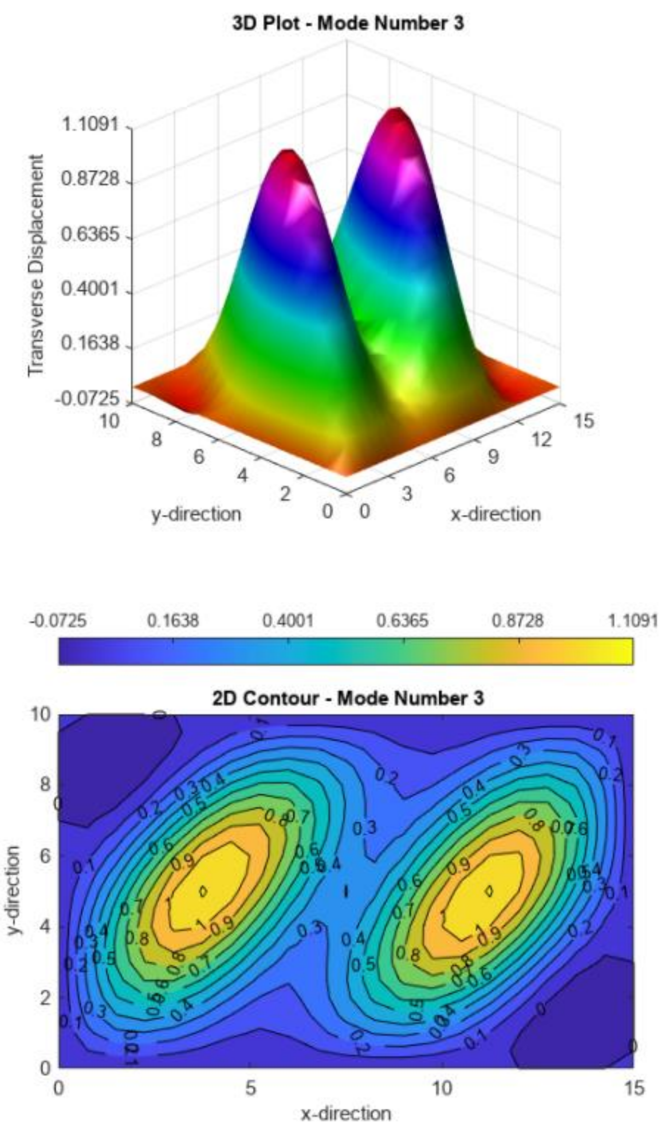




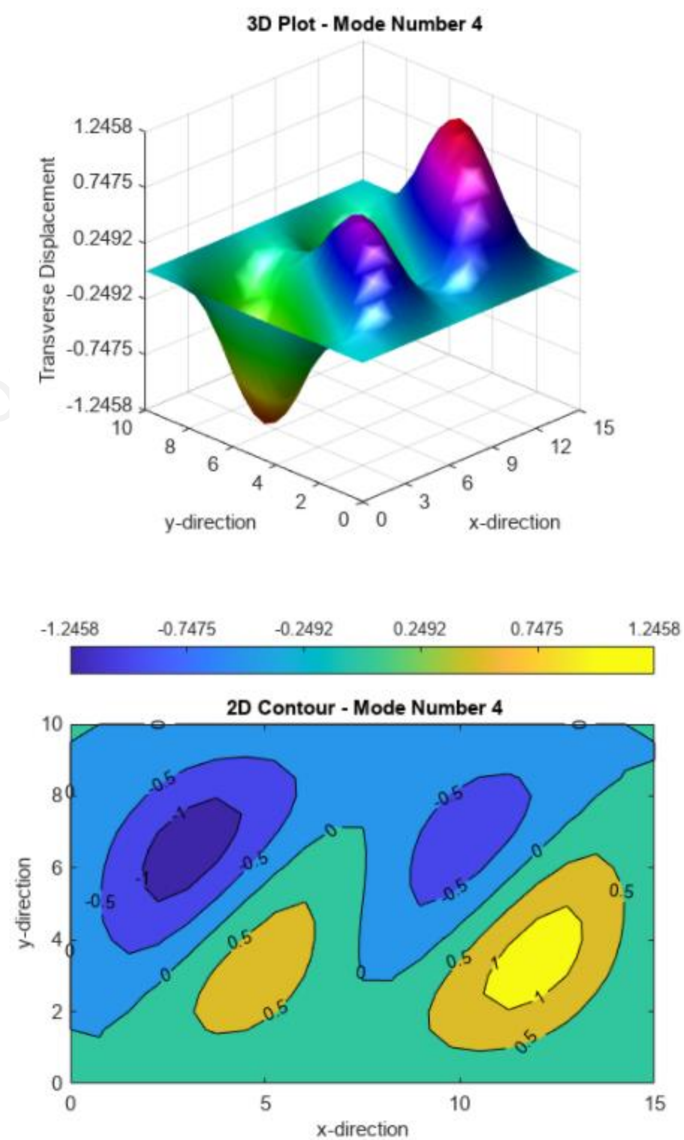


Case IV: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

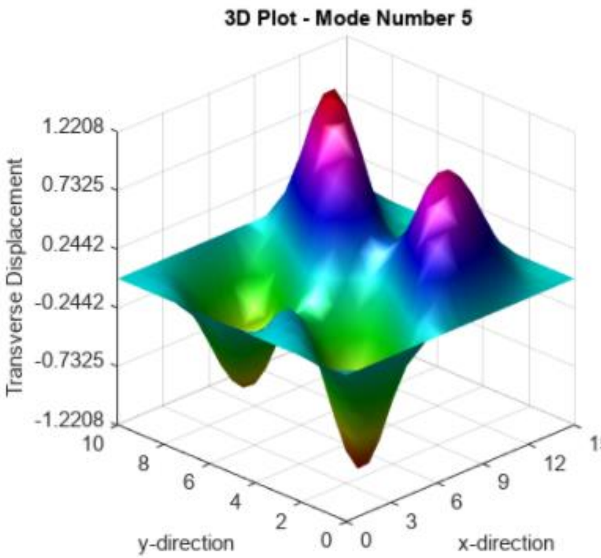
Third Buckling Factor: 5.767



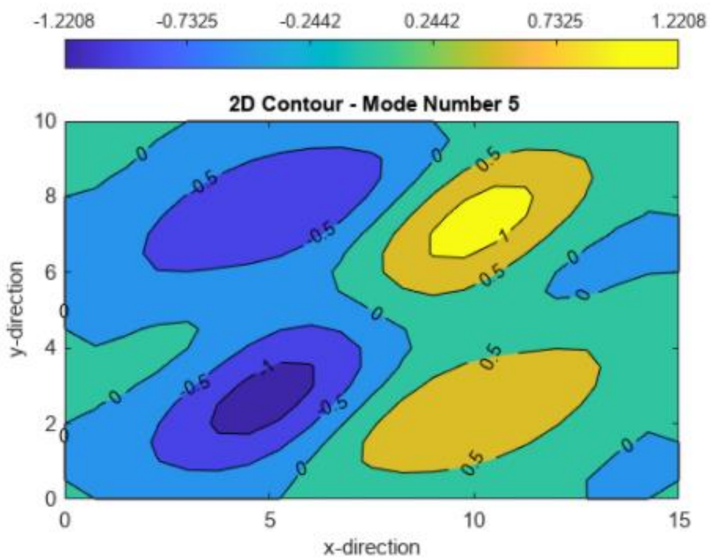
Fourth Buckling Factor: 6.292



