3PC SOLVER009

Strength Prediction of Laminated Composite Plates with Circular and Elliptical Cutouts

3P COMPOSITES

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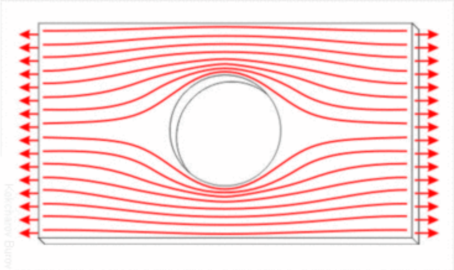
Contents: 3pcsolver009

Stress Concentration in Structures

- Notches or holes or cutouts are commonly created in industrial structures for many reasons such as to
  - Provide easy access to various systems such as fuel, hydraulic, electrical, electronics, pneumatic, waste, flight, flight controls, etc.
  - Allow passing through of various cables and pipes for many systems mentioned above through other structural elements such as shells, plates, ribs, frames, beams, etc.
  - Lighten the structure by removing material from shells, plates, ribs, frames, beams, etc.
  - Damage repairs – cleaning of damage by removing material
  - ..

- Penetrations (holes or cutouts) in the structures create localized stress concentrations resulting in considerably higher stresses and stress gradients around the penetrations than the surrounding structure (or material around them). In simple terms, stress concentrations occur when the irregularities in the structural component cause an interruption to the flow of stress as shown below



Stress Concentration in Structures

- Stress concentrations can cause premature static and/or fatigue failure of any structure, increase the static instability (i.e., reduce critical buckling loads), decrease natural frequencies of vibrations or alter the dynamic response. Hence, the effects of stress concentrations should be addressed early in the product design phase

- Most practical penetrations are either circular or elliptical in shape. A circular hole/penetration in metallic material structures subjected to uniaxial tension loading results in a stress concentration factor of 3, a well-known fact. However, due to anisotropic behavior of composite materials and structures, obtaining stress concentration factors for typical penetrations such as circular or elliptical is not straight forward unless the laminate is quasi-isotropic

- Finite element analysis using standard commercial software such as NASTRAN, ANSYS or ABAQUS, etc. is commonly employed to compute stress concentration factor(s) in composite structures with cutouts. Any significant change in hole dimensions relative to the structure's dimensions require re-analyses which can be very time consuming

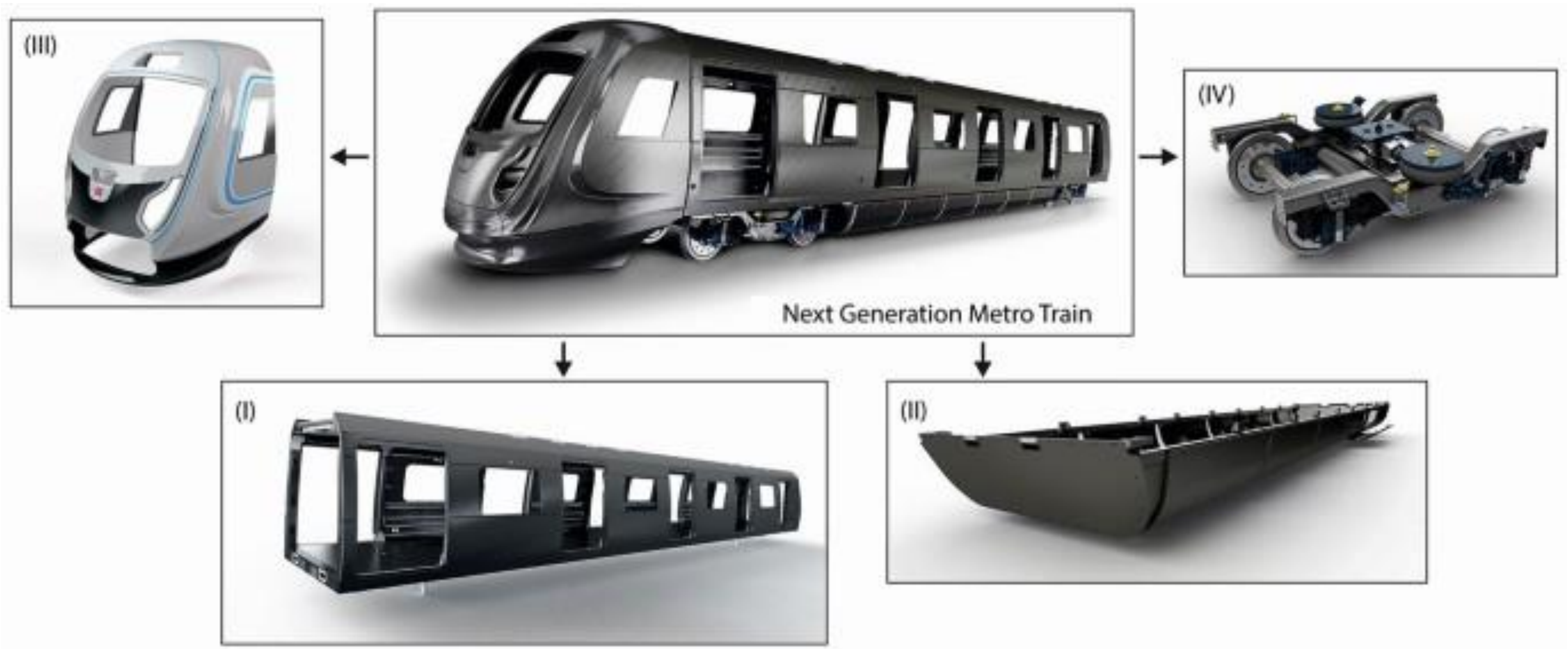
Stress Concentration in Composites

- Regardless of the availability of powerful finite element analyses tools, development of methods and computational tools that provide approximate solutions to stress concentration in composite structures in relatively simpler and computationally efficient way will be very useful in overall product design process

- A few examples of composite structures with cutouts are shown below

Picture Ref: https://www.cnet.com/pictures/in-kansas-where-787-dreamliners-are-born-pictures/11/

Stress Concentration in Composites



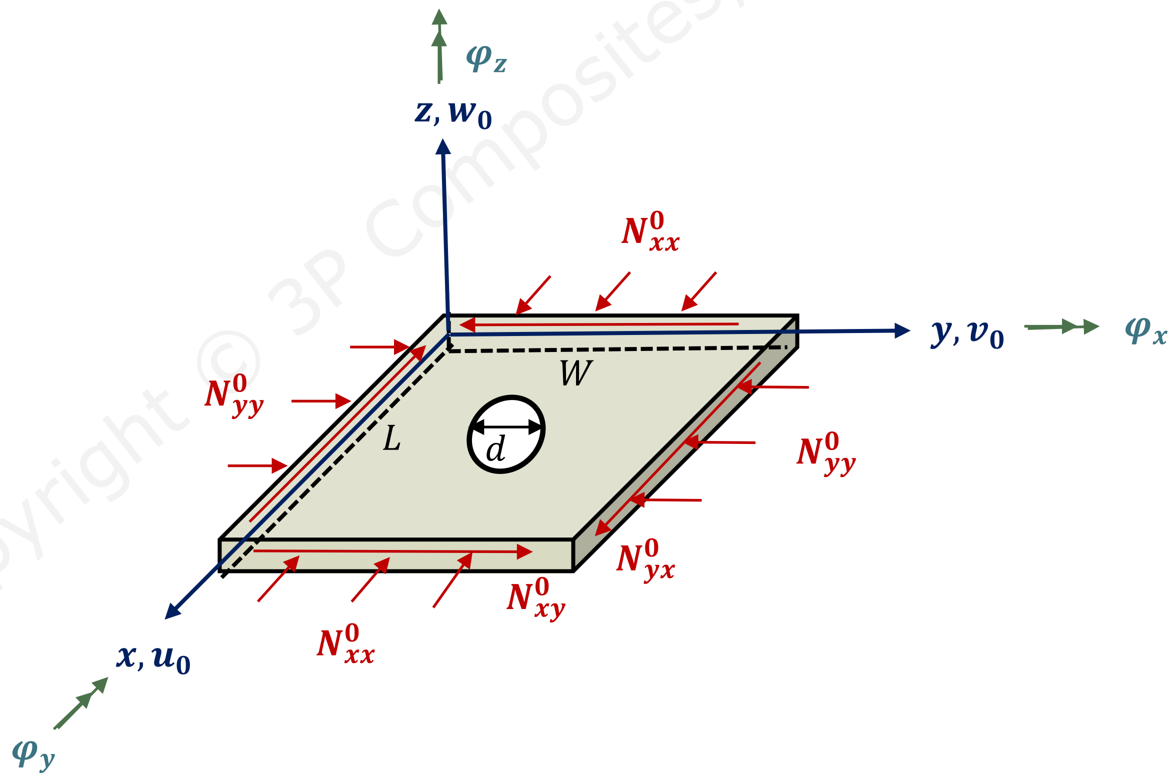
Picture Ref: Ulbricht, A. Rail Vehicle in CFRP-intensive Design. Lightweight des worldw 12, 36–41 (2019). https://doi.org/10.1007/s41777-019-0009-4

Stress Concentration in Composites

Picture Ref: https://sendcutsend-frontend.s3.amazonaws.com/wp-content/uploads /2022/02/23010517/carbon-fiber-header.jpg

Overview

- 3pcsolver009 performs stress analysis of laminated composite plates having circular or elliptical holes (or cutouts) subjected to in-plane mechanical and hygrothermal tension/compression and/or shear edge loads. Furthermore, the solver predicts failure strength (or failure load) of the laminated composite plates with circular or elliptical holes using standard composite failure theories



Applications

- Stress concertation and strength prediction analysis performed by 3pcsolver009 solver is applicable to the laminates built-up (or fabricated) from a LAMINA that
  - has any kind of FIBER such as boron, carbon, graphite, glass, Kevlar, Aramid, polyester, natural fibers, etc.,
  - is in any type of broad form such as short and long continuous, unidirectional, bi-directional 2D textile weaves like plain weave, twill and harness, biaxial and triaxial braids, chopped random fibers, non-crimp, nonwoven fabrics, etc.
  - Is impregnated with any RESIN/MATRIX, thermoset or thermoplastic systems such as epoxy, polyester, vinyl ester, polyurethane, phenolic, cyanate ester, bis-maleimide, polyimides, benzoxazine, Acrylic, ABS, Polylactic acid PLA, Polybenzimidazole PBI, Polyether sulfone PES, Polyoxymethylene POM, Polyether ether ketone PEEK, Polyetherimide PEI, Polyphenylene oxide PPO, Polyphenylene sulfide PPS, Polystyrene PS, Polypropylene PP, Polyvinyl chloride PVC, Teflon PTFE, etc.
  - is cured using any MANUFACTURING PROCESS such as Autoclave, Resin Transfer Molding like VARTM, SQRTM, RIM, SRIM, Filament Winding, Pultrusion, Compression Molding, Wet-lay up, etc.

- The analysis is equally applicable to Hybrid Laminates manufactured from a single or multiple types of lamina materials and/or ply broad forms or fiber types or single or multiple materials systems or their combinations

Theoretical Background

- **3pcsolver009** solver is based on a powerful complex variable analysis approach developed by Muskhelishvili, Savin and Lekhnitskii to obtain stresses in infinite laminated composite plates with cutout(s) or hole(s) subjected to applied far-field axial, transverse and shear loads. The laminated plate stresses and strains obtained using the complex variable based analytical approach are independent of laminated plate length, width and the hole diameter around the edge contour of the hole. However, the an empirical finite width correction factor can be used for uniaxial loading to account for W/d or L/d effects. For $\frac{W}{d}\, or \frac{L}{d} > 8$, the error in the stresses and strains obtained with or without correction factor are less than 2%.

- The stresses and strains can also be obtained at a characteristic distance d_0 away from the contour of the hole. Strength or failure load for the laminated composite plate with circular or elliptical hole can be predicted using standard composite failure theories

- Analytical solutions obtained here using the complex potential approach are valid for linear elastic homogeneous materials only

Theoretical Background

- In-plane strains of the reference surface (usually mid-plane) of a laminate in terms of in-plane force resultants can be written as

$$\begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}$$

- Using stress-function approach, the laminate in-plane force resultants are assumed in terms of stress function $F(x, y)$, a real function, as

$$N_{xx} = \frac{\partial^2 F}{\partial y^2} \qquad N_{yy} = \frac{\partial^2 F}{\partial x^2} \qquad N_{xy} = N_{yx} = -\frac{\partial^2 F}{\partial x \partial y}$$

- Using stress-function approach, equation of equilibriums are satisfied, and the constitutive law and the compatibility equations can be used to get the following

$$a_{22}\frac{\partial^4 F}{\partial x^4} - 2a_{26}\frac{\partial^4 F}{\partial x^3 \partial y} + (2a_{12} + a_{66})\frac{\partial^4 F}{\partial x^2 \partial y^2} - 2a_{16}\frac{\partial^4 F}{\partial x \partial y^3} + a_{11}\frac{\partial^4 F}{\partial y^4} = 0$$

- Further assuming $F(x, y) = F(z) = F(x + \mu y)$, a characteristic equation for the laminated plate can be obtained as

$$a_{11}\mu^4 - 2a_{16}\mu^3 + (2a_{12} + a_{66})\mu^2 - 2a_{26}\mu + a_{22} = 0$$

Theoretical Background

- Noting that a_{ij} are real, the solution to the characteristic equation for the laminated plate provides two pairs of complex conjugate roots: (i) all roots are different (ii) Roots are pairwise equal

$$\mu_1 = \alpha + i\beta; \overline{\mu_1} = \alpha - i\beta; \mu_2 = \gamma + i\delta, \overline{\mu_2} = \gamma - i\delta; \beta, \delta > 0$$

- Next, the stress functions are assumed in terms of complex potential $\varphi_j\left(z_j\right)$. Solution to the two complex potentials $\varphi_1\left(z_1\right)$ and $\varphi_2\left(z_2\right)$ are obtained by satisfying the static and/or kinematic boundary conditions at the internal boundary such as contour of the opening, and at the external boundary of the plate at infinity. Furthermore, the contour of a hole or cutout is transformed into a unit circle using conformal mapping approach. Once the complex functions $\varphi_j\left(z_j\right)$ are known, the in-plane force resultants and displacements around the hole in the laminated plate are obtained as

$$N_{\theta\theta} = N_{\theta\theta}^0 + 2Re[(\mu_1^2\sin^2\theta + \cos^2\theta + \mu_1\sin 2\theta)\varphi_{01}'(z_1) + (\mu_2^2\sin^2\theta + \cos^2\theta + \mu_2\sin 2\theta)\varphi_{02}'(z_2)]$$

$$N_{rr} = N_{rr}^0 + 2Re[(\mu_1^2\cos^2\theta + \sin^2\theta - \mu_1\sin 2\theta)\varphi_{01}'(z_1) + (\mu_2^2\cos^2\theta + \sin^2\theta - \mu_2\sin 2\theta)\varphi_{02}'(z_2)]$$

$$N_{r\theta} = N_{r\theta}^0 + 2Re\begin{bmatrix} \left((1-\mu_1^2)\,sin\,\theta\,cos\,\theta - \mu_1(\cos^2\theta - \sin^2\theta)\right)\varphi_{01}'(z_1) \\ + \left((1-\mu_2^2)\,sin\,\theta\,cos\,\theta - \mu_2(\cos^2\theta - \sin^2\theta)\right)\varphi_{02}'(z_2) \end{bmatrix}$$

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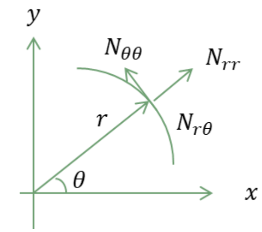
Theoretical Background