Case IV: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

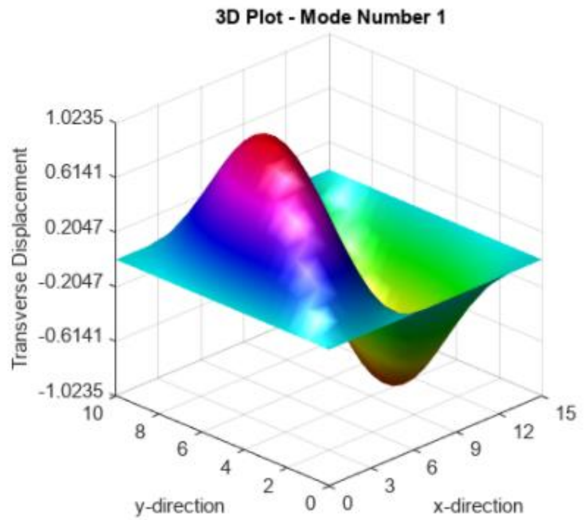


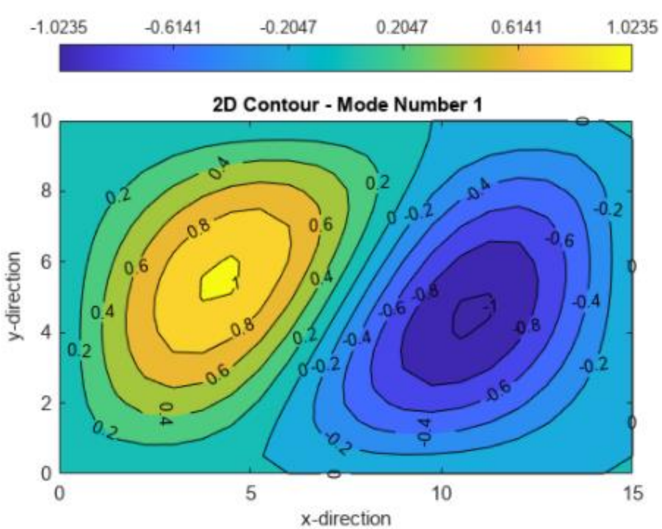
Fifth Buckling Factor: 9.991



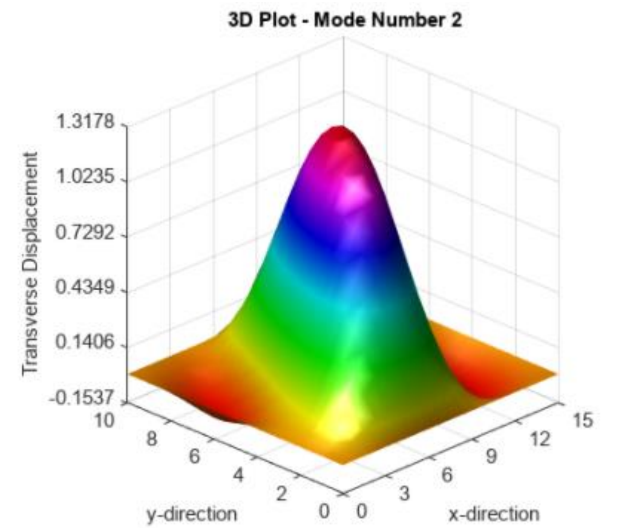
Case V: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

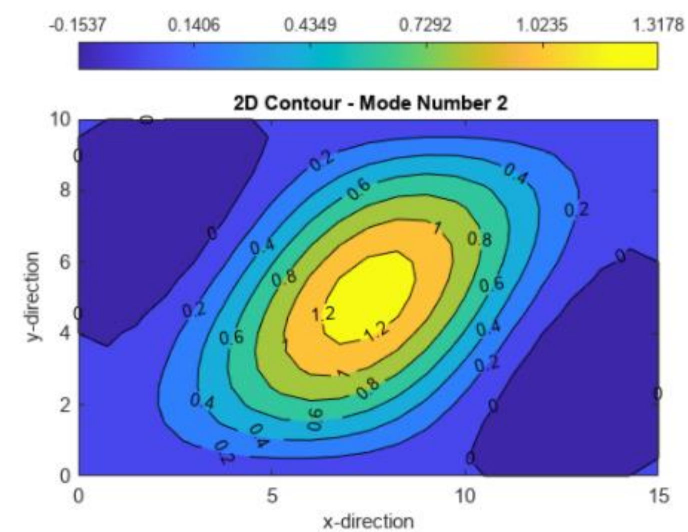
First Buckling Factor: 1.84





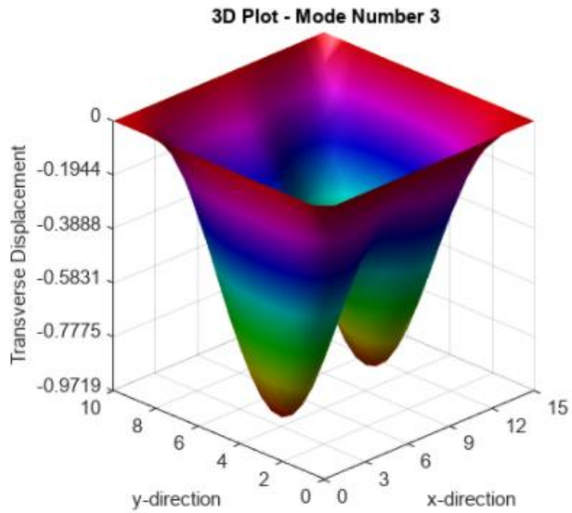
Second Buckling Factor: 1.999

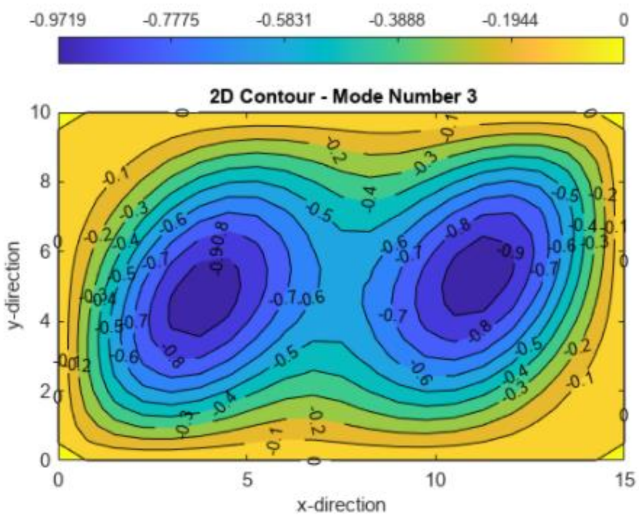




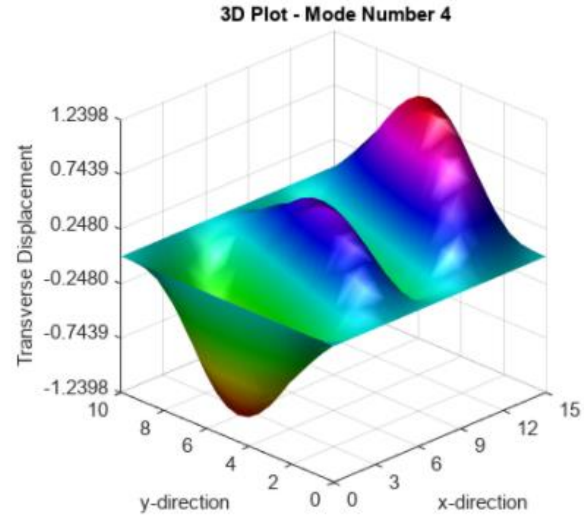
Case V: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