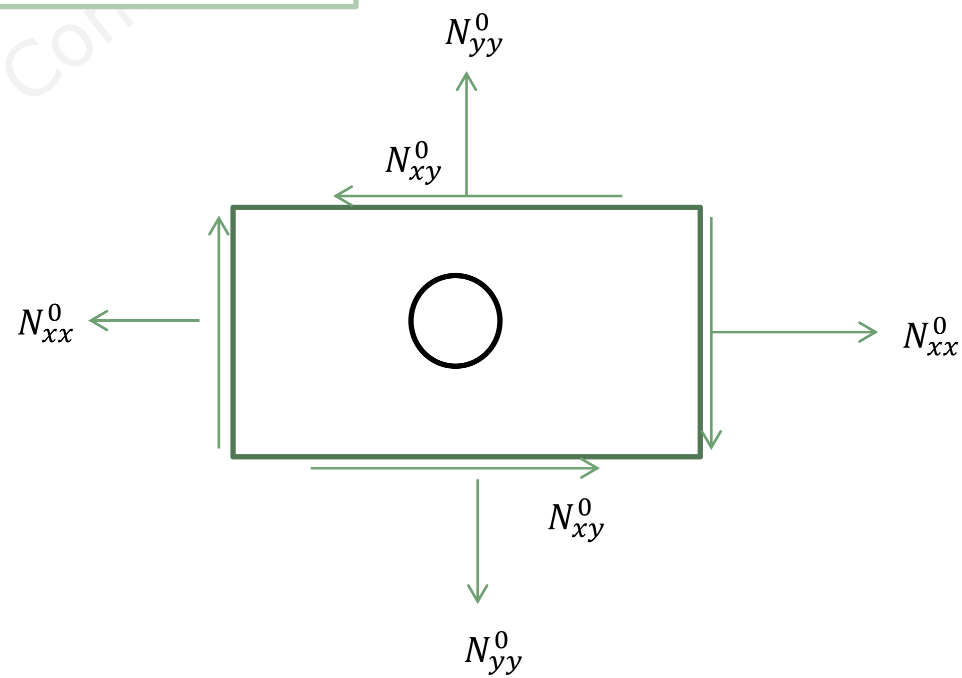
$$N_{rr}^0 = N_{xx}^0 \cos^2\theta + N_{yy}^0 \sin^2\theta + N_{xy}^0 \sin 2\theta$$
$$N_{\theta\theta}^0 = N_{xx}^0 \sin^2\theta + N_{yy}^0 \cos^2\theta - N_{xy}^0 \sin 2\theta$$
$$N_{r\theta}^0 = \left(N_{yy}^0 - N_{xx}^0\right) \sin\theta \cos\theta + N_{xy}^0(\cos^2\theta - \sin^2\theta)$$



$$u_r(r,\theta) = 2Re\left[(p_1 cos\theta + q_1 sin\theta)\varphi_1\ (z_1) + (p_2 cos\theta + q_2 sin\theta)\varphi_2\ (z_2)\right]$$
$$u_\theta(r,\theta) = 2Re\left[(q_1 cos\theta - p_1 sin\theta)\varphi_1\ (z_1) + (q_2 cos\theta - p_2 sin\theta)\varphi_2\ (z_2)\right]$$

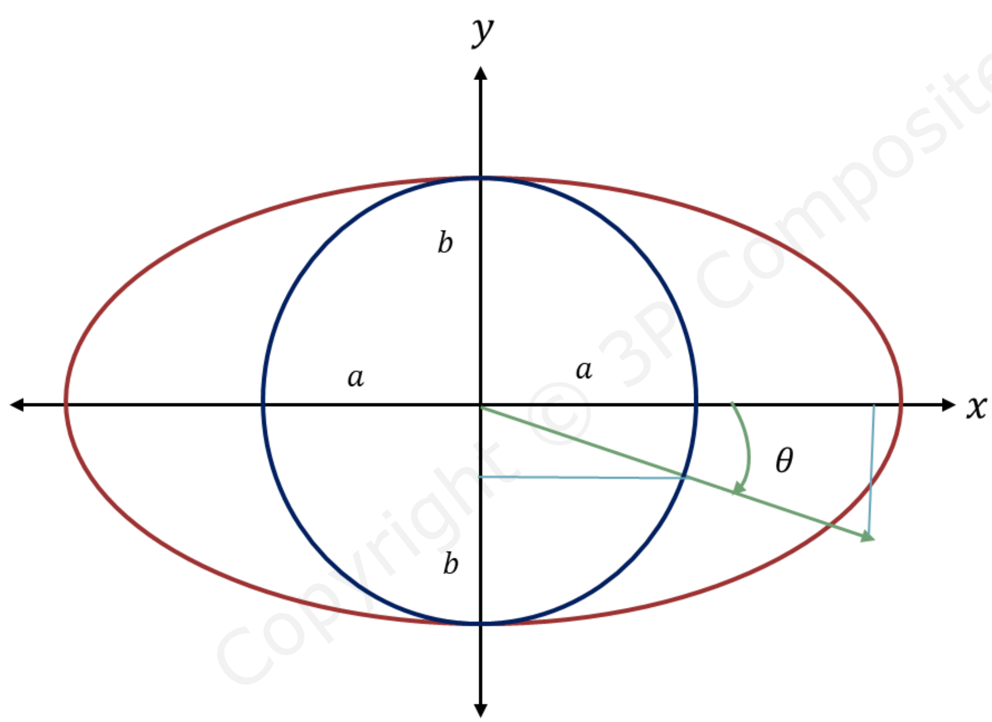
$$p_1 = a_{11}\mu_1^2 + a_{12} - a_{16}\mu_1; p_2 = a_{11}\mu_2^2 + a_{12} - a_{16}\mu_2$$
$$q_1 = a_{12}\mu_1 + \frac{a_{22}}{\mu_1} - a_{26}; q_2 = a_{12}\mu_2 + \frac{a_{22}}{\mu_2} - a_{26}$$

$$N_{xx} = N_{xx}^0 + 2Re[\mu_1^2\varphi_{01}'(z_1) + \mu_2^2\varphi_{02}'(z_2)]$$
$$N_{yy} = N_{yy}^0 + 2Re[\varphi_{01}'(z_1) + \varphi_{02}'(z_2)]$$
$$N_{xy} = N_{xy}^0 - 2Re[\mu_1\varphi_{01}'(z_1) + \mu_2\varphi_{02}'(z_2)]$$
$$u(x,y) = 2Re\left[p_1\varphi_1\ (z_1) + p_2\varphi_2\ (z_2)\right],$$
$$v(x,y) = 2Re\left[q_1\varphi_1\ (z_1) + q_2\varphi_2\ (z_2)\right]$$



Theoretical Background

- The expressions for complex functions $\varphi_j\left(z_j\right)$ and its derivatives are complex and are not provided here. The angle θ is measured positive clockwise as shown. For elliptical hole, a is semi-major-axis and b is semi-minor-axis, and for circular hole, $a = b = r$



θ	x	y
0, 360	a	0
90	0	-b
180	-a	0
270	0	b

Theoretical Background

- Hygrothermal effects can be accounted for the given difference in temperature ΔT and difference in moisture content ΔC. Laminated plate theory can be used to compute thermal force resultants N_{xx}^{T}, N_{yy}^{T} & N_{xy}^{T} and/or moisture force resultants N_{xx}^{M}, N_{yy}^{M} & N_{xy}^{M} as shown below

$$\begin{Bmatrix} N_{xx}^{T\,or M} \\ N_{yy}^{T\,or\,M} \\ N_{xy}^{T\,or\,M} \end{Bmatrix} = [A] \begin{Bmatrix} \varepsilon_{xx}^{T\,or\,M} \\ \varepsilon_{yy}^{T\,or\,M} \\ \gamma_{xy}^{T\,or\,M} \end{Bmatrix}, [A] = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix}^{-1}$$

- The **total applied** in-plane edge (or buckling) loads are then obtained as summation of mechanical and hydrothermal force resultants as

$$\begin{Bmatrix} N_{xx}^{Total} \\ N_{yy}^{Total} \\ N_{xy}^{Total} \end{Bmatrix} = \begin{Bmatrix} N_{xx}^{T} \\ N_{yy}^{T} \\ N_{xy}^{T} \end{Bmatrix} + \begin{Bmatrix} N_{xx}^{M} \\ N_{yy}^{M} \\ N_{xy}^{M} \end{Bmatrix} + \begin{Bmatrix} N_{xx}^{0} \\ N_{yy}^{0} \\ N_{xy}^{0} \end{Bmatrix}$$

Theoretical Background

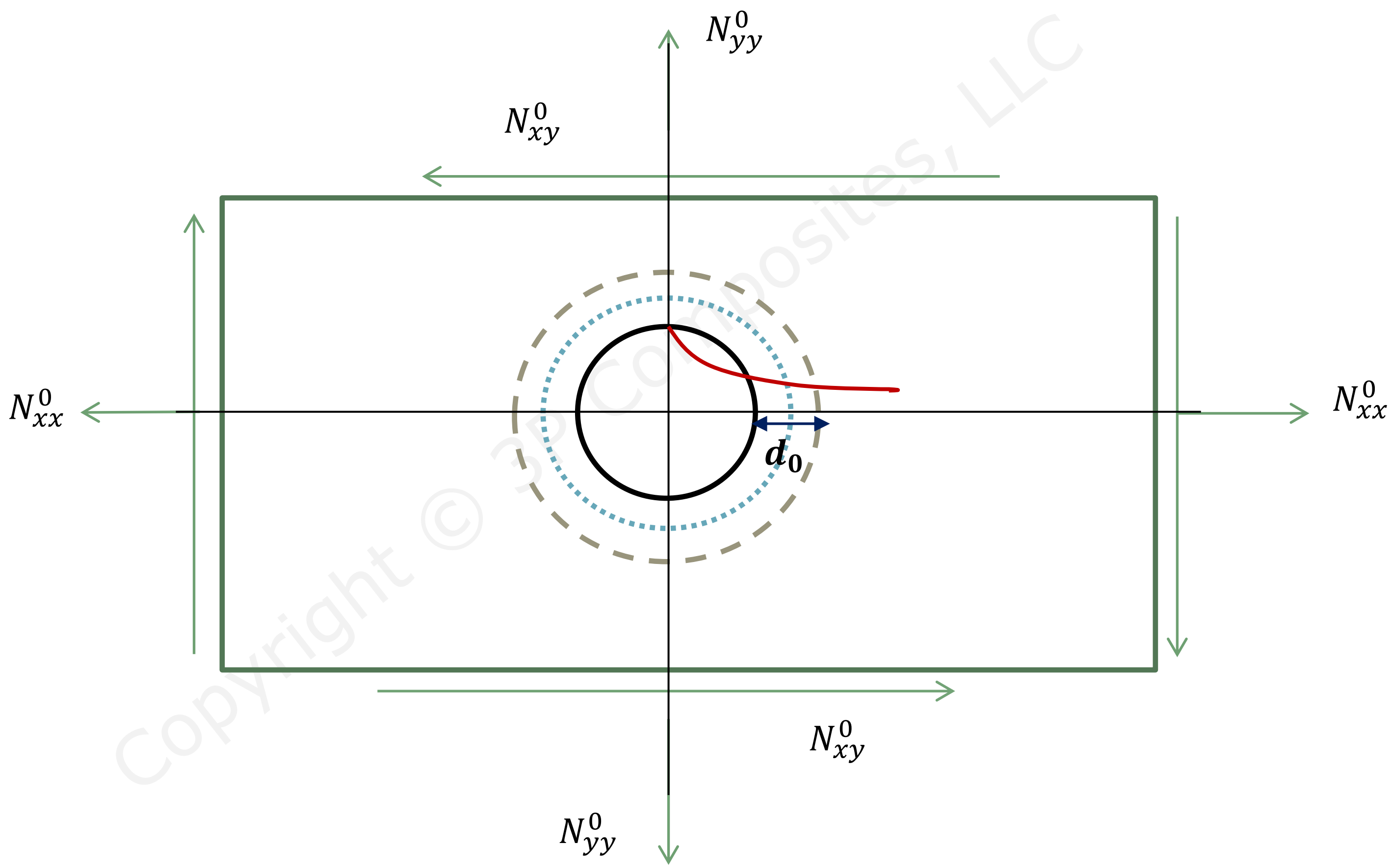
- Given the laminate material, ply stack up information, plate and cutout dimensions and far-field applied in-plane mechanical and hygrothermal tension/compression and/or shear edge loads, **3pcsolver009** solver calculates in-plane force resultants and displacements going around the hole $0^o \leq \theta \leq 360^o$ in clockwise direction, in both polar and Cartesian systems. Furthermore, stresses and strains, margins of safety/failure indices in each ply are obtained from the calculated in-plane force resultants using laminate theory

- There is high stress gradients at the contour of the cutout (stress concentration effects) as shown in Figure on next Slide. Hence, it is prudent to apply the selected failure criteria away from the hole boundary to get a good estimate of the notched strength (or failure loads) for laminated plates with cutouts. Typically, an experimentally validated characteristic distance d_0 away from the contour of the hole is used to calculate the quantities mentioned above in the step increments. Both the characteristic distance d_0 and the number of steps are provided by the user

- Details of the theoretical approach along with verification and application examples are provided in the training module **3pcmodule009**

Theoretical Background

❖ High gradients of in-plane force resultants reaching far-filed applied loads away from the hole



Inputs

- All inputs should be in consistent units. Use either (N, m, kg, Celsius, N/mm N-m/m) OR (N, mm, Kg, Celsius, N/mm, N-mm/mm) or (lbs, in, Fahrenheit, lb/in, lb-in/in) consistently. Inputs in scientific notation (0.0+e) are acceptable

- Input process is intuitive and uses the following logical order:

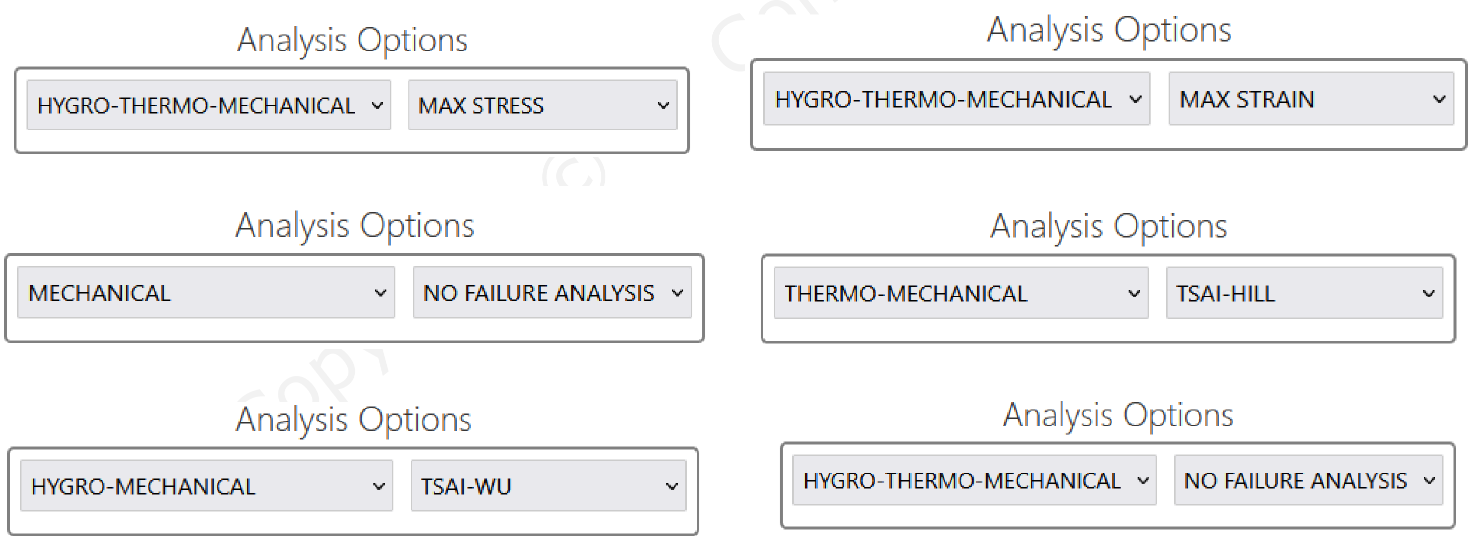
  - Analysis Options
  - Materials
  - Plies / Laminae
  - Laminates
  - Panels
  - Loads
  - Output Options

- The type of analysis selected dictates the required inputs. In general, the loads are applied to the panels consisting of laminates, which are built from specifically oriented plies/laminae fabricated from individual materials

Inputs: Analysis Options

❖ Analysis Options:

Four types of analyses can be performed using this solver, viz. (i) Mechanical (ii) Thermo-mechanical (iii) Hygro-mechanical, and (iv) Hygro-thermo-mechanical, In addition, Failure analysis of the laminates can be performed based on First-ply failure criteria using one of the four commonly used composite failure theories, viz. (i) Maximum Stress (ii) Maximum Strain (iii) Tsai-Hill, and (iv) Tsai-Wu. In total there are **Twenty** (20) possible ways of using this solver to perform stress concentration analysis of laminated composite plates subjected to Hygro-thermo-mechanical loads. A few combination of analyses are shown below:



Inputs: Materials

❖ Material Properties:

In the SI system, MPa or Pa, and in the US system Psi are used to input the orthotropic lamina Moduli E_1, E_2, G_{12}, G_{13} and G_{23}. Coefficient of thermal expansions α_1 and α_2 are expressed in mm/mm or m/m per degree Celsius in SI system, and in/in per Fahrenheit in the US system. Similarly, Coefficient of moisture expansions β_1 and β_2 are in mm/mm per Kg/Kg or m/m per Kg/Kg in SI system and in/in per lb/lb in the US system. Coefficients of Thermal and Moisture expansions are required to perform hygrothermal analysis due change in temperature and/or moisture content. ν_{12} is major Poison's ratio. Multiple lamina types and lamina materials can be input by simply clicking the '+' sign on the extreme right. Based on the type of analyses selected, the required material inputs for an orthotopic Lamina can vary as shown below:

Materials

ID	E_1	E_2	G_{12}	G_{23}	G_{13}	ν_{12}	
1	0	0	0	0	0	0	+ −
	MUST BE +VE REAL	MUST BE +VE REAL	MUST BE +VE REAL	MUST BE +VE REAL	MUST BE +VE REAL	MUST BE +VE REAL	

Materials

ID	E_1	E_2	G_{12}	G_{23}	G_{13}	ν_{12}	α_1	α_2	β_1	β_2	
1	0	0	0	0	0	0	0	0	0	0	+ −
	MUST BE +VE REAL	MUST BE +VE REAL	MUST BE +VE REAL	MUST BE +VE REAL	MUST BE +VE REAL	MUST BE +VE REAL					

Inputs: Materials

❖ Material Allowables:

Additional inputs are required to perform laminate failure analysis. Depending upon the type of failure theory selected for an orthotropic lamina, either strength ($\sigma_{11}^{T}, \sigma_{11}^{C}, \sigma_{22}^{T}, \sigma_{22}^{C}$ and τ_{12}) or strain ($\varepsilon_{11}^{T}, \varepsilon_{11}^{C}, \varepsilon_{22}^{T}, \varepsilon_{22}^{C}$ and γ_{12}) based material allowables, in tension, compression and shear should be input as shown below:

Allowables ⓘ

ID	σ_{11}^{T}	σ_{11}^{C}	σ_{22}^{T}	σ_{22}^{C}	τ_{12}
1	0	0	0	0	0

Allowables ⓘ

ID	ε_{11}^{T}	ε_{11}^{C}	ε_{22}^{T}	ε_{22}^{C}	γ_{12}
1	0	0	0	0	0

Allowables ⓘ

ID	σ_{11}^{T}	σ_{22}^{T}	τ_{12}
1	0	0	0

Strength allowables are input as MPa or Pa in SI system, and in Psi in the US systems, and should be consistent with the unit system used for input of Moduli

Inputs: Plies

❖ Plies/Laminae:

Types of plies in a laminate are required as input. Each ply type is defined by its angle (or orientation) in degrees, material type and its thickness. Material of a ply/lamina can be selected from a predefined list of materials that are input in the Material Properties Section above. The thickness of the ply or lamina is in mm or m in the SI system or inch in the US system. Multiple ply or lamina types can be input by simply clicking the '+' sign on the extreme right. The required ply/lamina type inputs with few examples are shown below:

Plies

ID	Angle (deg)	Material	Thickness
1	0	Uni	0.005
2	45	PW	0.010
3	90	Uni	0.005

Plies

ID	Angle (deg)	Material	Thickness
1	0	CEP	0.005
2	30	Flax	0.010
3	60	CEP	0.005

Plies

ID	Angle (deg)	Material	Thickness
1	0	CEP	0.005
2	45	CEP	0.005
3	-45	CEP	0.005
4	90	CEP	0.005
5	0	Flax	0.01

Inputs: Laminates

❖ Laminates:

Multiple laminates can be quickly created by defining their stacking sequences using the plies defined in the previous step. Hybrid laminates can be defined using different ply and material combinations established in the previous steps. Additional laminates can be added by simply clicking the '+' sign on the extreme right. A few examples of laminates and their inputs are shown below:

Laminates

ID	Stacking Sequence	Stacking Sequence (Angle)	Offset		
CEP-QI	2,3,1,4,4,1,3,2	45,-45,0,90,90,0,-45	Middle	+	–
CEP-Cross Ply	1,4,1,4,1,4,1,4	0,90,0,90,0,90,0,90	Bottom	+	–
CEP-Angle Ply	2,3,2,3,2,3,2,3	45,-45,45,-45,45,-45	Top	+	–
CEP-Flax Hybrid	1,2,3,4,5	0,45,-45,90,0	Middle	+	–

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Inputs: Panels

❖ Panels:

Panels are defined using the definitions of laminates, plate length and width dimensions, hole/cutout type and its dimensions. A laminated plate has a unique ID that facilitates its analyses for multiple load cases. Laminated plate analysis is based on middle surface being the reference plane. Additional laminated composite plates can be added by simply clicking the '+' sign on the extreme right (see below):

Panels

ID	Length	Width	Laminate	Cutout	Major Axis Length	Minor Axis Length	
1	10	10	1	Circle	0.25	0.25	+ −
2	6	6	2	Ellipse	0.2	0.2	+ −
3	6	6	3	Circle	0.2	0.2	+ −

Inputs: Loads

❖ Loads:

Hygrothermomechanical loads can be applied to the laminated plates with cutouts. Single or multiple panels (or laminates) can be analyzed for single or multiple load cases (upto 100 max). For analyses of laminates subjected to mechanical loads, in-plane force resultants are provided as inputs. For thermomechanical analysis, temperature change is required as input. For hygromechanical analysis, change in moisture absorption is required as input. Examples of the load inputs for typical mechanical or full hygrothermomechanical analyses are shown below. Additional load cases can be added by simply clicking the '+' sign on the extreme right as shown below:

Loads

ID	Panel	N_{xx}	N_{yy}	N_{xy}		
1	1	10000	0	0	+	−
2	1	-10000	0	0	+	−
3	1	0	10000	0	+	−
4	1	0	-10000	0	+	−
5	1	0	0	10000	+	−
6	1	0	0	-10000	+	−
7	2	0	0	-10000	+	−

Inputs: Loads

Loads

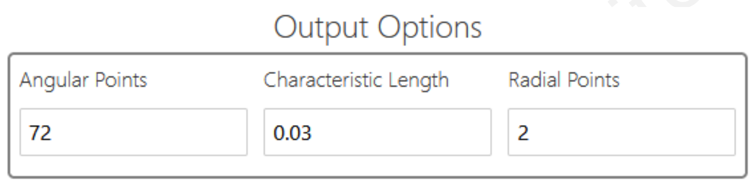
ID	Panel	N_{xx}	N_{yy}	N_{xy}	Δ T	Δ C
1	1	10000	0	0	0	0
2	1	-10000	0	0	0	0
3	1	0	10000	0	0	0
4	1	0	-10000	0	0	0
5	1	0	0	10000	0	0
6	1	0	0	-10000	0	0
7	2	0	0	-10000	0	0

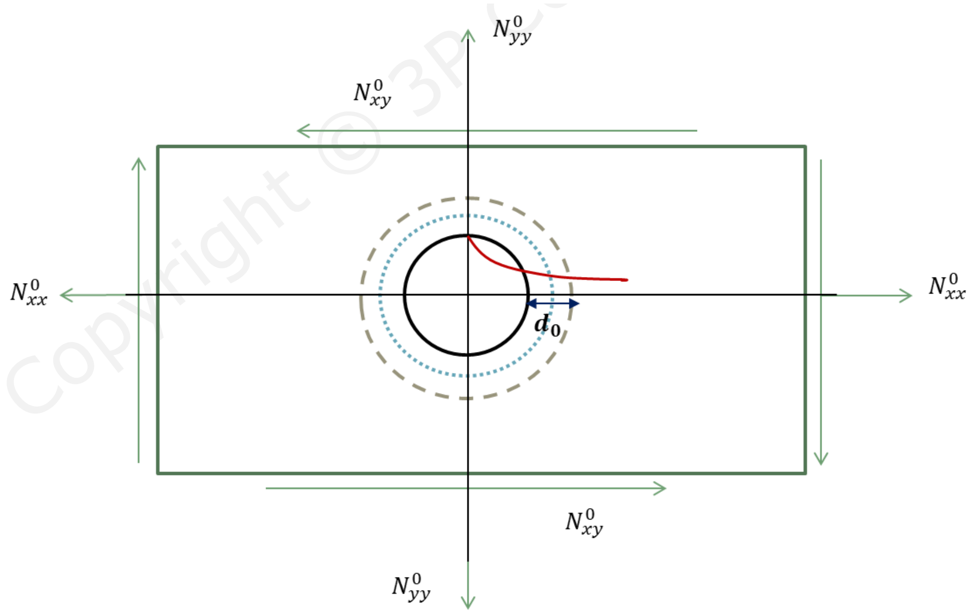
In-plane force force resultants are input as N/mm or N/m in SI system, and lb/in in provided as inputs. Differential Temperature is input in degree Celsius in SI system, and in Fahrenheit in the US system. Difference in moisture absorption is prescribed as Kg/Kg in SI system and lb/lb in the US system

Inputs: Output Options

❖ Output Options:

User can request ALL output quantities (see next Slide) along the contour of the cutout or hole $0^o \leq \theta \leq 360^o$ in clockwise direction at minimum 8 and maximum 72 points or subdivisions. As mentioned earlier, a characteristic distance d_0 and number of radial points (maximum 4) can be provided by the user to obtain ALL output quantities along multiple contour(s) or concentric points of the cutout or hole as one moves away from the hole boundary





Outputs

❖ **Analysis Outputs:**

Once all the Input steps viz., Analysis Options, Materials, Plies / Laminae, Laminates , Panels and Loads are completed, analyses can be run by clicking the "submit" button. Maximum 100 Load Cases can be analyzed at one time

Submit

Upon completion of analyses, an output is displayed for each Load ID in the window underneath

Output

Depending upon the analysis option selected, the analyses output contains the following information at minimum.

- Material Properties and Laminate Information
- Laminate [A], [B], [D] stiffness matrices
- Effective laminate in-plane and flexural engineering constants
- Effective hygrothermal engineering constants
- Applied Hygro-thermo-mechanical loads
- Laminated Plate in-plane force resultants around cutout in both polar and Cartesian Systems
- Ply-by-Ply in-plane strains and Stresses (TOTAL, MECHANICAL OR RESIDUAL) in global and ply coordinate systems
- Laminate/Lamina failure analysis– Failure Indices or Margins of Safety

Outputs: Laminate and Stiffnesses

3pc-solver009, v1.0b1

LOADS ID PANEL ID
1 1

PANEL GEOMETRY
LENGTH: 10.00
WIDTH: 10.00
CUTOUT: circle
MAJOR AXIS LENGTH: 0.25
MINOR AXIS LENGTH: 0.25

ANALYSIS OPTIONS
NUMBER OF POINTS AROUND CUTOUT: 72
CHARACTERISTIC LENGTH: 0.0300
NUMBER OF ADDL POINTS ON CHARACTERISTIC LENGTH: 2

MATERIAL PROPERTIES

ID	E1	E2	G12	v12
Tuttle	2.25e+07	1.10e+06	6.40e+05	0.3400

HYGROTHERMAL MATERIAL PROPERTIES

ID	alpha1	alpha2	beta1	beta2
Tuttle	1.00e-06	1.00e-05	1.00e-05	1.00e-04

MATERIAL ALLOWABLES

ID	SIG11T	SIG22T	TAU12
Tuttle	+20000.00	+20000.00	+20000.00

LAMINATE GEOMETRY
STACKING SEQUENCE (PLY ANG): [+0.0]
STACKING SEQUENCE (PLY MAT): [Tuttle]
TOTAL THICKNESS: 1.0000

Outputs: Effective Stiffnesses and Applied Loads

LAMINATE PROPERTIES

A MATRIX

+11780785.28	+2039274.41	+0.00
+2039274.41	+4888001.95	+0.00
+0.00	+0.00	+2332000.00

B MATRIX

+0.00	+0.00	+0.00
+0.00	+0.00	+0.00
+0.00	+0.00	+0.00

D MATRIX

+1.00	+0.00	+0.00
+0.00	+1.00	+0.00
+0.00	+0.00	+1.00

EFFECTIVE LAMINATE INPLANE AND FLEXURAL ENGINEERING CONSTANTS

Ex	Ey	Gxy	vxy	vyx	Efx	Efy	Gfxy	vfxy	vfyx
+1.09e+07	+4.54e+06	+2.33e+06	+0.4172	+0.1731	+2.25e+07	+1.10e+06	+6.40e+05	+0.3400	+0.0166

EFFECTIVE LAMINATE HYGROTHERMAL ENGINEERING CONSTANTS

alphax	alphay	alphaxy	betax	betay	betaxy
+1.98e-06	+1.52e-06	+0.00e+00	+1.98e-05	+1.52e-05	+0.00e+00

MECHANICAL LOADS

NX	NY	NXY
+10000.00	+0.00	+0.00

THERMAL LOADS

DELTAT	NXT	NYT	NXYT
+10.00	+263.89	+114.39	+0.00

HYGRAL LOADS

DELTAC	NXH	NYH	NXYH
+10.00	+2638.91	+1143.86	+0.00

TOTAL LOADS

NX	NY	NXY
+12902.81	+1258.25	+0.00

Outputs: Force Resultants Around Cutout

LAMINATE RUNNING LOADS AROUND CUTOUT

DISTANCE	ANGLE	NXX	NYY	NXY	NRR	NTT	NRT
0.0000e+00	0.00	-0.00	-4915.10	+0.00	-0.00	-4915.10	+0.00
0.0000e+00	5.00	-36.09	-4714.45	-412.46	-0.00	-4750.53	-0.00
0.0000e+00	10.00	-128.34	-4128.00	-727.88	-0.00	-4256.35	-0.00
0.0000e+00	15.00	-229.81	-3200.88	-857.67	-0.00	-3430.70	-0.00
0.0000e+00	20.00	-265.44	-2003.70	-729.29	+0.00	-2269.14	+0.00
0.0000e+00	25.00	-136.26	-626.63	-292.20	-0.00	-762.88	+0.00
0.0000e+00	30.00	+275.77	+827.32	+477.65	+0.00	+1103.09	+0.00
0.0000e+00	35.00	+1102.62	+2248.92	+1574.71	+0.00	+3351.54	+0.00
0.0000e+00	40.00	+2484.55	+3528.75	+2960.98	-0.00	+6013.31	-0.00
0.0000e+00	310.00	+7466.86	+5257.33	-6265.44	+0.00	+12724.20	+0.00
0.0000e+00	315.00	+4562.64	+4562.64	-4562.64	-0.00	+9125.28	-0.00
0.0000e+00	320.00	+2484.55	+3528.75	-2960.98	-0.00	+6013.31	+0.00
0.0000e+00	325.00	+1102.62	+2248.92	-1574.71	+0.00	+3351.54	-0.00
0.0000e+00	330.00	+275.77	+827.32	-477.65	-0.00	+1103.09	+0.00
0.0000e+00	335.00	-136.26	-626.63	+292.20	+0.00	-762.88	+0.00
0.0000e+00	340.00	-265.44	-2003.70	+729.29	-0.00	-2269.14	+0.00
0.0000e+00	345.00	-229.81	-3200.88	+857.67	+0.00	-3430.70	-0.00
0.0000e+00	350.00	-128.34	-4128.00	+727.88	-0.00	-4256.35	+0.00
0.0000e+00	355.00	-36.09	-4714.45	+412.46	-0.00	-4750.53	-0.00
1.5000e-02	0.00	-141.21	-2442.38	+0.00	-141.21	-2442.38	+0.00
1.5000e-02	5.00	-75.63	-2335.56	+175.35	-123.25	-2287.94	+368.90
1.5000e-02	10.00	+131.73	-2023.15	+396.95	-69.01	-1822.41	+741.52
1.5000e-02	15.00	+512.31	-1529.10	+706.14	+22.49	-1039.28	+1121.88
1.5000e-02	20.00	+1116.79	-892.14	+1134.14	+152.78	+71.87	+1514.46
1.5000e-02	25.00	+2011.73	-163.84	+1696.51	+323.56	+1524.33	+1923.78
1.5000e-02	30.00	+3273.16	+594.77	+2387.24	+536.15	+3331.78	+2353.39
1.5000e-02	90.00	+31040.24	+2933.21	+0.00	+2933.21	+31040.24	+0.00
1.5000e-02	95.00	+30748.33	+2921.90	-1162.40	+2931.43	+30738.80	-1271.27
1.5000e-02	100.00	+29853.38	+2896.37	-2310.81	+2918.88	+29830.87	-2438.47
1.5000e-02	105.00	+28313.29	+2876.80	-3408.19	+2876.62	+28313.47	-3407.54
1.5000e-02	110.00	+26105.36	+2881.56	-4382.20	+2781.40	+26205.51	-4107.03
1.5000e-02	115.00	+23278.99	+2910.32	-5133.38	+2615.90	+23573.42	-4501.98
1.5000e-02	120.00	+19988.96	+2933.52	-5565.81	+2377.25	+20545.23	-4602.32
1.5000e-02	125.00	+16481.18	+2897.12	-5626.25	+2079.19	+17299.10	-4458.13
1.5000e-02	130.00	+13032.54	+2742.26	-5328.14	+1746.76	+14028.04	-4141.75
1.5000e-02	135.00	+9879.59	+2428.11	-4746.52	+1407.33	+10900.37	-3725.74
1.5000e-02	140.00	+7173.82	+1945.66	-3990.74	+1083.55	+8035.92	-3267.35
1.5000e-02	145.00	+4975.46	+1318.50	-3173.24	+790.49	+5503.48	-2803.52
1.5000e-02	150.00	+3273.16	+594.77	-2387.24	+536.15	+3331.78	-2353.39
1.5000e-02	155.00	+2011.73	-163.84	-1696.51	+323.56	+1524.33	-1923.78
1.5000e-02	160.00	+1116.79	-892.14	-1134.14	+152.78	+71.87	-1514.46
1.5000e-02	165.00	+512.31	-1529.10	-706.14	+22.49	-1039.28	-1121.88