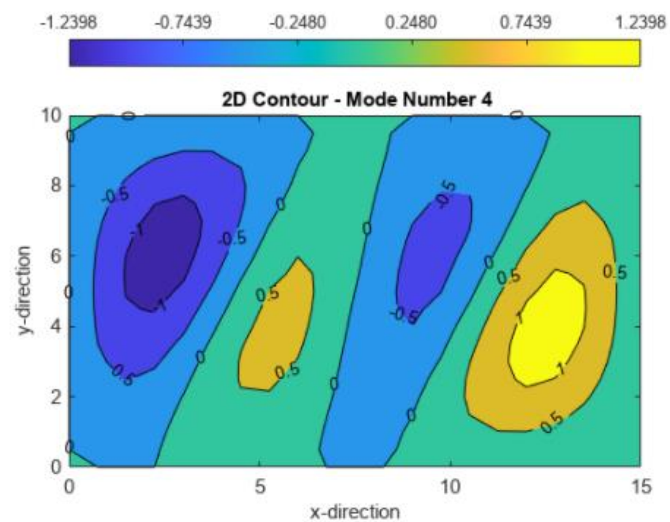
Third Buckling Factor: 2.649



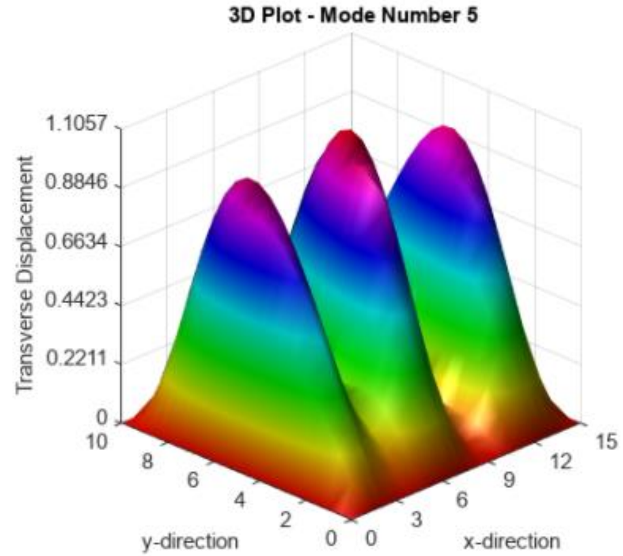


Fourth Buckling Factor: 2.978

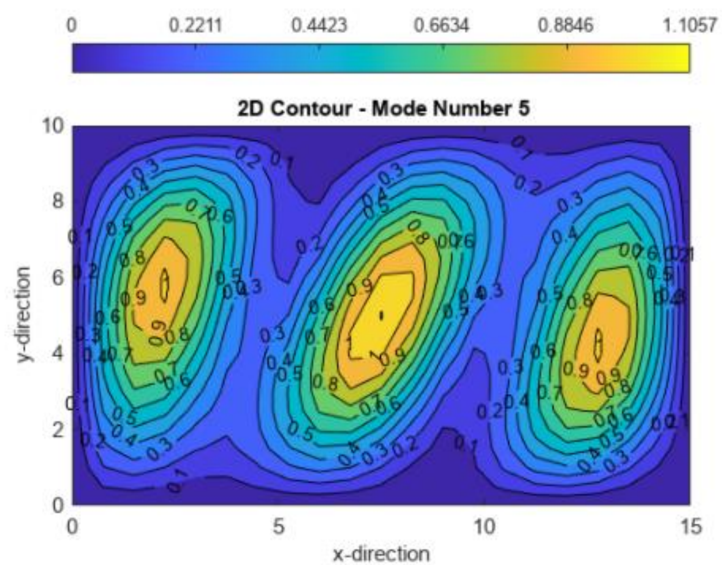




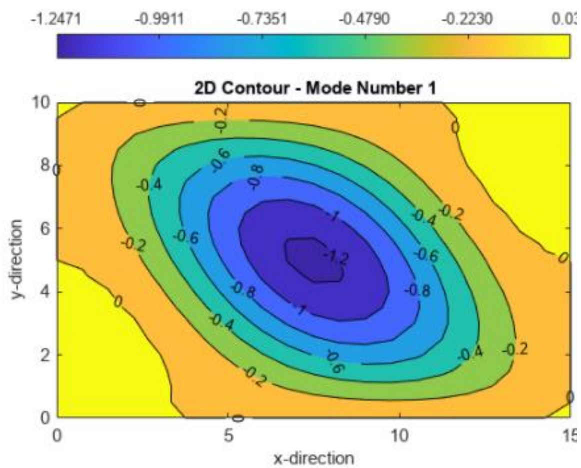
Case V: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_{2s}$

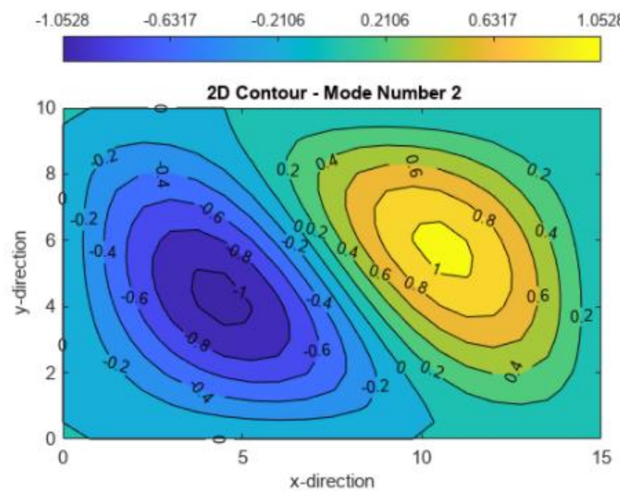


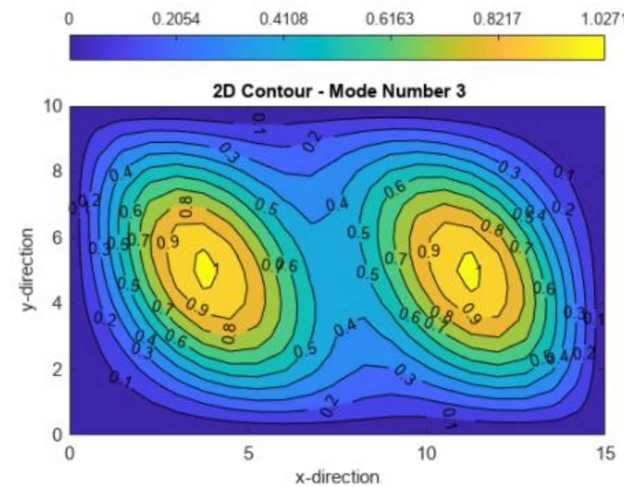
Fifth Buckling Factor: 4.099



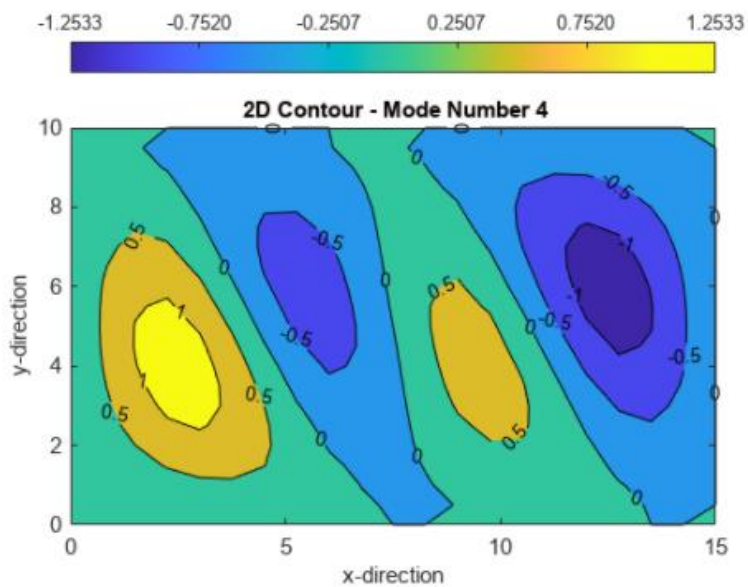
Case I: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_T$