Outputs: Force Resultants Around Cutout

LAMINATE RUNNING LOADS AROUND CUTOUT

DISTANCE	ANGLE	NXX	NYY	NXY	NRR	NTT	NRT
1.5000e-02	170.00	+131.73	-2023.15	-396.95	-69.01	-1822.41	-741.52
1.5000e-02	175.00	-75.63	-2335.56	-175.35	-123.25	-2287.94	-368.90
1.5000e-02	180.00	-141.21	-2442.38	-0.00	-141.21	-2442.38	-0.00
1.5000e-02	185.00	-75.63	-2335.56	+175.35	-123.25	-2287.94	+368.90
1.5000e-02	190.00	+131.73	-2023.15	+396.95	-69.01	-1822.41	+741.52
1.5000e-02	195.00	+512.31	-1529.10	+706.14	+22.49	-1039.28	+1121.88
1.5000e-02	200.00	+1116.79	-892.14	+1134.14	+152.78	+71.87	+1514.46
1.5000e-02	205.00	+2011.73	-163.84	+1696.51	+323.56	+1524.33	+1923.78
1.5000e-02	210.00	+3273.16	+594.77	+2387.24	+536.15	+3331.78	+2353.39
1.5000e-02	215.00	+4975.46	+1318.50	+3173.24	+790.49	+5503.48	+2803.52
1.5000e-02	220.00	+7173.82	+1945.66	+3990.74	+1083.55	+8035.92	+3267.35
1.5000e-02	225.00	+9879.59	+2428.11	+4746.52	+1407.33	+10900.37	+3725.74
1.5000e-02	230.00	+13032.54	+2742.26	+5328.14	+1746.76	+14028.04	+4141.75
1.5000e-02	335.00	+2011.73	-163.84	-1696.51	+323.56	+1524.33	-1923.78
1.5000e-02	340.00	+1116.79	-892.14	-1134.14	+152.78	+71.87	-1514.46
1.5000e-02	345.00	+512.31	-1529.10	-706.14	+22.49	-1039.28	-1121.88
1.5000e-02	350.00	+131.73	-2023.15	-396.95	-69.01	-1822.41	-741.52
1.5000e-02	355.00	-75.63	-2335.56	-175.35	-123.25	-2287.94	-368.90
3.0000e-02	0.00	+271.06	-935.72	+0.00	+271.06	-935.72	+0.00
3.0000e-02	5.00	+404.86	-885.92	+517.42	+305.21	-786.26	+621.63
3.0000e-02	10.00	+813.65	-741.20	+1048.31	+408.22	-335.77	+1250.99
3.0000e-02	15.00	+1518.53	-515.38	+1601.45	+581.56	+421.59	+1895.37
3.0000e-02	20.00	+2550.74	-230.40	+2175.97	+826.72	+1493.61	+2560.73
3.0000e-02	25.00	+3944.44	+85.89	+2756.46	+1143.71	+2886.62	+3249.73
3.0000e-02	30.00	+5724.22	+403.80	+3308.91	+1528.52	+4599.50	+3958.26
3.0000e-02	35.00	+7886.60	+698.10	+3779.89	+1969.73	+6614.97	+4670.29
3.0000e-02	40.00	+10378.06	+955.81	+4102.39	+2444.95	+8888.92	+5351.93
3.0000e-02	45.00	+13077.97	+1182.80	+4211.31	+2919.07	+11341.70	+5947.59
3.0000e-02	50.00	+15801.30	+1404.25	+4067.10	+3347.45	+13858.09	+6382.92
3.0000e-02	285.00	+24078.99	+3118.08	-1206.63	+3918.88	+23278.19	-4195.25
3.0000e-02	290.00	+23325.94	+2729.60	-1787.14	+3990.16	+22065.37	-5250.51
3.0000e-02	295.00	+22151.09	+2328.97	-2442.26	+3998.44	+20481.62	-6022.46
3.0000e-02	300.00	+20484.45	+1962.27	-3104.45	+3904.28	+18542.44	-6468.11
3.0000e-02	305.00	+18332.57	+1655.06	-3677.80	+3685.79	+16301.84	-6577.99
3.0000e-02	310.00	+15801.30	+1404.25	-4067.10	+3347.45	+13858.09	-6382.92
3.0000e-02	315.00	+13077.97	+1182.80	-4211.31	+2919.07	+11341.70	-5947.59
3.0000e-02	320.00	+10378.06	+955.81	-4102.39	+2444.95	+8888.92	-5351.93
3.0000e-02	325.00	+7886.60	+698.10	-3779.89	+1969.73	+6614.97	-4670.29
3.0000e-02	330.00	+5724.22	+403.80	-3308.91	+1528.52	+4599.50	-3958.26
3.0000e-02	335.00	+3944.44	+85.89	-2756.46	+1143.71	+2886.62	-3249.73
3.0000e-02	340.00	+2550.74	-230.40	-2175.97	+826.72	+1493.61	-2560.73
3.0000e-02	345.00	+1518.53	-515.38	-1601.45	+581.56	+421.59	-1895.37
3.0000e-02	350.00	+813.65	-741.20	-1048.31	+408.22	-335.77	-1250.99
3.0000e-02	355.00	+404.86	-885.92	-517.42	+305.21	-786.26	-621.63

Outputs: Displacements Around Cutout

DISPLACEMENTS AROUND CUTOUT

DISTANCE	ANGLE	U	V	UR	UT
0.0000e+00	0.00	+5.1445e-04	+6.9496e-23	+5.1445e-04	+6.9496e-23
0.0000e+00	5.00	+5.1249e-04	+1.1808e-05	+5.1157e-04	-3.2904e-05
0.0000e+00	10.00	+5.0663e-04	+2.3525e-05	+5.0302e-04	-6.4808e-05
0.0000e+00	15.00	+4.9692e-04	+3.5064e-05	+4.8906e-04	-9.4742e-05
0.0000e+00	20.00	+4.8342e-04	+4.6336e-05	+4.7012e-04	-1.2180e-04
0.0000e+00	25.00	+4.6625e-04	+5.7255e-05	+4.4676e-04	-1.4515e-04
0.0000e+00	75.00	+1.3315e-04	+1.3086e-04	+1.6086e-04	-9.4742e-05
0.0000e+00	80.00	+8.9333e-05	+1.3342e-04	+1.4690e-04	-6.4808e-05
0.0000e+00	85.00	+4.4837e-05	+1.3496e-04	+1.3836e-04	-3.2904e-05
0.0000e+00	90.00	+2.9472e-20	+1.3548e-04	+1.3548e-04	-2.1176e-20
0.0000e+00	95.00	-4.4837e-05	+1.3496e-04	+1.3836e-04	+3.2904e-05
0.0000e+00	100.00	-8.9333e-05	+1.3342e-04	+1.4690e-04	+6.4808e-05
0.0000e+00	105.00	-1.3315e-04	+1.3086e-04	+1.6086e-04	+9.4742e-05
0.0000e+00	110.00	-1.7595e-04	+1.2731e-04	+1.7981e-04	+1.2180e-04
0.0000e+00	115.00	-2.1741e-04	+1.2278e-04	+2.0316e-04	+1.4515e-04
0.0000e+00	120.00	-2.5722e-04	+1.1733e-04	+2.3022e-04	+1.6410e-04
1.5000e-02	135.00	-3.3058e-04	+7.3418e-05	+2.8567e-04	+1.8184e-04
1.5000e-02	140.00	-3.6628e-04	+6.2892e-05	+3.2101e-04	+1.8726e-04
1.5000e-02	145.00	-3.9929e-04	+5.3048e-05	+3.5750e-04	+1.8557e-04
1.5000e-02	150.00	-4.2900e-04	+4.3900e-05	+3.9348e-04	+1.7648e-04
1.5000e-02	155.00	-4.5491e-04	+3.5420e-05	+4.2726e-04	+1.6015e-04
1.5000e-02	160.00	-4.7662e-04	+2.7542e-05	+4.5730e-04	+1.3713e-04
1.5000e-02	165.00	-4.9381e-04	+2.0178e-05	+4.8221e-04	+1.0832e-04
1.5000e-02	170.00	-5.0626e-04	+1.3218e-05	+5.0086e-04	+7.4893e-05
1.5000e-02	175.00	-5.1378e-04	+6.5379e-06	+5.1240e-04	+3.8266e-05
1.5000e-02	180.00	-5.1631e-04	+9.3448e-21	+5.1631e-04	+5.3884e-20
1.5000e-02	185.00	-5.1378e-04	-6.5379e-06	+5.1240e-04	-3.8266e-05
1.5000e-02	190.00	-5.0626e-04	-1.3218e-05	+5.0086e-04	-7.4893e-05
1.5000e-02	195.00	-4.9381e-04	-2.0178e-05	+4.8221e-04	-1.0832e-04
1.5000e-02	200.00	-4.7662e-04	-2.7542e-05	+4.5730e-04	-1.3713e-04
1.5000e-02	205.00	-4.5491e-04	-3.5420e-05	+4.2726e-04	-1.6015e-04
3.0000e-02	295.00	+1.5525e-04	-1.1726e-04	+1.7189e-04	+9.1150e-05
3.0000e-02	300.00	+1.9136e-04	-1.0326e-04	+1.8510e-04	+1.1409e-04
3.0000e-02	305.00	+2.2919e-04	-8.8689e-05	+2.0411e-04	+1.3687e-04
3.0000e-02	310.00	+2.6815e-04	-7.4305e-05	+2.2928e-04	+1.5765e-04
3.0000e-02	315.00	+3.0735e-04	-6.0749e-05	+2.6029e-04	+1.7437e-04
3.0000e-02	320.00	+3.4574e-04	-4.8480e-05	+2.9601e-04	+1.8510e-04
3.0000e-02	325.00	+3.8220e-04	-3.7765e-05	+3.3474e-04	+1.8829e-04
3.0000e-02	330.00	+4.1574e-04	-2.8685e-05	+3.7438e-04	+1.8303e-04
3.0000e-02	335.00	+4.4545e-04	-2.1186e-05	+4.1267e-04	+1.6906e-04
3.0000e-02	340.00	+4.7065e-04	-1.5110e-05	+4.4743e-04	+1.4677e-04

Outputs: Ply-by-ply Strains

PLY BY PLY INPLANE STRAINS (TOTAL)

DISTANCE	ANGLE	ANGLE	Z	EPS1	EPS2	GAMMA12	EPSX	EPSY	GAMMAXY
			(x 1E-6)	(x 1E-6)	(x 1E-6)	(x 1E-6)	(x 1E-6)	(x 1E-6)	
0.0000e+00	0.00	+0.0	+0.00000	+187.61	-1083.82	+0.00	+187.61	-1083.82	+0.00
0.0000e+00	5.00	+0.0	+0.00000	+176.65	-1038.19	-176.87	+176.65	-1038.19	-176.87
0.0000e+00	10.00	+0.0	+0.00000	+145.82	-905.36	-312.13	+145.82	-905.36	-312.13
0.0000e+00	15.00	+0.0	+0.00000	+101.15	-697.05	-367.78	+101.15	-697.05	-367.78
0.0000e+00	20.00	+0.0	+0.00000	+52.20	-431.70	-312.73	+52.20	-431.70	-312.73
0.0000e+00	25.00	+0.0	+0.00000	+11.45	-132.98	-125.30	+11.45	-132.98	-125.30
0.0000e+00	30.00	+0.0	+0.00000	-6.35	+171.90	+204.82	-6.35	+171.90	+204.82
0.0000e+00	35.00	+0.0	+0.00000	+15.04	+453.82	+675.26	+15.04	+453.82	+675.26
0.0000e+00	40.00	+0.0	+0.00000	+92.62	+683.28	+1269.72	+92.62	+683.28	+1269.72
0.0000e+00	45.00	+0.0	+0.00000	+243.29	+831.94	+1956.53	+243.29	+831.94	+1956.53
1.5000e-02	150.00	+0.0	+0.00000	+276.76	+6.21	-1023.69	+276.76	+6.21	-1023.69
1.5000e-02	155.00	+0.0	+0.00000	+190.31	-112.92	-727.49	+190.31	-112.92	-727.49
1.5000e-02	160.00	+0.0	+0.00000	+136.23	-239.35	-486.34	+136.23	-239.35	-486.34
1.5000e-02	165.00	+0.0	+0.00000	+105.24	-356.73	-302.80	+105.24	-356.73	-302.80
1.5000e-02	170.00	+0.0	+0.00000	+89.28	-451.15	-170.22	+89.28	-451.15	-170.22
1.5000e-02	175.00	+0.0	+0.00000	+82.23	-512.12	-75.19	+82.23	-512.12	-75.19
1.5000e-02	180.00	+0.0	+0.00000	+80.31	-533.17	-0.00	+80.31	-533.17	-0.00
1.5000e-02	185.00	+0.0	+0.00000	+82.23	-512.12	+75.19	+82.23	-512.12	+75.19
1.5000e-02	190.00	+0.0	+0.00000	+89.28	-451.15	+170.22	+89.28	-451.15	+170.22
1.5000e-02	195.00	+0.0	+0.00000	+105.24	-356.73	+302.80	+105.24	-356.73	+302.80
1.5000e-02	200.00	+0.0	+0.00000	+136.23	-239.35	+486.34	+136.23	-239.35	+486.34
1.5000e-02	205.00	+0.0	+0.00000	+190.31	-112.92	+727.49	+190.31	-112.92	+727.49
1.5000e-02	210.00	+0.0	+0.00000	+276.76	+6.21	+1023.69	+276.76	+6.21	+1023.69
1.5000e-02	215.00	+0.0	+0.00000	+404.88	+100.82	+1360.74	+404.88	+100.82	+1360.74
1.5000e-02	220.00	+0.0	+0.00000	+582.08	+155.21	+1711.30	+582.08	+155.21	+1711.30
1.5000e-02	225.00	+0.0	+0.00000	+811.21	+158.31	+2035.39	+811.21	+158.31	+2035.39
1.5000e-02	230.00	+0.0	+0.00000	+1087.69	+107.23	+2284.80	+1087.69	+107.23	+2284.80
1.5000e-02	235.00	+0.0	+0.00000	+1397.30	+9.75	+2412.63	+1397.30	+9.75	+2412.63
.0000e-02	270.00	+0.0	+0.00000	+2126.03	-123.56	+0.00	+2126.03	-123.56	+0.00
3.0000e-02	275.00	+0.0	+0.00000	+2123.02	-137.63	-144.89	+2123.02	-137.63	-144.89
3.0000e-02	280.00	+0.0	+0.00000	+2111.45	-176.68	-311.90	+2111.45	-176.68	-311.90
3.0000e-02	285.00	+0.0	+0.00000	+2084.00	-231.54	-517.42	+2084.00	-231.54	-517.42
3.0000e-02	290.00	+0.0	+0.00000	+2029.93	-288.46	-766.35	+2029.93	-288.46	-766.35
3.0000e-02	295.00	+0.0	+0.00000	+1937.74	-331.96	-1047.28	+1937.74	-331.96	-1047.28
3.0000e-02	300.00	+0.0	+0.00000	+1799.25	-349.20	-1331.24	+1799.25	-349.20	-1331.24
3.0000e-02	305.00	+0.0	+0.00000	+1614.10	-334.81	-1577.10	+1614.10	-334.81	-1577.10
3.0000e-02	310.00	+0.0	+0.00000	+1392.08	-293.49	-1744.04	+1392.08	-293.49	-1744.04
3.0000e-02	315.00	+0.0	+0.00000	+1151.37	-238.37	-1805.88	+1151.37	-238.37	-1805.88
3.0000e-02	320.00	+0.0	+0.00000	+913.02	-185.37	-1759.17	+913.02	-185.37	-1759.17