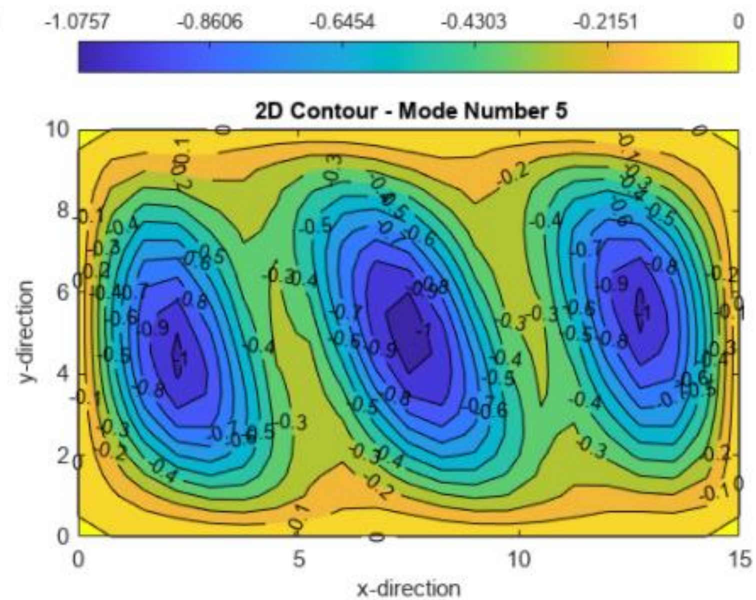






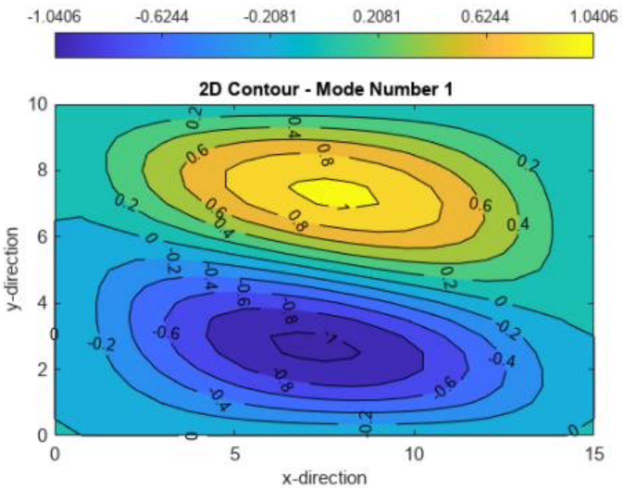
Buckling Load Factor
0.196
0.201
0.293
0.322
0.441

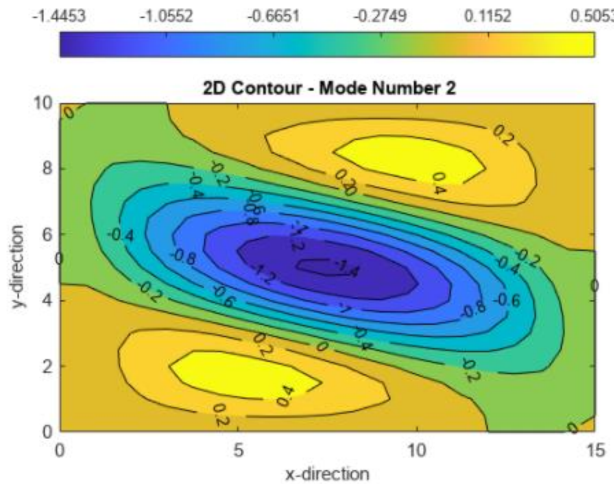


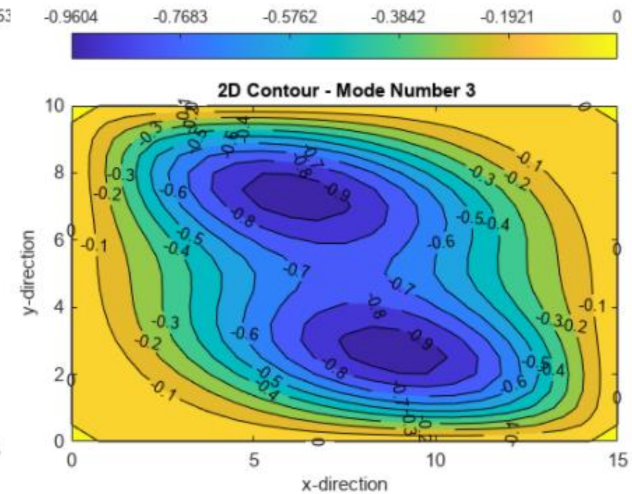


3P COMPOSITES
PRODUCIBILITY PERFORMANCE PROFITABILITY

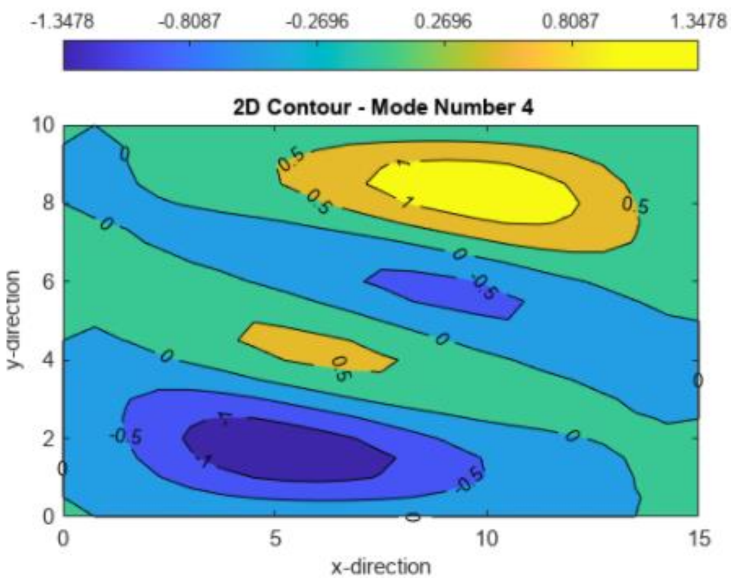
Case II: Critical Buckling Load Factors And Mode Shapes $[\pm 45]_T$