Outputs: Ply Stresses and FIs

PLY BY PLY INPLANE STRESSES (TOTAL) — FAILURE ANALYSIS - TSAI-HILL

DISTANCE	ANGLE	ANGLE	Z	SIG1	SIG2	TAU12	SIGX	SIGY	TAUXY	FI	MS
0.0000e+00	0.00	+0.0	+0.00000	+3837.57	-1128.41	+0.00	+3837.57	-1128.41	+0.00	+0.05	+3.44
0.0000e+00	5.00	+0.0	+0.00000	+3606.72	-1082.06	-113.20	+3606.72	-1082.06	-113.20	+0.05	+3.70
0.0000e+00	10.00	+0.0	+0.00000	+2959.16	-946.70	-199.76	+2959.16	-946.70	-199.76	+0.03	+4.66
0.0000e+00	15.00	+0.0	+0.00000	+2026.69	-733.06	-235.38	+2026.69	-733.06	-235.38	+0.02	+7.04
0.0000e+00	20.00	+0.0	+0.00000	+1018.72	-457.93	-200.15	+1018.72	-457.93	-200.15	+0.00	+14.10
0.0000e+00	25.00	+0.0	+0.00000	+209.13	-142.80	-80.19	+209.13	-142.80	-80.19	+0.00	NA
0.0000e+00	30.00	+0.0	+0.00000	-78.99	+187.78	+131.09	-78.99	+187.78	+131.09	+0.00	NA
0.0000e+00	35.00	+0.0	+0.00000	+510.99	+507.69	+432.17	+510.99	+507.69	+432.17	+0.00	+28.94
0.0000e+00	40.00	+0.0	+0.00000	+2352.84	+790.72	+812.62	+2352.84	+790.72	+812.62	+0.01	+7.98
0.0000e+00	45.00	+0.0	+0.00000	+5817.94	+1011.84	+1252.18	+5817.94	+1011.84	+1252.18	+0.08	+2.62
0.0000e+00	50.00	+0.0	+0.00000	+11246.33	+1148.63	+1719.50	+11246.33	+1148.63	+1719.50	+0.29	+0.84
0.0000e+00	55.00	+0.0	+0.00000	+18898.28	+1183.16	+2170.60	+18898.28	+1183.16	+2170.60	+0.85	+0.08
0.0000e+00	60.00	+0.0	+0.00000	+28872.95	+1104.87	+2547.78	+28872.95	+1104.87	+2547.78	+2.02	-0.30
0.0000e+00	65.00	+0.0	+0.00000	+40983.20	+914.93	+2780.73	+40983.20	+914.93	+2780.73	+4.13	-0.51
0.0000e+00	70.00	+0.0	+0.00000	+54592.79	+631.91	+2792.99	+54592.79	+631.91	+2792.99	+7.39	-0.63
0.0000e+00	75.00	+0.0	+0.00000	+68469.09	+296.82	+2517.69	+68469.09	+296.82	+2517.69	+11.69	-0.71
0.0000e+00	80.00	+0.0	+0.00000	+80776.94	-26.87	+1924.04	+80776.94	-26.87	+1924.04	+16.33	-0.75
0.0000e+00	85.00	+0.0	+0.00000	+89377.95	-264.06	+1046.88	+89377.95	-264.06	+1046.88	+20.03	-0.78
1.5000e-02	60.00	+0.0	+0.00000	+38804.84	+517.29	+1527.49	+38804.84	+517.29	+1527.49	+3.72	-0.48
1.5000e-02	65.00	+0.0	+0.00000	+45586.93	+486.26	+1408.82	+45586.93	+486.26	+1408.82	+5.15	-0.56
1.5000e-02	70.00	+0.0	+0.00000	+51420.11	+457.57	+1202.66	+51420.11	+457.57	+1202.66	+6.56	-0.61
1.5000e-02	75.00	+0.0	+0.00000	+55963.11	+439.23	+935.35	+55963.11	+439.23	+935.35	+7.77	-0.64
1.5000e-02	80.00	+0.0	+0.00000	+59114.09	+431.69	+634.18	+59114.09	+431.69	+634.18	+8.67	-0.66
1.5000e-02	85.00	+0.0	+0.00000	+60934.08	+430.55	+319.01	+60934.08	+430.55	+319.01	+9.22	-0.67
1.5000e-02	90.00	+0.0	+0.00000	+61525.40	+430.87	+0.00	+61525.40	+430.87	+0.00	+9.40	-0.67
1.5000e-02	95.00	+0.0	+0.00000	+60934.08	+430.55	-319.01	+60934.08	+430.55	-319.01	+9.22	-0.67
1.5000e-02	100.00	+0.0	+0.00000	+59114.09	+431.69	-634.18	+59114.09	+431.69	-634.18	+8.67	-0.66
1.5000e-02	105.00	+0.0	+0.00000	+55963.11	+439.23	-935.35	+55963.11	+439.23	-935.35	+7.77	-0.64
1.5000e-02	110.00	+0.0	+0.00000	+51420.11	+457.57	-1202.66	+51420.11	+457.57	-1202.66	+6.56	-0.61
1.5000e-02	115.00	+0.0	+0.00000	+45586.93	+486.26	-1408.82	+45586.93	+486.26	-1408.82	+5.15	-0.56
1.5000e-02	120.00	+0.0	+0.00000	+38804.84	+517.29	-1527.49	+38804.84	+517.29	-1527.49	+3.72	-0.48
1.5000e-02	125.00	+0.0	+0.00000	+31621.62	+536.34	-1544.08	+31621.62	+536.34	-1544.08	+2.46	-0.36
1.5000e-02	130.00	+0.0	+0.00000	+24652.50	+527.74	-1462.27	+24652.50	+527.74	-1462.27	+1.49	-0.18
1.5000e-02	135.00	+0.0	+0.00000	+18415.62	+480.25	-1302.65	+18415.62	+480.25	-1302.65	+0.83	+0.10
1.5000e-02	140.00	+0.0	+0.00000	+13229.52	+390.63	-1095.23	+13229.52	+390.63	-1095.23	+0.43	+0.53
1.5000e-02	145.00	+0.0	+0.00000	+9199.60	+263.82	-870.87	+9199.60	+263.82	-870.87	+0.21	+1.19
3.0000e-02	285.00	+0.0	+0.00000	+47069.43	+527.70	-331.15	+47069.43	+527.70	-331.15	+5.48	-0.57
3.0000e-02	290.00	+0.0	+0.00000	+45824.55	+444.40	-490.47	+45824.55	+444.40	-490.47	+5.20	-0.56
3.0000e-02	295.00	+0.0	+0.00000	+43721.99	+361.60	-670.26	+43721.99	+361.60	-670.26	+4.74	-0.54
3.0000e-02	300.00	+0.0	+0.00000	+40581.85	+290.44	-851.99	+40581.85	+290.44	-851.99	+4.09	-0.51
3.0000e-02	305.00	+0.0	+0.00000	+36397.68	+236.72	-1009.35	+36397.68	+236.72	-1009.35	+3.29	-0.45
3.0000e-02	310.00	+0.0	+0.00000	+31389.45	+198.92	-1116.19	+31389.45	+198.92	-1116.19	+2.45	-0.36
3.0000e-02	315.00	+0.0	+0.00000	+25963.47	+169.36	-1155.76	+25963.47	+169.36	-1155.76	+1.68	-0.23
3.0000e-02	320.00	+0.0	+0.00000	+20589.96	+138.34	-1125.87	+20589.96	+138.34	-1125.87	+1.06	-0.03

Inputs and Outputs: Consistent Units

Quantity	SI System 1	SI system 2	US System
$E_1, E_2, G_{12}, G_{13}, G_{23}$ $E_x, E_y, G_{xy}, E_{fx}, E_{fy}, G_{fxy}$	$MPa\ (N/mm^2)$	$Pa\ (N/m^2)$	$Psi\ (lb/in^2)$
$\alpha_1\ ,\alpha_2\ ,\alpha_x\ ,\alpha_y, \alpha_{xy}$	$mm/mm/$°C	$m/m/$°C	$in/in/$°F
$\beta_1, \beta_2\ , \beta_x\ , \beta_y, \beta_{xy}$	$mm/mm/Kg/Kg$	$m/m/Kg/Kg$	$in/in/lb/lb$
$\sigma_{11}^T, \sigma_{11}^C, \sigma_{22}^T, \sigma_{22}^C\ \tau_{12}^S, \sigma_1, \sigma_2, \tau_{12}, \tau_{23}, \tau_{13},$ $\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz}$	$MPa\ (N/mm^2)$	$Pa\ (N/m^2)$	$Psi\ (lb/in^2)$
$\varepsilon_{11}^T, \varepsilon_{11}^C, \varepsilon_{22}^T, \varepsilon_{22}^C, \gamma_{12}, \varepsilon_1, \varepsilon_2, \gamma_{12}, \gamma_{13}, \gamma_{23}, \varepsilon_{x0},$ $\varepsilon_{y0}, \gamma_{xy0}, \gamma_{yz0}, \gamma_{xz0}\ \varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}$	mm/mm	m/m	$in/$in
Ply Angle, θ	Degree	Degree	Degree
Ply or Laminate thickness or d_0	mm	m	in
$N_{xx}, N_{yy}, N_{xy}, Q_{yz}, Q_{xz},$ $N_{xx}^T, N_{yy}^T, N_{xy}^T, N_{xx}^H, N_{yy}^H, N_{xy}^H$ [A]	N/mm	N/m	lb/in
$M_{xx}, M_{yy}, M_{xy}, M_{xx}^T, M_{yy}^T, M_{xy}^T,$ $M_{xx}^H, M_{yy}^H, M_{xy}^H$ [B]	$N - mm/mm$	$N - m/m$	$lb - in/in$
ΔT	°C	°C	°F
ΔC	Kg/Kg	Kg/Kg	lb/lb
[D]	$N - mm$	$N - m$	$lb - in$
$\kappa_{x0}, \kappa_{y0}, \kappa_{xy0}$	$1/mm$	$1/m$	$1/in$

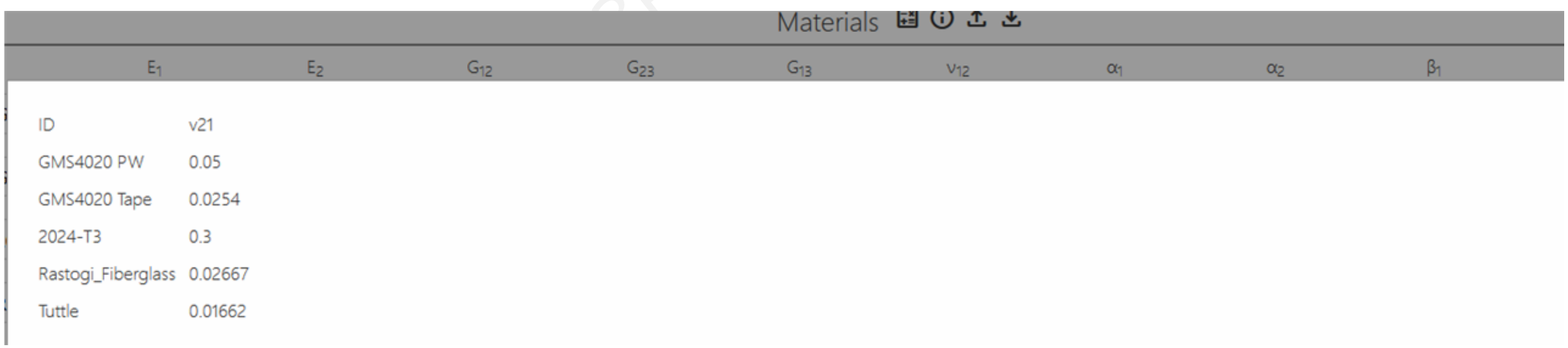
Other Features

❖ Upload/Download:

Users can upload and download material properties, Plies, Laminates, Panels and Loads data files (*.json) using the upload and download buttons next to these inputs. Sample input and output files can be downloaded from the 3p Composites website at www.3pcomposites.com

❖ Additional Output:

Users can review a few intermediate calculations such as minor Poison's ratios ν_{21}, Q_{ij} for each ply type and laminate ABD by using the calculation button . Examples are shown below:


Materials

E1	E2	G12	G23	G13	V12	α1	α2	β1

ID	v21
GMS4020 PW	0.05
GMS4020 Tape	0.0254
2024-T3	0.3
Rastogi_Fiberglass	0.02667
Tuttle	0.01662

Other Features

Plies

ID	Angle (deg)	Material	Thickness
1	0	Tuttle	0.0075
2	90	Tuttle	0.0075

ID	Q	Q44	Q55	Qbar	Q44bar	Q45bar	Q55bar
1	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	0	640000
2	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[1106252.04, 376125.7, 0.0], [376125.7, 22627882.74, 0.0], [0.0, 0.0, 640000.0]]	640000	0	640000
3	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[6761596.54, 5481596.54, 5380407.67], [5481596.54, 6761596.54, 5380407.67], [5380407.67, 5380407.67, 5745470.85]]	640000	0	640000
4	[[22627882.74, 376125.7, 0.0], [376125.7, 1106252.04, 0.0], [0.0, 0.0, 640000.0]]	640000	640000	[[6761596.54, 5481596.54, -5380407.67], [5481596.54, 6761596.54, -5380407.67], [-5380407.67, -5380407.67, 5745470.85]]	640000	0	640000

Laminates

ID	Stacking Sequence	Stacking Sequence (Angle)	Offset
1	1,1,1,1,1,1,1,1	0,0,0,0,0,0,0,0	Middle
2	1,2,1,2,2,1,2,1	0,90,0,90,90,0,90,0	Middle
3	3,4,3,4,4,3,4,3	45,-45,45,-45,-45,45	Middle

ID	Thickness	A	B	D	A44	A45	A55
1	0.06	[[1357672.96, 22567.54, 0.0], [22567.54, 66375.12, 0.0], [0.0, 0.0, 38400.0]]	[[0.0, 0.0, 0.0], [0.0, -0.0, 0.0], [0.0, 0.0, 0.0]]	[[407.3, 6.77, 0.0], [6.77, 19.91, 0.0], [0.0, 0.0, 11.52]]	38400	0	38400
2	0.06	[[712024.04, 22567.54, 0.0], [22567.54, 712024.04, 0.0], [0.0, 0.0, 38400.0]]	[[0.0, 0.0, 0.0], [0.0, -0.0, 0.0], [0.0, 0.0, 0.0]]	[[286.24, 6.77, 0.0], [6.77, 140.97, 0.0], [0.0, 0.0, 11.52]]	38400	0	38400
3	0.06	[[405695.79, 328895.79, 0.0], [328895.79, 405695.79, 0.0], [0.0, 0.0, 344728.25]]	[[0.0, -0.0, 0.0], [-0.0, -0.0, 0.0], [0.0, 0.0, -0.0]]	[[121.71, 98.67, 36.32], [98.67, 121.71, 36.32], [36.32, 36.32, 103.42]]	38400	0	38400

PRODUCIBILITY PERFORMANCE PROFITABILITY 3P COMPOSITES

General Information

- Subscription fee is $49/year per single-login license to access 3pcsolver009

- Training module 3pcmodule009 supports the solver 3pcsolver009. Users' can buy the training module 3pcmodule009 online at

  https://www.3pcomposites.com/etraining.

- 3P Composites, LLC can conduct online or in-class trainings, for the 3pcsolver009 and 3pcmodule009. The training can be adapted to meet the requirements of individual company and/or industrial applications

- For questions, issues, comments, suggestions, trainings, please contact us at 3pcomps@gmail.com. Your feedback is always appreciated, and helps us continuously improve the product and services

Strength Analysis of a Laminate with a hole

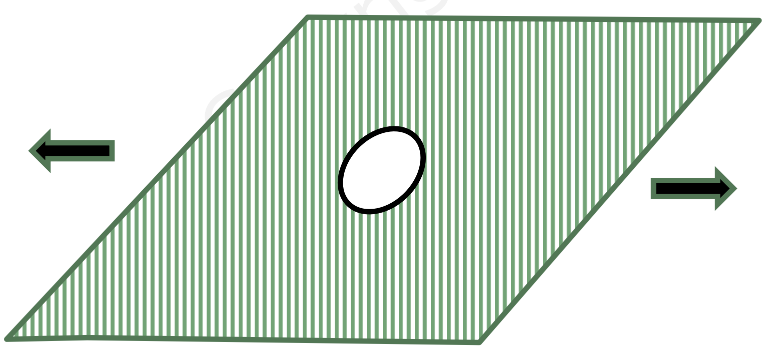
- Unidirectional Lamina Properties:

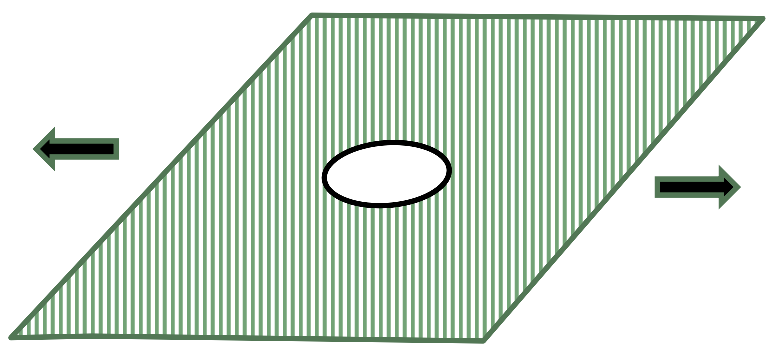
$$E_1 = 1.68e7\,psi, E_2 = 1.16e6\,psi\,;\; G_{12} = G_{13} = 8.0e5\,psi, G_{23} = 6.0e5\,psi;$$
$$\nu_{21} = 0.35, t_{ply} = 0.00525\,inch,\; \alpha_1 = 1.0 \times 10^{-8}\,/\mathrm{F}, \alpha_2 = 12.5 \times 10^{-6}\,/\mathrm{F}$$
$$\varepsilon_{11}^{T} = 5000\,\mu\varepsilon, \varepsilon_{11}^{C} = -5000\,\mu\varepsilon, \varepsilon_{22}^{T} = 20000\,\mu\varepsilon, \varepsilon_{22}^{C} = -20000\,\mu\varepsilon\,, \gamma_{12} = 20000\,\mu\varepsilon;$$

- Laminate Stack-up $[0/90/\pm45/90/0]_T$, Max Strain Failure Criteria, Characteristic Distance: d_0 = 0.05 in, number of radial points = 2

- Panel Geometry: Length = Width = 12.00 in,
  - Case I: Circular Cutout , Diameter = 1.0 in
  - Case II: Elliptical Cutout, Major Axis = 1.5 in, Minor Axis = 0.75 in

- Applied Far Field Load:
  - N_{xx} = 100 lb/in for Case I and II
  - ΔT = -225 F for Case I





Strength Analysis of a Laminate with a hole

- Laminate [A] MATRIX

+248285.53	+49895.58	+0.00
+49895.58	+248285.53	+0.00
+0.00	+0.00	+62197.49

- Equivalent or smeared properties of laminate

$E_x = 7.56e6\ psi, E_y = 7.56e6\ psi, G_{xy} = 1.97e6\ psi, \nu_{yx} = 0.201, t_{lam} = 0.0315\ inch$

$\alpha_x = \alpha_y = 1.05 \times 10^{-6}$ /F

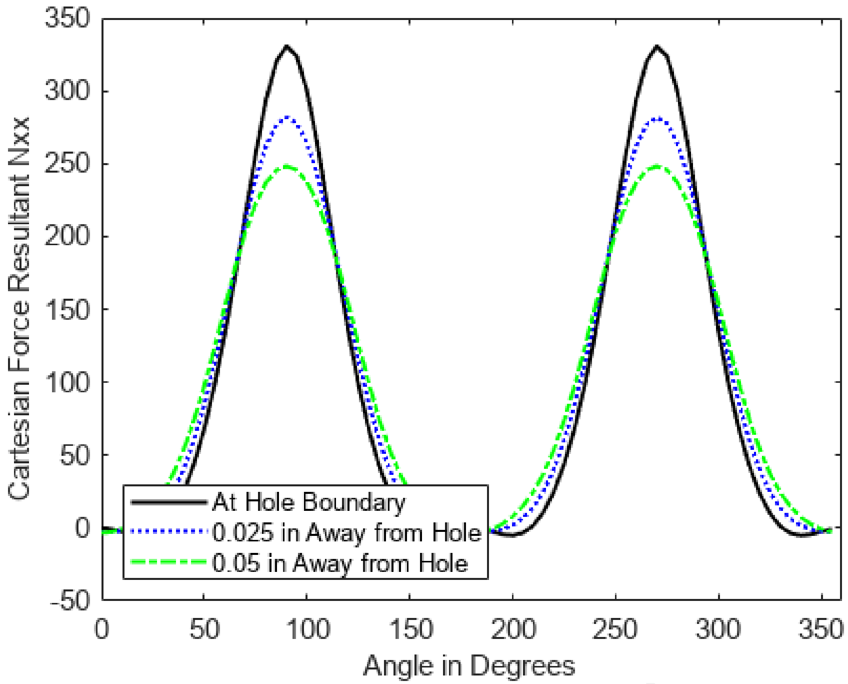
- Mid Plane Strains and Curvatures:

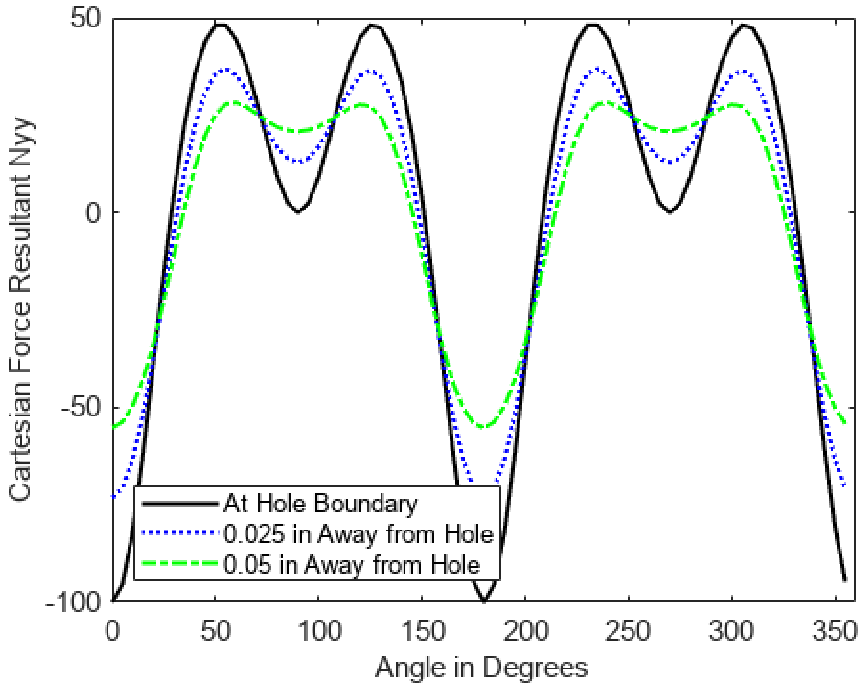
ε_{x0} = 419.7 $\mu\varepsilon$, ε_{y0} = -84.4 $\mu\varepsilon$,, $\gamma_{xy0} = \kappa_{x0} = \kappa_{y0} = \kappa_{xy0} = 0$

- Applied Thermal Loads due to temperature ΔT = -225 F

$N_{xx}^T = N_{yy}^T$ = - 70.58 lb/in

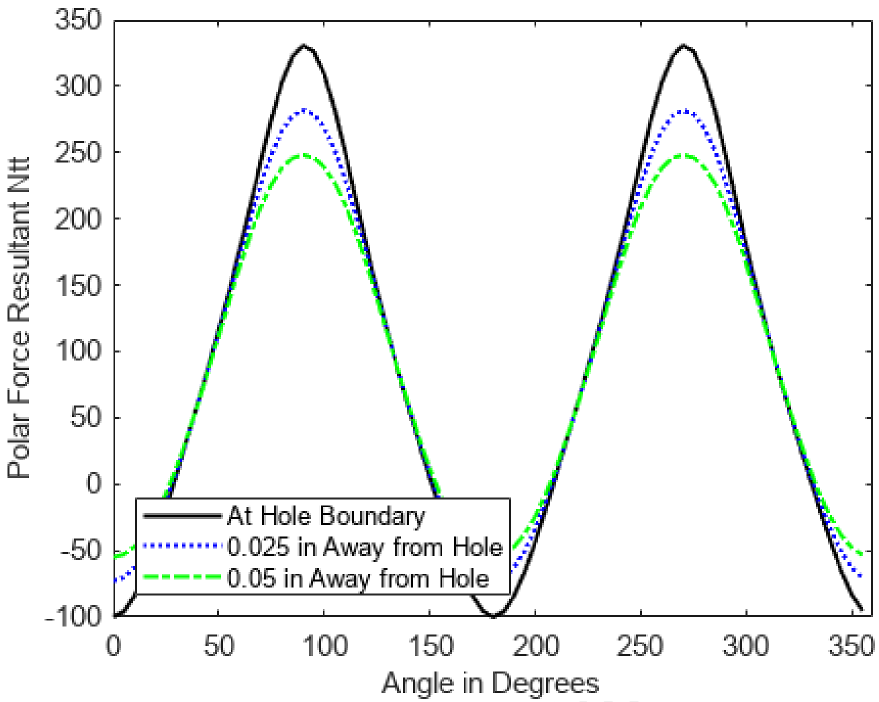
Circular Hole : Force Resultants N_{xx} and N_{yy}

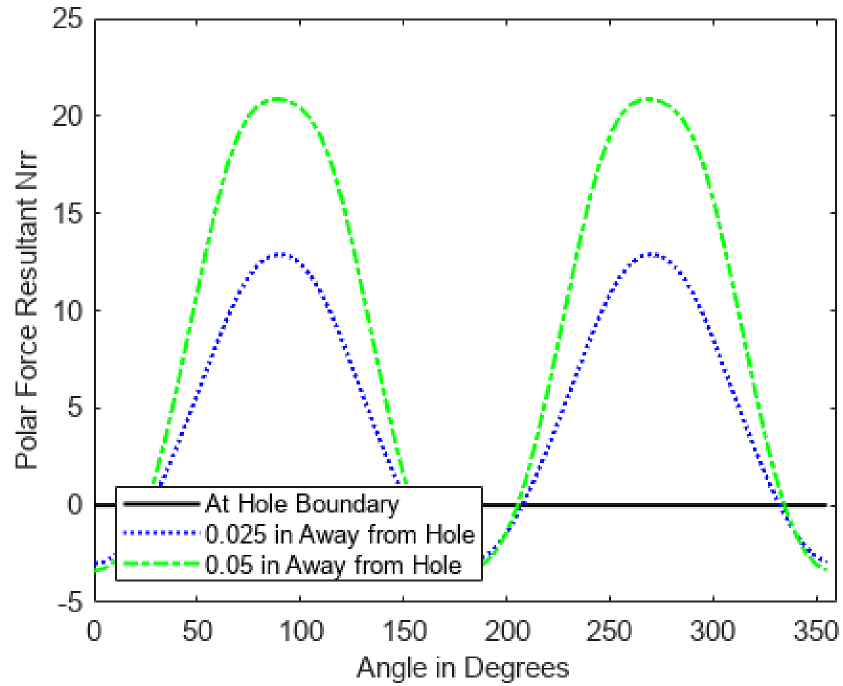




- Applied Far Field Load: N_{xx} = 100 lb/in

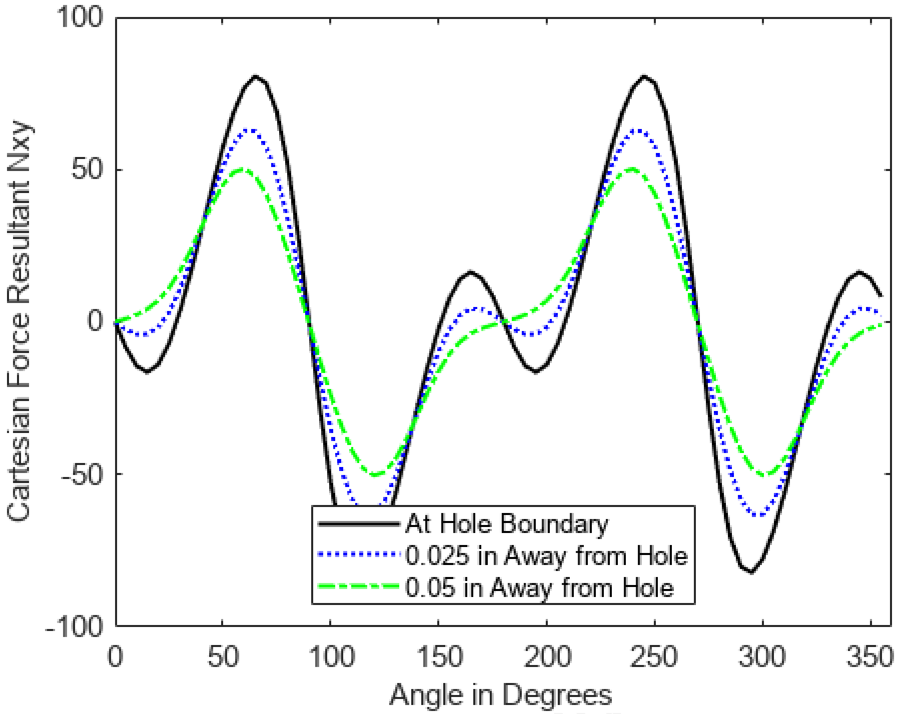
Circular Hole : Force Resultants $N_{\theta\theta}$ and N_{rr}

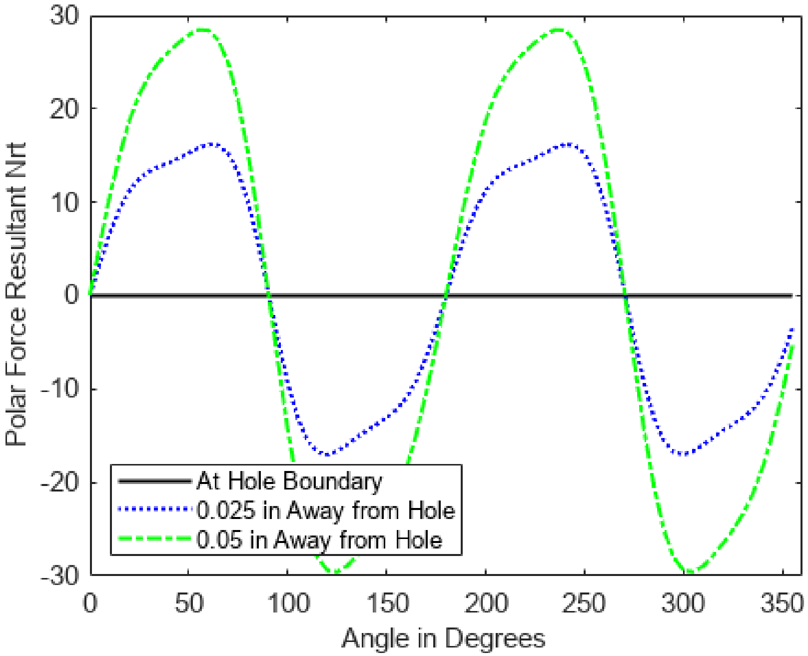




- Applied Far Field Load: N_{xx} = 100 lb/in

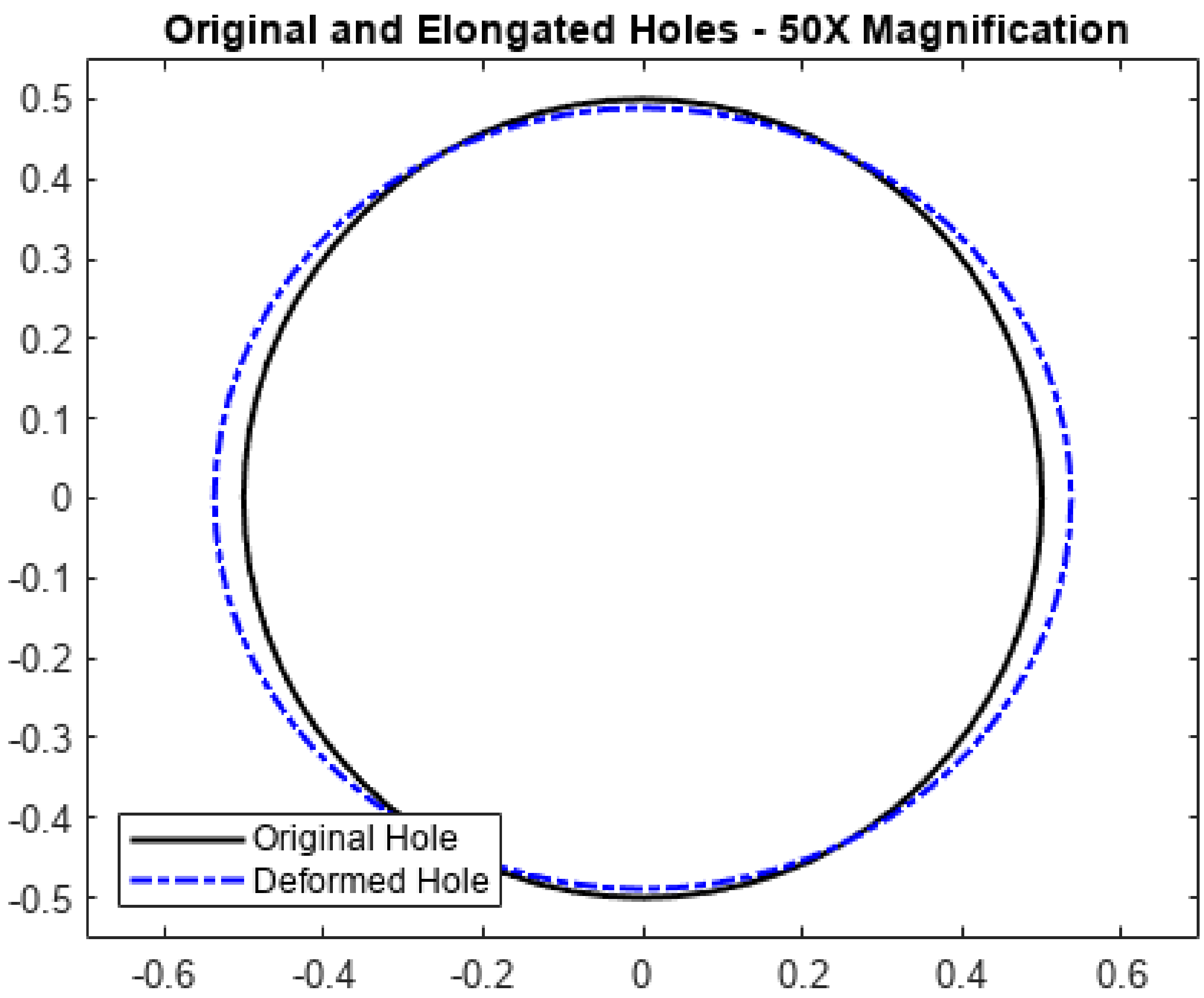
Circular Hole : Force Resultants N_{xy} and $N_{r\theta}$