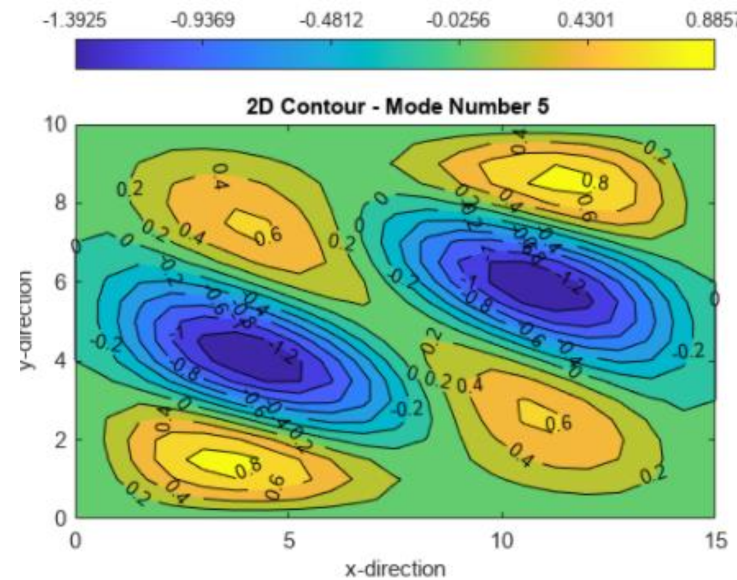




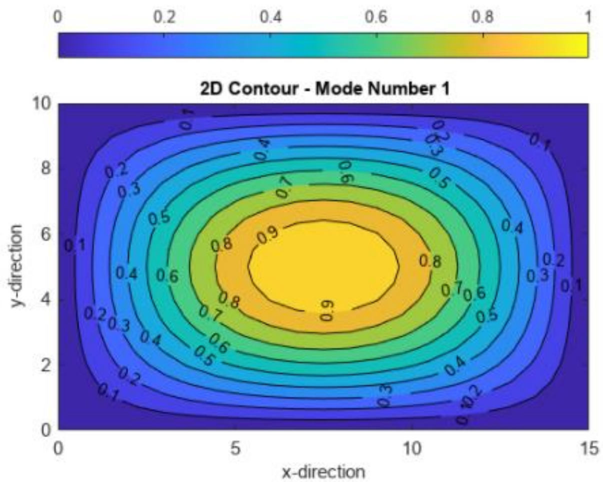


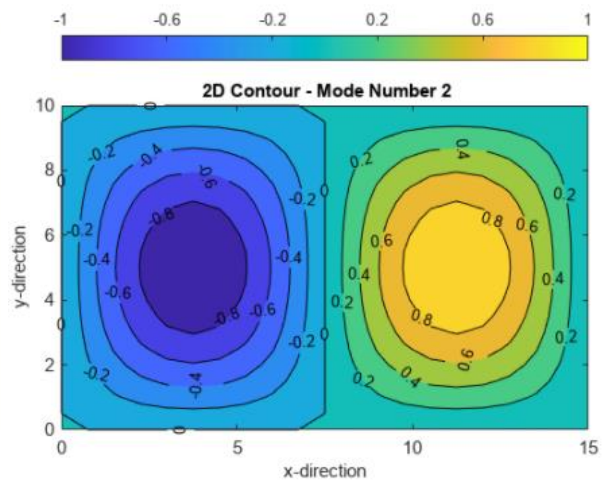
Buckling Load Factor
0.298
0.361
0.476
0.518
0.665

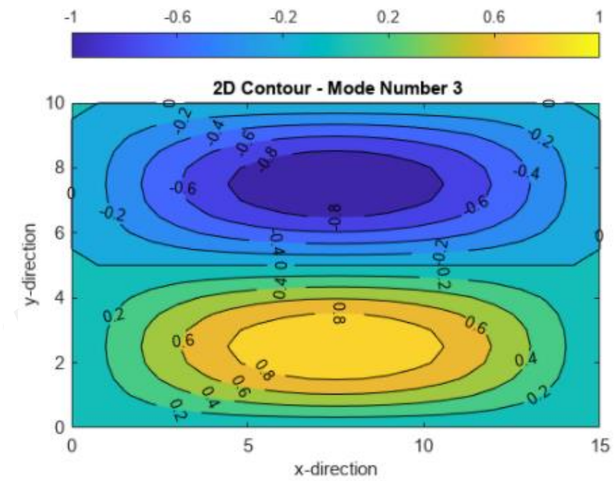




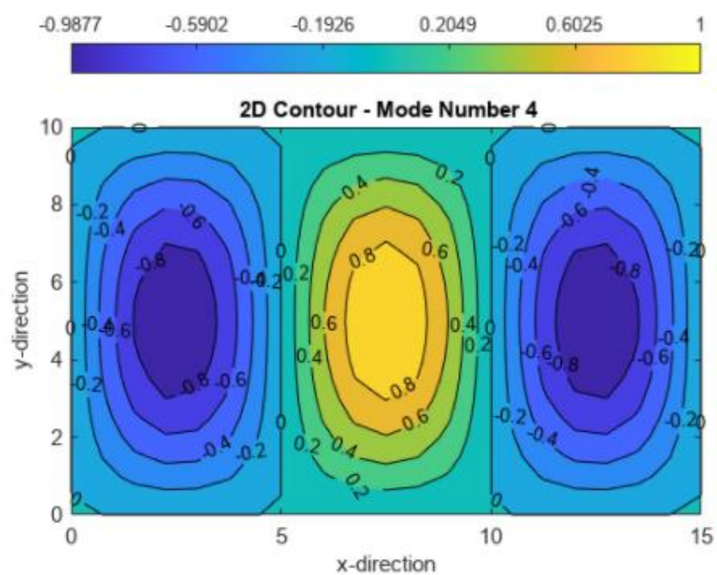
Case I: Critical Buckling Load Factors And Mode Shapes $[0/90]_T$