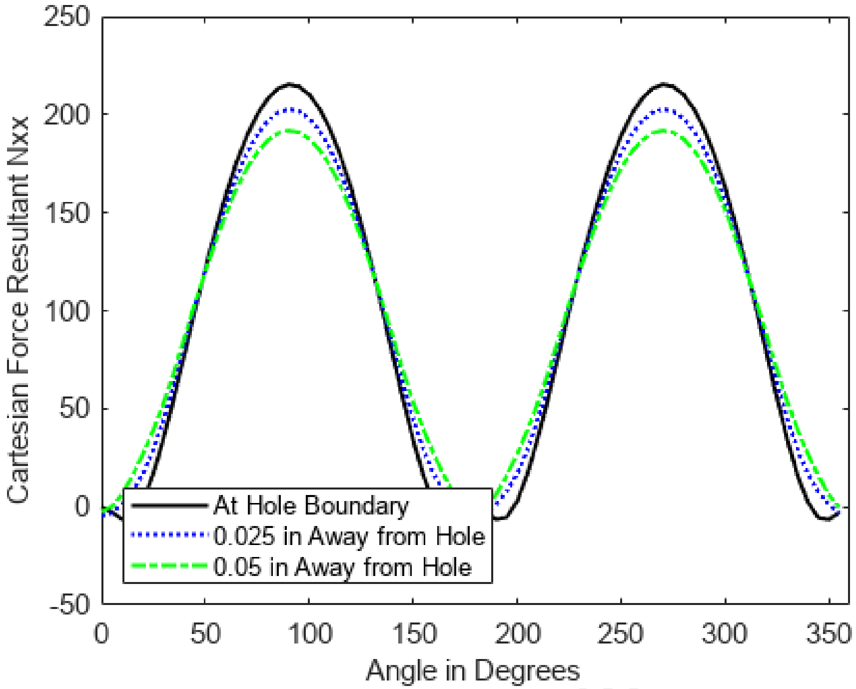
❖ Applied Far Field Load: N_{xx} = 100 lb/in

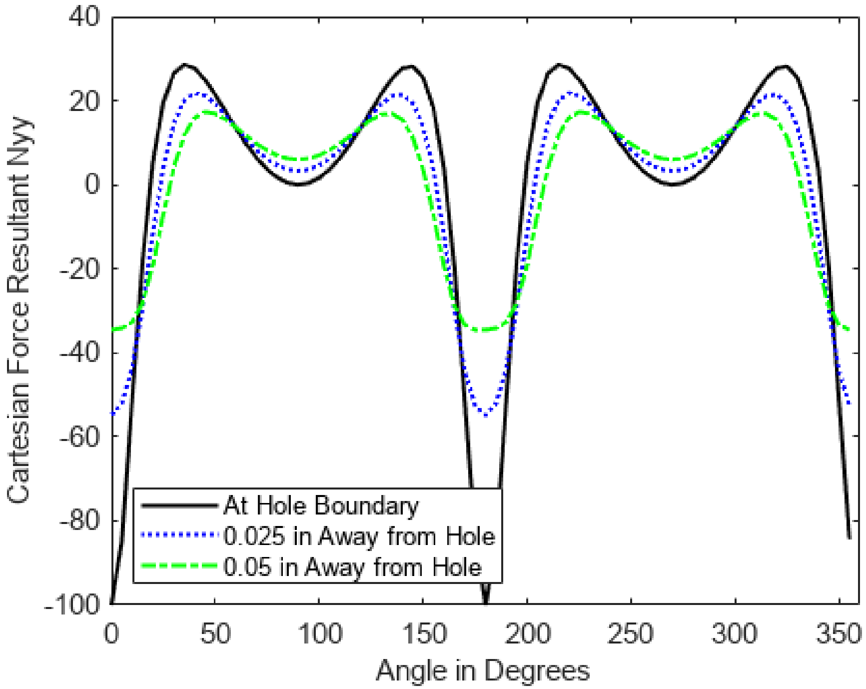
Deformation of Circular Hole



❖ Applied Far Field Load: N_{xx} = 100 lb/in

Elliptical Hole : Force Resultants N_{xx} and N_{yy}

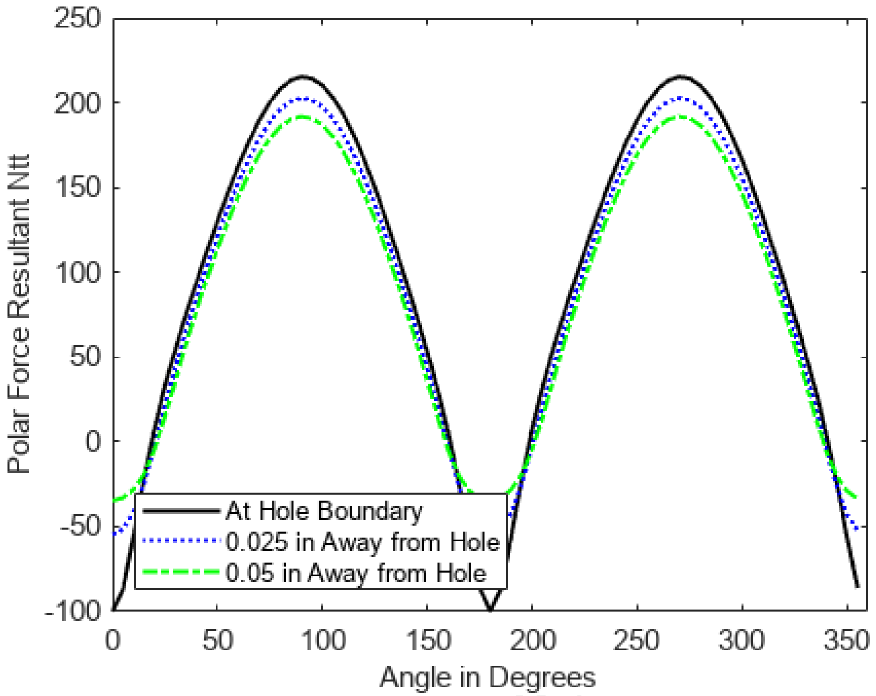


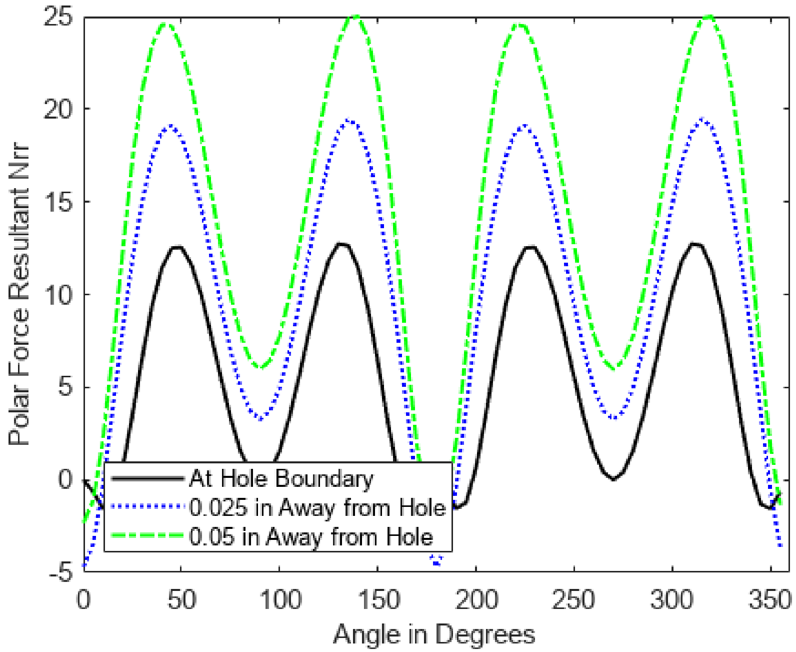


- Applied Far Field Load: N_{xx} = 100 lb/in

3P COMPOSITES PRODUCIBILITY PERFORMANCE PROFITABILITY

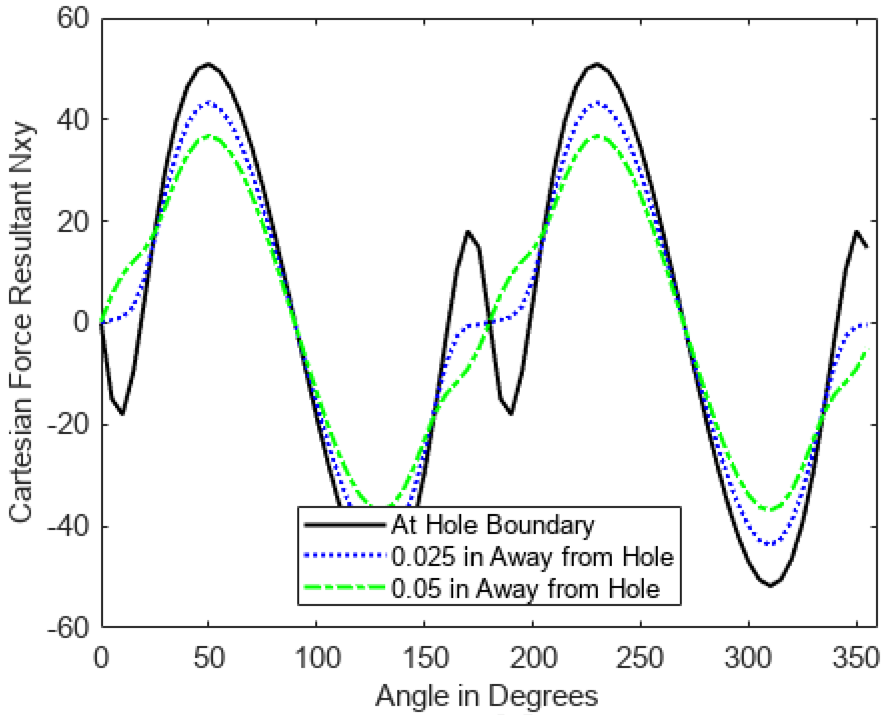
Elliptical Hole : Force Resultants $N_{\theta\theta}$ and N_{rr}

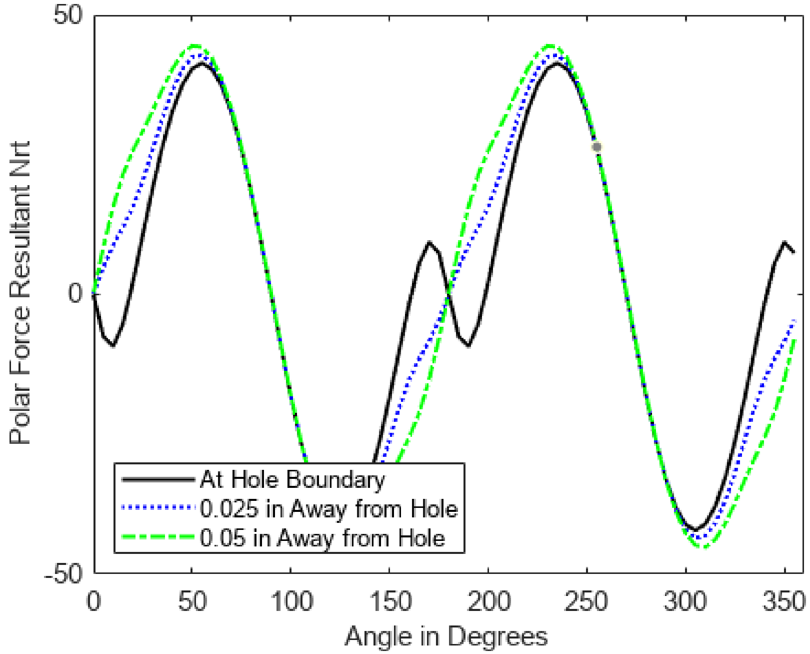




❖ Applied Far Field Load: N_{xx} = 100 lb/in

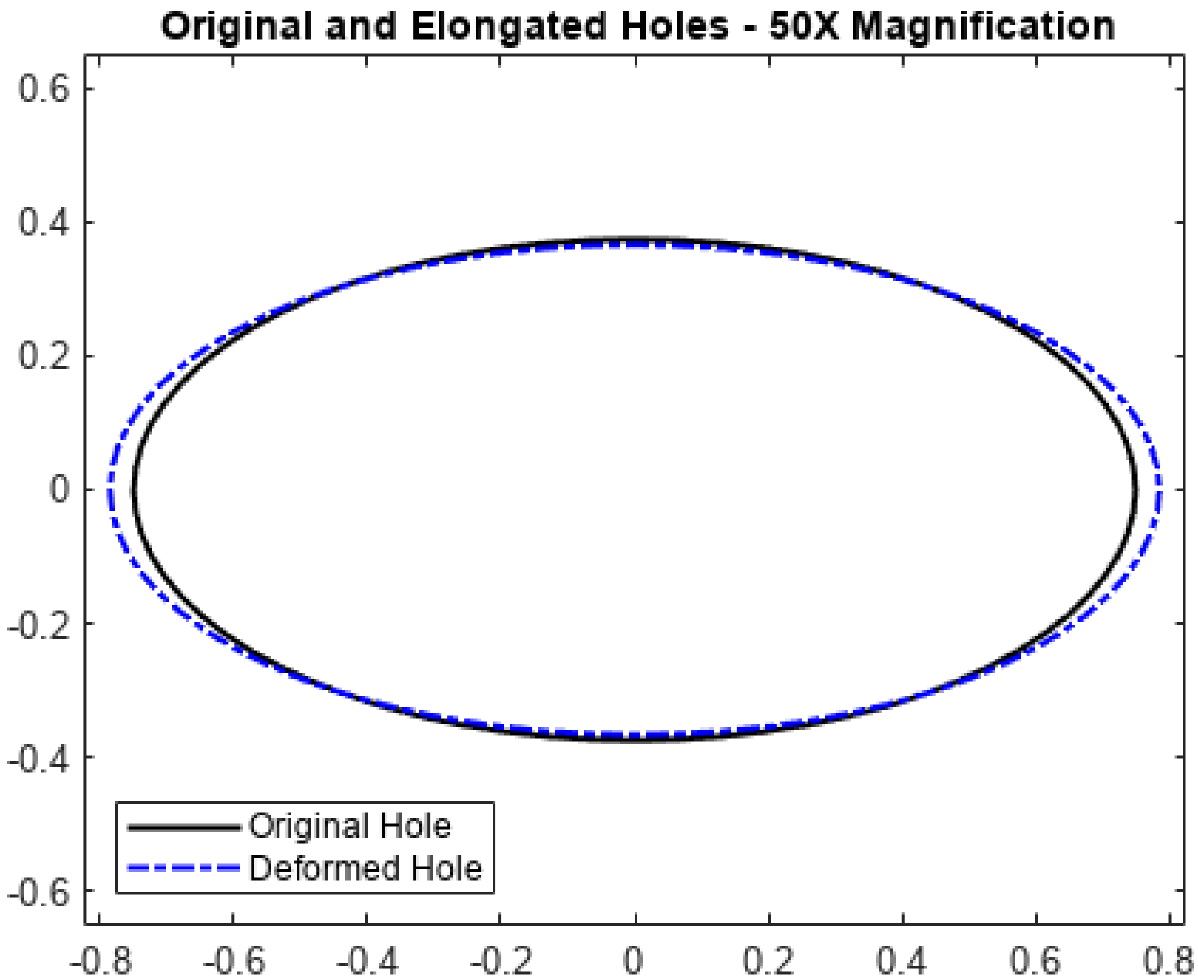
Elliptical Hole : Force Resultants N_{xy} and $N_{r\theta}$