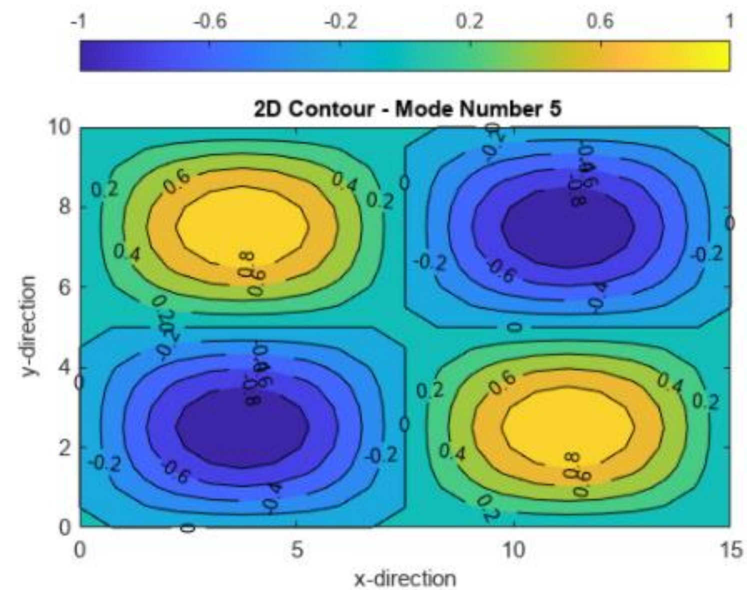




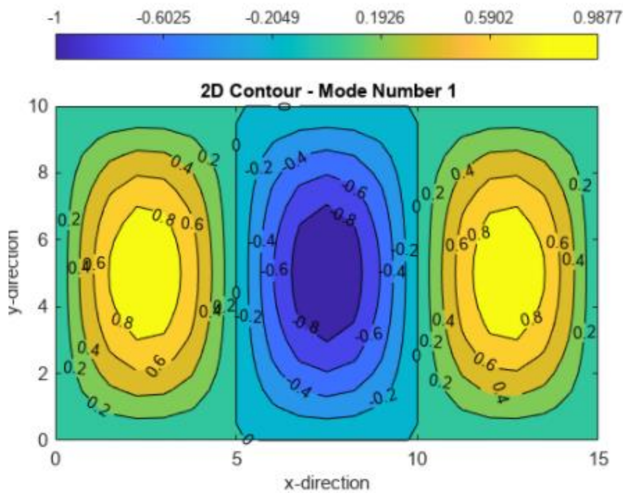


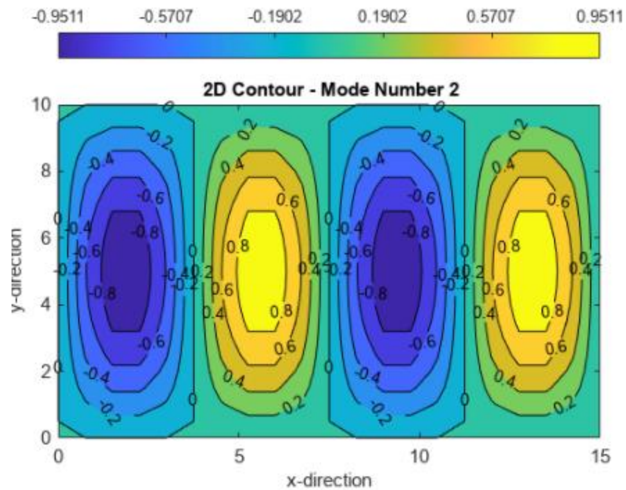
Buckling Load Factor
0.0485
0.0907
0.1775
0.1816
0.1939

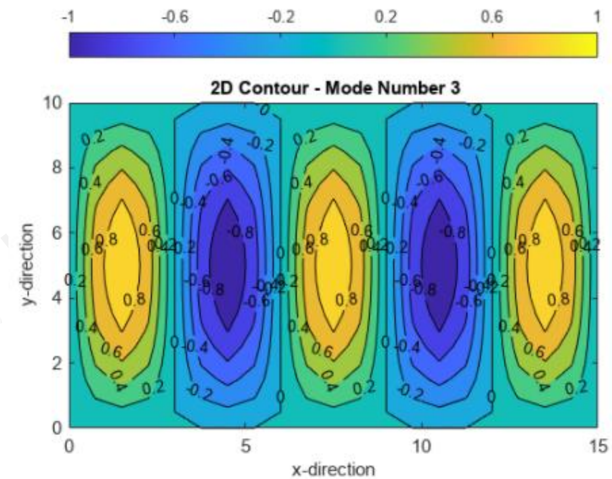




Case II: Critical Buckling Load Factors And Mode Shapes $[0/90]_T$







Buckling Load Factor
0.477
0.502
0.627
0.704
0.817