❖ Applied Far Field Load: N_{xx} = 100 lb/in

Deformation of Elliptical Hole



❖ Applied Far Field Load: N_{xx} = 100 lb/in

Failure Prediction : Max Strain Criteria

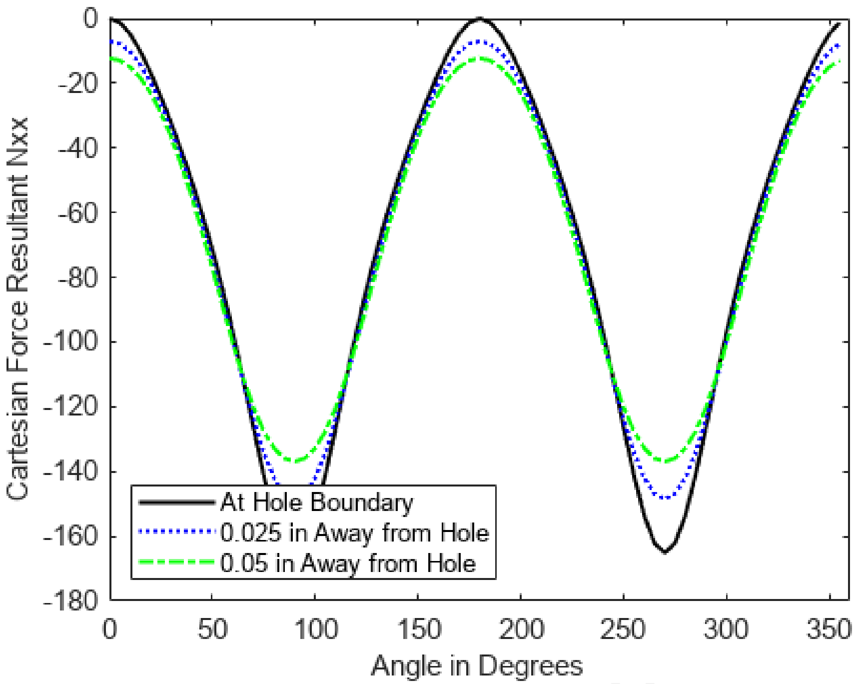
Circular Hole				
Distance from Hole, in	**Location around Hole, degrees**	**Ply Angle, degrees**	**Fiber Strain, Micro strains**	**Margin of Safety**
0.0	0, 270	0	1674	1.99
0.025	0, 270	0	1414	2.54
0.05	0, 270	0	1237	3.04

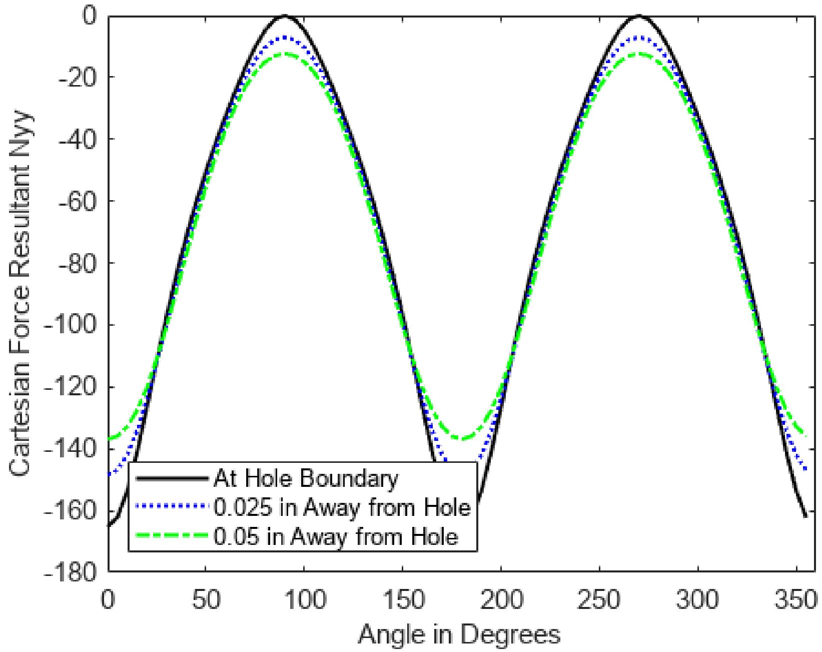
Elliptical Hole				
Distance from Hole, in	**Location around Hole, degrees**	**Ply Angle, degrees**	**Fiber Strain, Micro strains**	**Margin of Safety**
0.0	0, 270	0	1090	3.59
0.025	0, 270	0	1023	3.89
0.05	0, 270	0	965	4.18

❖ Applied Far Field Load: N_{xx} = 100 lb/in

- Magnitudes of strains (and stress) decrease away from the hole boundary
- Generally, Elliptical holes have lower strain (or stress) concentration than circular holes

Circular Hole : Force Resultants N_{xx} and N_{yy}

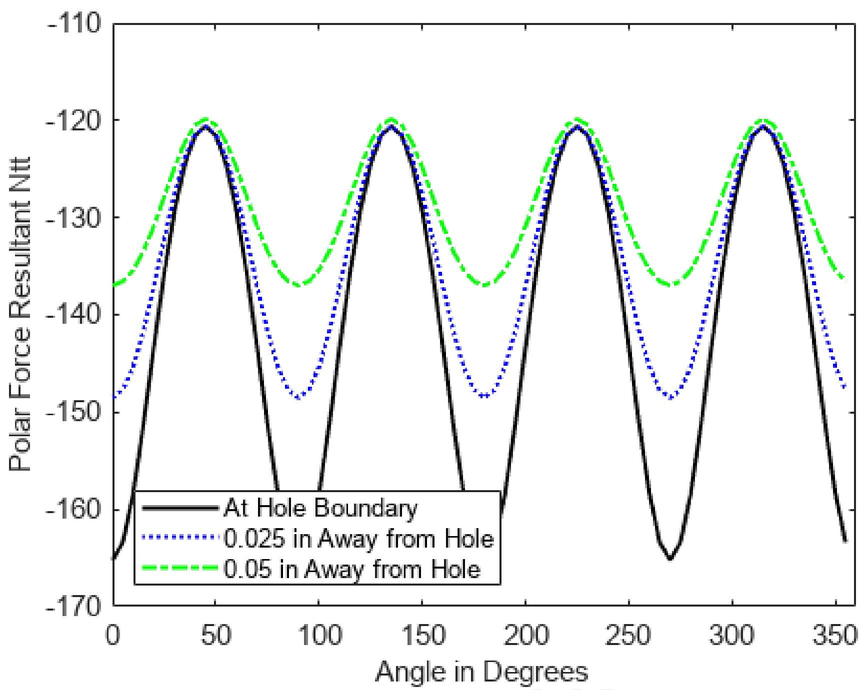


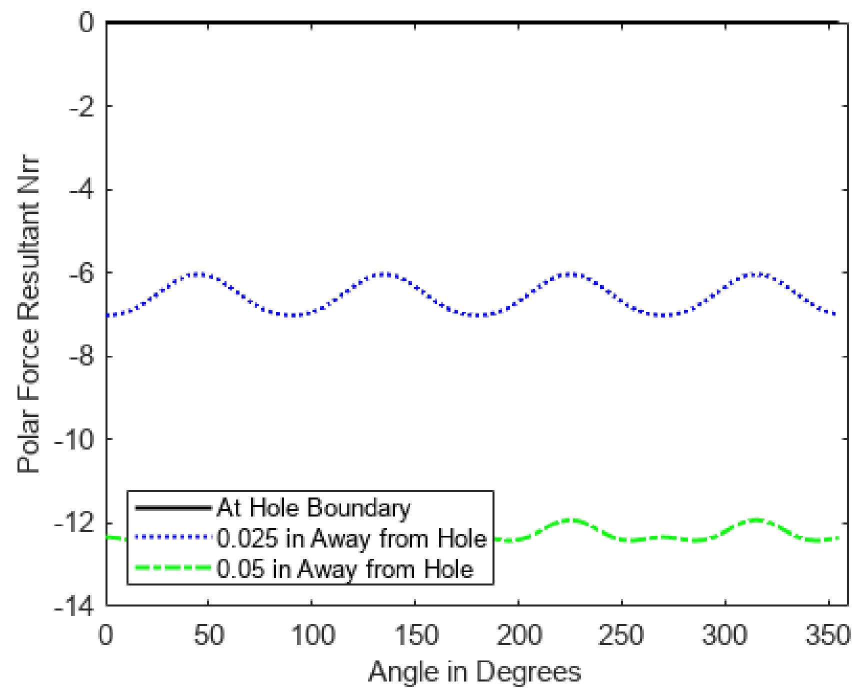


❖ Applied Thermal Loads due to temperature ΔT = -225 F

$N_{xx}^T = N_{yy}^T =$ - 70.58 lb/in

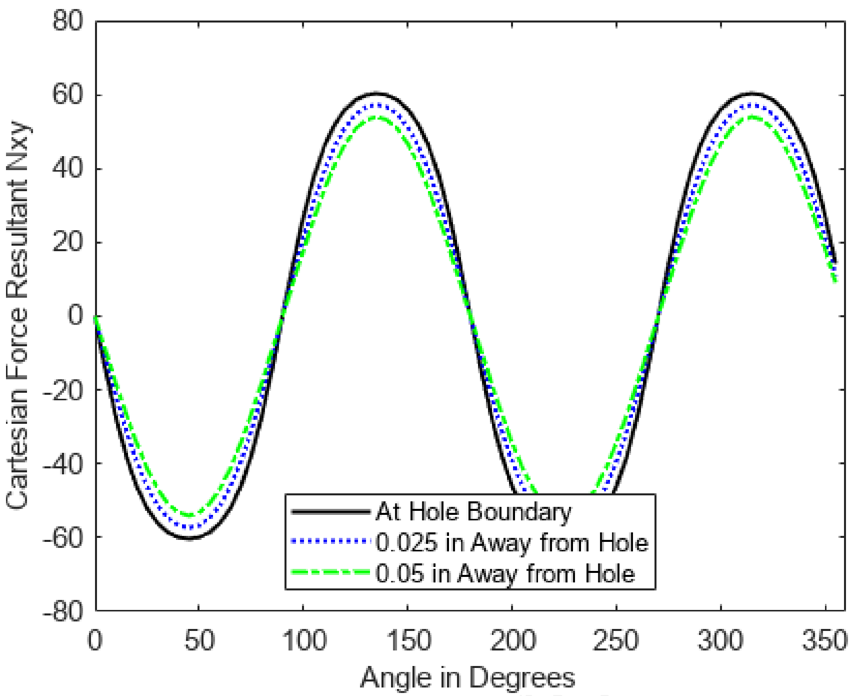
Circular Hole : Force Resultants $N_{\theta\theta}$ and N_{rr}

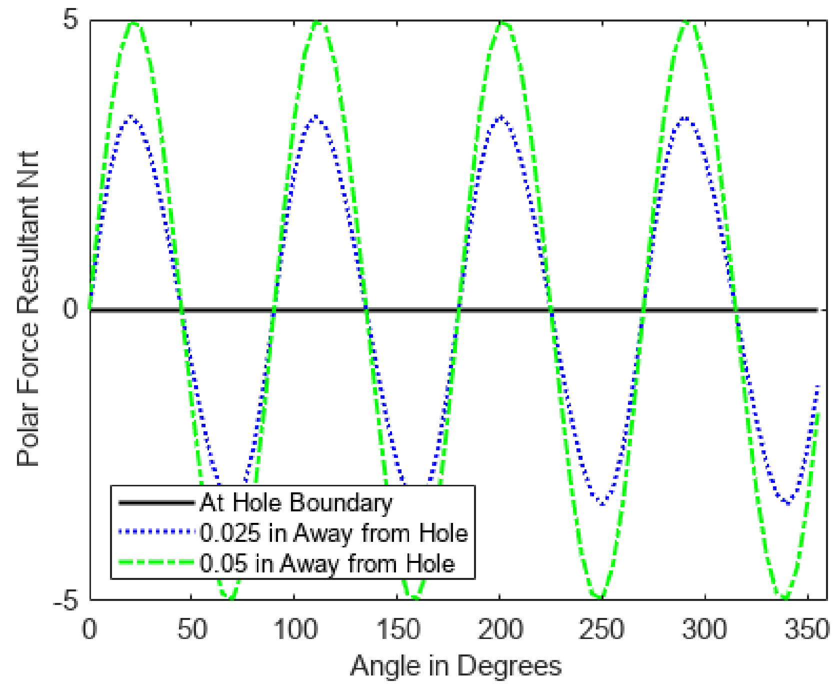




- Applied Thermal Loads N_{xx}^{T} = N_{yy}^{T} = - 70.58 lb/in

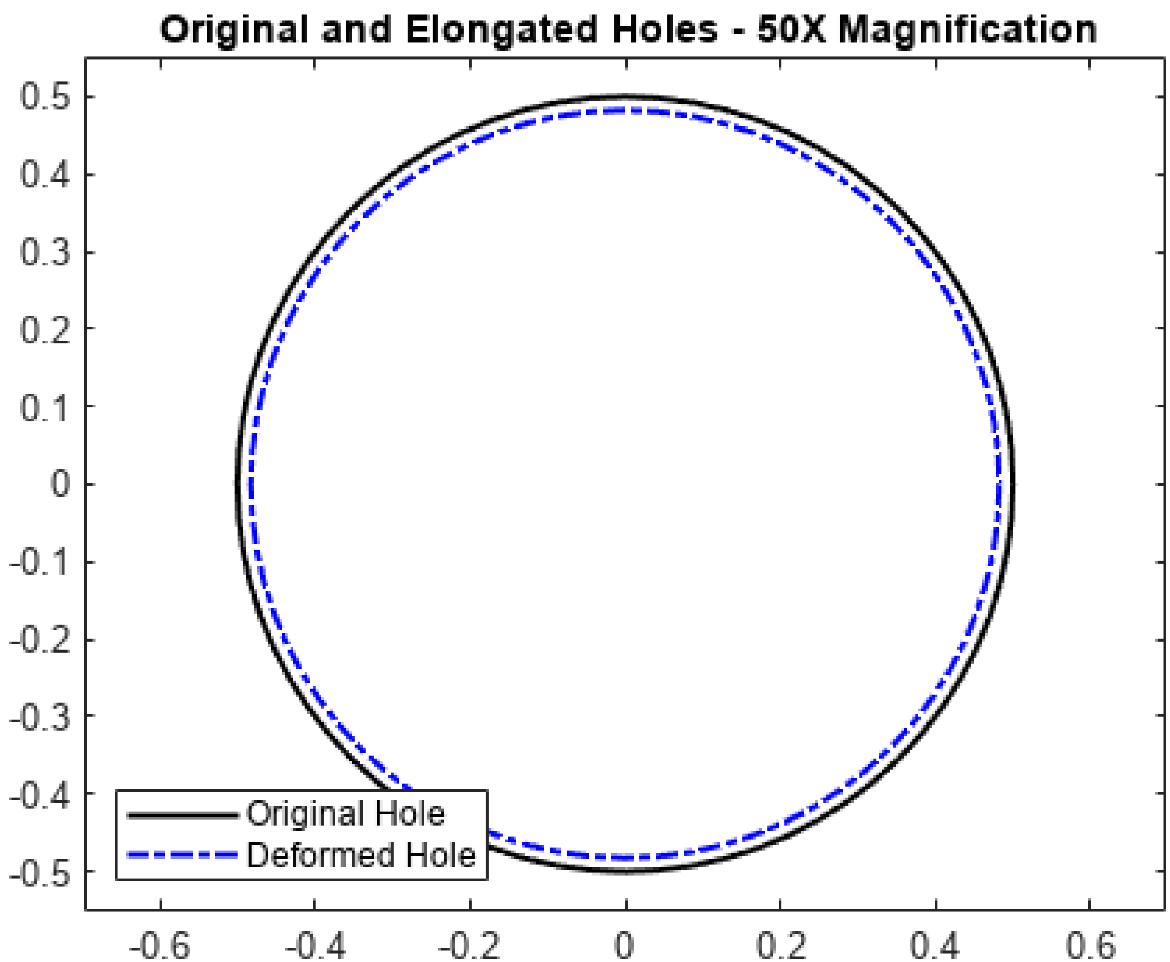
Circular Hole : Force Resultants N_{xy} and $N_{r\theta}$





- Applied Thermal Loads $N_{xx}^{T} = N_{yy}^{T} =$ - 70.58 lb/in

Deformation of Circular Hole



❖ Applied Thermal Loads $N_{xx}^{T} = N_{yy}^{T}$ = - 70.58 lb/in